Kolmogorov and non-Kolmogorov scalings

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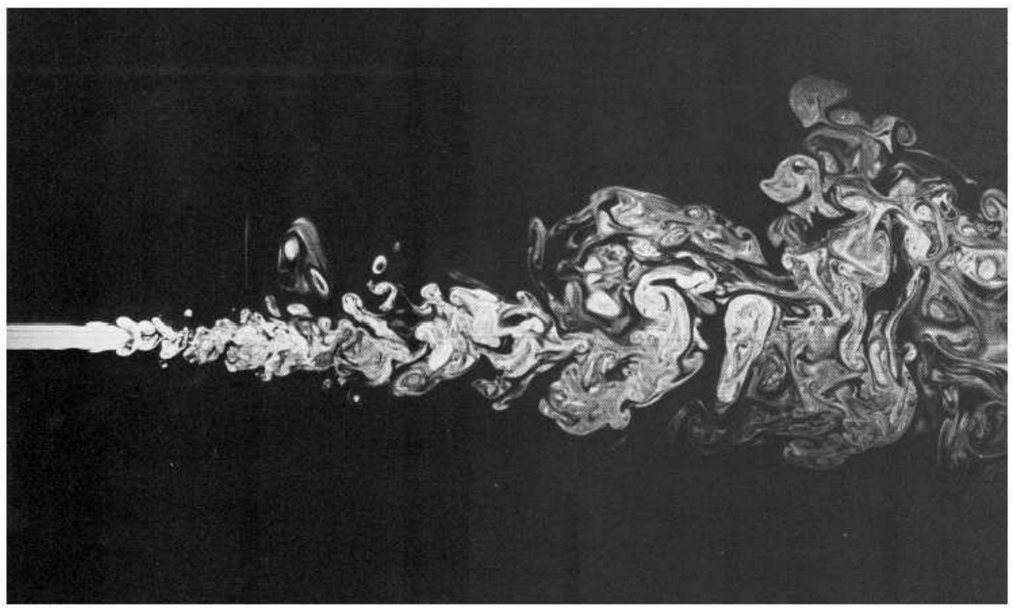
Turbulent jets

(Picture from album of fluid motion)

Jets



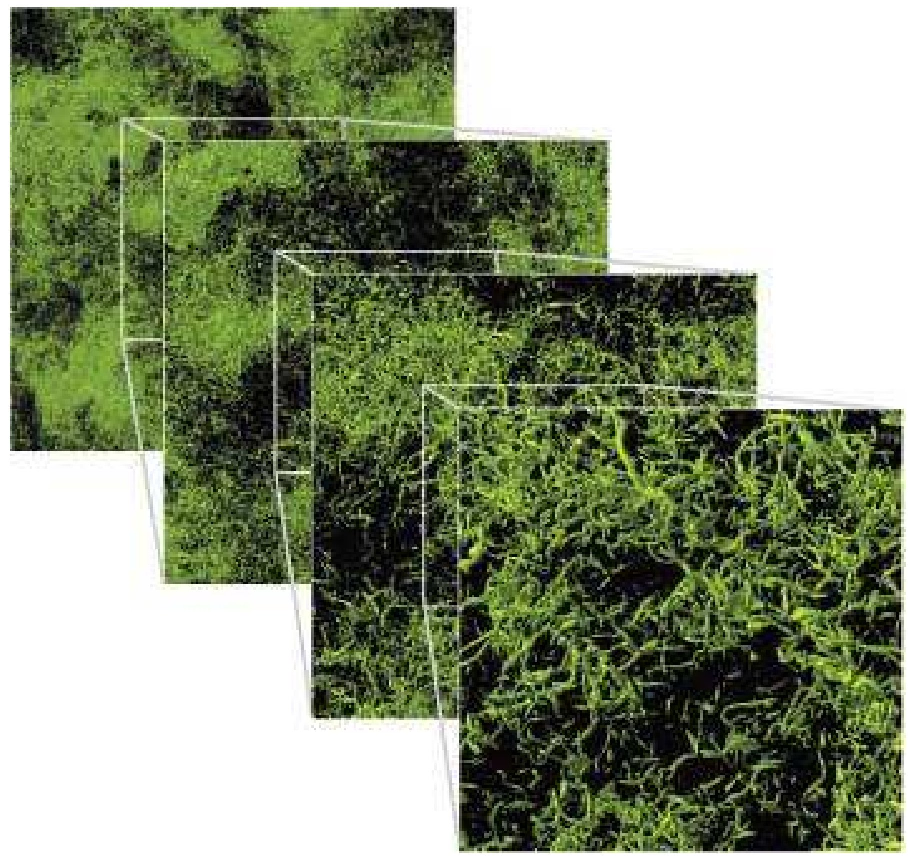
Turbulent wakes

(Picture downloaded from the web)

FIG. 1.

Homogeneous turbulence

(from Ishihara et al, early/mid 2000s, Japan, Earth Simulator calculations)



Some background

	WT HT	Axisym Jet	Plane Jet
u'	$U_\infty(\frac{x-x_0}{L_b})^{-p}$, $1/2 < p < 3/4$	$U_\infty(\frac{x-x_0}{L_b})^{-1}$	$U_\infty(\frac{x-x_0}{L_b})^{-1/2}$
L_u	$L_b(\frac{x-x_0}{L_b})^{q}$, $0 < q < 1/2$	$x - x_0$	$x - x_0$

	Axisym Wake	Plane Wake	Mixing Layer
u'	$U_\infty(\frac{x-x_0}{L_b})^{-2/3}$	$U_\infty(\frac{x-x_0}{L_b})^{-1/2}$	U_∞
L_u	$L_b^{2/3}(x-x_0)^{1/3}$	$L_b^{1/2}(x-x_0)^{1/2}$	$(x-x_0)$

L_b: characteristic cross-stream length-scale of inlet (e.g. mesh or nozzle or bluff body size...)
U_∞: characteristic inlet mean flow velocity or mean flow velocity cross-stream variation

Some background

	WT HT	Axisym Jet	Plane Jet
L_u/λ	$Re_0^{1/2}(\frac{x-x_0}{L_b})^{-p}$	$Re_0^{1/2}(\frac{x-x_0}{L_b})^{0}$	$Re_0^{1/2}(\frac{x-x_0}{L_b})^{1/4}$
Re_λ	$Re_0^{1/2}(\frac{x-x_0}{L_b})^{-p}$	$Re_0^{1/2}(\frac{x-x_0}{L_b})^{0}$	$Re_0^{1/2}(\frac{x-x_0}{L_b})^{1/4}$

	Axisym Wake	Plane Wake	Mixing Layer
L_u/λ	$Re_I^{1/2}(\frac{x-x_0}{L_b})^{-1/6}$	$Re_I^{1/2}(\frac{x-x_0}{L_b})^{0}$	$Re_I^{1/2}(\frac{x-x_0}{L_b})^{1/2}$
Re_λ	$Re_I^{1/2}(\frac{x-x_0}{L_b})^{-1/6}$	$Re_I^{1/2}(\frac{x-x_0}{L_b})^{0}$	$Re_I^{1/2}(\frac{x-x_0}{L_b})^{1/2}$

if λ obtained from $\epsilon \sim u'^3/L_u \sim \nu u'^2/\lambda^2$

then $L_u/\lambda \sim Re_I^{1/2} f(\frac{x-x_0}{L_b}) \sim Re_\lambda$ in all cases

$Re_I \equiv \frac{U_\infty L_b}{\nu}$: inlet Reynolds number

$L/\lambda \sim C_\epsilon Re_\lambda$

$\epsilon \sim \nu u'^2/\lambda^2$ (Taylor 1935)

$\epsilon = C_\epsilon u'^3/L$ (definition of C_ϵ introduced by Taylor 1935)

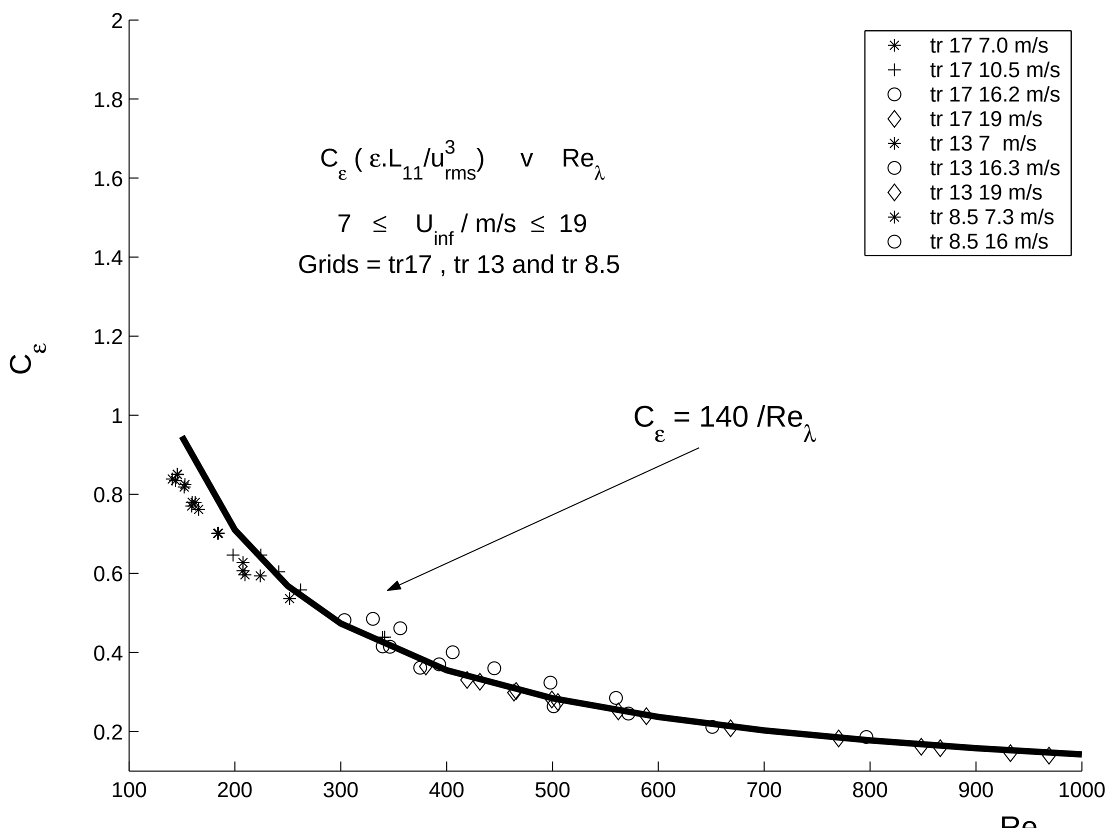
These two relations imply $L/\lambda \sim C_\epsilon Re_\lambda$.

$C_\epsilon = Const$ IFF $L/\lambda \sim Re_\lambda$

in which case,
if $Re_\lambda \sim Re_I^{1/2} f(\frac{x-x_0}{L_b})$
then $L/\lambda \sim Re_I^{1/2} f(\frac{x-x_0}{L_b})$.

However...

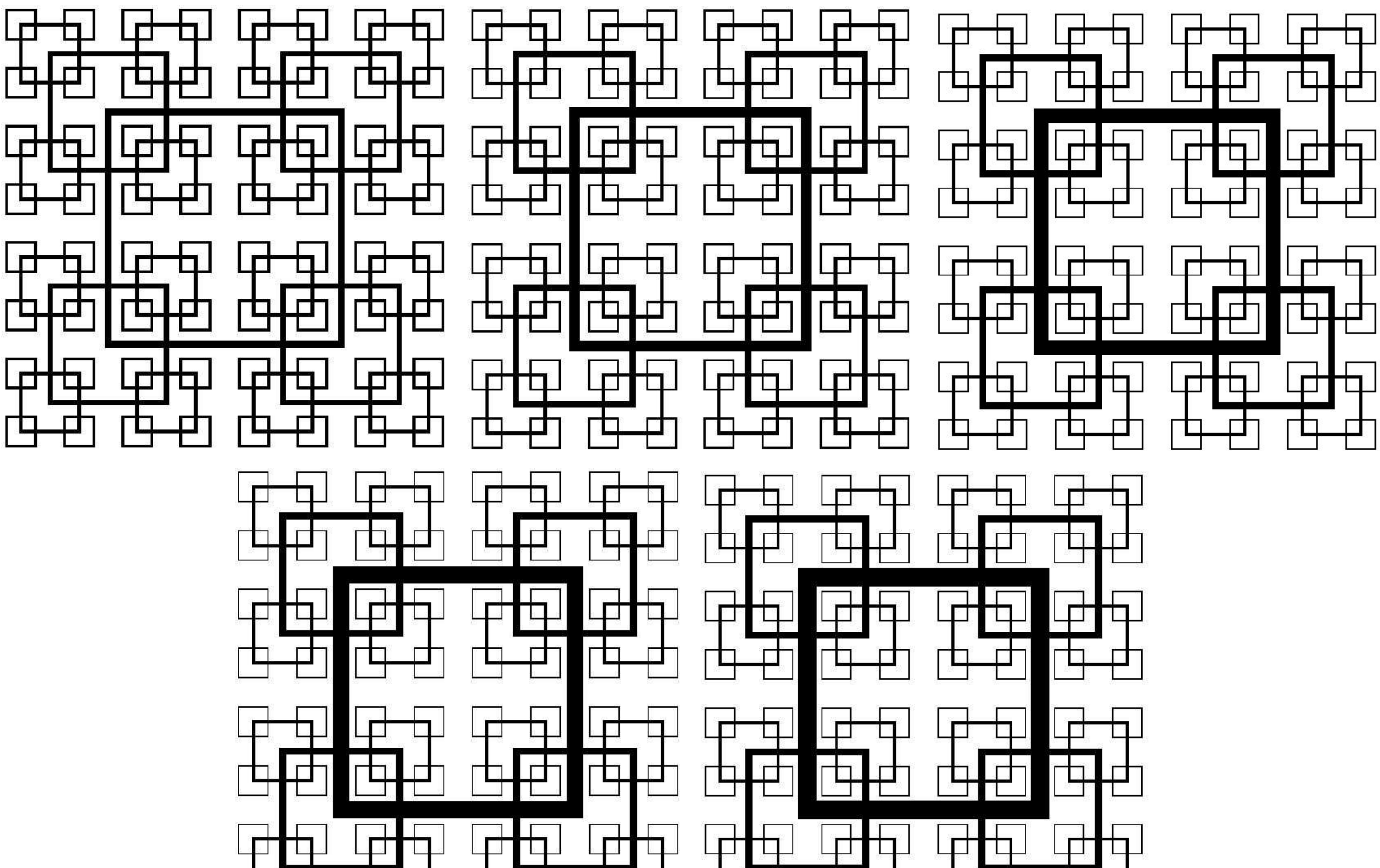
using fractal square grids, Seoud & V (PoF, 2007) showed that L/λ remains about constant as Re_λ decays when moving in the streamwise downstream direction in the part of the flow where the turbulence decays and which could be probed in the tunnel. $L/\lambda \sim C_\epsilon Re_\lambda$ then implies $C_\epsilon \sim Re_\lambda^{-1}$.



$D_f = 2$, $\sigma = 25\%$ fractal square grids

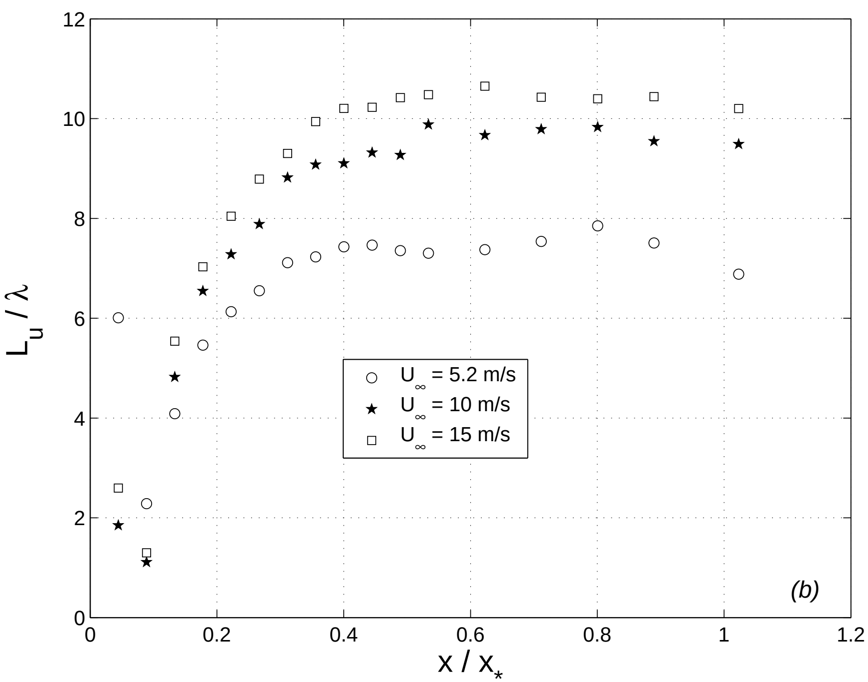
and equal $M_{eff} \approx 2.6cm$, $L_{max} \approx 24cm$, $L_{min} \approx 3cm$, $N = 4$, $T = 0.46m$.

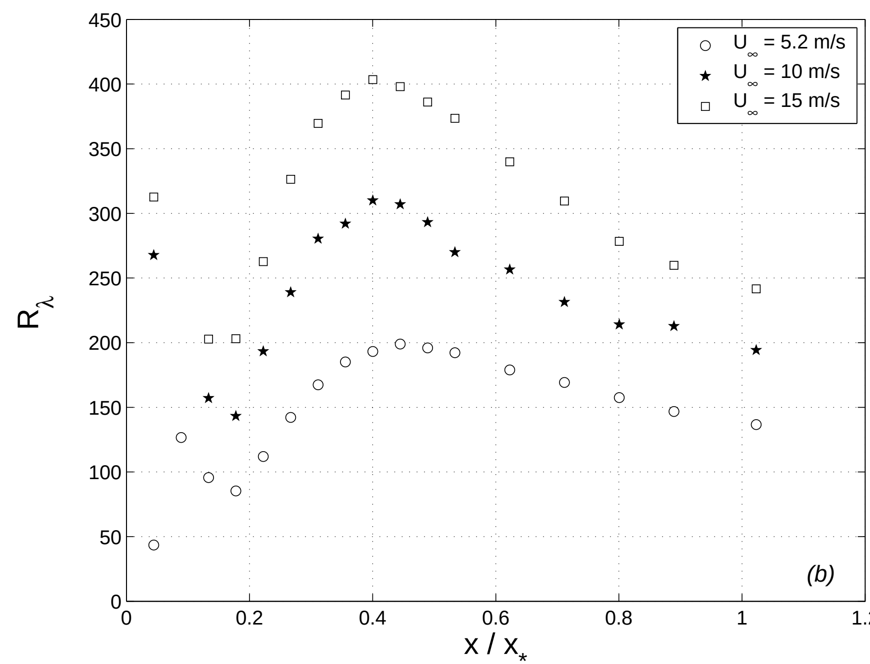
BUT $t_r = 2.5, 5.0, 8.5, 13.0, 17.0$



Mazellier & V (PoF 2010)

confirmed that L_u/λ is about constant where Re_λ decays and introduced the wake-interaction length-scale x_*.





L_u/λ indep of Re_λ but they also found that L_u/λ is increasing fct of Re_I, e.g. $L_u/\lambda \propto Re_I^\gamma$, $\gamma > 0$, (e.g. $\gamma = 1/2$?)

$L_u/\lambda \sim C_\epsilon Re_\lambda \propto Re_I^\gamma$ indep of x means $C_\epsilon \sim \frac{Re_I^\gamma}{Re_\lambda}$.

George (PoF 1992)

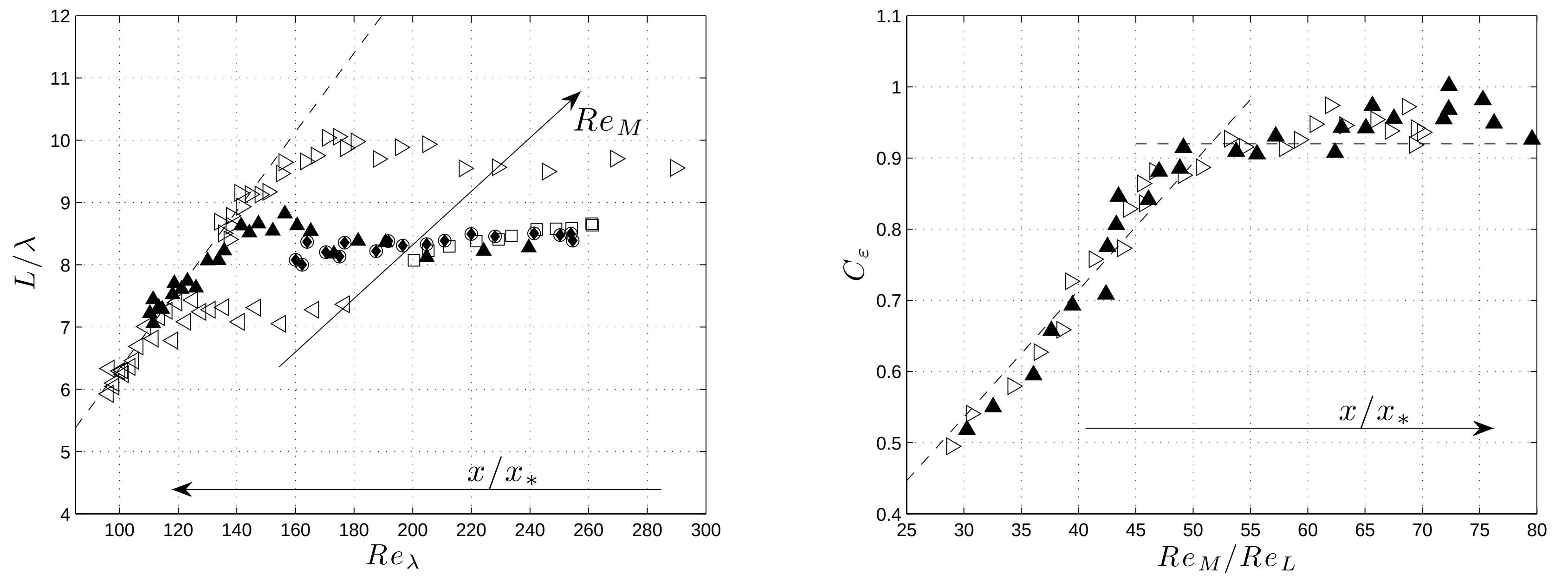
That L_u/λ may be independent of the local turbulence Reynolds number Re_λ but nevertheless an increasing function of the inlet Reynolds number Re_I, e.g. as $L_u/\lambda \sim Re_I^{1/2}$ had been anticipated by George (1992) but for homogeneous decaying turbulence, not specifically decaying turbulence downstream of fractal square grids.

Valente & V (JFM 2011; PRL 2012)

confirmed in JFM, October 2011 the findings of Mazellier & V (PoF 2010) over a much longer downstream decay region and carried our a very detailed documentation of various turbulence profiles etc.

In PRL, April 2012, Valente demonstrated that L_u/λ is independent of Re_λ and an increasing function of Re_I in the near decay region of regular grid turbulence too, a region which had been mostly neglected till now. The trick was to apply to regular grid turbulence the scaling of downstream distance based on x_* and find the equivalent region where $C_\epsilon \sim Re_\lambda^{-1}$ for regular grids appropriately designed to be able to detect this region.

$C_\epsilon \sim Re_I^\gamma / Re_\lambda \sim Re_I^{2\gamma} / Re_L$



$C_\epsilon \sim Re_I^\gamma / Re_\lambda \sim Re_I^{2\gamma} / Re_L$ (Valente's PRL plots above with $\gamma = 1/2$) in the high-Re_L decay region followed by $C_\epsilon \sim const$ in the further downstream low-Re_L decay region

For the rest of the talk, consider $C_\epsilon \sim Re_I^\beta / Re_L^\alpha$ for more generality and greater sweep of concepts.

Batchelor (1953) and $C_\epsilon \sim Re_I^\beta / Re_L^\alpha$

Batchelor (1953) provides evidence from grid-generated turbulence which seems to suggest that C_ϵ is independent of both Re_I and streamwise distance x, i.e. local turbulence Reynolds number $Re_L \equiv u'L/\nu$. Hence $\beta = \alpha = 0$.

He interprets this observation as being a "demonstration that changes in ν, which will be accompanied by changes in the motion associated with the dissipation range of wave-numbers, have no effect on the rate of transfer of energy from the lower wavenumbers".

$C_\epsilon \sim Re_I^\beta / Re_L^\alpha$

Batchelor's (1953) comment implies that if $\alpha - \beta$ is different from 0 then viscosity has an effect on the rate of energy transfer. We now look closer into this by a re-examination of $< \delta u^3(r) > \approx -\frac{4}{5}\epsilon r$ in homogeneous isotropic turbulence.

Quoting Batchelor (1953): “the statistical quantities determined by the equilibrium range are independent of the properties of the large-scale components of the turbulence and do not require the turbulence to be accurately homogeneous.”

$$< \delta u^3(r) > \approx -\frac{4}{5}\epsilon r$$

Derivations of this relation in the inertial range of scales r have been proposed for

(i) statistically stationary homogeneous isotropic turbulence forced at the large scales by a number of authors, e.g. Frisch (1995) following Kolmogorov (1941), under the assumption that $C_\epsilon = const$, specifically $\alpha = \beta = 0$

(ii) decaying homogeneous isotropic turbulence by Lundgren (PoF 2002, 2003) under the assumption that $C_\epsilon = const$, specifically $\alpha = \beta = 0$

(iii) decaying homogeneous isotropic turbulence by Tchoufag, Sagaut & Cambon (PoF 2012) under the assumption that the spectral transfer term $T(k) \approx 0$ in the assumed inertial range.

$< \delta u^3(r) > \approx -\frac{4}{5}\epsilon r$

It is possible to derive $T(k) \approx 0$ in an intermediate range of scales for decaying homogeneous isotropic turbulence under the assumption that energy and its time derivative are finite and various forms of assumption on dissipation, one of them being that $C_\epsilon \sim Re_L(0)^\beta / Re_L^\alpha$ with $\alpha < 1$.

It is then also possible to rigorously use the following relation of Tchoufag, Sagaut & Cambon (2012) to derive $< \delta u^3(r) > \approx -\frac{4}{5}\epsilon r$ in a rigorously defined sufficient intermediate range of scales r:

$$< \delta u^3(r) > = 12r \int_0^{+\infty} g_5(kr) T(k) dk$$

where $g_5(kr) = \frac{3(\sin kr - kr \cos kr) - (kr)^3 \sin kr}{(kr)^5}$

Decaying HIT

Lin (1949) equation:

$$\frac{\partial}{\partial t}E(k,t) = T(k,t) - 2\nu k^2 E(k,t)$$

ASSUMPTION 1: An outer lenth-scale $L(t)$ exists such that

$$\int_0^{k'} E(k,t) \approx \frac{3}{2}u'^2 \text{ \& } \frac{d}{dt}\int_0^{k'} E(k,t) \approx \epsilon$$

for $k'L \gg 1$.

Integrate the Lin equation from 0 to k' and apply assumption 1 to get:

$$-\epsilon \approx \int_0^{k'} T(k,t)dk - 2\nu \int_0^{k'} k^2 E(k,t)dk$$

Next, non-dimensionalise with $u'(t)$ and $L(t)$ using $\epsilon = C_\epsilon u'^3/L$ as definition of C_ϵ (no assumptions).

Non-dimensional approximate Lin eq.

$$-C_\epsilon \approx \int_0^{k'L} \tau(\kappa,t)d\kappa - \frac{2}{Re_L}\int_0^{k'L} \kappa^2 e(\kappa,t)d\kappa$$

where $\kappa \equiv kL$, $\tau(\kappa,t) \equiv \frac{T(k,t)}{u'^3(t)}$, $e(\kappa,t) \equiv \frac{E(k,t)}{u'^2(t)L(t)}$ and $Re_L \equiv u'L/\nu$.

A sufficient condition for $-C_\epsilon \approx \int_0^{k'L} \tau(\kappa,t)d\kappa$, $k'L \gg 1$ is

$$C_\epsilon \gg \frac{2}{Re_L}(k'L)^2 \int_0^{k'L} e(\kappa,t)d\kappa$$

because $(k'L)^2 \int_0^{k'L} e(\kappa,t)d\kappa > \int_0^{k'L} \kappa^2 e(\kappa,t)d\kappa$, which is the neglected viscous term.

Using assumption 1, the sufficient condition for

$$-C_\epsilon \approx \int_0^{k'L} \tau(\kappa,t)d\kappa \text{ at } k'L \gg 1$$

is $C_\epsilon \gg \frac{3}{Re_L}(k'L)^2$

An assumption on dissipation

A sufficient range where $-\epsilon \approx \int_0^{k'} T(k,t)dk$ is

$$L^{-1} \ll k' \ll L^{-1}(\frac{C_\epsilon Re_L}{3})^{1/2} \sim \lambda^{-1}$$

Now chose an assumption on dissipation, for example $C_\epsilon \sim \frac{Re_L(0)^\beta}{Re_L^\alpha}$.

The sufficent range becomes

$$L^{-1} \ll k' \ll L^{-1} Re_L(0)^\beta Re_L^{(1-\alpha)/2}$$

This is a sufficient range of wavenumbers where the interscale energy flux $\int_0^{k'} T(k,t)dk$ is independent of k', hence $T(k',t) \approx 0$. One can then obtain $< \delta u^3(r) > \approx -\frac{4}{5}\epsilon r$ in a well-defined sufficient range of length-scales r.

$L^{-1} \ll k' \ll L^{-1} Re_L(0)^\beta Re_L^{(1-\alpha)/2}$

This is not a range, however, where the interscale energy flux is independent of viscosity except in the particular case where $\alpha = \beta$ because of the ansatz $C_\epsilon \sim \frac{Re_L(0)^\beta}{Re_L^\alpha}$.

So, except if $\beta = \alpha$, the usual K41 dimensional arguments which require ϵ to be independent of ν to yield $< \delta u^n(r) > \sim (\epsilon r)^{n/3}$ in an appropriate intermediate range cannot be used. Nevertheless, $< \delta u^3(r) > \sim \epsilon r$ ($n = 3$).

However, one can always argue that $E(k)$ can only depend on k and interscale energy flux, thus leading to $E(k) \sim \epsilon^{2/3} k^{-5/3}$, and therefore $< \delta u^2(r) > \sim (\epsilon r)^{2/3}$ in the appropriate ranges. But this energy flux and ϵ can now depend on viscosity.

$L^{-1} \ll k' \ll L^{-1} Re_L(0)^{\beta} Re_L^{(1-\alpha)/2} \sim \lambda^{-1}$

Sufficient condition range for $\int_0^{k'} T(k,t)dk = -\epsilon$ under assumption 1 and $C_\epsilon \sim \frac{Re_L(0)^{\beta}}{Re_L^{\alpha}}$

Note 1: This range decreases with time if Re_L decreases with time and $\alpha < 1$.

Note 2: $\alpha = 1$ is special as the range L^{-1} to λ^{-1} remains constant during turbulence decay in that case.

Note 3: The range where $\int_0^{k'} T(k,t)dk = -\epsilon$ may in fact be wider as the range we found is only sufficient, not necessary.

Vorticity and strain

In homogeneous isotropic turbulence,

$$\frac{<\omega\cdot\mathbf{s}\omega>}{<\omega^2>} = \frac{1}{2}\Big(\frac{15}{2}\Big)^{1/2}\frac{\int_0^\infty k^2 T(k)dk}{(\int_0^\infty k^2 E(k)dk)^{3/2}}$$

In general $\mathbf{u} =< \mathbf{u} > +\mathbf{u}'$ and

$$\frac{\partial}{\partial t}\mathbf{u}' + \mathbf{u}' \cdot \nabla\mathbf{u}' = -\nabla p'/\rho + \nu\nabla^2\mathbf{u}' + \frac{1}{\rho}\mathbf{f}$$

where

$$\frac{1}{\rho}\mathbf{f} = \mathbf{u}' \cdot \nabla < \mathbf{u} > + \frac{\partial}{\partial x_k} < u_k'\mathbf{u}' > - < \mathbf{u} > \cdot\nabla\mathbf{u}'$$

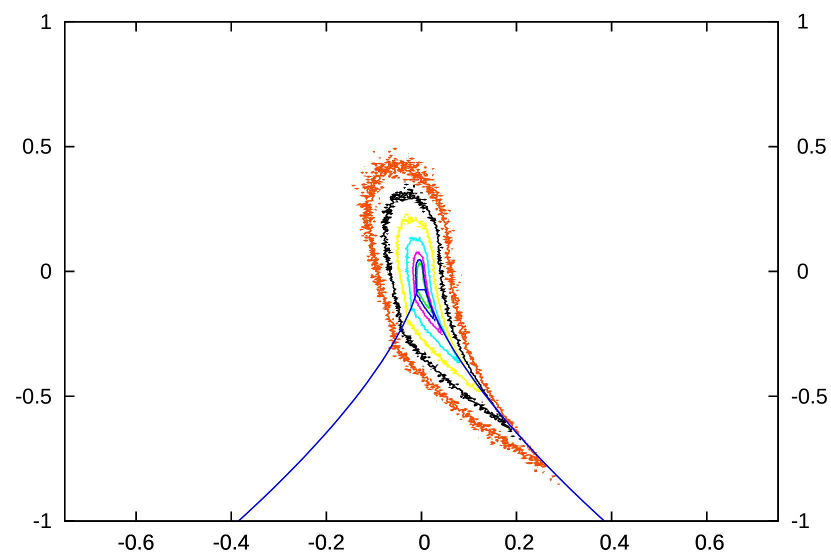
Vorticity and strain rate of fluctuating velocity:

$$\omega \equiv \nabla \times \mathbf{u}' \text{ and } s_{ij} \equiv \frac{1}{2}\Big(\frac{\partial}{\partial x_i}u_j' + \frac{\partial}{\partial x_j}u_i'\Big)$$

$Q - R$ diagram

$$Q \equiv \frac{1}{4}(\omega^2 - 2s_{ij}s_{ji}) \text{ and } R \equiv -\frac{1}{3}(s_{ij}s_{jk}s_{ki} + \frac{3}{4}\omega_i s_{ij}\omega_j)$$

The Q-R diagram (below for periodic DNS turbulence, courtesy of Ryo Onishi) has this tear-drop shape in many turbulent flows: turbulent boundary layers, mixing layers, grid turbulence, jet turbulence (see Tsinober 2009). For homogeneous/periodic turbulence it can be proved that $< Q >= 0$, $< R >= 0$.



Direct Numerical Simulations

We use Incompact3D which originates from Eric Lamballais (Poitiers) and his coworkers: solves incompressible Navier-Stokes on a cartesian mesh with 6th-order compact finite differen schemes for spatial discretization, 3d-order Adams-Bashforth for time advancement and a fractional step method for incompressibility which involves solving the Poisson pressure equation in spectral space and use of the concept of modified wave number.

Immersed Boundary Method for modelling the grid in the flow.

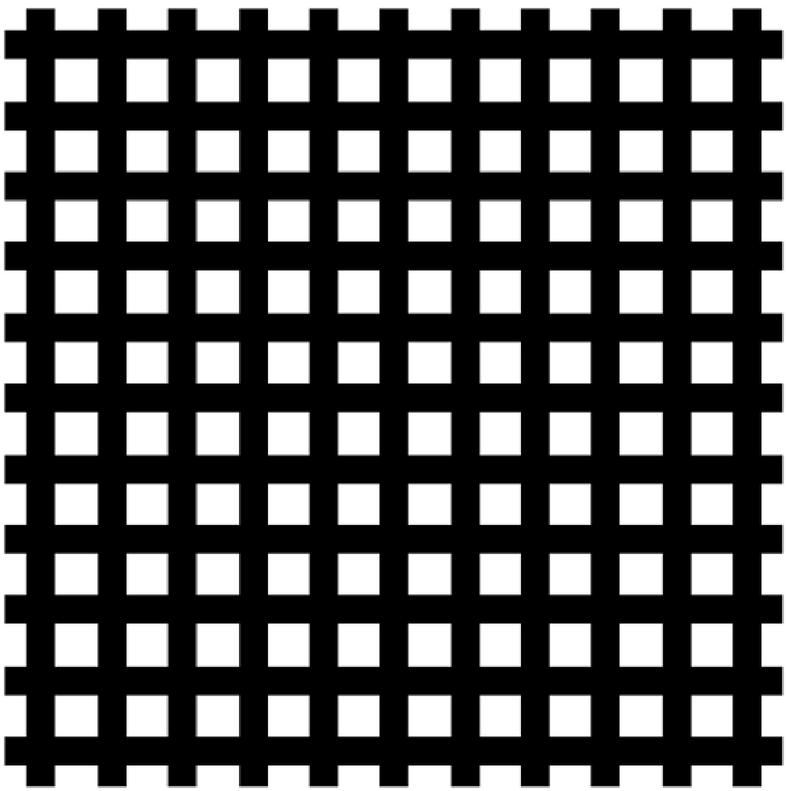
Inflow/outflow boundary conditions in the streamwise direction, periodic boundary conditions in the lateral directions.

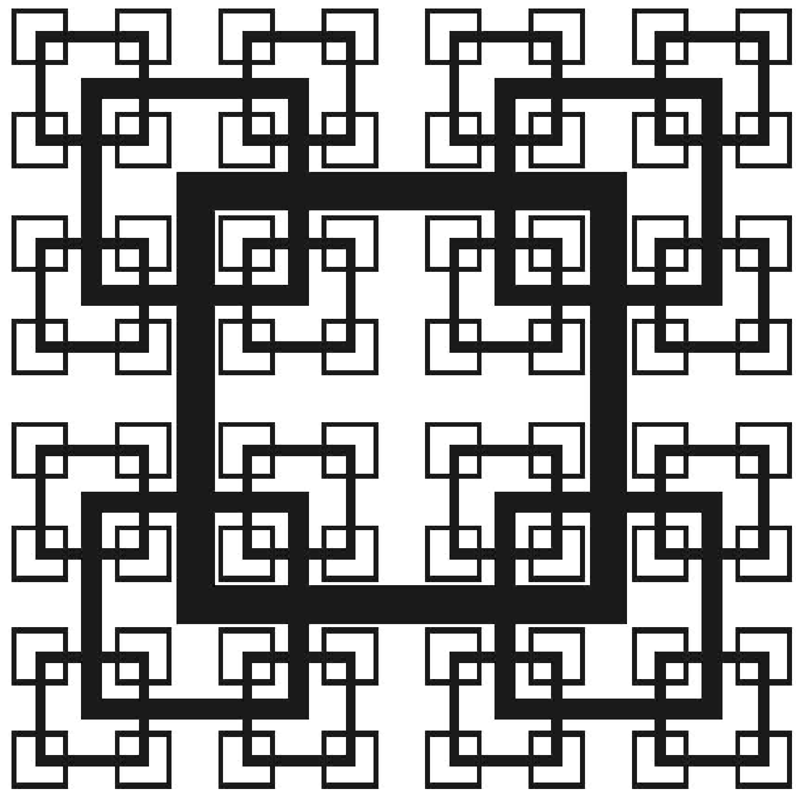
Grids with same $\sigma = 50$%

same $M_{eff} \equiv \frac{4T^2}{P}\sqrt{1-\sigma}$ and $U_\infty M_{eff}/\nu$.

Fractal grid: $D_f = 2$, $N = 4$, $t_r = 8.5$

Regular grid: $b = 2t_{min}$

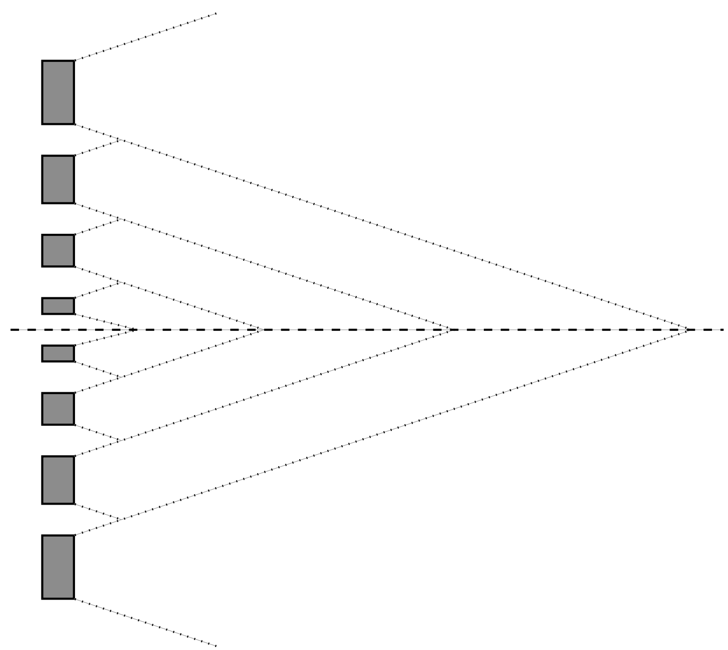




Wake-interaction length-scales

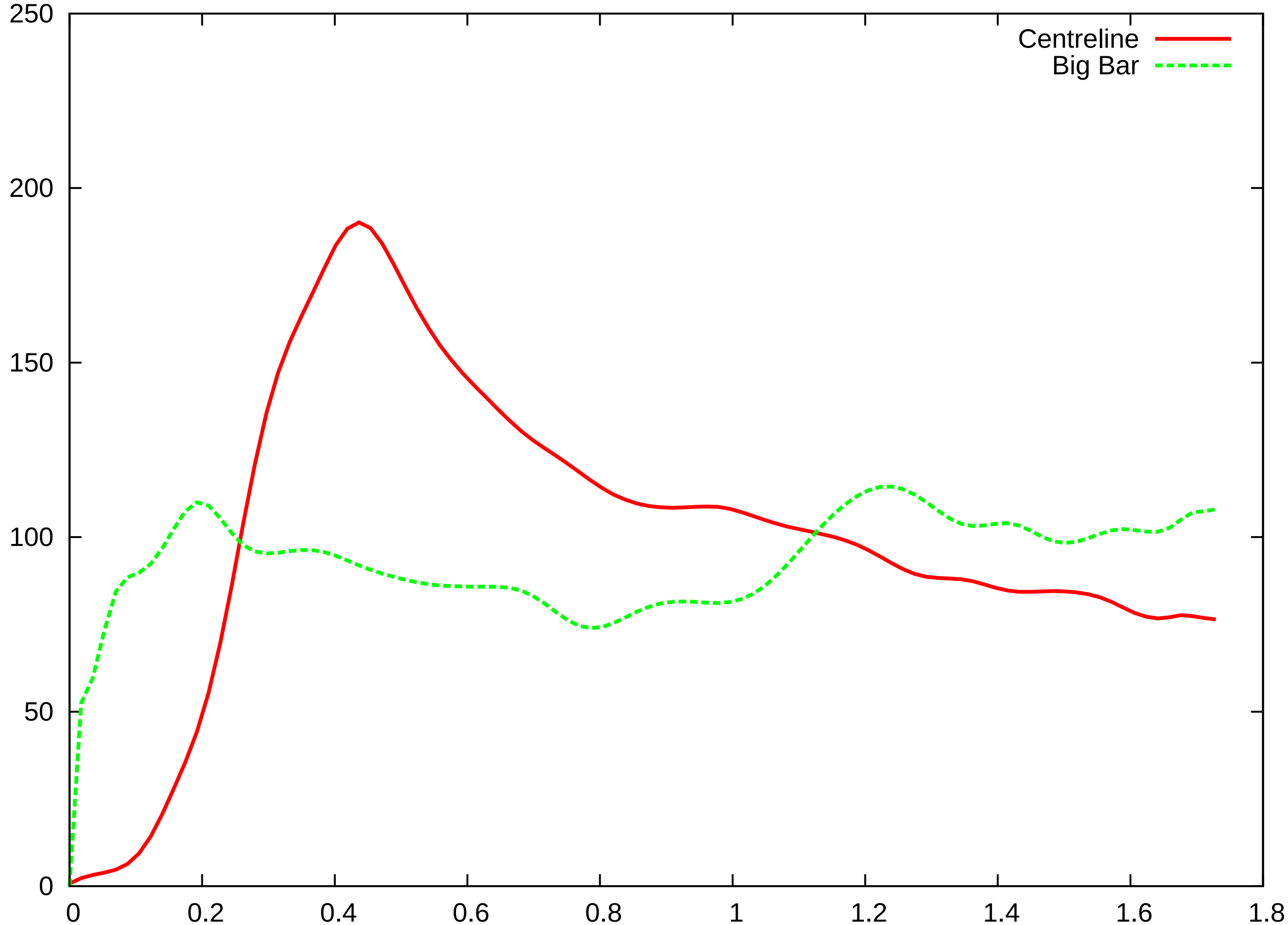
Regular grid: $\frac{1}{2}M^2/b$

Fractal grid: $\frac{1}{2}L_3^2/t_3 \approx 6.25 M_{eff} < \frac{1}{2}L_2^2/t_2 \approx 12.255 M_{eff} < \frac{1}{2}L_1^2/t_1 \approx 24 M_{eff} < \frac{1}{2}L_0^2/t_0 \approx 47 M_{eff} = \frac{1}{2}x_*$



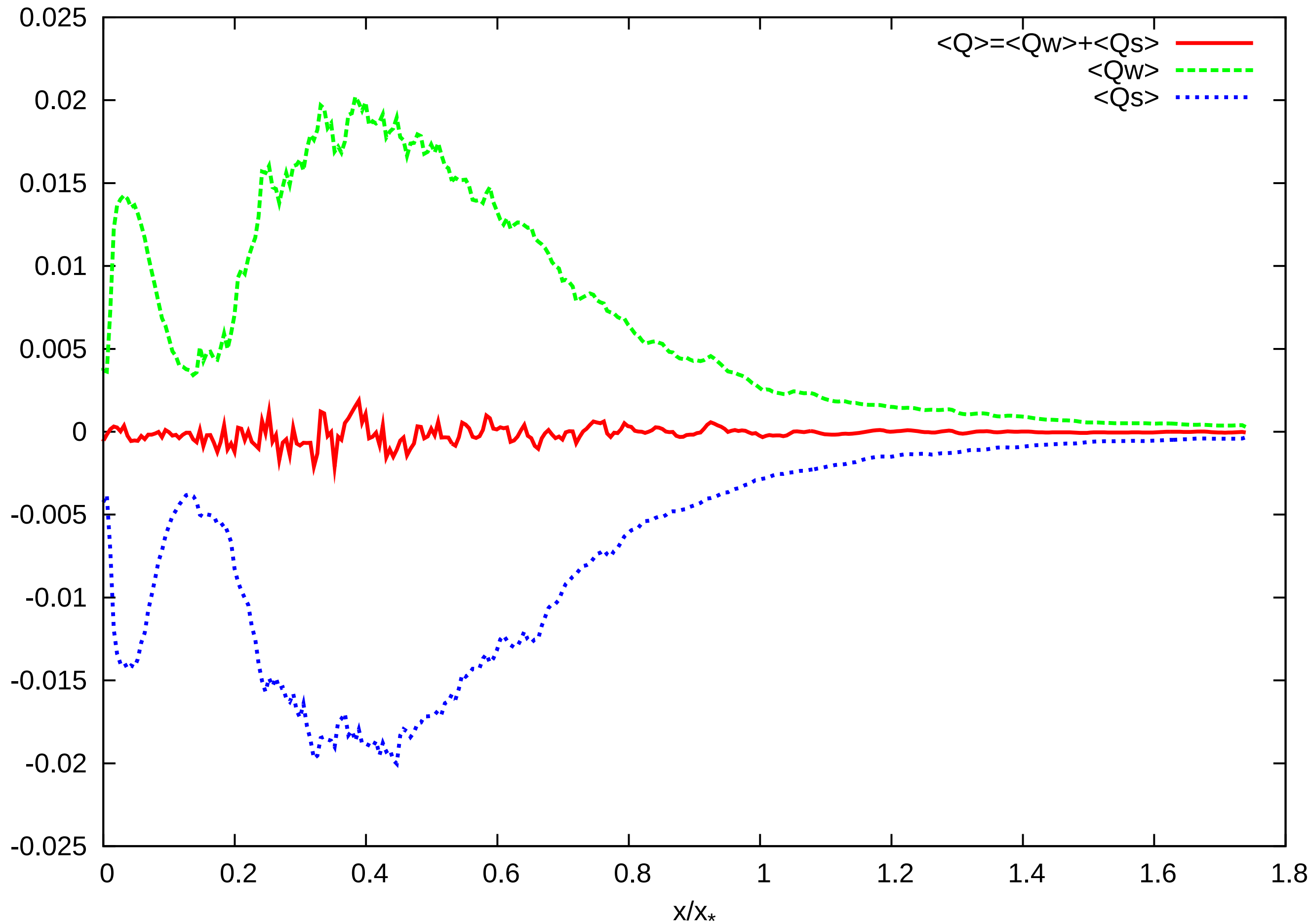
The wake-interaction length-scale x_* has been generalised for a wider range of grids and flow conditions by Gomes Fernandes (his talk in this workshop).

Re_λ as fct of x/x_*



$Q_w \equiv \frac{1}{4}\omega^2$ and $Q_s \equiv -\frac{1}{2}s_{ij}s_{ij}$

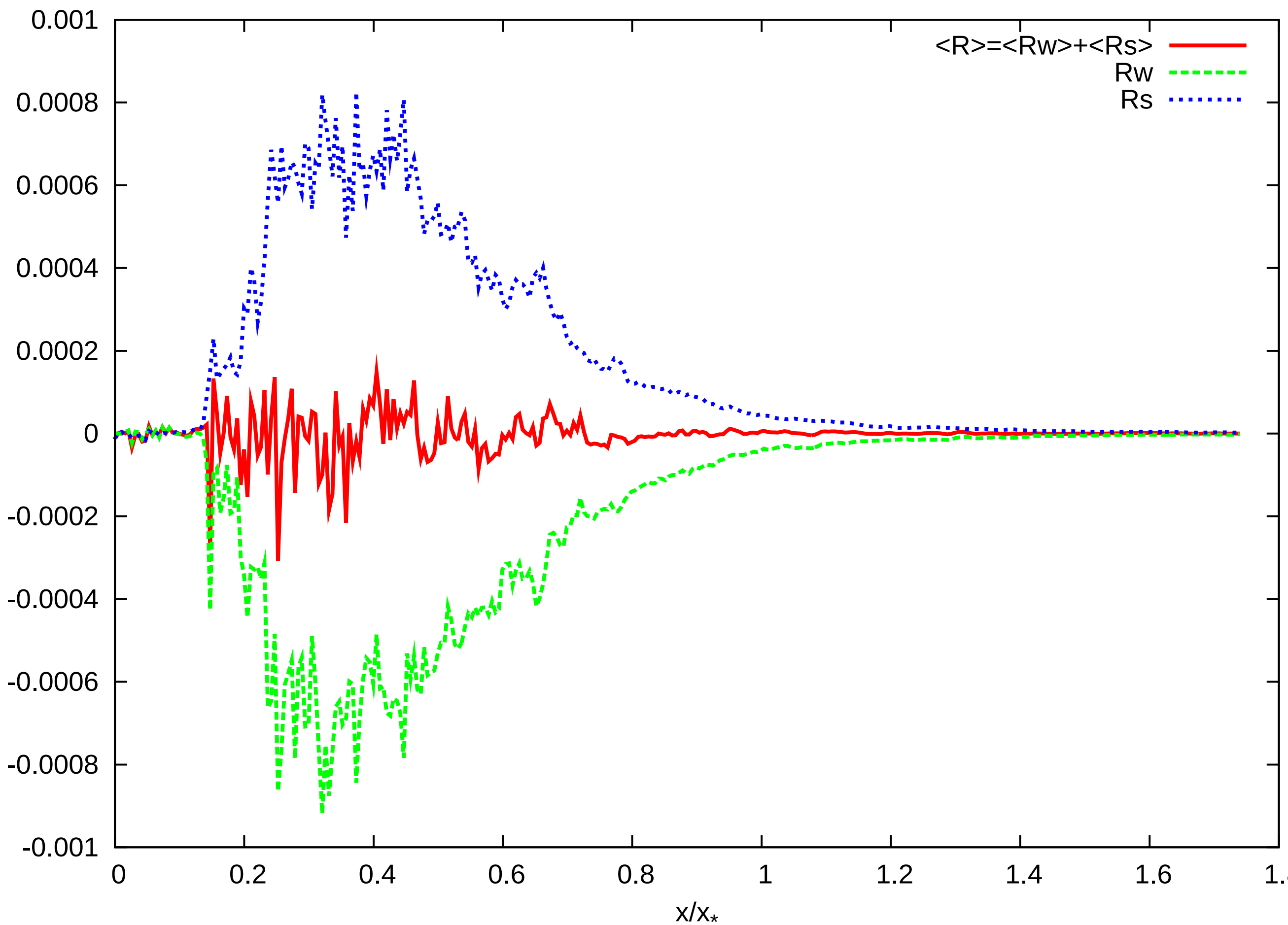
$< Q_w >$ and $< Q_s >$ as fcts of x/x_* along the centerline



$$Q = Q_w + Q_s$$

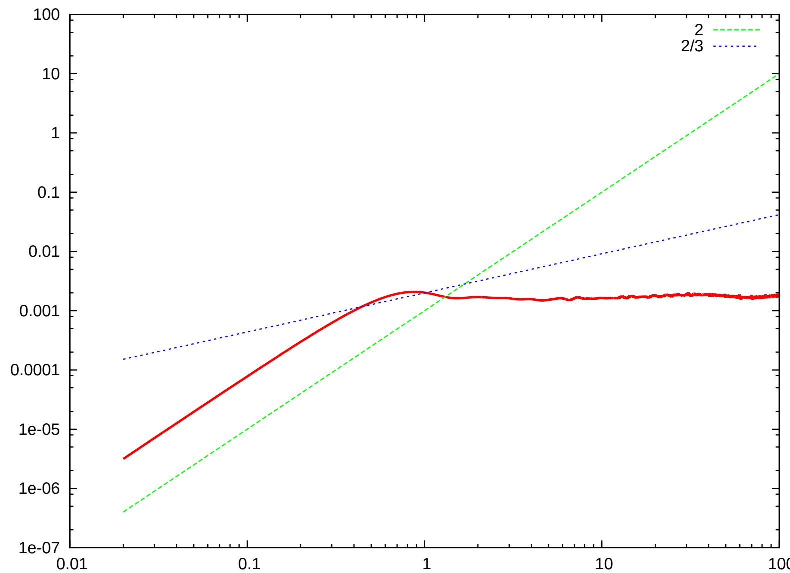
$$R_w \equiv -\frac{1}{4}\omega_i\omega_j s_{ij} \textbf{ and } R_s \equiv -\frac{1}{3}s_{ij}s_{jk}s_{ki}$$

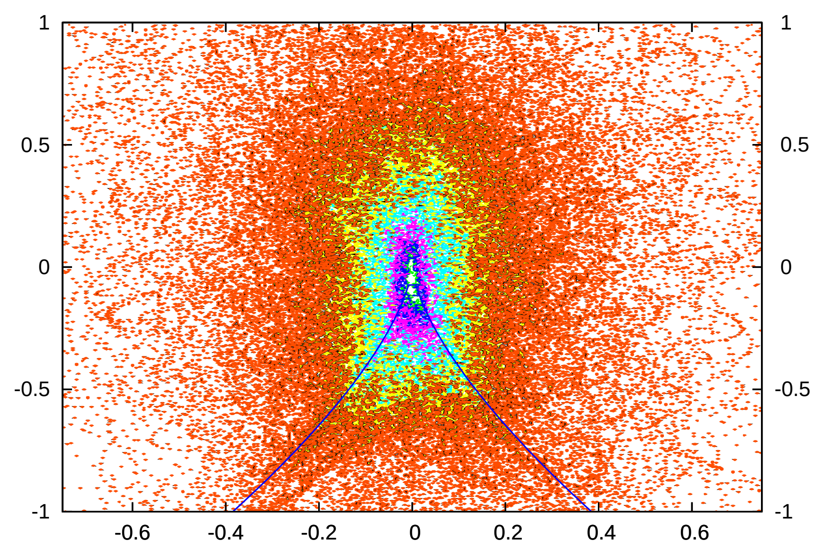
$< R_w >$ and $< R_s >$ as fcts of x/x_* along the centerline



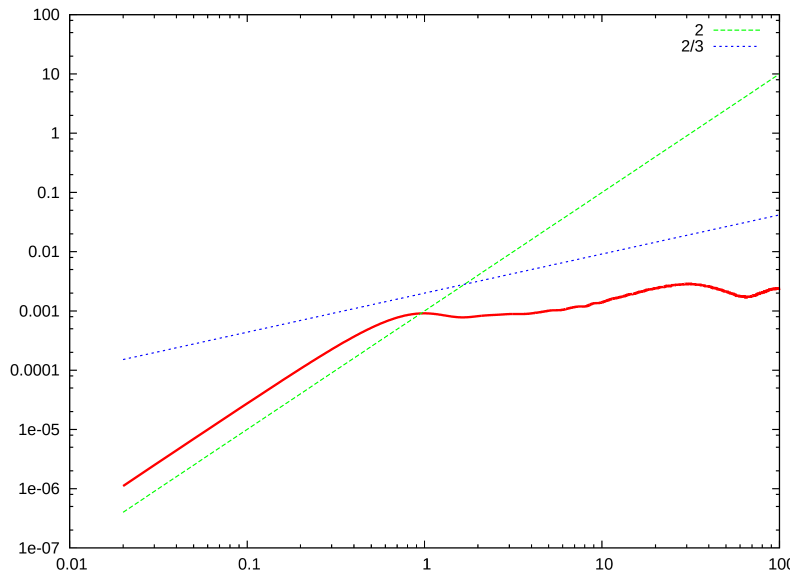
$$R = R_w + R_s$$

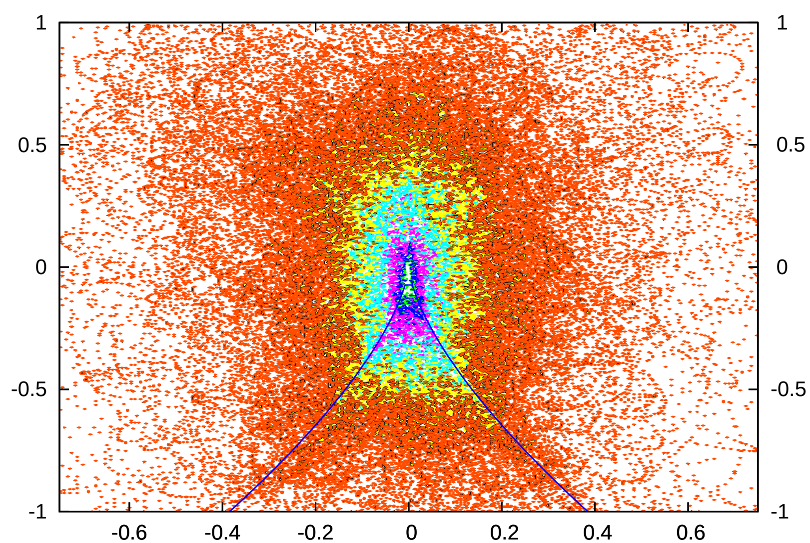
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 0.052$



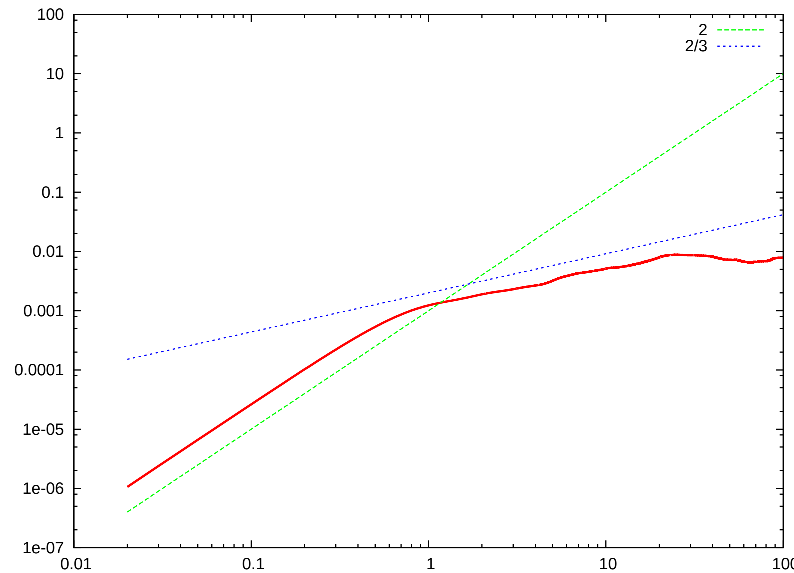


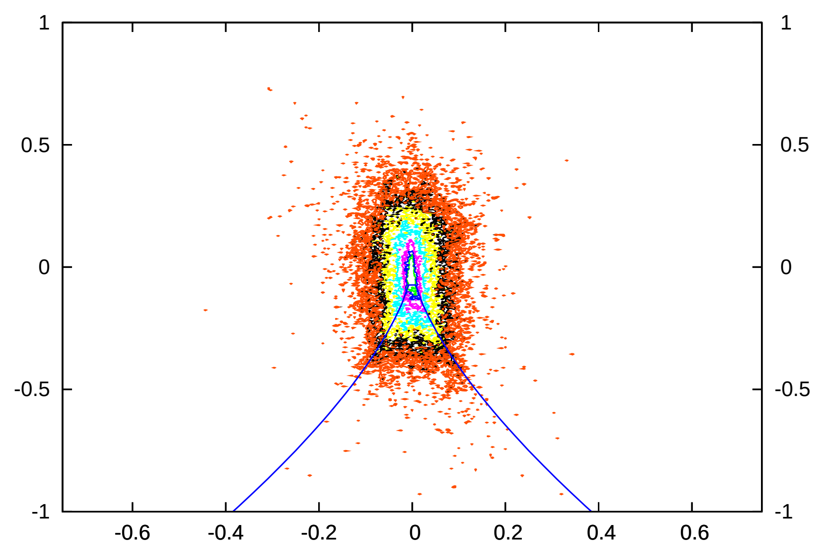
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 0.105$



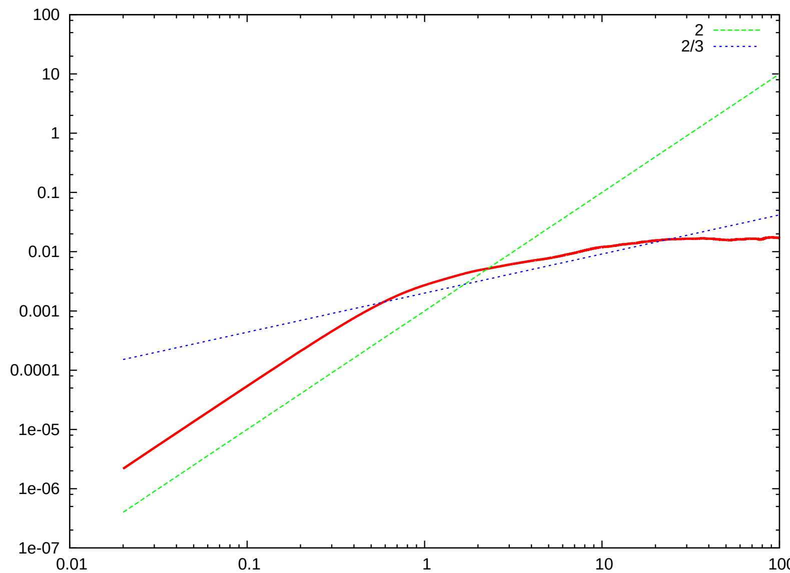


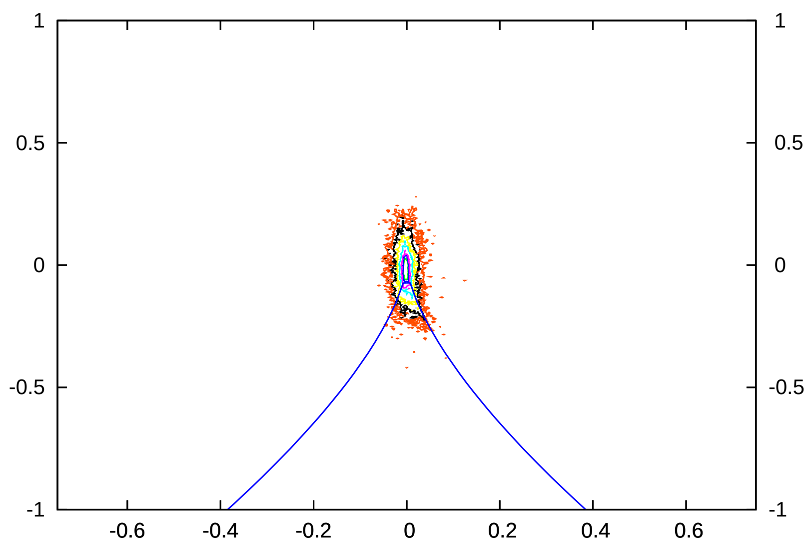
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 0.157$



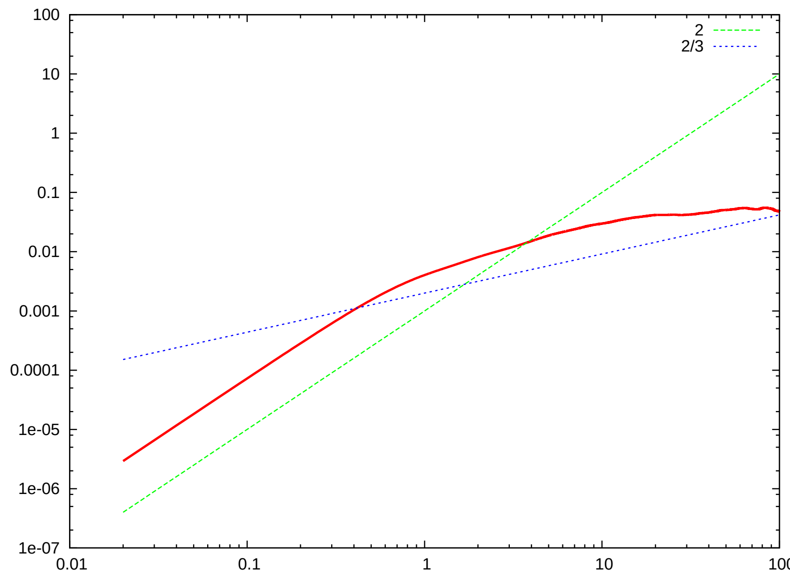


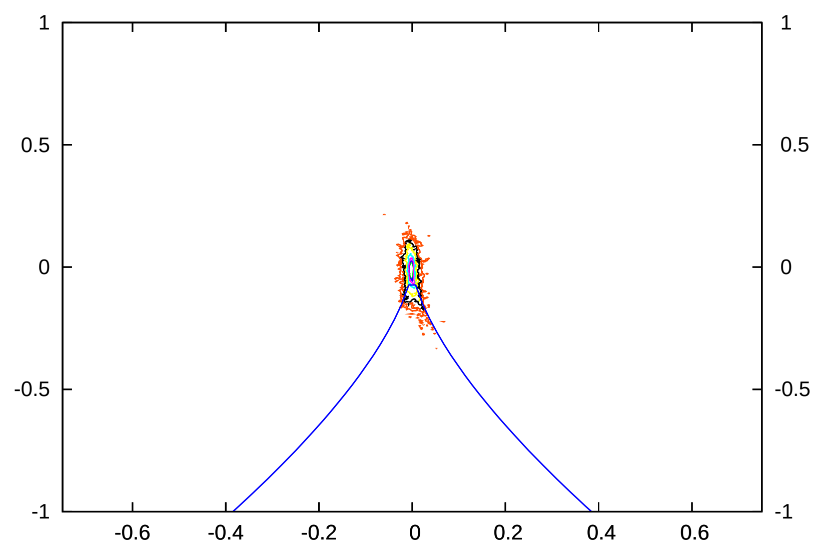
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 0.210$



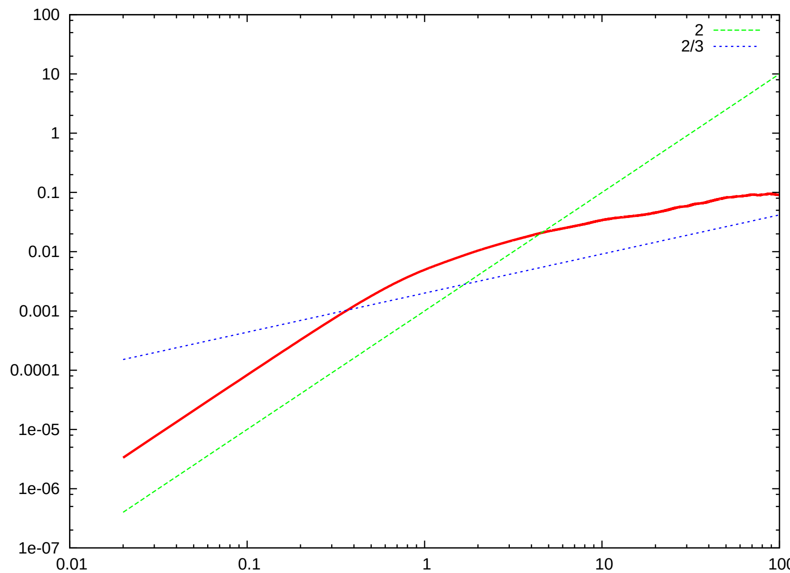


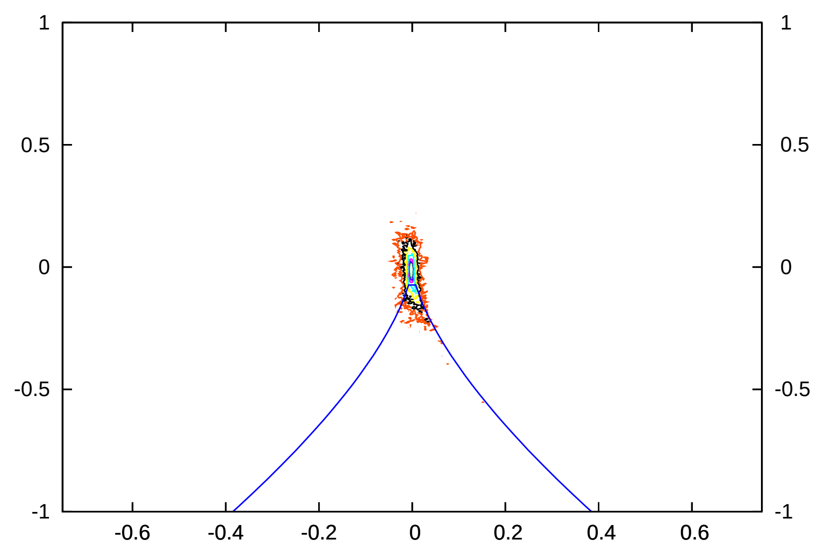
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 0.262$



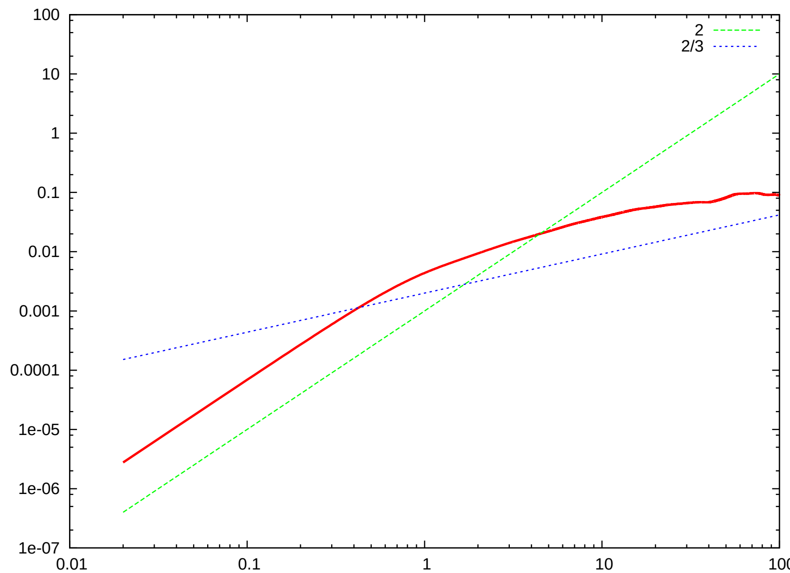


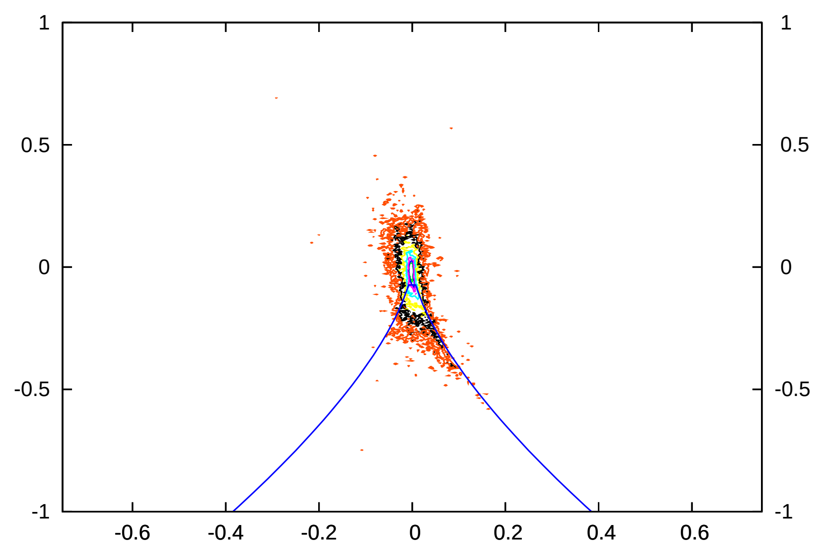
$<\delta u^2(\tau)>$ and $Q-R$ at $x/x_* = 0.315$



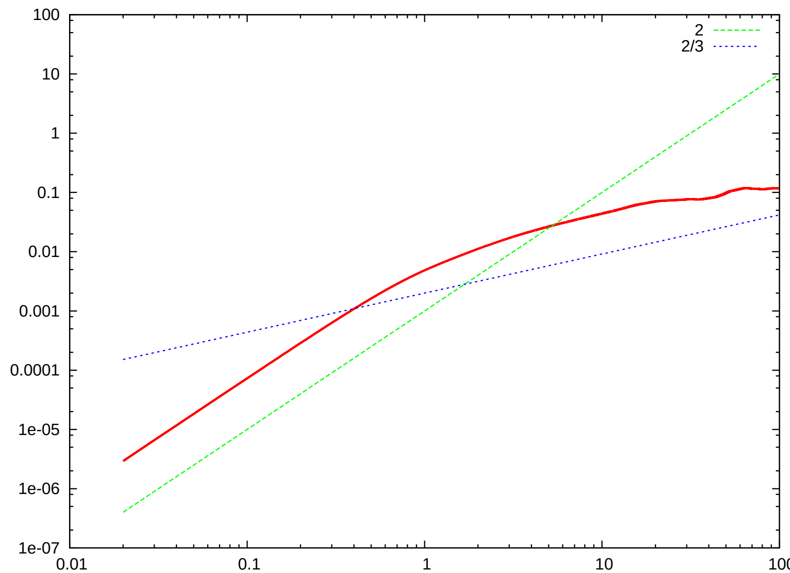


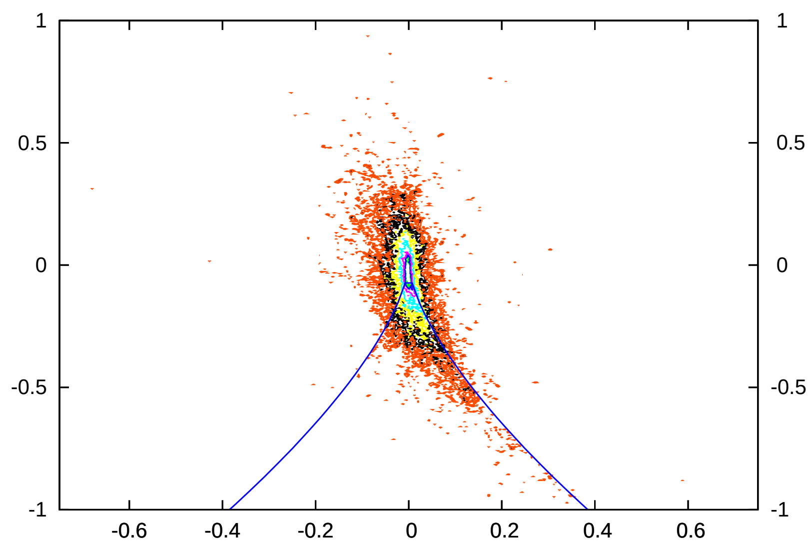
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 0.367$



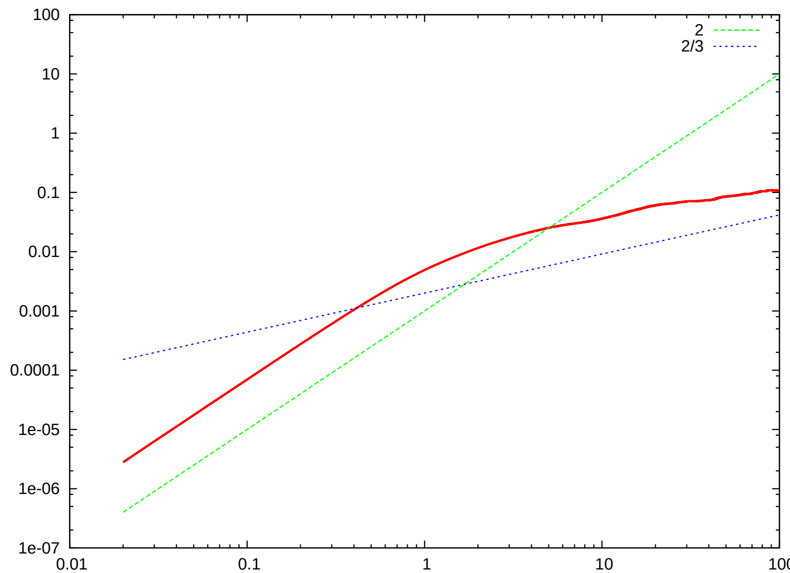


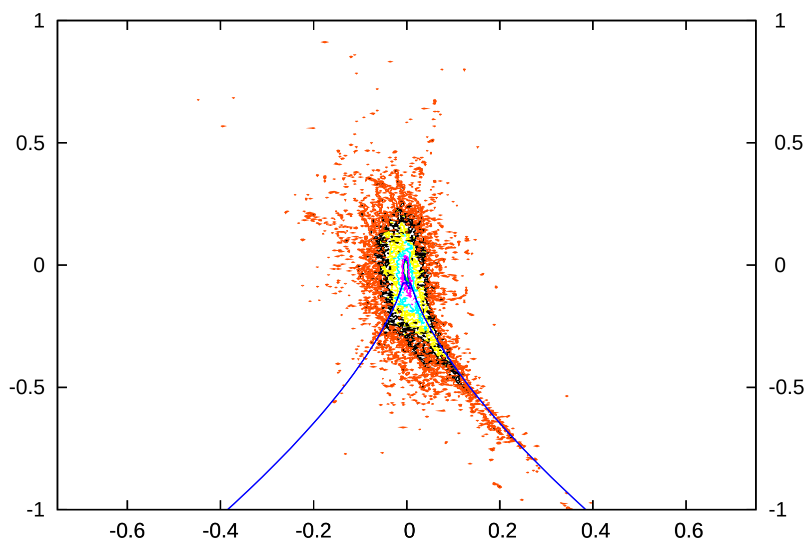
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 0.420$



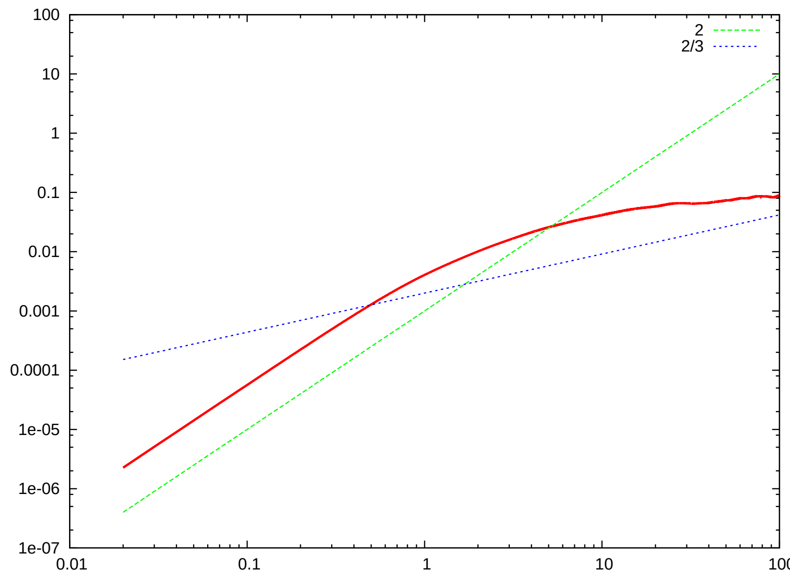


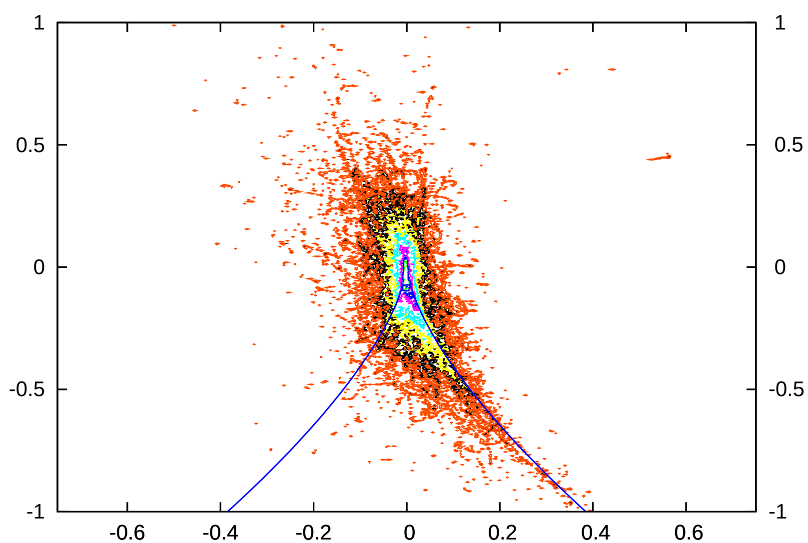
$<\delta u^2(\tau)>$ **and** $Q-R$ **at** $x/x_* = 0.472$



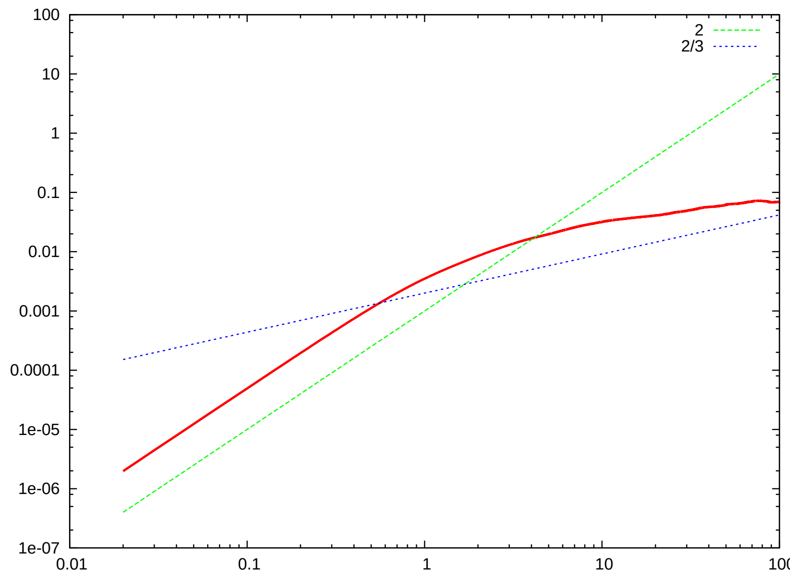


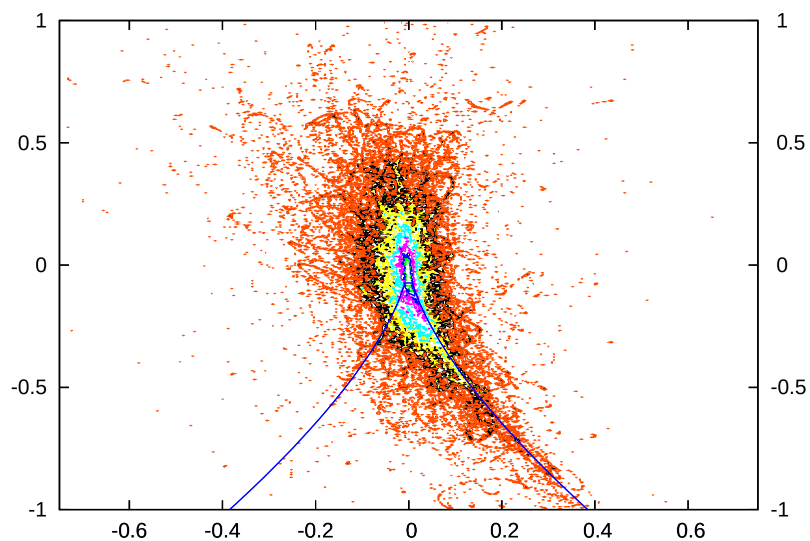
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 0.525$



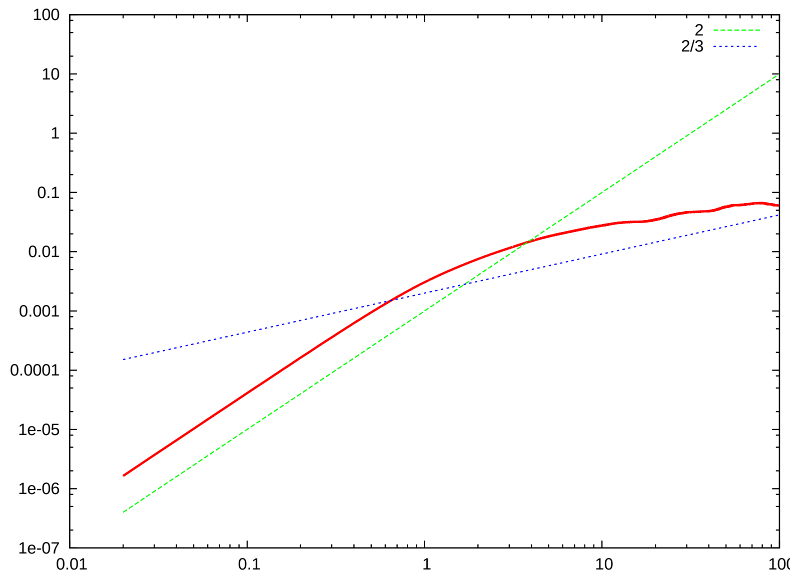


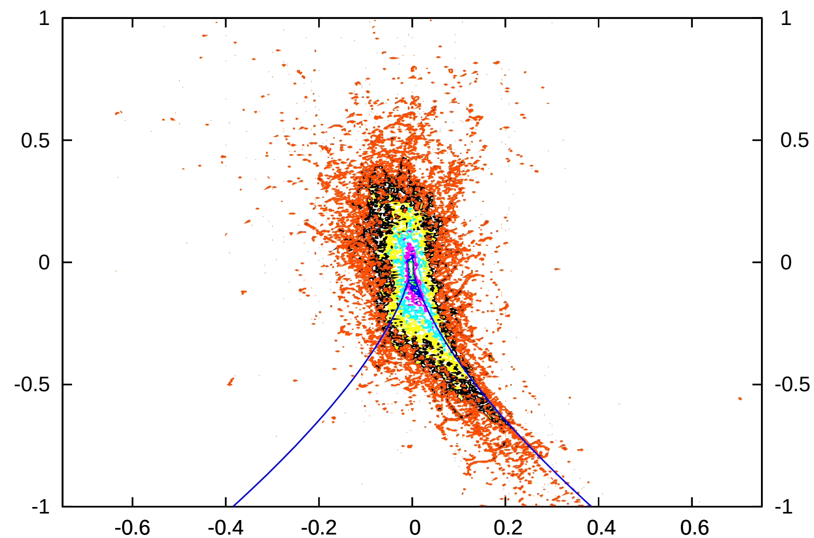
$< \delta u^2(\tau) >$ **and** $Q - R$ **at** $x/x_* = 0.577$



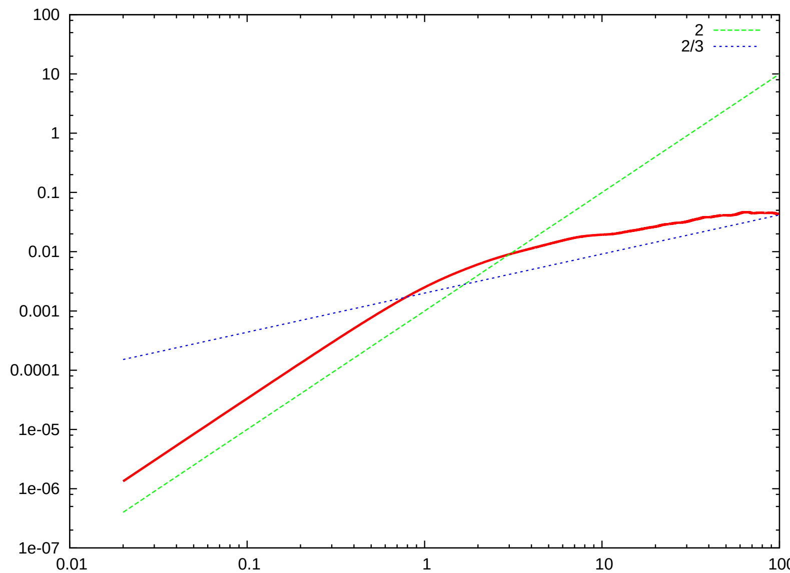


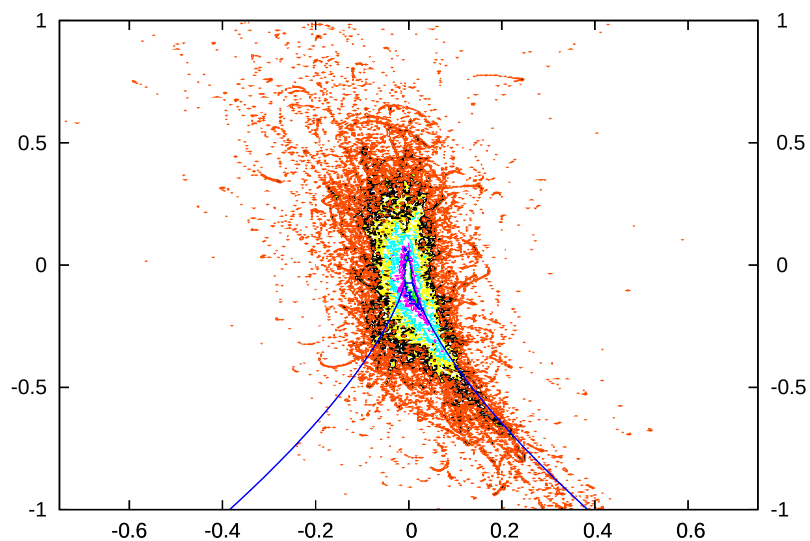
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 0.630$



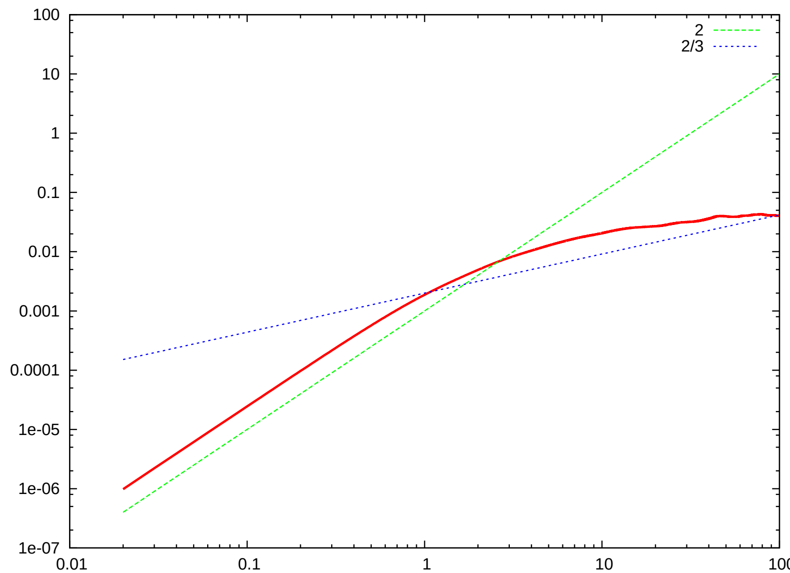


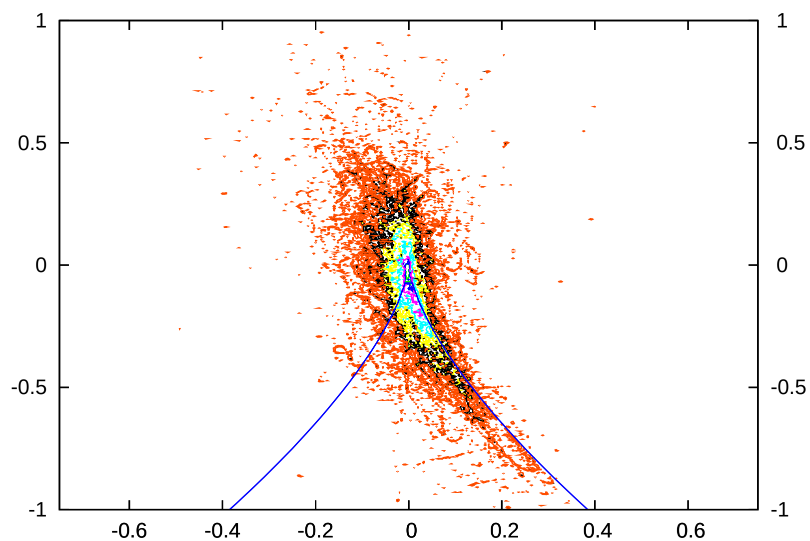
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 0.683$



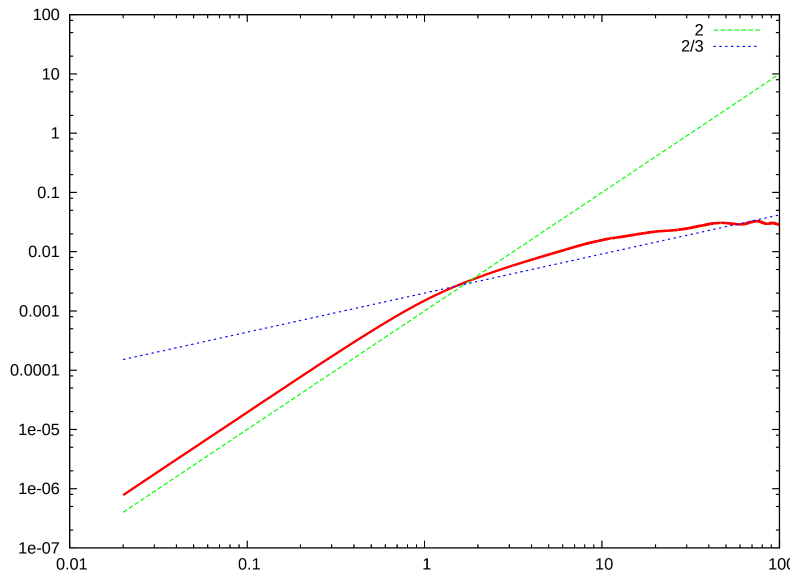


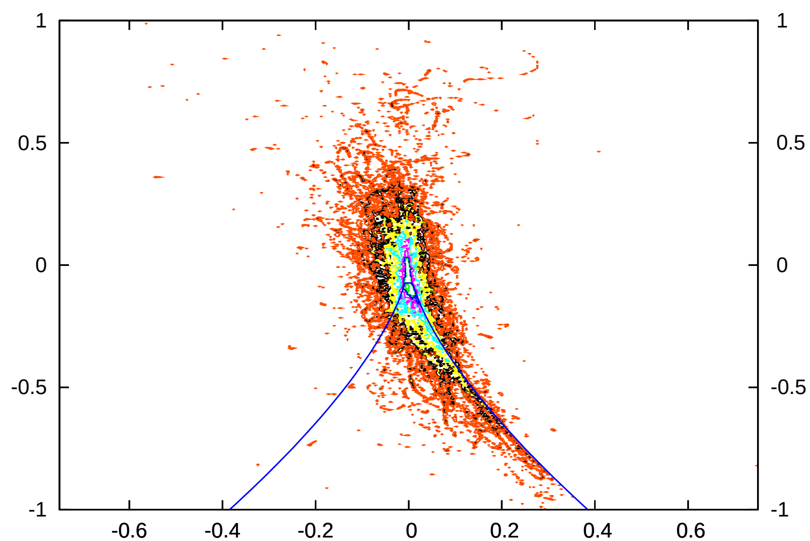
$< \delta u^2(\tau) >$ **and** $Q - R$ **at** $x/x_* = 0.735$



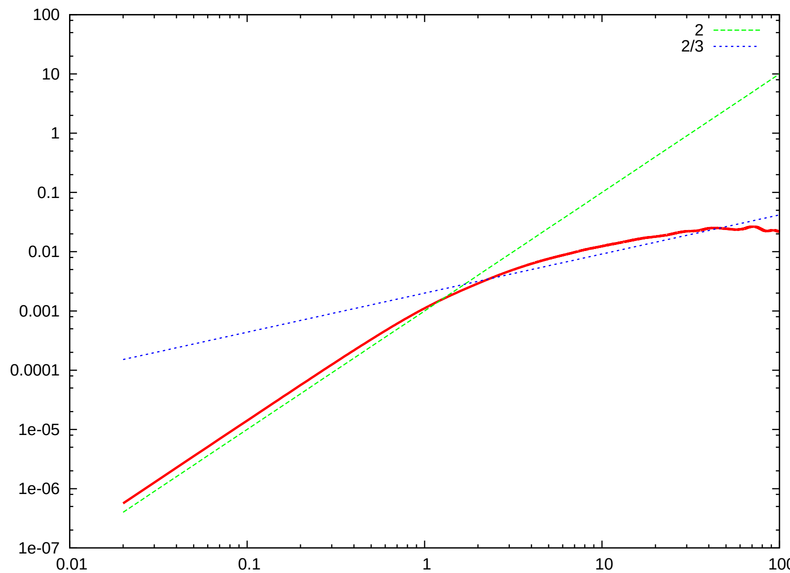


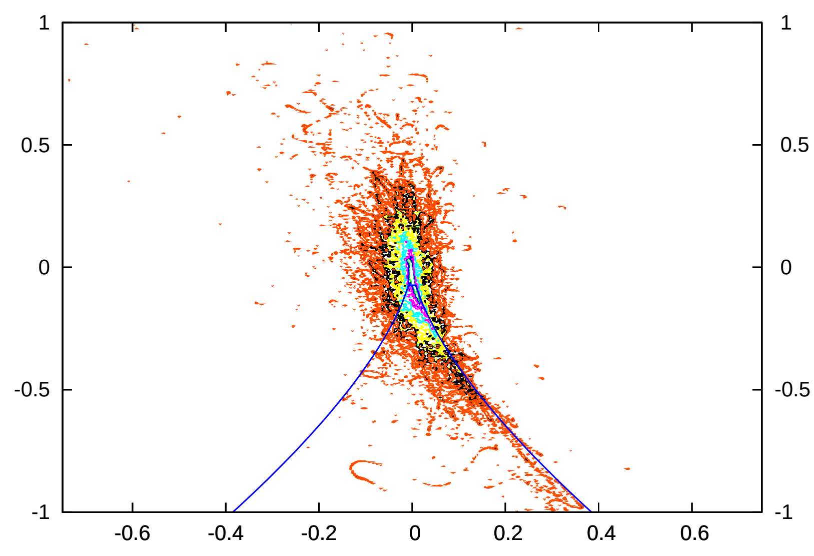
$< \delta u^2(\tau) >$ **and** $Q - R$ **at** $x/x_* = 0.788$



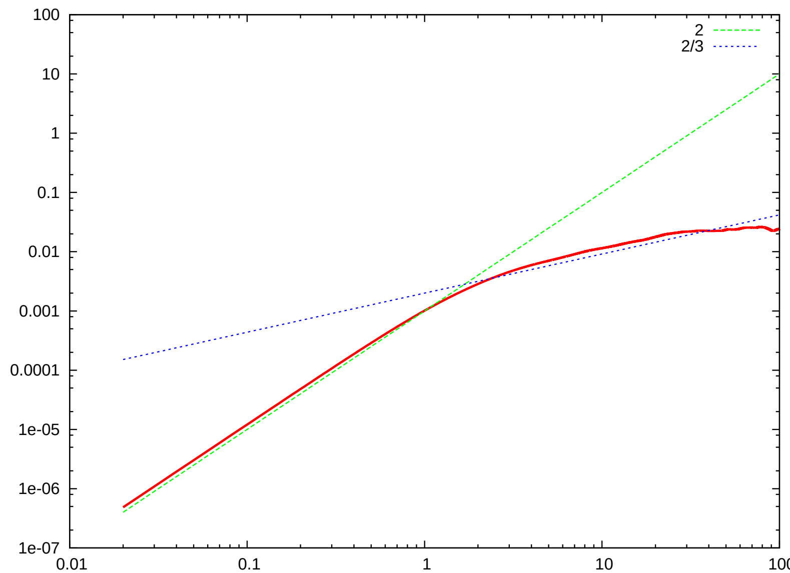


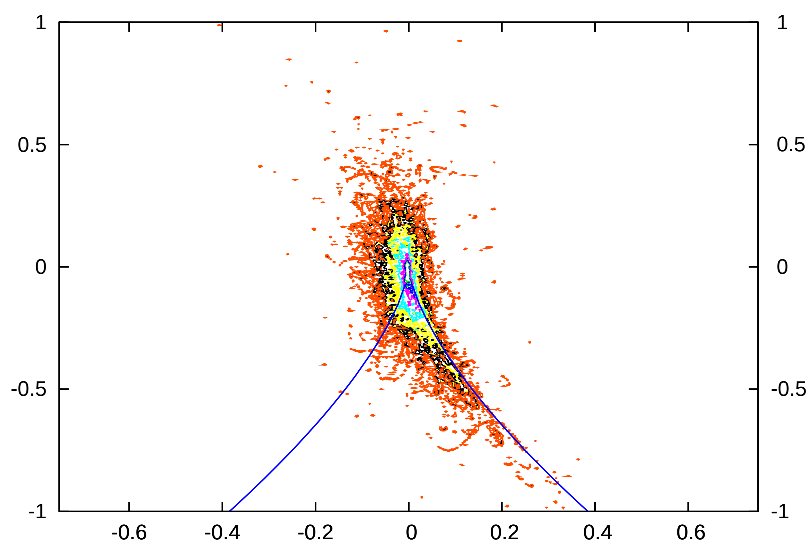
$< \delta u^2(\tau) >$ **and** $Q - R$ **at** $x/x_* = 0.840$



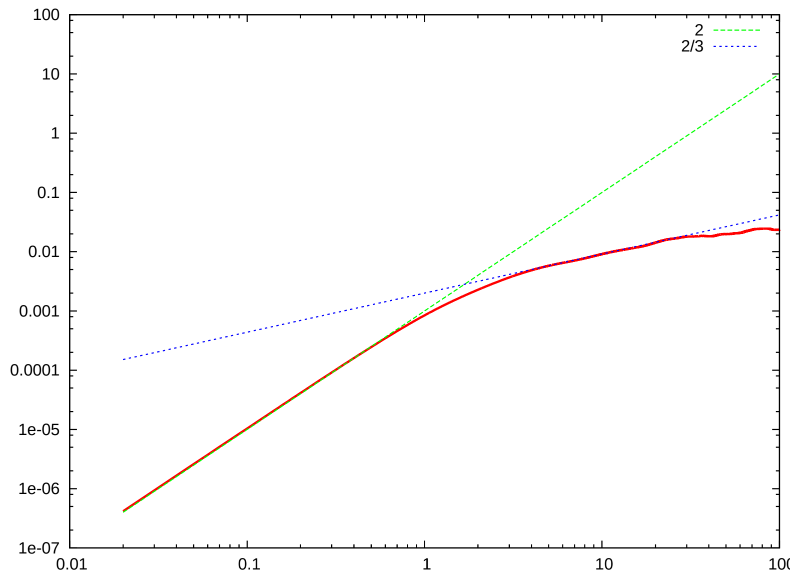


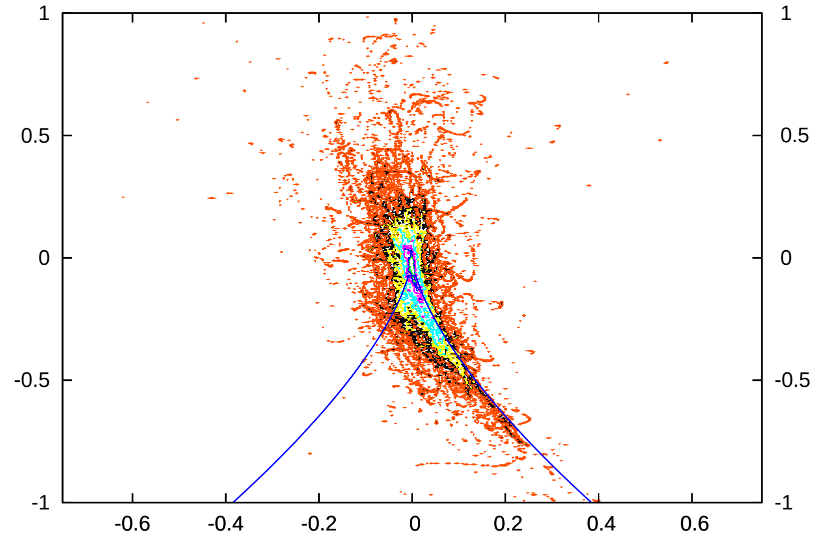
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 0.893$



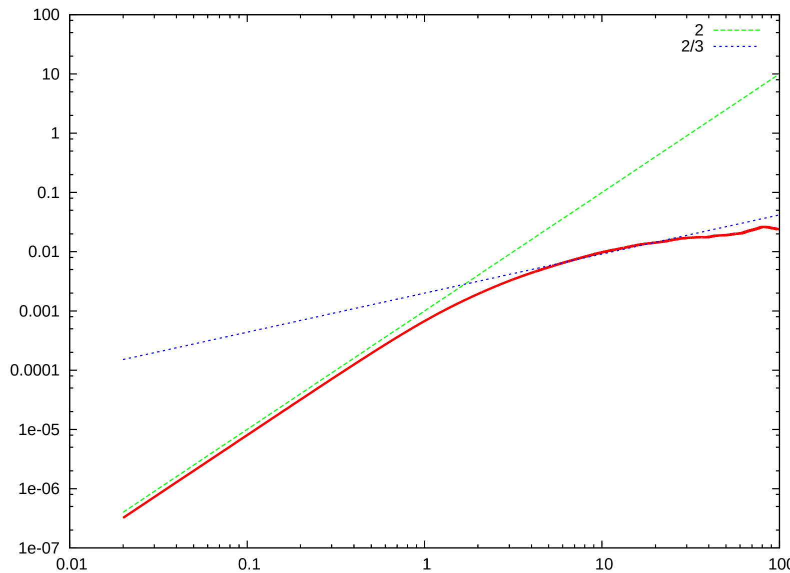


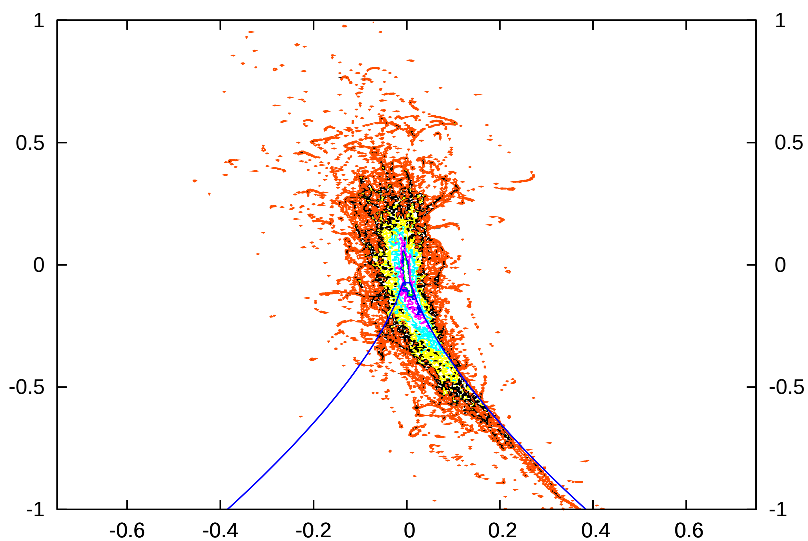
$< \delta u^2(\tau) >$ **and** $Q - R$ **at** $x/x_* = 0.945$



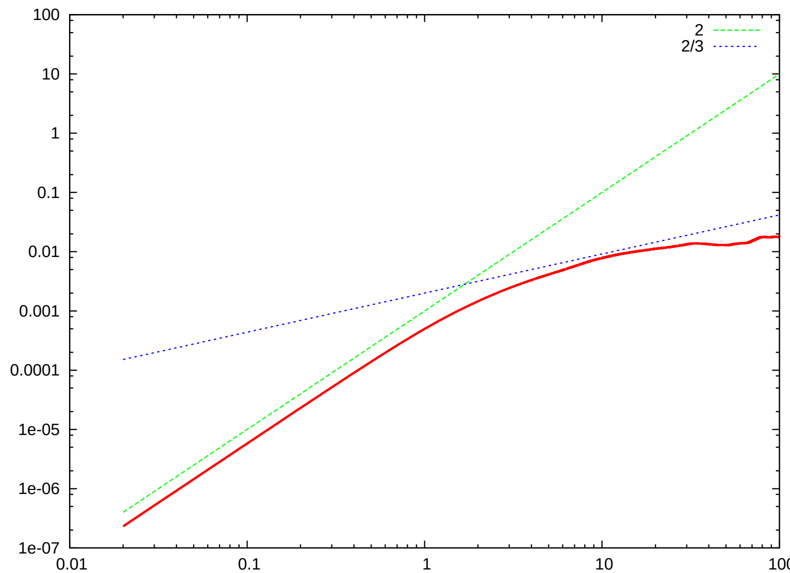


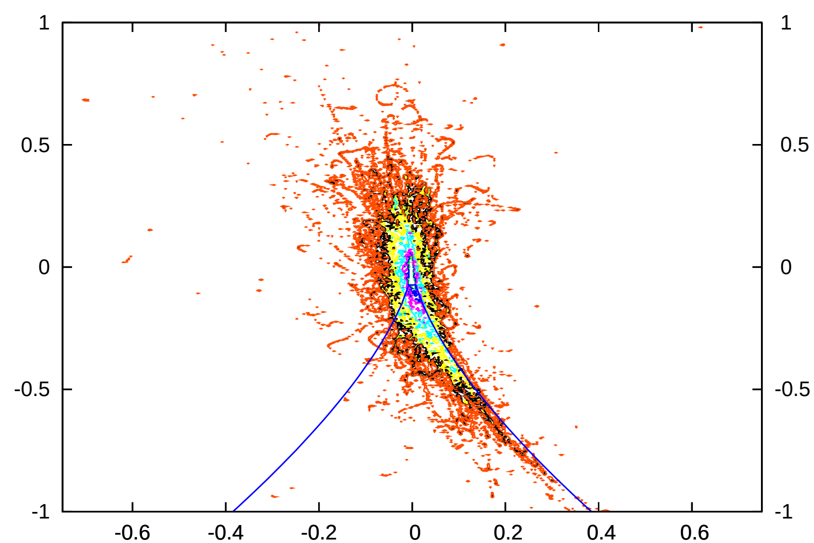
$<\delta u^2(\tau)>$ and $Q-R$ at $x/x_* = 0.998$



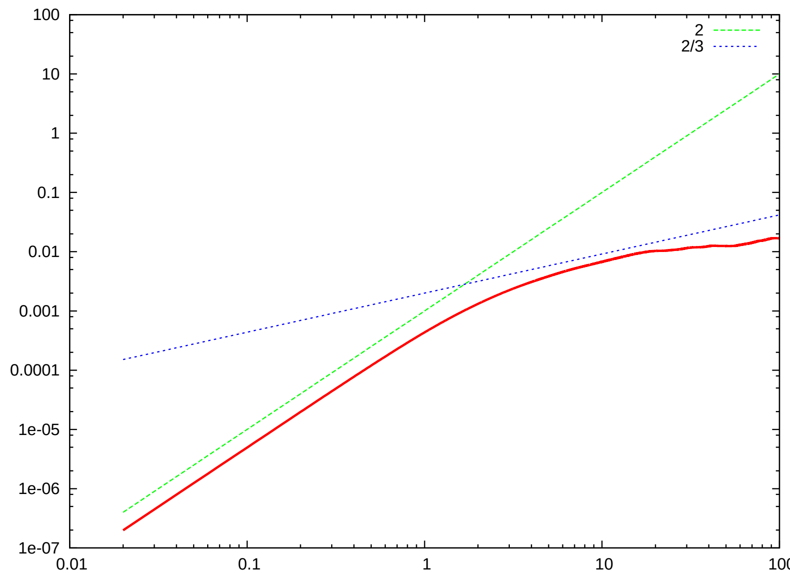


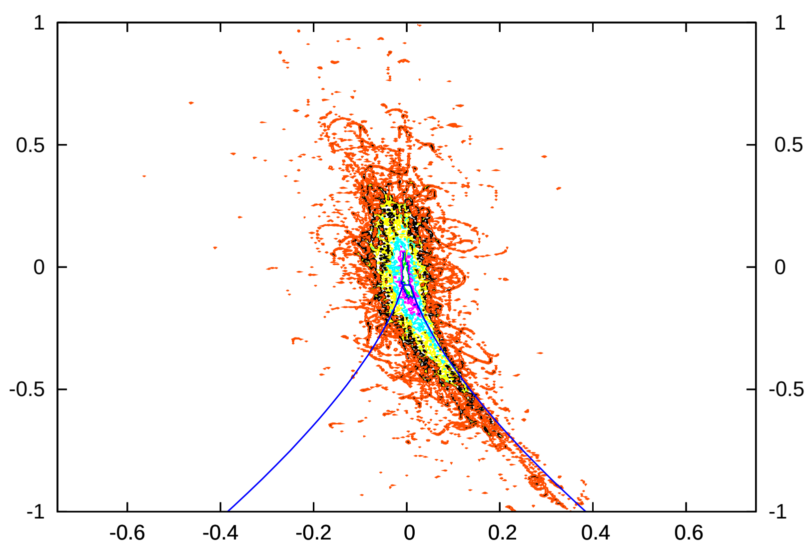
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 1.050$



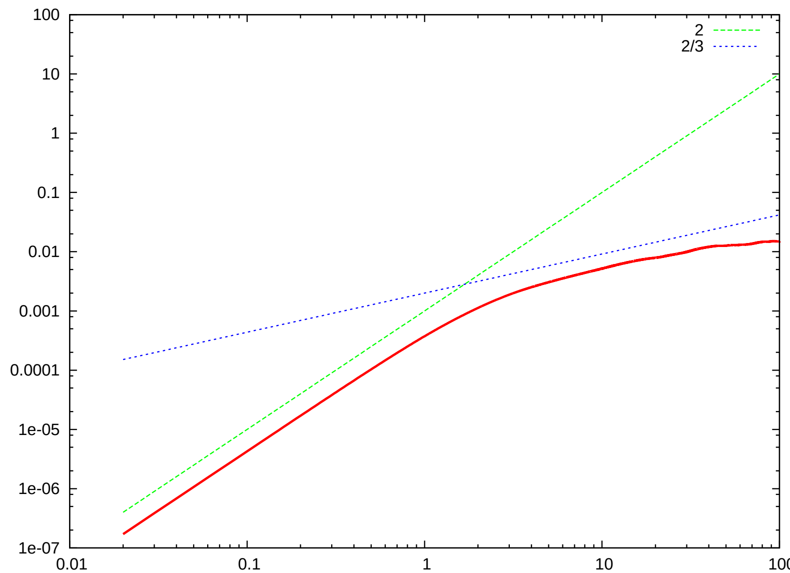


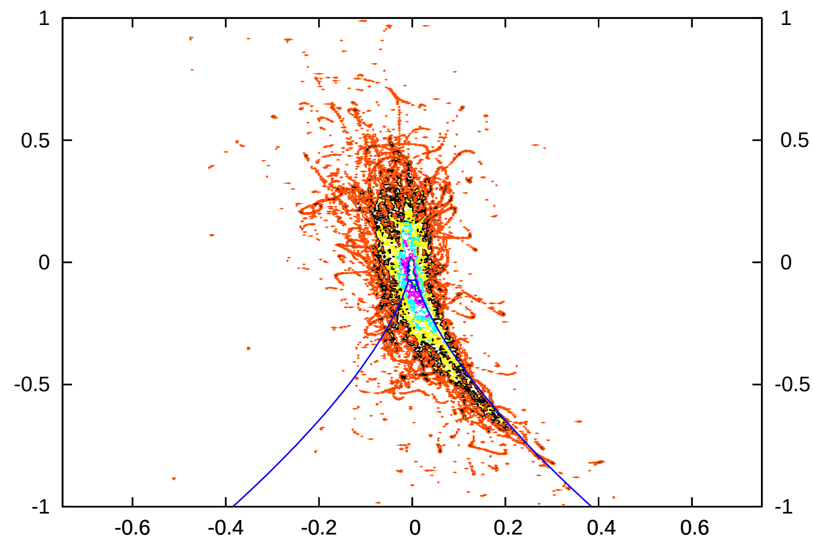
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 1.103$



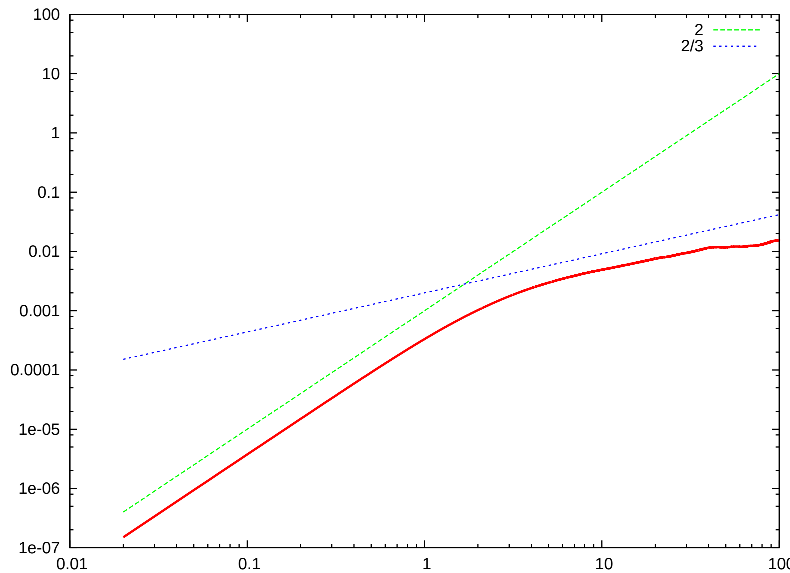


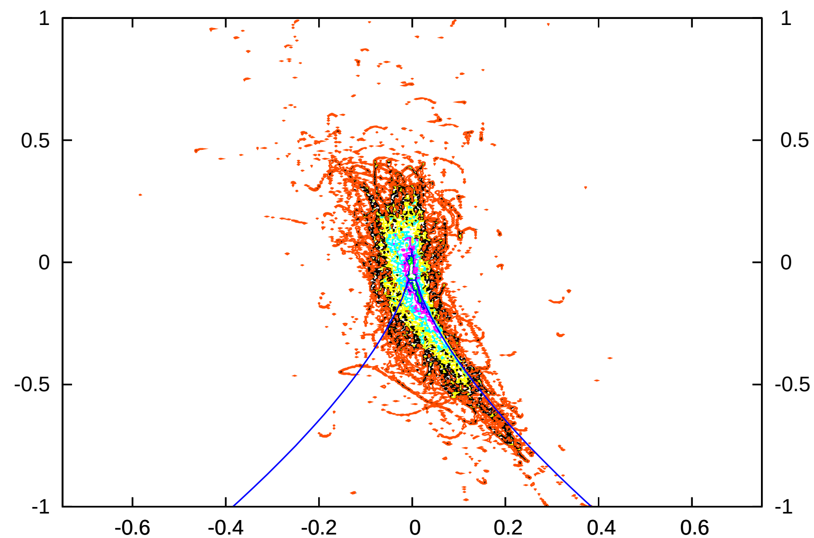
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 1.155$



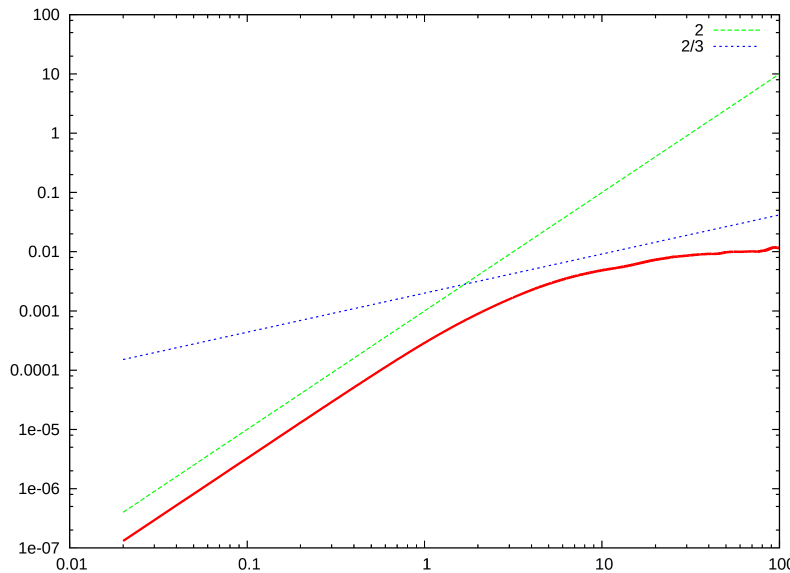


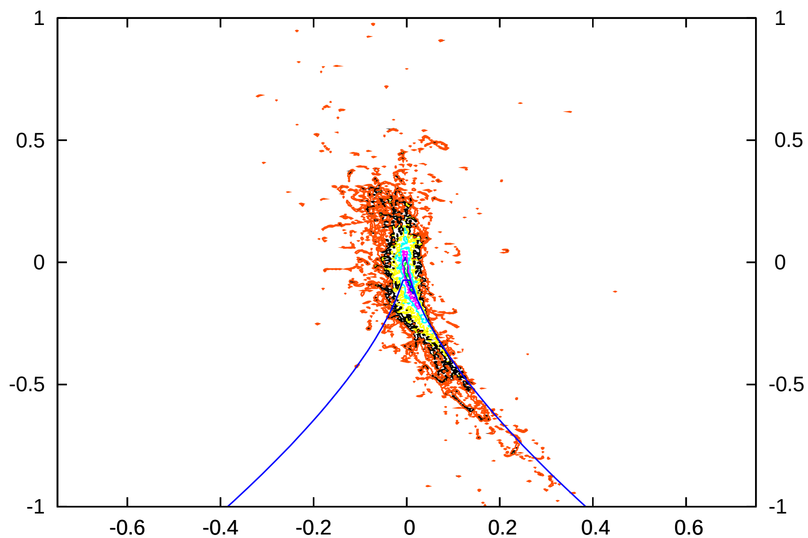
$< \delta u^2(\tau) >$ **and** $Q - R$ **at** $x/x_* = 1.208$



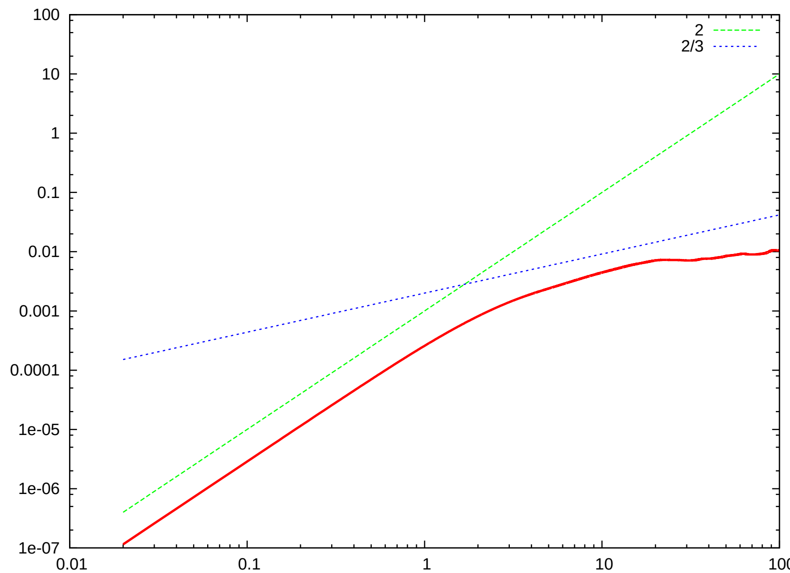


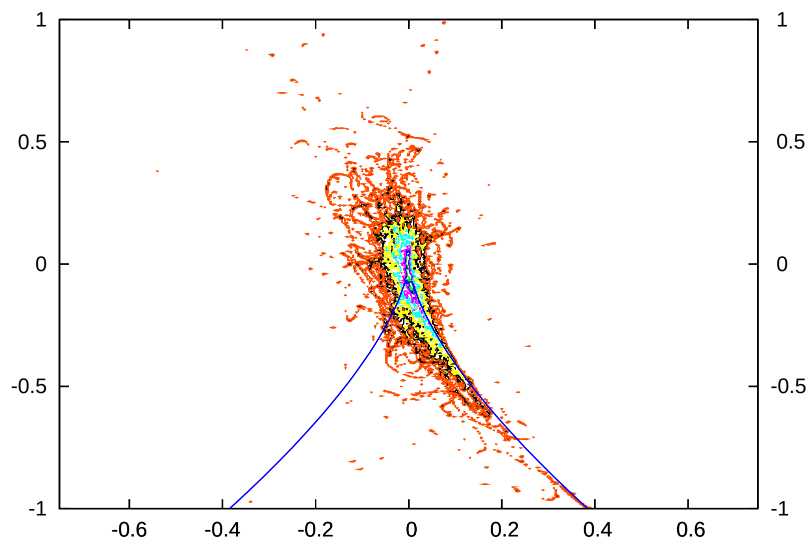
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 1.261$



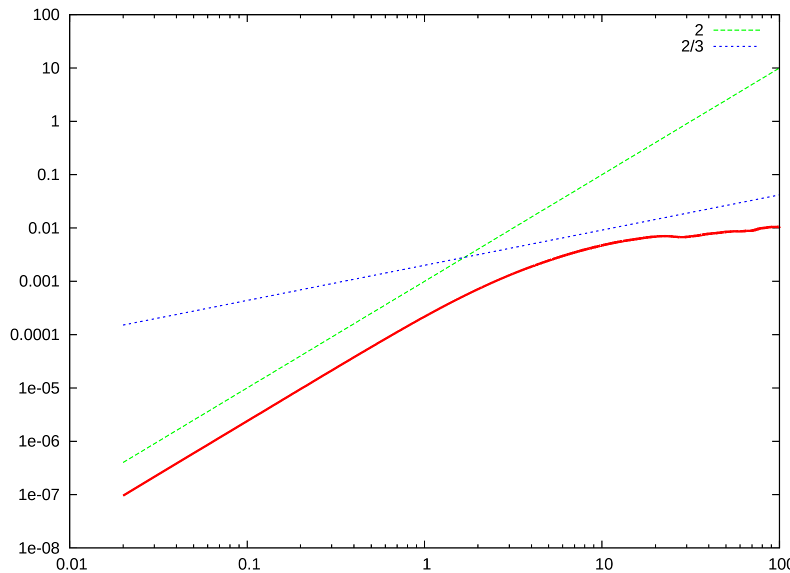


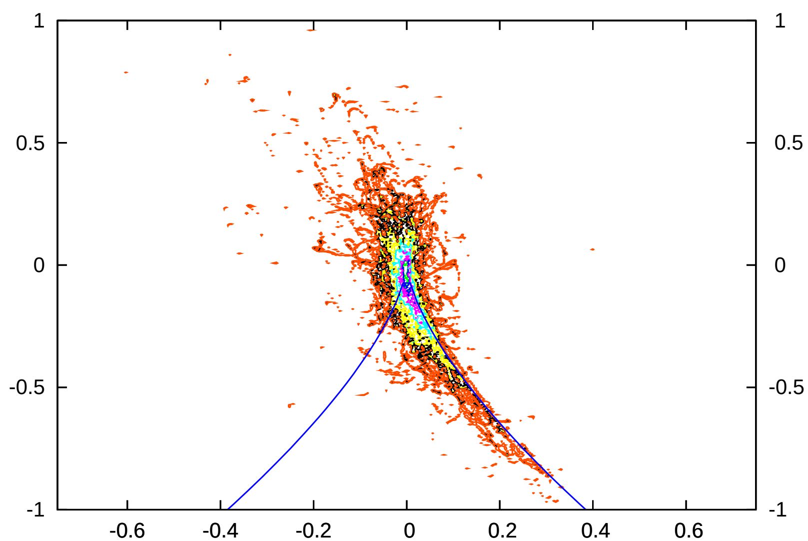
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 1.313$



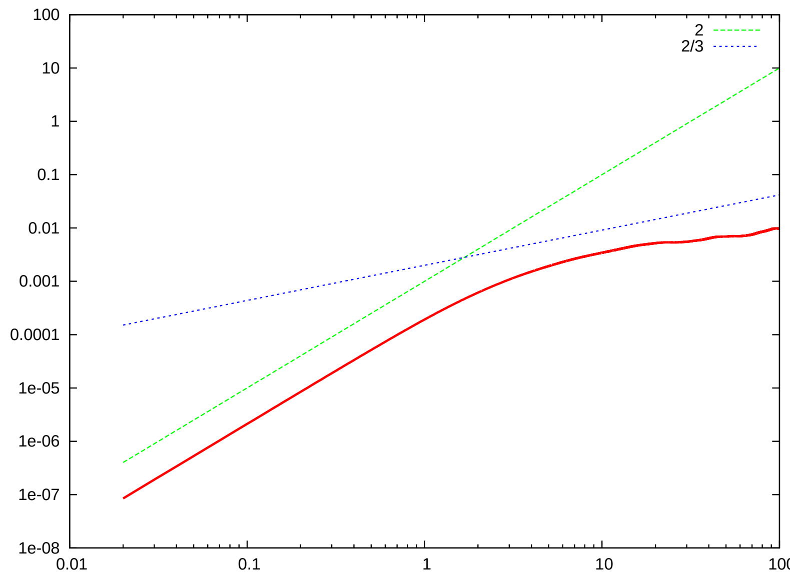


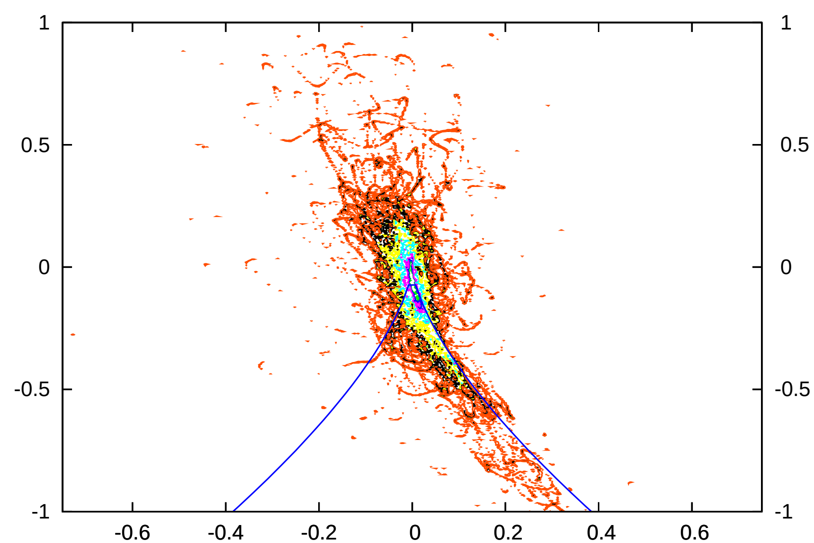
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 1.366$



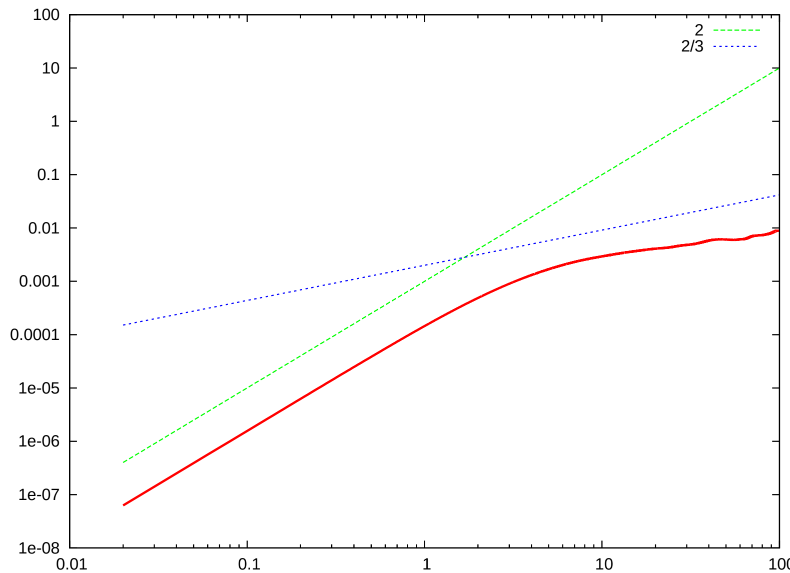


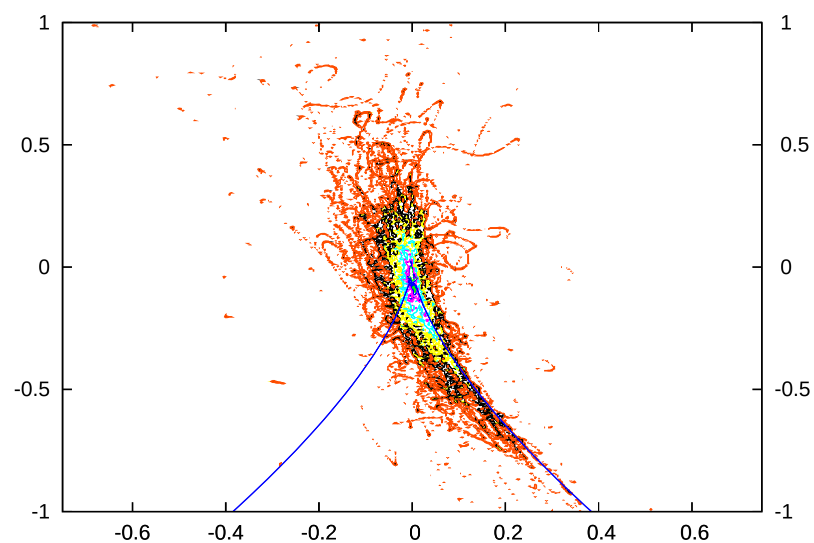
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 1.418$



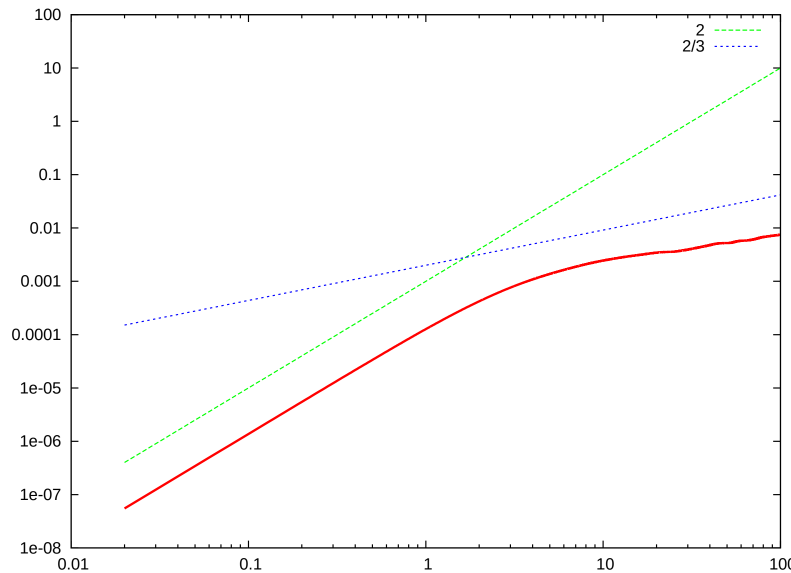


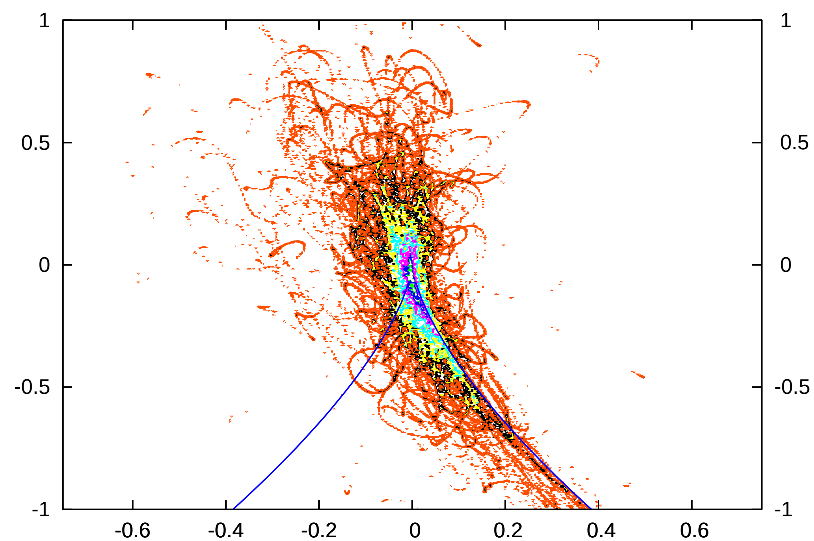
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 1.471$



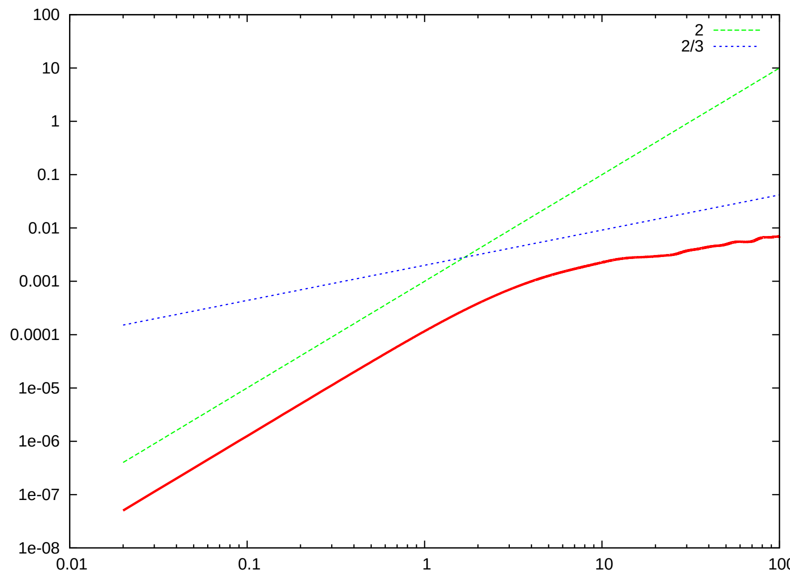


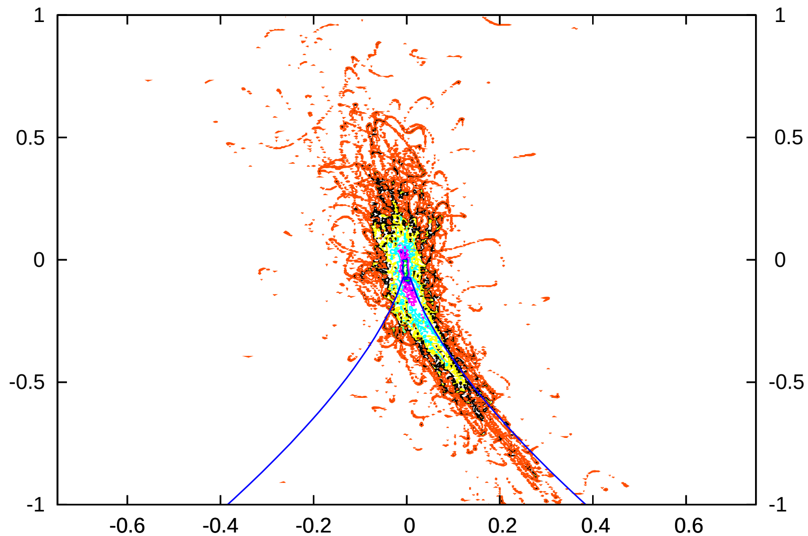
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 1.523$



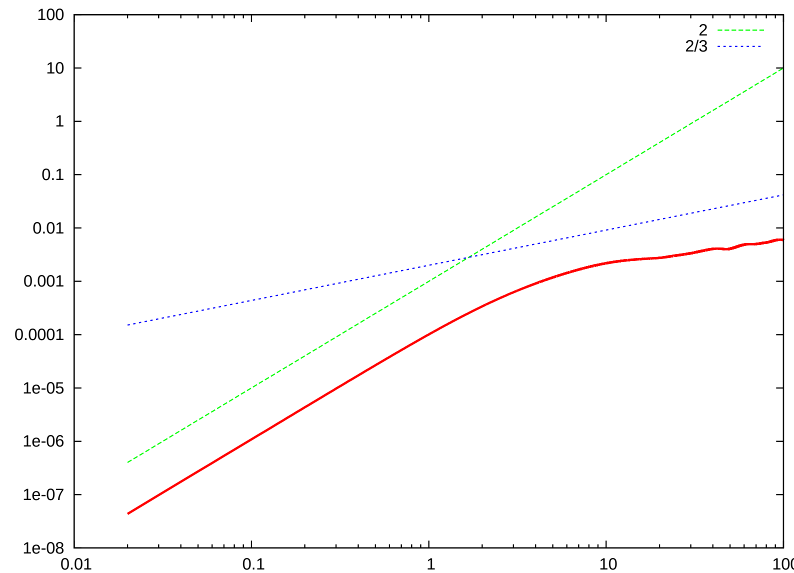


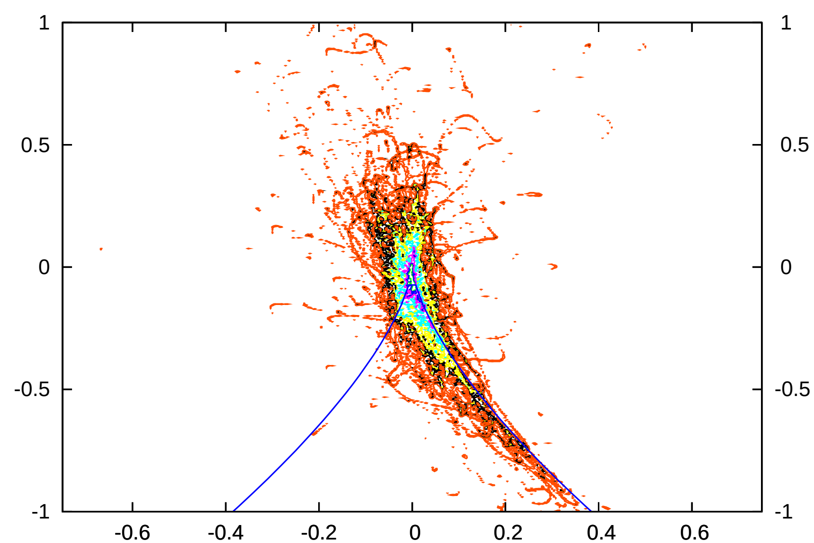
$< \delta u^2(\tau) >$ and $Q-R$ at $x/x_* = 1.576$



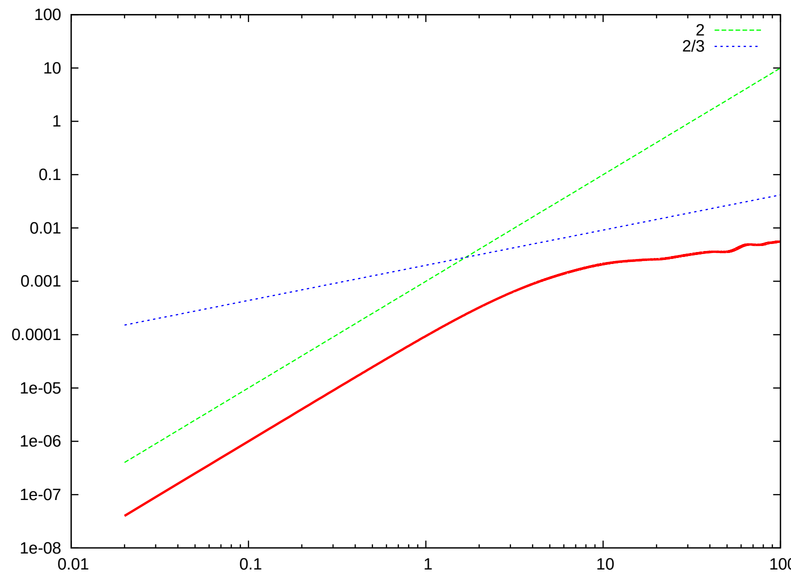


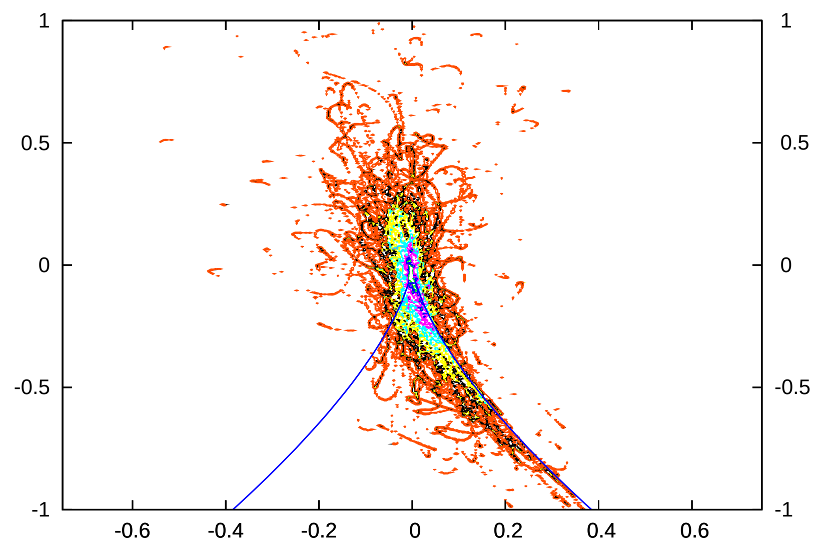
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 1.628$



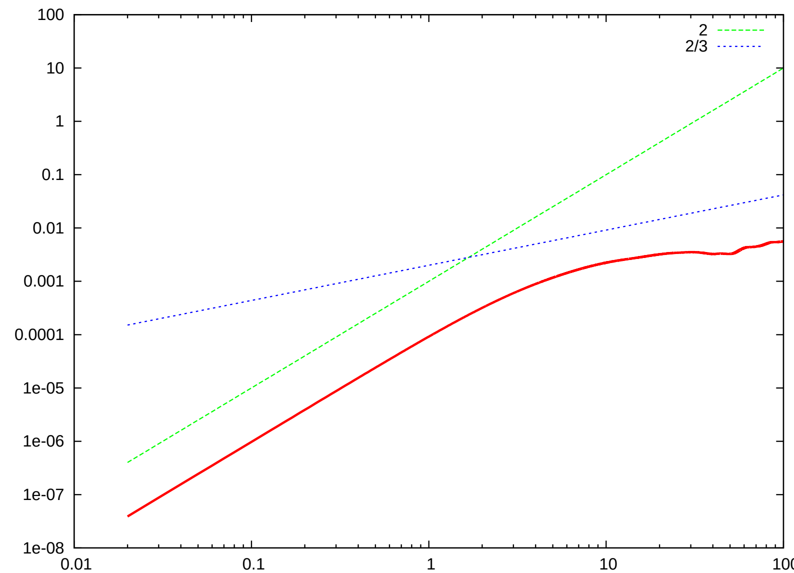


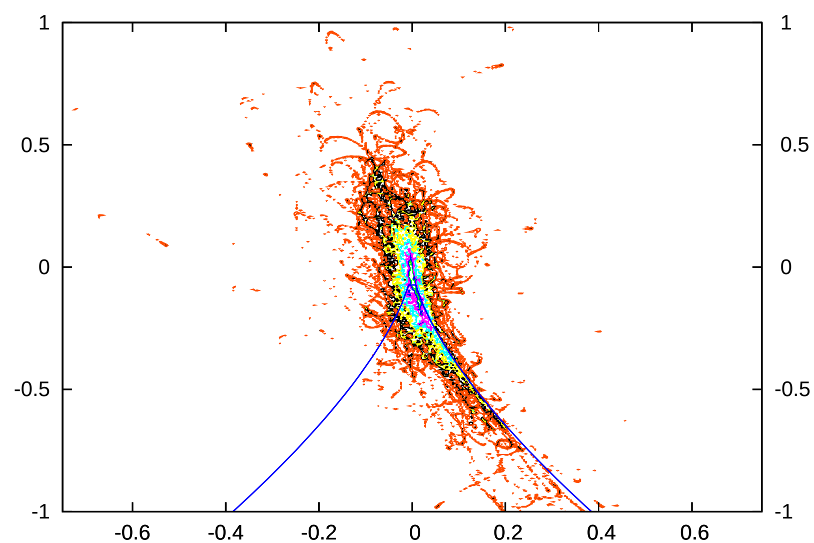
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 1.681$





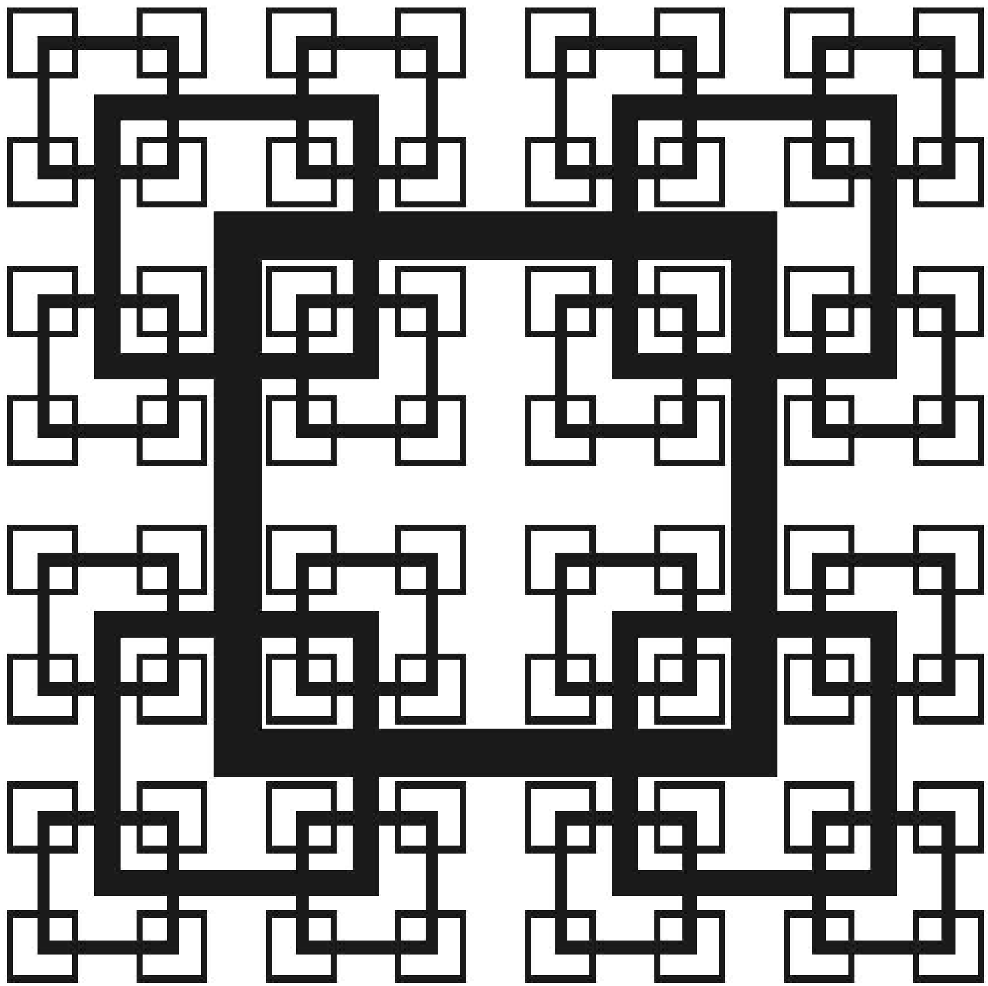
$< \delta u^2(\tau) >$ **and** $Q - R$ **at** $x/x_* = 1.733$



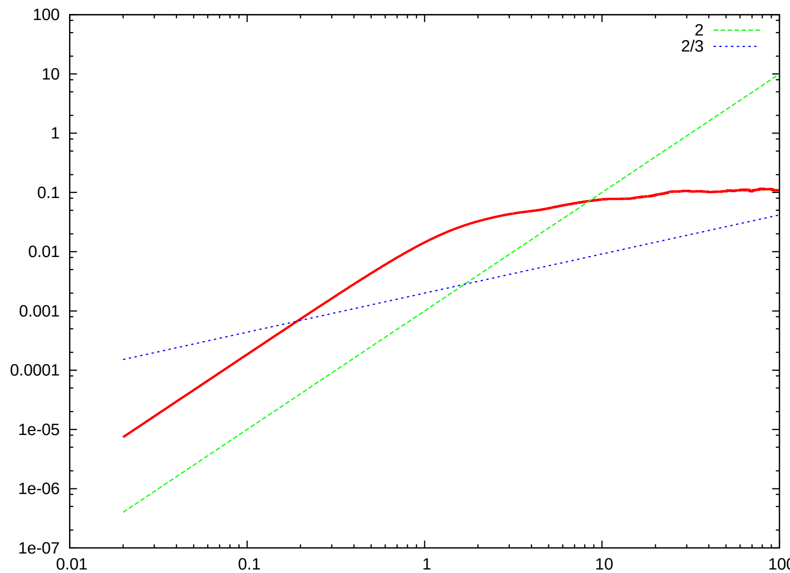


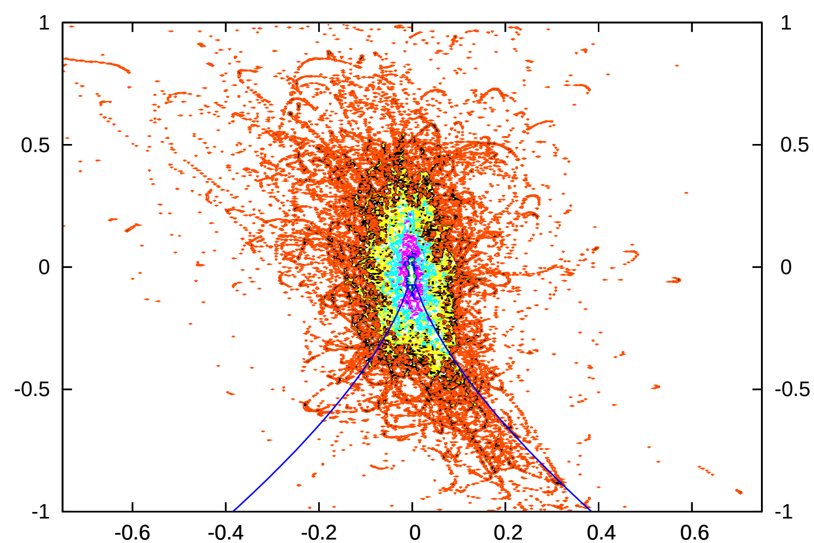
Next inside wake of big bars

Previous plots where taken along the centreline.
Next plots are taken along the line normal to the grid and crossing a biggest bar at the middle.

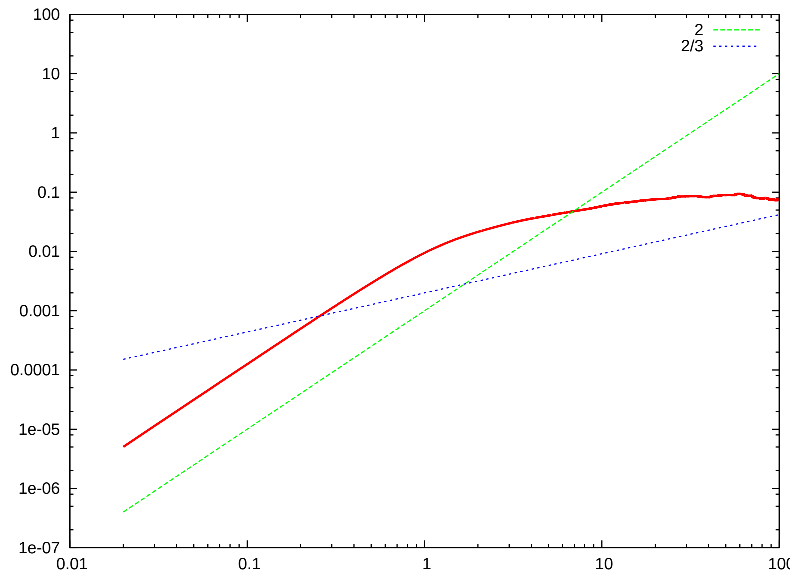


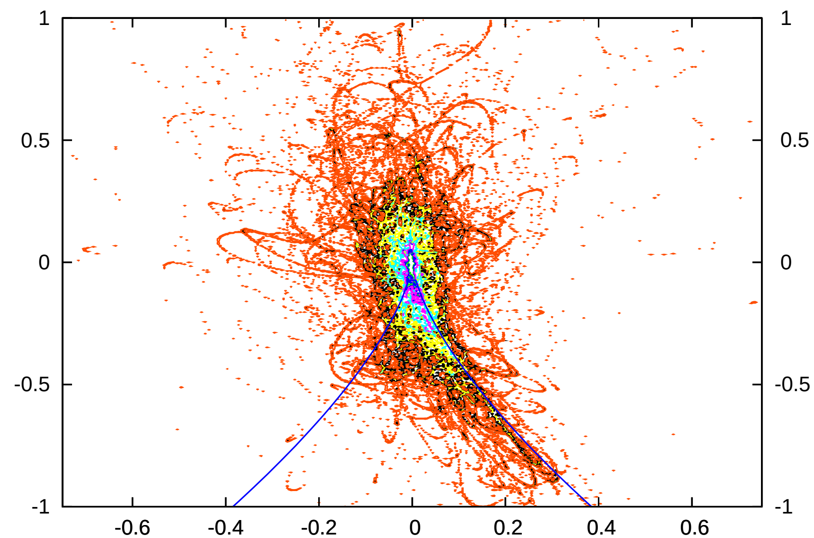
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 0.052$



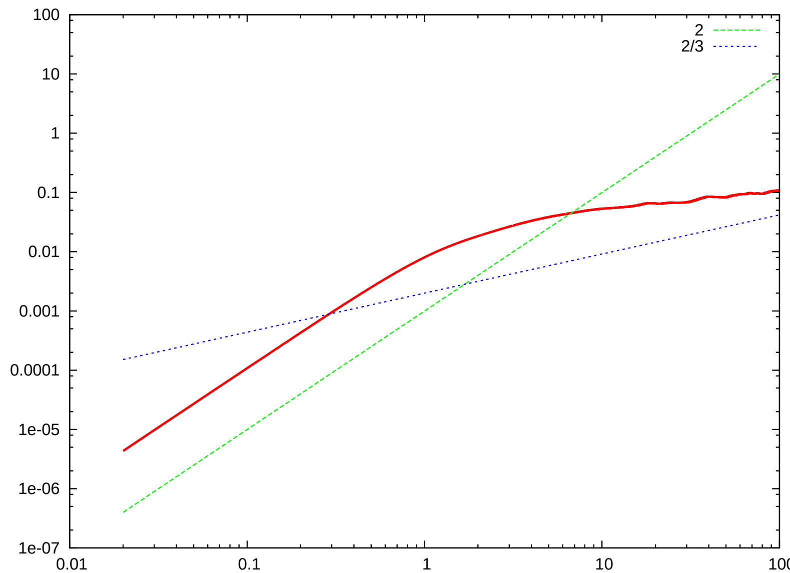


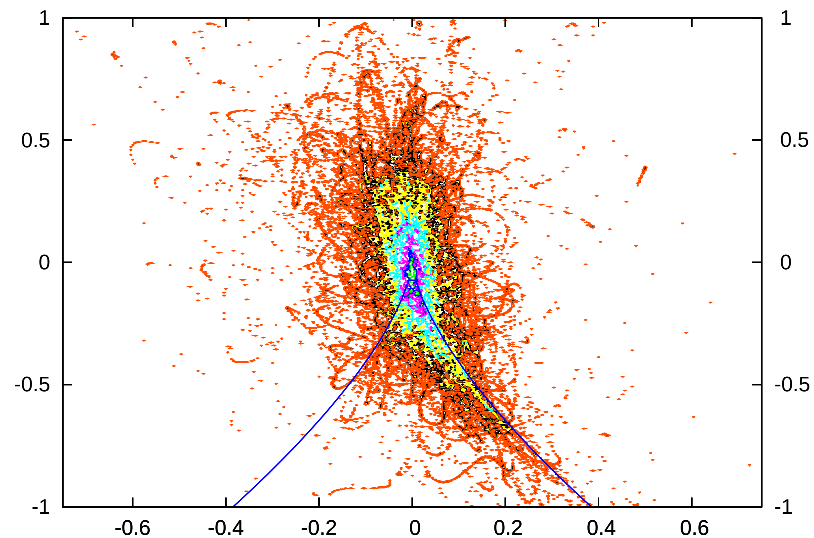
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 0.105$



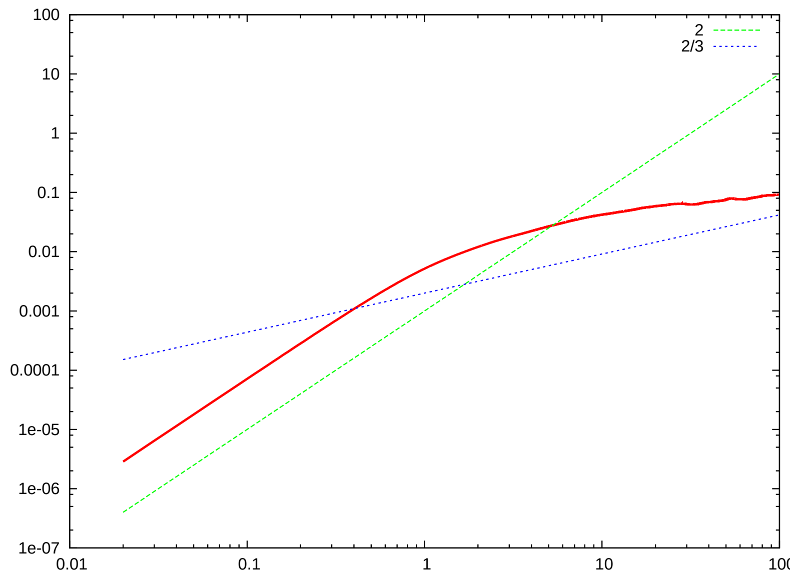


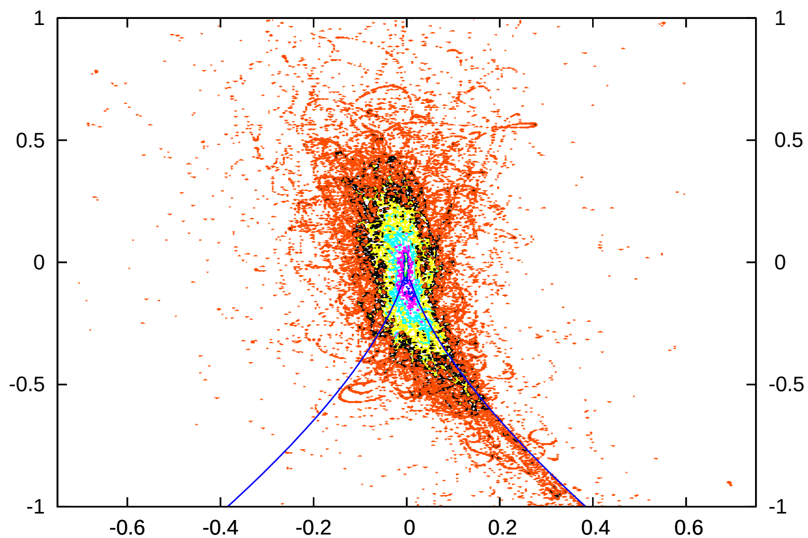
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 0.157$



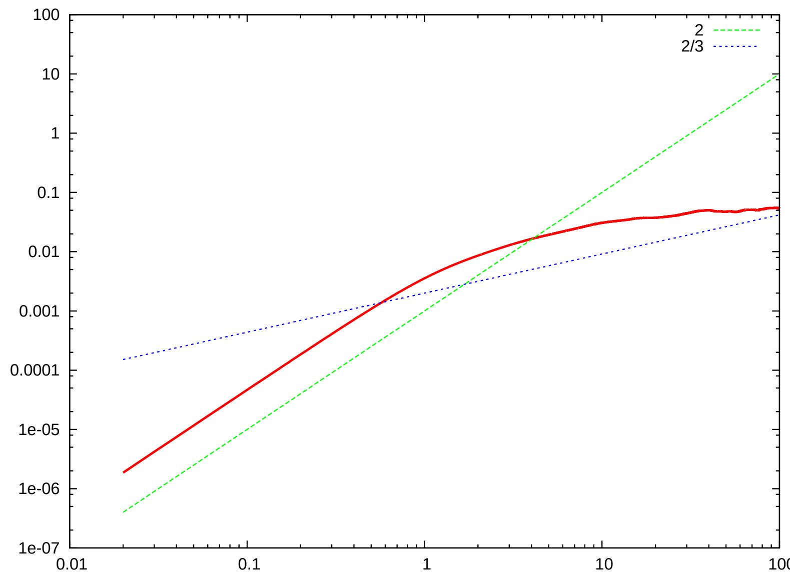


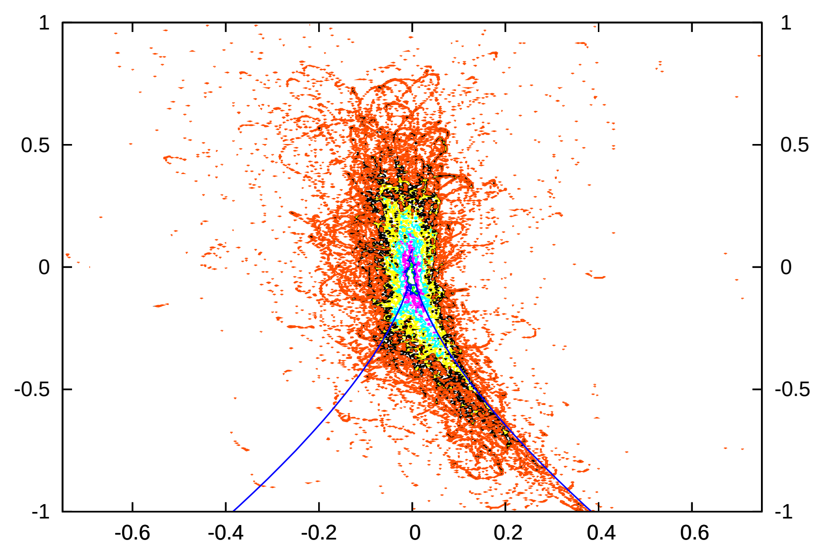
$< \delta u^2(\tau) >$ **and** $Q - R$ **at** $x/x_* = 0.210$



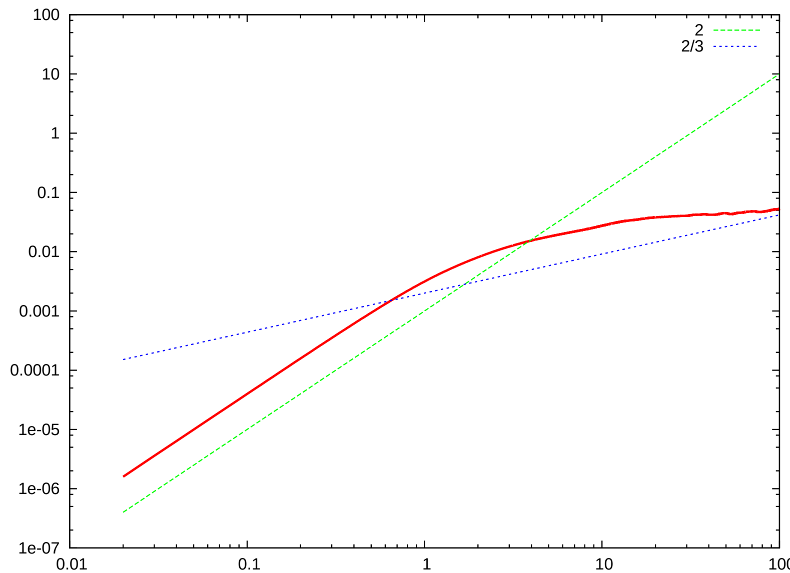


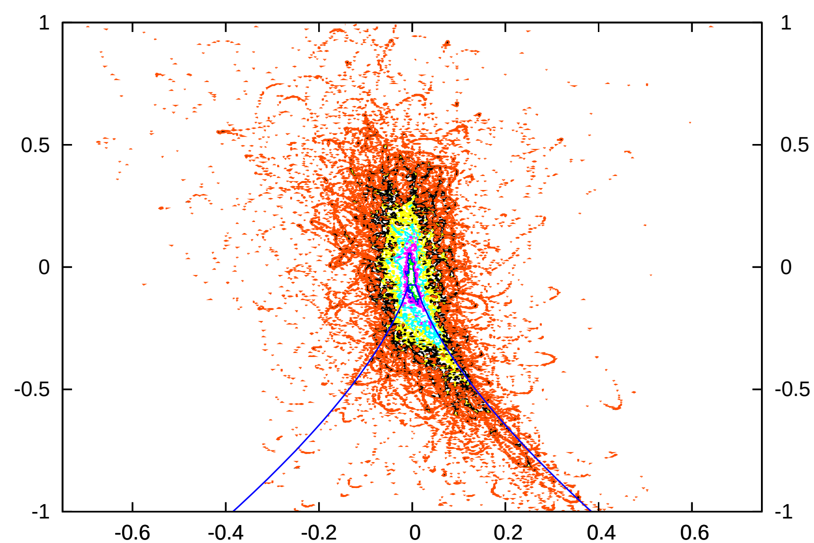
$< \delta u^2(\tau) >$ **and** $Q - R$ **at** $x/x_* = 0.262$



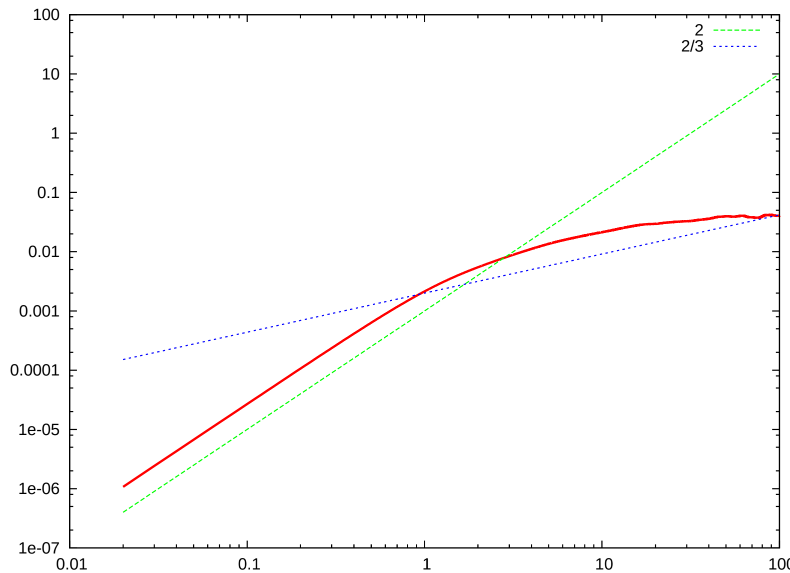


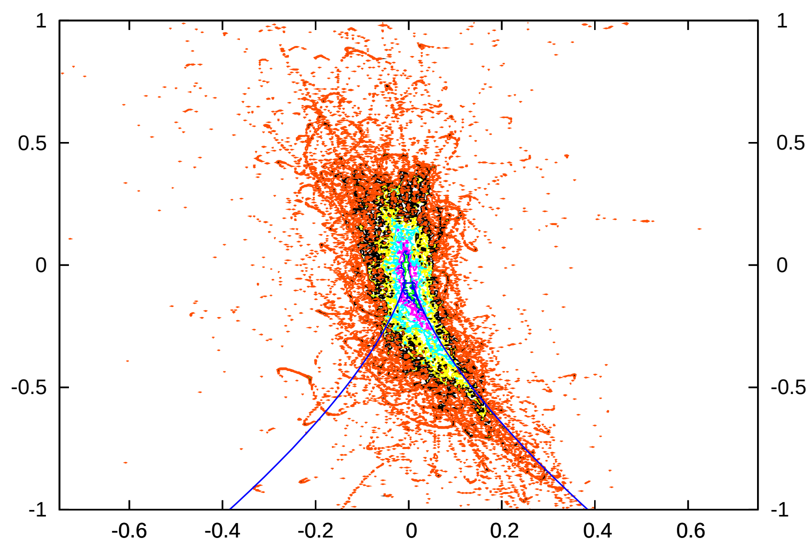
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 0.315$



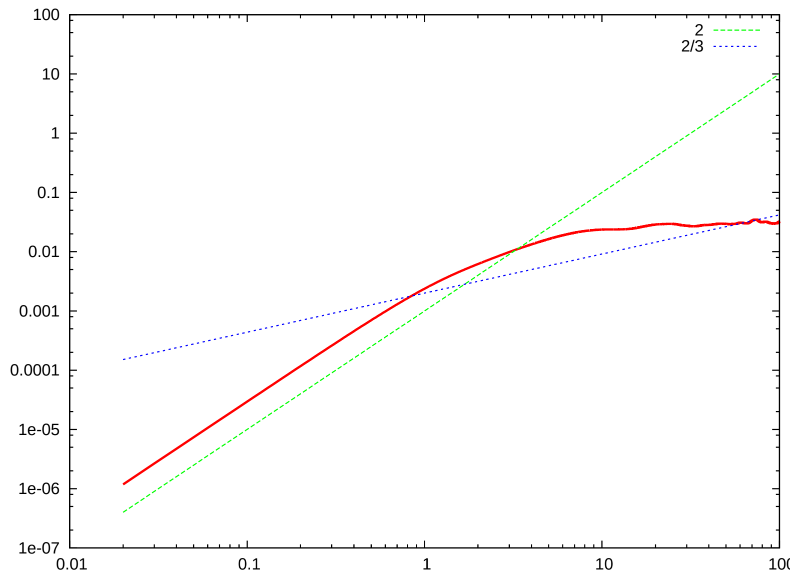


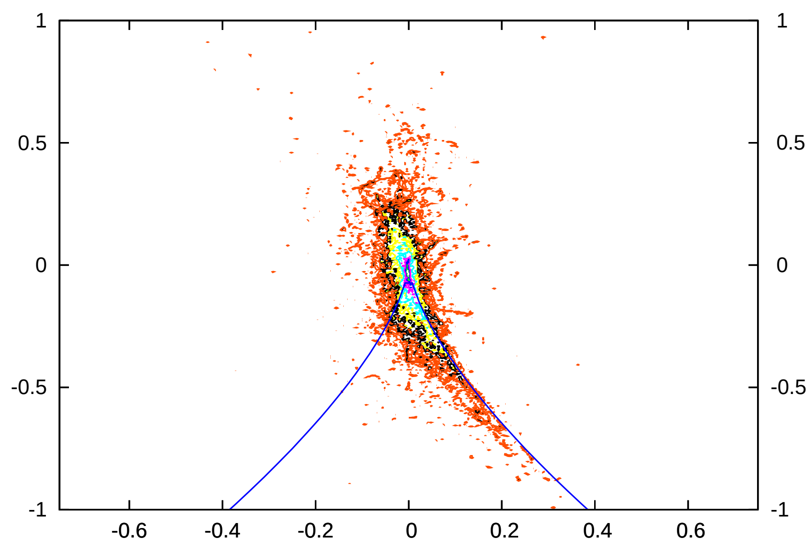
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 0.367$



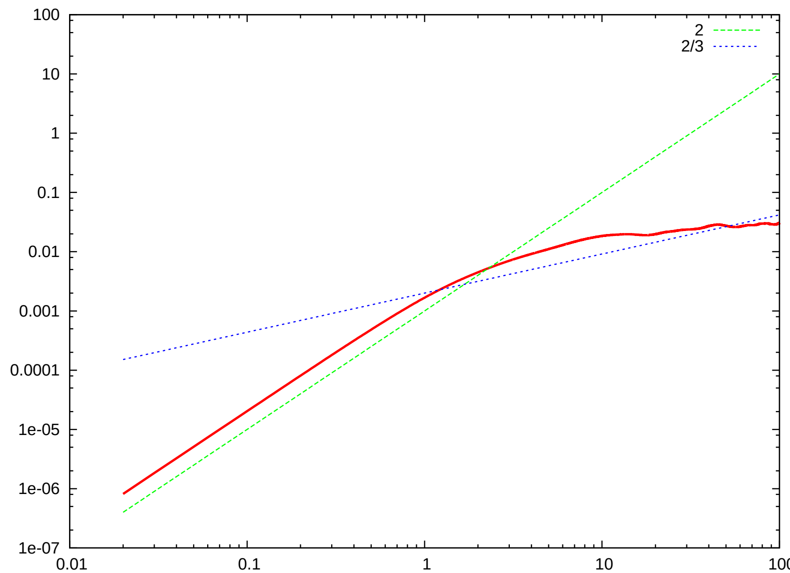


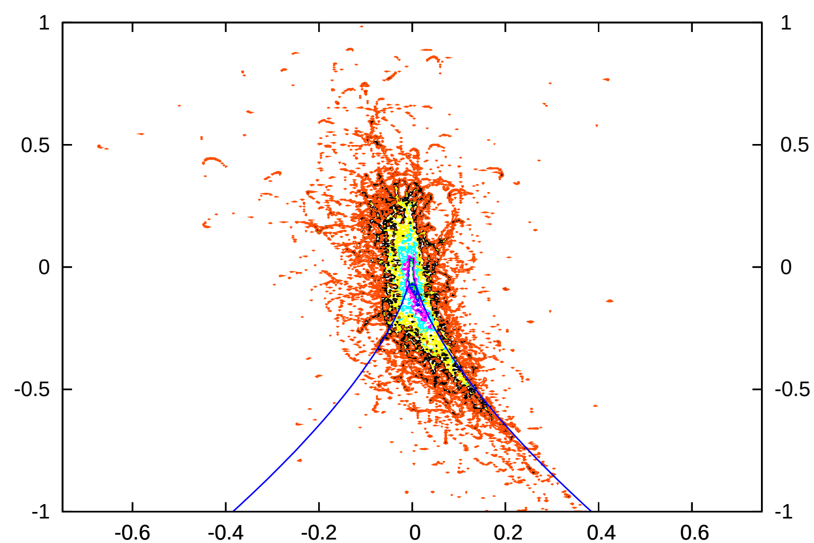
$< \delta u^2(\tau) >$ **and** $Q - R$ **at** $x/x_* = 0.420$



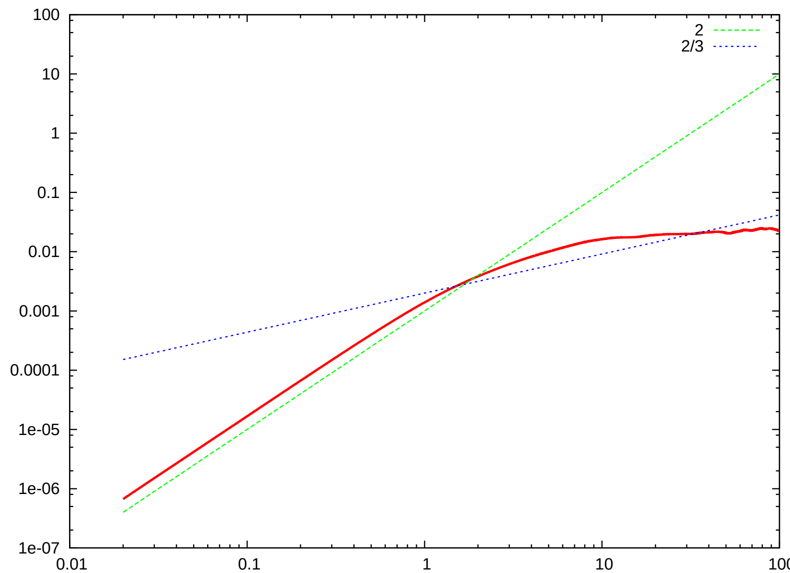


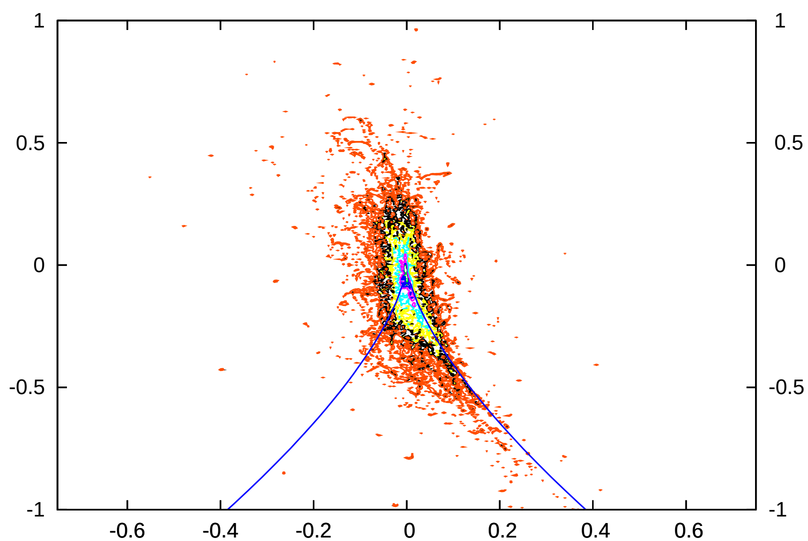
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 0.472$



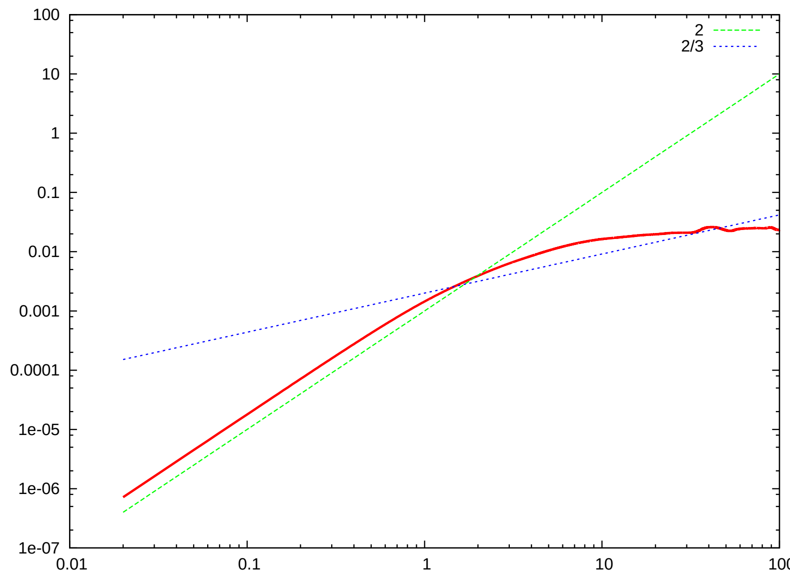


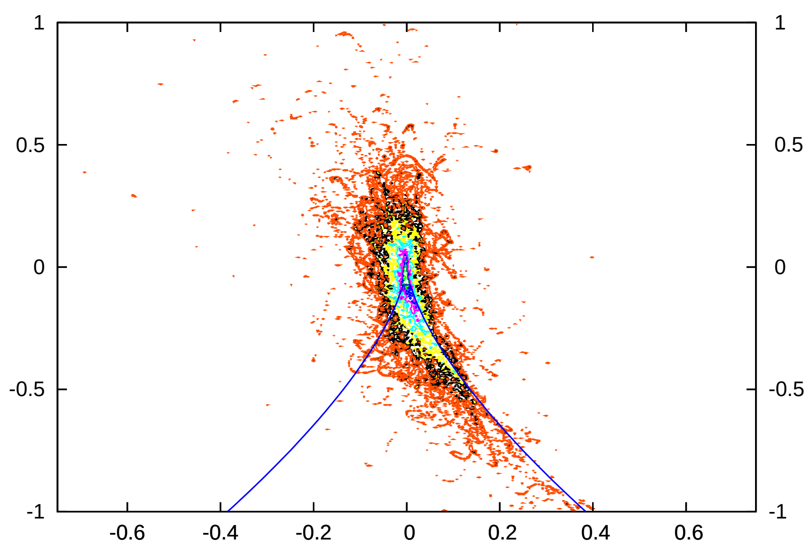
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 0.525$



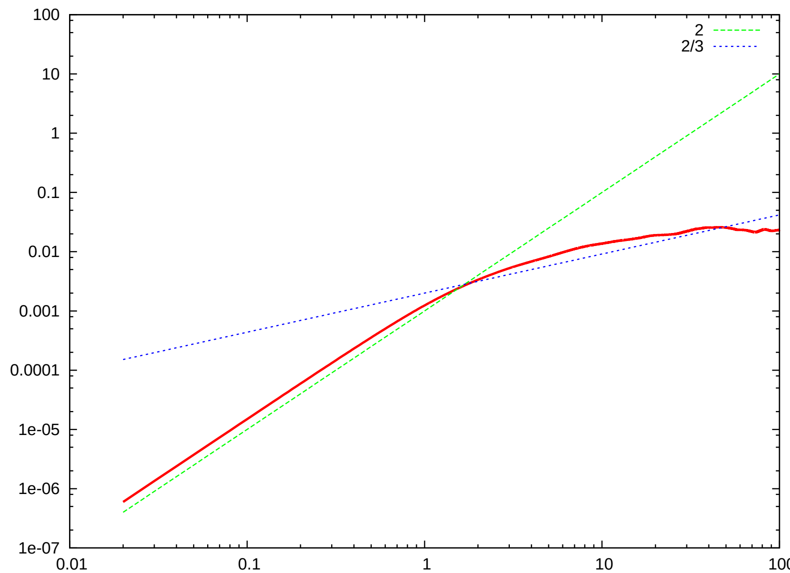


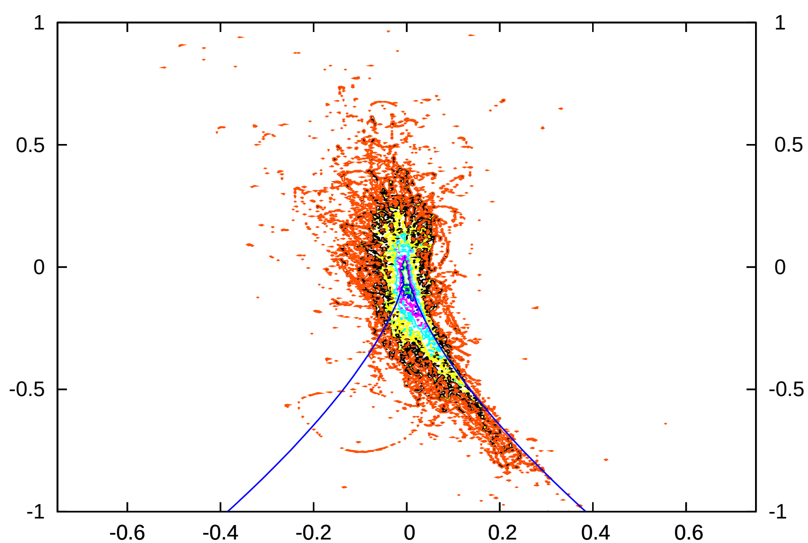
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 0.577$



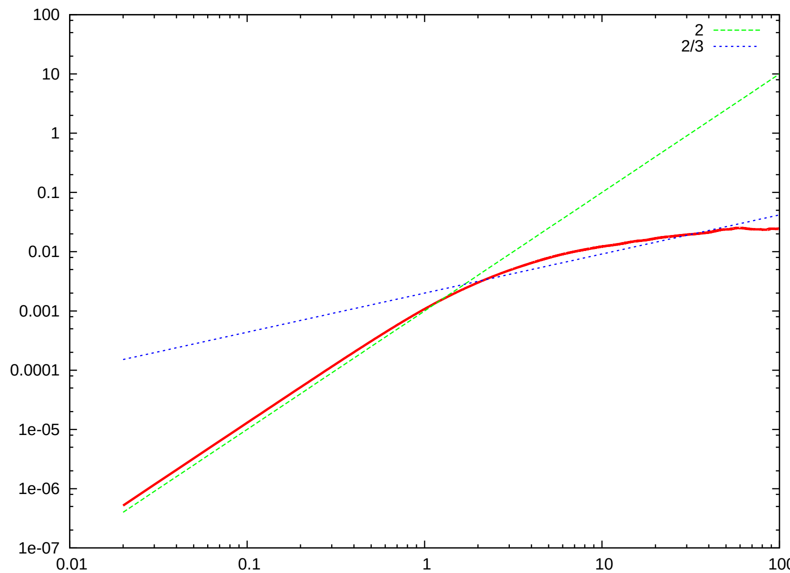


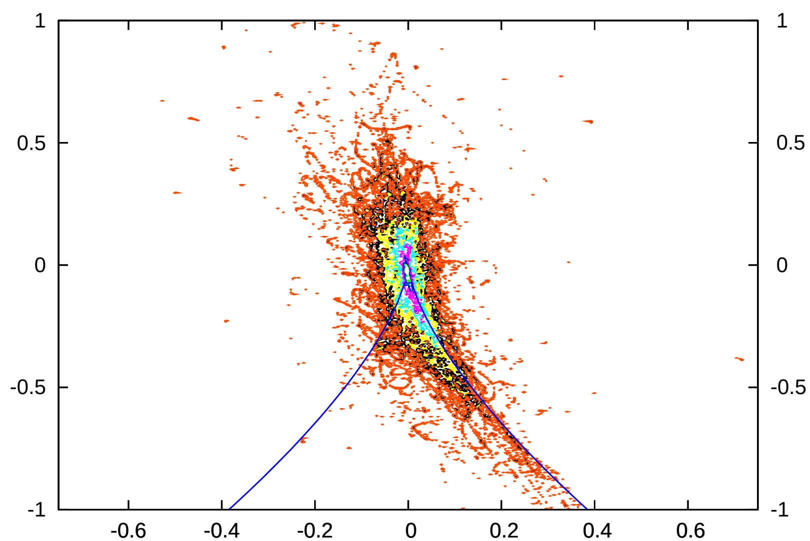
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 0.630$



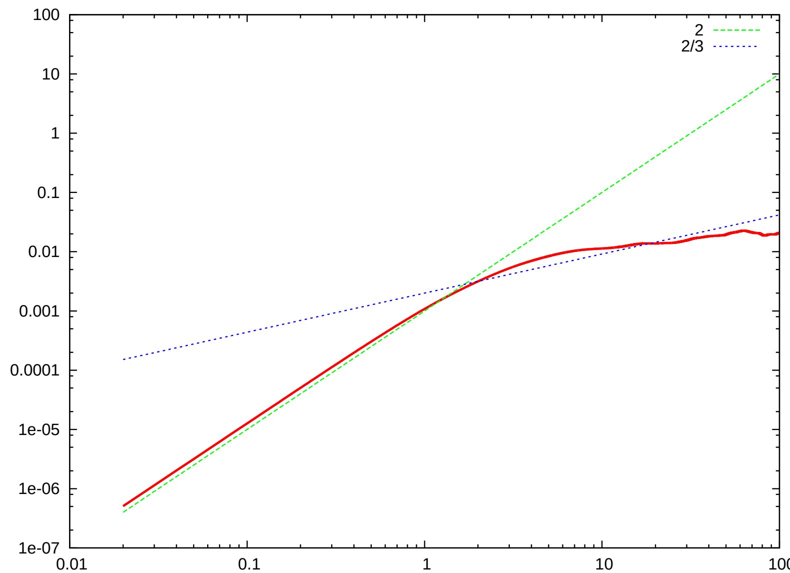


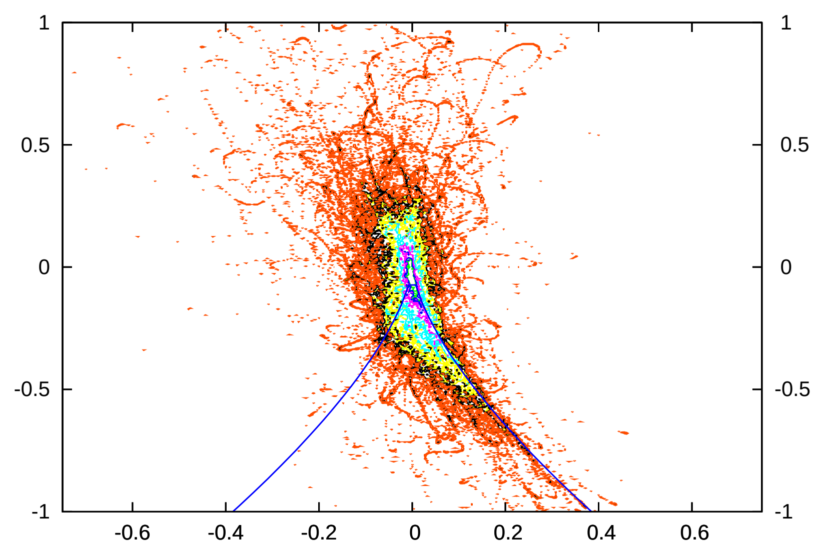
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 0.683$



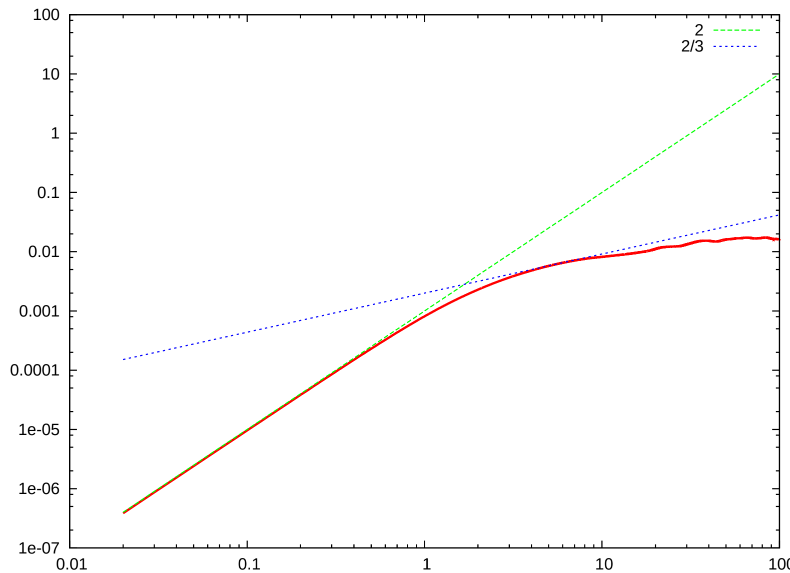


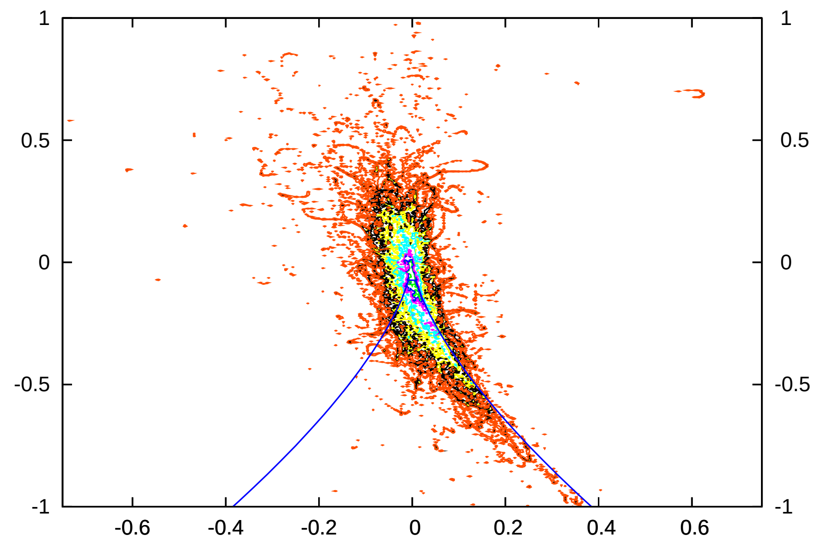
$< \delta u^2(\tau) >$ **and** $Q - R$ **at** $x/x_* = 0.735$



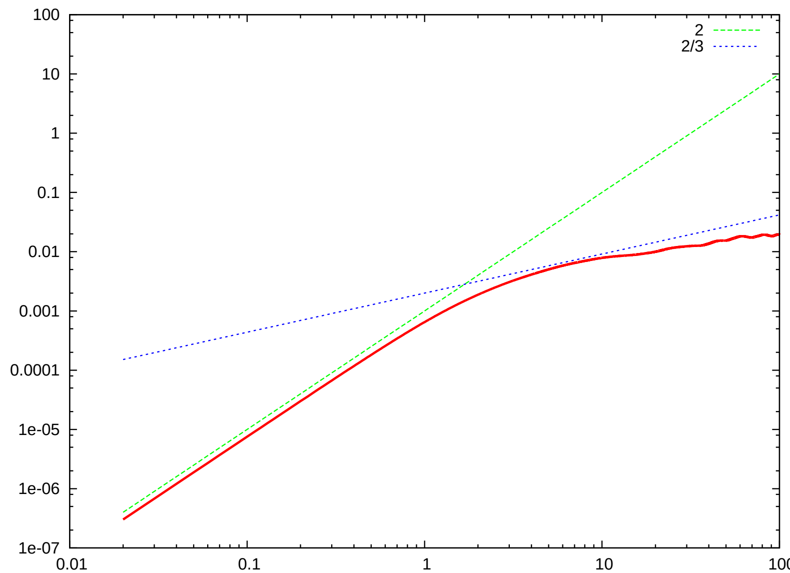


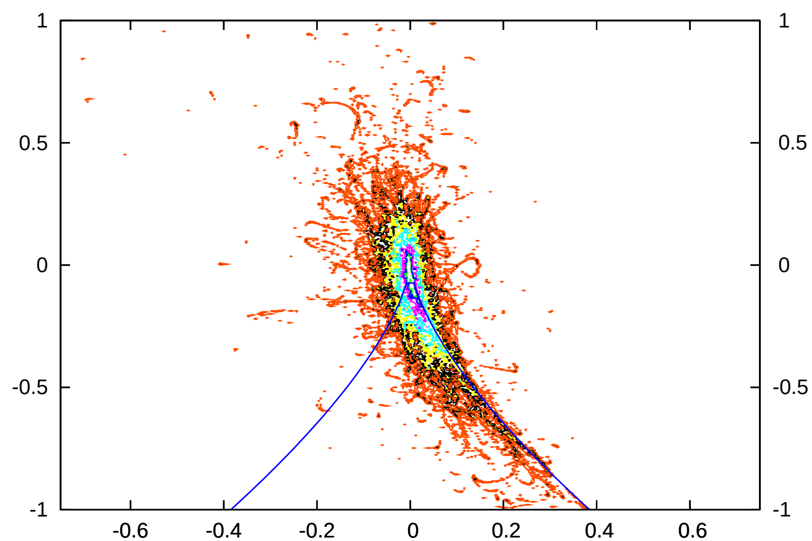
$< \delta u^2(\tau) >$ **and** $Q - R$ **at** $x/x_* = 0.788$



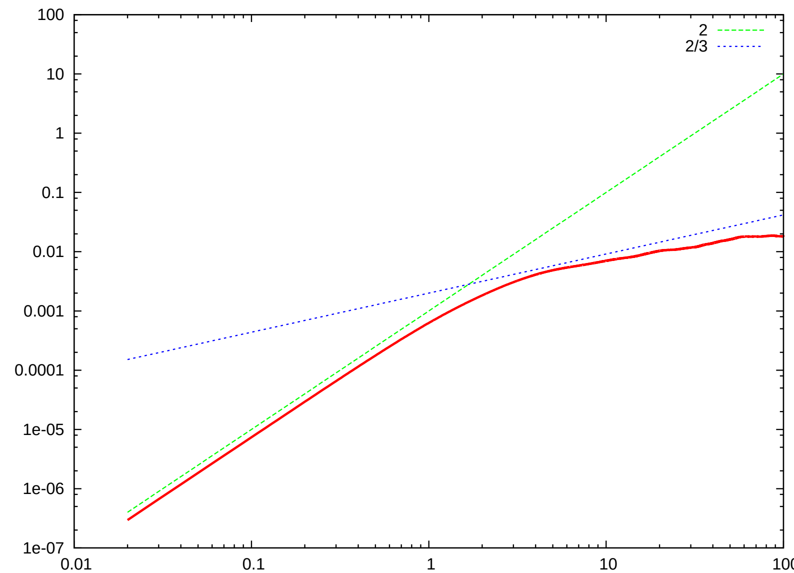


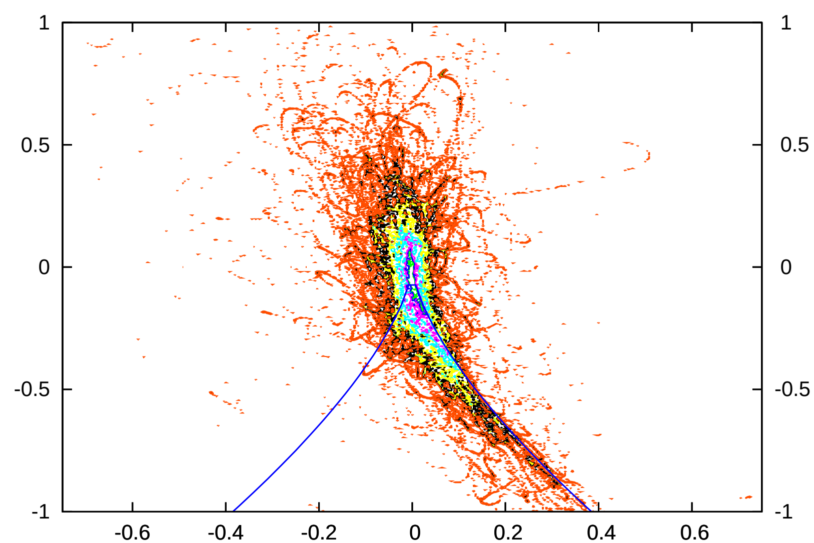
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 0.840$



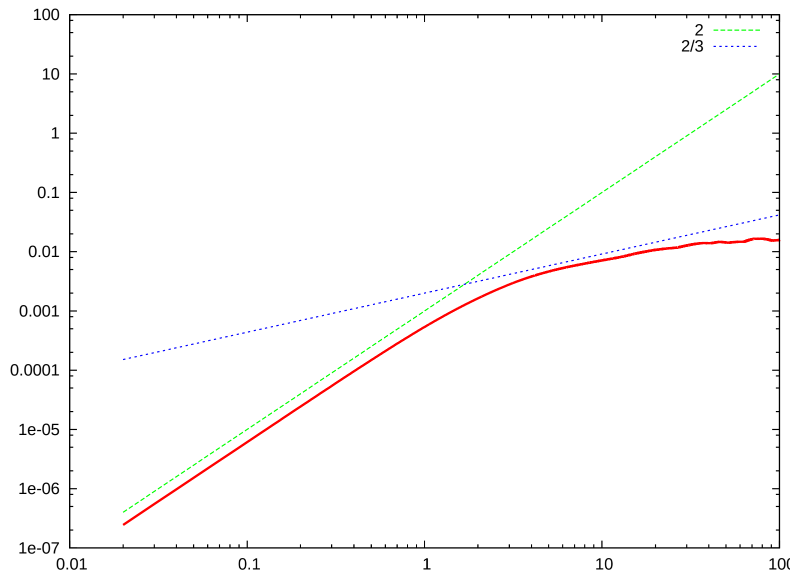


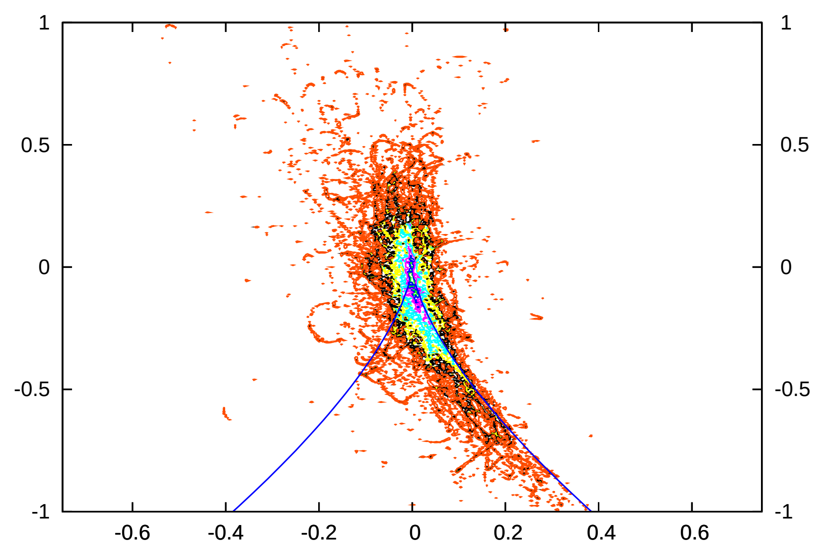
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 0.893$



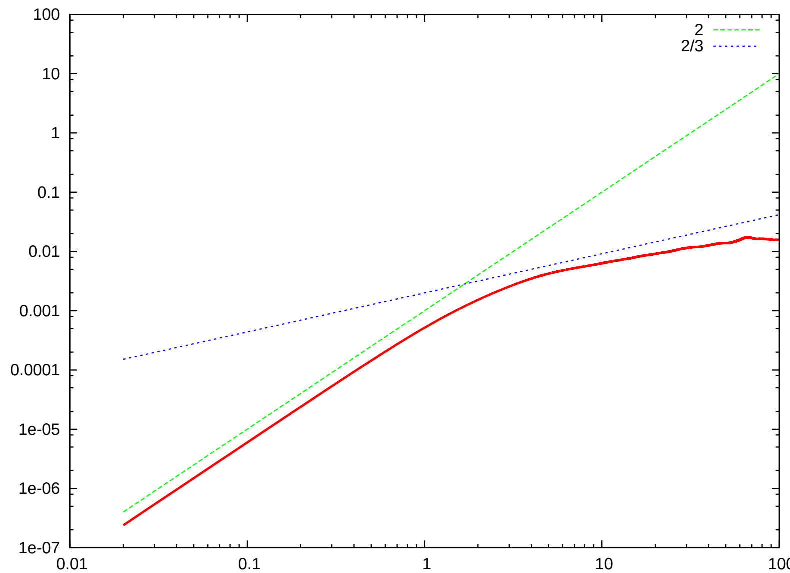


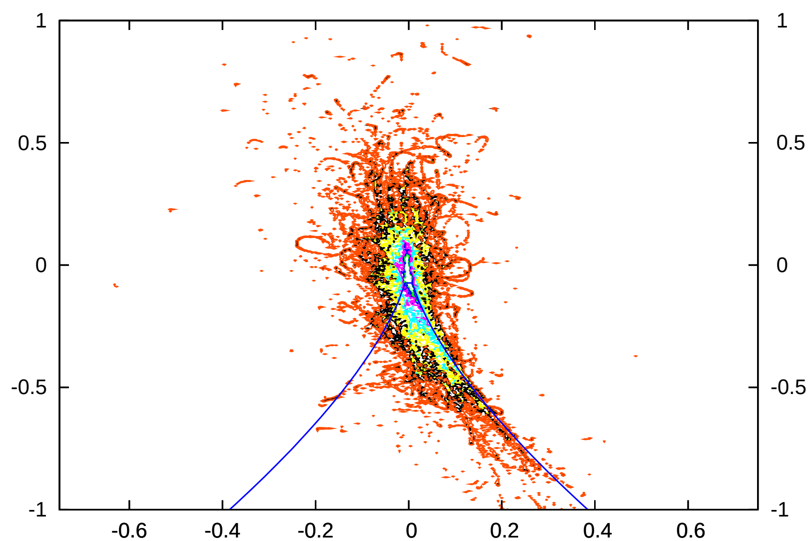
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 0.945$



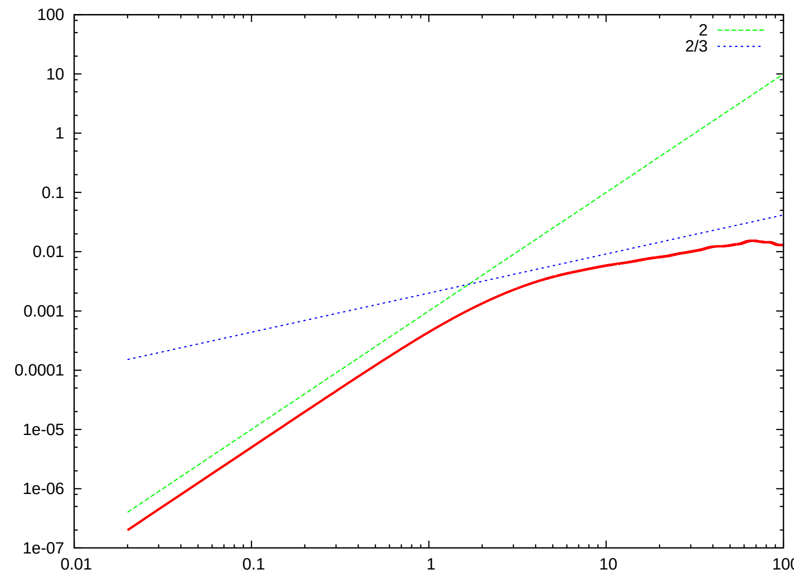


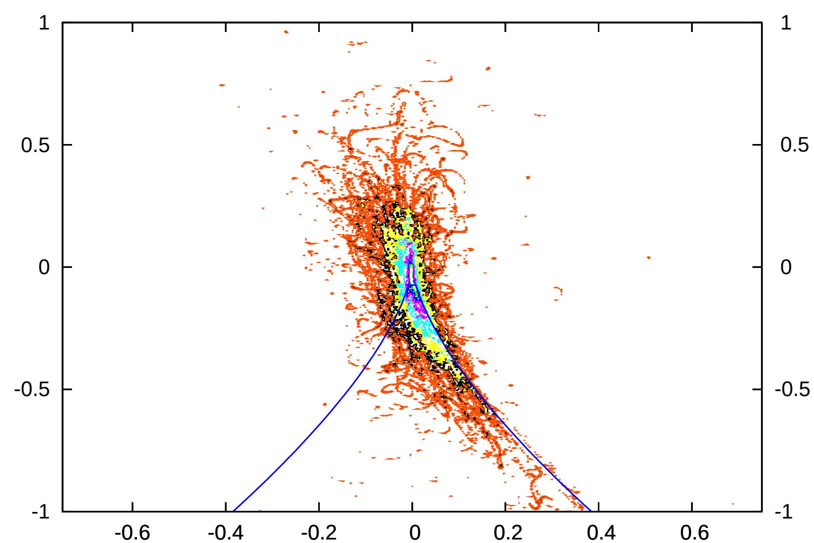
$< \delta u^2(\tau) >$ **and** $Q - R$ **at** $x/x_* = 0.998$



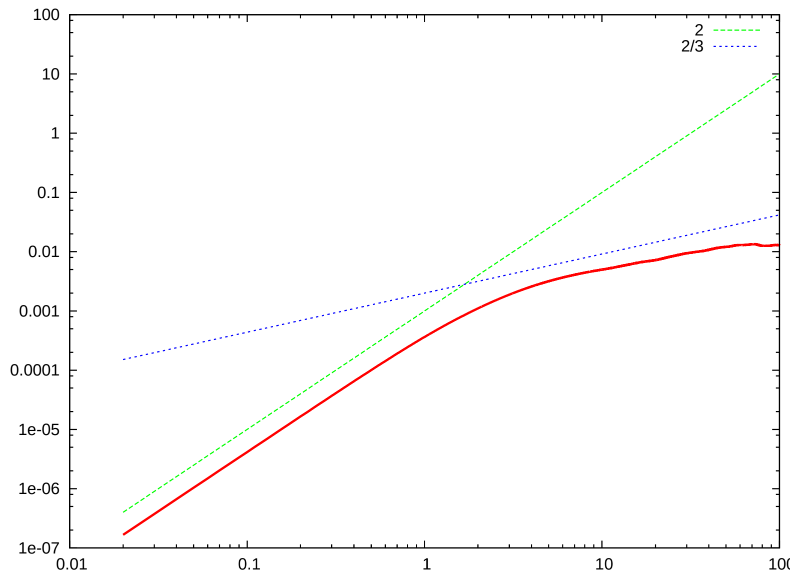


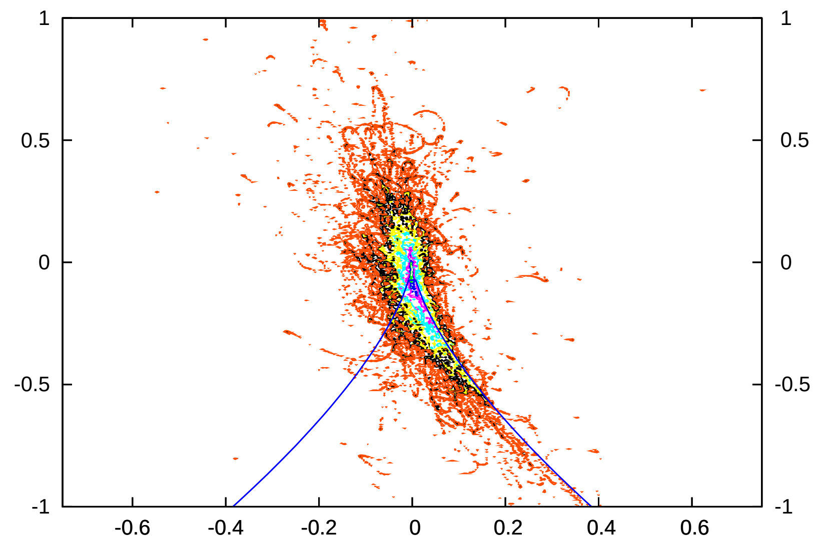
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 1.050$



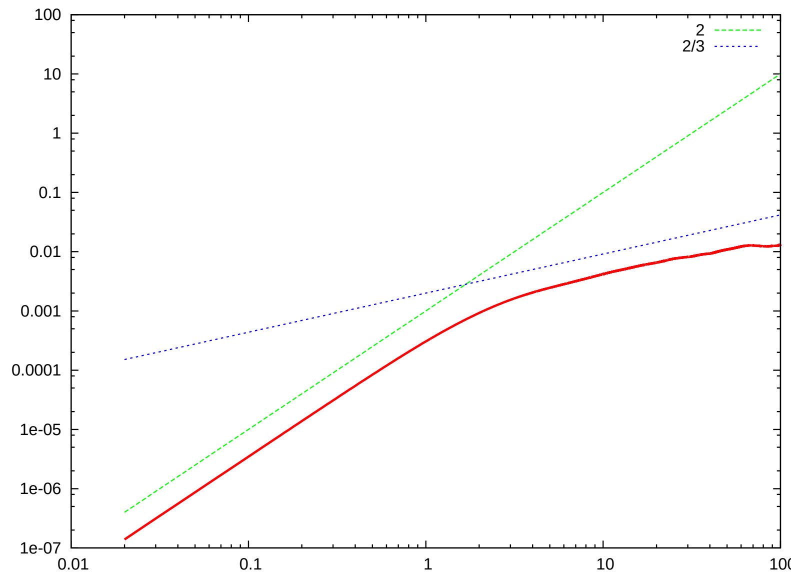


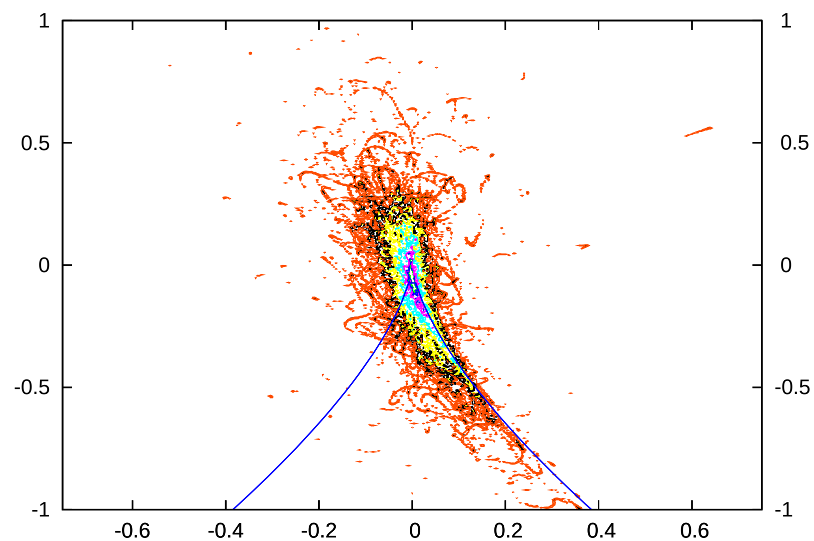
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 1.103$



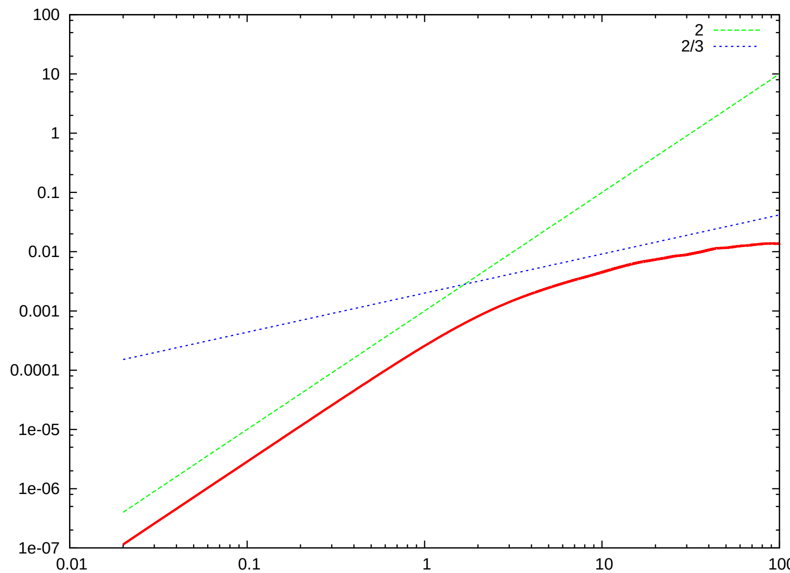


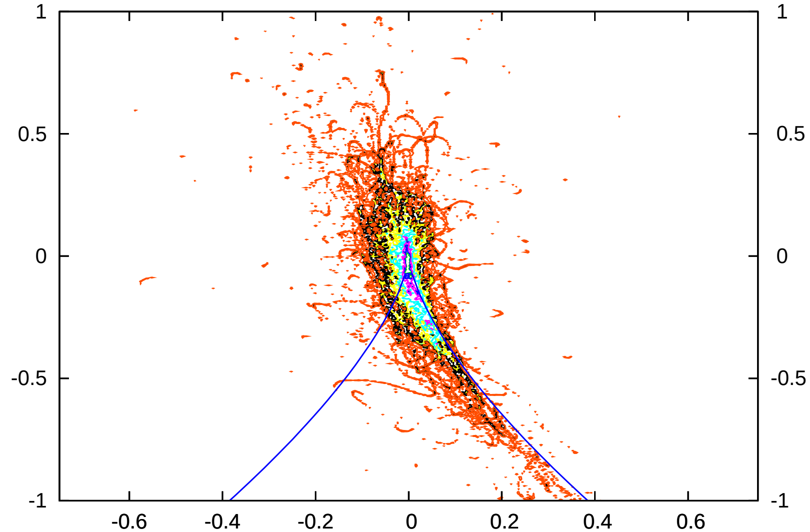
$< \delta u^2(\tau) >$ **and** $Q - R$ **at** $x/x_* = 1.155$



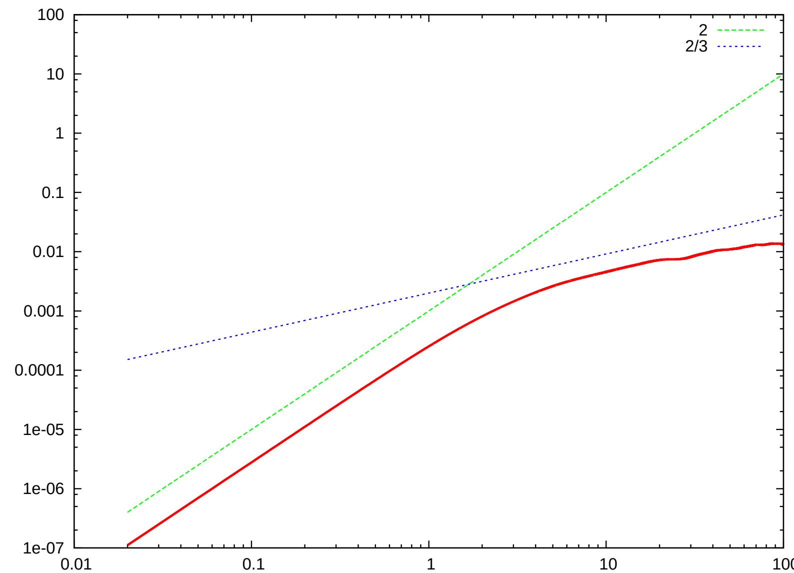


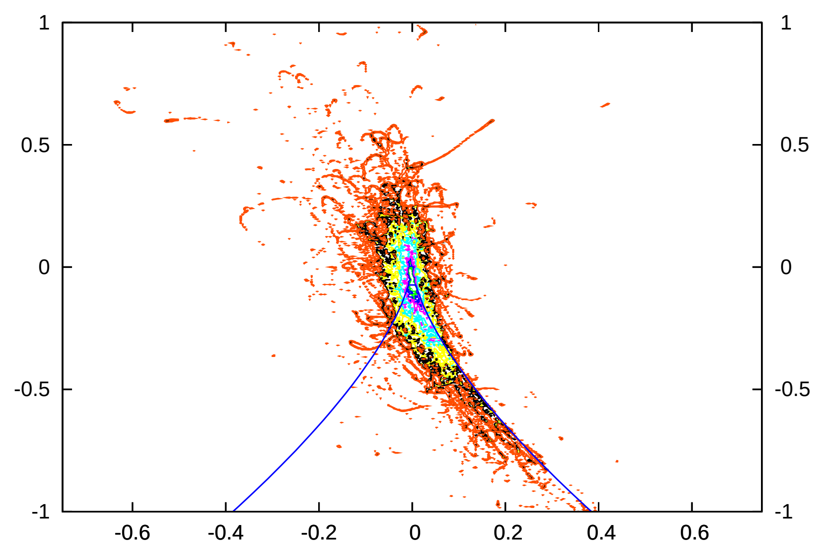
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 1.208$



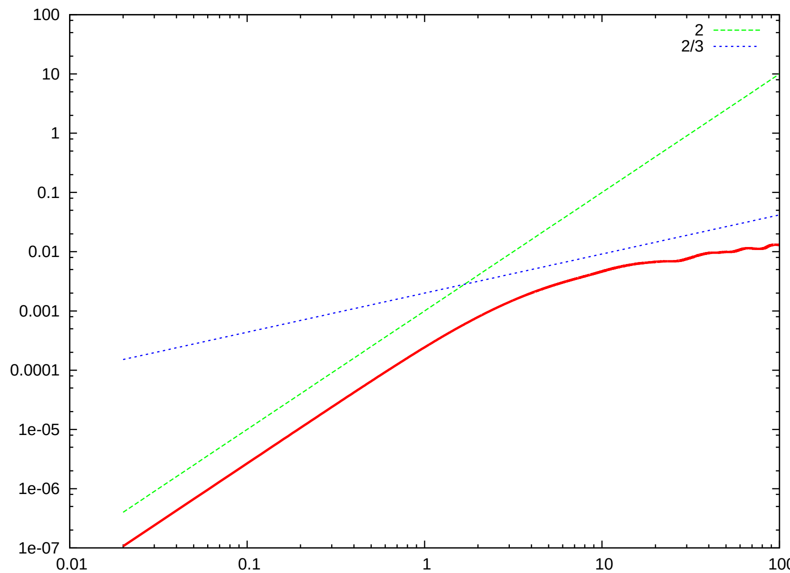


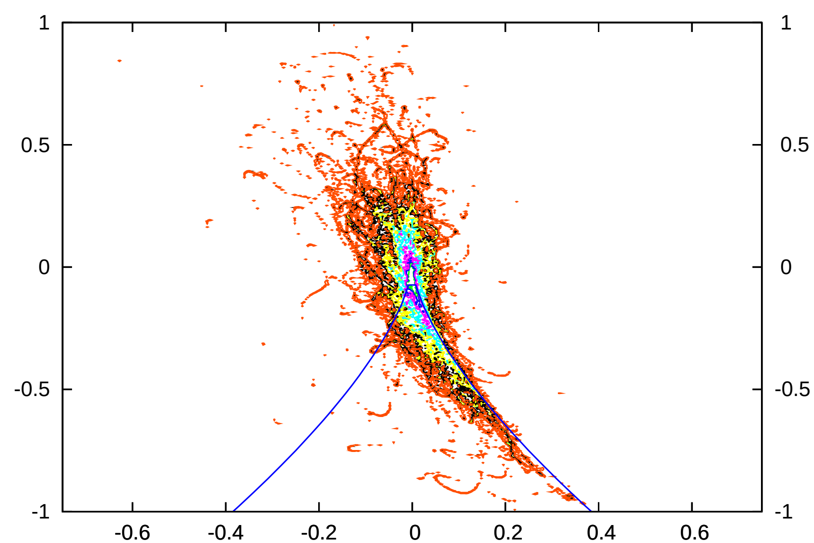
$< \delta u^2(\tau) >$ and $Q-R$ at $x/x_* = 1.261$



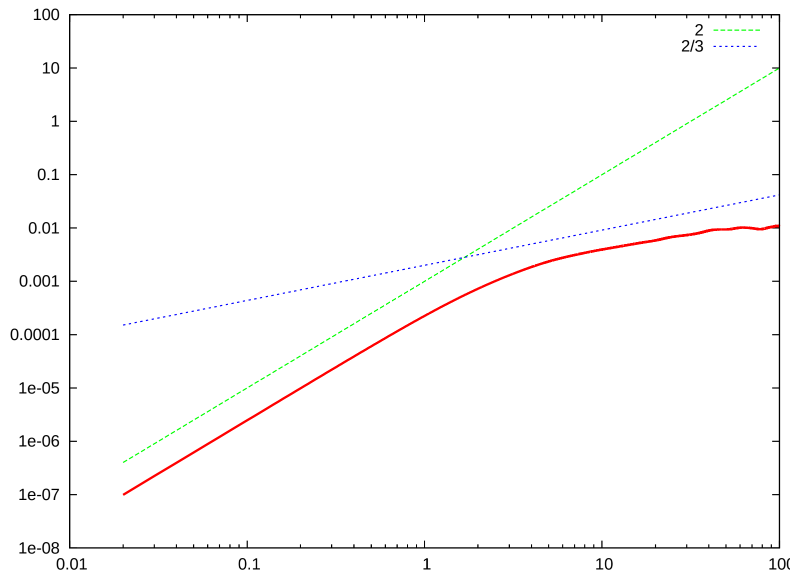


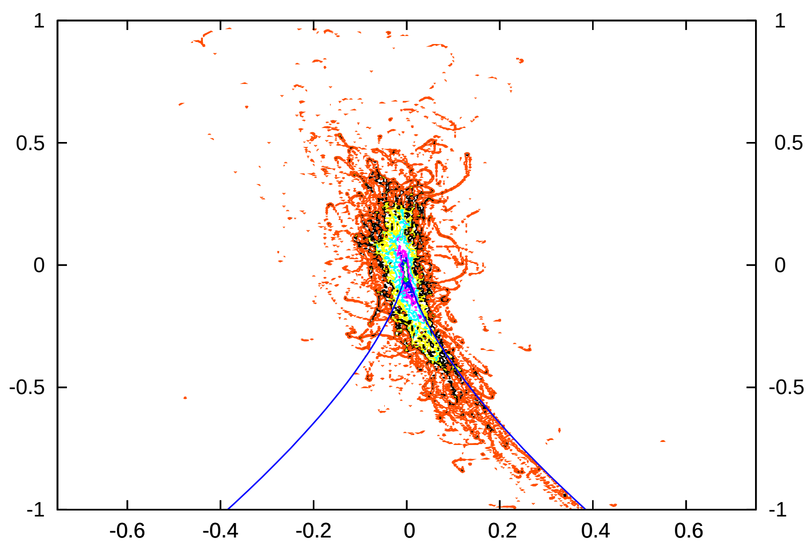
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 1.313$



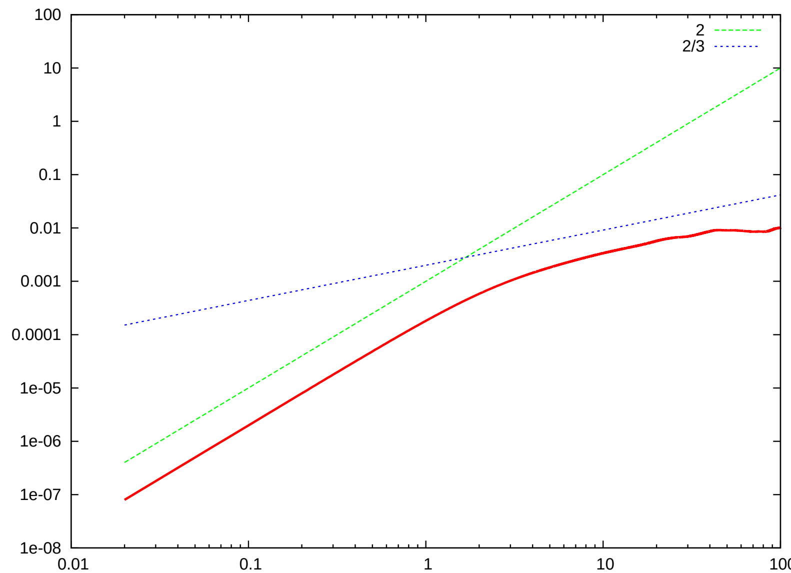


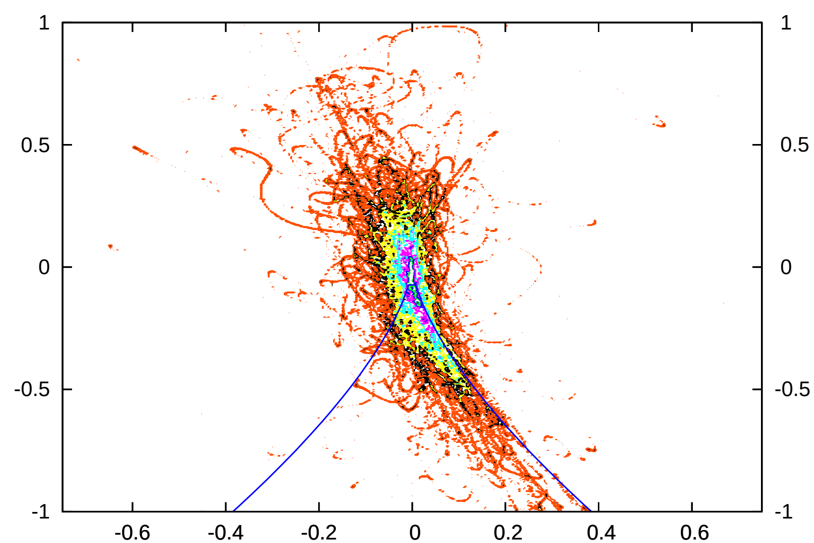
$< \delta u^2(\tau) >$ **and** $Q - R$ **at** $x/x_* = 1.366$



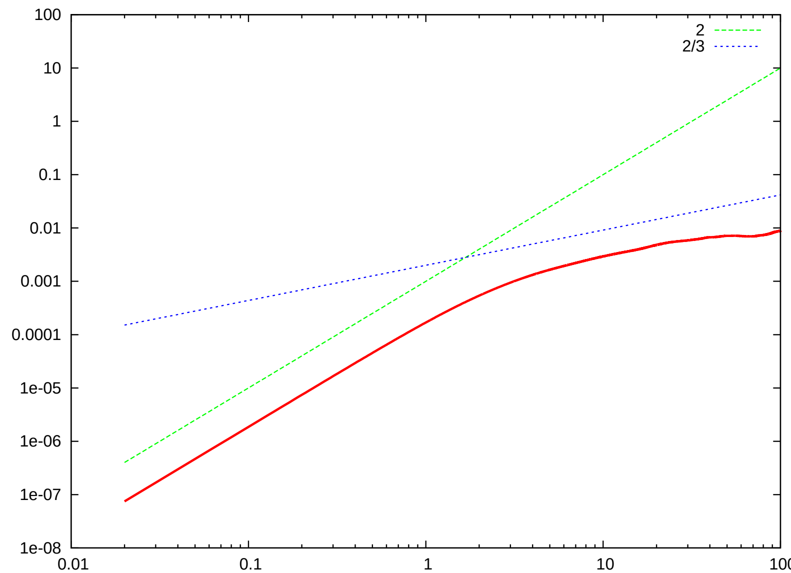


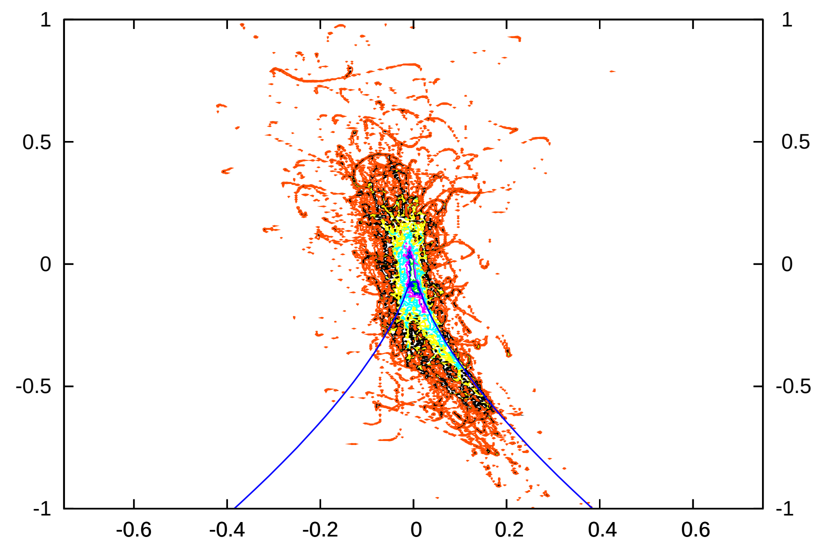
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 1.418$



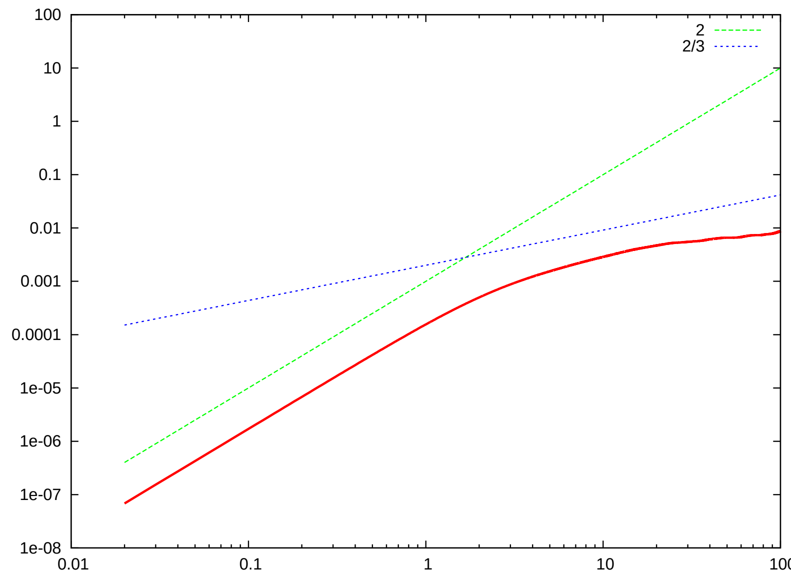


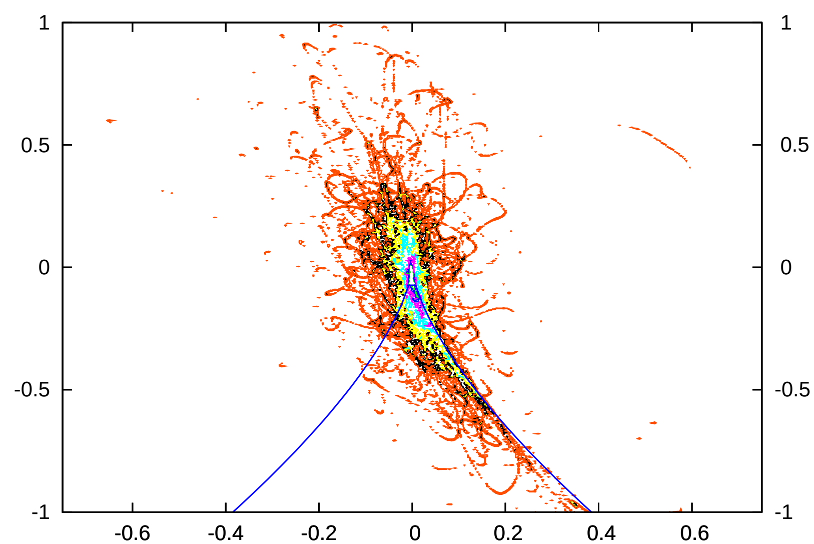
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 1.471$



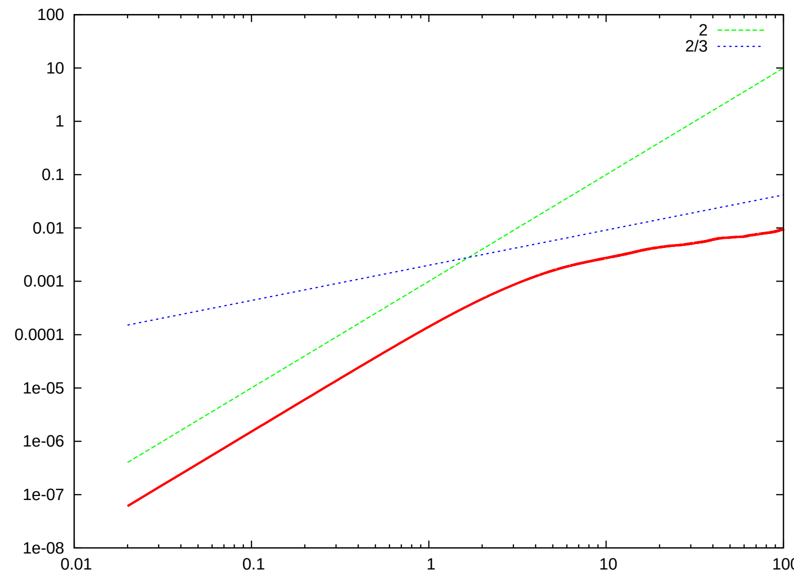


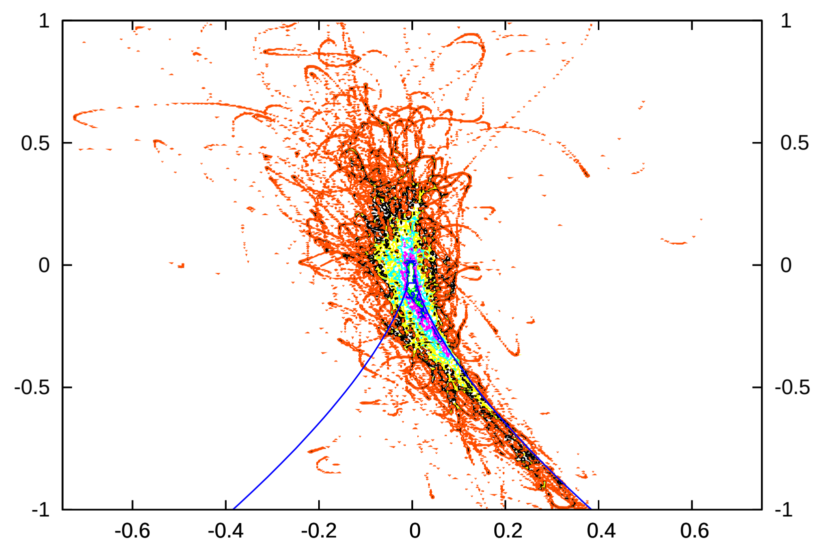
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 1.523$



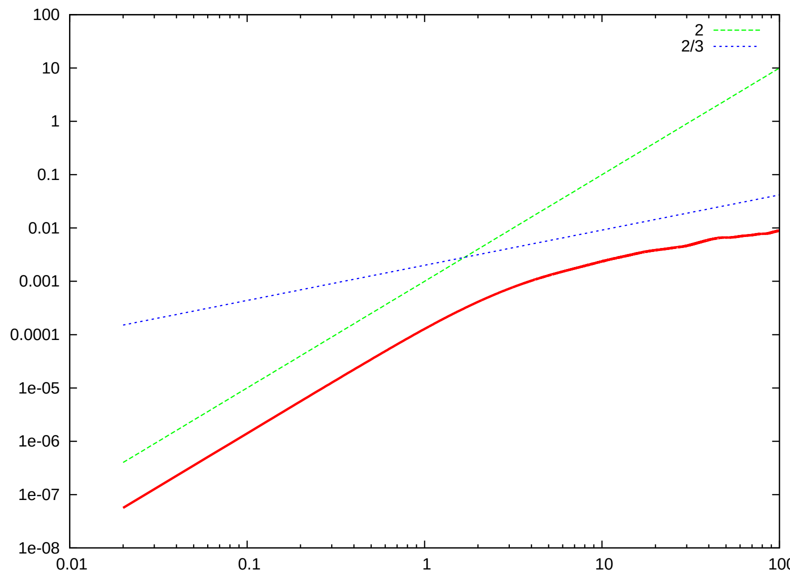


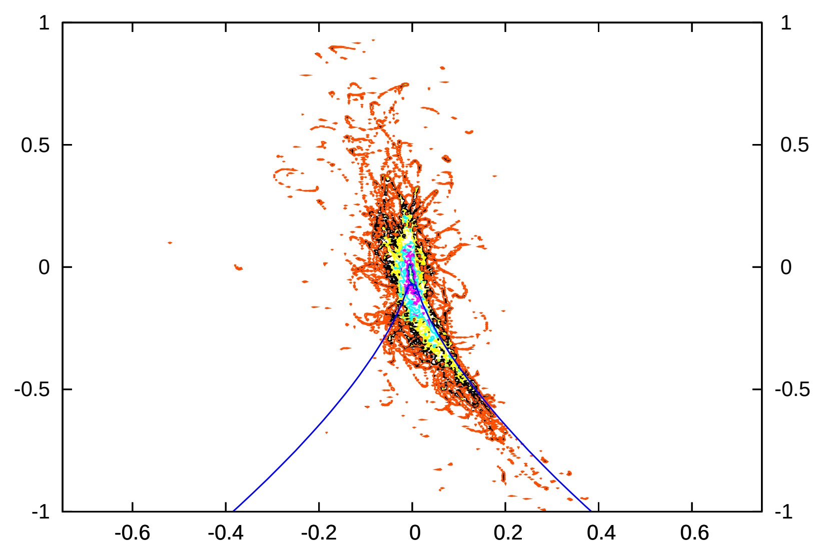
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 1.576$



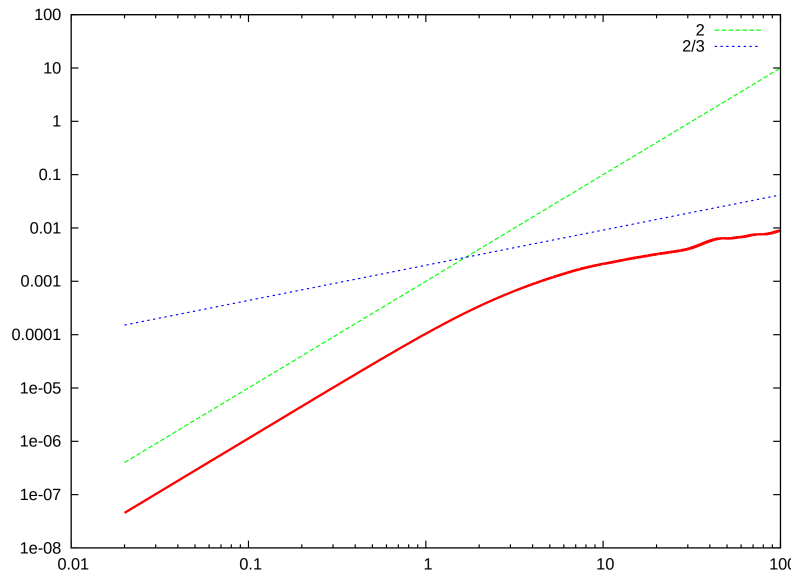


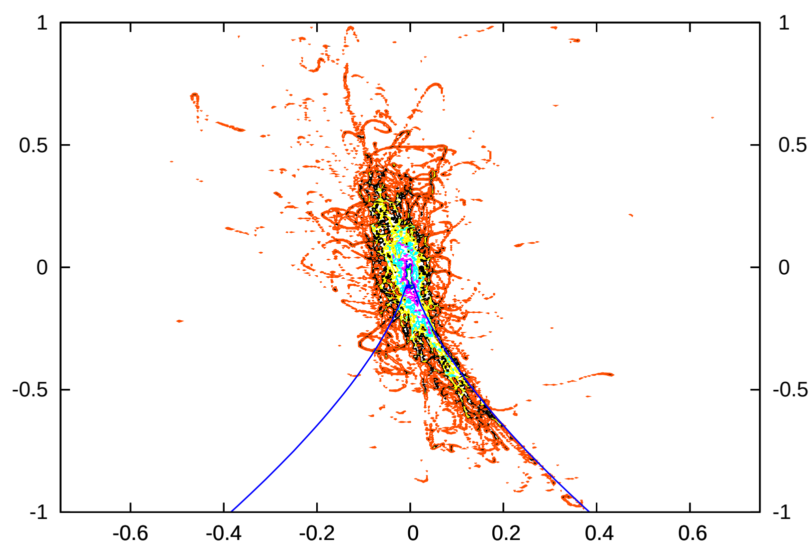
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 1.628$



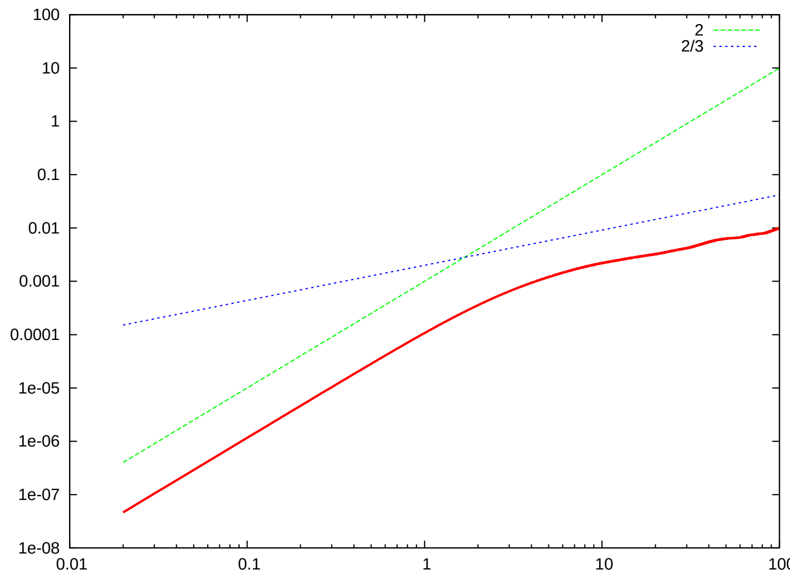


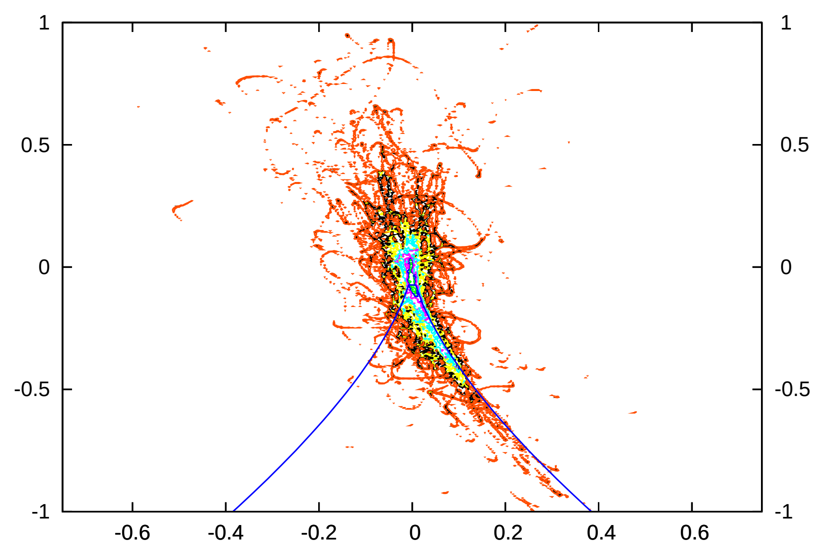
$< \delta u^2(\tau) >$ **and** $Q - R$ **at** $x/x_* = 1.681$





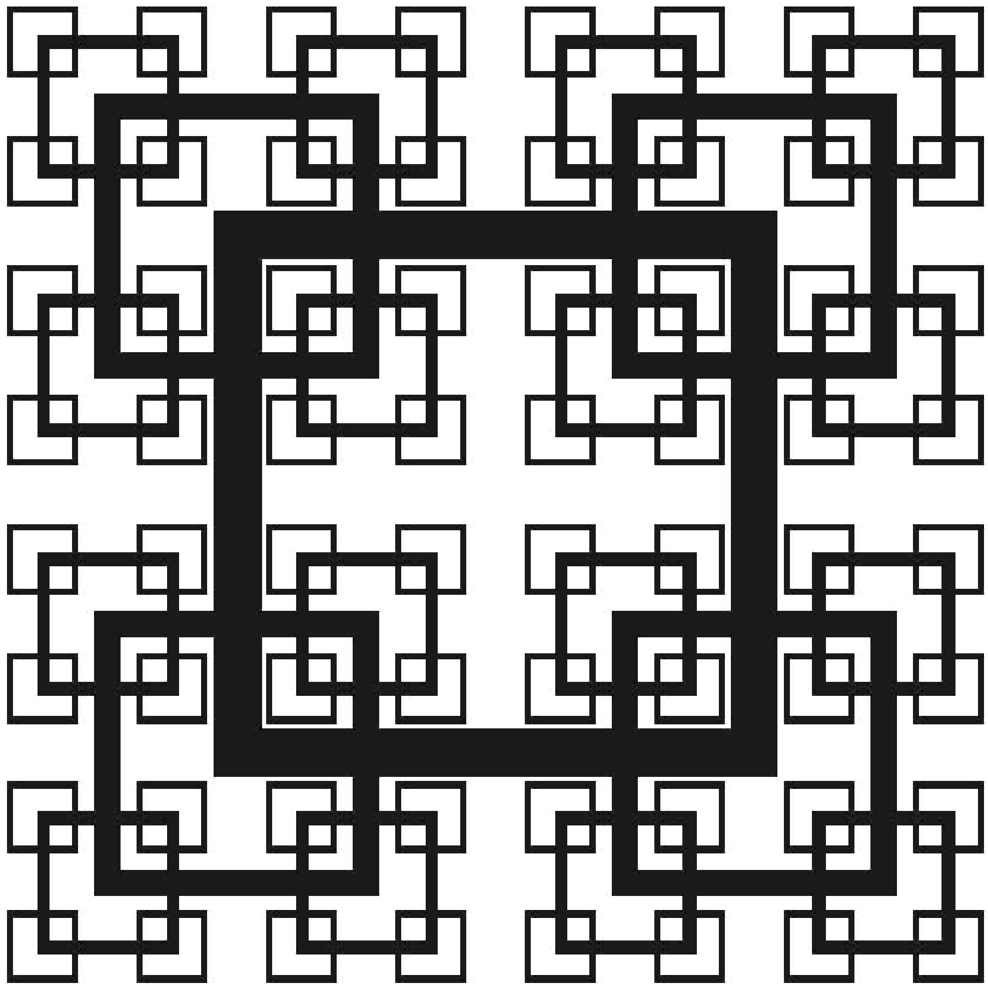
$< \delta u^2(\tau) >$ and $Q - R$ at $x/x_* = 1.733$



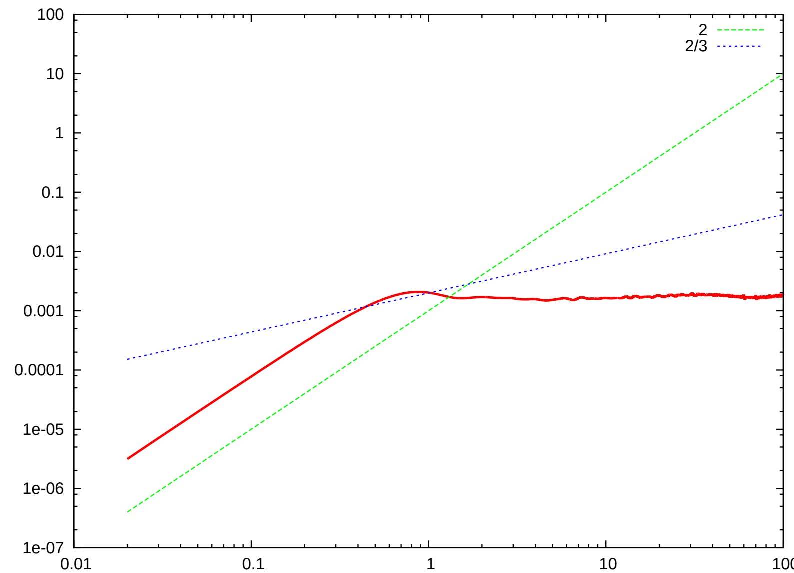


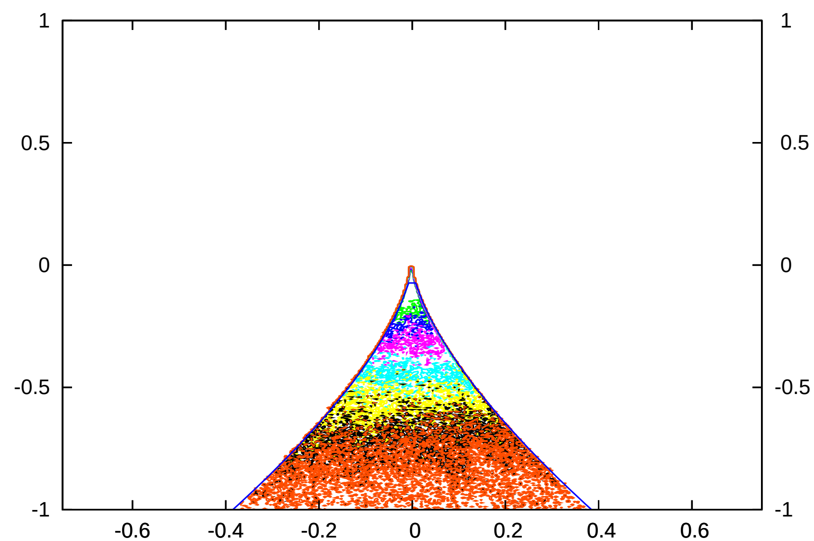
Back along centerline

But now $Q_s - R_s$ plots along the centreline.

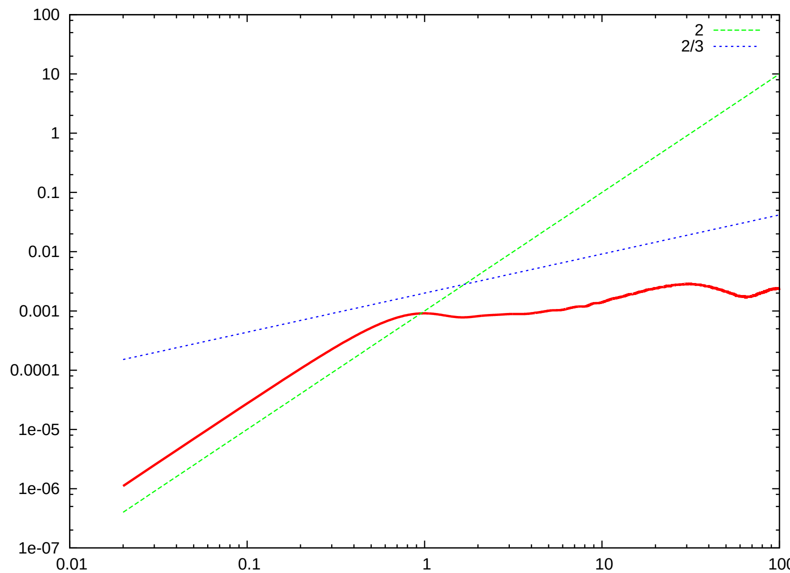


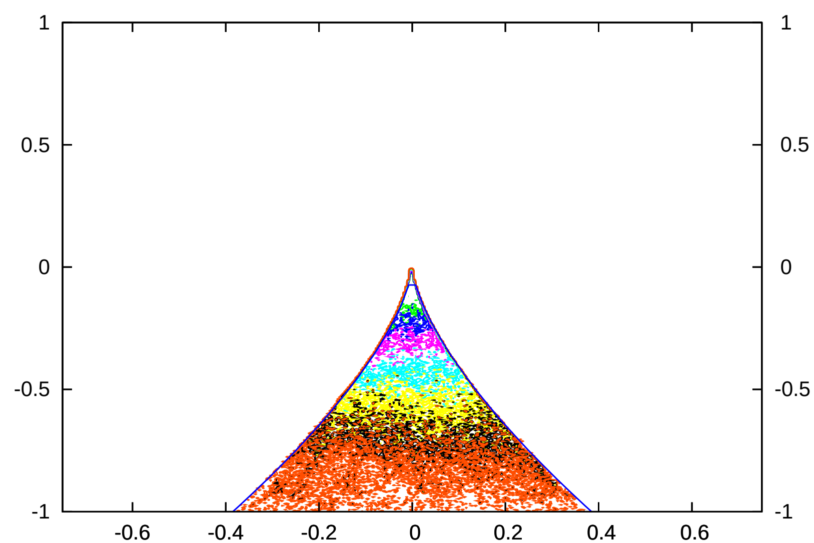
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.052$



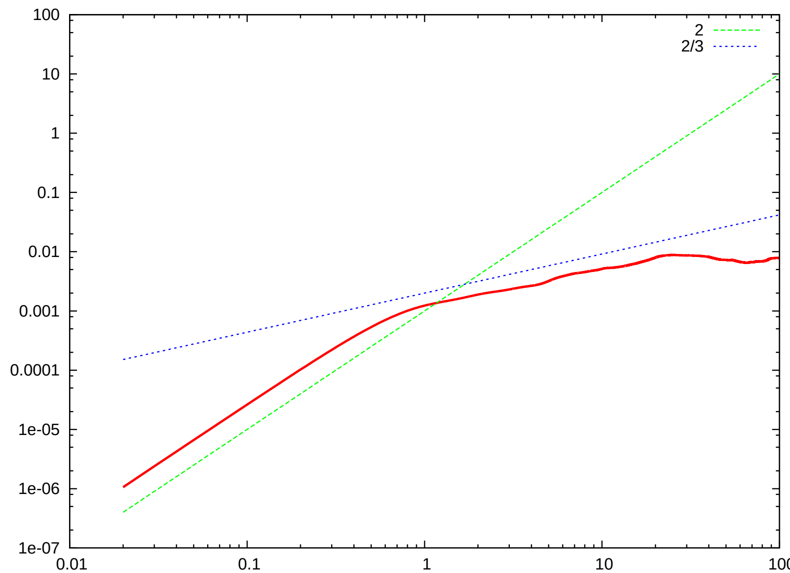


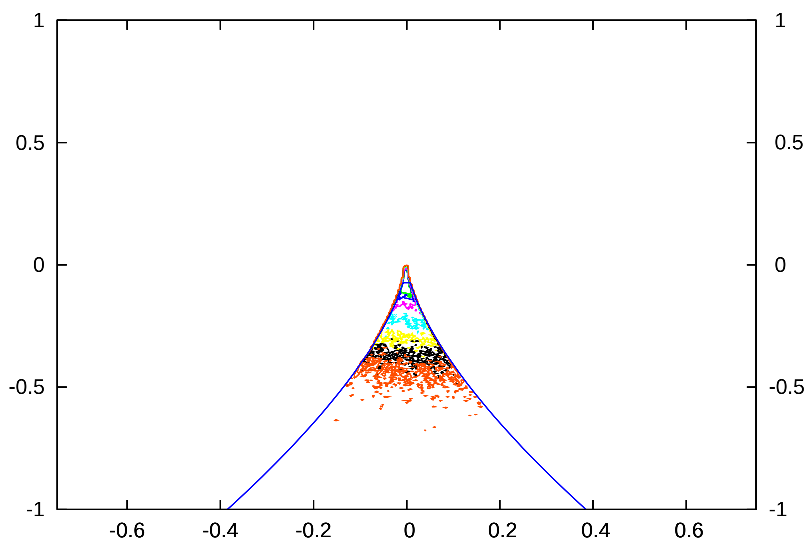
$< \delta u^2(\tau) >$ **and** $Q_s - R_s$ **at** $x/x_* = 0.105$



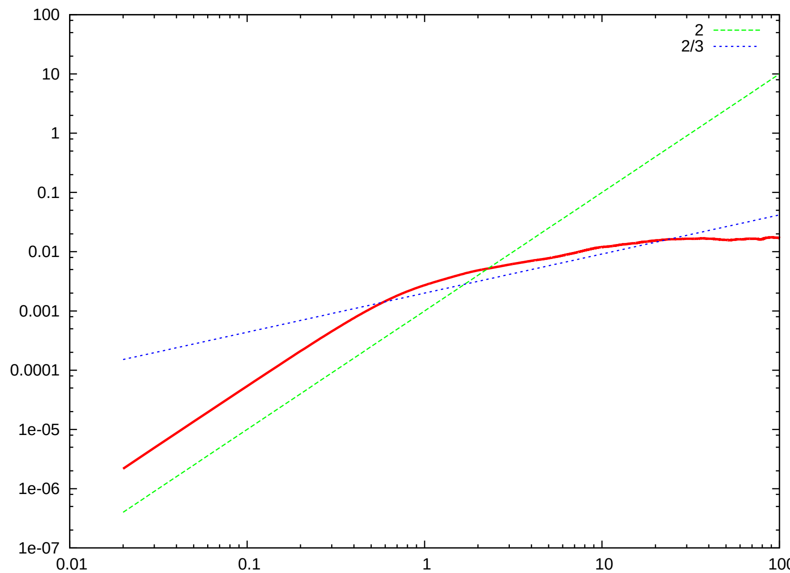


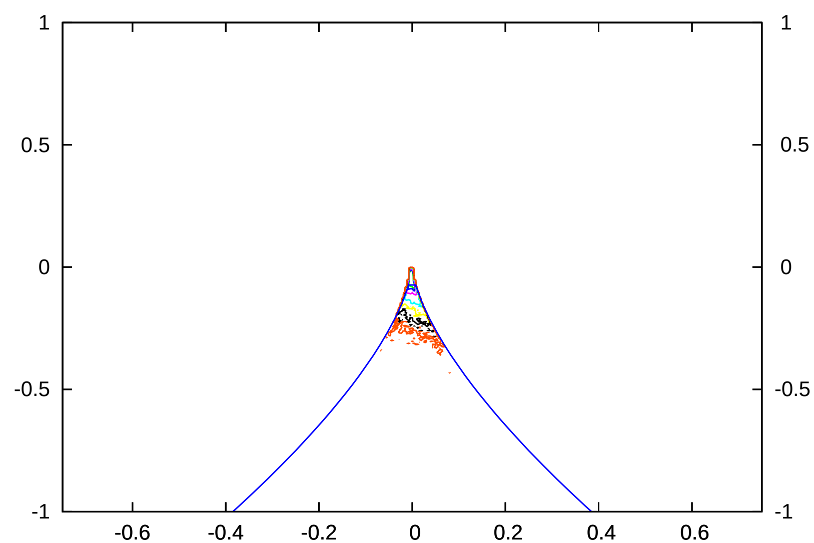
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.157$



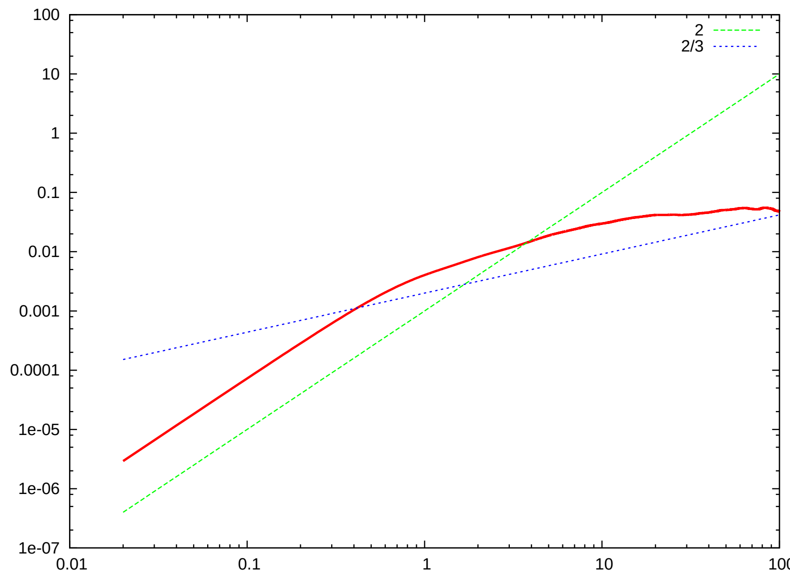


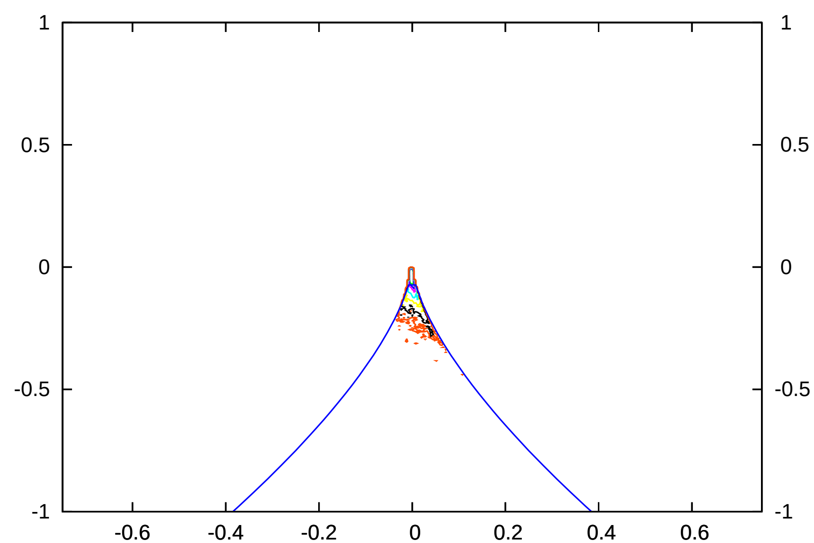
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.210$



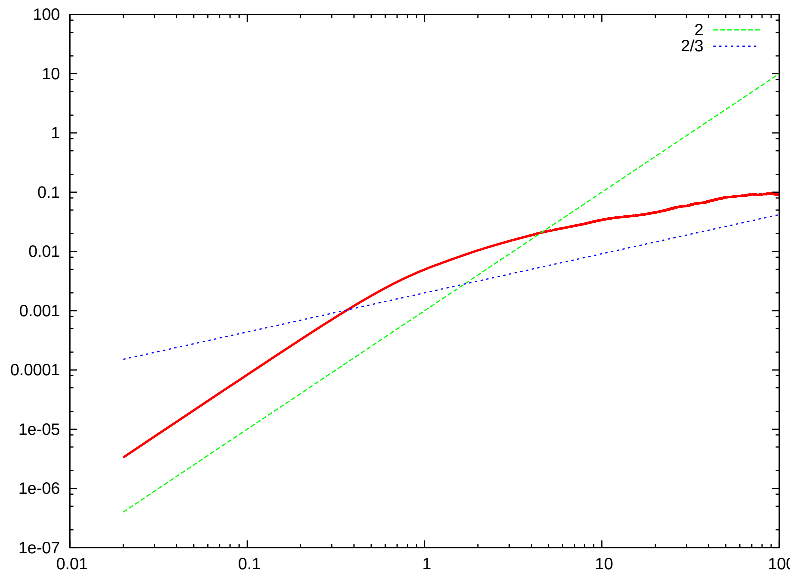


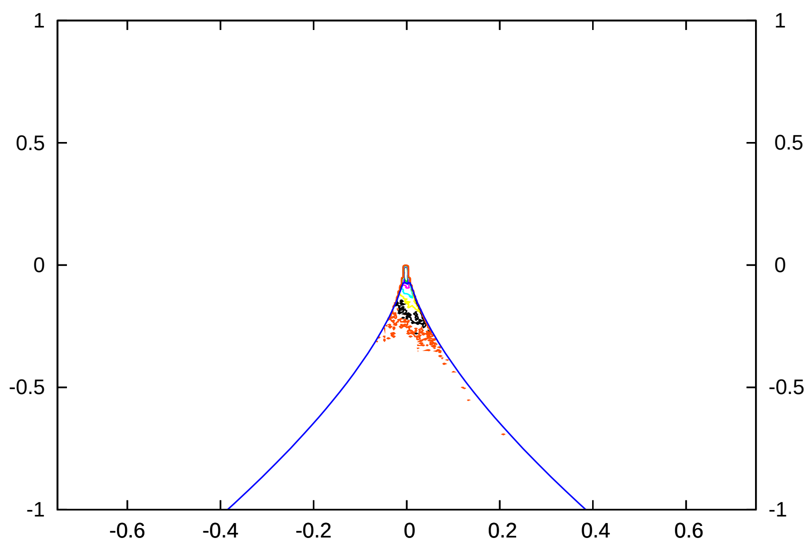
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.262$



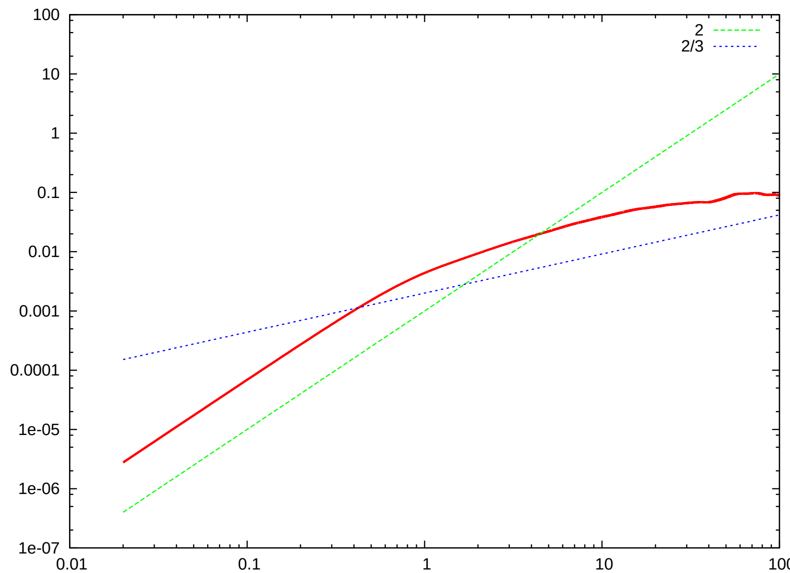


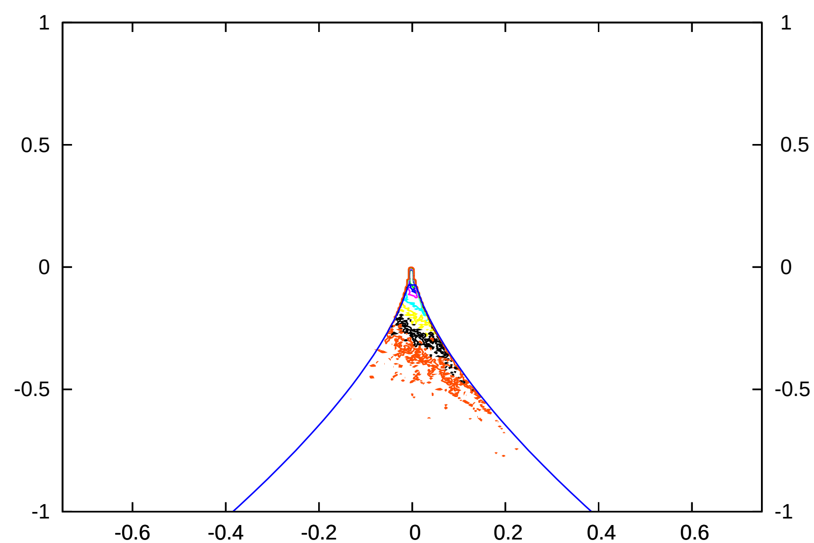
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.315$



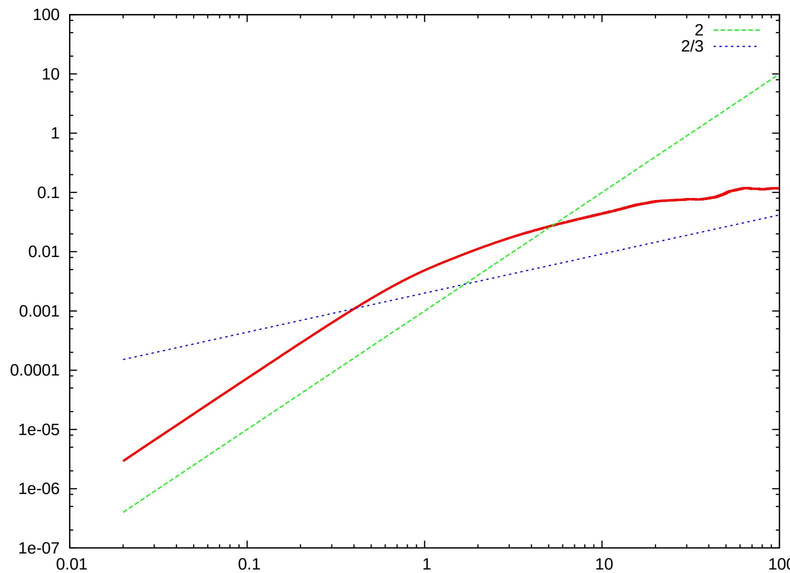


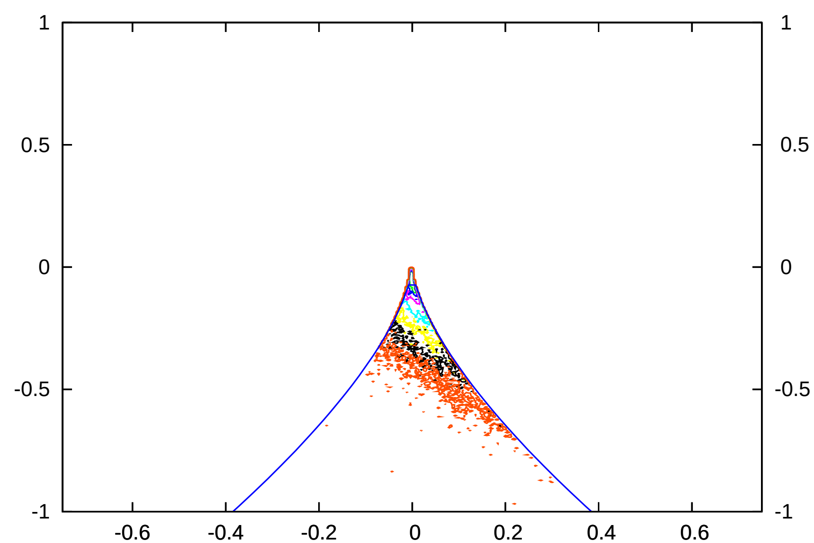
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.367$



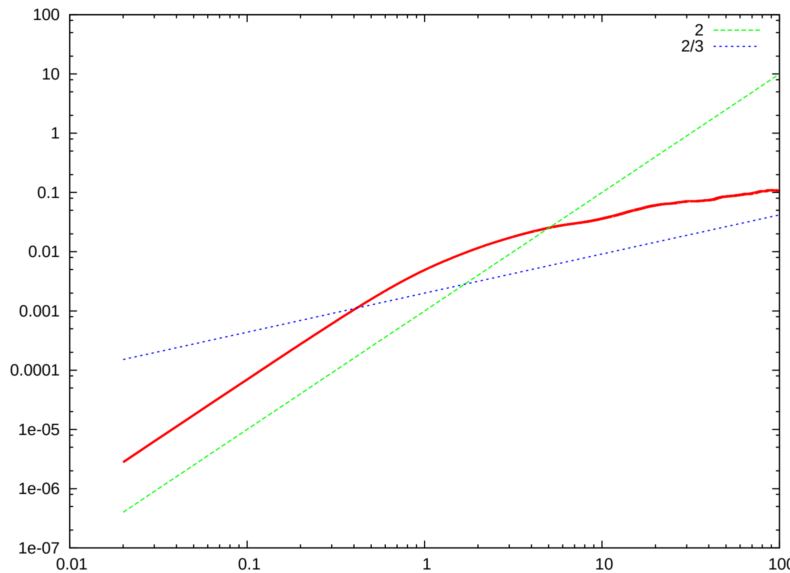


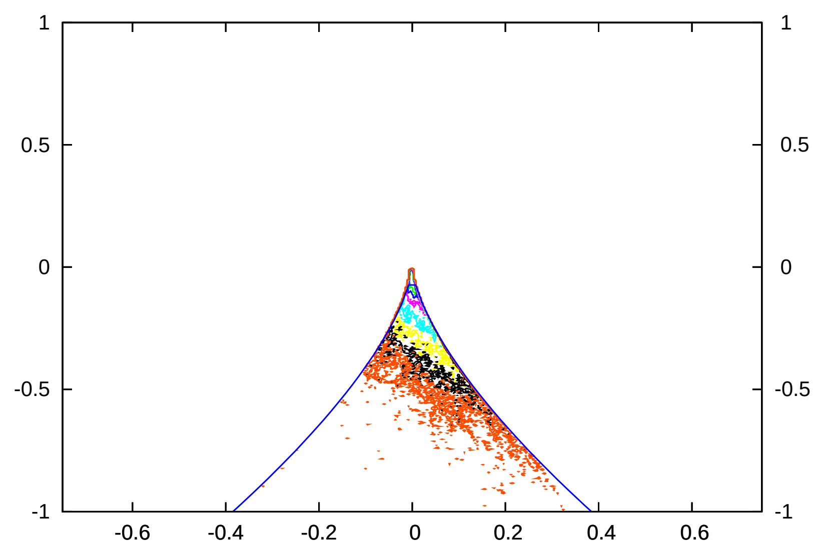
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.420$



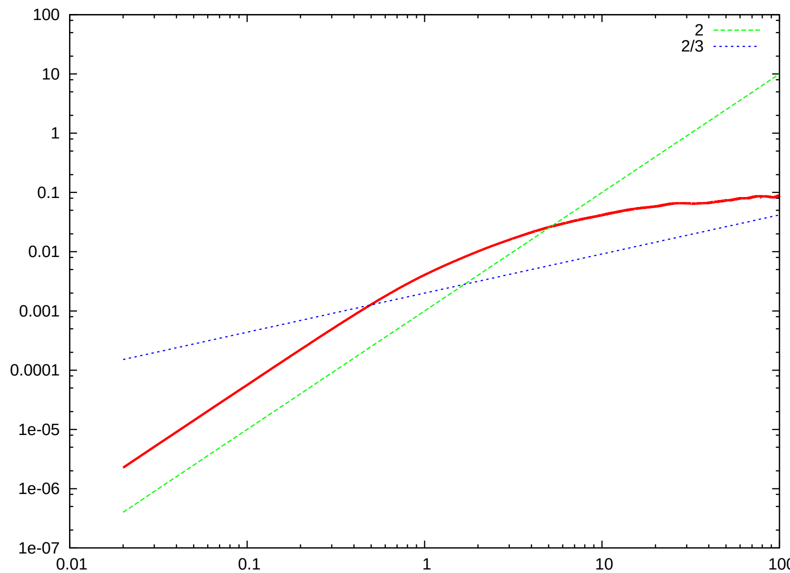


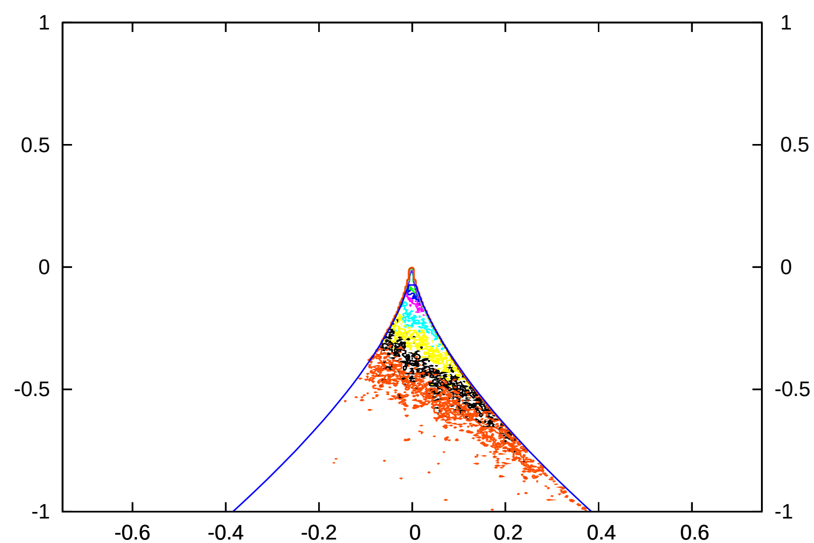
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.472$



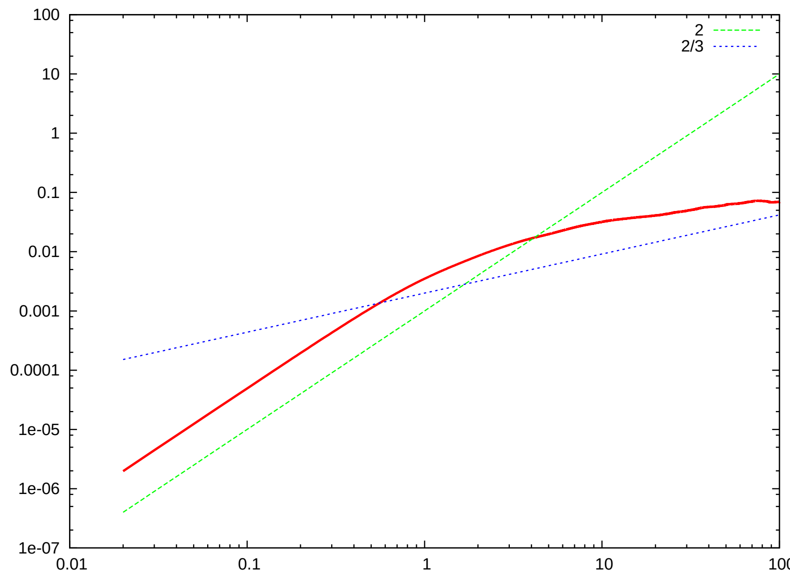


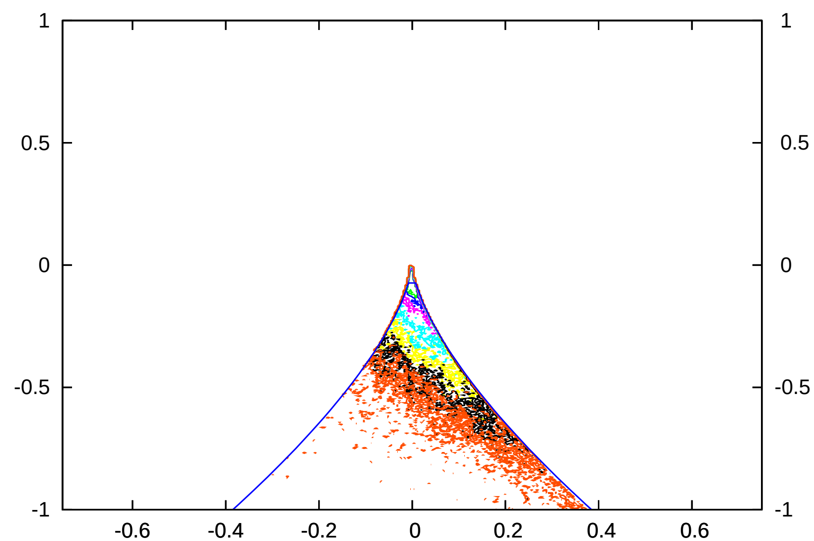
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.525$



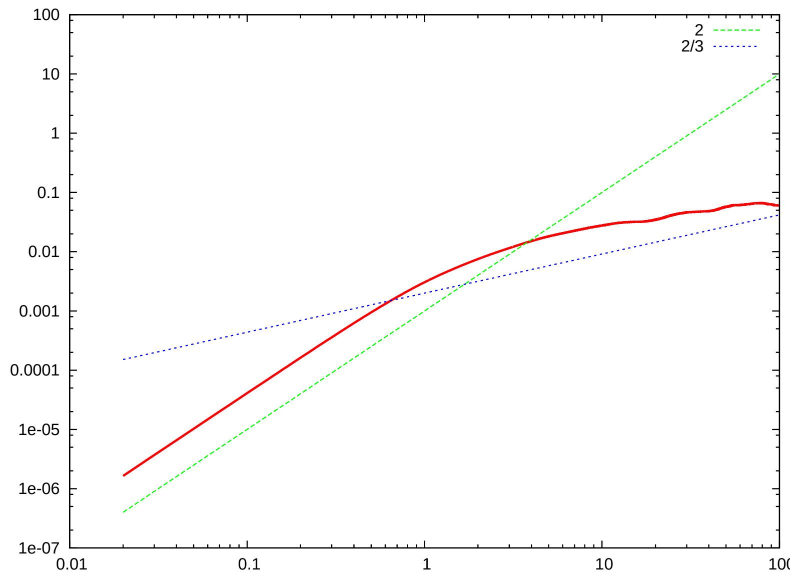


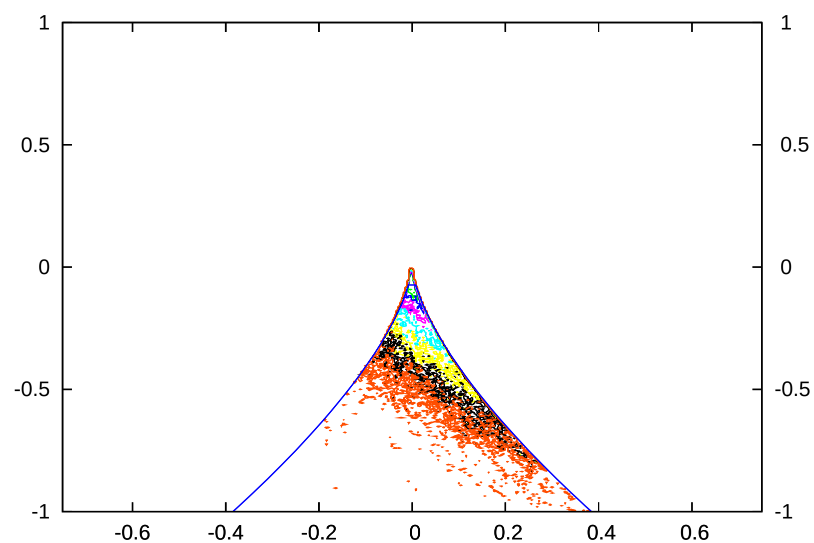
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.577$



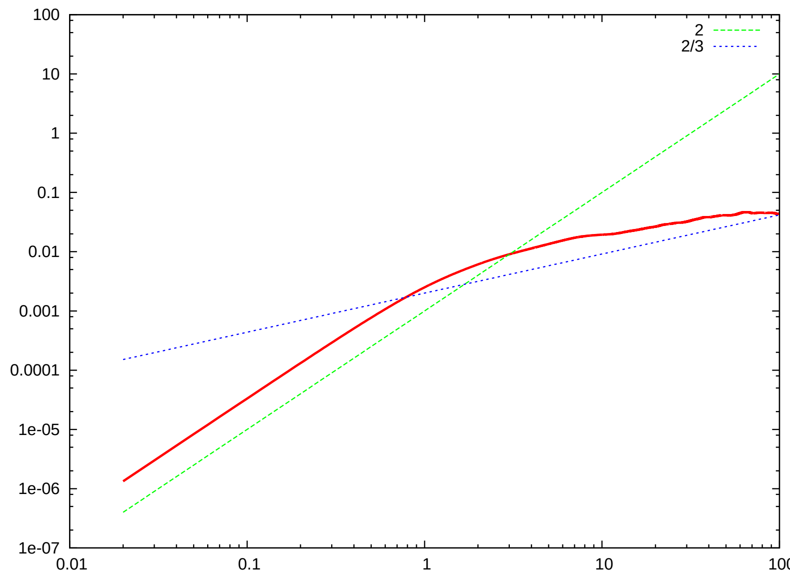


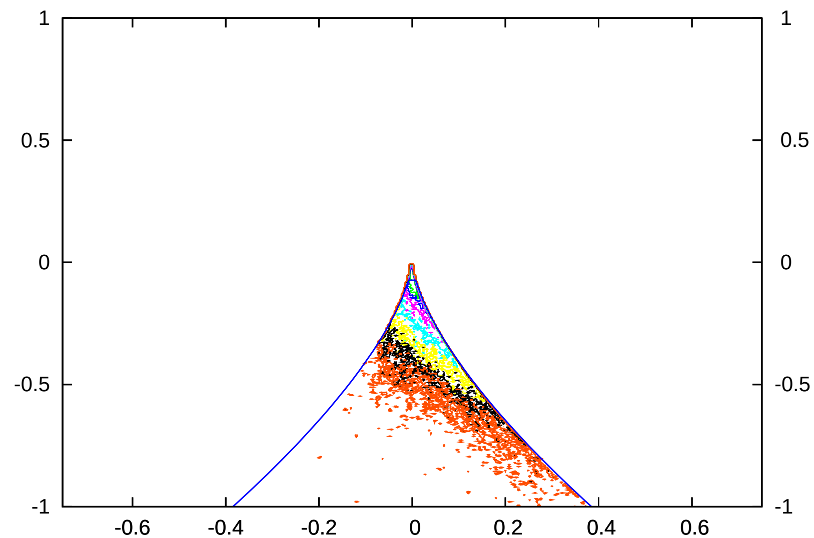
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.630$



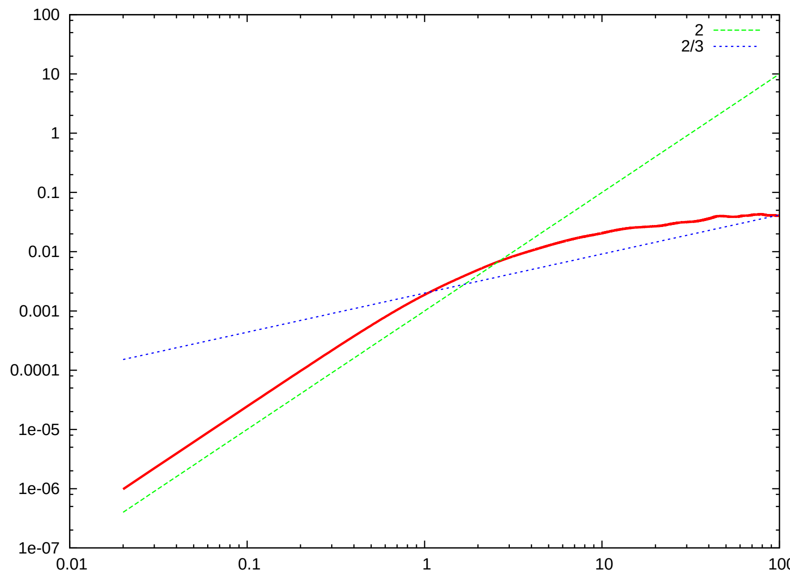


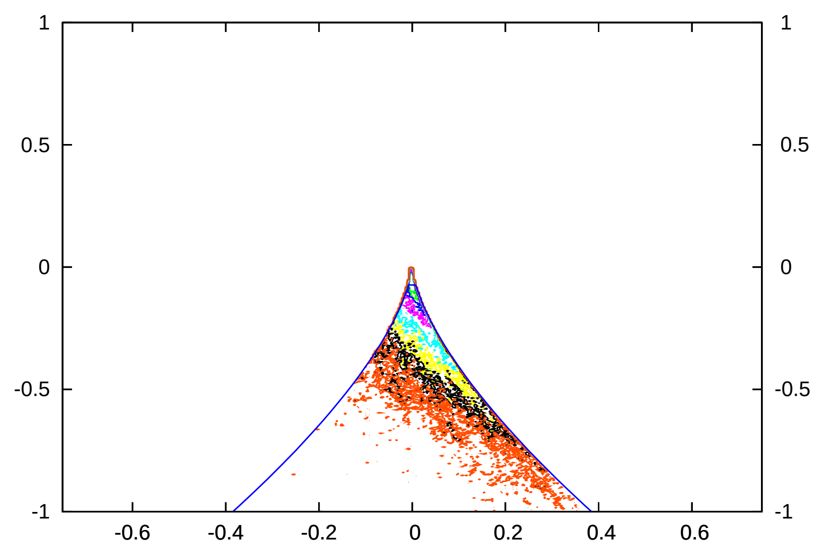
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.683$



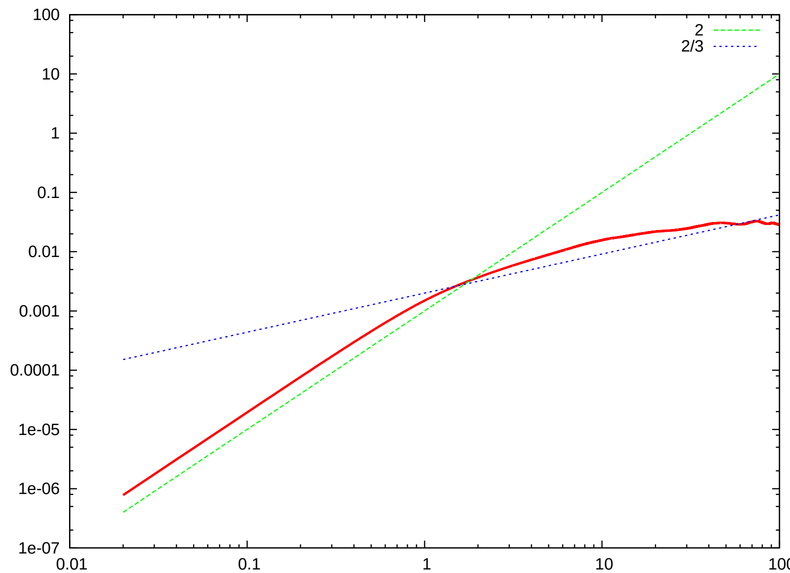


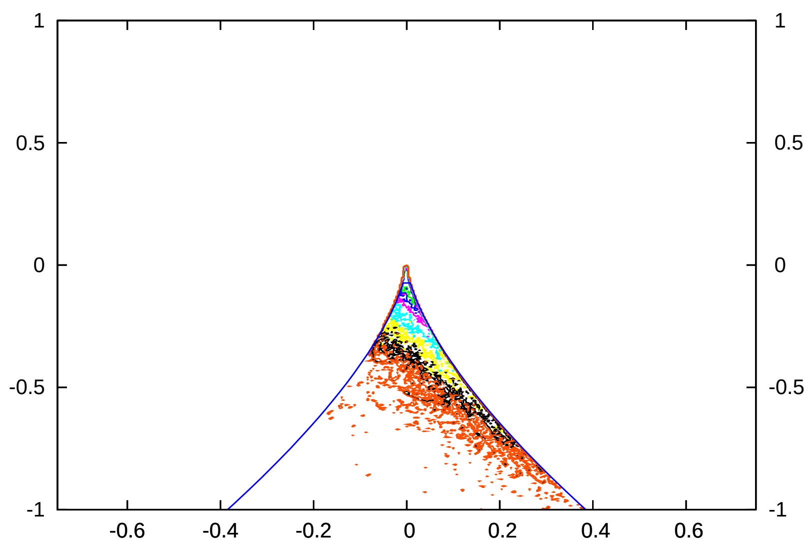
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.735$



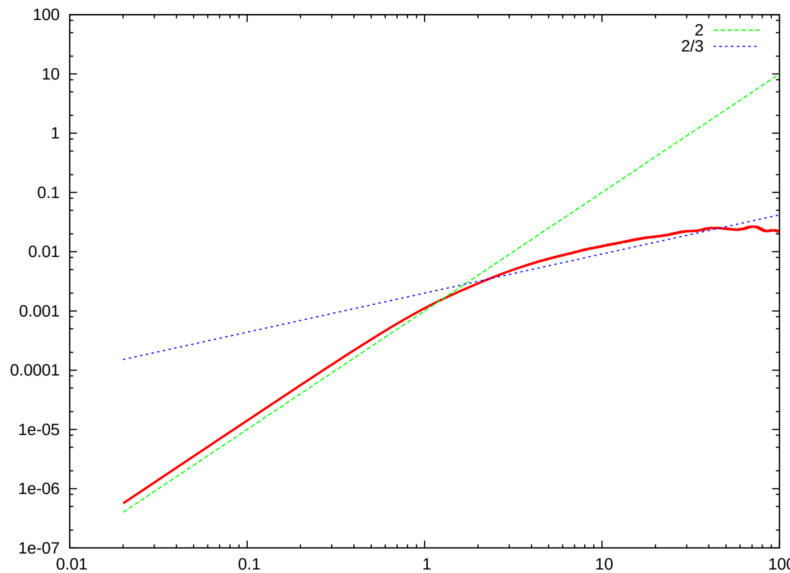


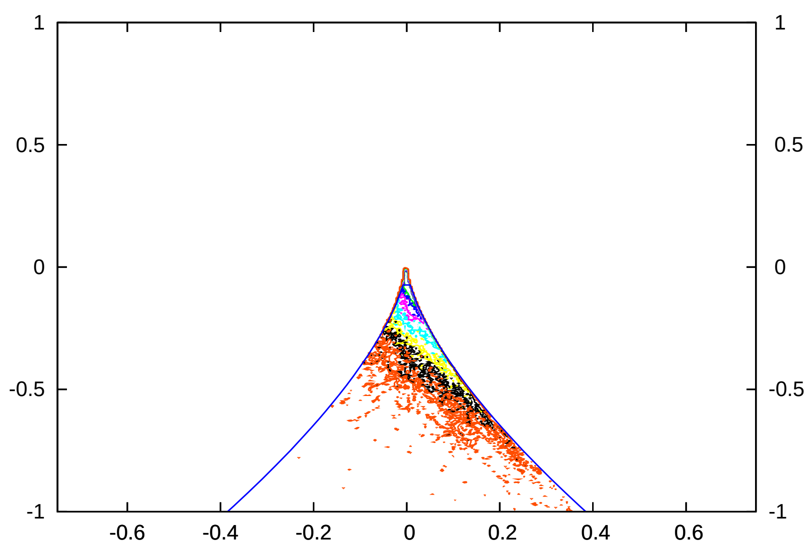
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.788$



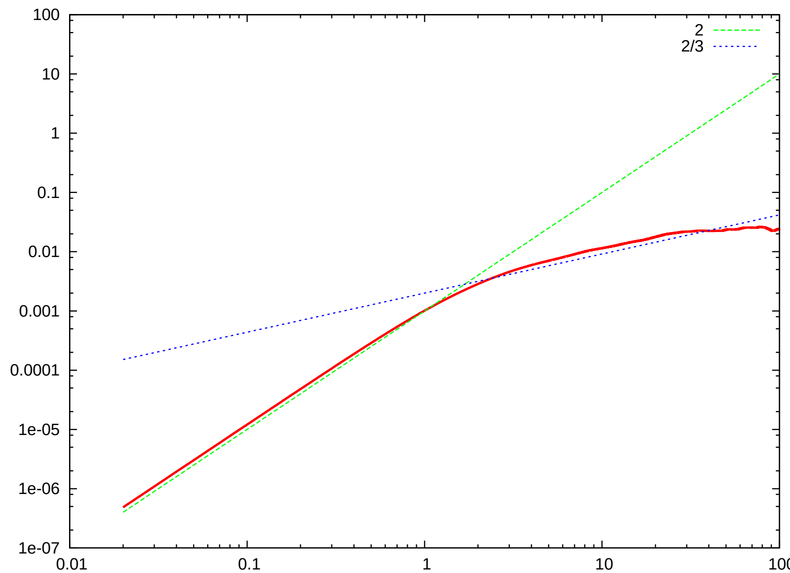


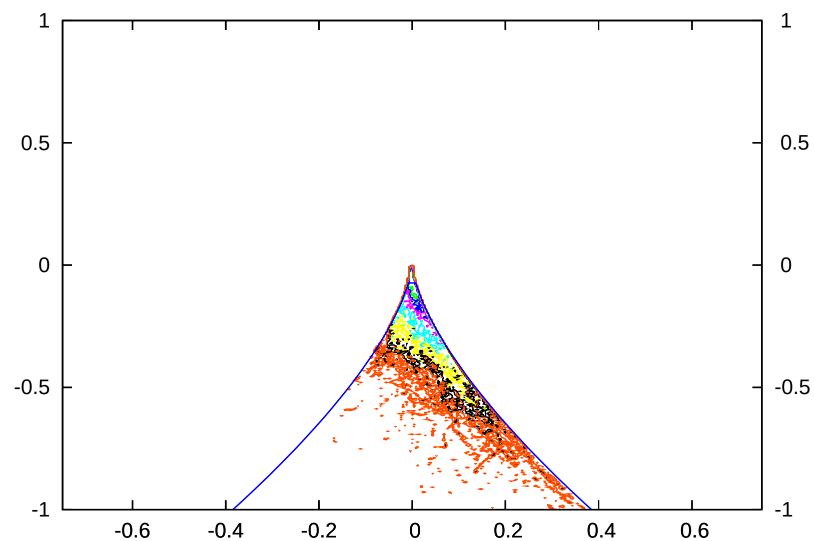
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.840$



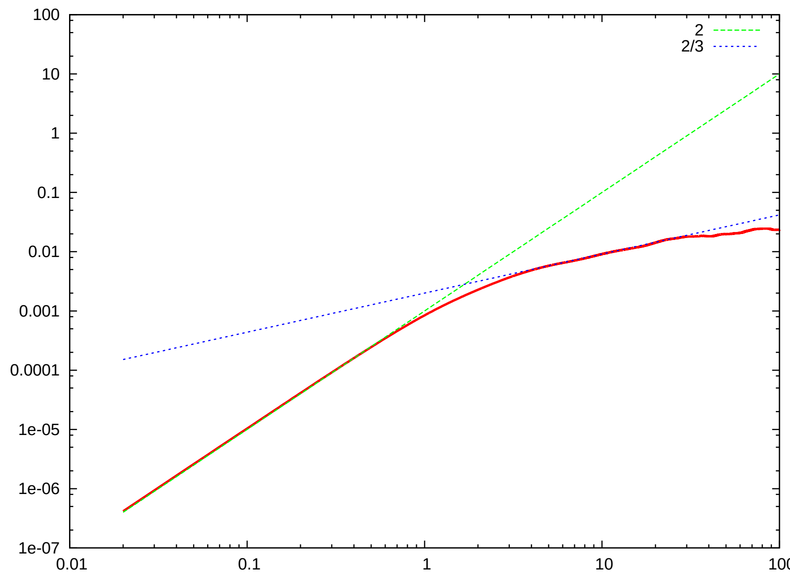


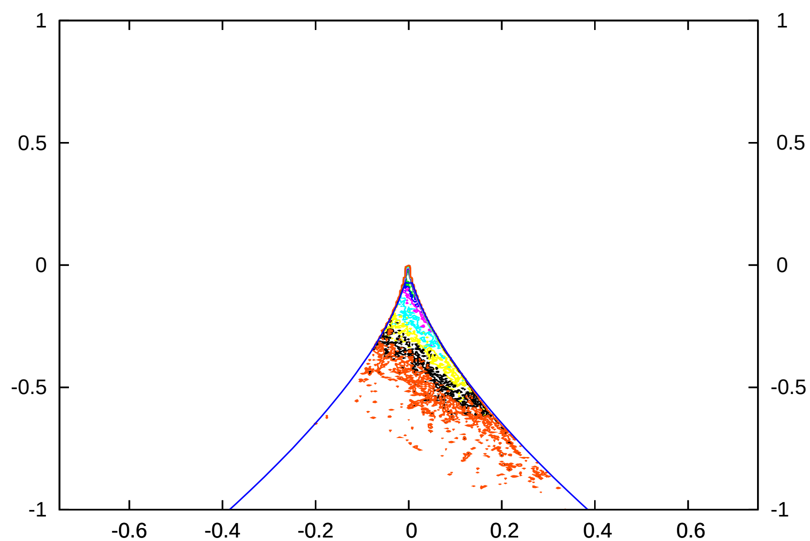
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.893$



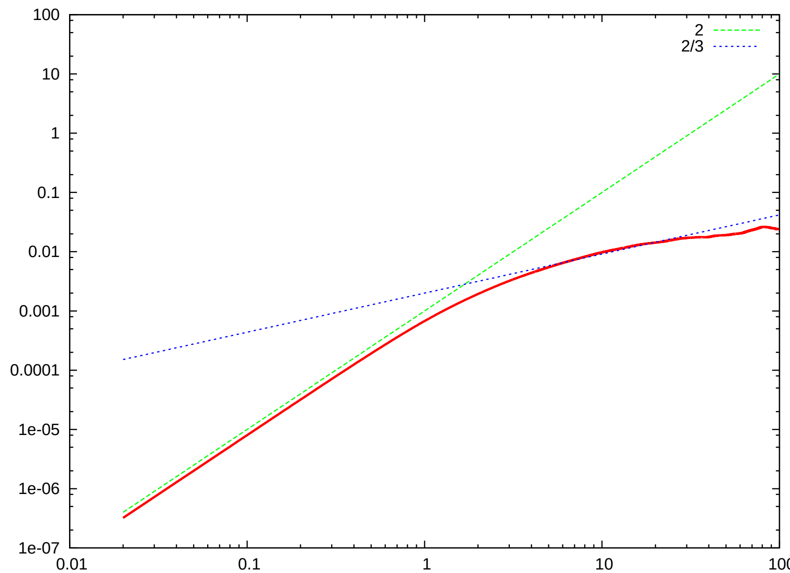


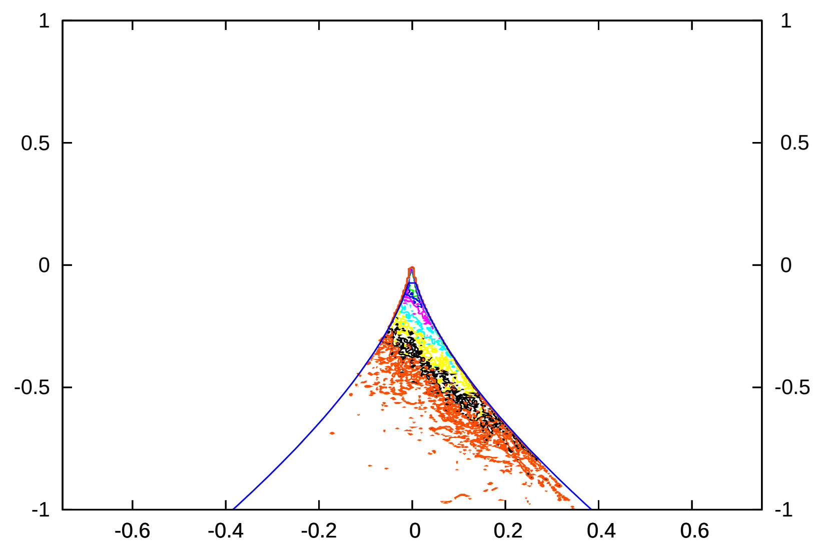
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.945$



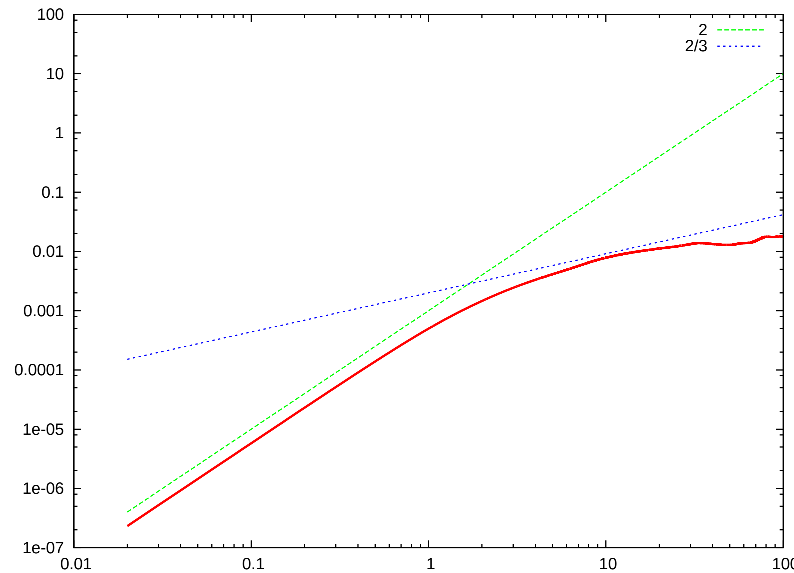


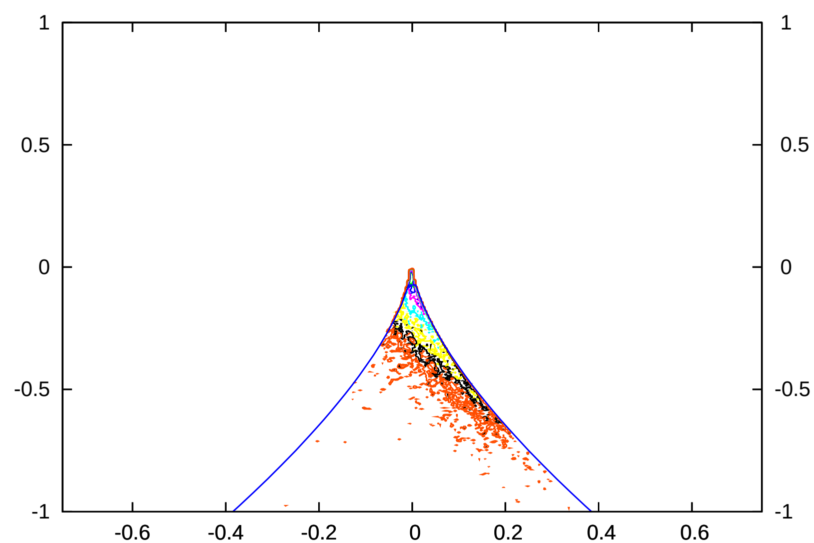
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.998$



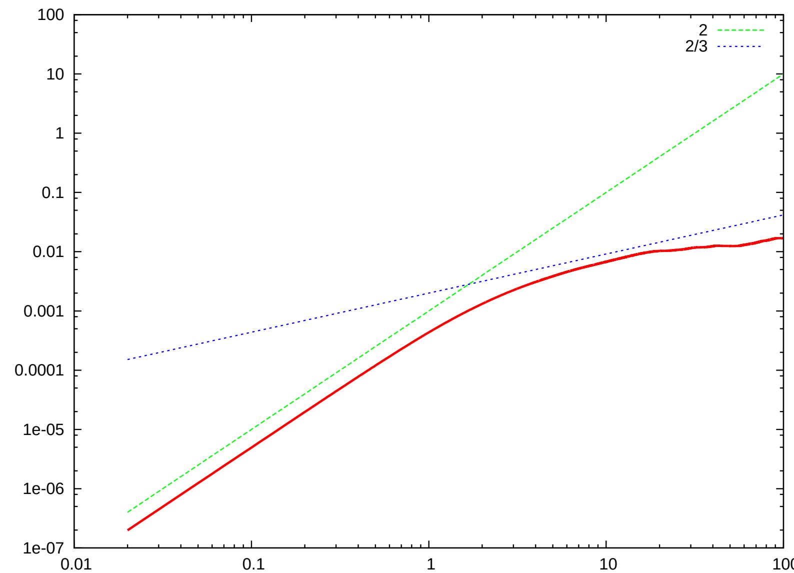


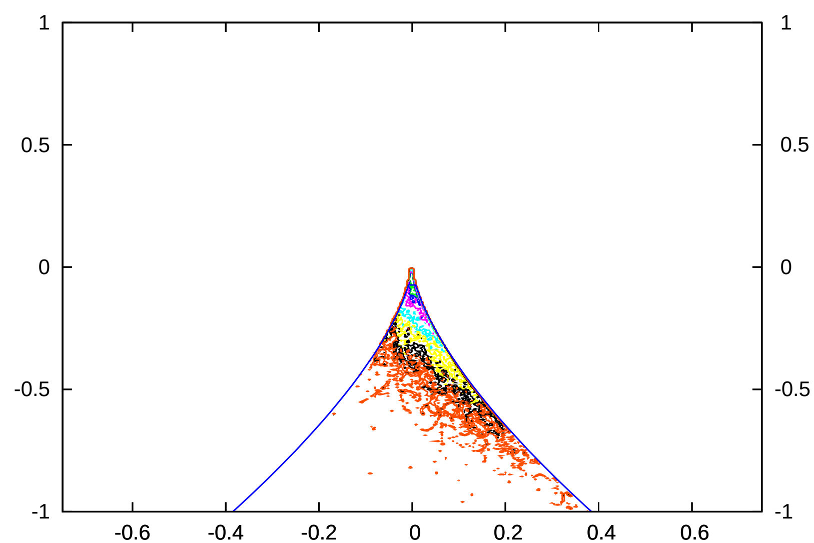
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 1.050$



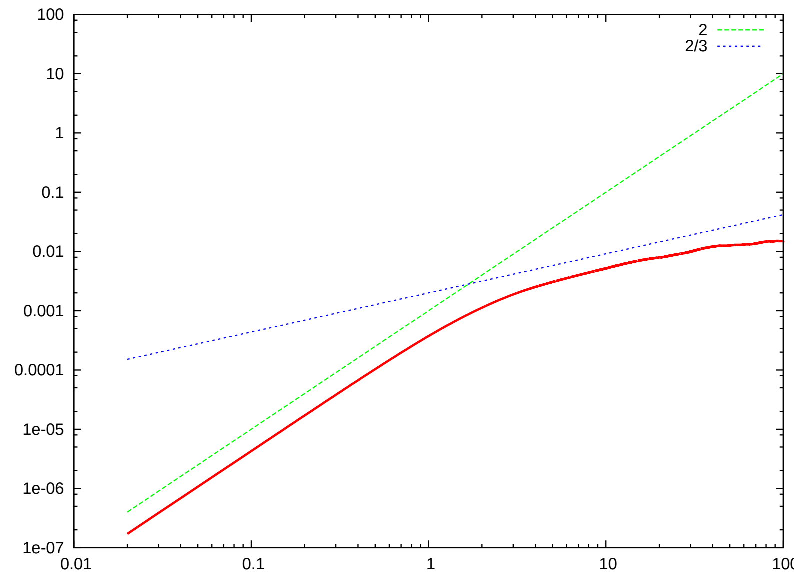


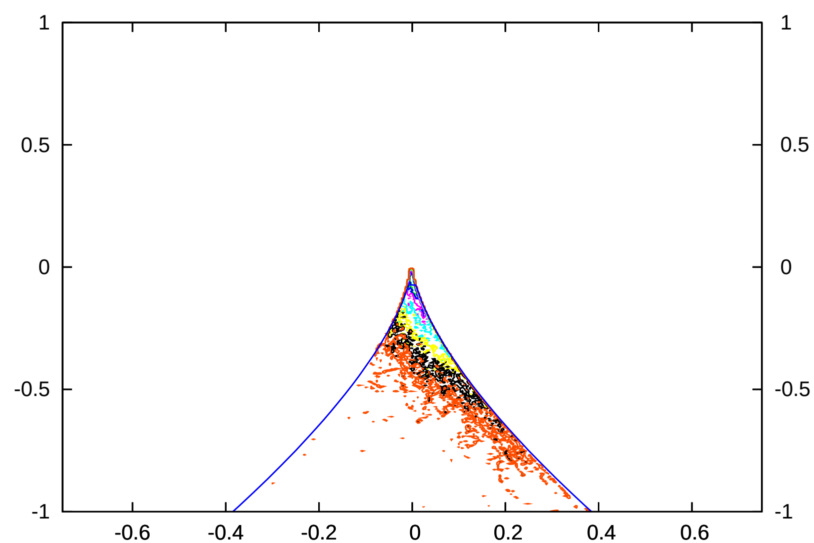
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 1.103$



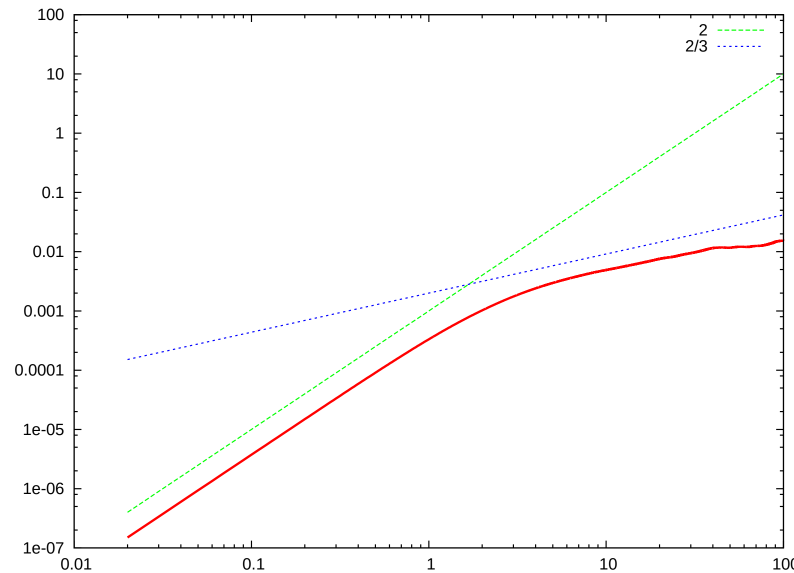


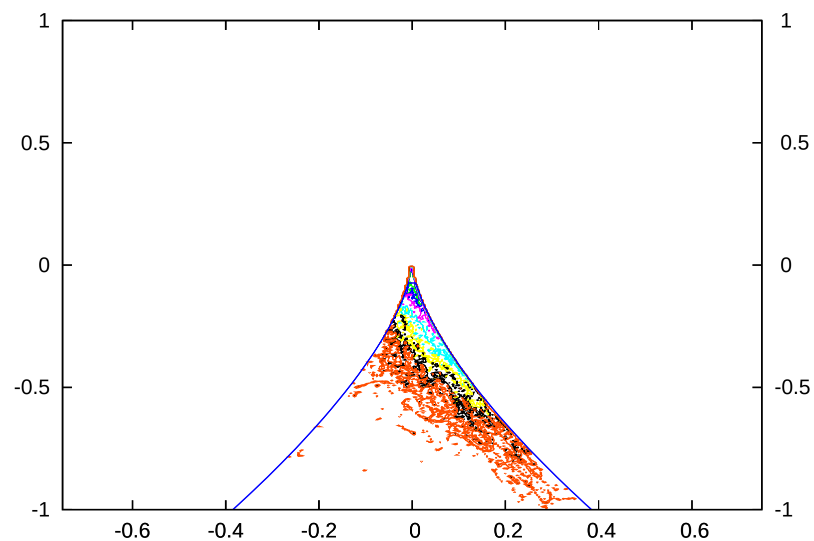
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 1.155$



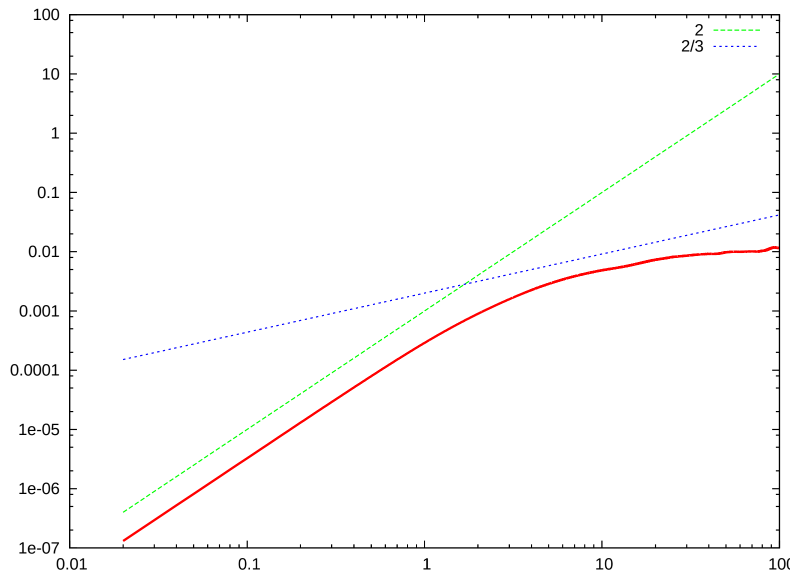


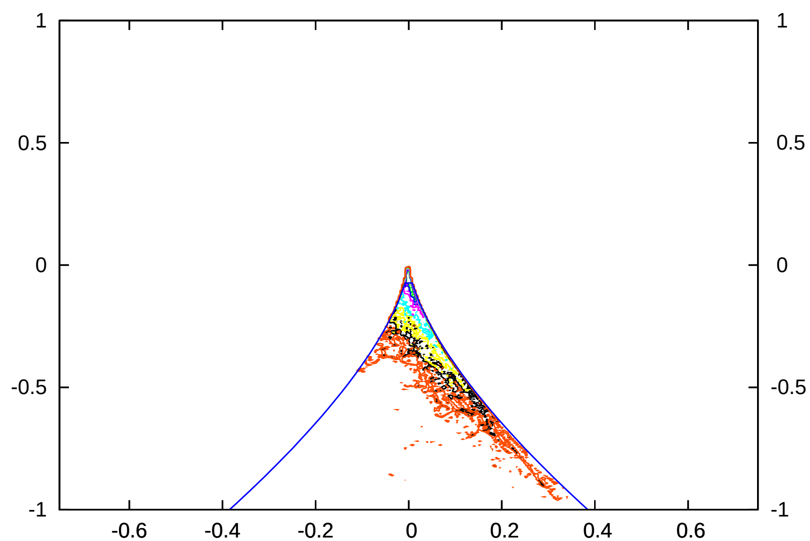
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 1.208$



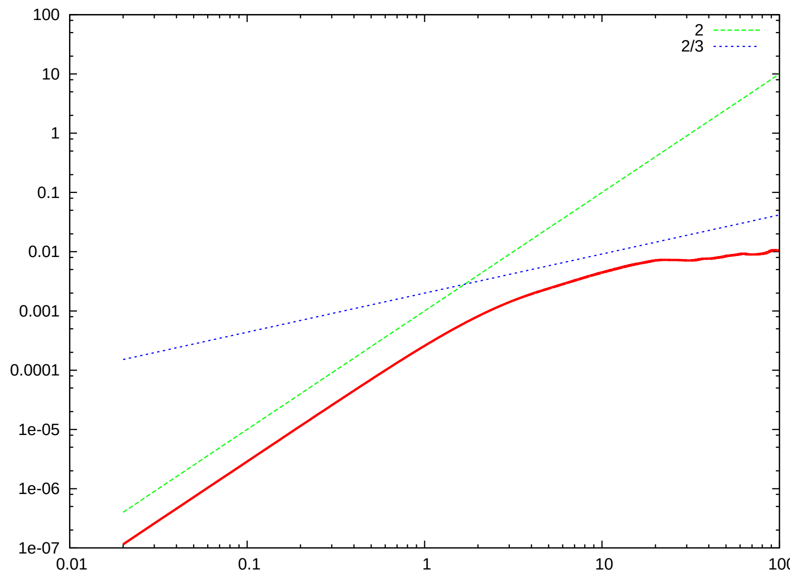


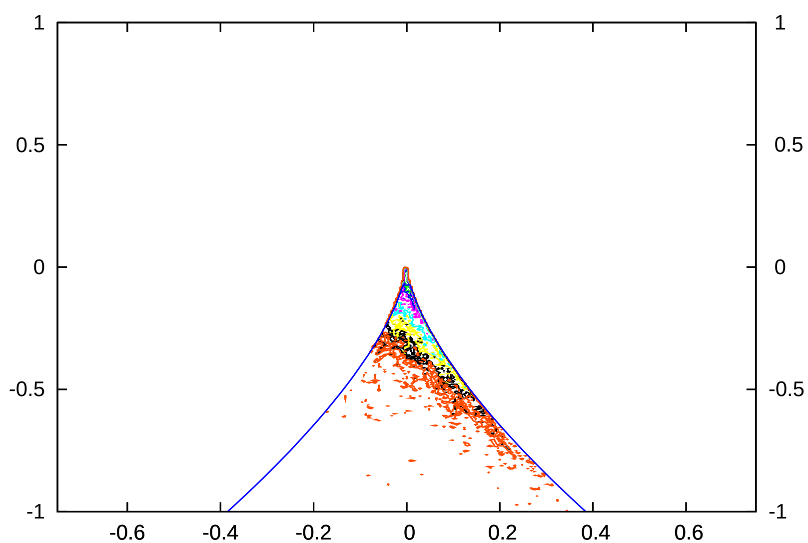
$< \delta u^2(\tau) >$ **and** $Q_s - R_s$ **at** $x/x_* = 1.261$



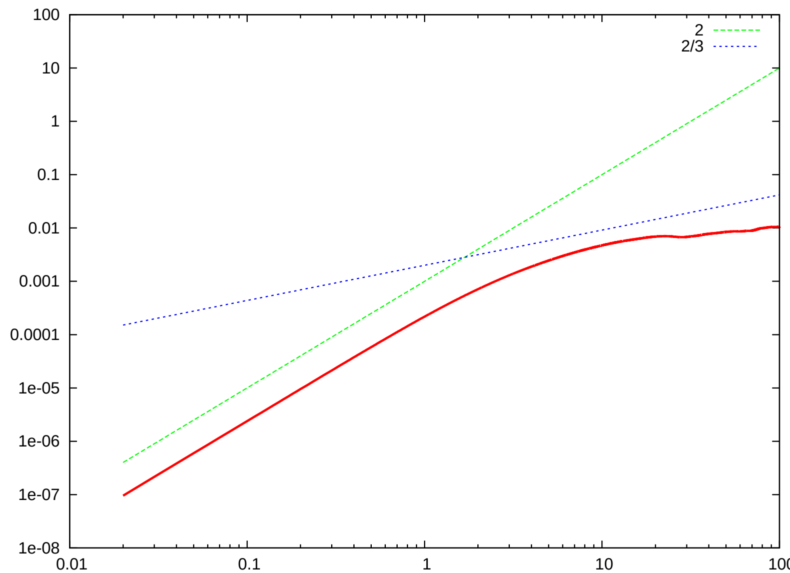


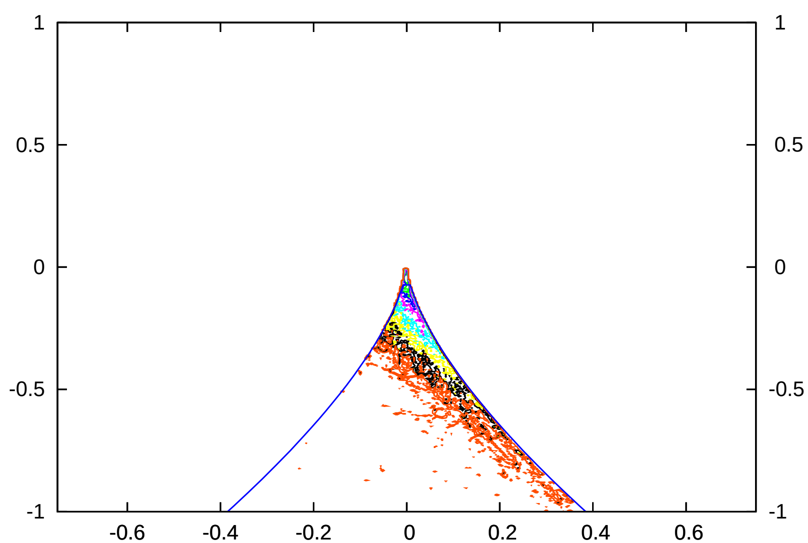
$<\delta u^2(\tau)>$ and $Q_s - R_s$ at $x/x_* = 1.313$



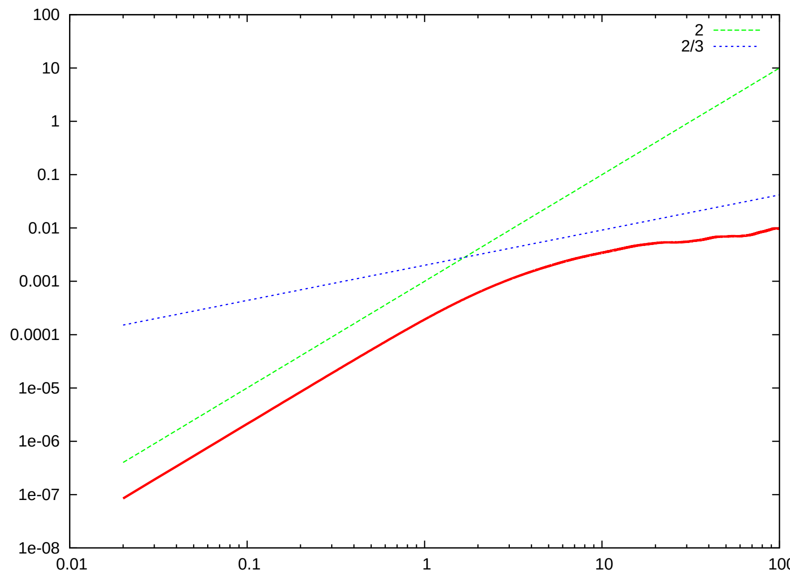


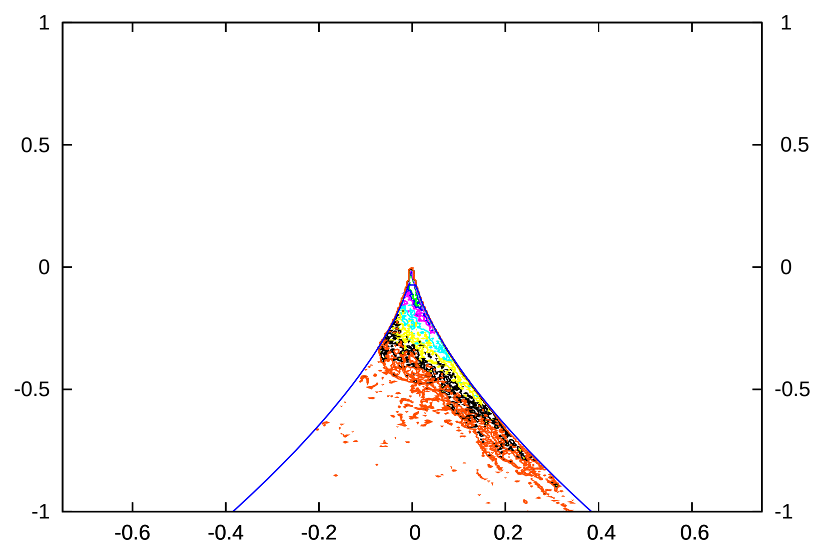
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 1.366$



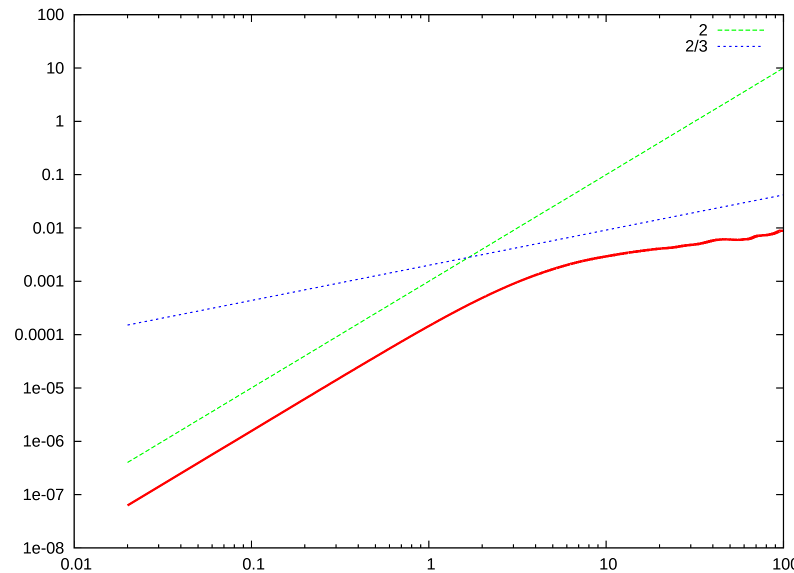


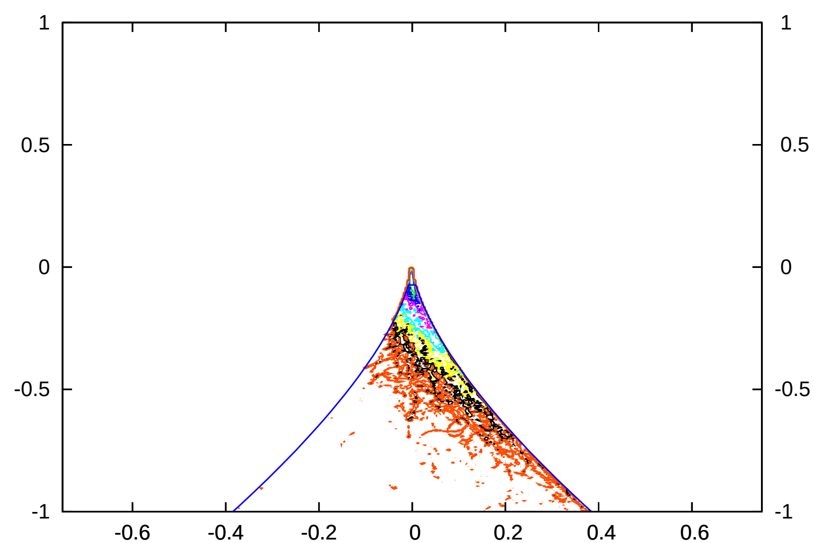
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 1.418$



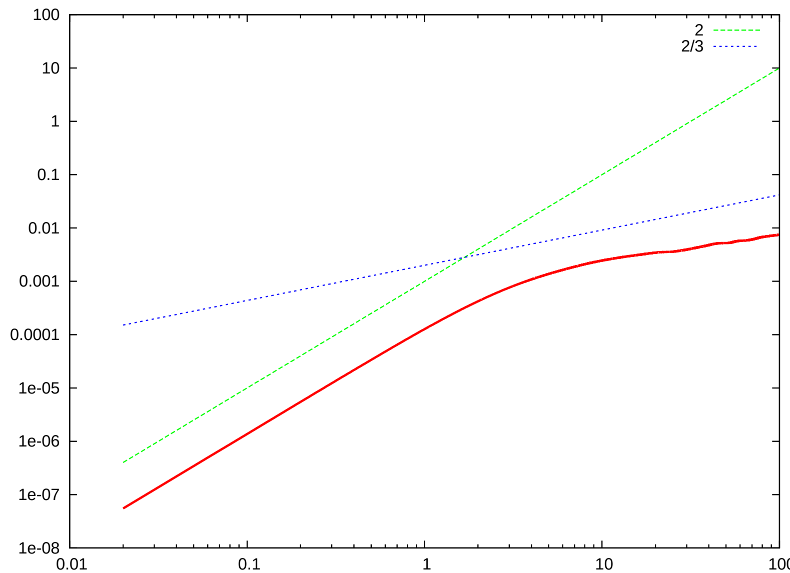


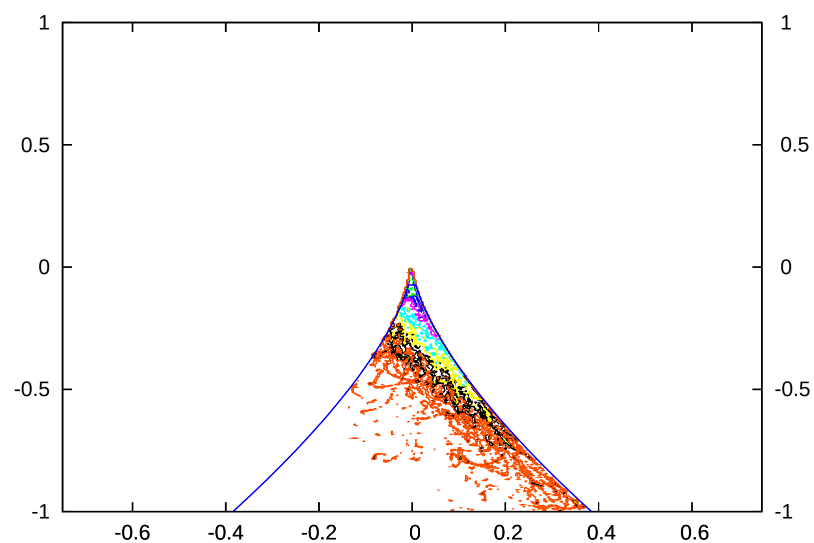
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 1.471$



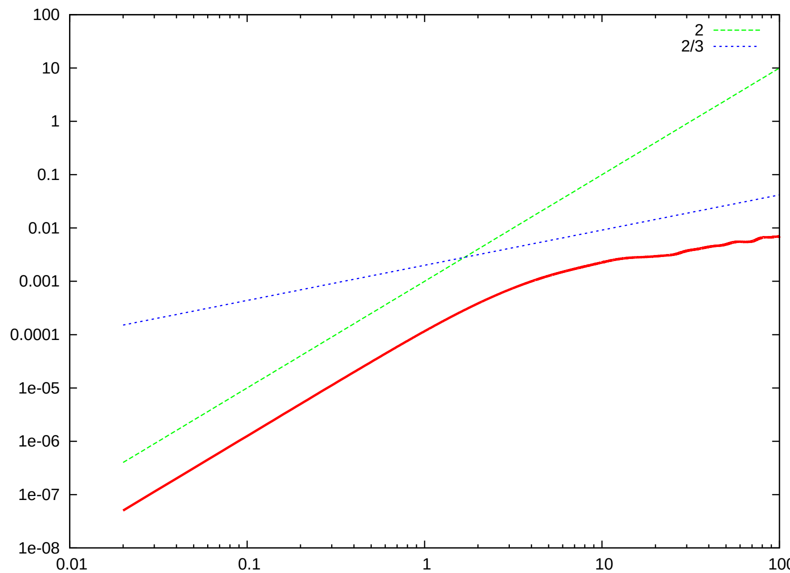


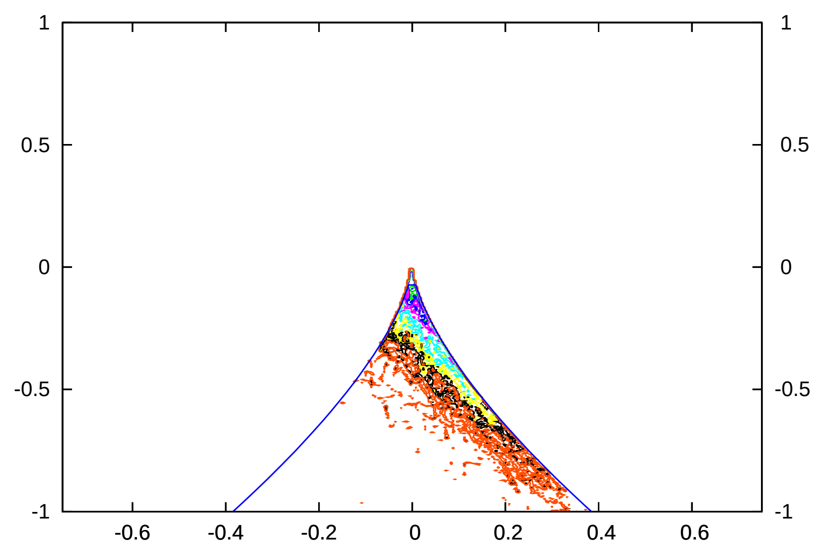
$< \delta u^2(\tau) >$ **and** $Q_s - R_s$ **at** $x/x_* = 1.523$



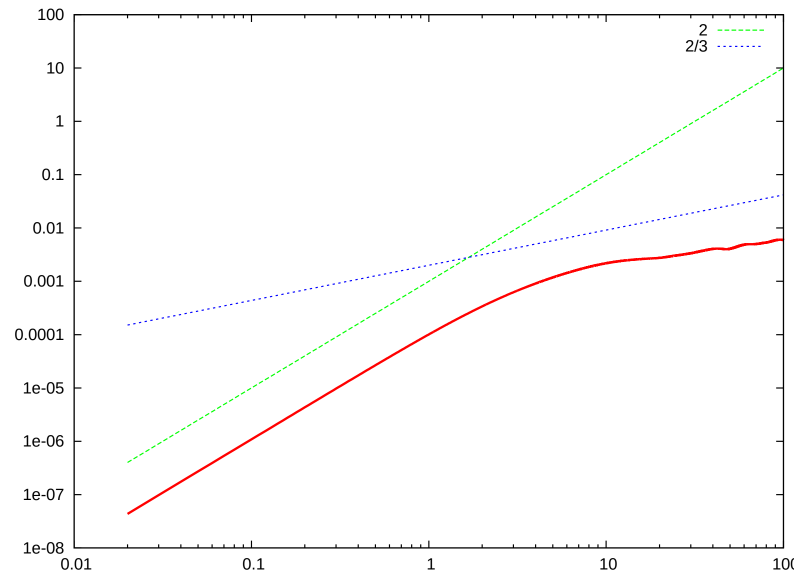


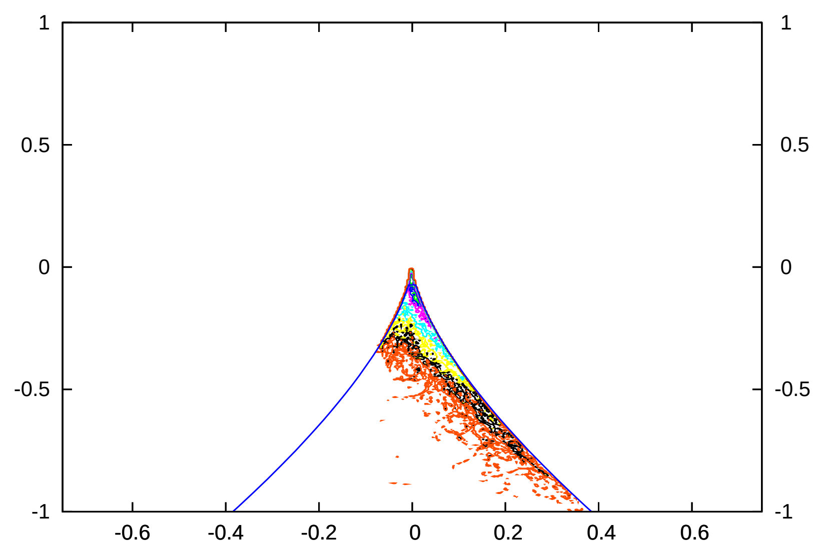
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 1.576$



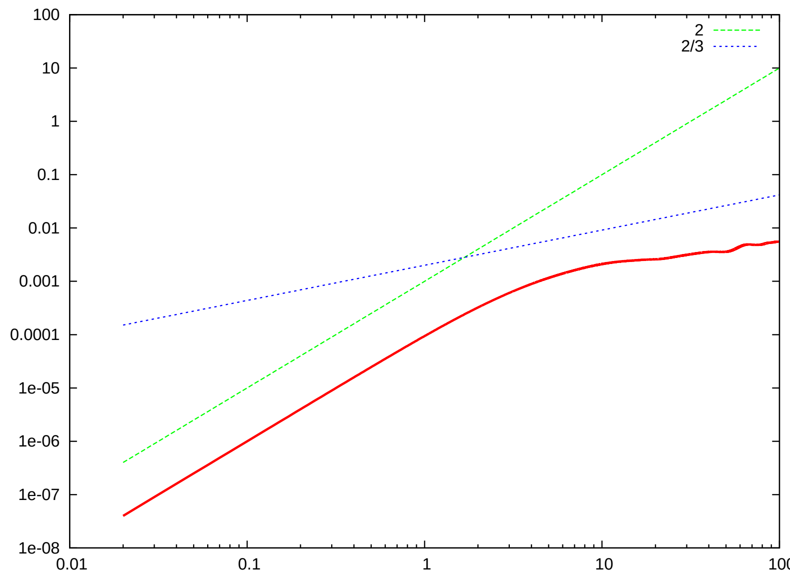


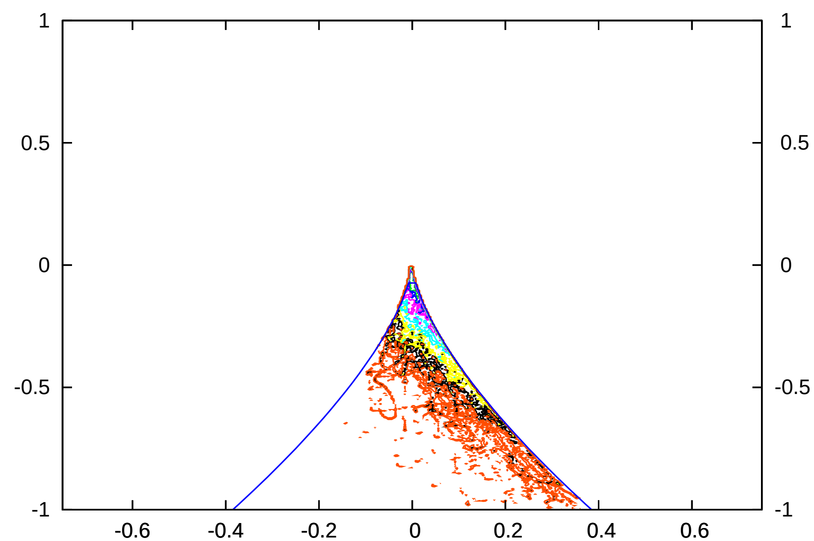
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 1.628$



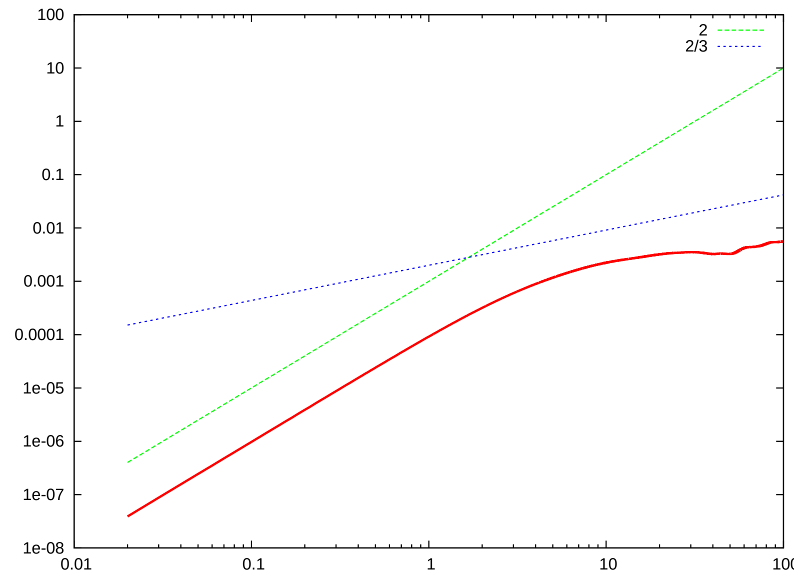


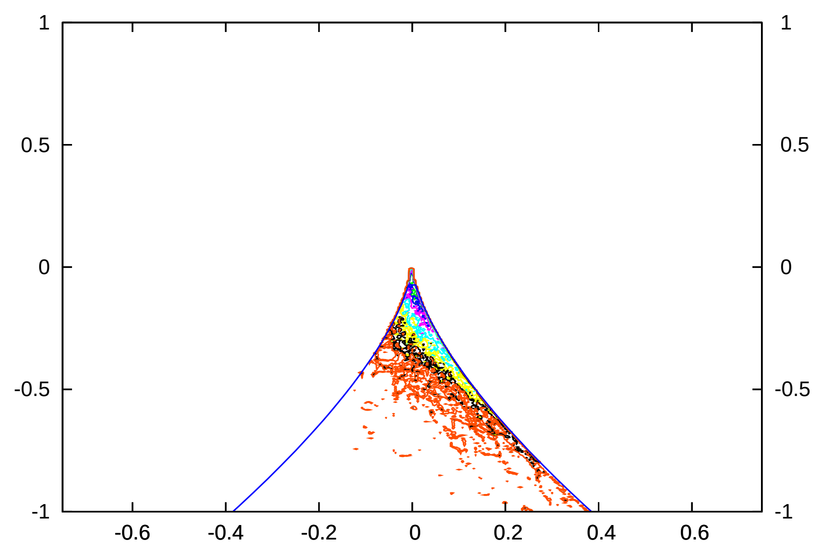
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 1.681$





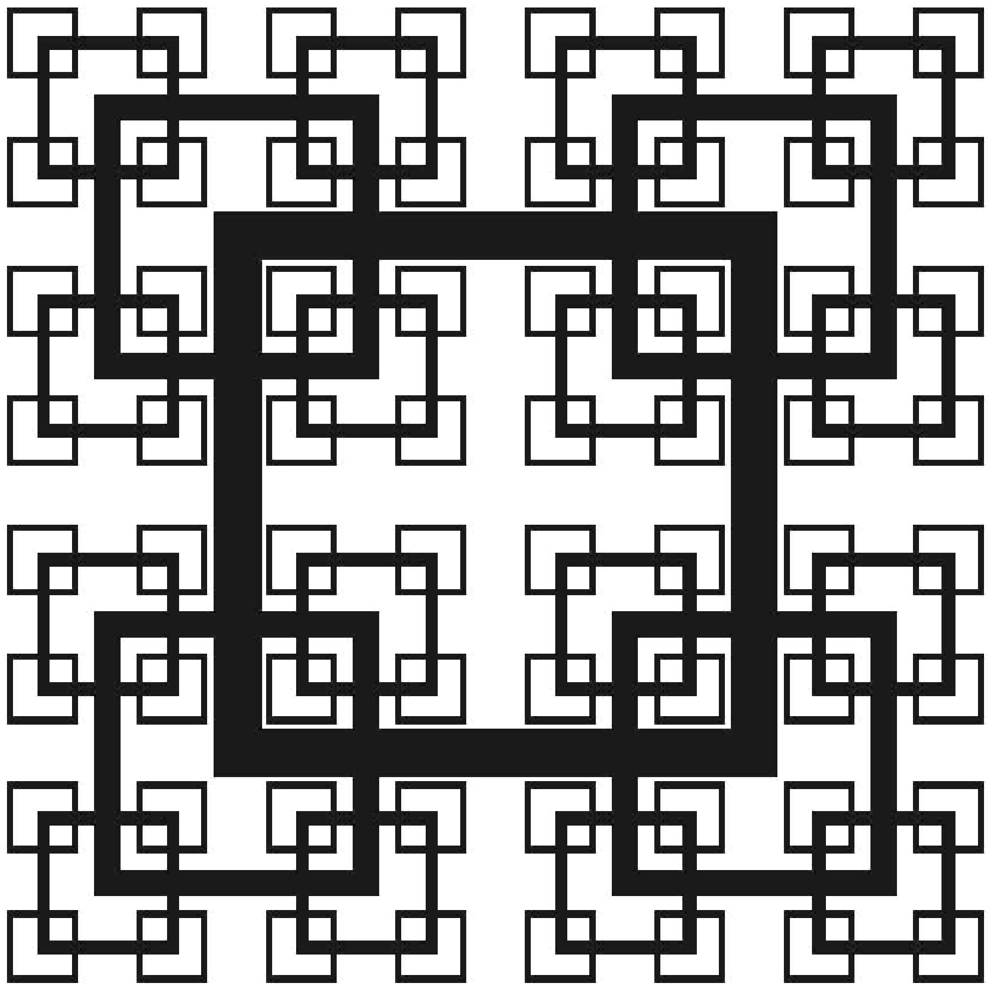
$< \delta u^2(\tau) >$ **and** $Q_s - R_s$ **at** $x/x_* = 1.733$



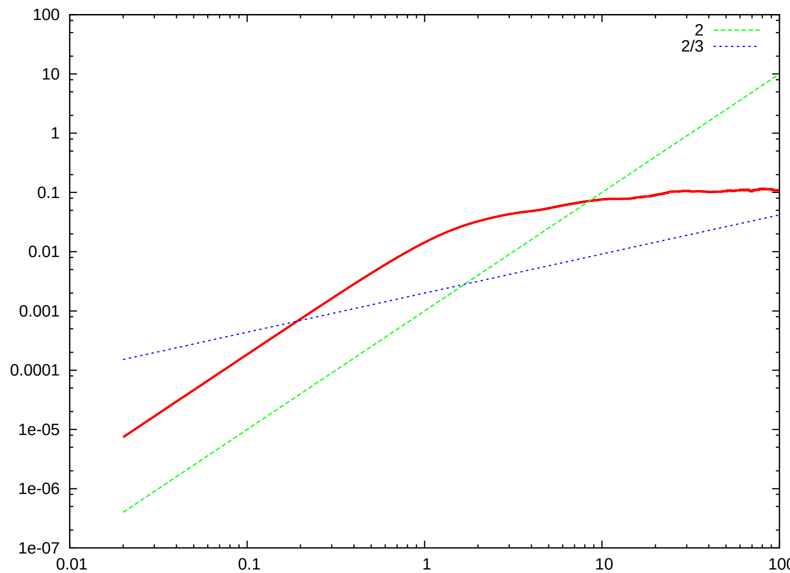


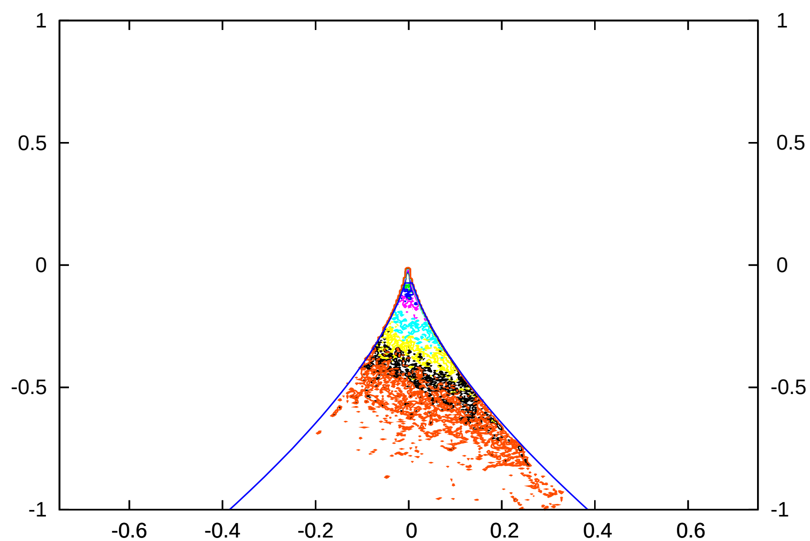
Next inside wake of big bars

Previous plots where taken along the centreline.
Next plots are taken along the line normal to the grid and crossing a biggest bar at the middle.

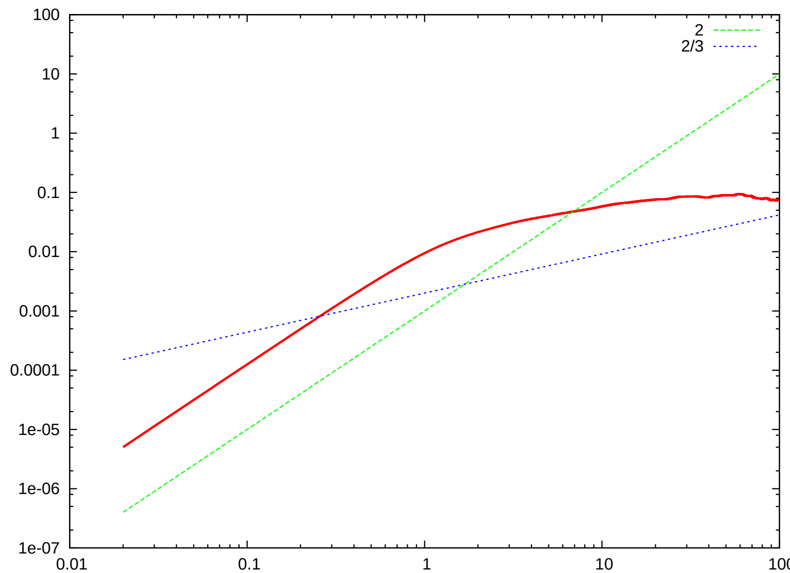


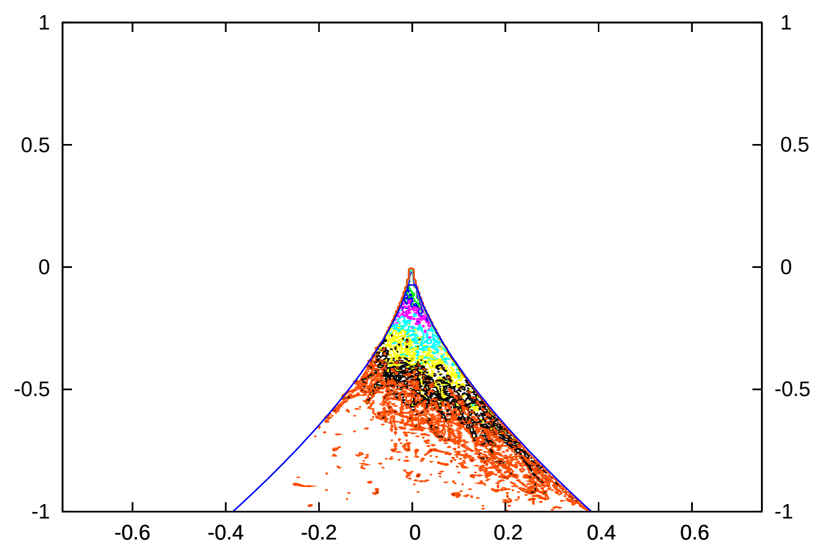
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.052$



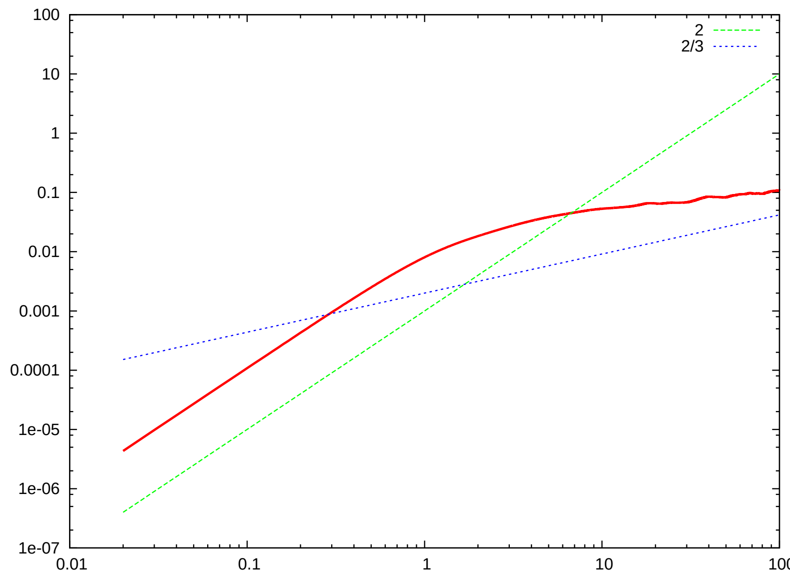


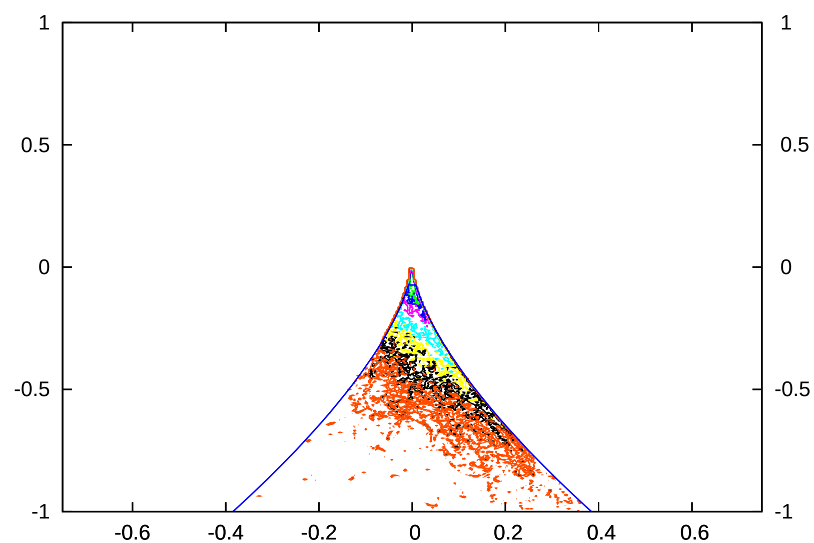
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.105$



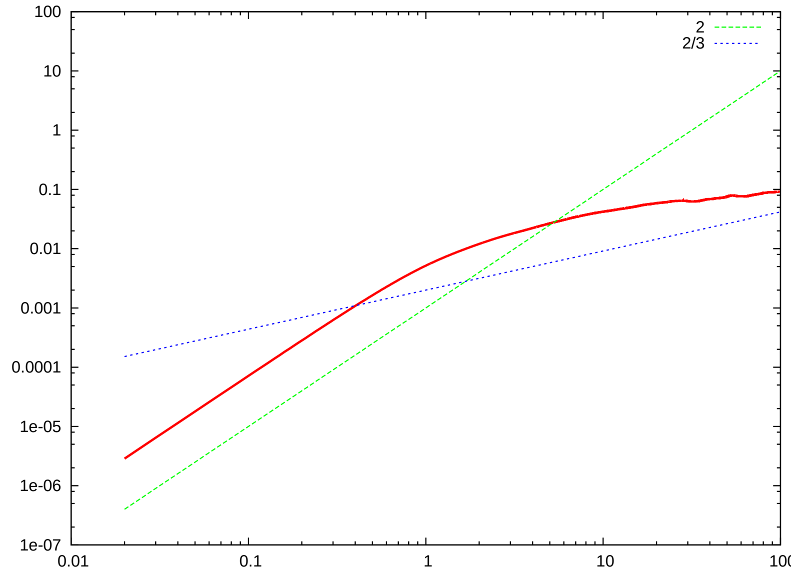


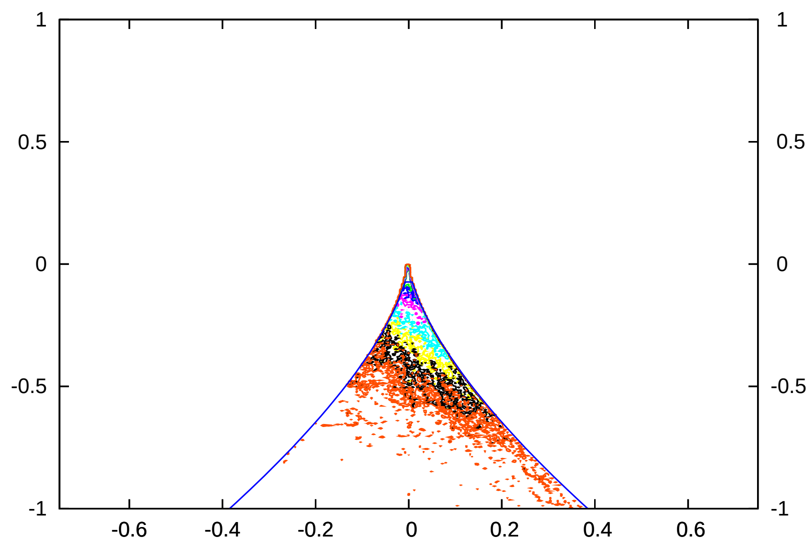
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.157$



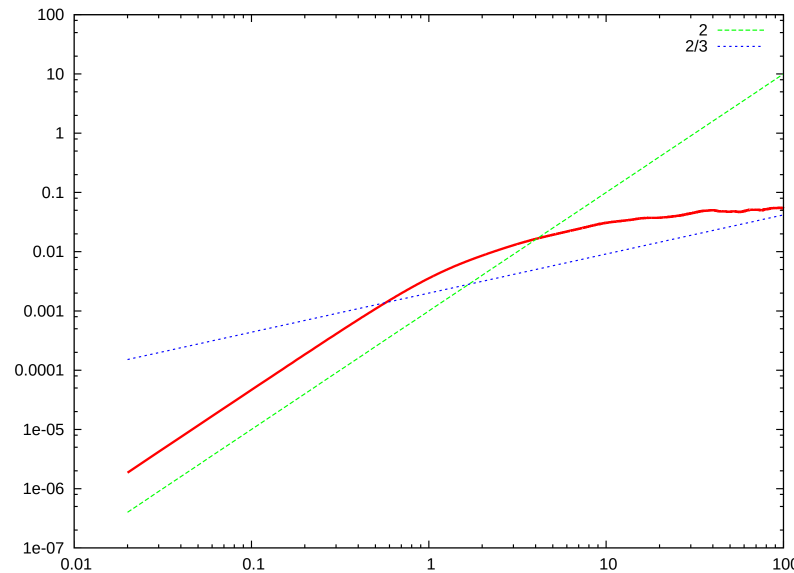


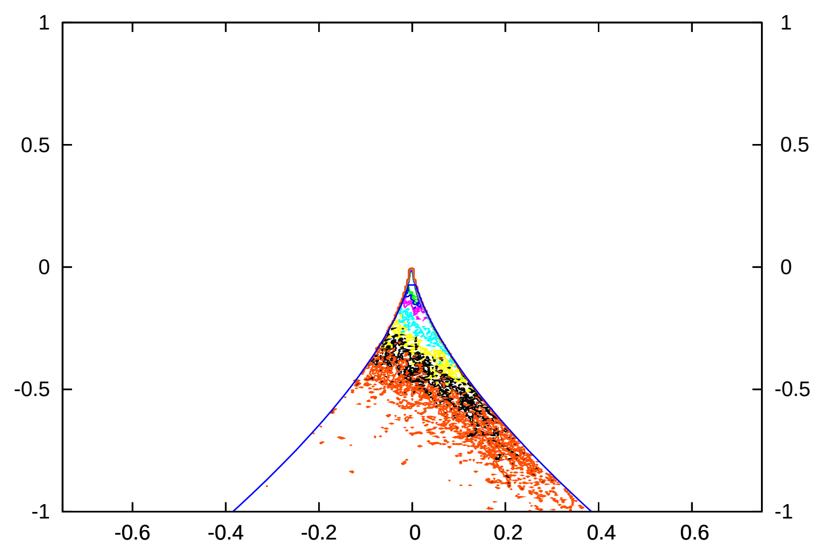
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.210$



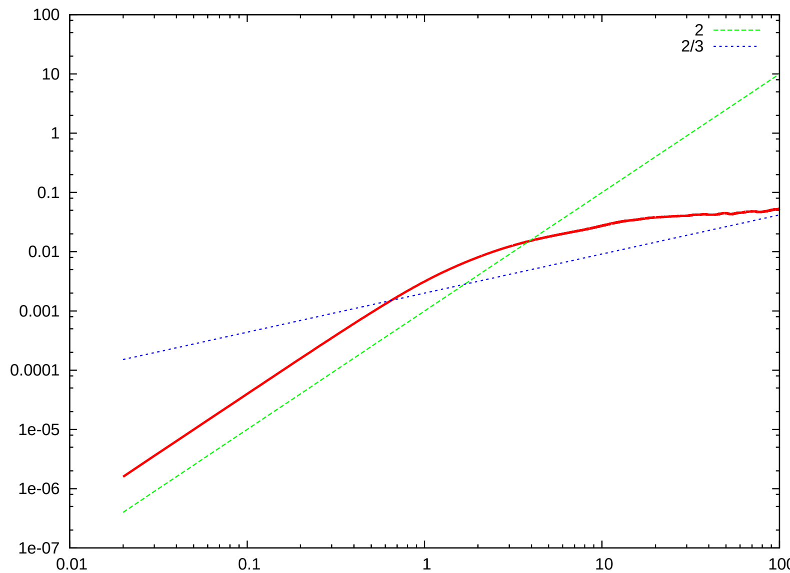


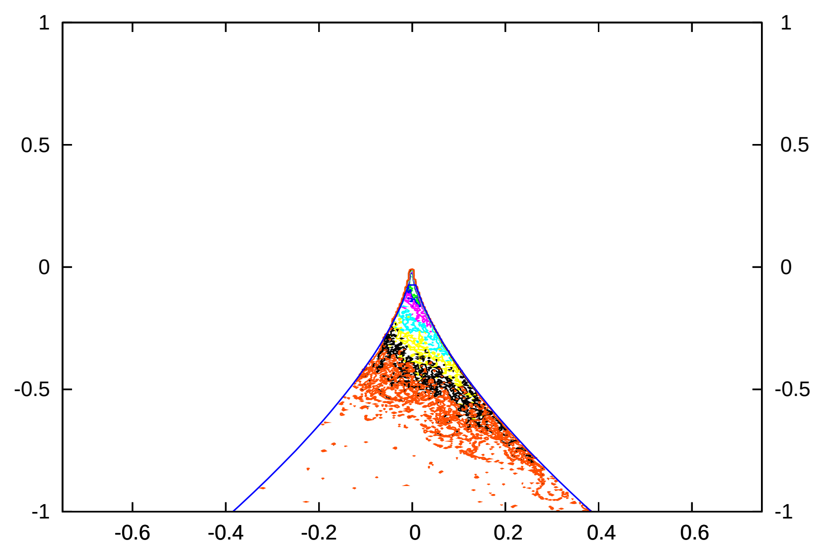
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.262$



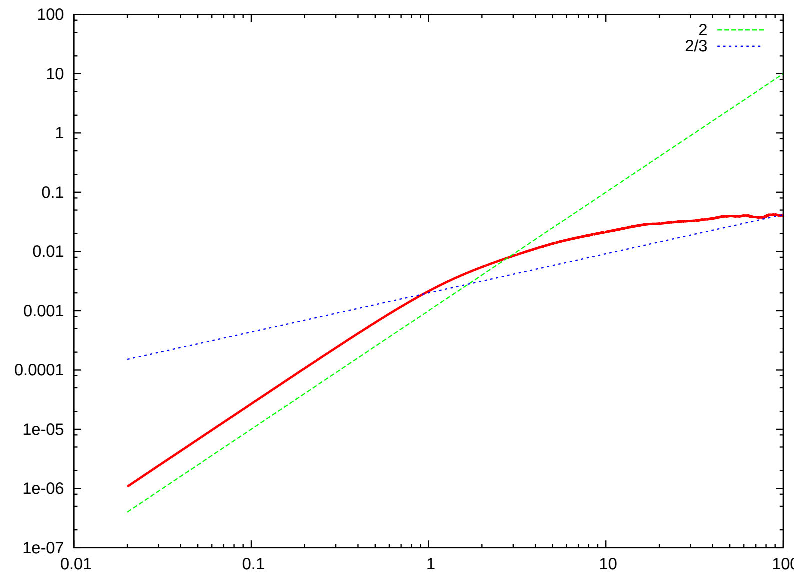


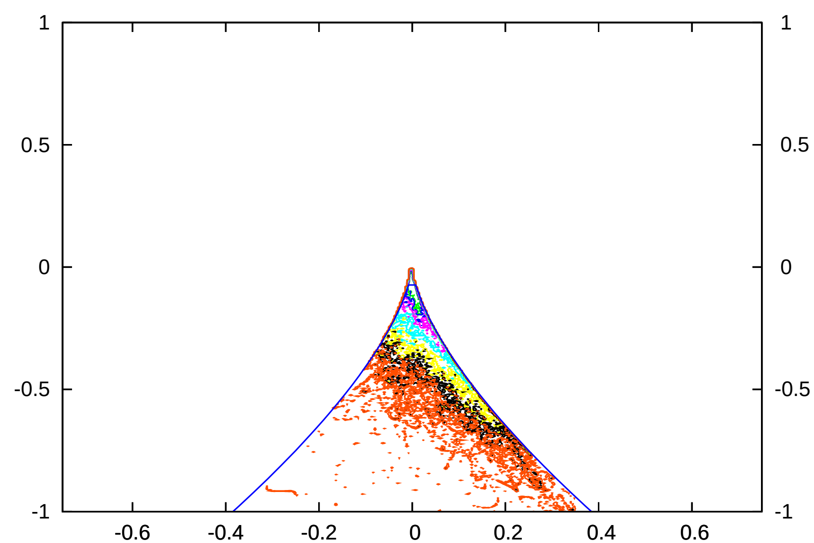
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.315$



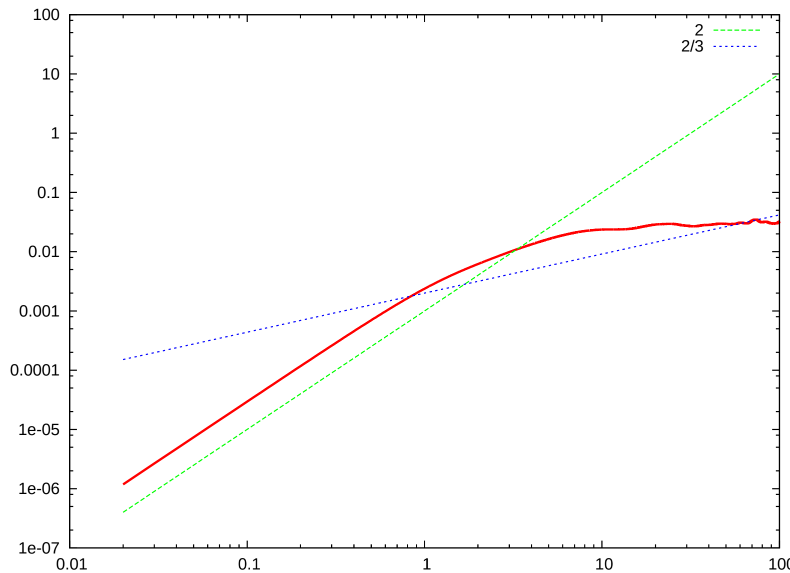


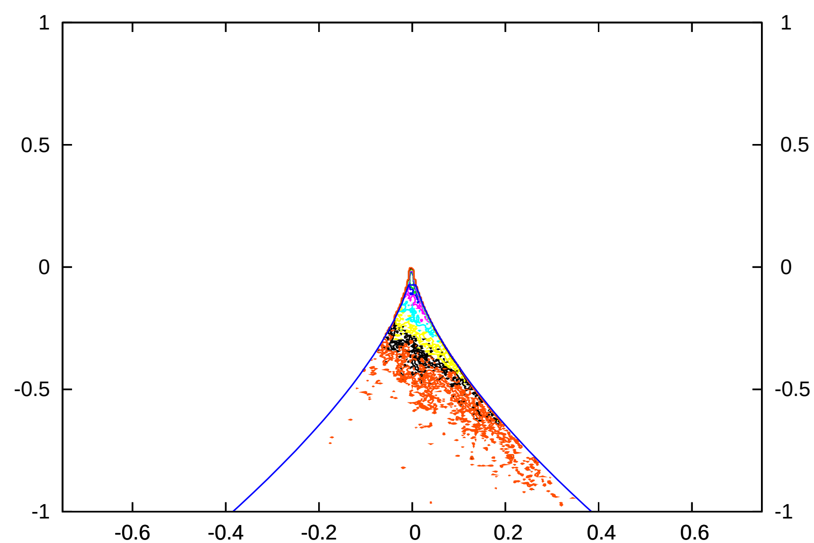
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.367$



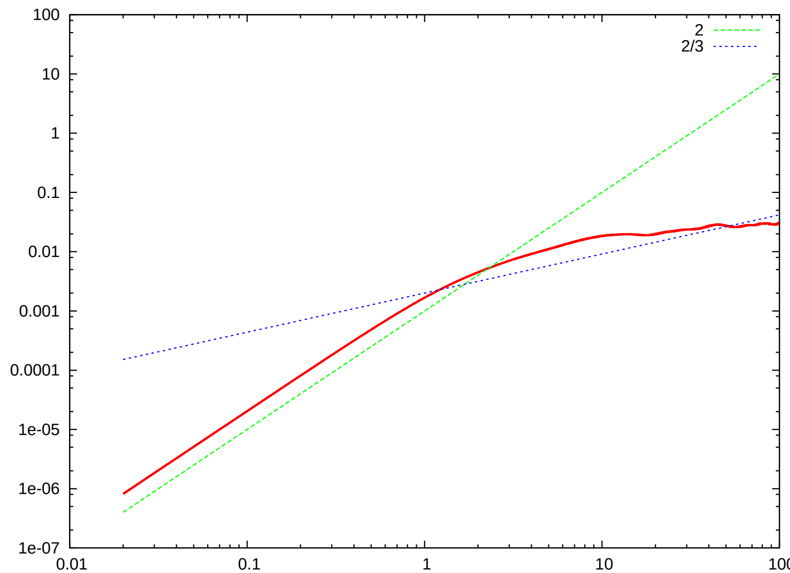


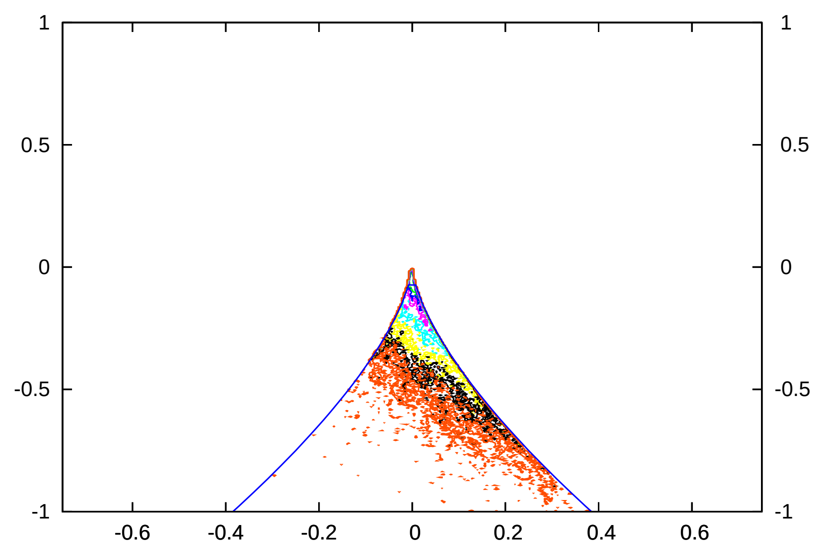
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.420$



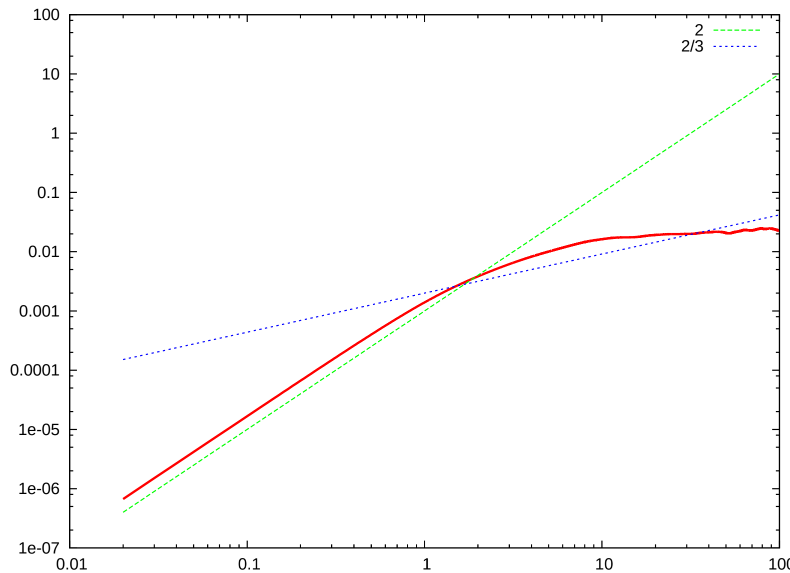


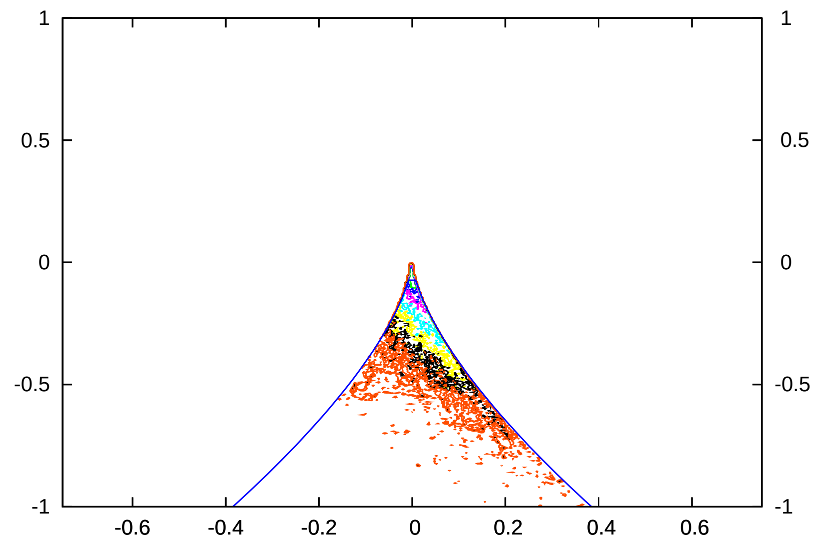
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.472$



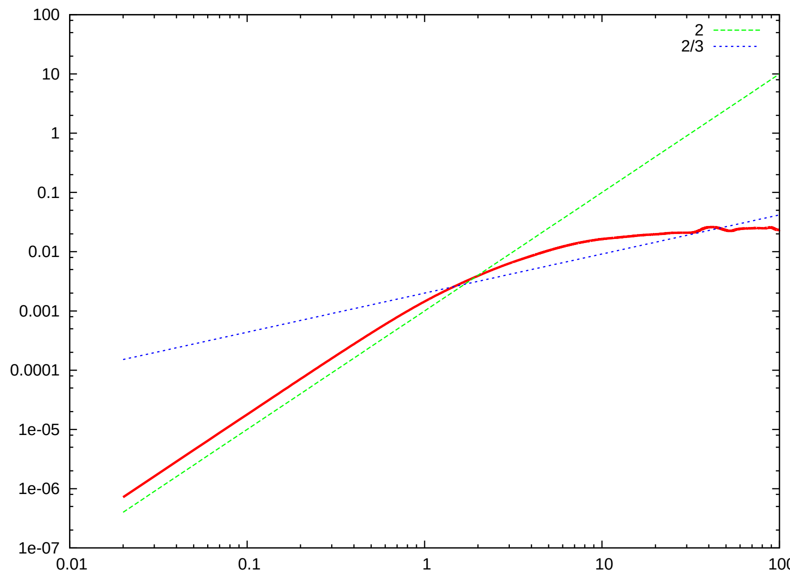


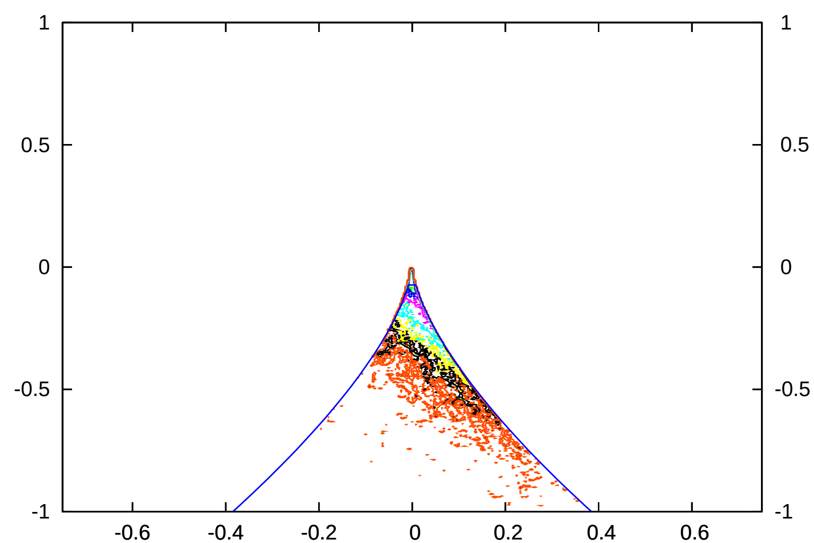
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.525$



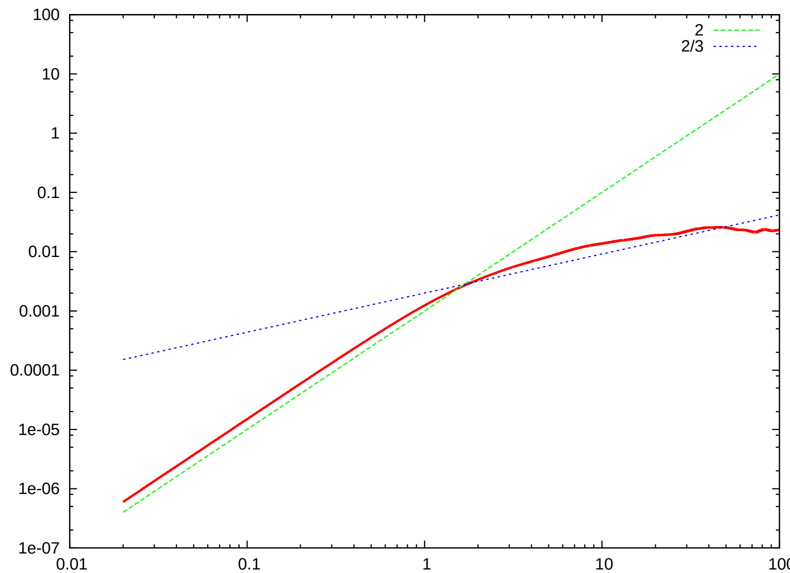


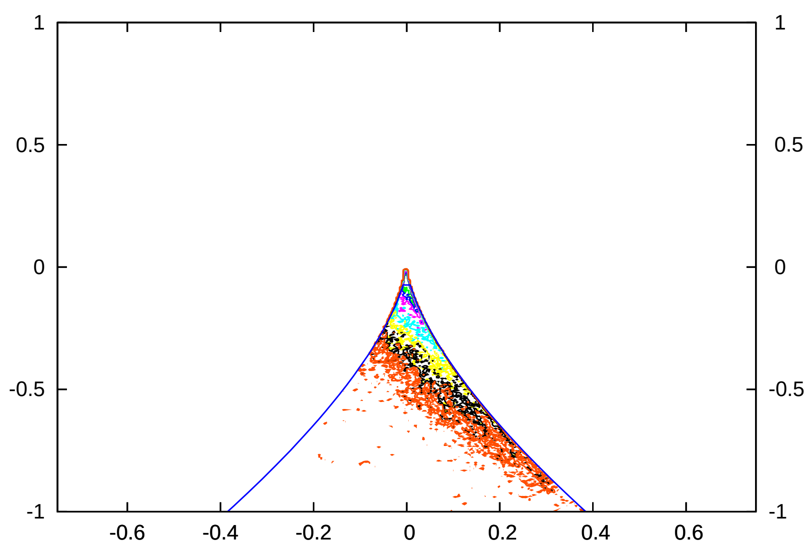
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.577$



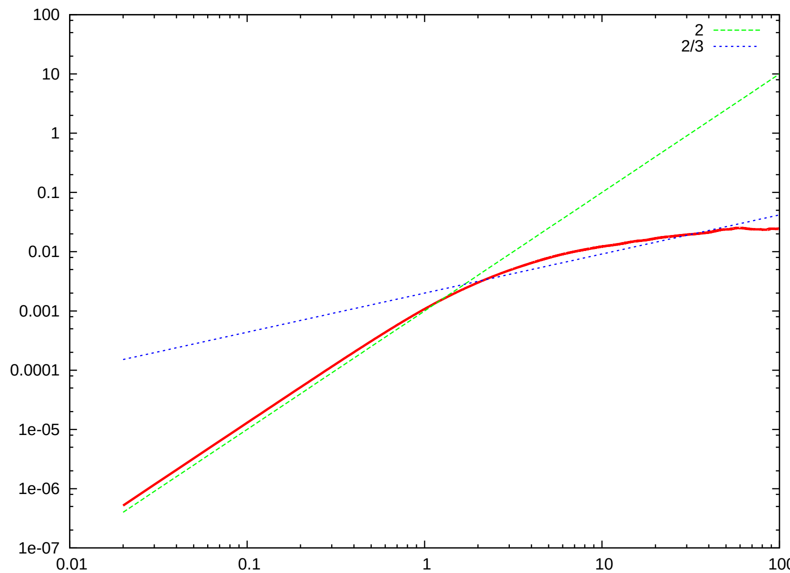


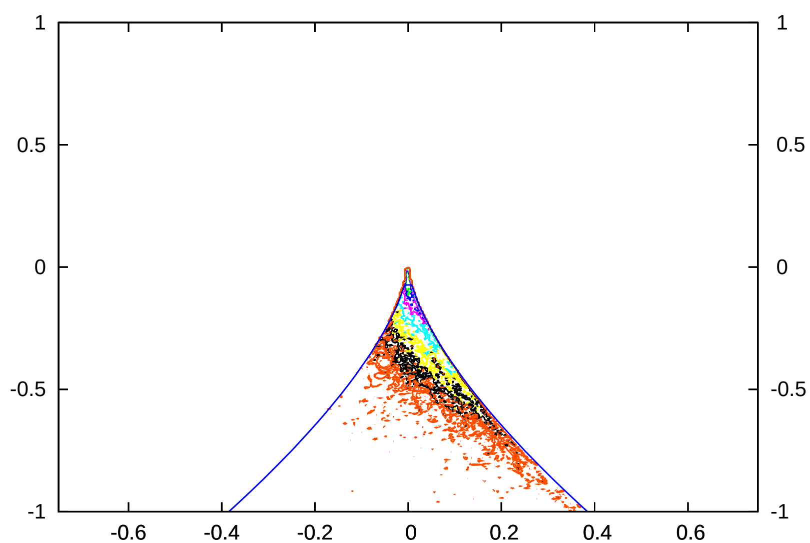
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.630$



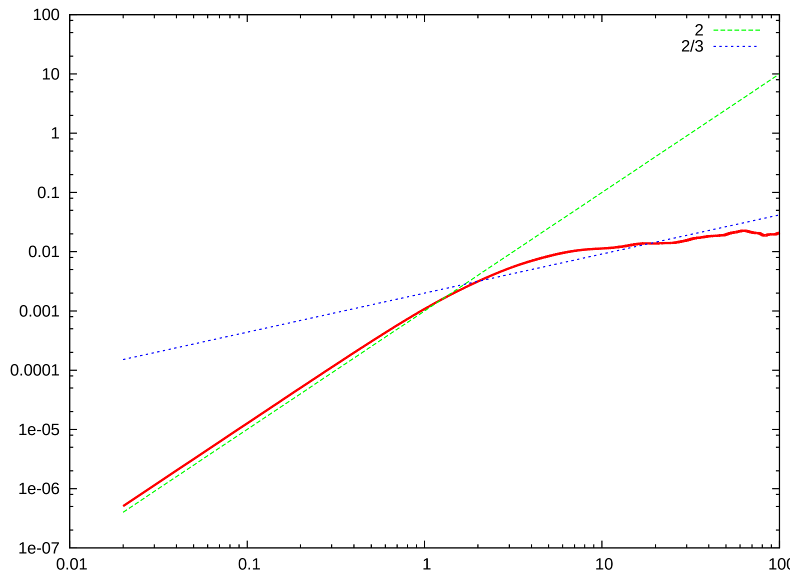


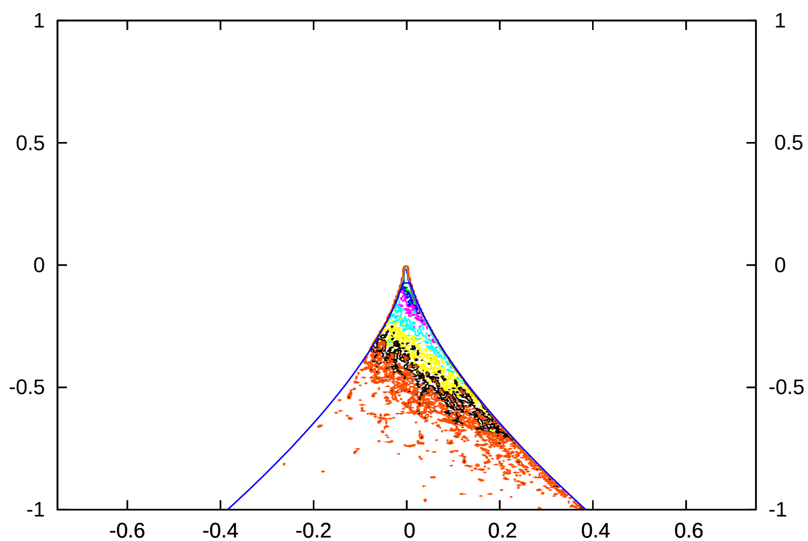
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.683$



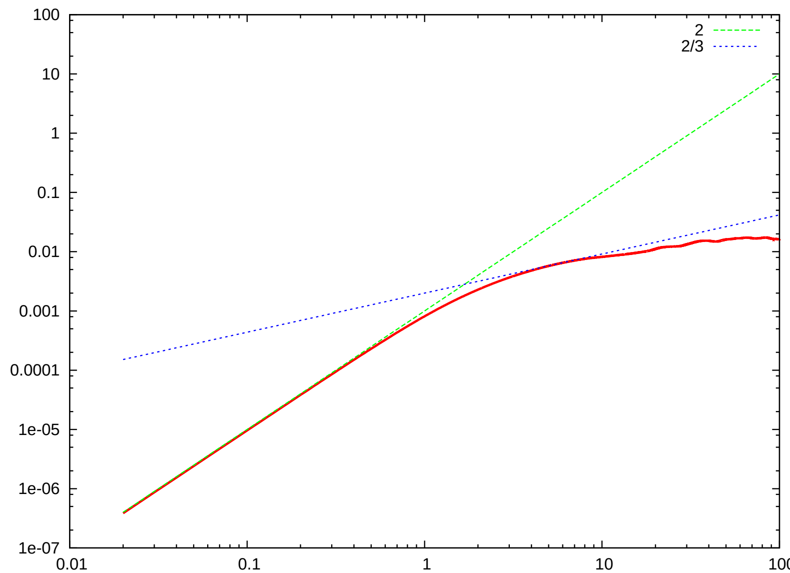


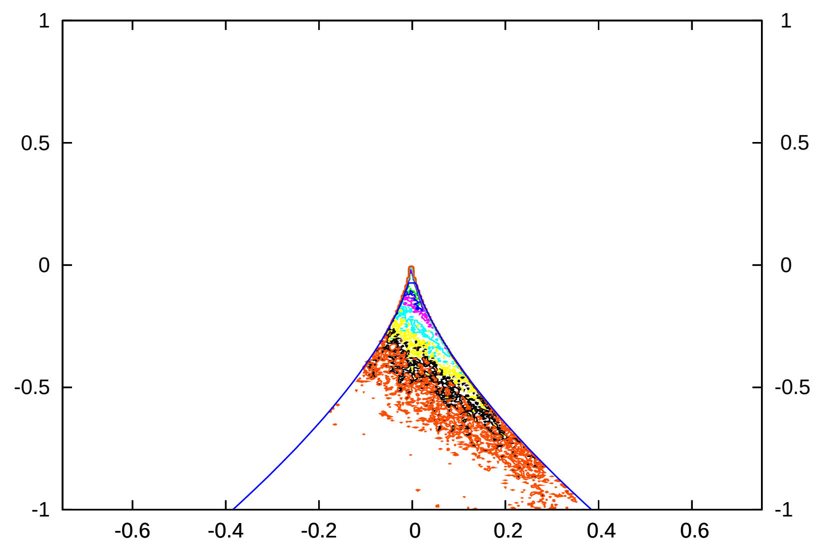
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.735$



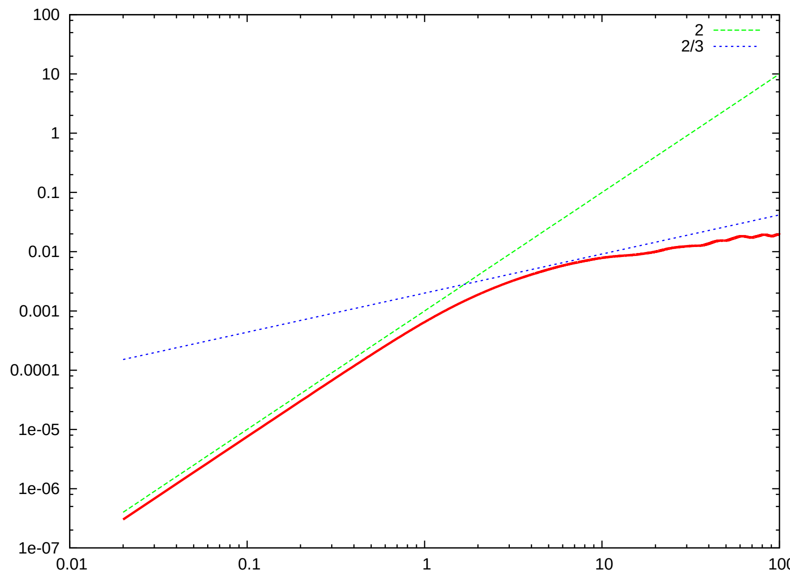


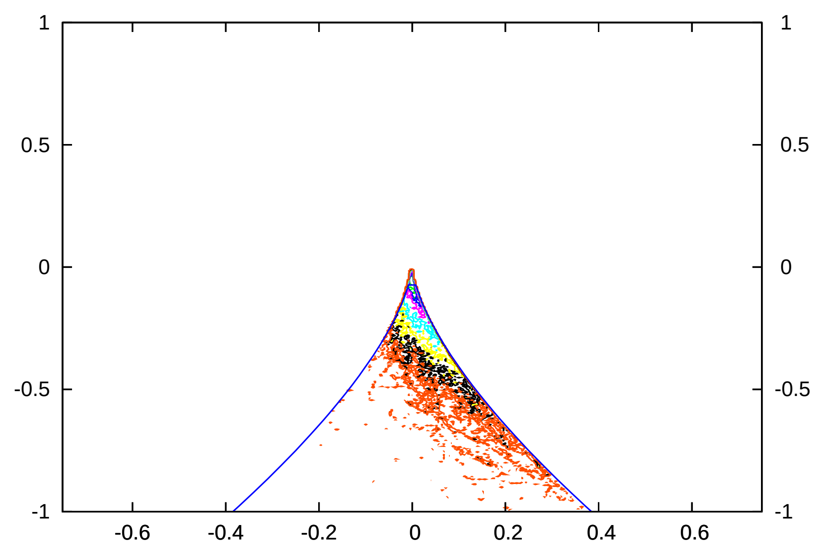
$< \delta u^2(\tau) >$ **and** $Q_s - R_s$ **at** $x/x_* = 0.788$



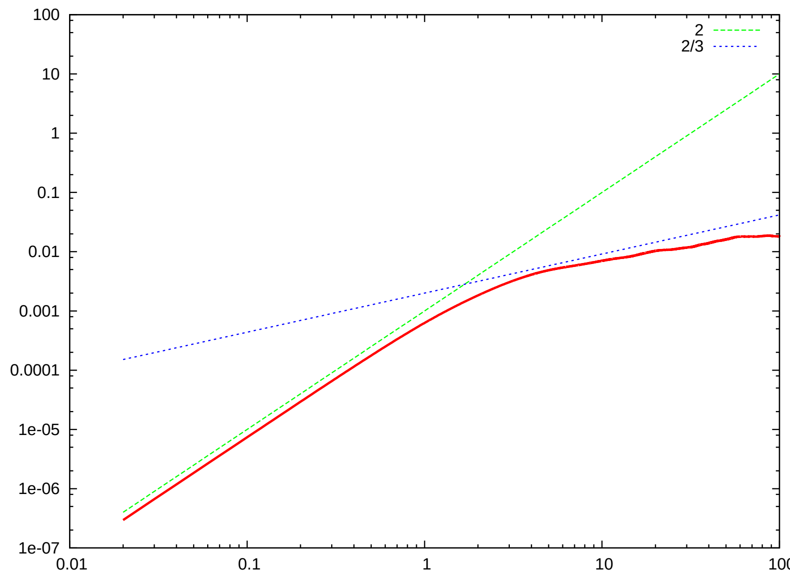


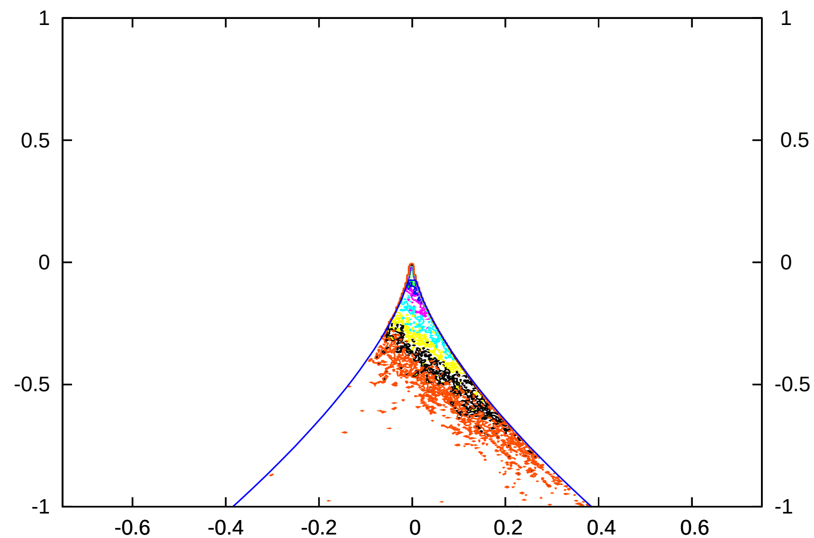
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.840$



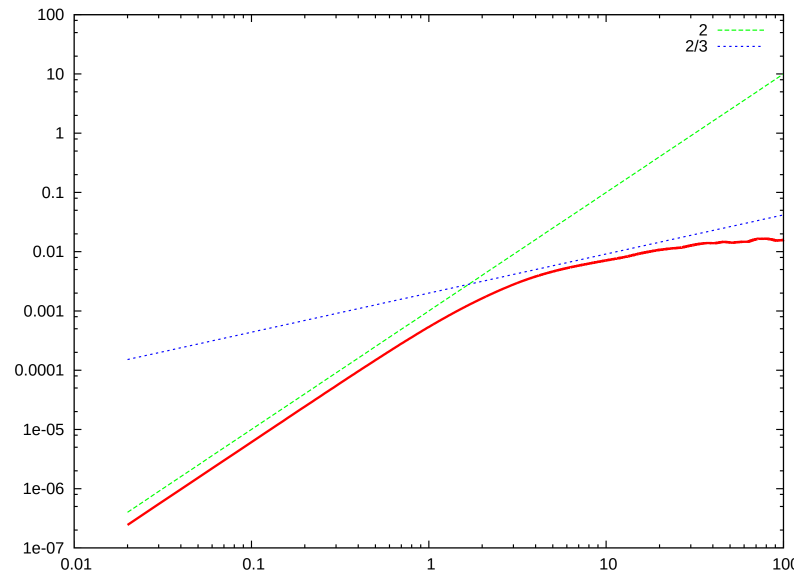


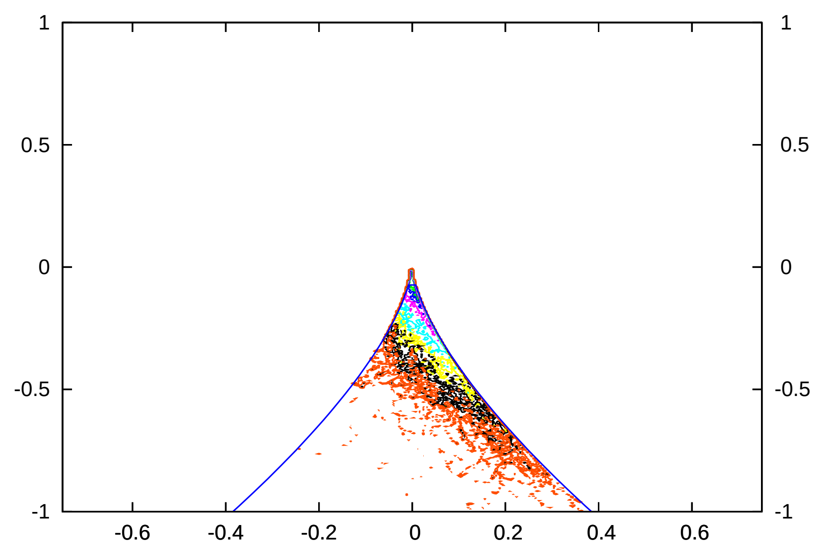
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.893$



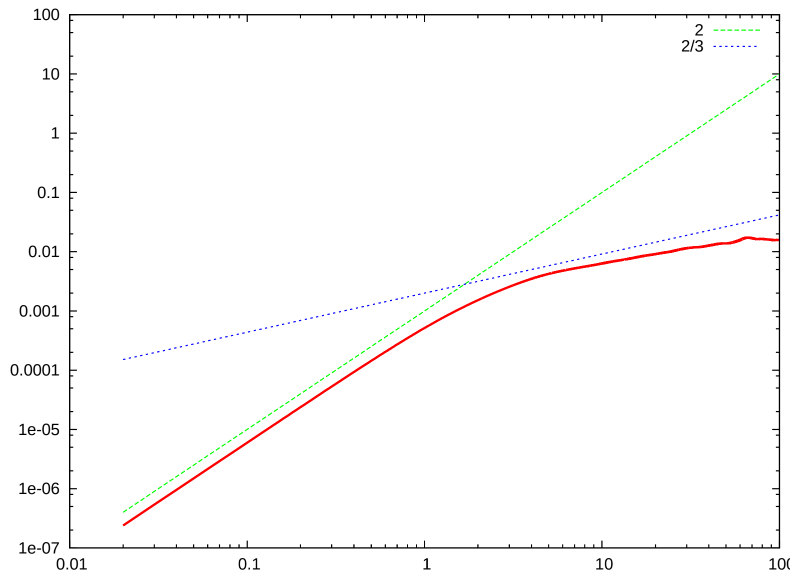


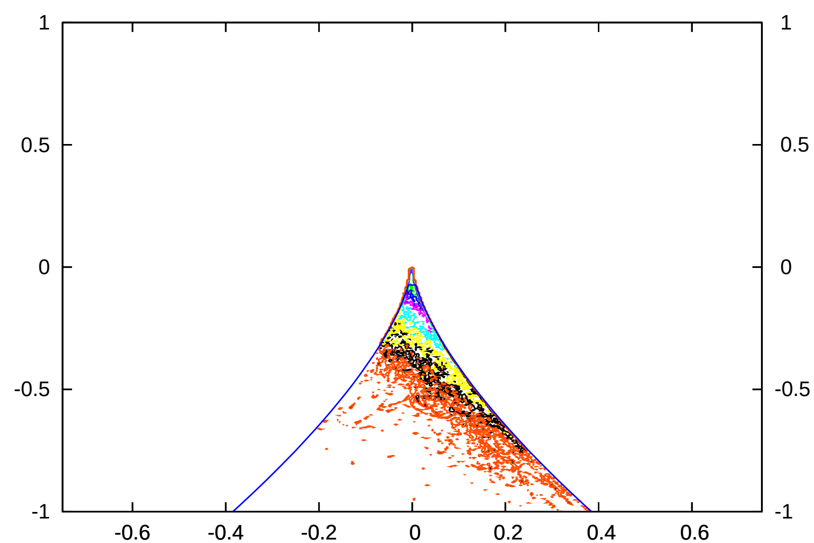
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.945$



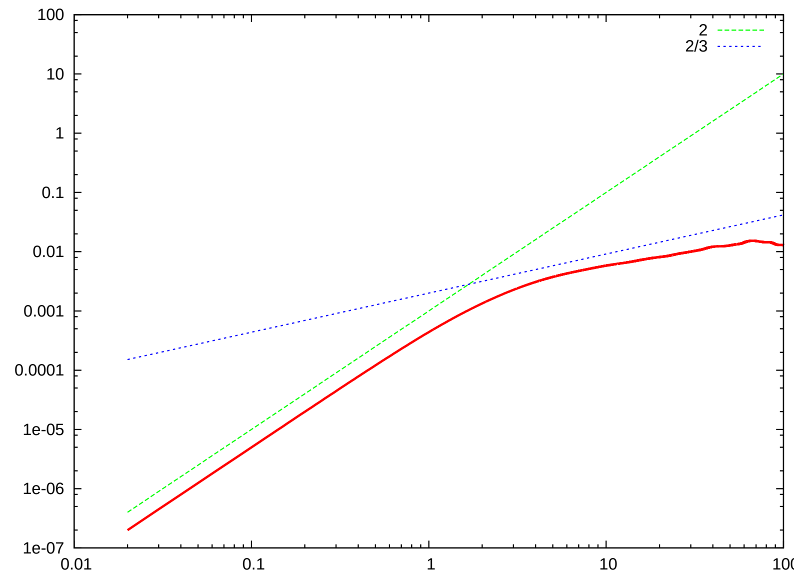


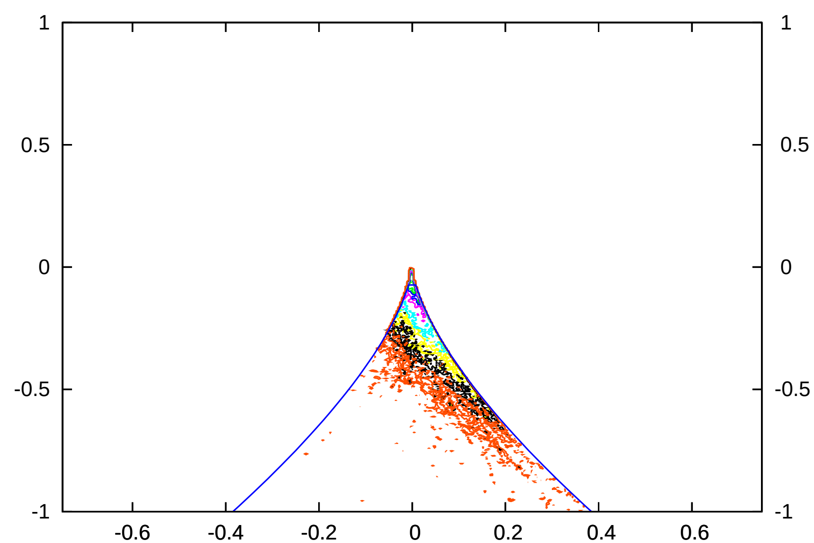
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 0.998$



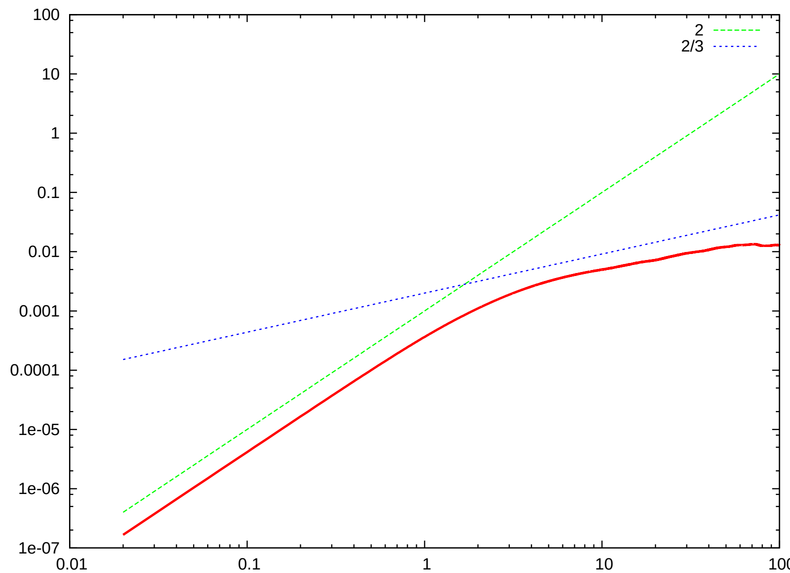


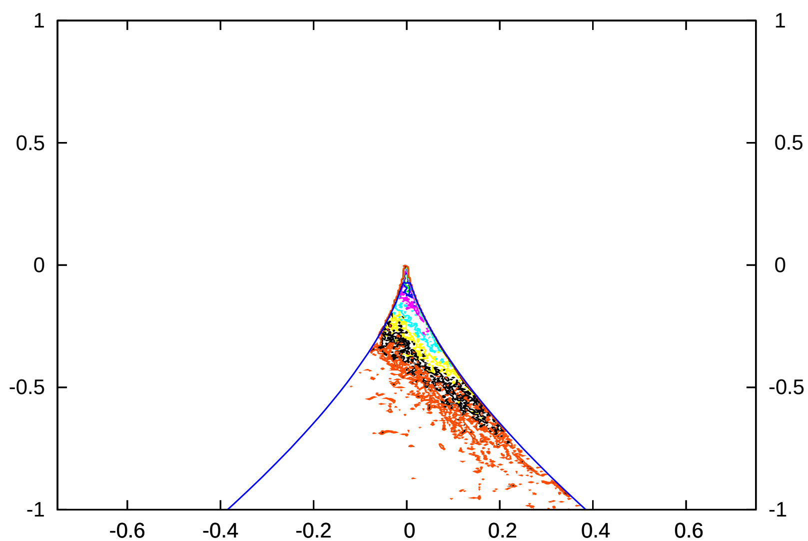
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 1.050$



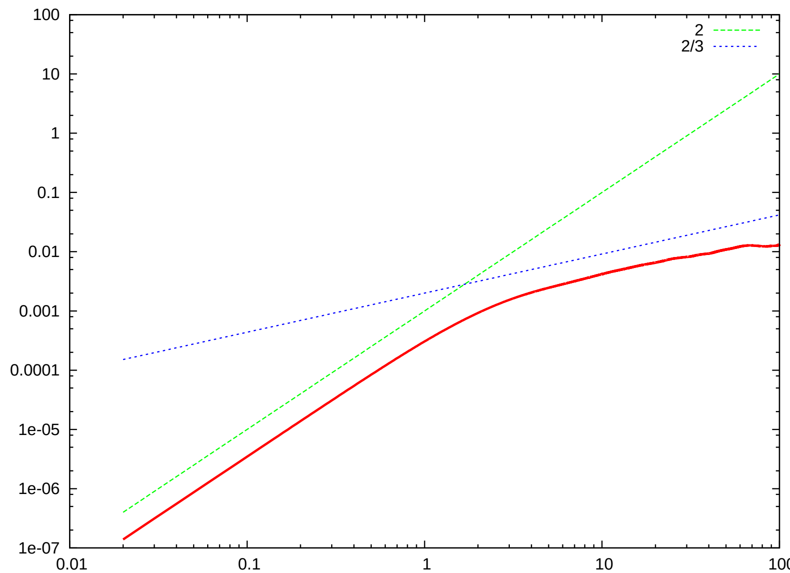


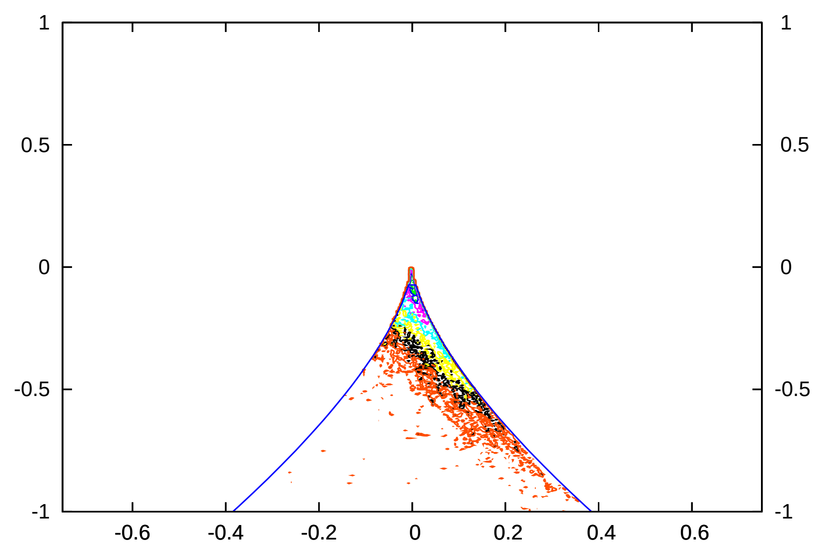
$< \delta u^2(\tau) >$ **and** $Q_s - R_s$ **at** $x/x_* = 1.103$



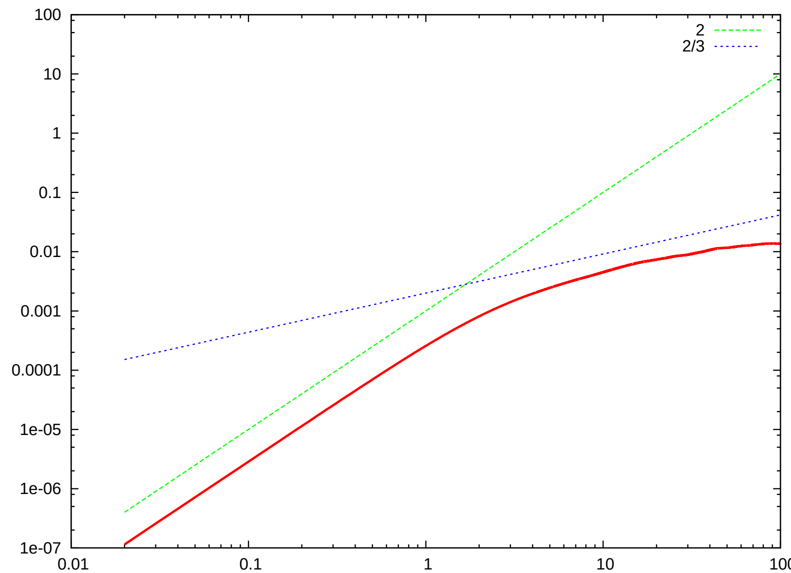


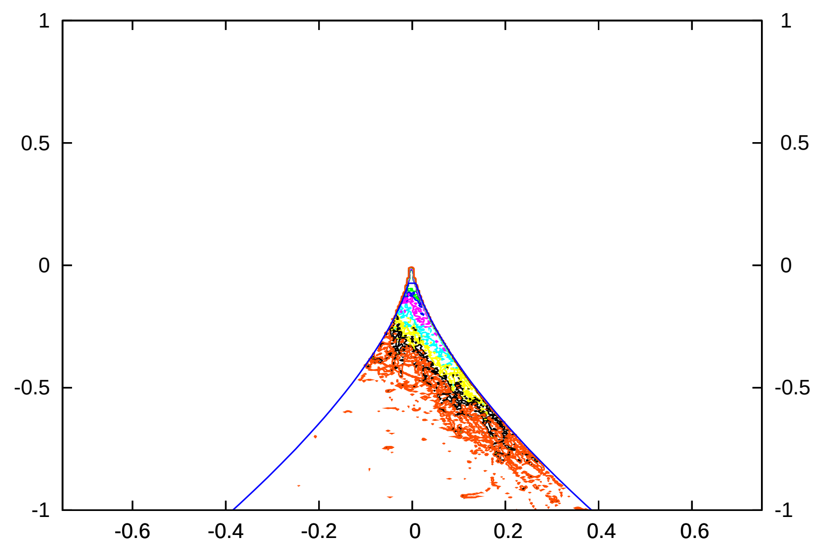
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 1.155$



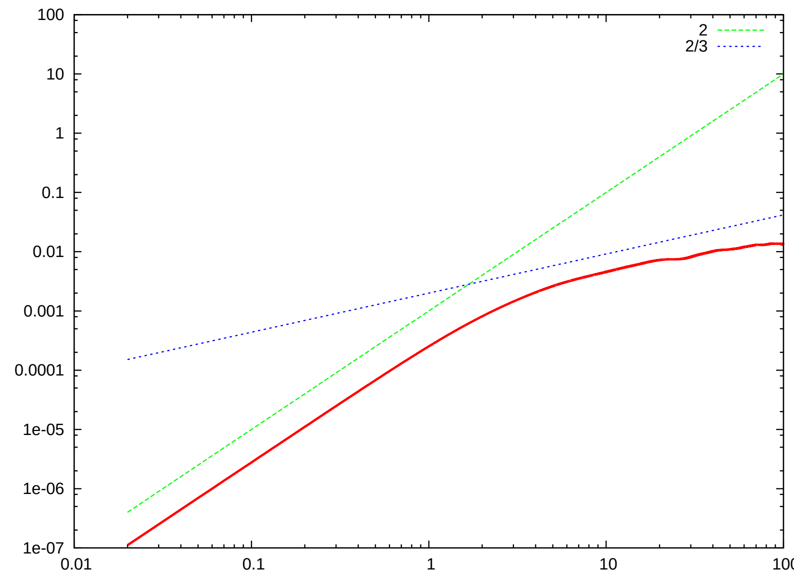


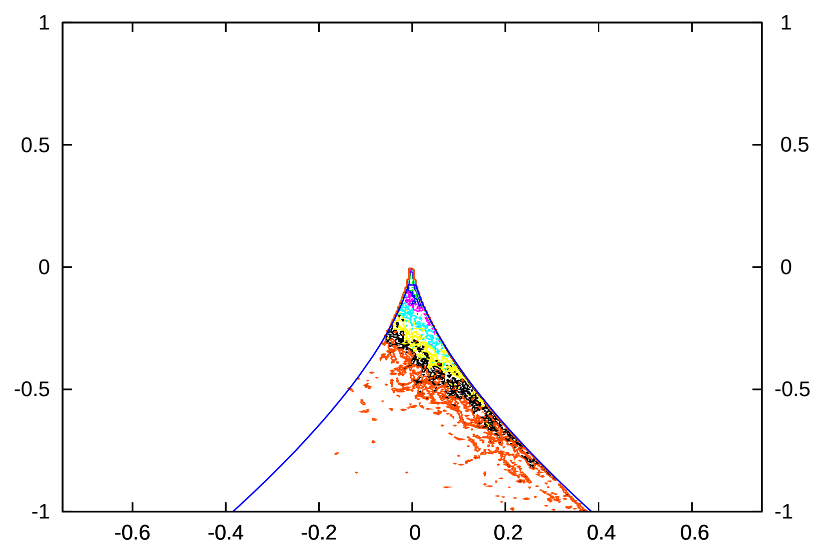
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 1.208$



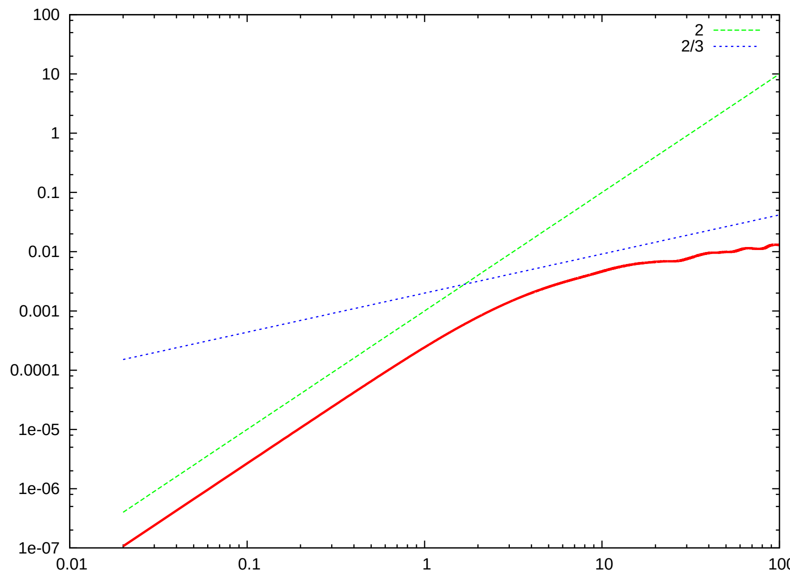


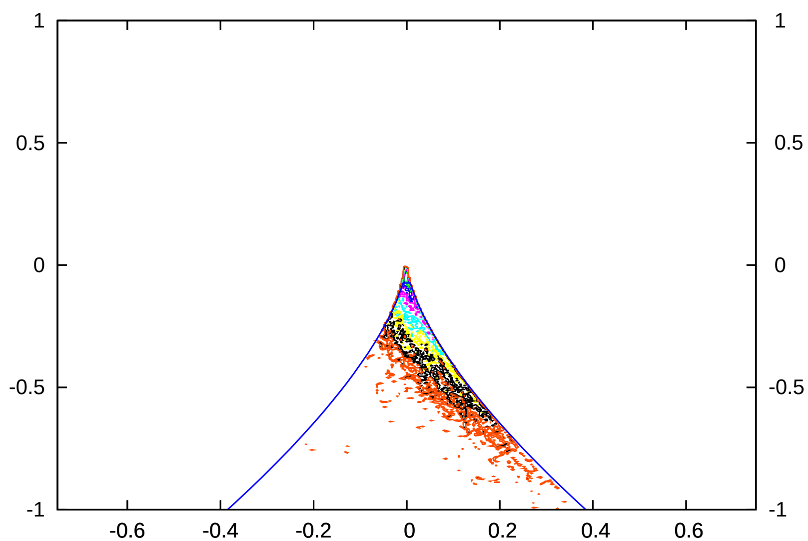
$<\delta u^2(\tau)>$ and $Q_s - R_s$ at $x/x_* = 1.261$



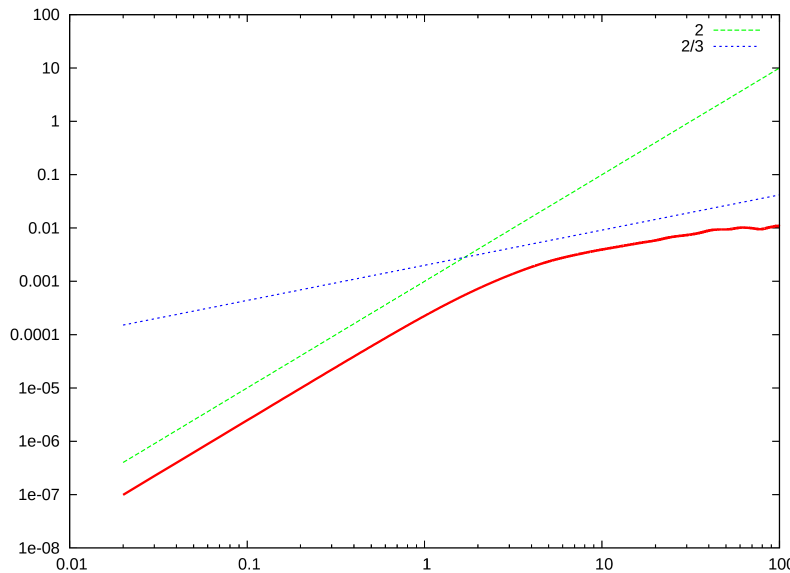


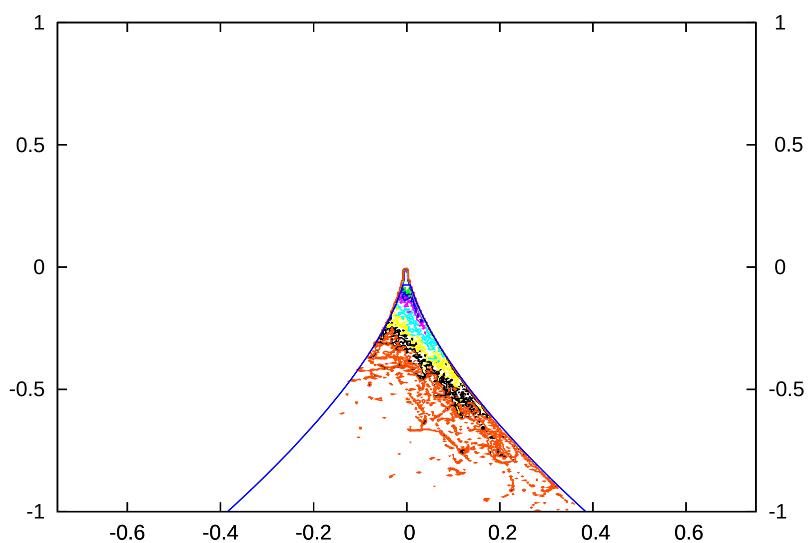
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 1.313$



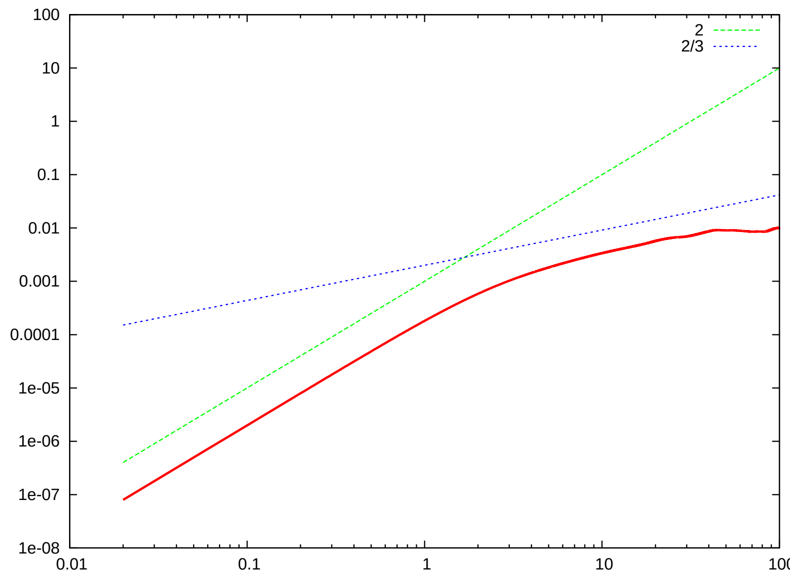


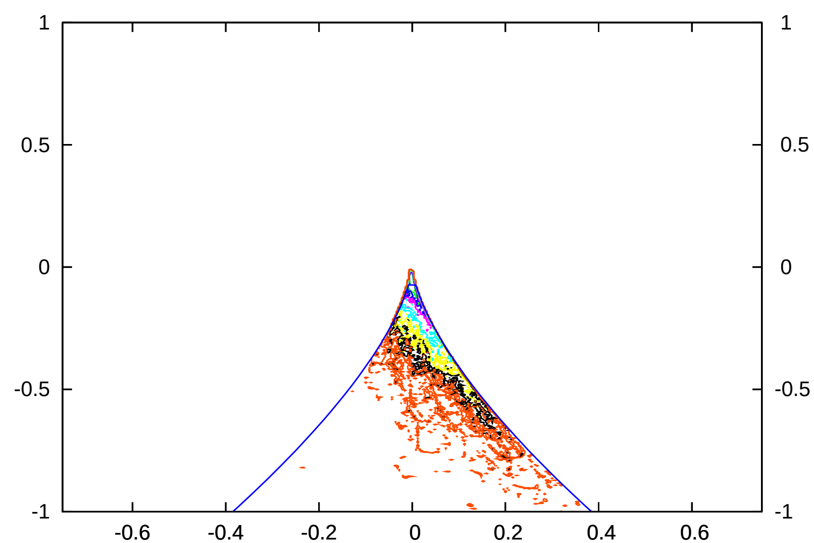
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 1.366$



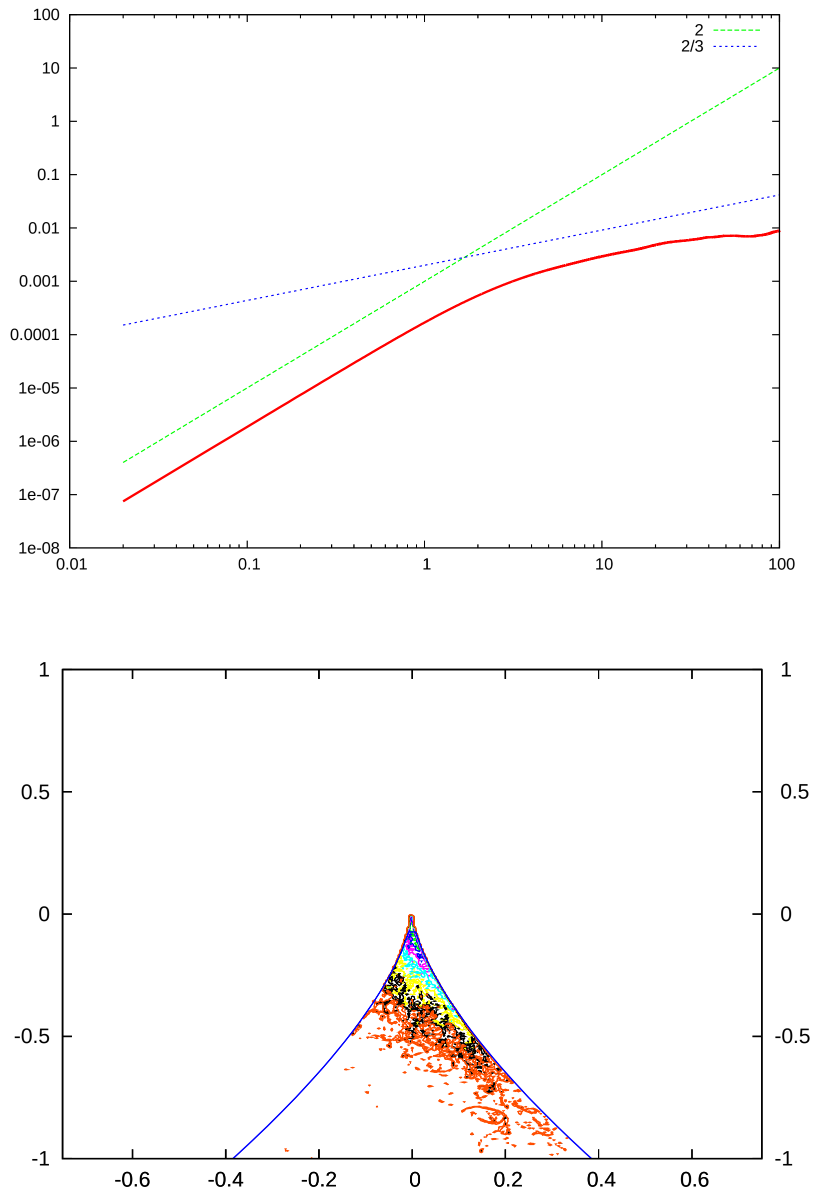


$<\delta u^2(\tau)>$ and $Q_s - R_s$ at $x/x_* = 1.418$

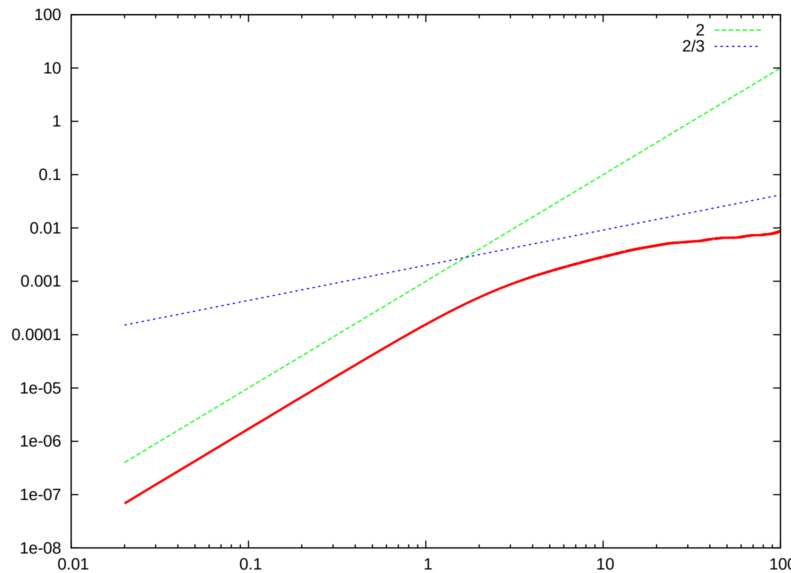


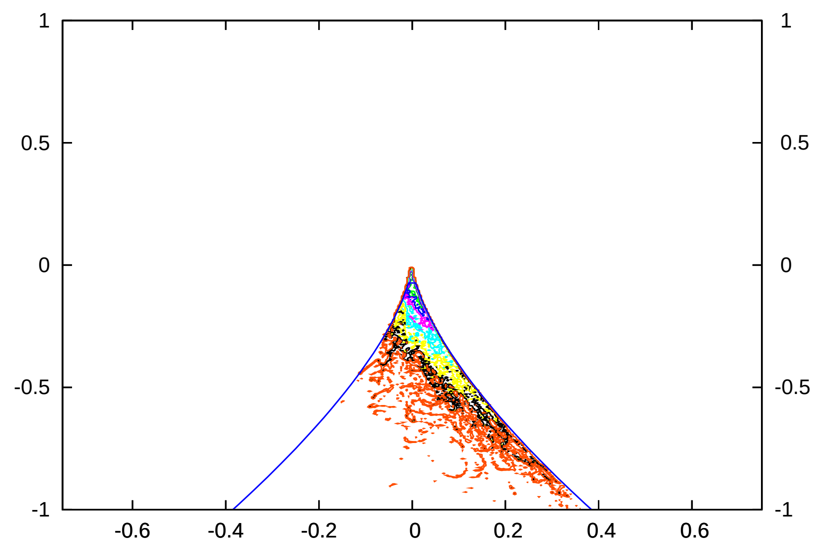


$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 1.471$

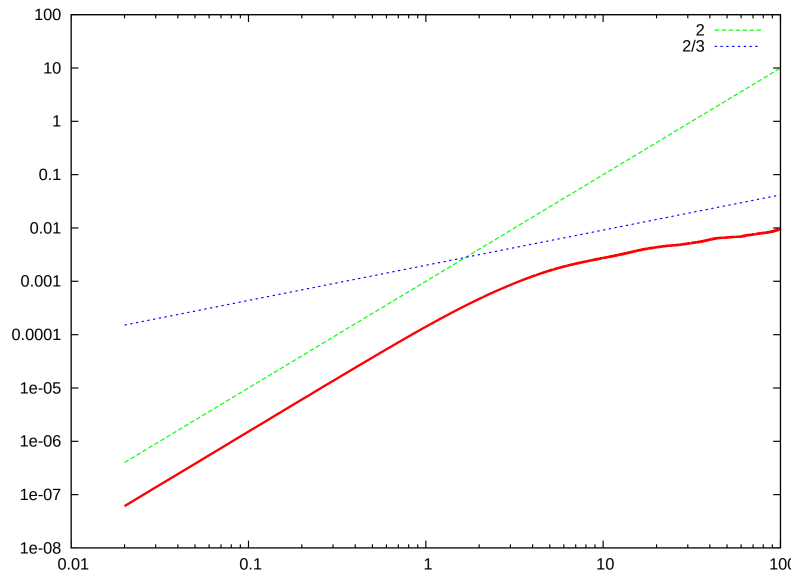


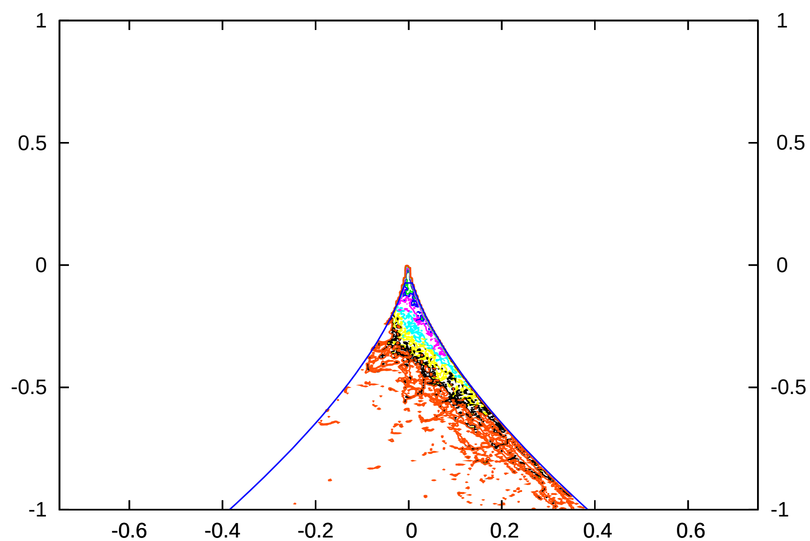
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 1.523$



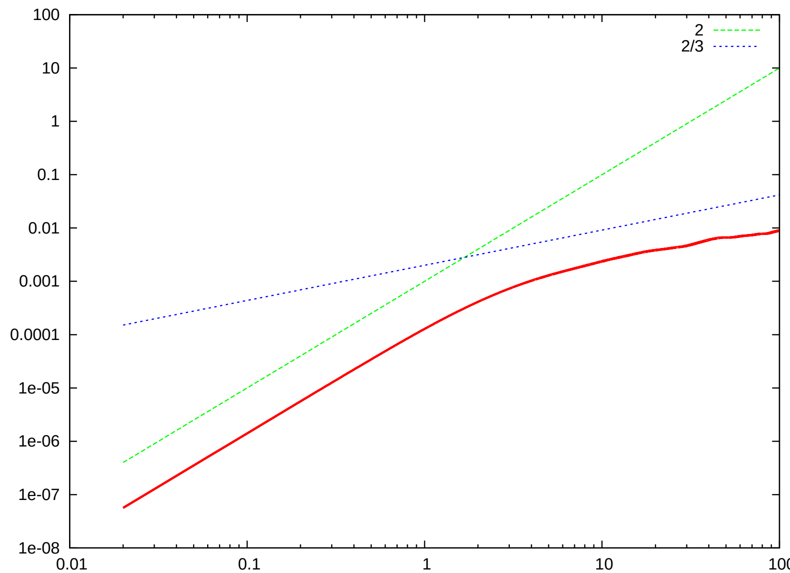


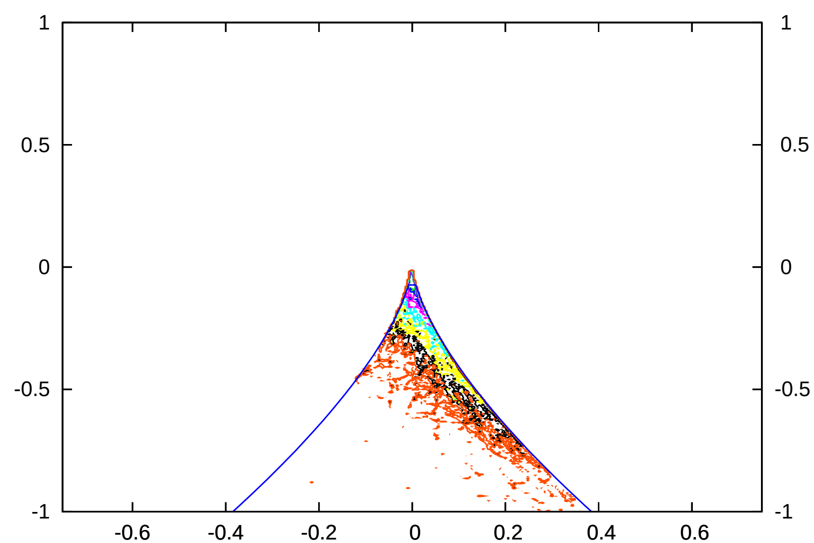
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 1.576$



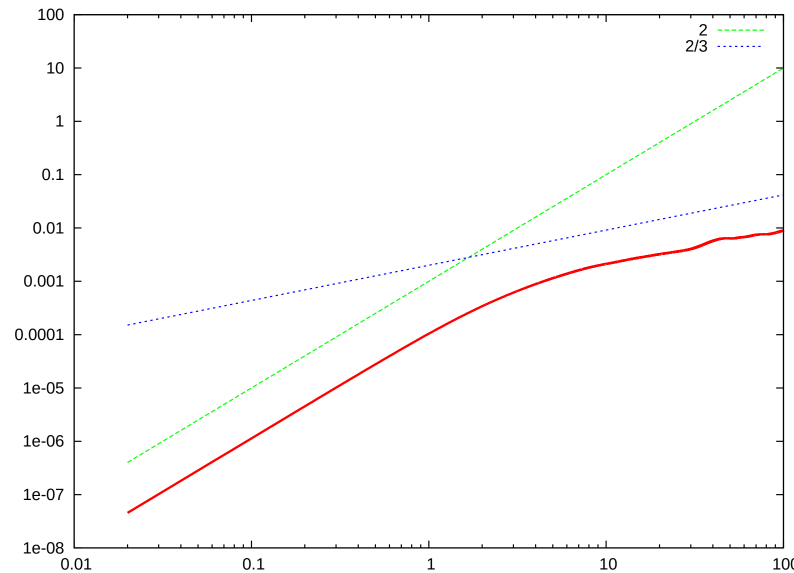


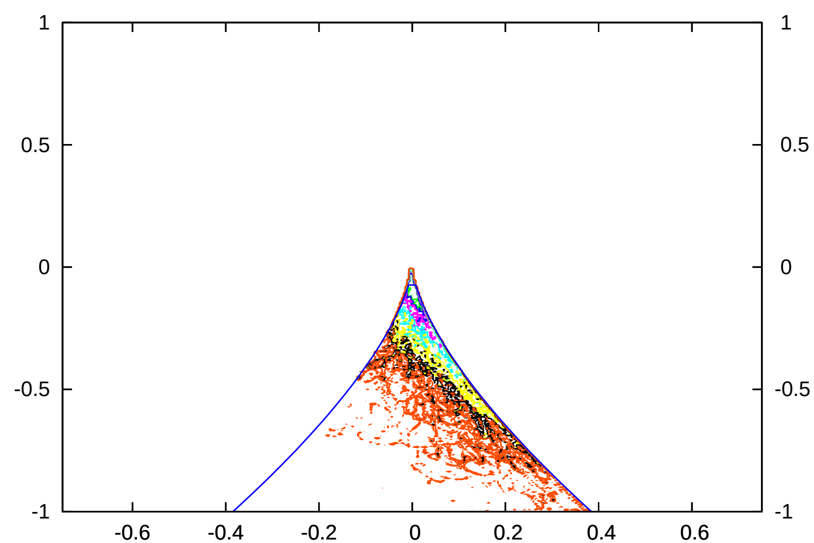
$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 1.628$



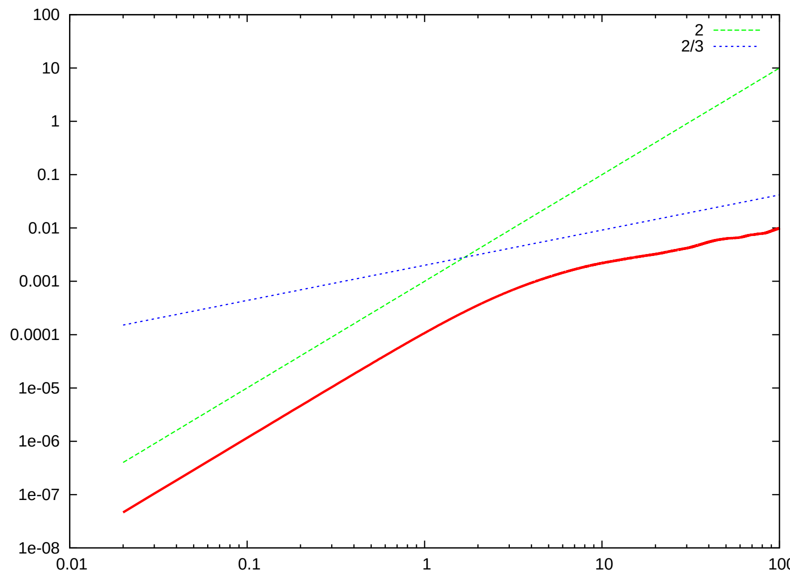


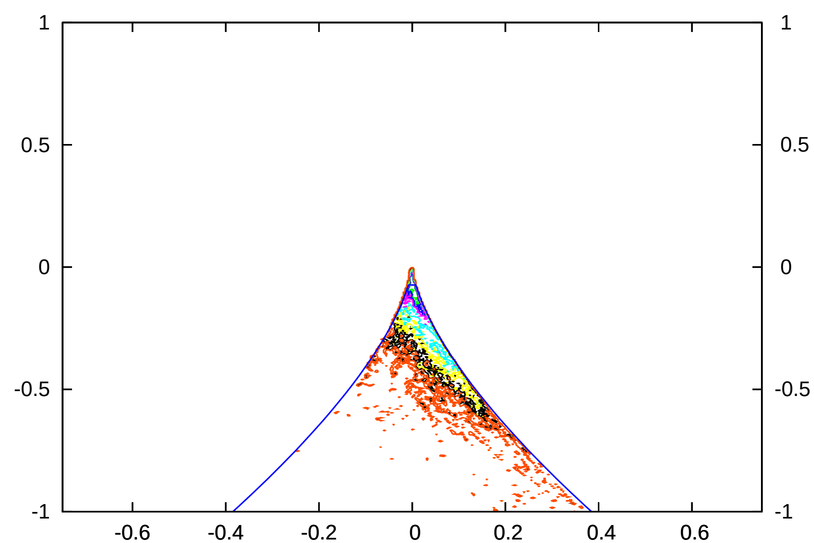
$< \delta u^2(\tau) >$ **and** $Q_s - R_s$ **at** $x/x_* = 1.681$





$< \delta u^2(\tau) >$ and $Q_s - R_s$ at $x/x_* = 1.733$





Conclusion of Q-R study

$< \delta u^2(\tau) > \sim \tau^{2/3}$ appears in the lee of the less blocked part of the grid and well before the appearance of the usual Q-R tear drop shape.