Turbulence and Energy Transfer in Strongly-Stratified Flows

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Outline

- Define, discuss ‘stratified turbulence’
  - potentially relevant to strongly stable regions of atmosphere, oceans
- Some numerical simulations of ‘stratified turbulence’
  - direct numerical simulations – some evolving flows
  - large eddy simulations – forced flows
- Scaling arguments
  - possible ‘stratified turbulence’ inertial range
- Field data
  - mainly ocean results

Stratified Turbulence (Lilly, 1983)

- Controlling parameters
  - Reynolds number: $R_\ell = u'\ell_H/\nu$
    - u' – characteristic rms velocity
    - ℓ_H – horizontal scale of energy-containing motions
  - Froude number: $F_\ell = u'/N\ell_H \sim T_B/T_{FM}$
  - Gradient Richardson Number: $Ri = N^2 \Big/ \left(\frac{\partial u}{\partial z}\right)^2$

- Typically, for strongly stable atmospheric boundary layers, for the atmosphere near and above the tropopause, for much of the ocean, etc.
  - for $\ell_H \sim 200\,\text{m}$, $F_\ell < \mathcal{O}(1)$, $R_\ell \gg 1$ (e.g., 10^8 or more)
  - $Ro = u'/\Omega\ell_H \gg 1$, no effect of rotation

Stratified Turbulence (cont'd)

- Definition of Stratified Turbulence
  - atmospheric/oceanic motions such that

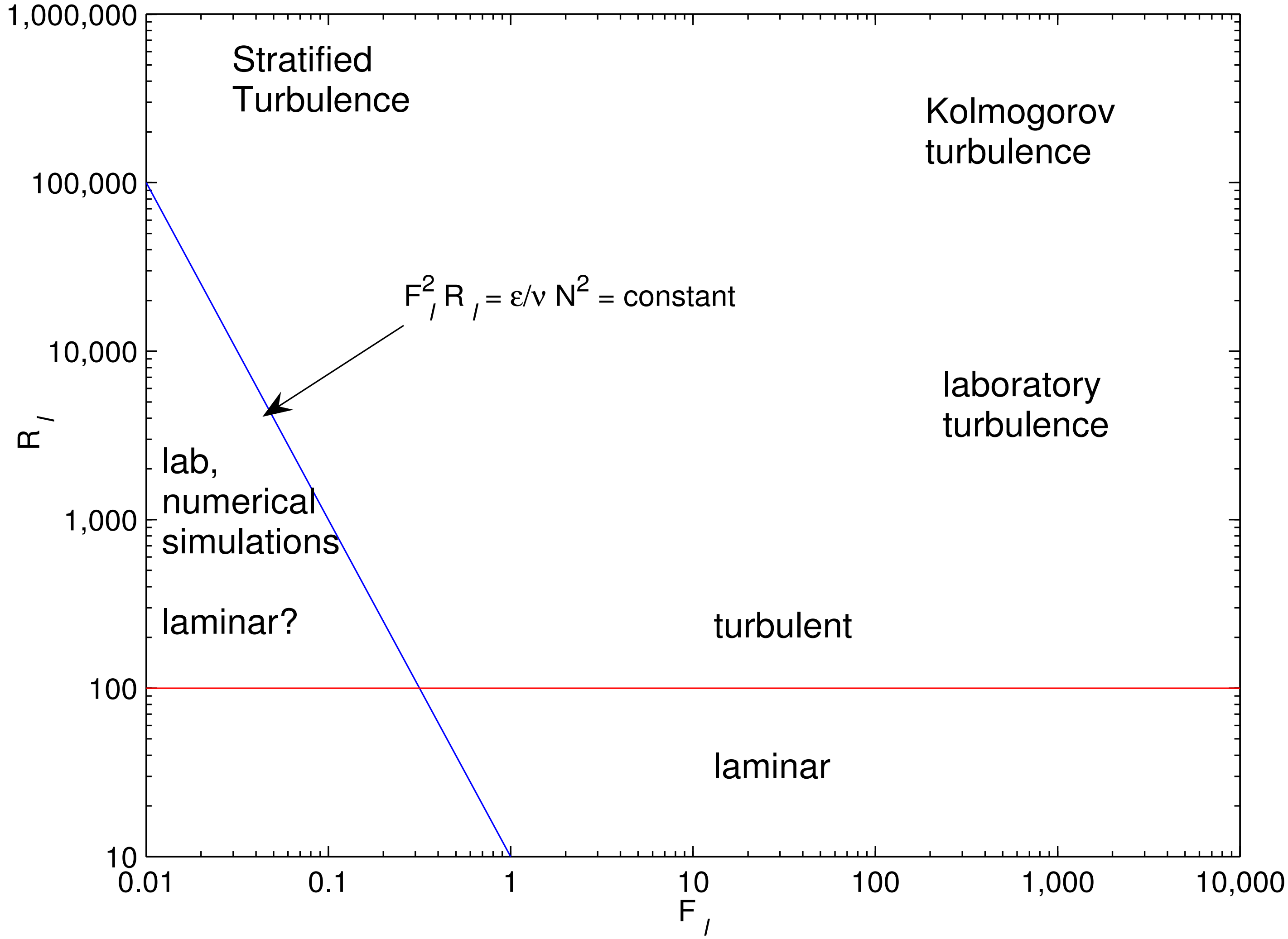
$$F_\ell < \mathcal{O}(1)\,, \quad Ri \sim \mathcal{O}(1)\,, \quad R_\ell \gg 1$$

  - contains both internal gravity waves and quasi-horizontal motions
    * potential vorticity is of importance
  - scaling arguments suggest that 'classical' turbulence will exist when

$$R_b \sim F_\ell^2 R_\ell \sim \epsilon/\nu N^2 > O(10)$$

    * R_b is called the 'activity parameter', 'buoyancy Reynolds number'

Stratified Turbulence (cont'd)



Laboratory Results – Stratified Turbulence

- Laboratory experiments, e.g., wake of sphere, wake of grid, jets (Flow Research, USC, ASU, Toulouse, Grenoble, Eindhoven, . . .)
  - usually when turbulence is generated, $F_\ell \gg 1$
    - but flow decays, F_ℓ and R_ℓ both decay
  - when $F_\ell \leq \mathcal{O}(1)$,
    - development of quasi-horizontal vortices
    - simultaneous with propagating internal waves
  - but generally R_ℓ is low; R_b is low; $Ri > \mathcal{O}(1)$
  - smaller-scale turbulence usually does not develop
  - scaling of full dynamics to geophysical turbulence unclear

Laboratory Results (cont'd)

Lin & Pao, 1979

Field Results

- Field experiments
  - usually an internal wave component
  - often meandering motions are observed
  - ‘classical’ turbulence is very intermittent, sporadic
  - effects of stratification ‘strong’ for $\ell > \ell_O \sim 1\,\mathrm{m}$ (ocean)
    - where $\ell_O = \left(\epsilon/N^3\right)^{1/2}$, the Ozmidov scale
  - for the strongly stable atmosphere, $\epsilon \sim 5 \cdot 10^{-4}\,\mathrm{m}^2/\mathrm{s}^3$, $\ell_O \sim 3\,\mathrm{m}$
  - component velocities highly non-isotropic

Theoretical Arguments – Stratified Turbulence

- Lilly (1983) used scaling arguments to suggest, for $F_\ell \leq \mathcal{O}(1)$:
  - flows in 'adjacent' horizontal layers are somewhat decoupled
  - leads to increasing vertical shearing of horizontal flow
  - and to decreasing Richardson numbers
- Billant and Chomaz (1999)
  - induced velocities lead to strong vertical inhomogeneities and layering
- Even though strong, stable stratification,
  at high Reynolds numbers, both mechanisms lead to
  - smaller vertical scales continually developing
  - local instabilities and turbulence intermittently occurring

Questions – Stratified Turbulence

- What are the dynamics of turbulent motions when $F_\ell \leq \mathcal{O}(1)$, especially with $R_\ell \gg 1$, $F_\ell^2 R_\ell \gg \mathcal{O}(1)$?
  - upscale or downscale transfer of energy?
- What are the effects of strong, stable stratification on:
  - turbulence structure, decay rates, dispersion, mixing rates, etc.,
  - turbulence modeling issues?
- Do the results from laboratory and numerical experiments scale up to high Reynolds numbers characteristic of the atmosphere and oceans?

Research Approach – Stratified Turbulence

- Numerical simulation
  - solve the 3-D, time-dependent Navier-Stokes equations subject to the Boussinesq approximation
  - uniform stratification, no ambient shear
  - consider flows with $F_\ell \leq \mathcal{O}(1)$

    i. initial value problems; time evolving flows
      - initiate 'late-stage' turbulence for a range of R_ℓ, F_ℓ

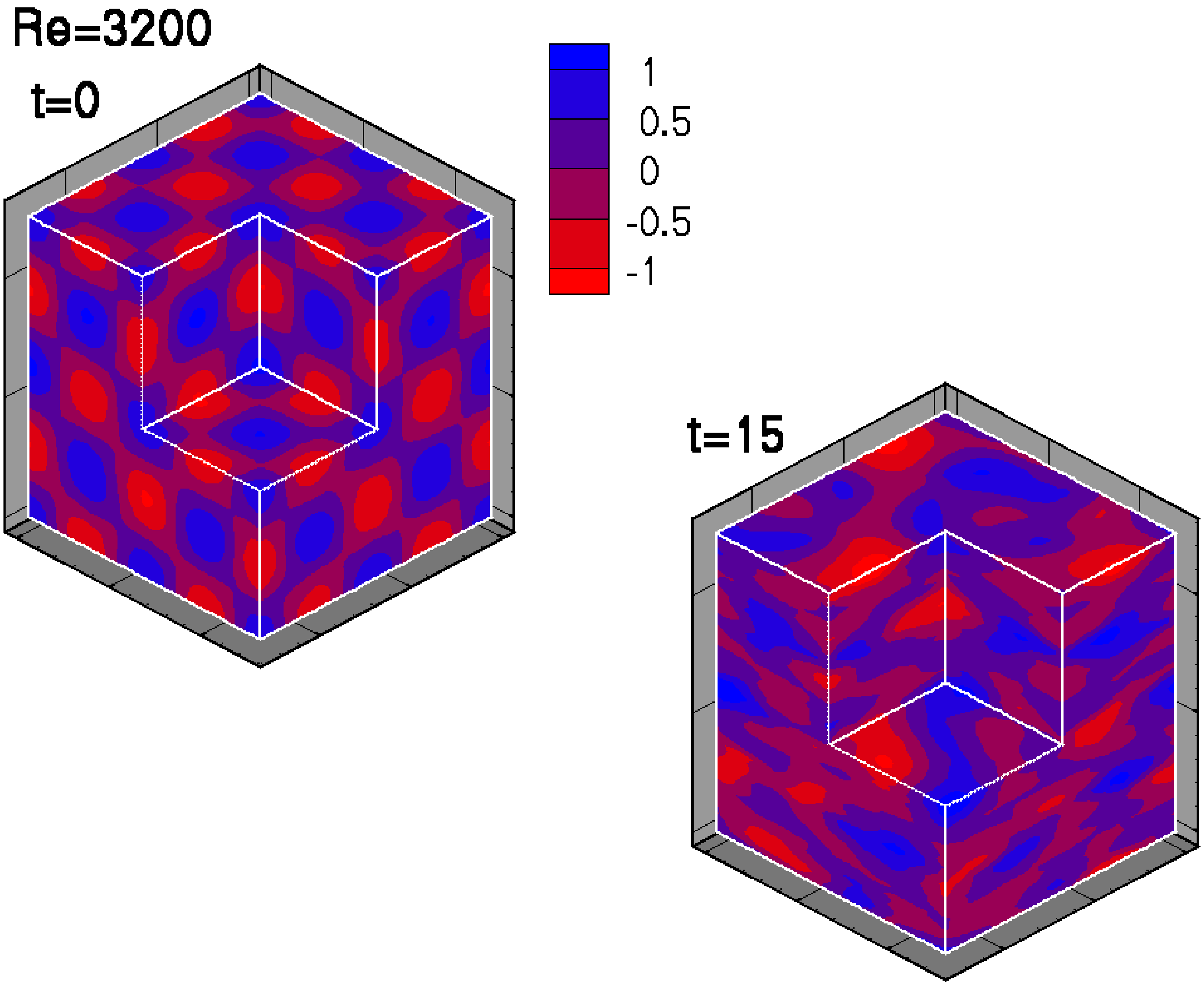
        enables higher Reynolds number simulations
      - direct numerical simulation; no subgrid modeling

    ii. forced turbulence; statistically stationary flow
      - large eddy simulation; subgrid model

- Accompanying scaling analysis

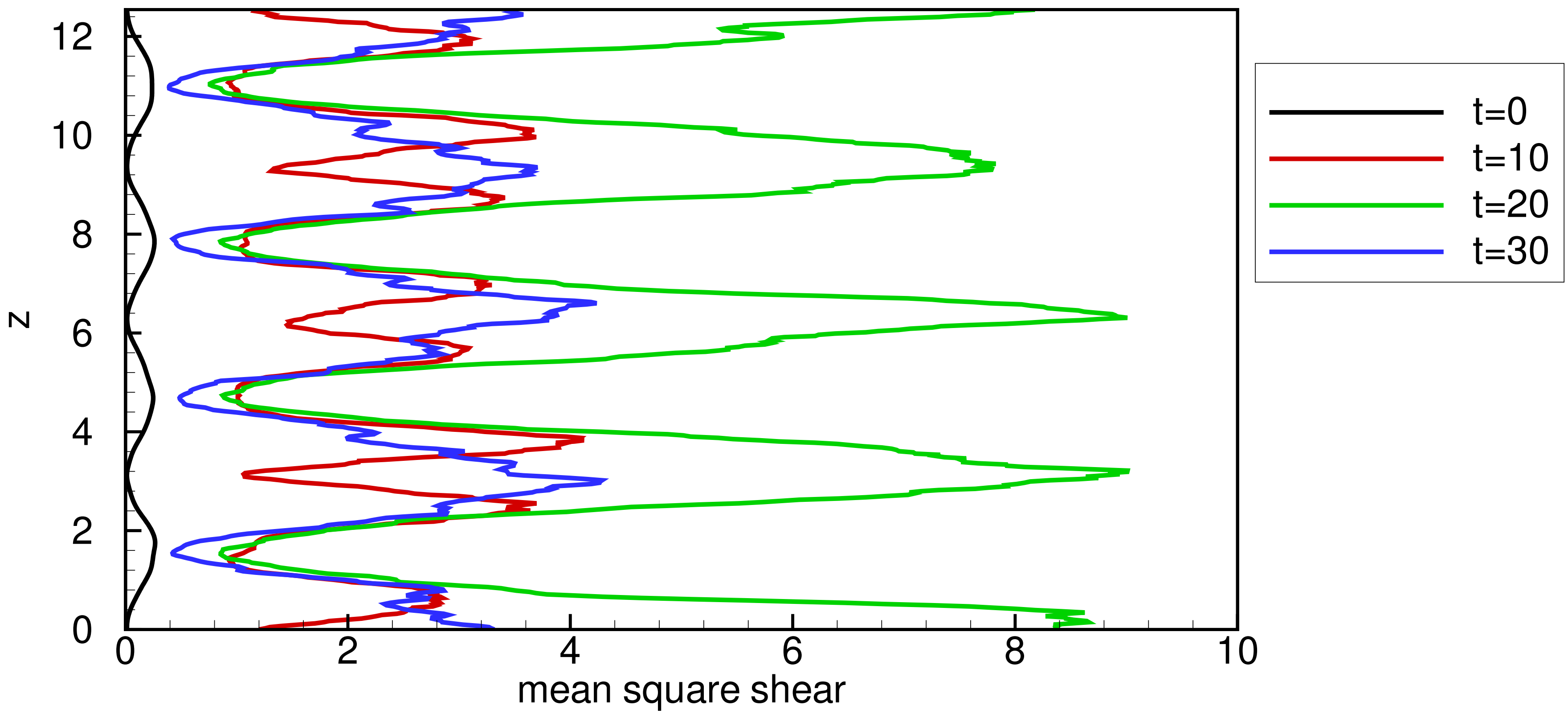
Initial Value Problems

- Two specific flows considered
  - defined by initial conditions (initial value problems; not forced)
    - ∗ Taylor-Green flow + low-level, broad-banded noise, and
    - ∗ quasi-horizontal array of ‘Karman’-street vortices
  - for all cases $\rho = 0$ initially
  - for each case, exact same initial conditions, except for F_ℓ and R_ℓ
  - $F_\ell = 4 \ (u'/N\ell_H \approx 0.6)\,, \quad 200 \leq R_\ell \leq 9600$
- Both flows have some properties of the late-time vortices observed in the laboratory studies
- Discuss mainly the Taylor-Green results today;
  the ‘Karman’-street results are qualitatively consistent with these



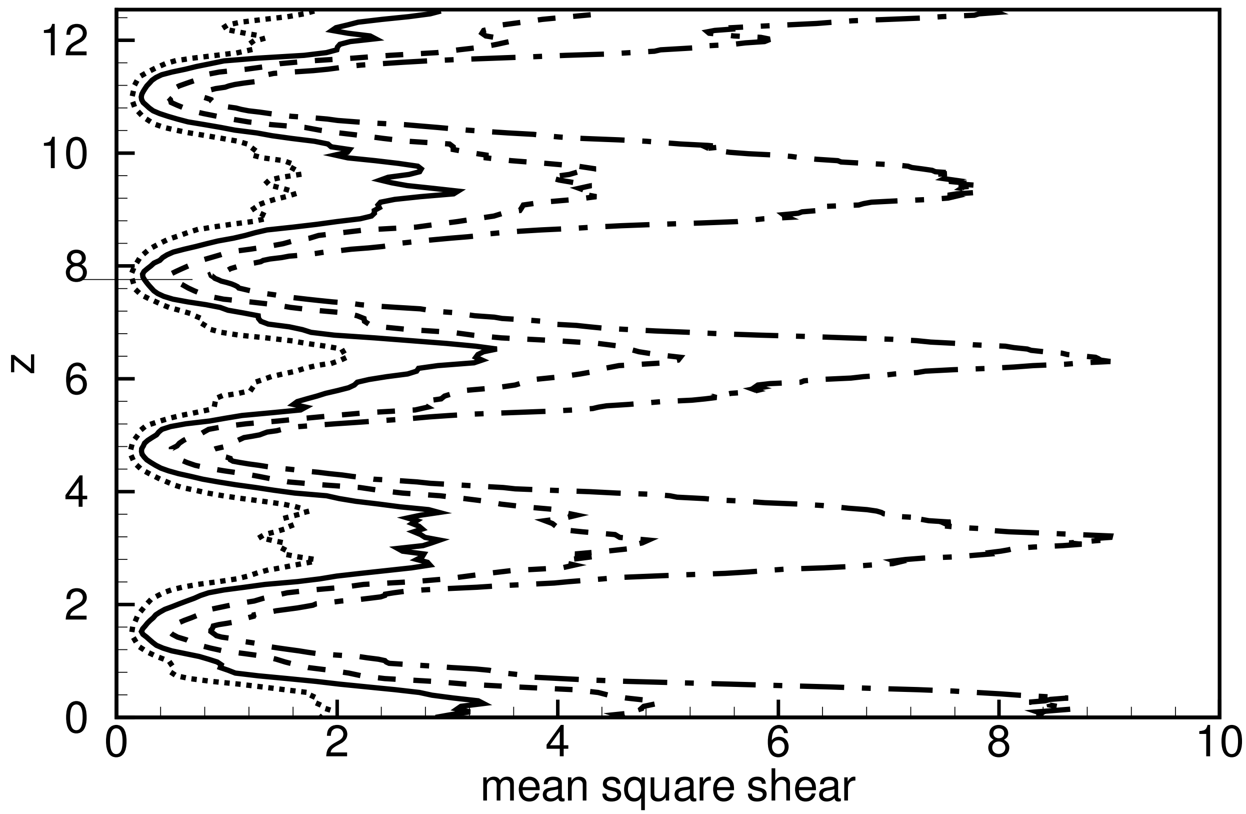
Three-dimensional contour plots of the stream function for the case with $F_\ell = 4$, $R_\ell = 3200$ at $t = 0$ (left) and $t = 15$ (right).

Mean Square Shear $\left\langle \left(\frac{\partial u}{\partial z}\right)^2 \right\rangle_H$ versus z



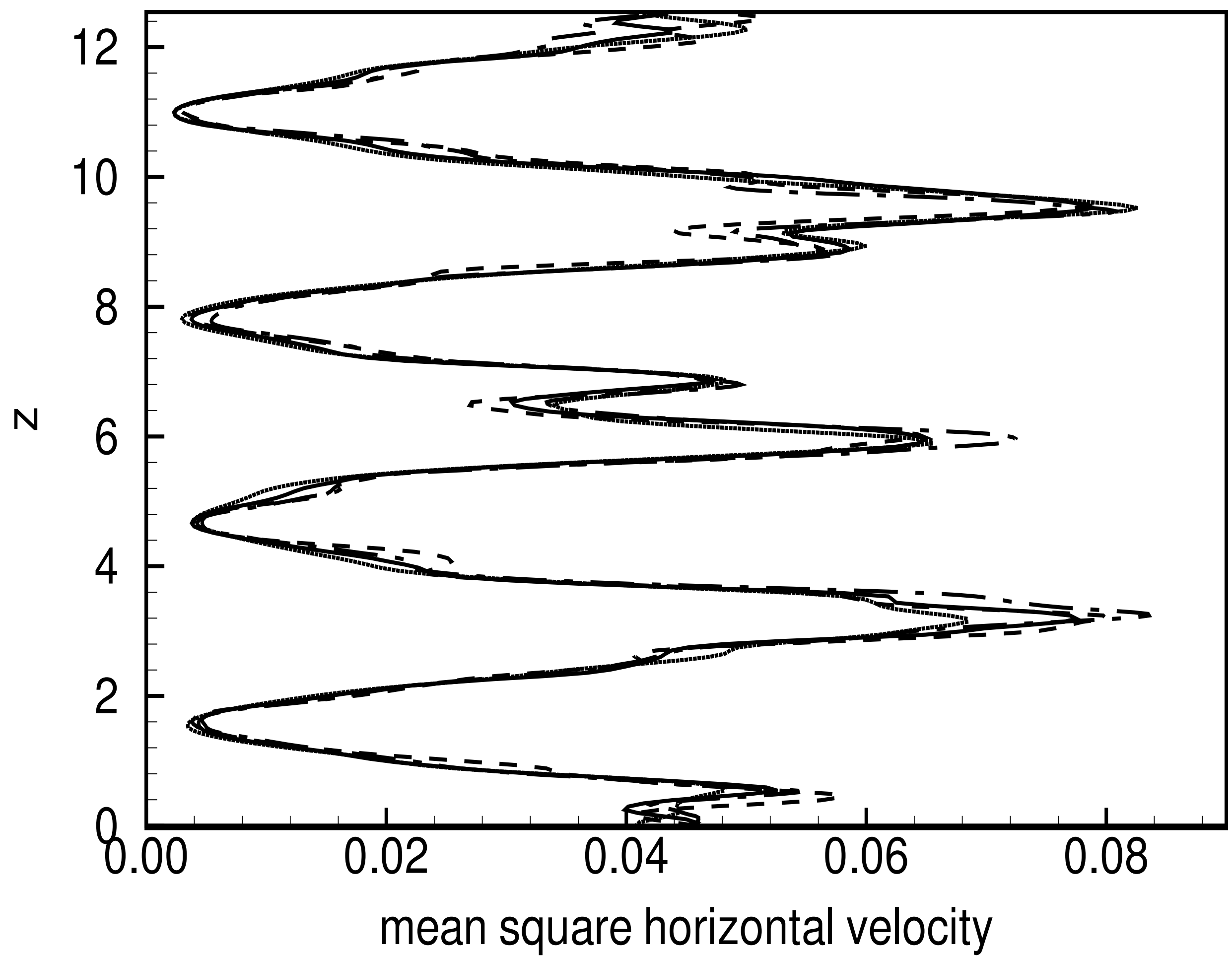
Mean square vertical shearing of the horizontal velocity vs z.
$F_\ell = 4$ and $R_\ell = 6400$ at different times.

Mean Square Shear $\left\langle \left(\frac{\partial u}{\partial z}\right)^2 \right\rangle_H$ versus z



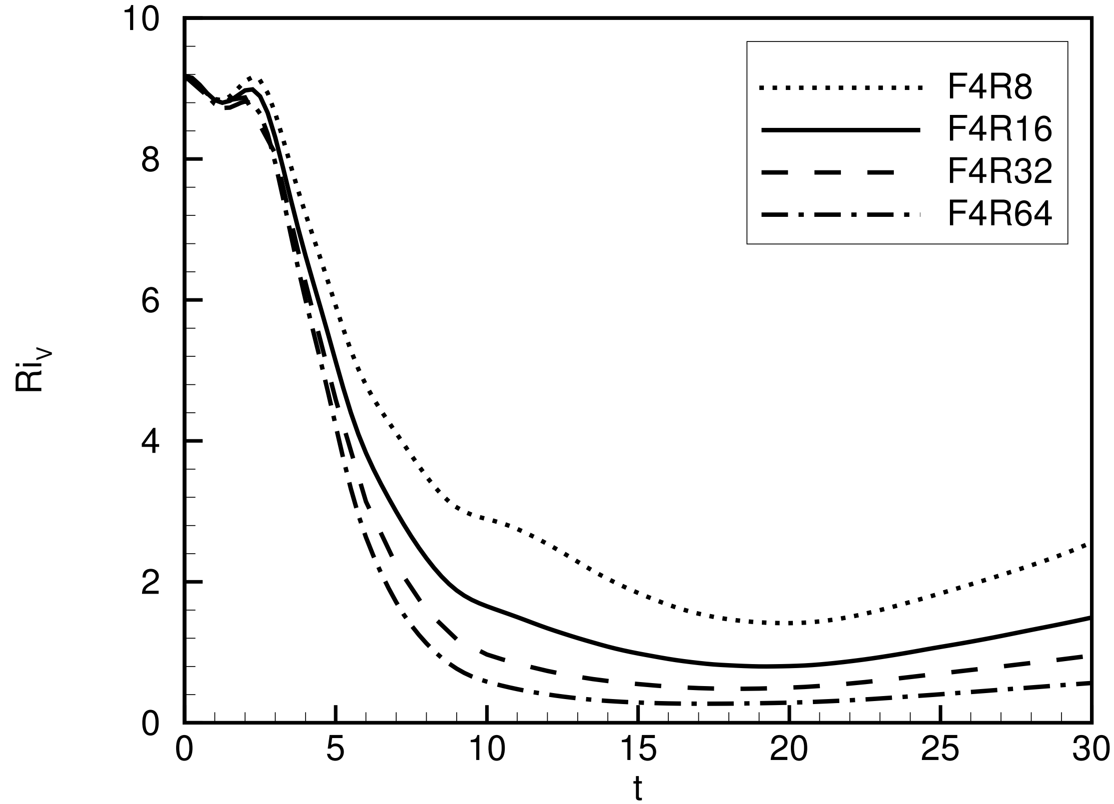
Mean square vertical shearing of the horizontal velocity vs z.
$F_\ell = 4$ and $t = 20$ for $R_\ell = 800, 1600, 3200, 6400$.

Mean Square Horizontal Velocity $\langle u^2 \rangle_H$ versus z



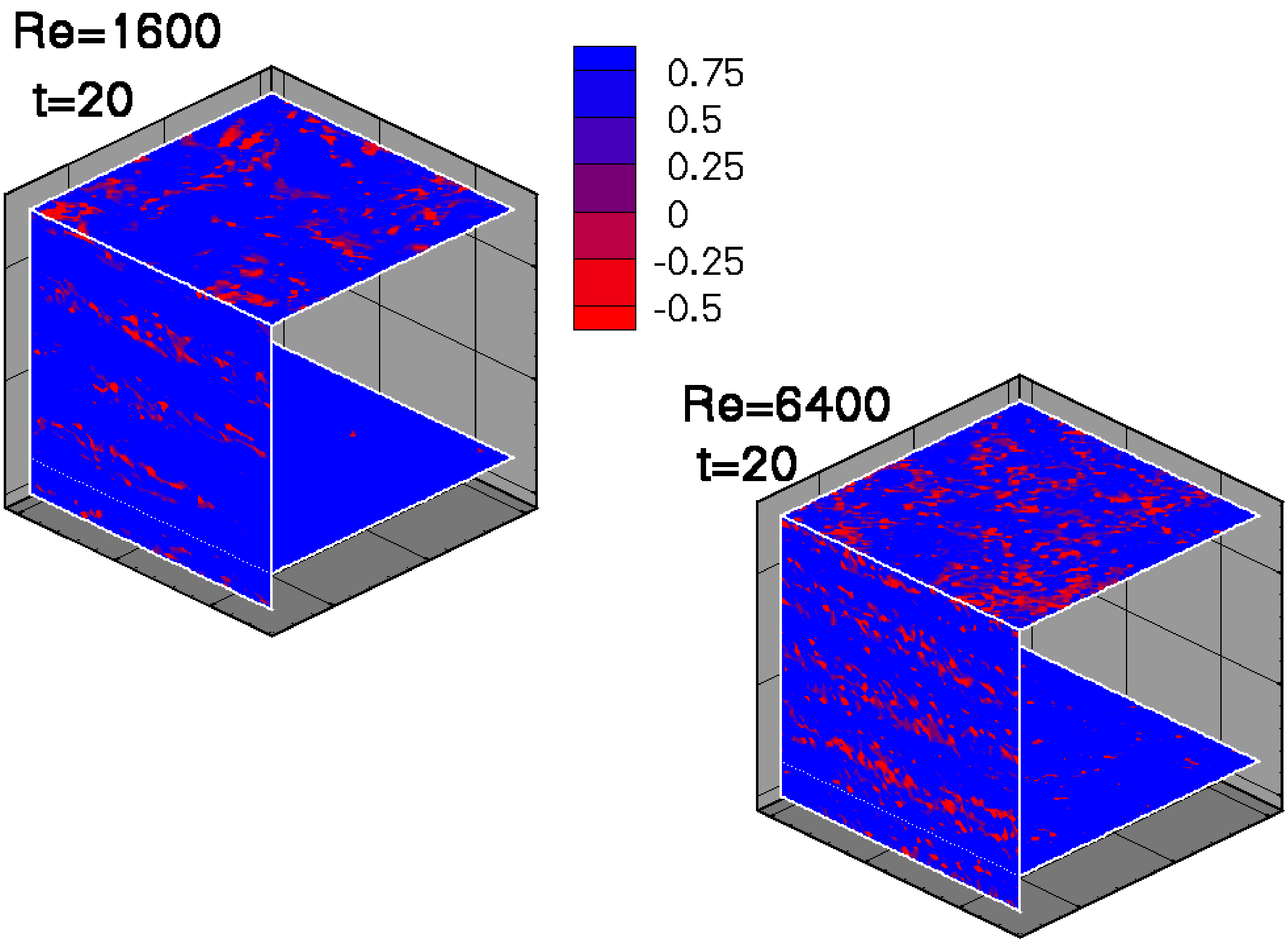
Mean square velocity vs z at $t = 30$, $F_\ell = 4$, various R_ℓ.

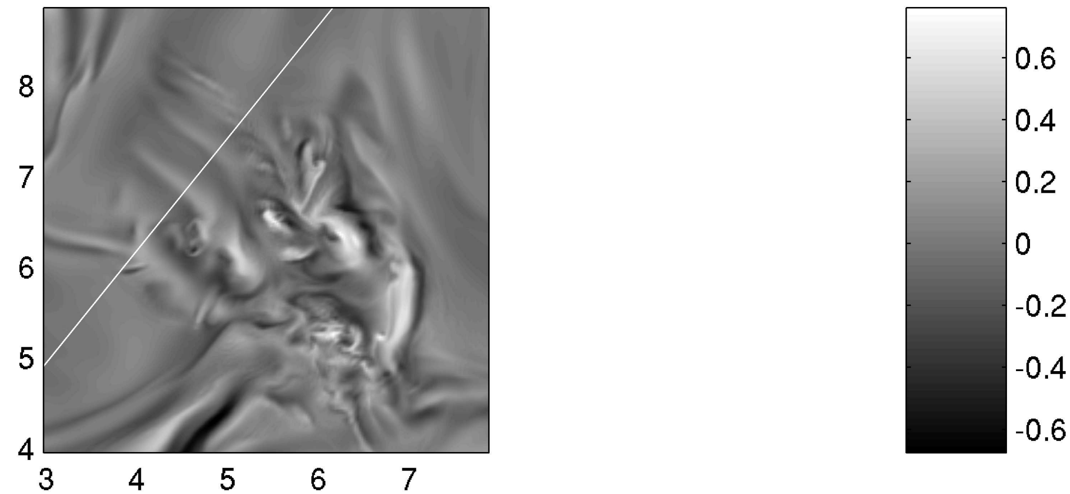
Volume-Averaged Gradient Richardson Number versus t



$$Ri_V = \frac{N^2}{\left\langle \left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 \right\rangle}$$

Local Gradient Richardson Number




8
7
6
5
4
3
4
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6
7
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0.4
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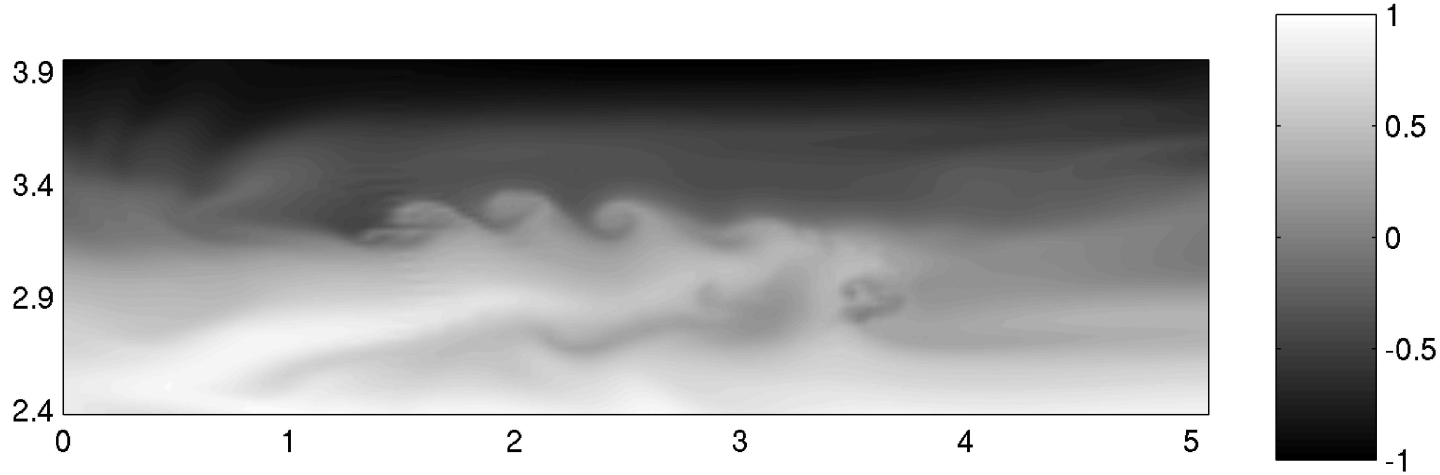
0
-0.2
-0.4
-0.6


3.9
3.4
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0
1
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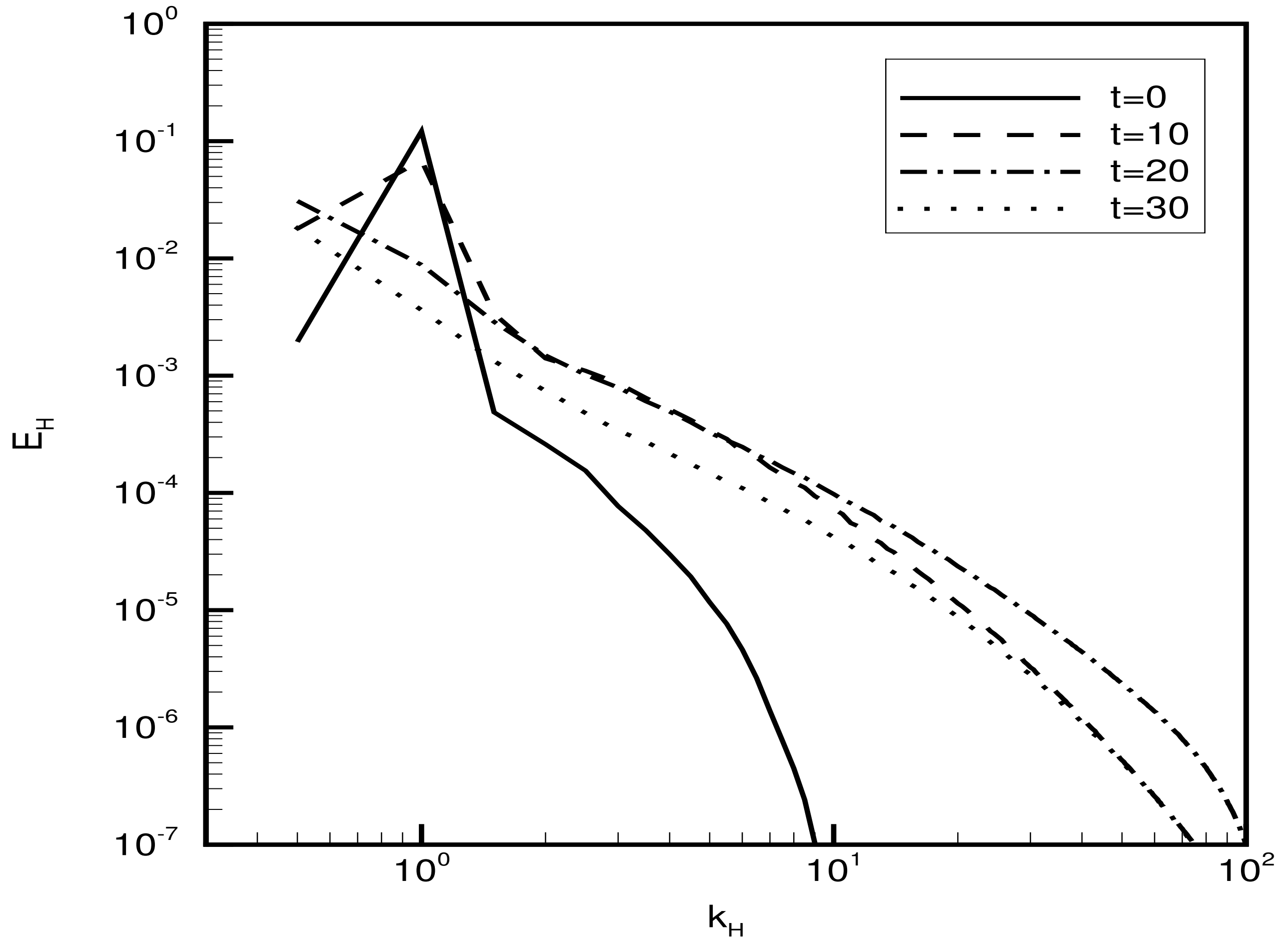
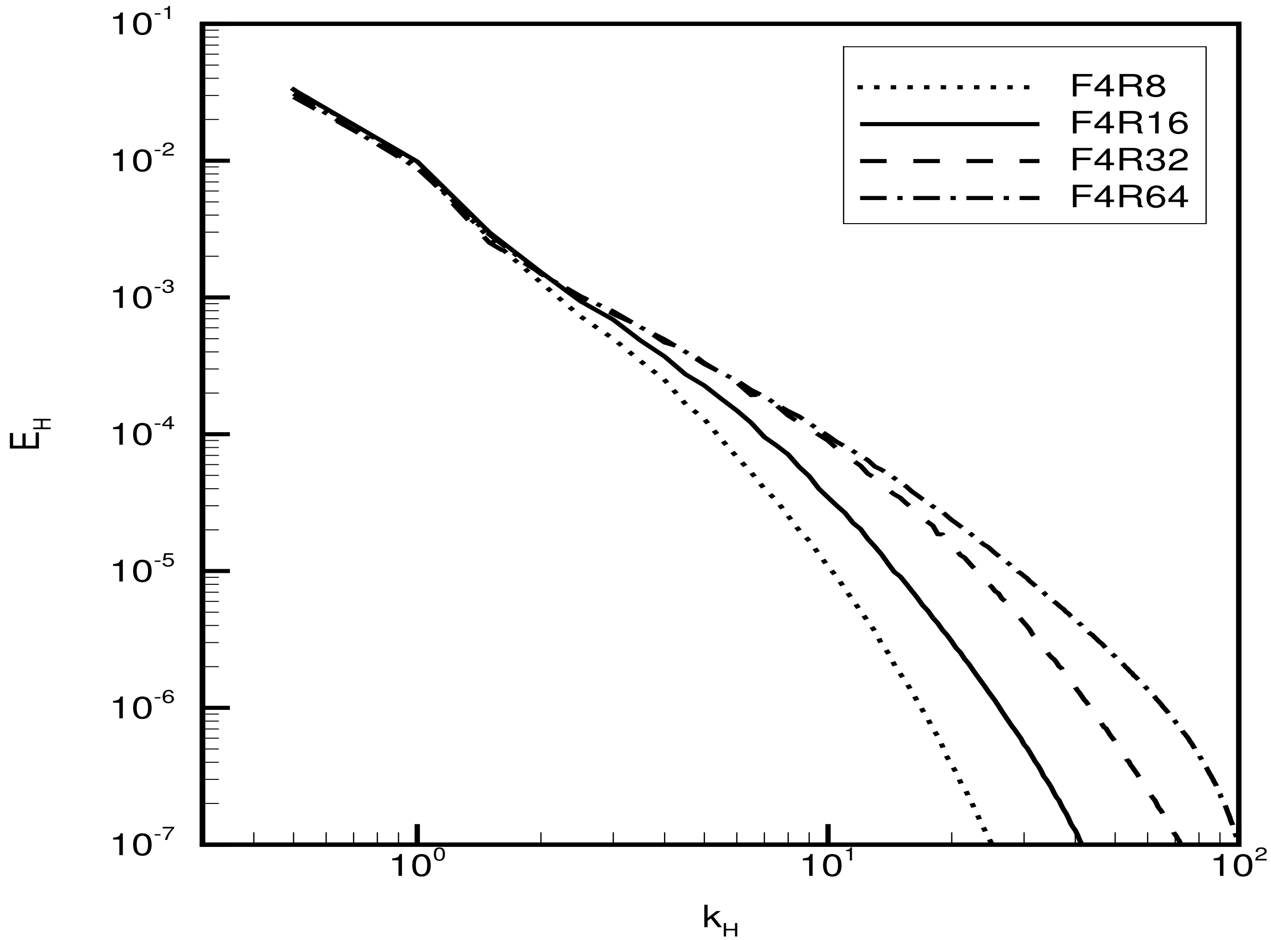
3
4
5
1
0.5
0
-0.5
-1

Horizontal Kinetic Energy Spectra



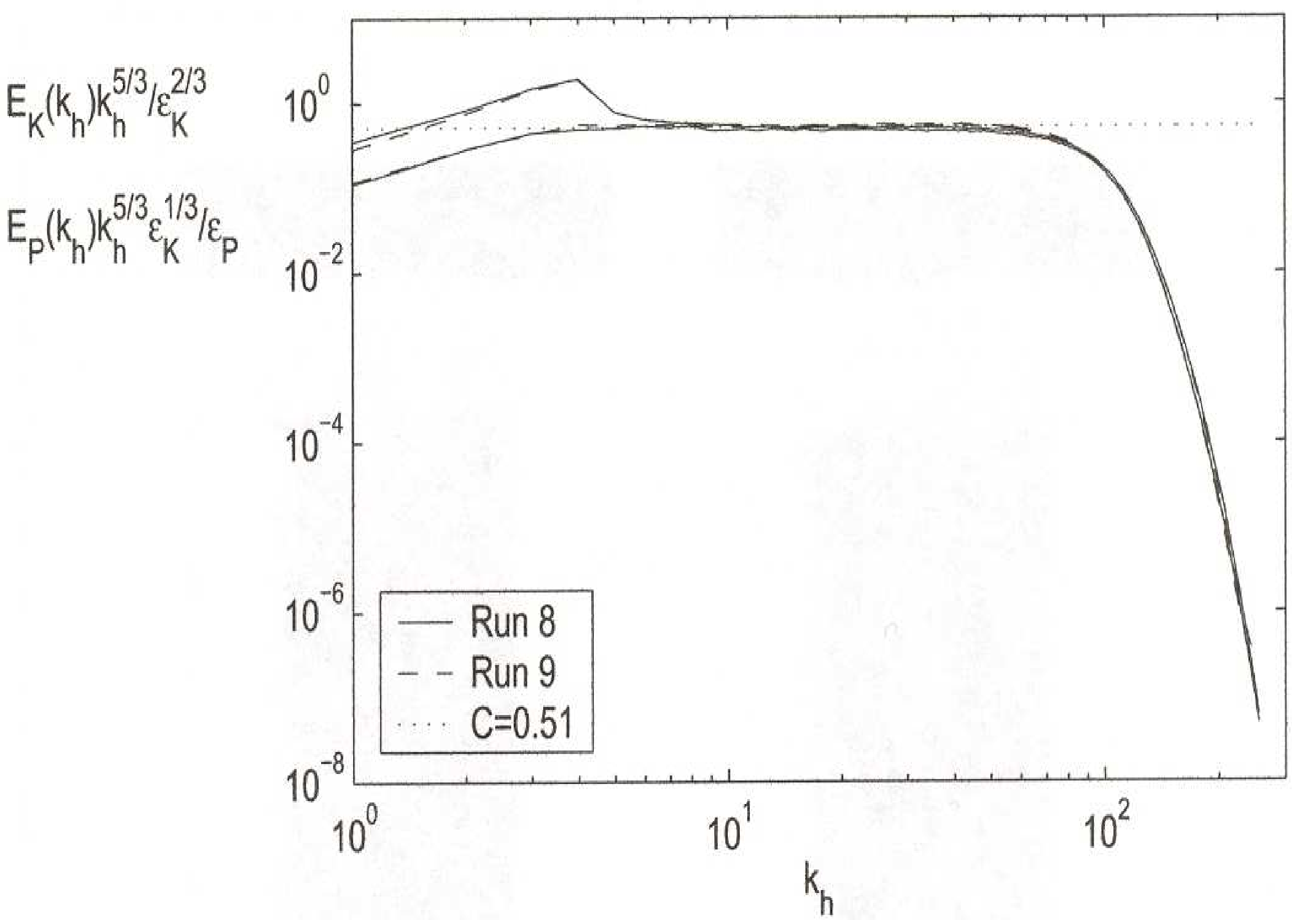
Horizontal kinetic energy spectra at $t = 0, 10, 20, 30$; $F_\ell = 4$, $R_\ell = 6400$.

Horizontal Kinetic Energy Spectra



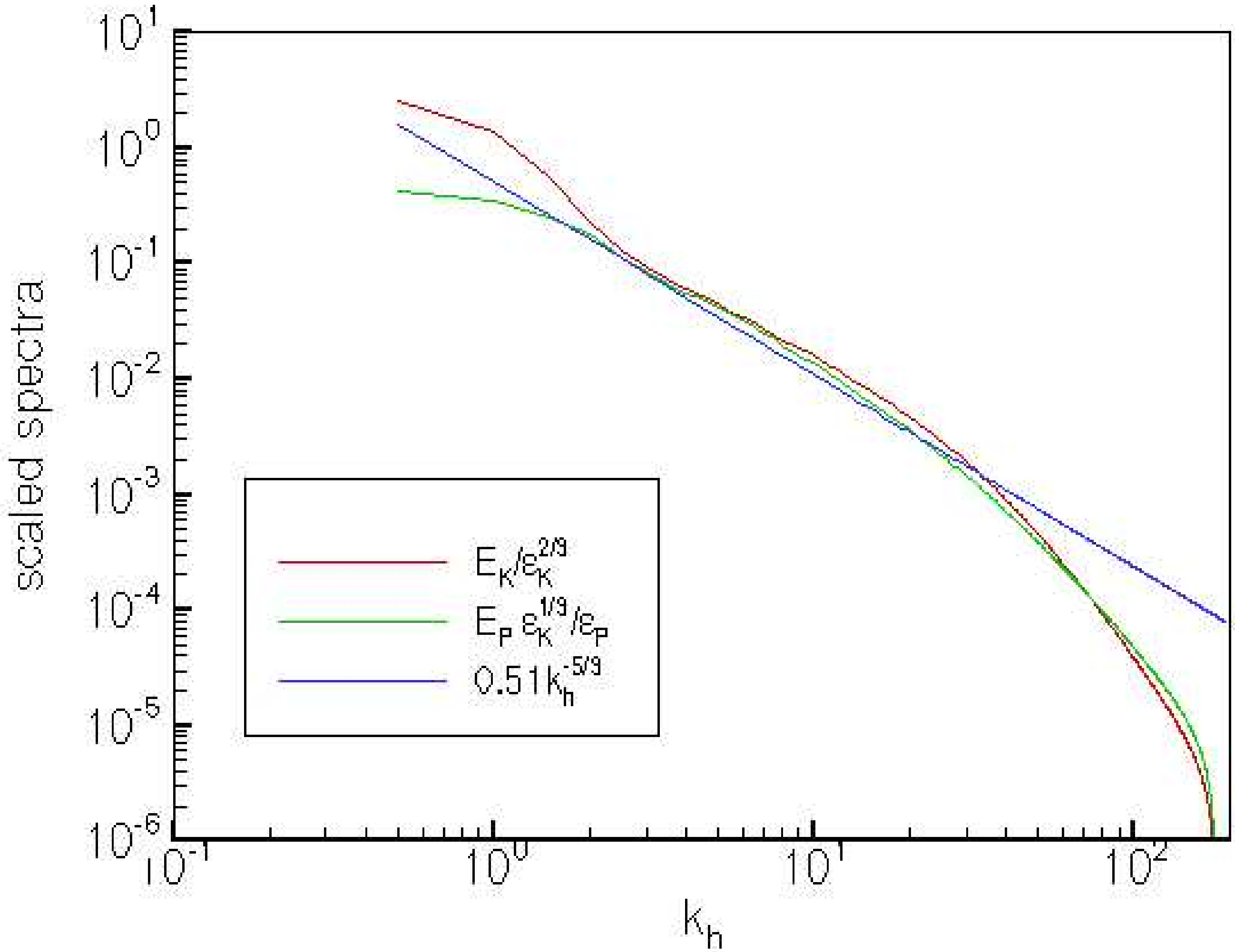
Horizontal kinetic energy spectra at $t = 20$ for four different R_ℓ cases.

Scaled Horizontal Energy Spectra



Scaled horizontal energy spectra, Lindborg (2005).

Horizontal Kinetic Energy Spectra – Lindborg Scaling



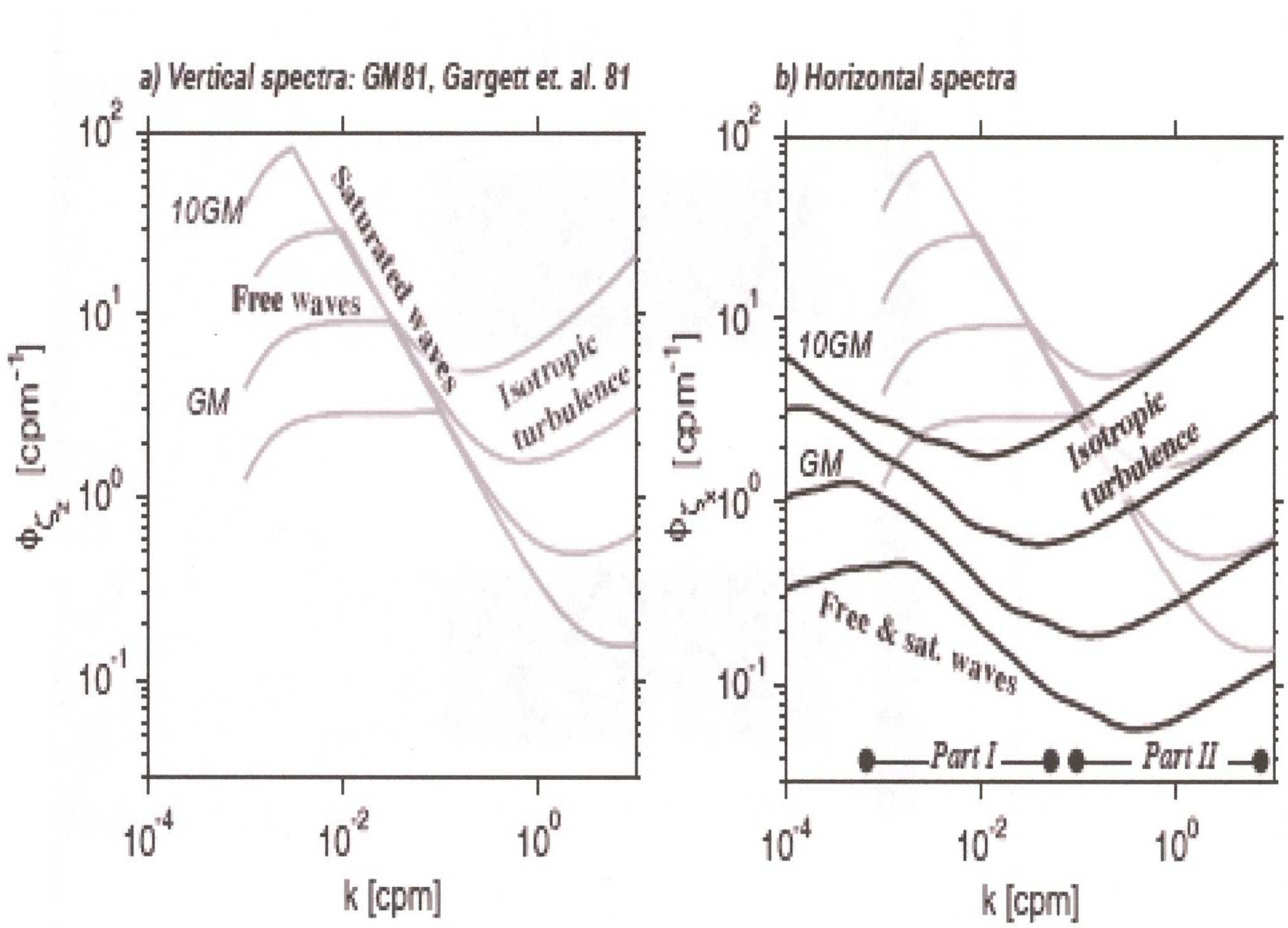
Horizontal kinetic energy spectra at $t = 18.5$ for $R_\ell = 9600$ case.

Implications

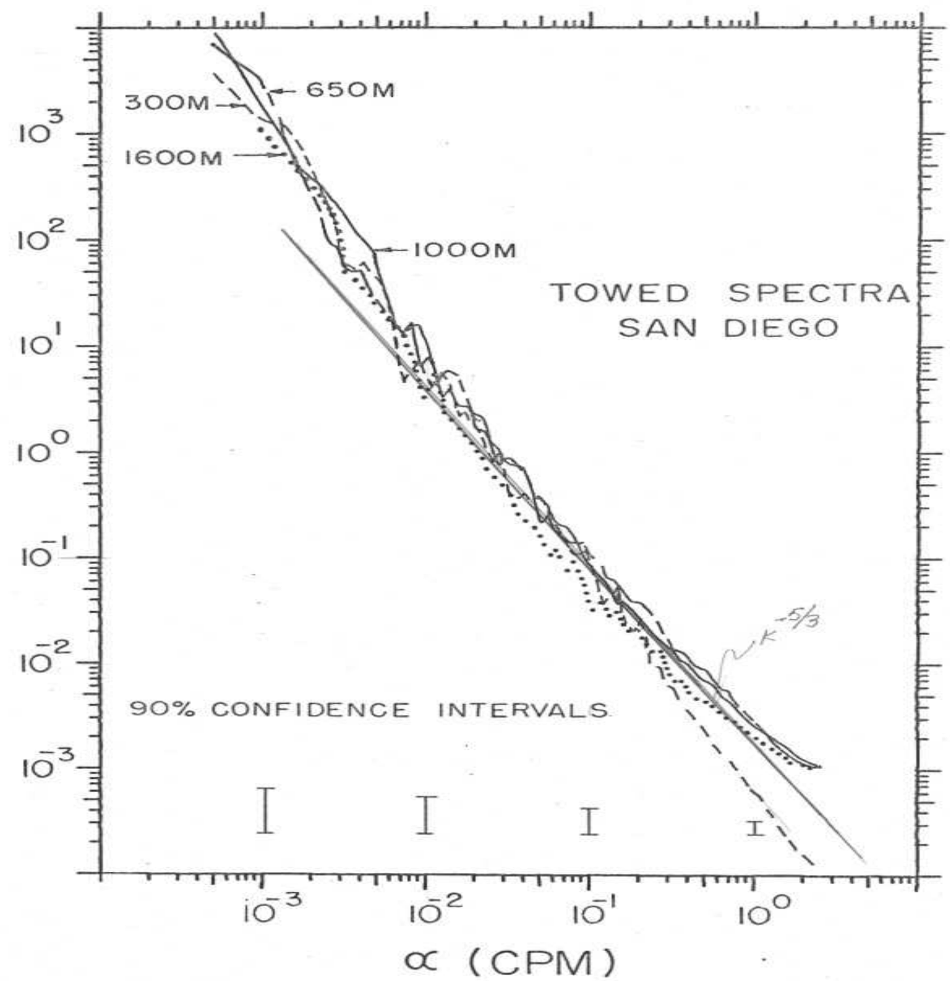
- Potential for stratified turbulence 'inertial cascade' for large R_ℓ (Riley and de Bruyn Kops, 2003; Lindborg, 2005)
  - if $F_\ell \ll 1$, with $\ell_i \sim u'/N$, then $\ell_H/\ell_i \sim \ell_H N/u' = 1/F_\ell$, so $\ell_H \gg \ell_i$
  - if $F_\ell \ll 1$, $R_\ell \gg 1$, $Ri \sim 1$
    * highly anisotropic 'inertial' subrange in the horizontal
      · spectral dependence only on ϵ, χ and k
    * $E_u(\kappa_H) = \mathcal{C}_u \, \epsilon^{2/3} \kappa_H^{-5/3}$
    * $E_\theta(\kappa_H) = \mathcal{C}_\theta \chi \epsilon^{-1/3} \kappa_H^{-5/3}$
    * $d_H^2(t) = \mathcal{C}_d \, \epsilon \, t^3$ (patch size; possibly)

- Potential for Kolmogorov cascade
  - if $F_\ell R_\ell^{3/4} \gg 1$, then $\ell_i \gg \eta$

Shear Spectra – Ocean (Klymak, 2005)

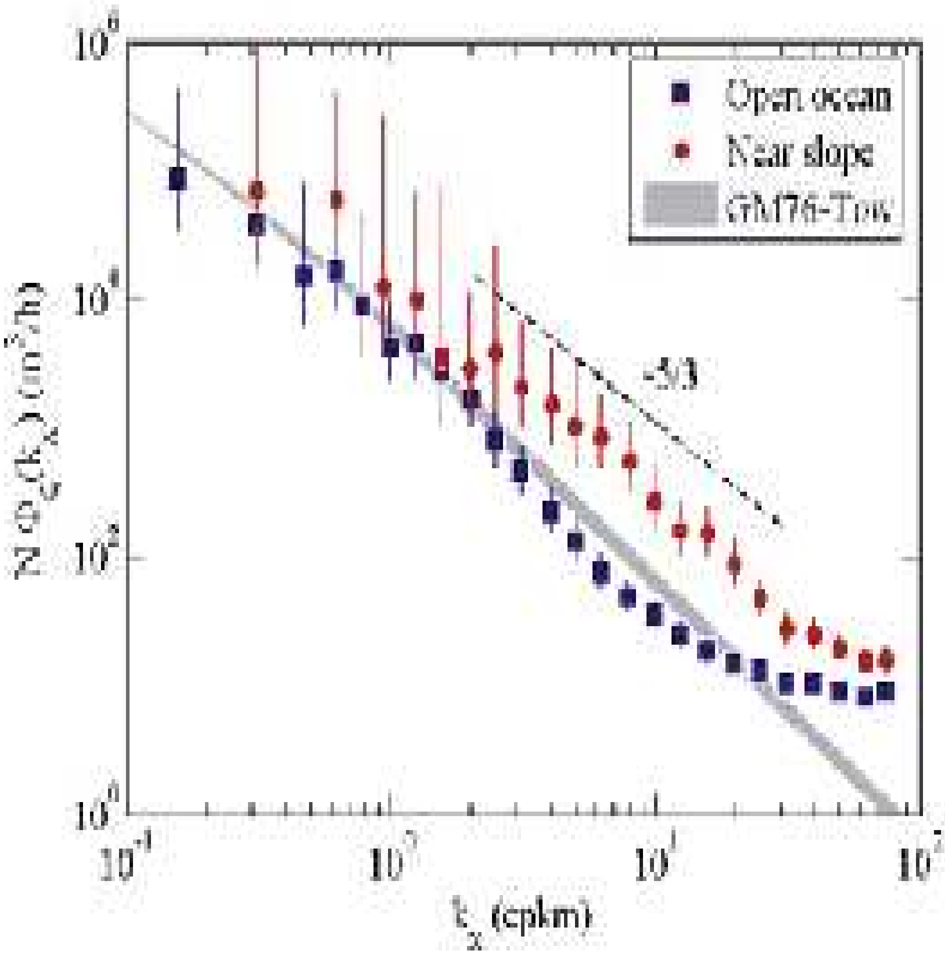


Temperature spectra – Ocean (Ewart, 1976)



Power spectra of temperature off the coast of San Diego (30^{o}N, 124^{o}W).

Displacement spectra – Ocean (Hollbrook & Fer, 2005)



Vertical displacement spectra from open ocean (squares) and near slope (dots)

Summary of Field Results

- Field experiments
  - 3-D turbulence is very intermittent, sporadic
  - Often observe in the oceans at scales $\ell_O < \ell_H < 100$'s m
    - horizontal spectra in velocity, temperature consistent with $\kappa_H^{-5/3}$
    - vertical spectra more consistent with κ_V^{-3}
  - not classical Kolmogorov-Oboukov-Corrsin spectra
    - highly nonisotropic
    - scales are much too large
    - influence of stable density stratification
  - consistent with numerical simulations, scaling arguments

Conclusions

- (At least) two types of dynamics are present – with Ri initially large
  - horizontal growth of larger-scale, quasi-horizontal motions
  - continual decrease in vertical scales (as suggested by Lilly, 1983)
    - there is strong tendency for vertical shearing of the horizontal velocity to develop
    - this leads to local instabilities, 'classical' turbulence and mixing
    - this process occurring intermittently in space causes a downscale transfer of energy
- Both upscale and downscale spectral transfer of energy in the horizontal
  - spectral transfer is very nonistropic

Conclusions (cont'd)

- Statistics of larger-scale motions relatively unaffected by changing R_ℓ, if R_ℓ is large enough
  - u', ϵ, χ and ℓ_H become approximately independent of R_ℓ
    - $\epsilon \sim u'^3/\ell_H, \quad \chi/\epsilon \simeq 0.43$
    - $\lambda \sim R_\ell^{-1/2}, \quad \left\langle \left(\frac{\partial u}{\partial z} \right)^2 \right\rangle \sim R_\ell$
  - smaller-scale motions adjust to the larger-scale ones
  - w', ρ' show more dependence on R_ℓ
    - their statistics depend more on smaller-scale motions

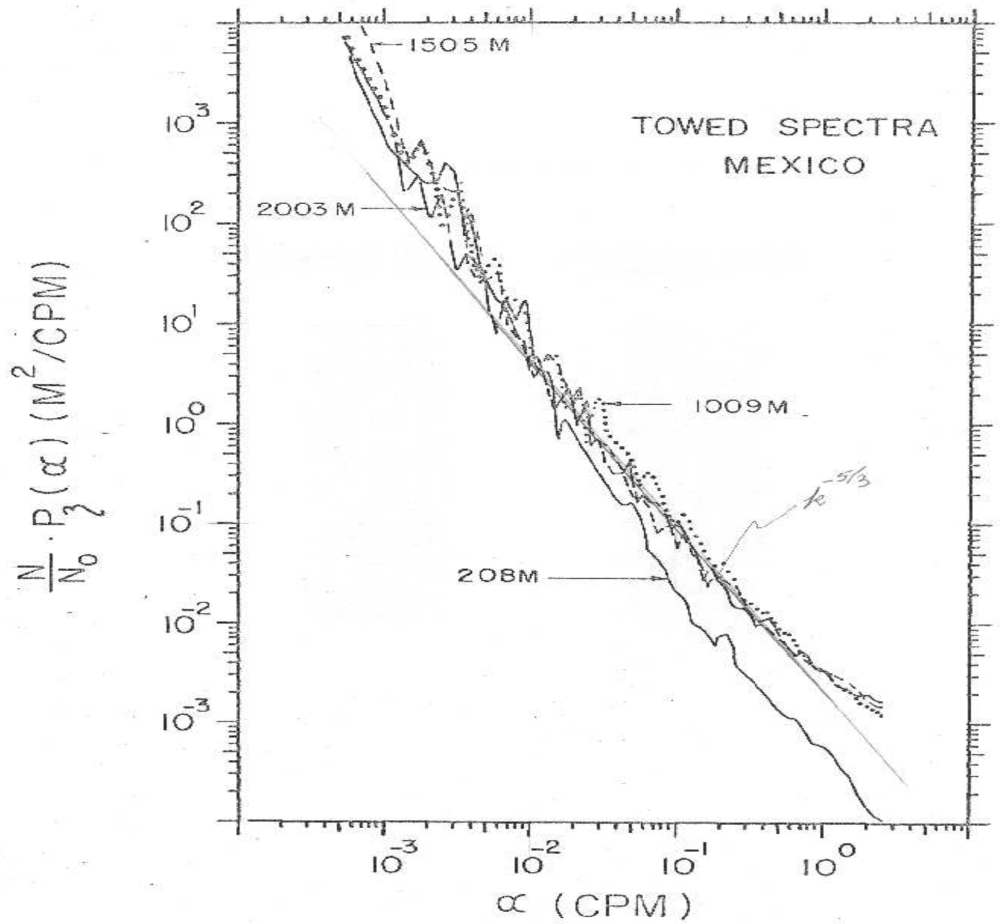
Conclusions (cont'd)

- There are several important scales in this problem
  - horizontal, energy-containing scales continue to grow (ℓ_H)
  - instability scale (ℓ_i) behaves as: $\ell_i/\ell_H \sim (u'/N\ell_H) \ll 1$ since $Ri \sim 1$
  - Ozmidov scale ℓ_O behaves as: $\ell_O/\ell_i \sim \left(u'/N\ell_H\right)^{1/2}$
    - stratification effects 'strong' for $\ell > \ell_O$
  - Taylor scale (λ):
    - decreases with time prior to appearance of 'classical' turbulence
    - behaves as: $\lambda/\ell_H \sim (u'\ell_H/\nu)^{-1/2}$ after flow becomes turbulent
  - Kolmogorov scale (η) behaves as: $\eta/\ell_H \sim (u'\ell_H/\nu)^{-3/4}$
  - Expect: $\ell_H \gg \ell_i > \ell_O > \lambda > \eta$

Conclusions (cont'd)

- Results suggest that, if the flows do not laminarize, they should approximately apply to geophysical flows
  - in laboratory experiments, numerical simulations
    - this could be a problem in the $F_\ell \leq \mathcal{O}(1)$ range
- Potential for stratified turbulence 'inertial cascade' (Lindborg; Riley and de Bruyn Kops)
  - if $F_\ell \ll 1$, then $\ell_H \gg \ell_i$
    - highly nonisotropic 'inertial' subrange
    - possible explanation for scaling range in field data

Temperature spectra – Ocean (Ewart, 1976)



Power spectra of temperature off the coast of Mexico (21^{o}N, 110^{o}W).

Temperature spectra – Ocean (Ewart, 1976)

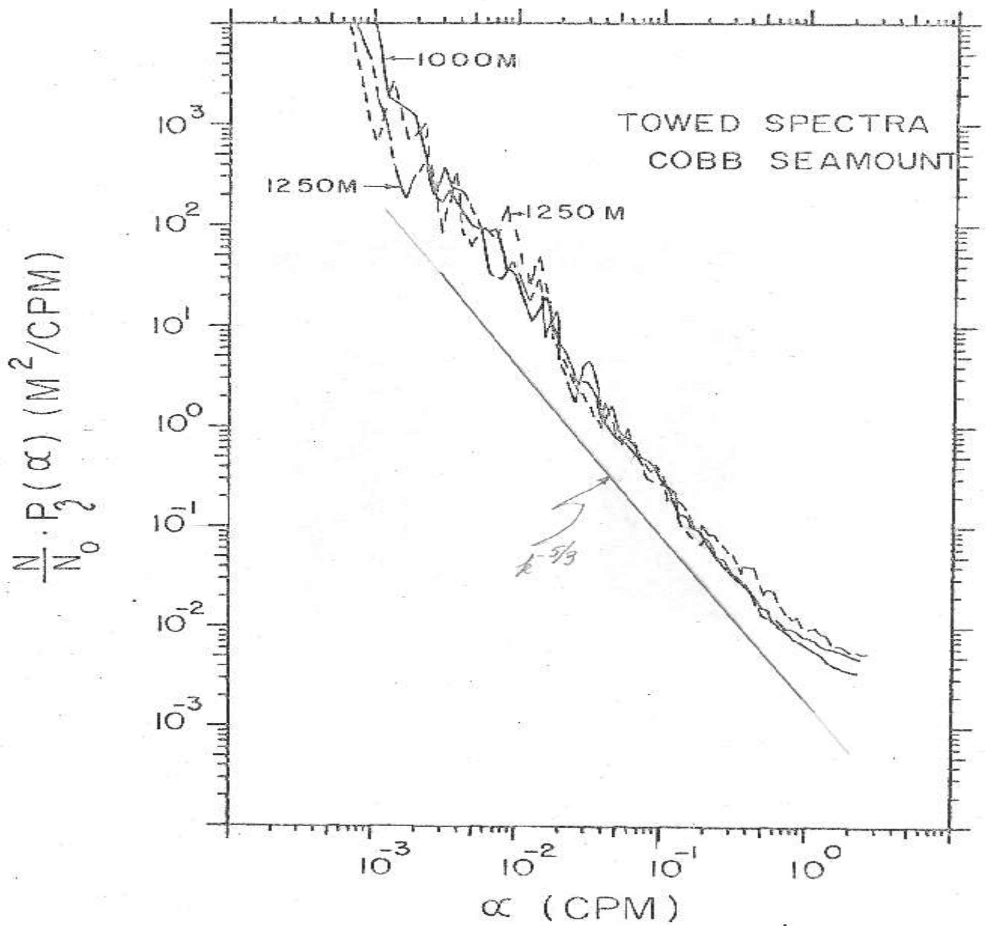
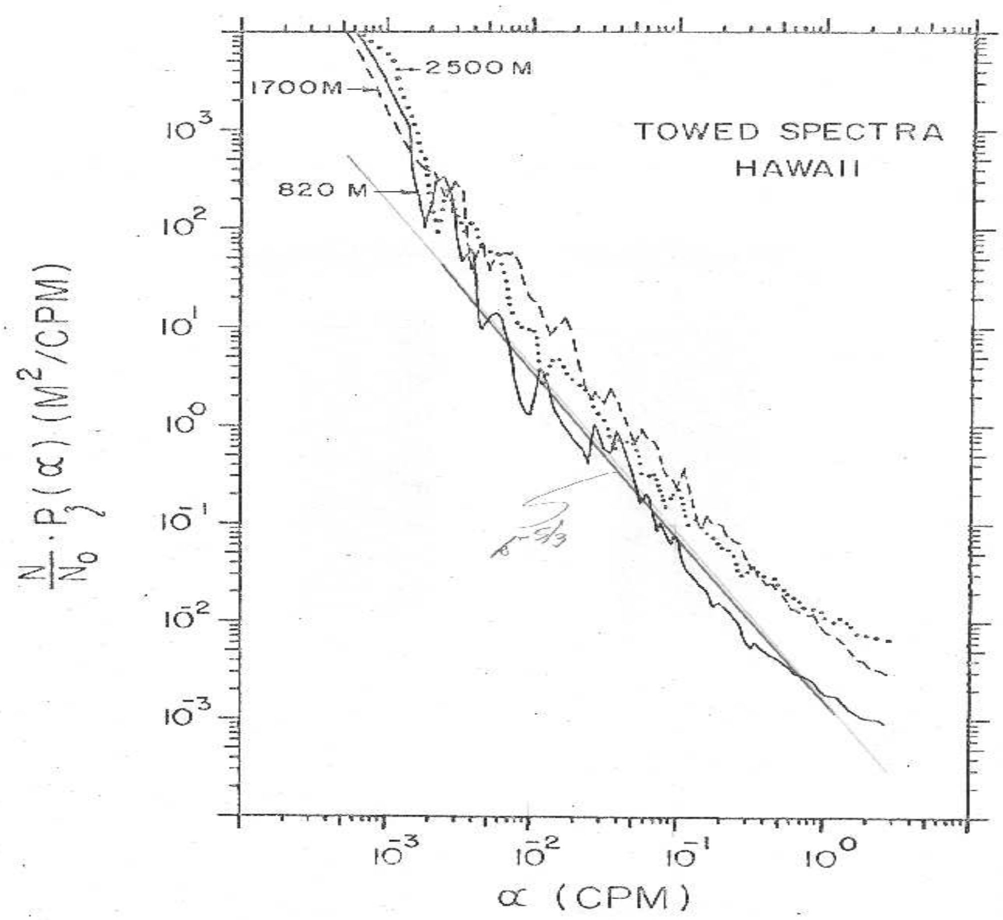


FIGURE 6.

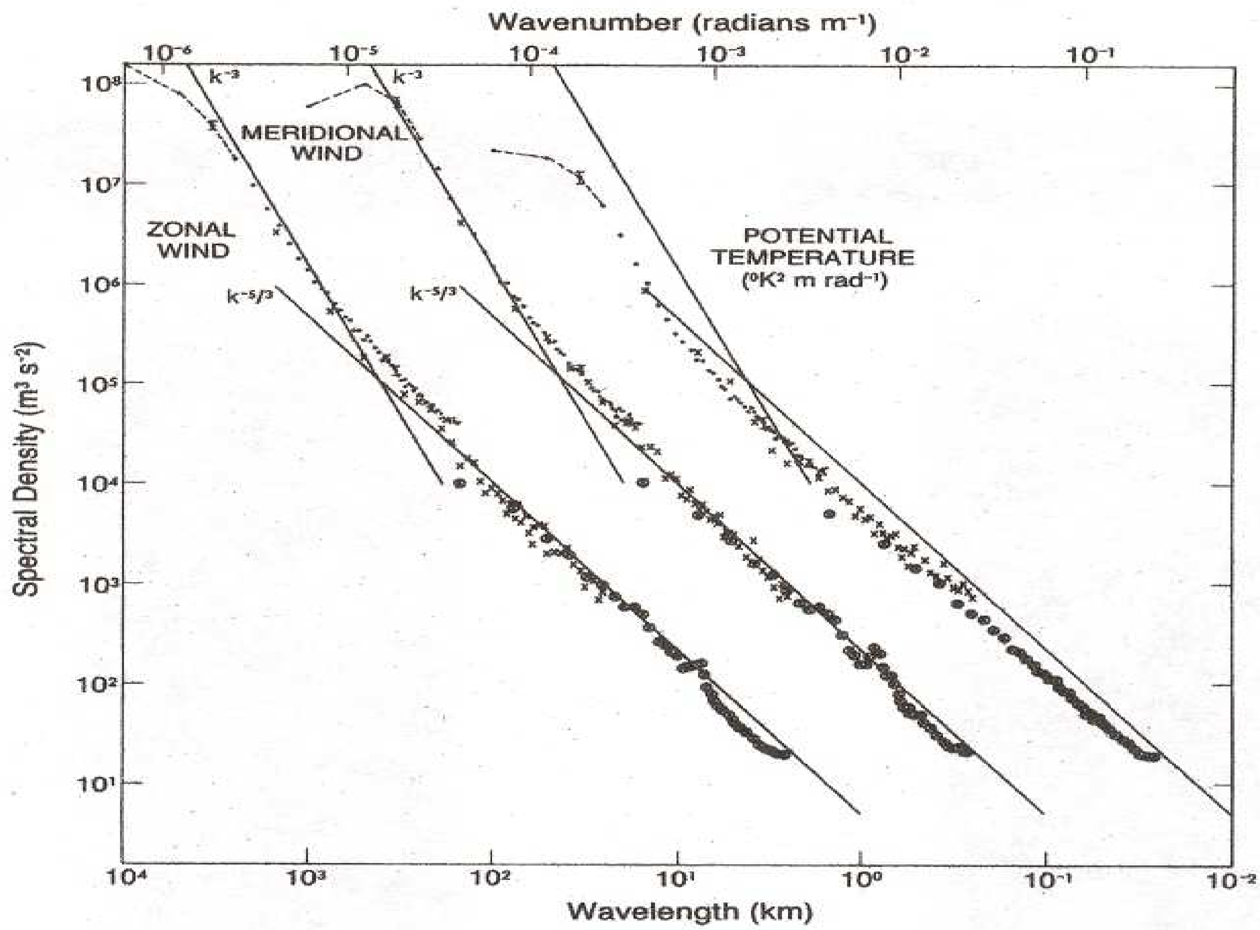
Power spectra of temperature near Cobb Seamount (47^{o}N, 131^{o}W).

Temperature spectra – Ocean (Ewart, 1976)



Power spectra of temperature near Hawaii (20^{o}N, 156^{o}W).

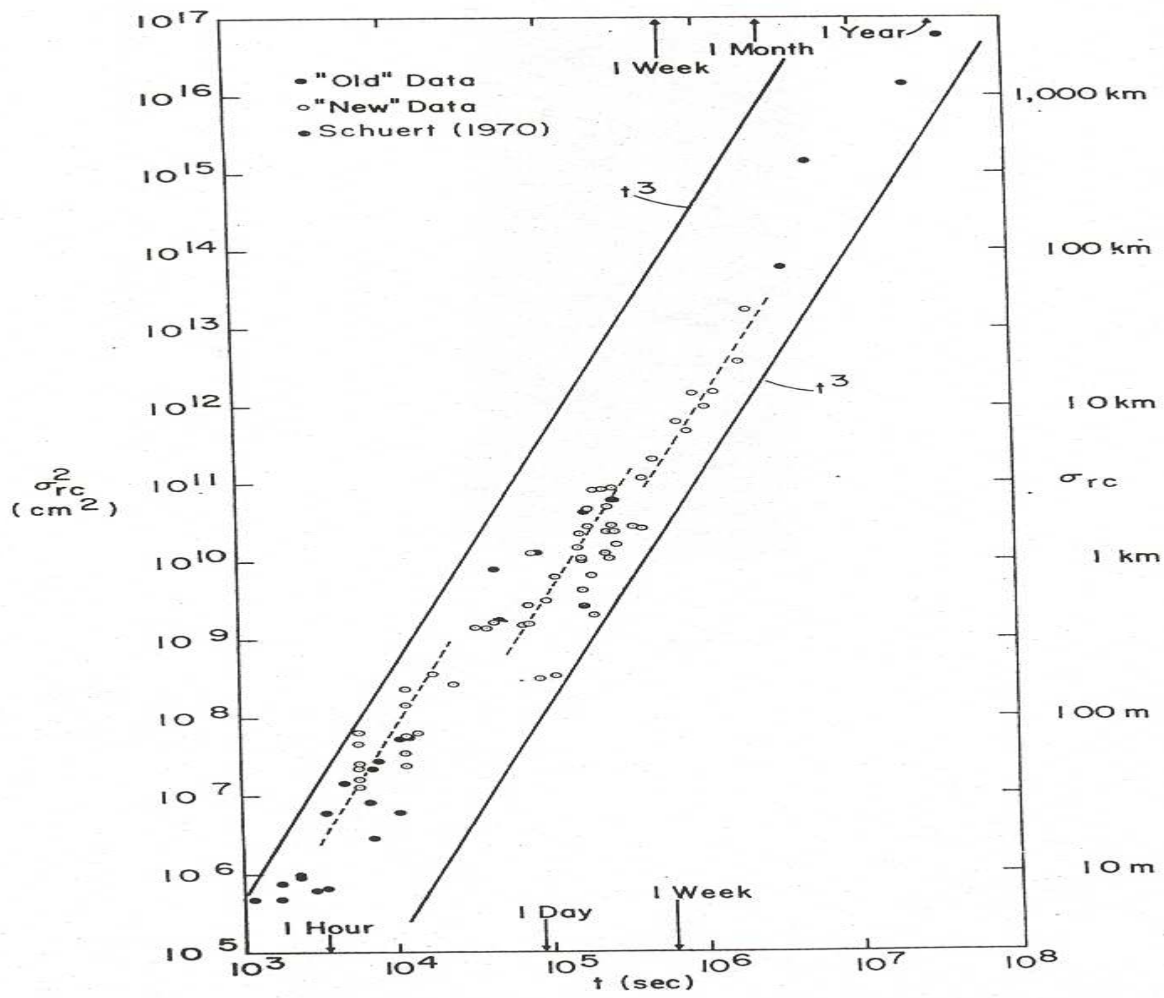
Power Spectra – Atmosphere



Zonal, meridional wind, and potential temperature (Nastrom and Gage, 1985)

Mean Square Patch Size – Ocean

Okubo, Deep-Sea Research, 1971



Spectra of Available Potential Energy – Ocean

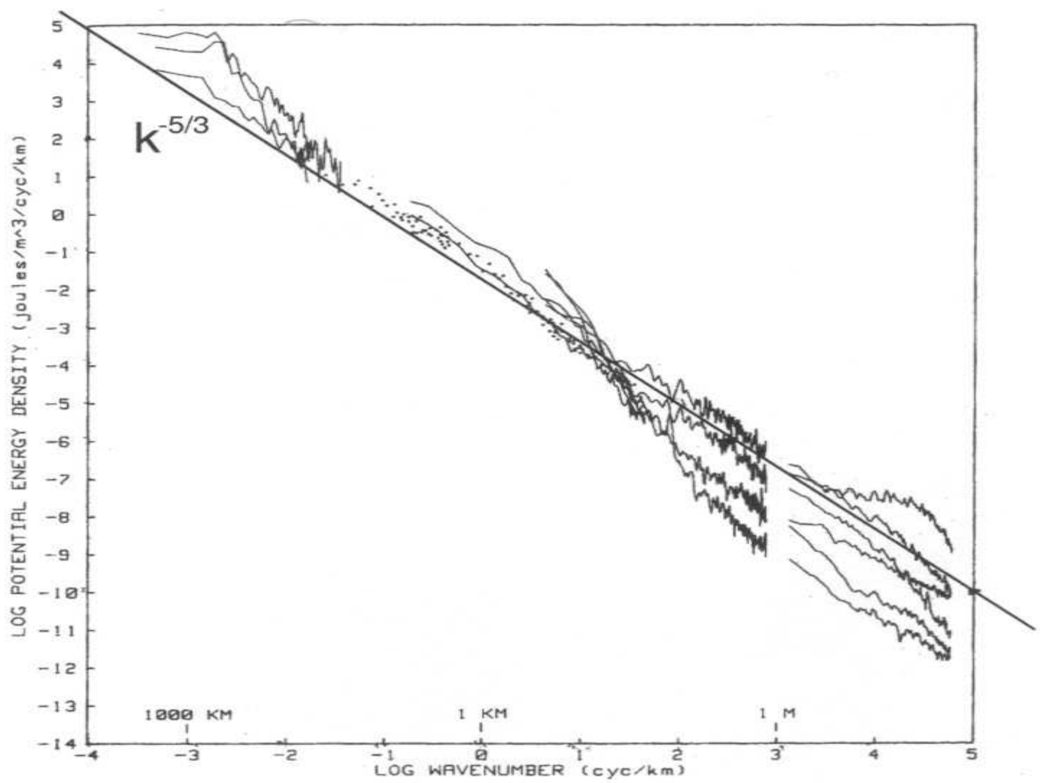


Figure 4: Horizontal wave number spectra of available potential energy in the ocean, collected from different observations. Reproduced from Dugan *et al.* (1986). We have inserted a straight line representing a $k_h^{-5/3}$-curve.

Spectra of available potential energy in horizontal (Dugan et al., 1986)

Temperature Structure Function – Ocean

Voorhis and Perkins, Deep-Sea Research, 1966

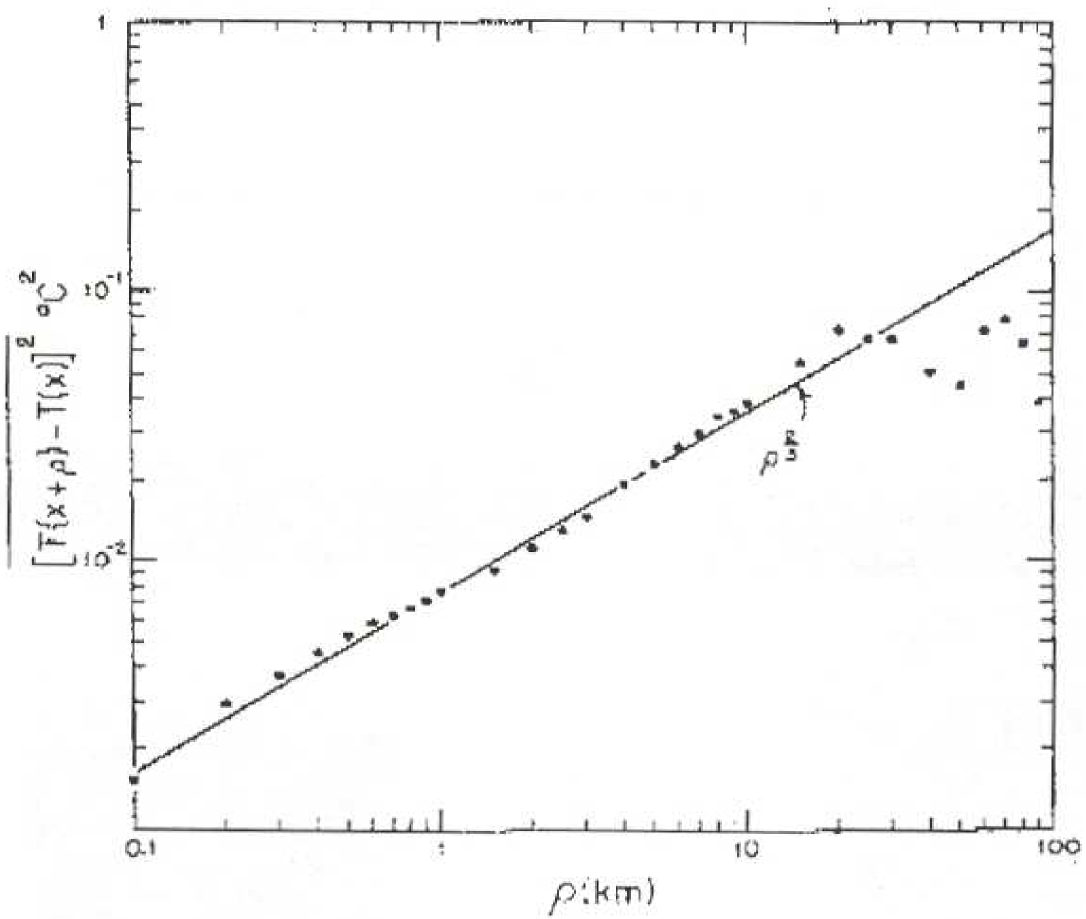


Fig. 10. Temperature structure function along the eastward tow track.

Temperature Spectrum – Ocean

Lafond and Lafond, 1967, Marine Technical Society

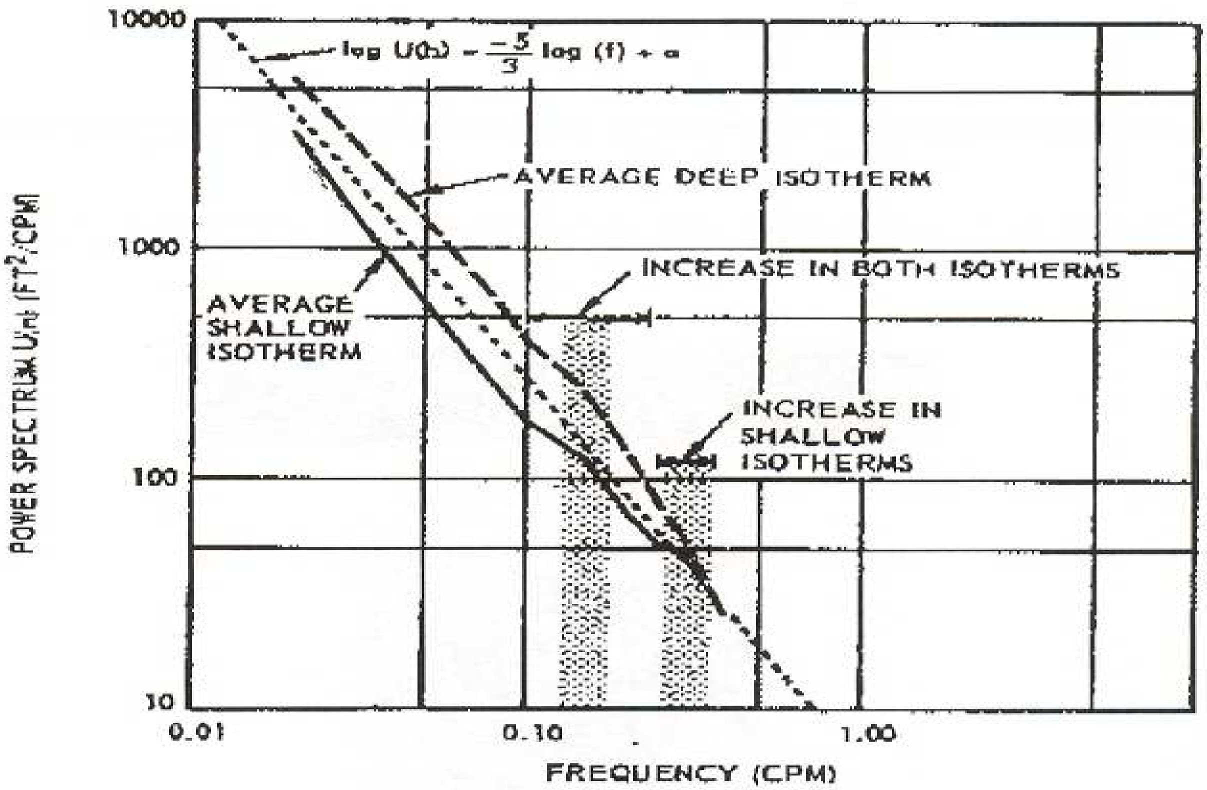
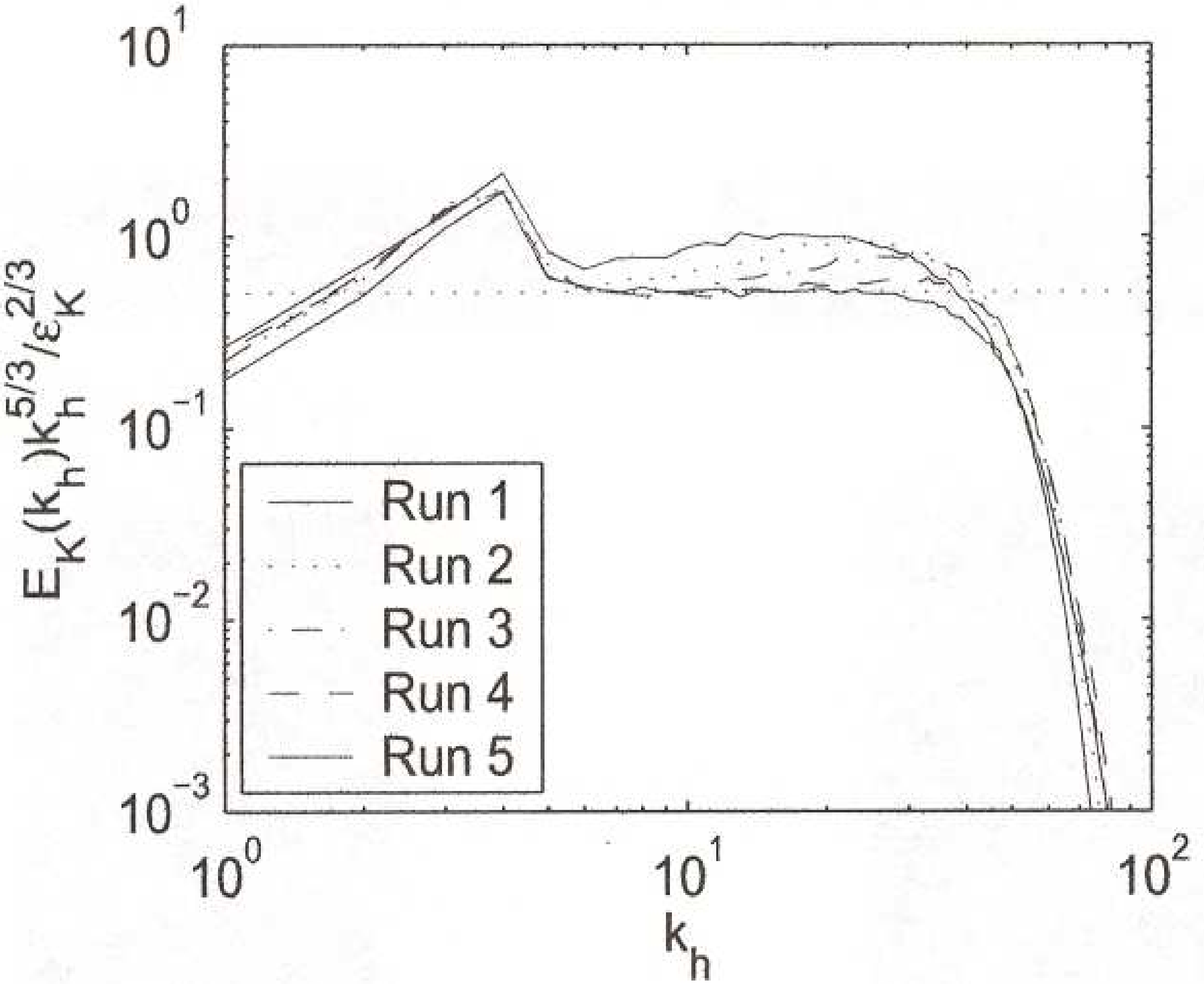


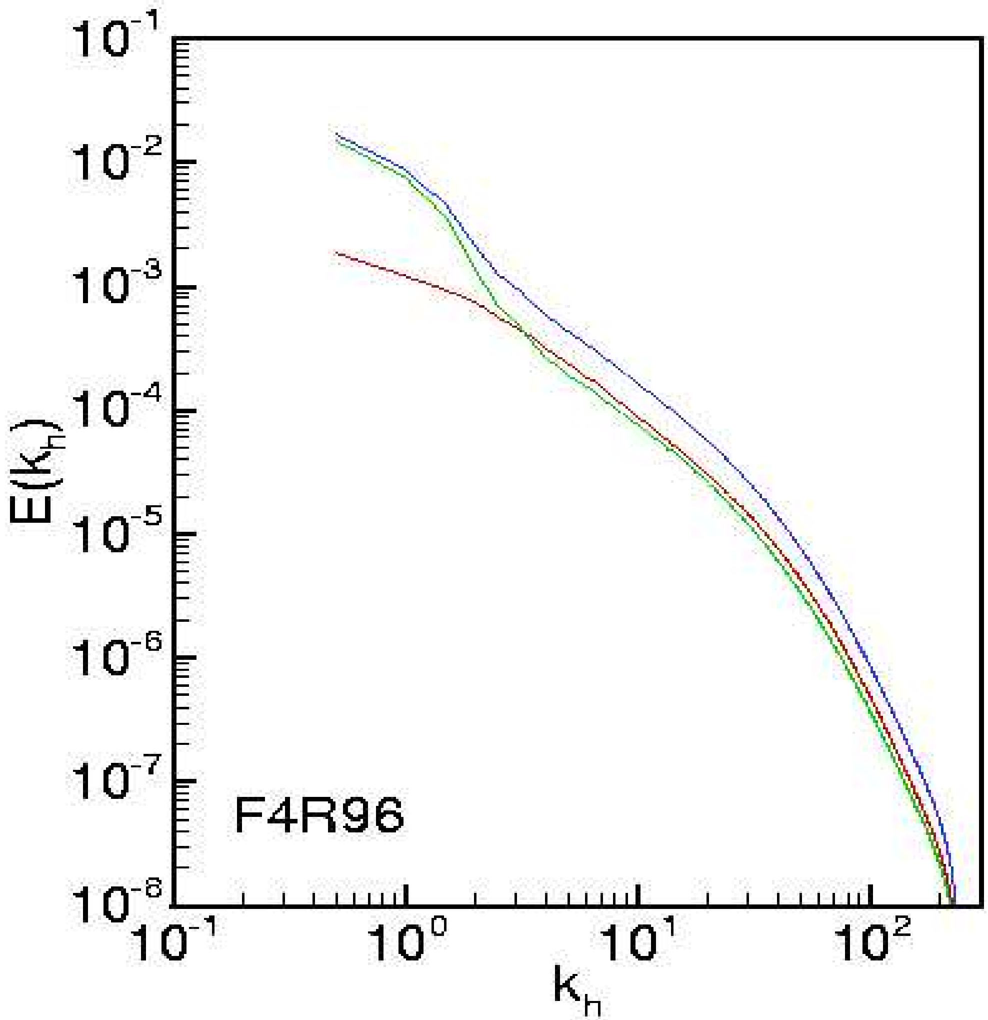
Figure 18. Average power spectrum of 25 sections of isotherm depths in the main thermocline and in isotherm depths just below the main thermocline. The slope of −5/3 in the log-log relationship is shown for comparison.

Scaled Horizontal Kinetic Energy Spectra



Scaled horizontal kinetic energy spectra, Lindborg (2005)

Wave/Vortex Kinetic Energy Decomposition



Horizontal kinetic energy spectra (Riley & deBruynKops, 2003)

Taylor-Green Flow

- Initial velocity and density fields:

$$\begin{aligned}\mathbf{v}(\mathbf{x},0) &= \mathcal{U}\cos(\kappa z)\Big[\cos(\kappa x)\sin(\kappa y), -\sin(\kappa x)\cos(\kappa y), 0\Big] \\ &+ \text{broad-banded, low-level noise} \\ \rho(\mathbf{x},0) &= 0\end{aligned}$$

  - In all cases, exact same initial conditions, except for F_ℓ and R_ℓ
  - For $N = 0$, flow develops into isotropic turbulence, with symmetries (without noise, Brachet et al., 1983)

Taylor-Green Flow (cont'd)

- Simulations discussed today: with $\ell = 1/\kappa$

$$F_\ell = \frac{2\pi\mathcal{U}}{N\ell} = 4\,, \quad R_\ell = 800,\ 1600,\ 3200,\ 6400$$

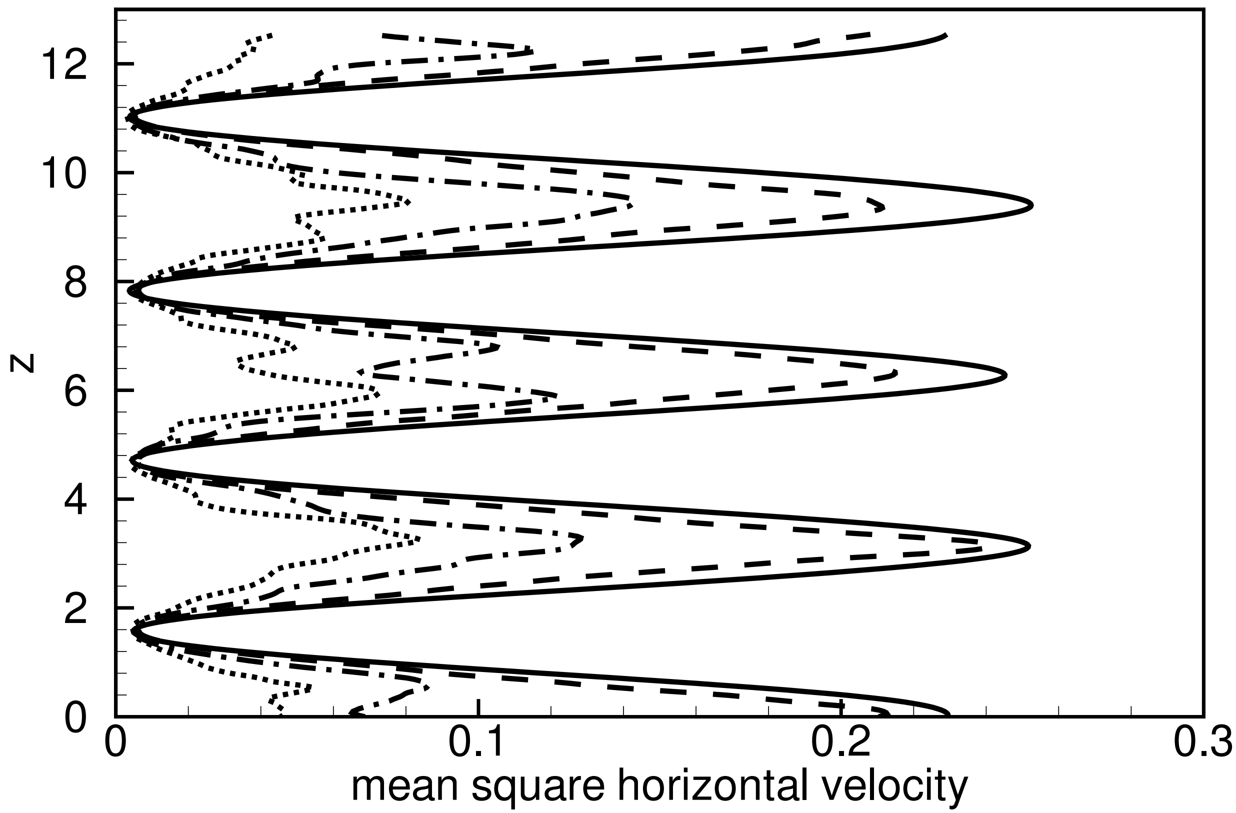
$$T_B = \frac{2\pi}{N} = 4\,, \quad T_A = \frac{\ell}{\mathcal{U}} = 1$$

  – similar results for $F_\ell = 2$

  – have now computer the range $200 \leq R_\ell \leq 9600$

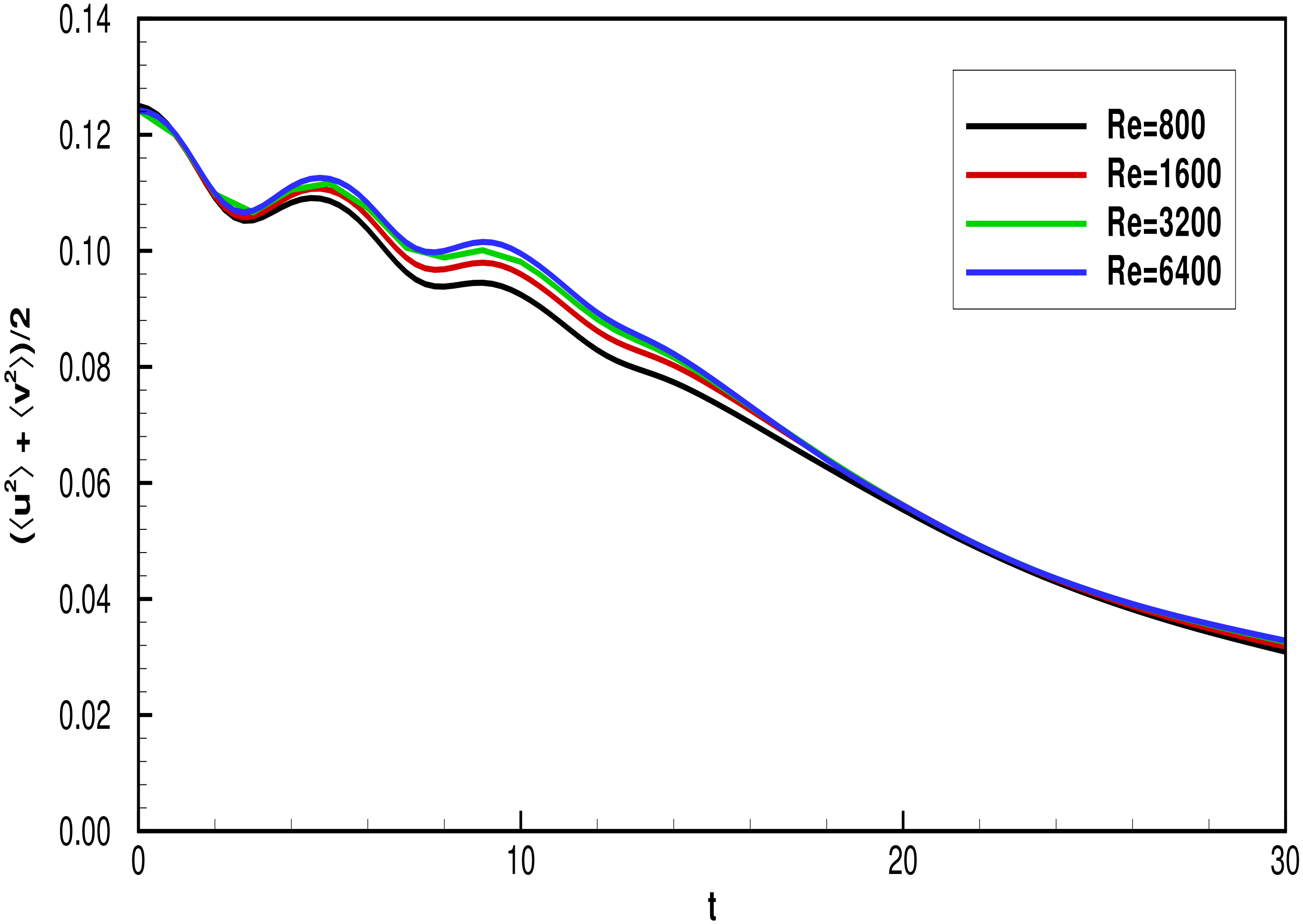
  – spans the range from laminar to very active turbulence

Mean Square Velocity $\langle u^2 \rangle_H$ versus z



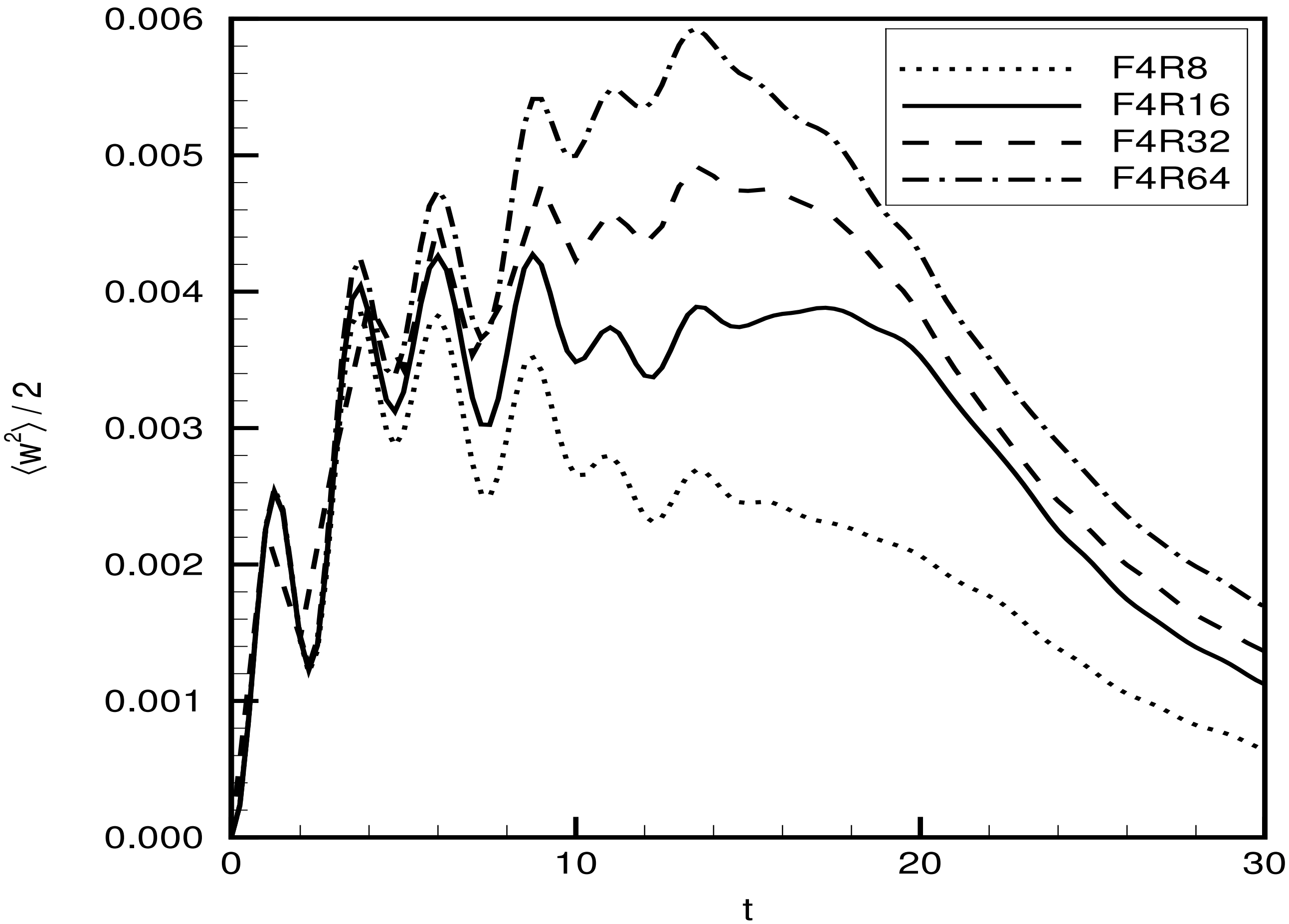
Mean square horizontal velocity vs z.
$F_\ell = 4$ and $R_\ell = 6400$ at $t = 0, 10, 20, 30$.

Horizontal Kinetic Energy versus Time



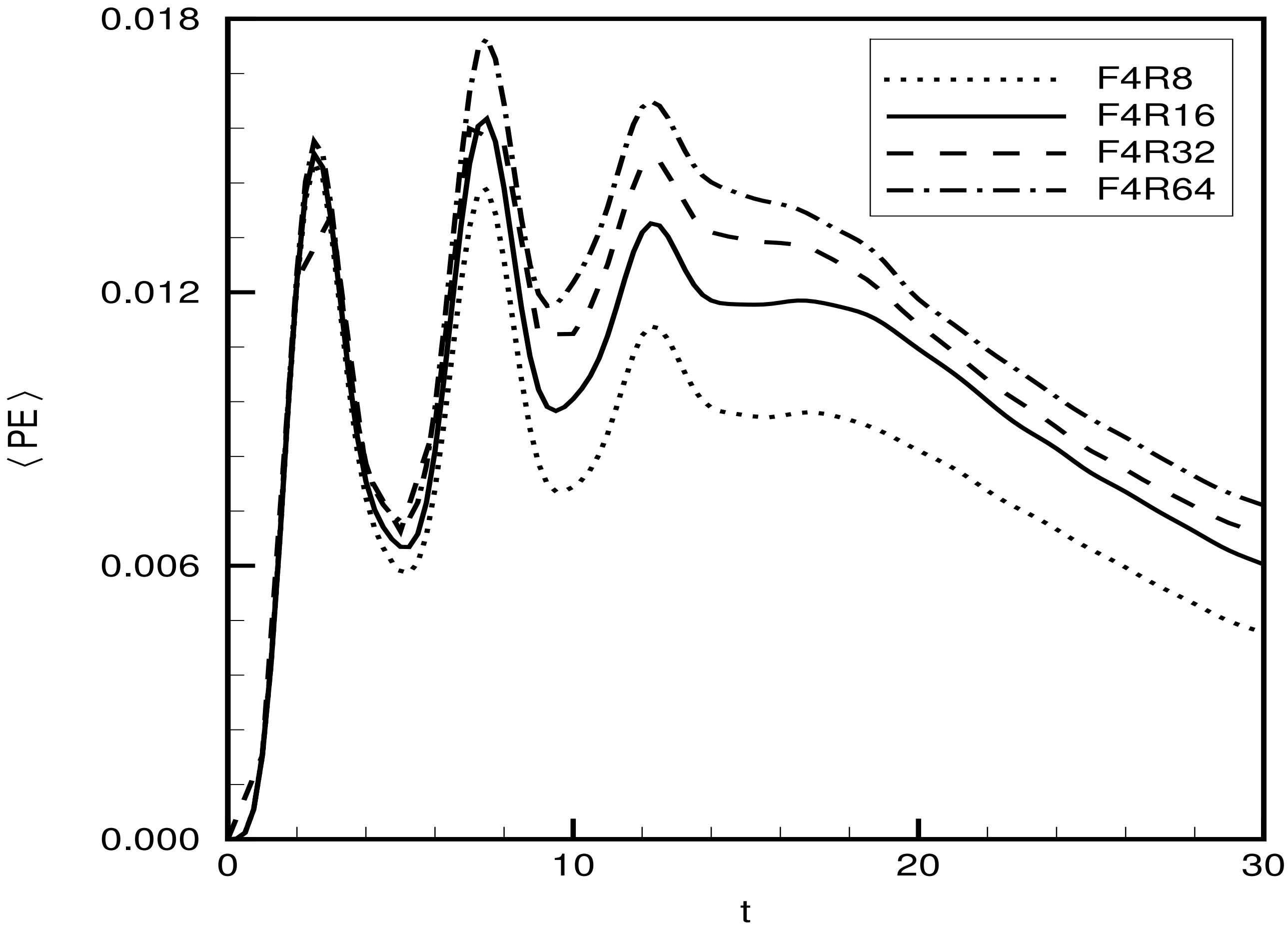
Volume-averaged horizontal kinetic energy vs t, $F_\ell = 4$, various R_ℓ.

Vertical Kinetic Energy versus Time



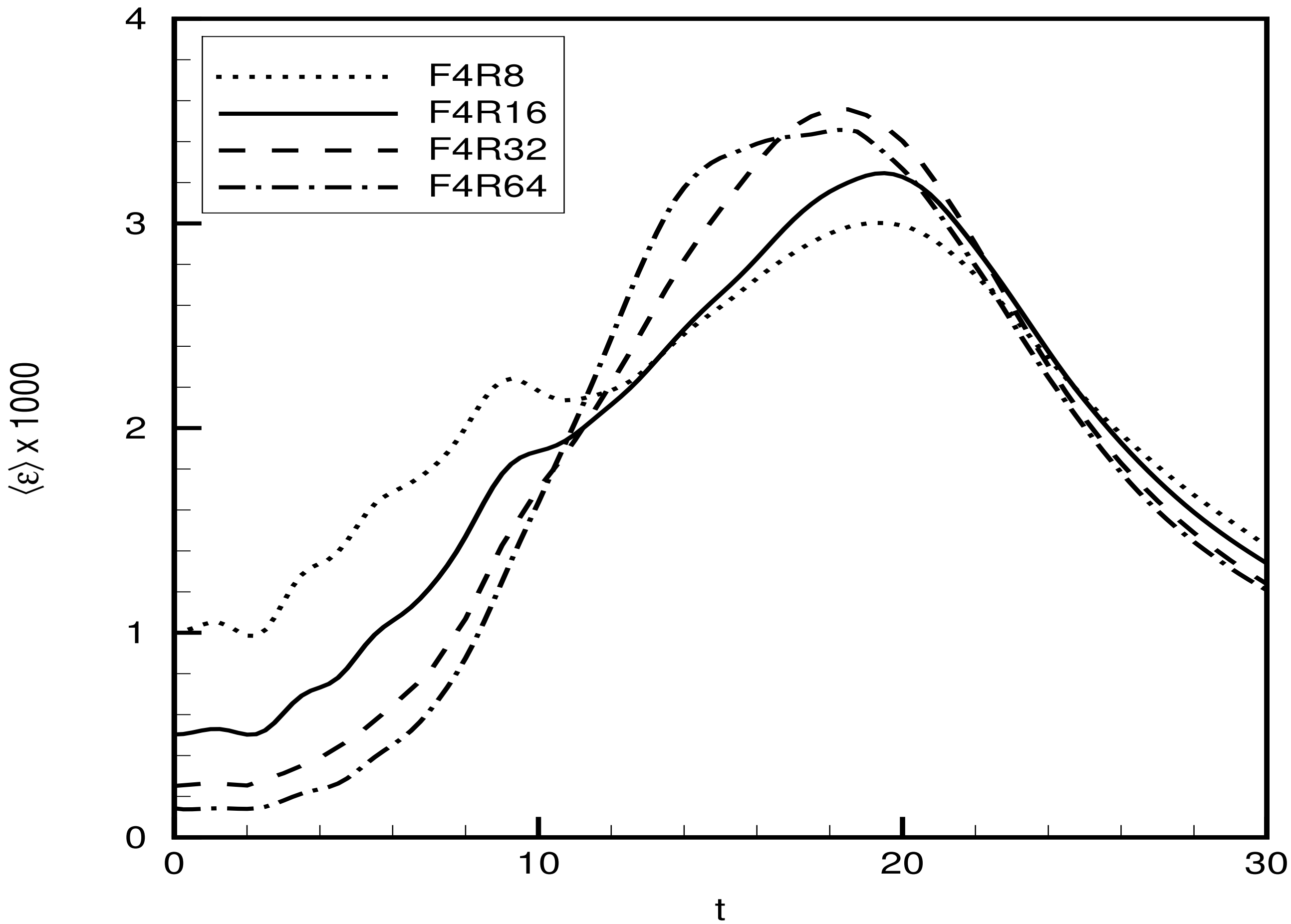
Volume-averaged vertical kinetic energy vs t, $F_\ell = 4$, various R_ℓ.

Potential Energy versus Time



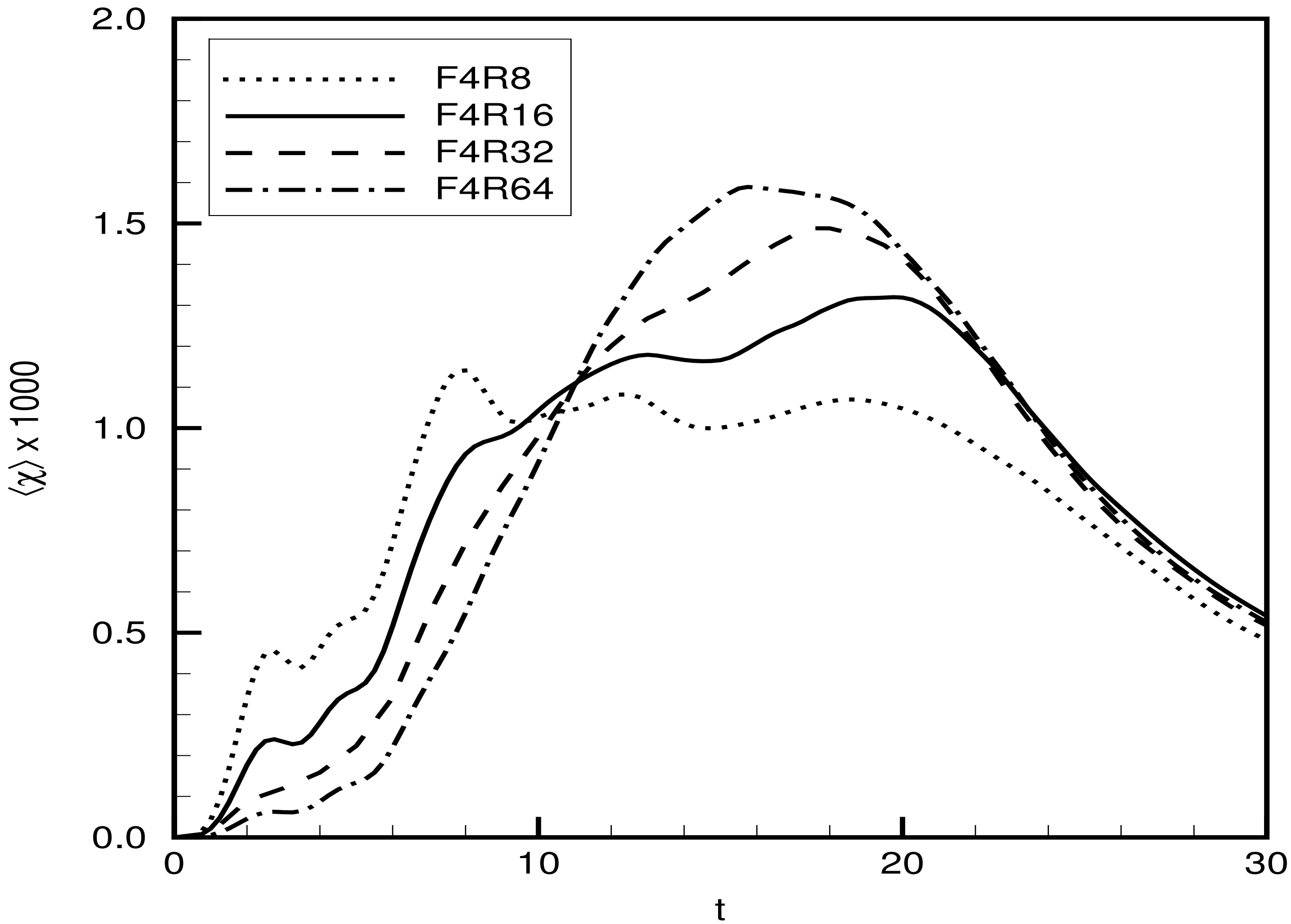
Volume-averaged potential energy vs t, $F_\ell = 4$, various R_ℓ.

Kinetic Energy Dissipation Rate versus Time



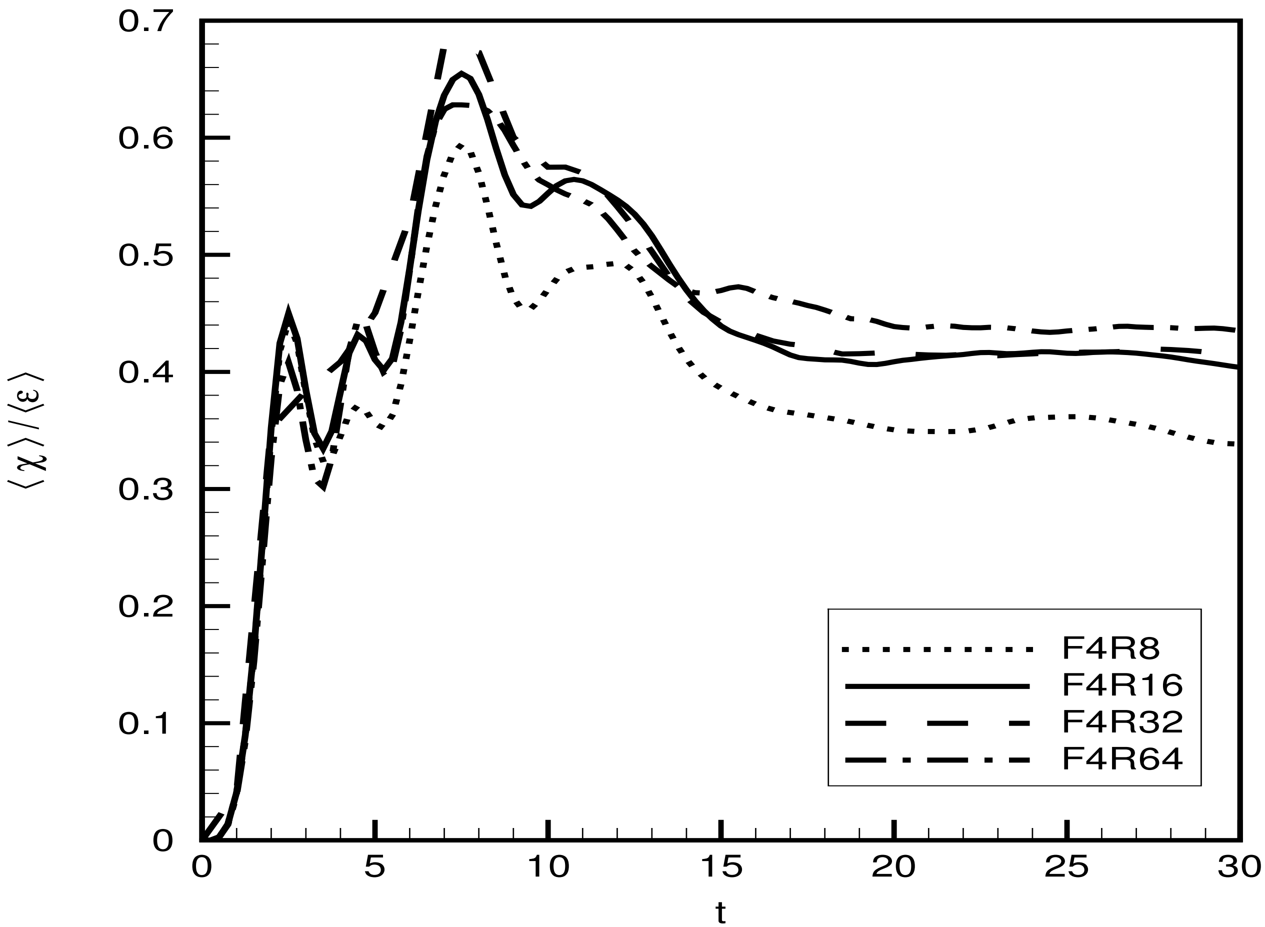
Volume-averaged kinetic energy dissipation rate vs t, $F_\ell = 4$, various R_ℓ.

Potential Energy Dissipation Rate versus Time

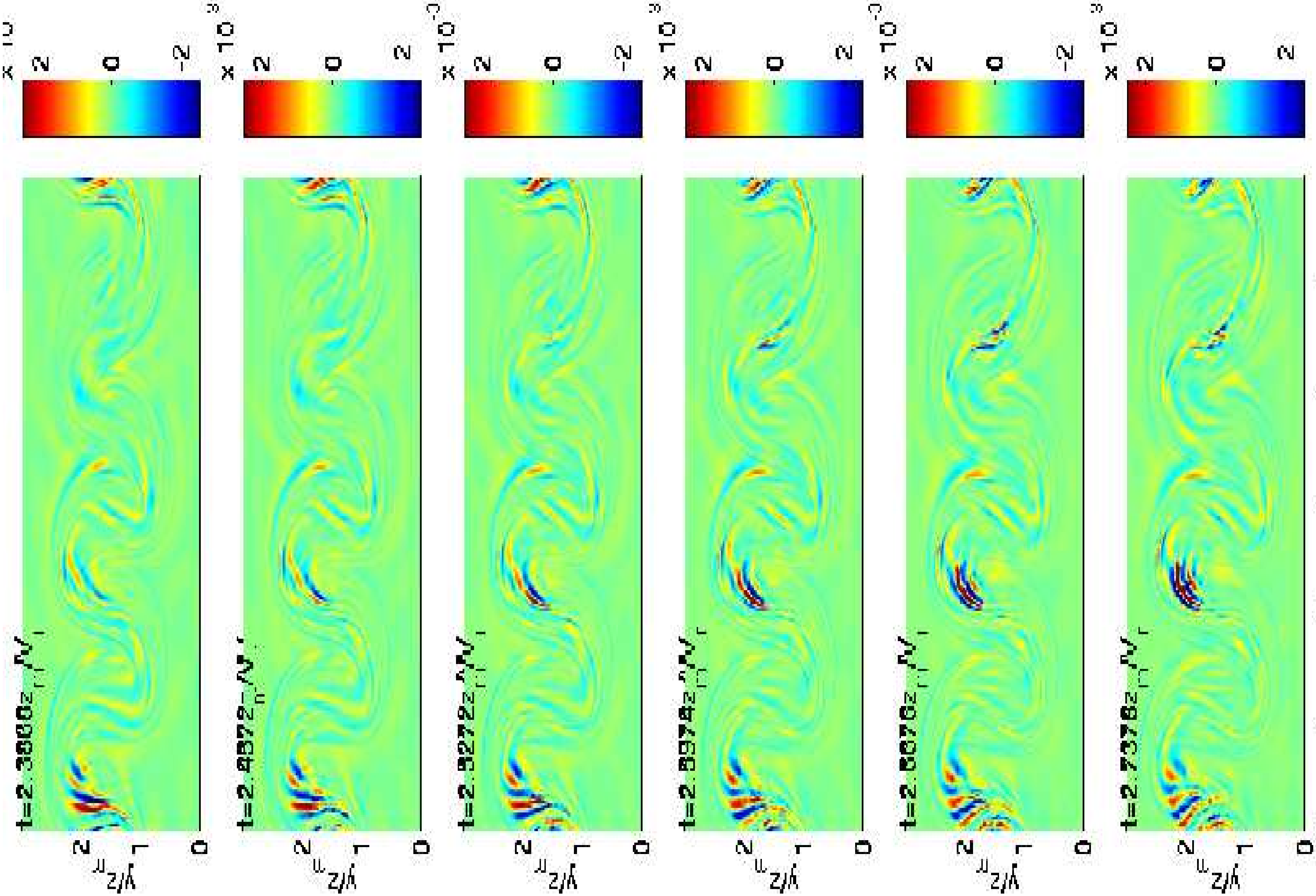


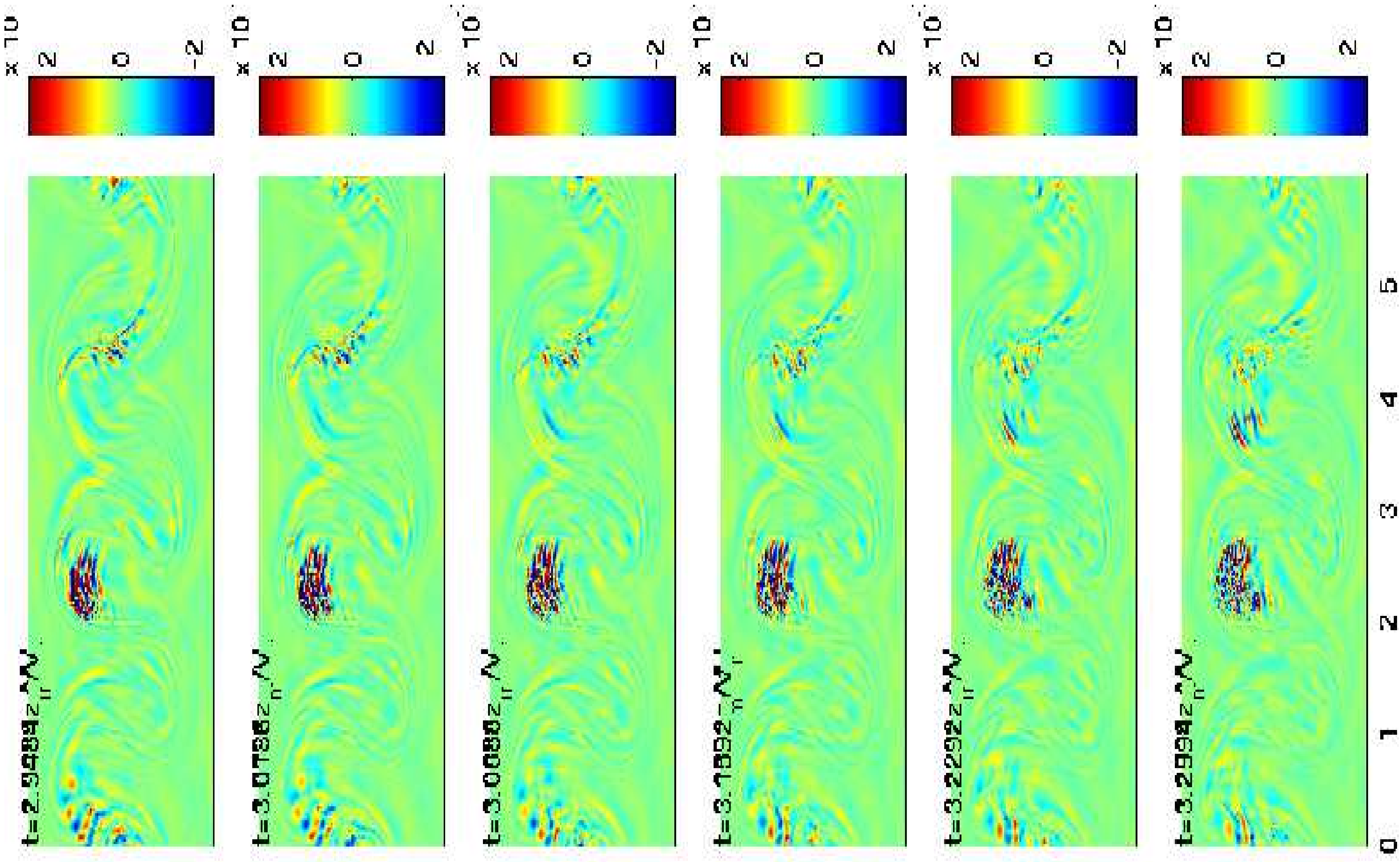
Volume-averaged potential energy dissipation rate vs t, $F_\ell = 4$, various R_ℓ.

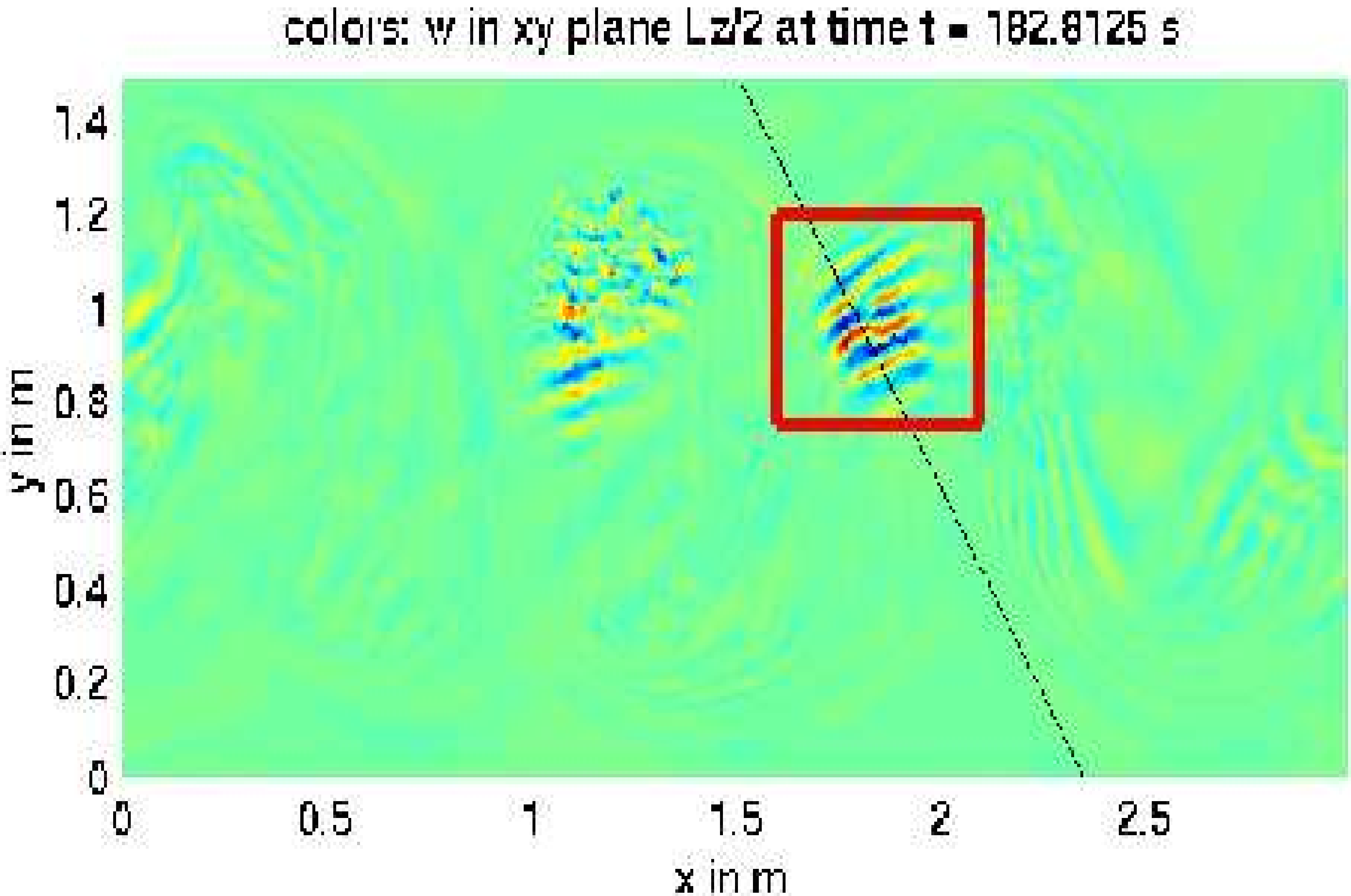
Mixing Efficiency $\langle\chi\rangle/\langle\epsilon\rangle$ versus Time



Mixing efficiency vs t, $F_\ell = 4$, various R_ℓ.






colors: w in xy plane Lz/2 at time t = 182.8125 s
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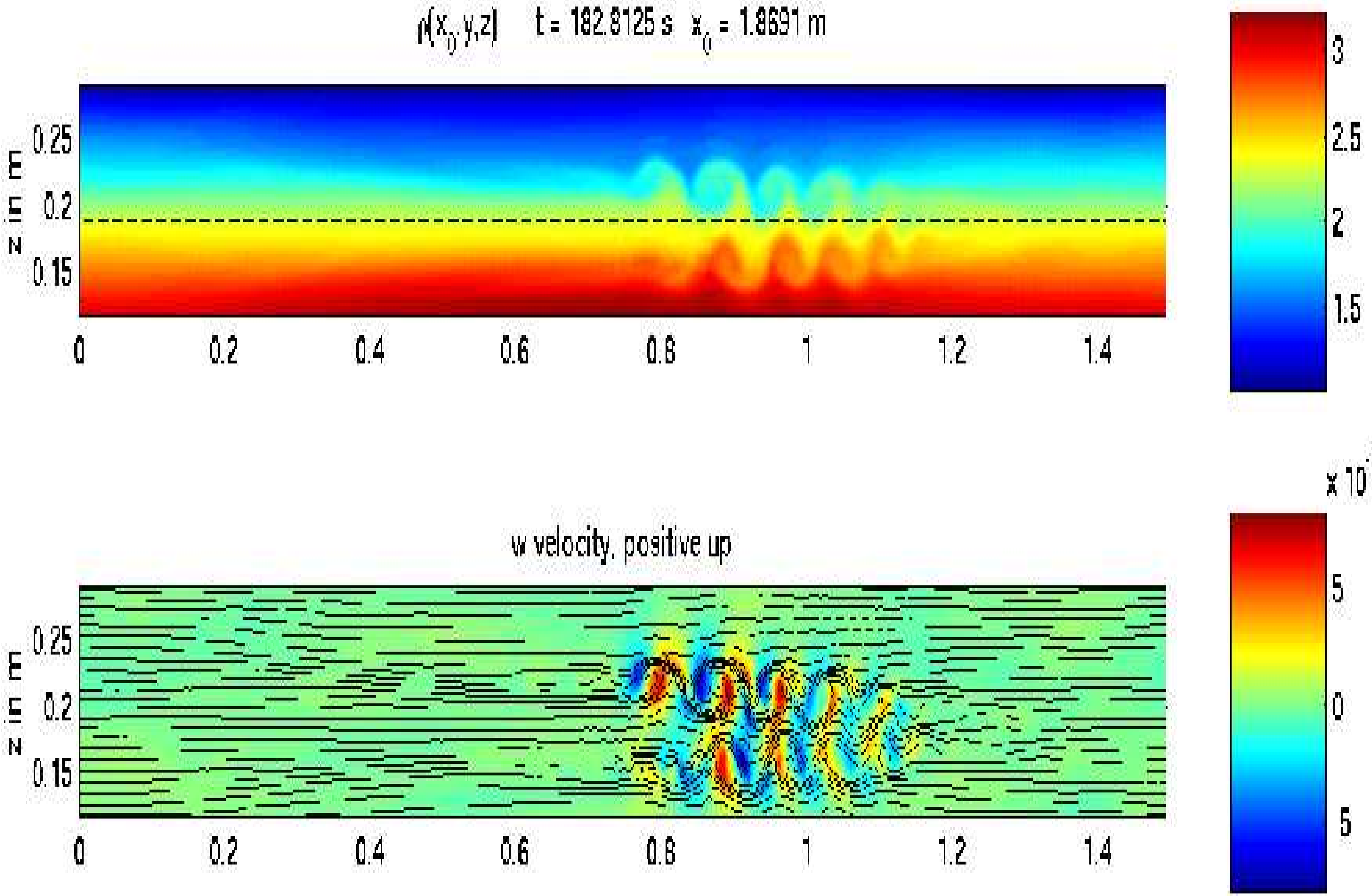
1
0.8
0.6
0.4
0.2
0
y in m
0
0.5
1
1.5
2
2.5
x in m



ρ(x0, y,z)   t = 182.8125 s   x0 = 1.8691 m
z in m
0.25
0.2
0.15
0
0.2
0.4
0.6
0.8
1
1.2
1.4
3
2.5
2
1.5
w velocity, positive up
z in m
0.25
0.2
0.15
0
0.2
0.4
0.6
0.8
1
1.2
1.4
x 10
5
0
5