INFERRING THE STRUCTURAL PROPERTIES OF EDDIES IN THE LOG LAYER FROM SPECTRAL STATISTICS

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Background

The structure of near-wall layers has been the subject of much research over many years, with conceptual descriptions of the Attached Eddy Hypothesis (AEH, henceforth) by Townsend (1980) and Perry & Chong (1982) being key historical fix points. The fact that major efforts have continued unabated over the past two to three decades reflects the exceptional structural complexity of near-wall layers awaiting insight, as well as new opportunities to investigate open questions as a consequence of outstandingly high-quality experimental and DNS data emerging over the past decade (Marusic *et al.* (2013); Smits *et al.* (2011); Hultmark *et al.* (2013); Rosenberg *et al.* (2013); Jiménez & Hoyas (2008); Lee & Moser (2015)). In particular, the availability of massive amounts of spatially and temporally fully-resolved raw DNS data for fairly high Reynolds numbers has opened new routes to investigating many statistical and structural properties of near-wall layers, with the objective of unravelling a variety of scale-interaction processes.

One subject of particular interest and attention has been the origin and significance of energetic large-scale structures present in the outer parts of the log layer, which are the cause of a plateau of, and – at sufficiently high Reynolds number – an outer (second) maximum in, the streamwise energy in a boundary layer. This behaviour is illustrated by the profiles in Fig. 1, derived from channel-flow DNS data created by Lozano-Durán & Jiménez (2014) and Lee & Moser (2015), respectively.
Among many processes associated with these structures, "footprinting" and "modulation" of small(er)-scale near-wall structures, and implications arising from both to the universality of near-wall turbulence, have been major focal point of recent studies, exploiting experimental data (Mathis *et al.* (2011); Ganapathisubramani *et al.* (2012)) and DNS fields (Agostini & Leschziner (2014) , Zhang & Chernyshenko (2016)). A question that is posed by the Reynolds-number-dependent elevation and distortion of the streamwise energy in the log-layer, depicted in Fig. 1, is whether the validity of the AEH in this layer can be defended, in view of the fact that the AEH is associated with a logarithmic decline of the streamwise energy away from the wall. In contrast to the variation in the log-layer, such a decay is observed, experimentally, in the remote outer region of boundary layers at very high Reynolds numbers (Hultmark *et al.* (2013); Rosenberg *et al.* (2013); Vassilicos *et al.* (2015); Vallikivi *et al.* (2015)), but this region is well beyond the log-layer and populated by "very large-scale structures". Also, in this outer region, the spectrum of the streamwise energy shows the variation $\phi_{uu}(k_x) \propto k_x^{-1}$, which is compatible with the logarithmic decay of the streamwise energy. On the other hand, recent studies by Davidson *et al.* (2006*a*); Hwang (2015) and observations by Jiménez & Hoyas (2008) suggest that the AEH is also valid in the intermediate (or meso-) layer covering the log-law region.

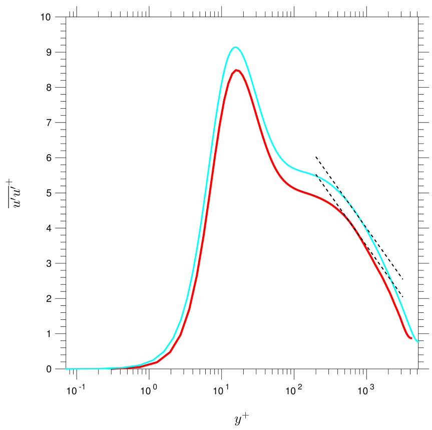
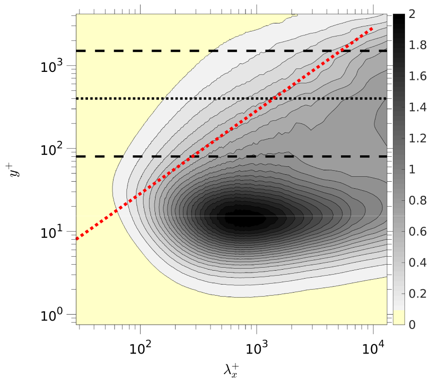


Figure 1. Wall-normal distribution of the streamwise stress at : $Re_\tau = 4200$ (red line) and $Re_\tau = 5200$ Lee & Moser (2015)). The dashed lines represents the variation $\overline{u'u'}^+ = -1.26 \log y^+ + B$, with $B = 12.2$ and 12.7 for $Re_\tau = 4200$ and $Re_\tau = 5200$, respectively.

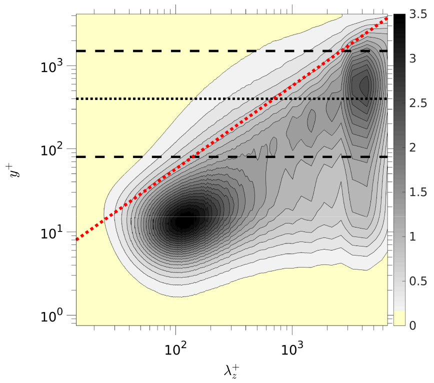
It is against the above background and controversy that the present study set out to examine closely the structural and spectral properties of a channel-flow boundary layer at $Re_\tau = 4200$, for which extensive DNS data were generated by Lozano-Durán & Jiménez (2014). The primary focus of the study is on an examination of the statistical properties of sub-ranges of scales within the pre-multiplied wall-normal spectra, and on related analysis of the pre-multiplied derivatives of the second-order structure function, the latter leading to a proposed alternative interpretation of the conventional AEH in the intermediate layer $100 < y^+ < 2000$.

Results

Fig. 2 shows pre-multiplied spectra $\phi_{uu}(\lambda_x)$ and $\phi_{uu}(\lambda_z)$, where λ_x and λ_z are the wave numbers in the streamwise and spanwise directions, respectively. The hor-



(a)



(b)

Figure 2. Pre-multiplied power spectrum of the streamwise fluctuations, in both streamwise (a) and spanwise direction (b); at $Re_\tau = 4200$. The dotted red lines show either the relation $\lambda_x^+ = 3.5y^+$ or $\lambda_z^+ = 7y^+ = 2\lambda_x^+$.

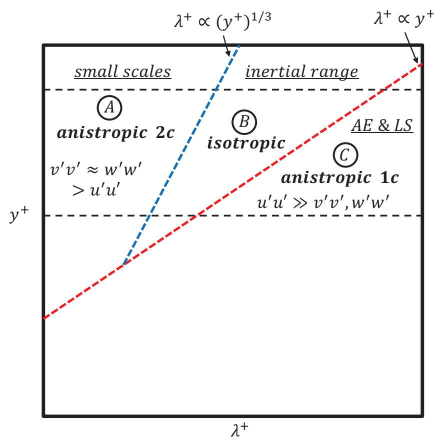


Figure 3. Sub-ranges in spectral map having distinct turbulence characteristics (1c: dominance of streamwise component; 2c: dominance of cross-flow components).

izontal dashed lines identify the meso-layer, on which attention focuses. Although there is an indication that the λ_x and λ_z locations at which the energy begins to rise steeply vary linearly with y^+, in consonance with the AEH, neither spectral map features a well-defined constant-value plateau within the meso-layer. However, the existence of such a plateau, implying the variation $\phi_{uu}(k_x) \propto k_x^{-1}$ or $\phi_{uu}(k_z) \propto k_z^{-1}$ within a triangular region that is bounded by $\lambda_{x,min}^+ = Cy^+$ and $\lambda_{x,max}^+ = cst$, can easily be shown to be necessary in order to satisfy the AEH-compatible logarithmic decay of the streamwise energy, such as indicated in the outer region of the meso-layer in Fig.1.

In a preliminary part of the present study, the spectra ϕ_{uu}, ϕ_{vv} and ϕ_{ww} were combined, in a manner identified below, to determine maps of scalar parameters in the $(\lambda_x^+ - y^+)$-plane that allowed distinct sub-ranges of eddy scales in the spectra $\phi_{uu}(\lambda_x)$ and $\phi_{uu}(\lambda_z)$ to be delineated, with each sub-range having distinct statistical characteristics. The newly proposed scalar parameters express, respectively, the degree of isotropy, via $\gamma^{3c} \equiv \frac{3|\Phi_{uu}||\Phi_{vv}||\Phi_{ww}|}{|\Phi_{uu}|^3+|\Phi_{vv}|^3+|\Phi_{ww}|^3}$, and the degree anisotropy, via $\gamma_u^{1c} \equiv \frac{|\Phi_{uu}||\Phi_{uu}|}{|\Phi_{uu}|^2+|\Phi_{vv}|^2+|\Phi_{ww}|^2}$, of the energy density. The eddy-scale sub-regions so identified are shown in Fig.3. Of primary relevance and interest is sub-region C, characterized by a dominance of the streamwise energy component, relative to the two others, in the region in which the length scales of the structures is relatively large. Eddies having smaller length scales, located within region B, are close to isotropic, in terms of energy, do not carry shear stress and are therefore deemed “detached”. In contrast, the state in region C is characterized by anisotropy, with streamwise energy dominating, and the eddies are affected by shear, carry shear stress and comply with the notion of “attached” eddies.

The main part of the study focused on an investigation of maps of the pre-multiplied derivatives of the second-order structure function $\delta \times \frac{dS_{2,u}(\delta)}{d\delta}$, where δ is the separation, either in x or z in $S_{2,u}(y,\delta) = \left\langle [u(y,x) - u(y,x+\delta)]^2 \right\rangle_{z,t}$, the subscripts z,t identifying homogeneous averaging directions. Such maps are shown in Fig. 4. The rationale of focusing on these maps arises from the observation by Davidson *et al.* (2006*b*,*a*) that there is a close relationship between $S_{2,u}$ and the pre-multiplied spectra – a subject that will be discussed and clarified in the full paper. However, the advantage of the pre-multiplied derivatives of the structure function is that plateau regions in these map are more pronounced and thus more readily identifiable as being associated with the AEH than in the corresponding spectral maps. A key argument advanced in this paper is that the trapezoidal regions, rather than the conventional triangular areas shown in Fig 4, allow an extended or modified interpretation of the AEH in the meso-layer, as is schematically indicated in the parts (c) and (d) of Fig. 4. This argument will be set out in detail in the full paper. A supporting observation is that profiles of the structure functions themselves, $S_{2,u}(\delta_x)$ and $S_{2,u}(\delta_z)$, at different y^+ levels within the meso-layer collapse onto an almost universal logarithmic variation when δ_x and δ_x are scaled with y. This connection will also be pursued in detail in the full paper.

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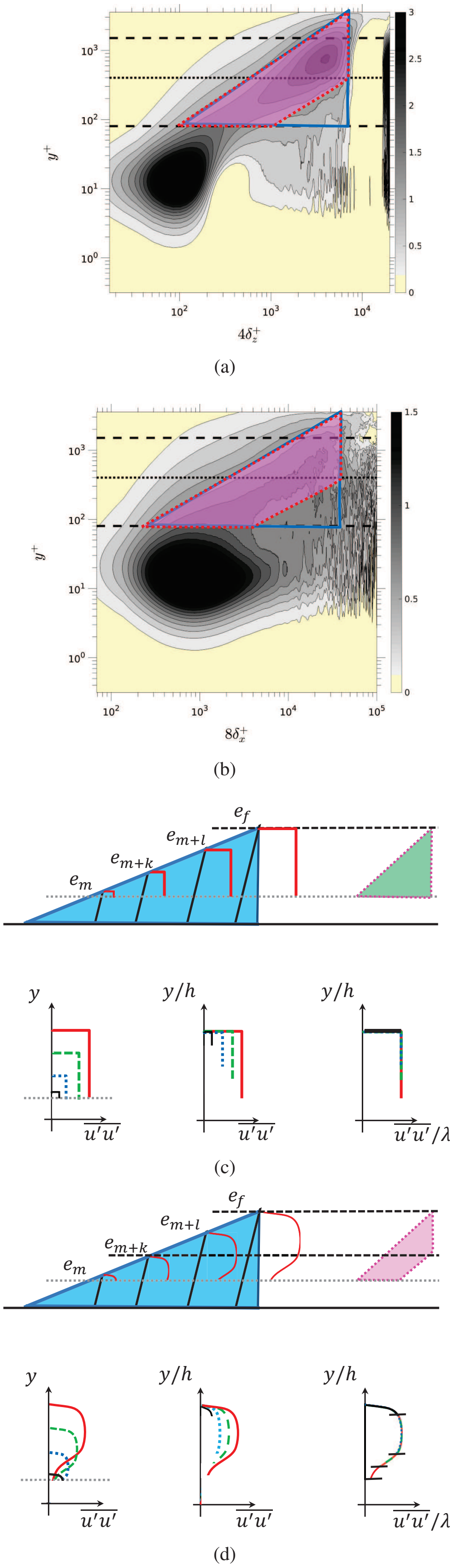

Figure 4. Conceptual representation of AE self-similarity through the meso-layer :(a) Maps of PMDS2 with δ taken in streamwise and spanwise direction, respectively. Conceptual velocity profile associated to (b) the original AEH and (c) revisited AEH.

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