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Interscale energy transfers in various turbulent flows,
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Kinematic Simulation and Rapid Distortion of diffusion in stably stratified and rotating turbulence

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Collaborations

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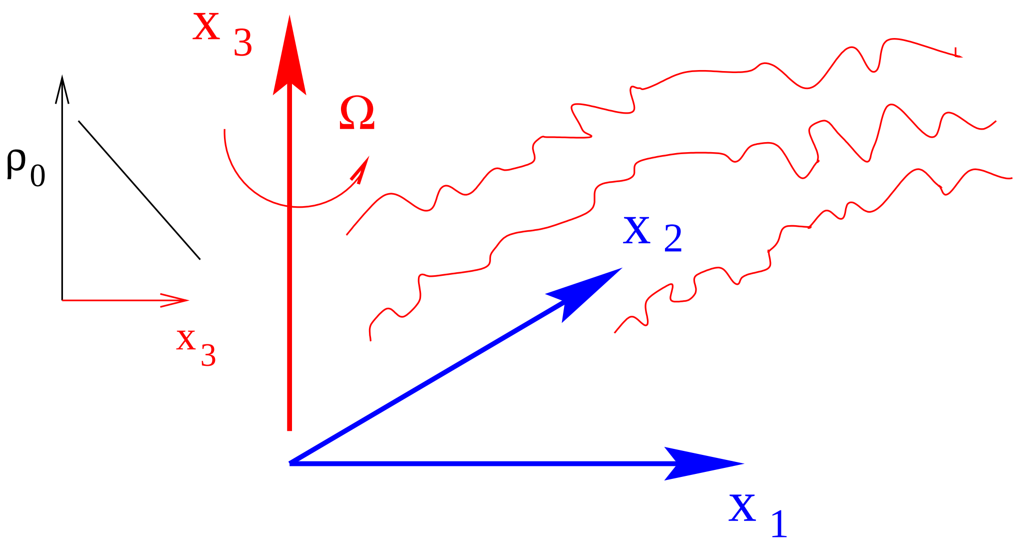
Content

Stably stratified turbulence with rotation, this study is limited to



- Homogeneous incompressible turbulence.

- We investigate the case of a flow submitted to a vertical gradient of mean density.

Boussinesq + rotation equations:

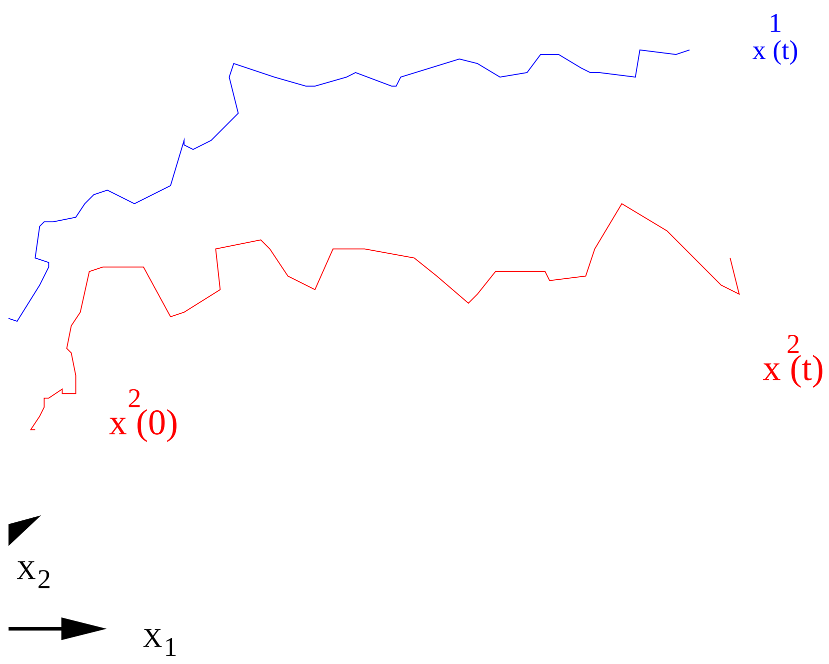
$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\nabla\mathbf{u} = -\frac{1}{\rho}\nabla p' - 2\mathbf{\Omega}\times\mathbf{u} + \nu\nabla^2\mathbf{u}$$

$$\frac{\partial}{\partial t}\frac{\rho' g}{\rho} + \mathbf{u}\nabla\frac{\rho' g}{\rho} = -u_3\frac{g}{\rho}\frac{d\rho}{dx_3} + \kappa\nabla^2\frac{\rho' g}{\rho}$$

- We neglect dissipative terms $\nu = \kappa \rightarrow 0$

- and terms non-linear in $\mathbf{u}$.

One and two particle dispersion



- One particle dispersion

$$< (x_i(t) - x_i(0))^2 >$$

- Two particle dispersion

$$< (x_i{}^1(t) - x_i{}^1(0) - x_i{}^2(t) + x_i{}^2(0))^2 >$$

We define $\Delta_0 = |\mathbf{x}^1(0) - \mathbf{x}^2(0)|$

KS: A Lagrangian model

- Generation of an EULERIAN velocity field

which incorporates turbulent-like flow structure.

$$\mathbf{u}(\mathbf{x}, t),$$

- Individual trajectories are then integrated

$$\frac{d}{dt}\mathbf{x}(t) = \mathbf{u}(\mathbf{x}(t), t),$$

from individual Eulerian flow fields and are therefore smooth, non-Markovian and comparable in character to experimental trajectories.

- The statistics are afterwards taken over many realisations of Eulerian flow fields.

KS velocity field for isotropic turbulence

$$\mathbf{u}(\mathbf{x},0) = \int\int\int \tilde{\mathbf{u}}(\mathbf{k},0)e^{i\mathbf{k}.\mathbf{x}}d\mathbf{k}$$

the KS velocity field is constructed by discretisation of the Fourier transform

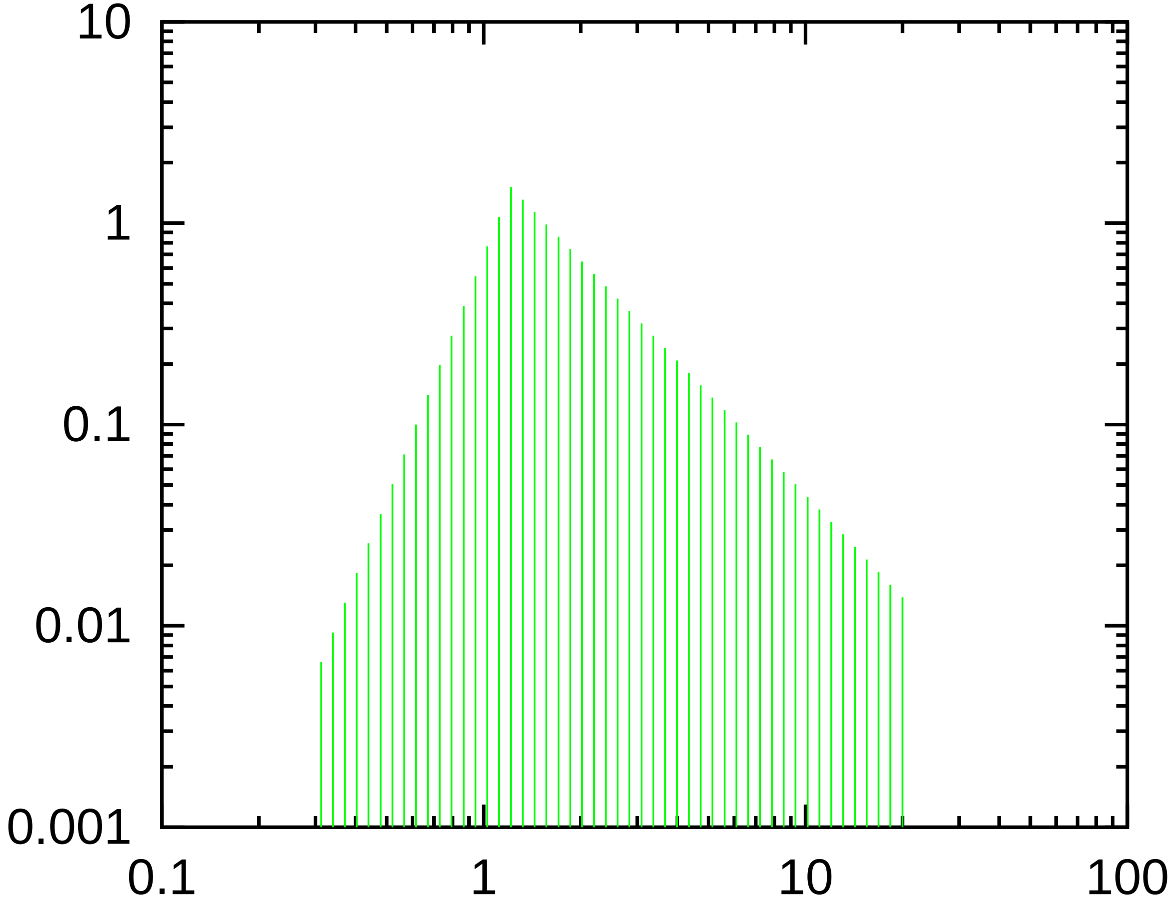
$$\mathbf{u}(\mathbf{x},0) = \sum_n \mathbf{A}_n \cos(\mathbf{k}.\mathbf{x}) + \mathbf{B}_n \sin(\mathbf{k}.\mathbf{x})$$

with

- incompressibility
  $\mathbf{A}_n.\mathbf{k} = \mathbf{B}_n.\mathbf{k} = 0$

- Prescribe energy spectrum $E(k) = k^{-p}$
  $|\mathbf{A}_n|^2 = |\mathbf{B}_n|^2 = E(k)\Delta k$

Energy spectrum



$$k_n = k_{min} \left(\frac{k_{max}}{k_{min}} \right)^{\frac{n-1}{n_{max}-1}}$$

Unsteadiness frequency ω_n

At time $t > 0$

$$\mathbf{u}(\mathbf{x}, t) = \sum_n \mathbf{A}_n cos(\mathbf{k}.\mathbf{x} + \omega_n t) + \mathbf{B}_n cos(\mathbf{k}.\mathbf{x} + \omega_n t)$$

with

$$\omega_n = \lambda \sqrt{{k_n}^3 E(k_n)}$$

and $\lambda \sim 1$

Flow Chart of a KS code

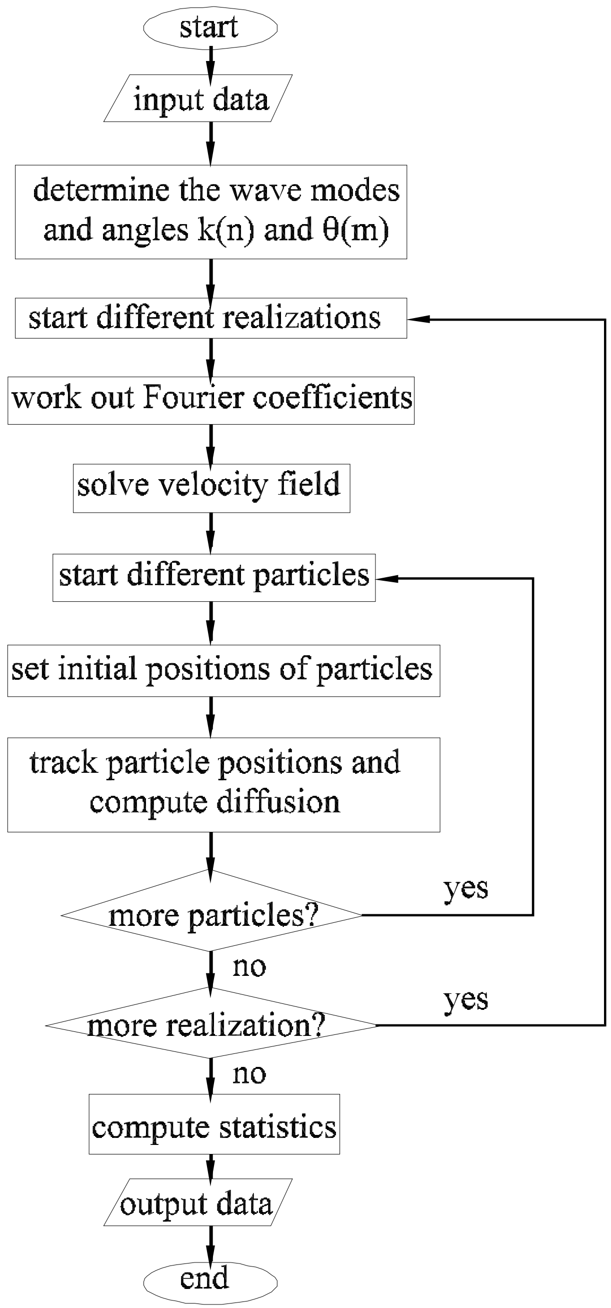
Most of the code is within two do-loops

each realisation

each particle trajectory

can be computed

on a separate processor



KS code and parallel computing

A code contains: a fraction F that can be parallelised

and a fraction $1-F$ that cannot

The total amount of computing time necessary for a run is

$$H_{tot} = (1-F)H_{tot} + FH_{tot}$$

on a single processor computer it is also the wall-clock time H_{wall}. Whereas on a computer with n_p processor

$$H_{wall} = (1-F)H_{tot} + \frac{F}{n_p}H_{tot}$$

KS code and parallel computing

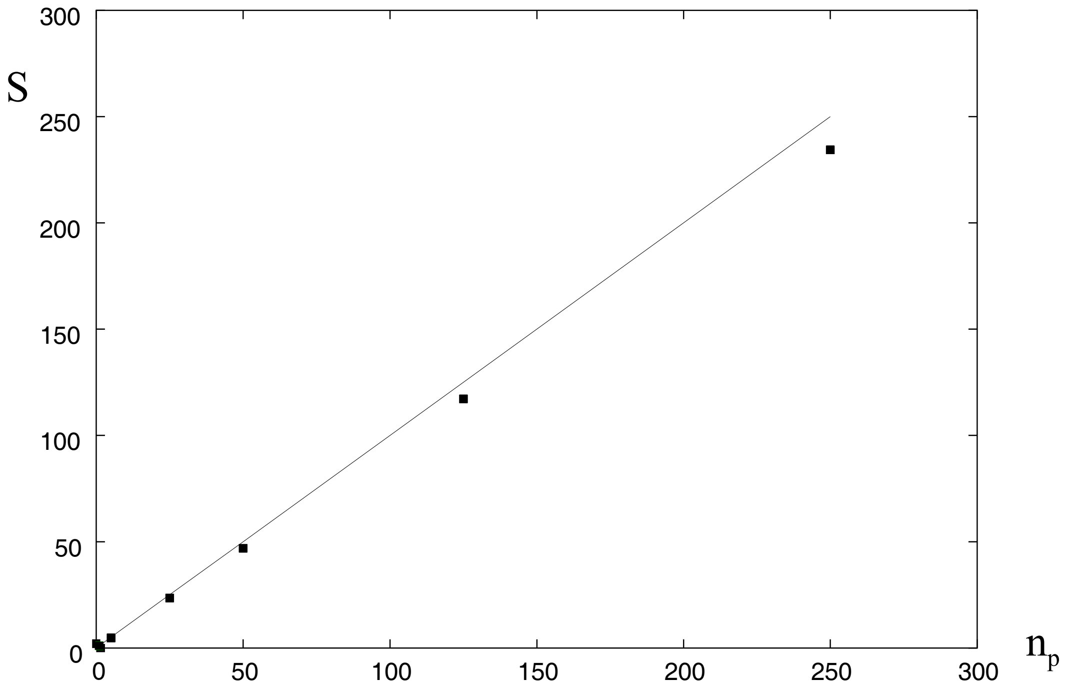
The efficiency of the parallelisation can be measured by comparing wall-clock times of serial and parallel codes:

$$S(n_p) = \frac{H_{wall}(serial)}{H_{wall}(parallel)} = \frac{H_{tot}}{(1 - F)H_{tot} + \frac{F}{n_p}H_{tot}}$$

This is known as Amdahl's law,

$$S(n_p) = 1/(1 - F + F/n_p)$$

Flow Chart of a KS code



$S(n_p)$: ratio of the serial and parallel wall-clock time

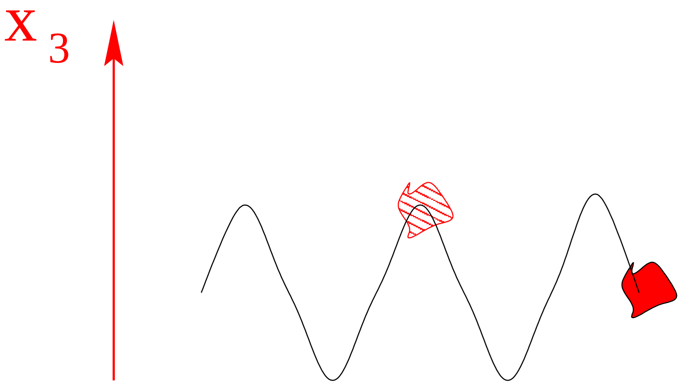
n_p : number of processors used for the computation.

From Amdahl's law, $S(n_p) = \frac{n_p}{(1-F)n_p+F}$

The parallelised code performs very efficiently. F in our KS code is estimated at least 99.6%.

KS model for stratified flows. KS of stratified flows differs from homogenous isotropic 3-D turbulence in two respects:

In time dependence: Within stably stratified flows, gravity and buoyancy forces act upon the fluid particule, their combined action generates internal waves known as intertio-gravity waves.



The frequency of these waves is

$$N = \sqrt{\beta \frac{\partial < \ominus >}{\partial x_3} g},$$

In Energy repartition: Due to anisotropy the energy spectrum is function of both wavenumber $|\mathbf{k}|$ and angle with the direction of anisotropy θ.

KS model for stratified flows.

Assumptions:

- **Time dependence**:

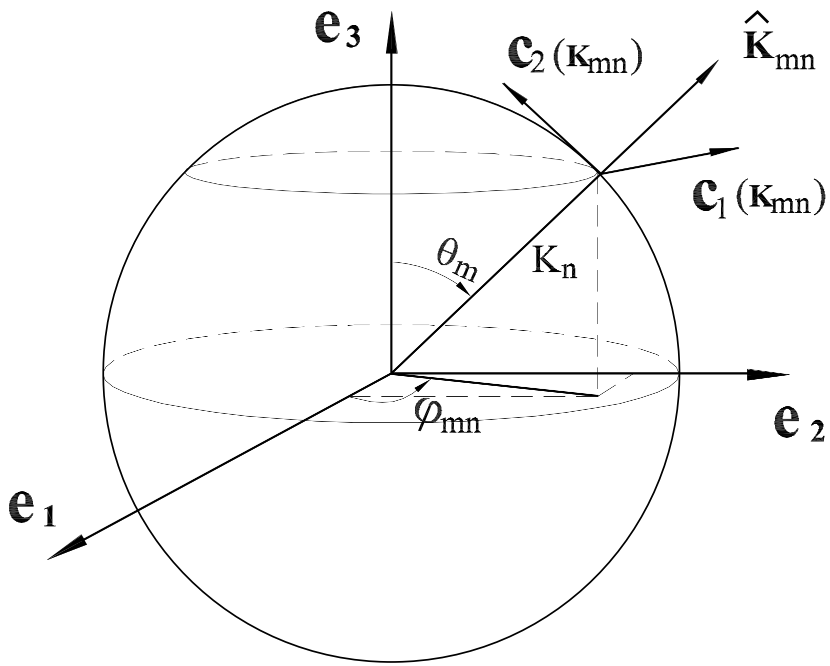
  It is the most important feature for particle diffusion.
  KS can incorporate most of the time dependence using RDT

- **Eulerian 'anisotropic structure'**:

  Perhaps not that important for particle's diffusion.
  Not present in KS.

Time dependence for Stratified KS

We thus have to use the local frame of Craya-Herring.



$$\mathbf{e}^1 = (\mathbf{k} \times \mathbf{n})/|\mathbf{k} \times \mathbf{x_3}|$$
$$\mathbf{e}^2 = (\mathbf{k} \times \mathbf{e}^1)/|\mathbf{k} \times \mathbf{e}^1|$$
$$\mathbf{e}^3 = \mathbf{k}/|\mathbf{k}|$$

Time dependence for Stratified KS

Time evolution is obtained using Green's operator:

$$\mathbf{u}^{KS}(\mathbf{x},t) = \sum_{m=1}^{m_s} \sum_{n=1}^{n_s} \mathbf{G}(\mathbf{k},t,t_0) : \mathbf{A}(\mathbf{k}_{mn},t_0) e^{i\mathbf{k}_{mn}.\mathbf{x}}.$$

If unsteadiness is allowed, a more general expression is obtained by

replacing $(\mathbf{k}_{mn}.\mathbf{x})$ by $(\mathbf{k}_{mn}.\mathbf{x} + \omega_{mn} t)$.

Linearised equations

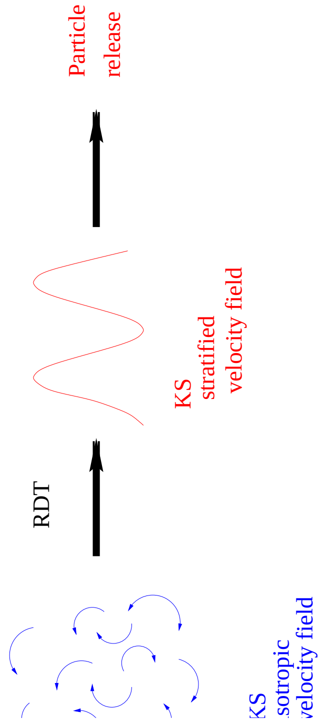
$$\tilde{\mathbf{u}}(\mathbf{k},t) = \tilde{u}_1(\mathbf{k},t)\mathbf{c_1} + \tilde{u}_2(\mathbf{k},t)\mathbf{c_2}$$

Linearised equations can be solved in Fourier space:

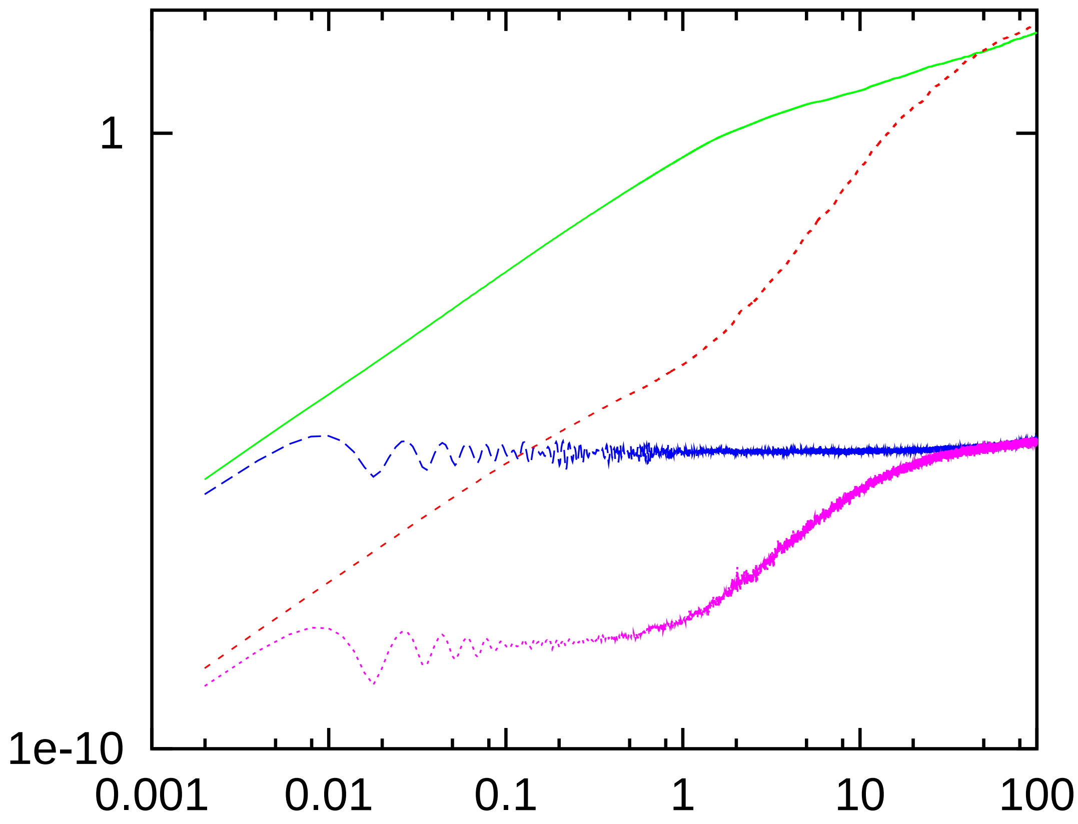
And the solutions of the linearised equations are

$$\begin{aligned}
\tilde{v}_1(\mathbf{k},t) &= \tilde{v}_1(\mathbf{k},0) \\
\tilde{v}_2(\mathbf{k},t) &= \tilde{v}_2(\mathbf{k},0)\cos(Nt\sin\theta) \\
\frac{1}{\rho}\tilde{\rho}'(\mathbf{k},t) &= \frac{N}{g}\tilde{v}_2(\mathbf{k},0)\sin(Nt\sin\theta)
\end{aligned}$$

KS for stratified flows



Main results: pure stratification



$$< {x_3}^2 > \sim \frac{u'^2}{N^2}$$

$$< \Delta {x_3}^2 > \sim \frac{\Delta {v_0}^2}{N^2}$$

— 1-particle horizontal diffusion $< (x_1 - x_1(0))^2 > /L^2$

— 1-particle vertical diffusion $< (x_3 - x_3(0))^2 > /L^2$

— 2-particle horizontal diffusion $< (\Delta x_1 - \Delta x_1(0))^2 > /L^2$

— 2-particle vertical diffusion $< (\Delta x_3 - \Delta x_3(0))^2 > /L^2$

as functions of $\frac{u'}{L}\tau$

KS for stratified flows

The Eulerian 'anisotropic structure'is not important for particle's diffusion.

- Time-correlation argument
- Energy argument

DNS pure stratification, 1-particle

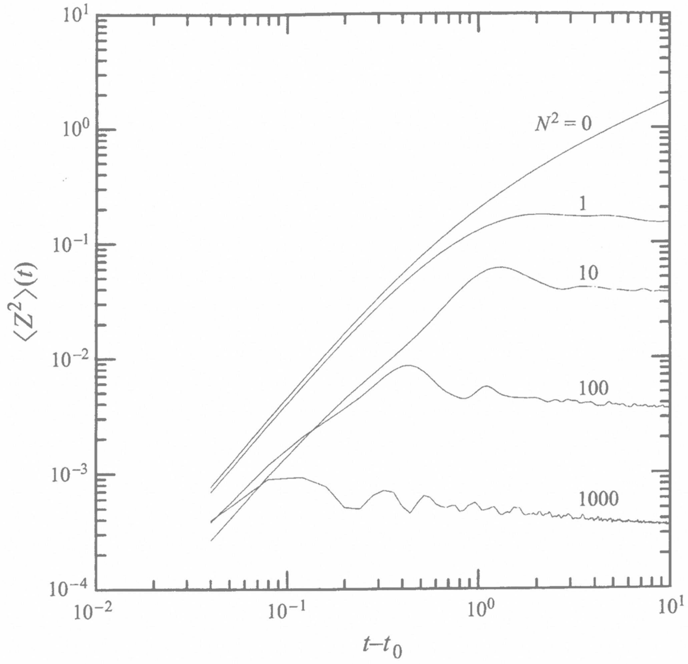
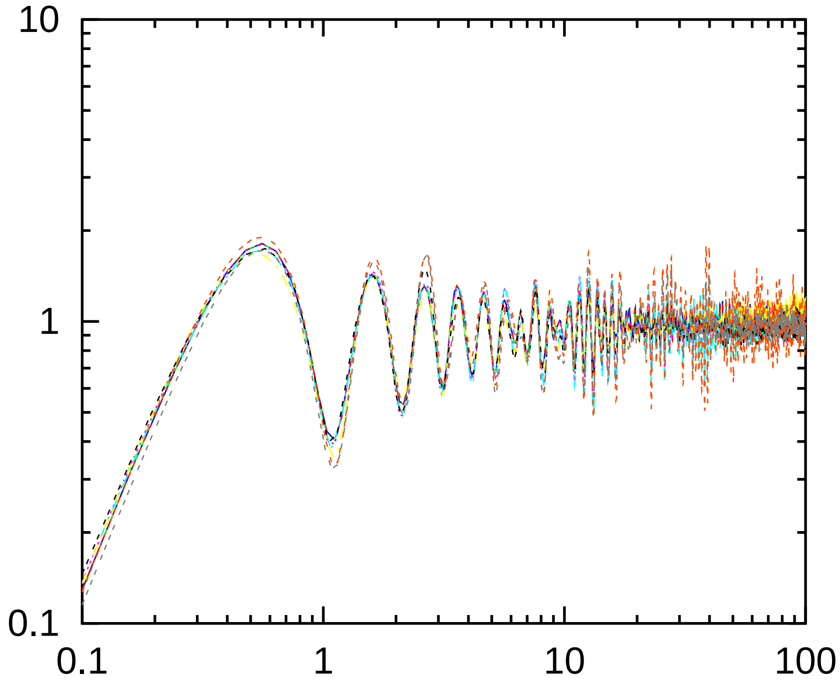


FIGURE 4. The mean square of the vertical displacement $\langle Z^2\rangle(t)$ for $N^2 = 0$, 1, 10, 100, 1000.



Y. Kimura & J. R. Herring
JFM (1996)

$$< {x_3}^2 > \sim \frac{u'^2}{N^2}$$

DNS pure stratification, 2-particle

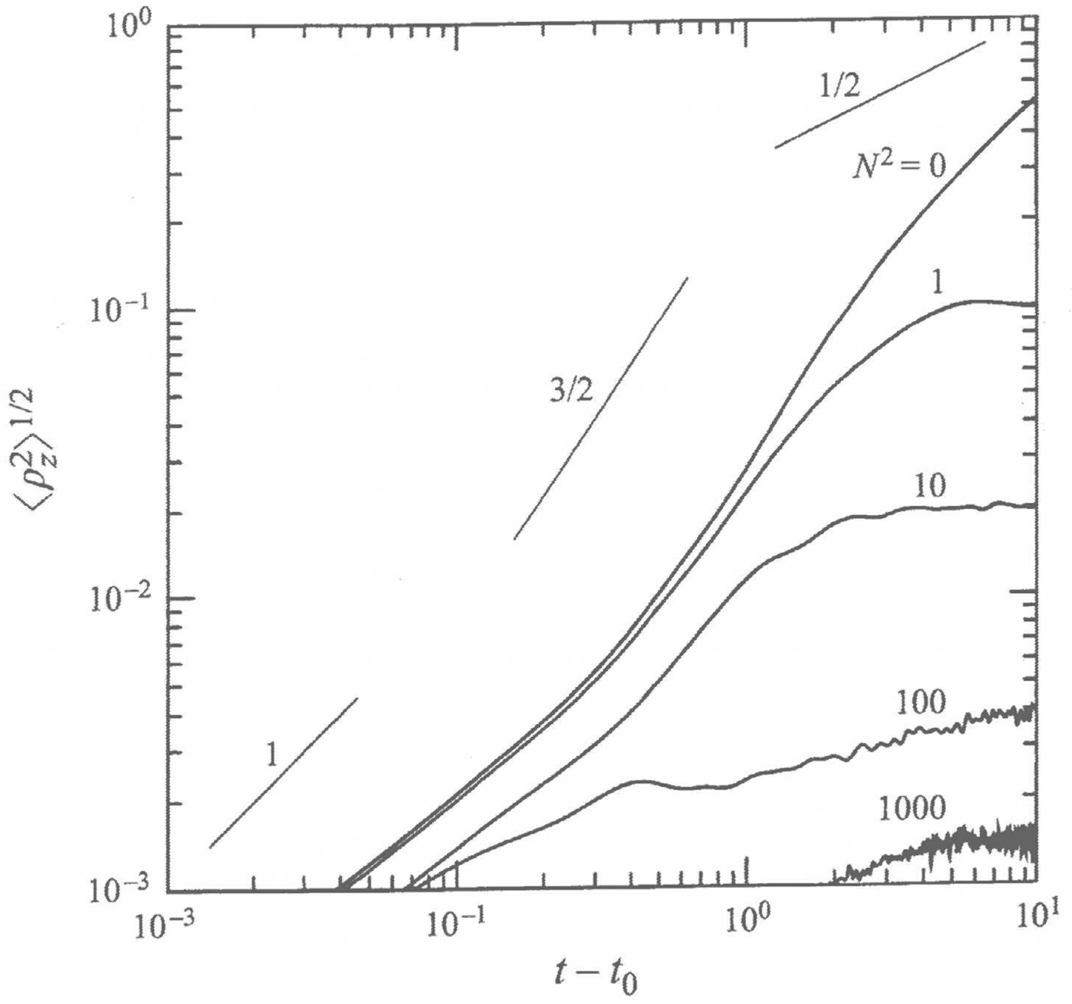


FIGURE 6. The root-mean-square of vertical dispersion of particle pairs, $\langle \rho_z^2 \rangle^{1/2}(t)$ for $N^2 = 0, 1, 10, 100, 1000$.

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&
J. R. Herring

JFM (1996)

DNS pure stratification, 1-particle

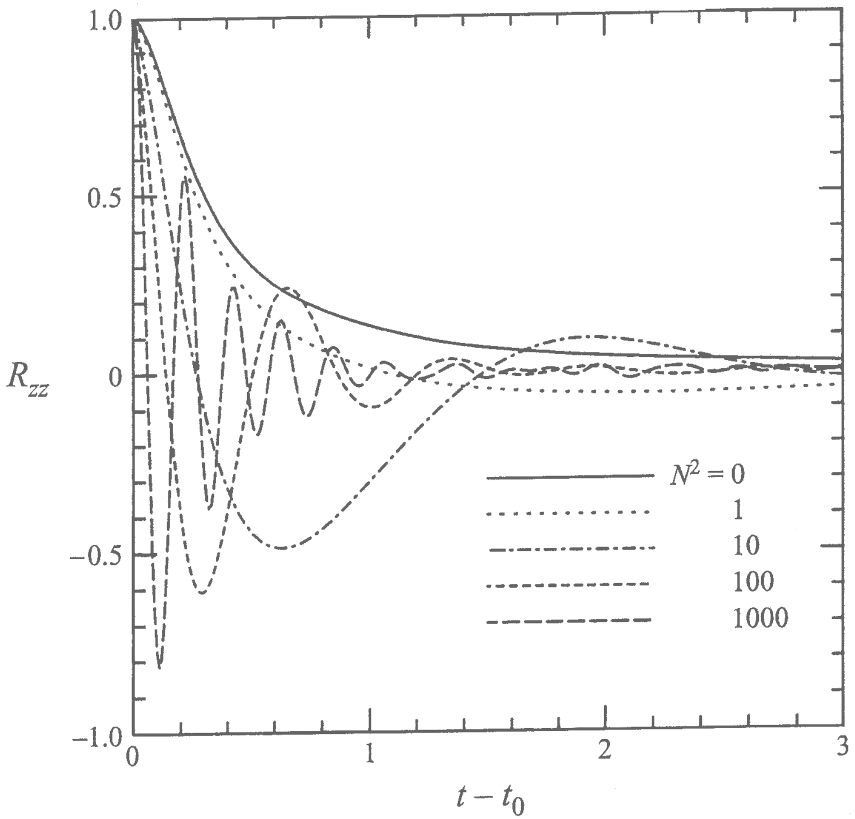
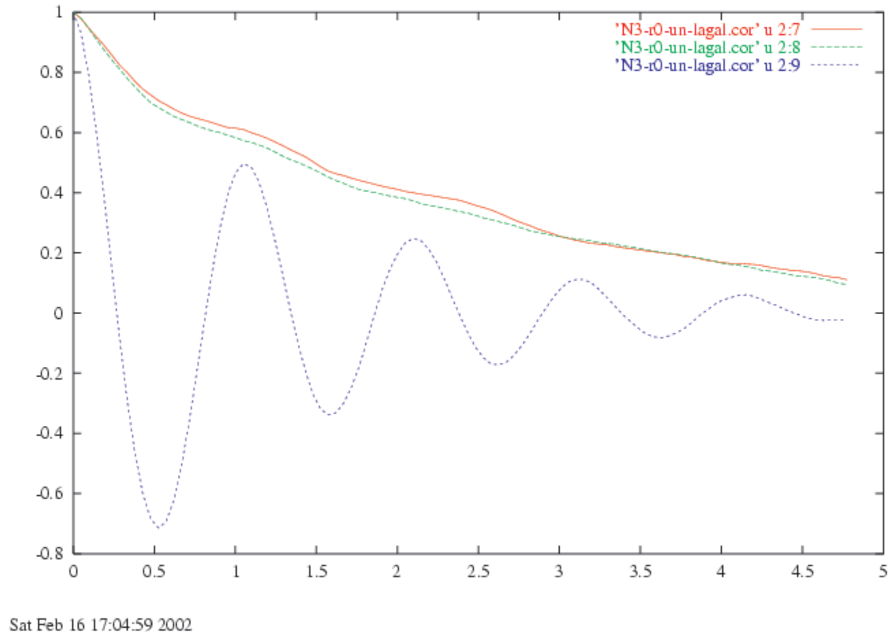
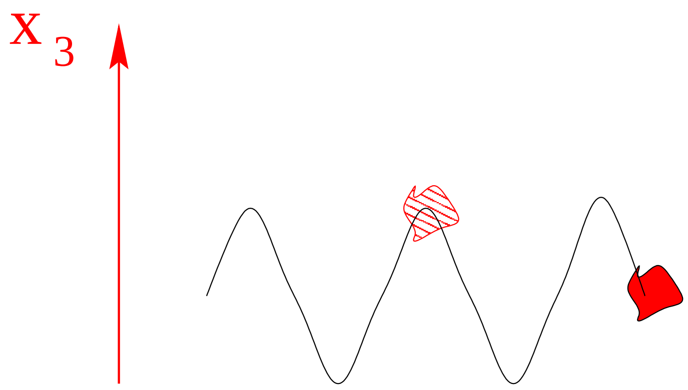


FIGURE 7. Lagrangian covariance for $N^2 = 0, 1, 10, 100, 1000$.



Y. Kimura & J. R. Herring JFM (1996)   Nicolleau & Vassilicos 2003

KS model for stratified flows.



$$\ddot{x}_1 = -\frac{1}{\rho}\frac{\partial p'}{\partial x_1}$$

$$\ddot{x}_2 = -\frac{1}{\rho}\frac{\partial p'}{\partial x_2}$$

$$\ddot{x}_3 = -\frac{1}{\rho}\frac{\partial p'}{\partial x_3} - \frac{\rho' g}{\rho}$$

$$\frac{\partial}{\partial t}\frac{\rho' g}{\rho} = -N^2 \dot{x}_3$$

$$\ddot{x}_3 + N^2 x_3 = -\frac{1}{\rho}\frac{\partial p'}{\partial x_3}$$

KS model for stratified flows.

$$\ddot{x}_3 + N^2 x_3 = -\frac{1}{\rho}\frac{\partial p'}{\partial x_3}$$

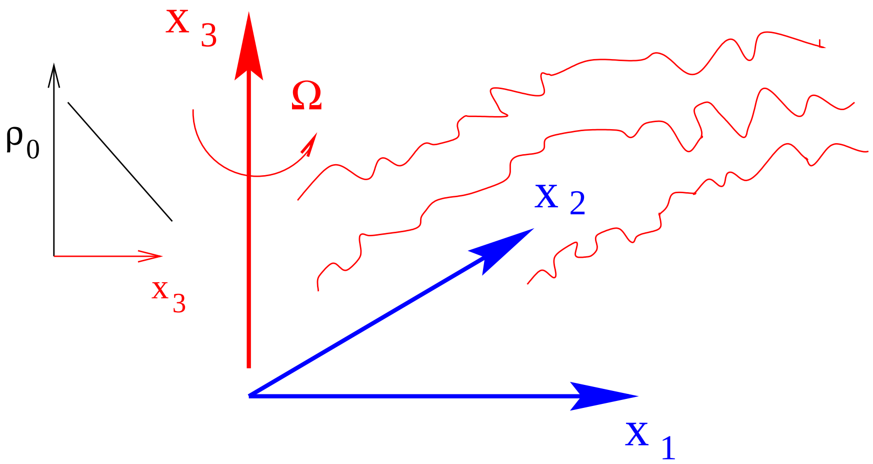
multiply by $\dot{x}_3$ and integrate:

$$\frac{1}{2}\dot{x}_3^2 + \frac{1}{2}N^2 {x_3}^2 = -\int \frac{1}{\rho}\frac{\partial p'}{\partial x_3}\dot{x}_3 + C$$

it can then be derived that

$< \mathbf{{x_3}^2} >$ is bounded

Rotation and stratified flows.



$$\ddot{x}_1(t) = -\frac{1}{\rho}\frac{\partial p'}{\partial x_1}(t) + 2\Omega u_2(\mathbf{x}, t)$$

$$\ddot{x}_2(t) = -\frac{1}{\rho}\frac{\partial p'}{\partial x_2}(t) - 2\Omega u_1(\mathbf{x}, t)$$

$$\ddot{x}_3 = -\frac{1}{\rho}\frac{\partial p'}{\partial x_3} - \frac{\rho' g}{\rho}$$

$$\frac{\partial}{\partial t}\frac{\rho' g}{\rho} = -N^2 \dot{x}_3$$

Note that only the equations for horizontal motion are affected by Ω.

The Lagrangian equation for the vertical diffusion does not explicitly depend on Ω:

$$\ddot{x}_3 = a_3 - N^2(x_3 - x_3(t_0)) - g\Theta(t_0)$$

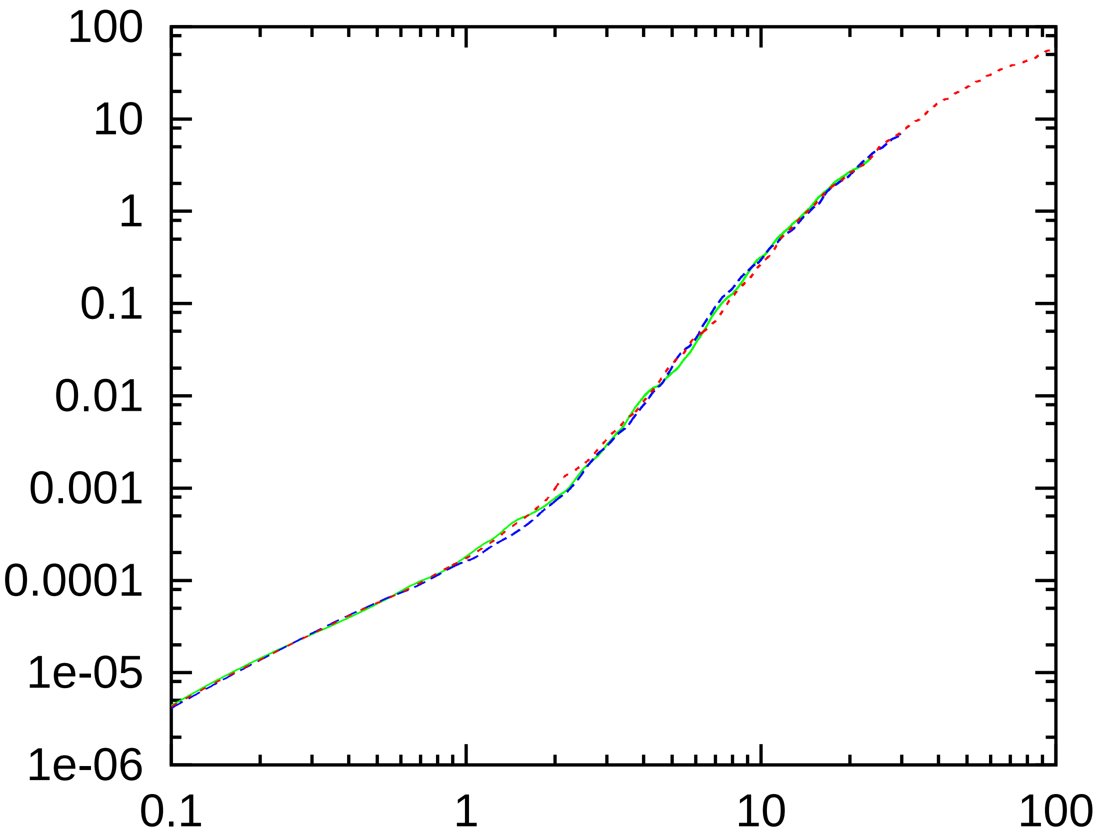
KS model for stratified flows.

$$\ddot{x}_3 + N^2 x_3 = -\frac{1}{\rho}\frac{\partial p'}{\partial x_3}$$

The Coriolis force does not work and it can then be derived that

$$< \mathbf{x_3}^{\mathbf{2}} > \quad \text{is bounded}$$

Froude number dependence: two particle horizontal



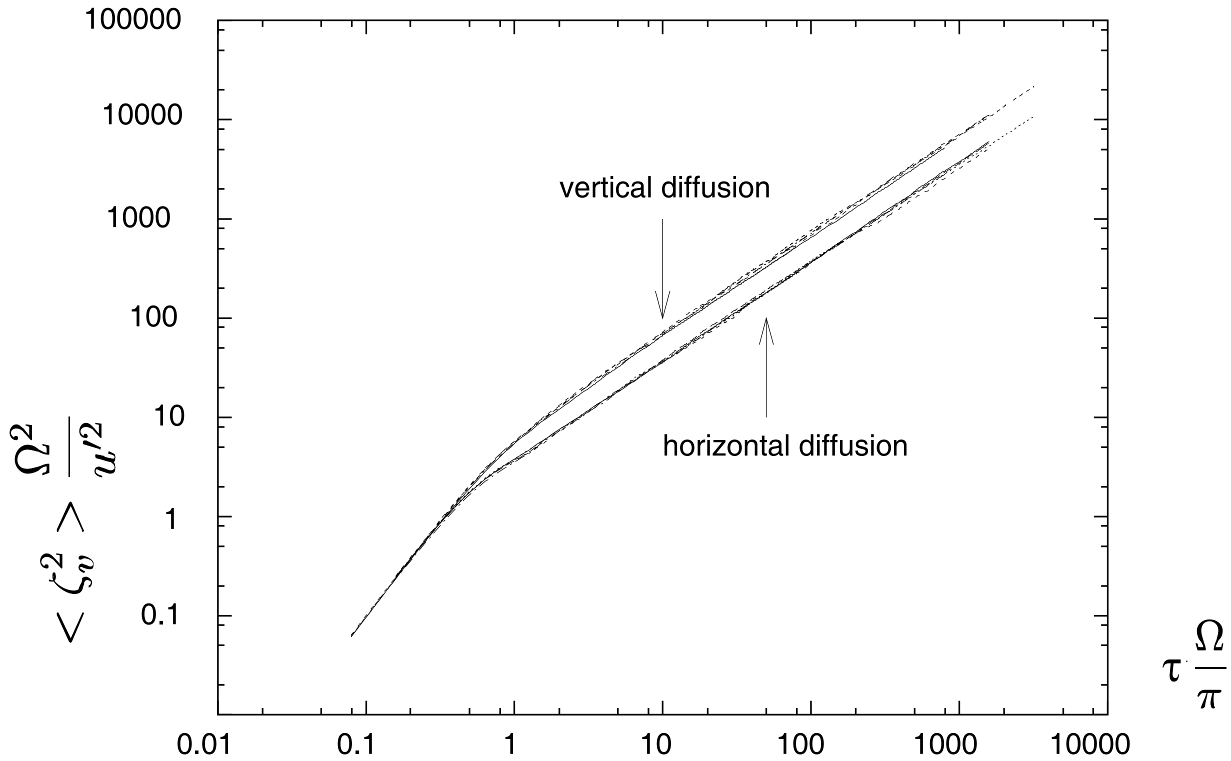
$< x_1{}^2 >$ and $< \Delta x_1{}^2 >$

do not depend on Fr

$\Delta_0 k_{max} = 0.2$ and $Lk_{max} = 20$ and different Froude numbers

$Fr =$ 0.008,
0.004 and
0.0026

KS pure rotation, 1-particle diffusion Nicolleau, Yu & Vassilicos 2004



$$< \zeta_h^2(\tau) > = \frac{u'^2}{\Omega^2} \, f\left(\frac{\Omega}{\pi}\tau\right)$$

at large times

$$< \delta_h^2 > = \frac{1}{2} < \delta_v^2 >$$

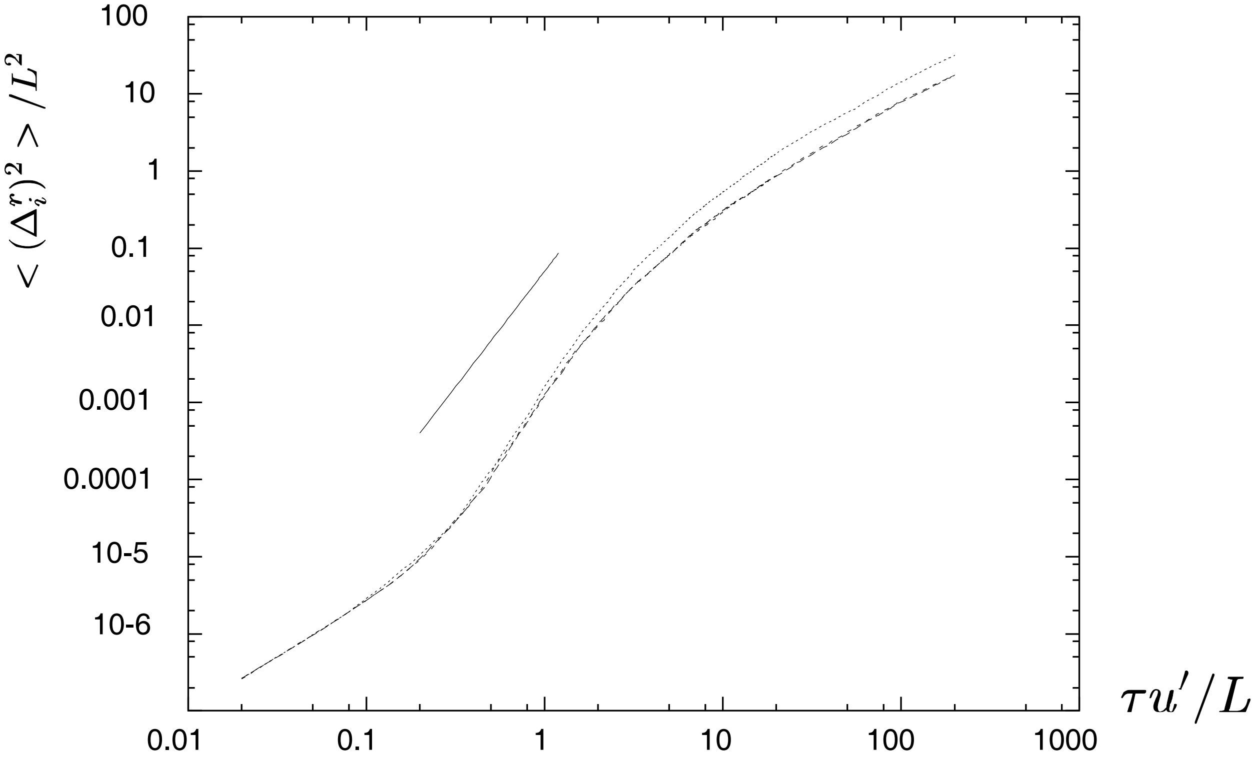
More precisely, this universal function is composed of two régimes:

- When $\tau \ll \pi/\Omega$: a t^2 ballistic régime is found

$$< \zeta_h^2(\tau) > = \frac{10}{\pi^2} u'^2 \tau^2$$

- Whereas when $\tau > \pi/\Omega$ a $t-$régime is found

$$< \zeta_h^2(\tau) > = \frac{10}{\pi} \frac{u'^2 \tau}{\Omega \pi} = \frac{20}{\pi^2} Ro\, L u'\, \tau$$



$$\frac{< (\Delta_i^r(\tau))^2 >}{L^2} = C_i \, Ro \, \frac{\tau}{t_d}$$

$$C_i = \begin{cases} 16 & \text{horizontal} \\ 32 & \text{vertical} \end{cases}$$

$$< (\Delta_i^r(\tau))^2 >= (\delta v_i)^2 \, \tau^2$$

ROTATION + STRATIFICATION

DNS rotation + stratification - horizontal 1-particle diffusion

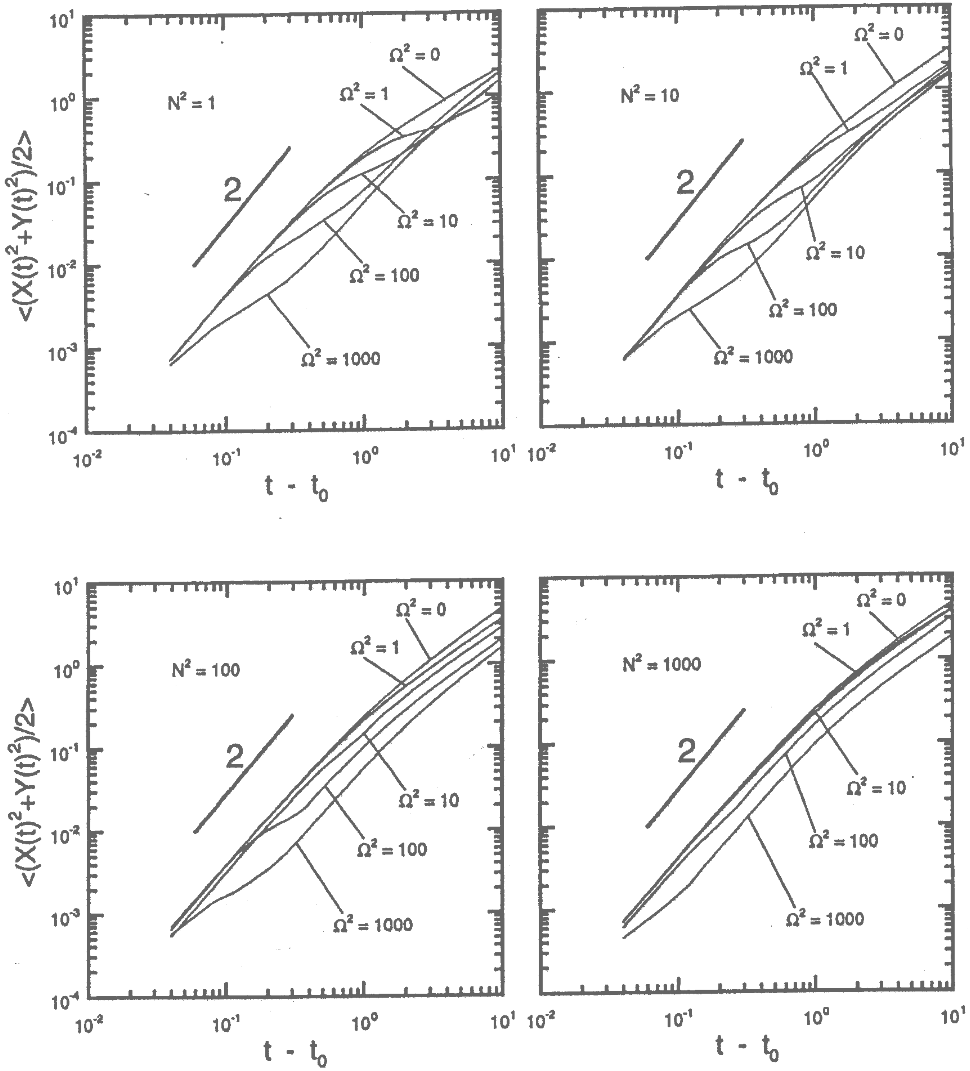


Figure 2. Single particle dispersion in the horizontal direction.

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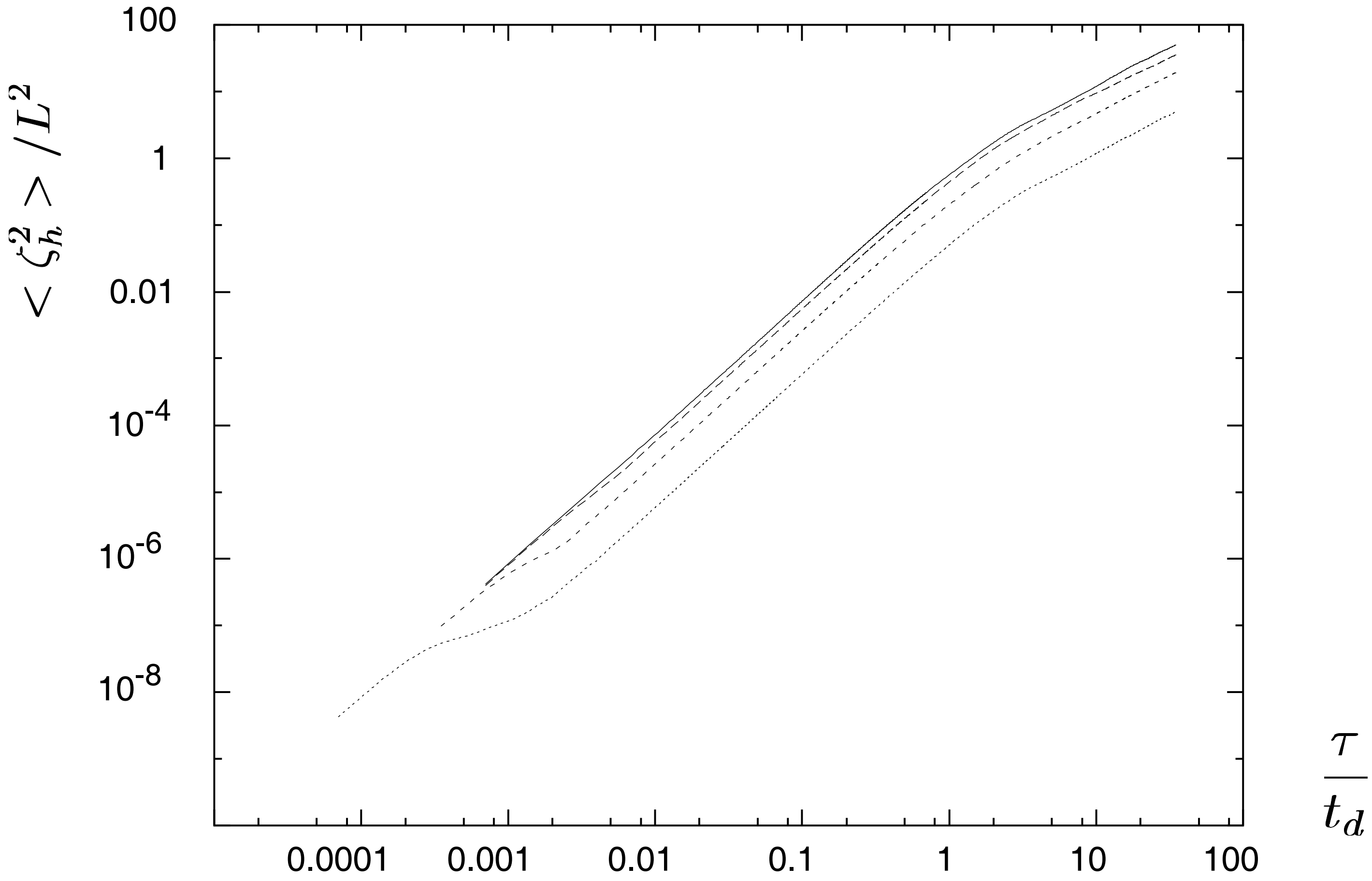
FEDSM99-7753 (1999)

KS, effect of rotation on stratification - 1-particle horizontal diffusion

(Nicolleau, Yu & Vassilicos 2004)

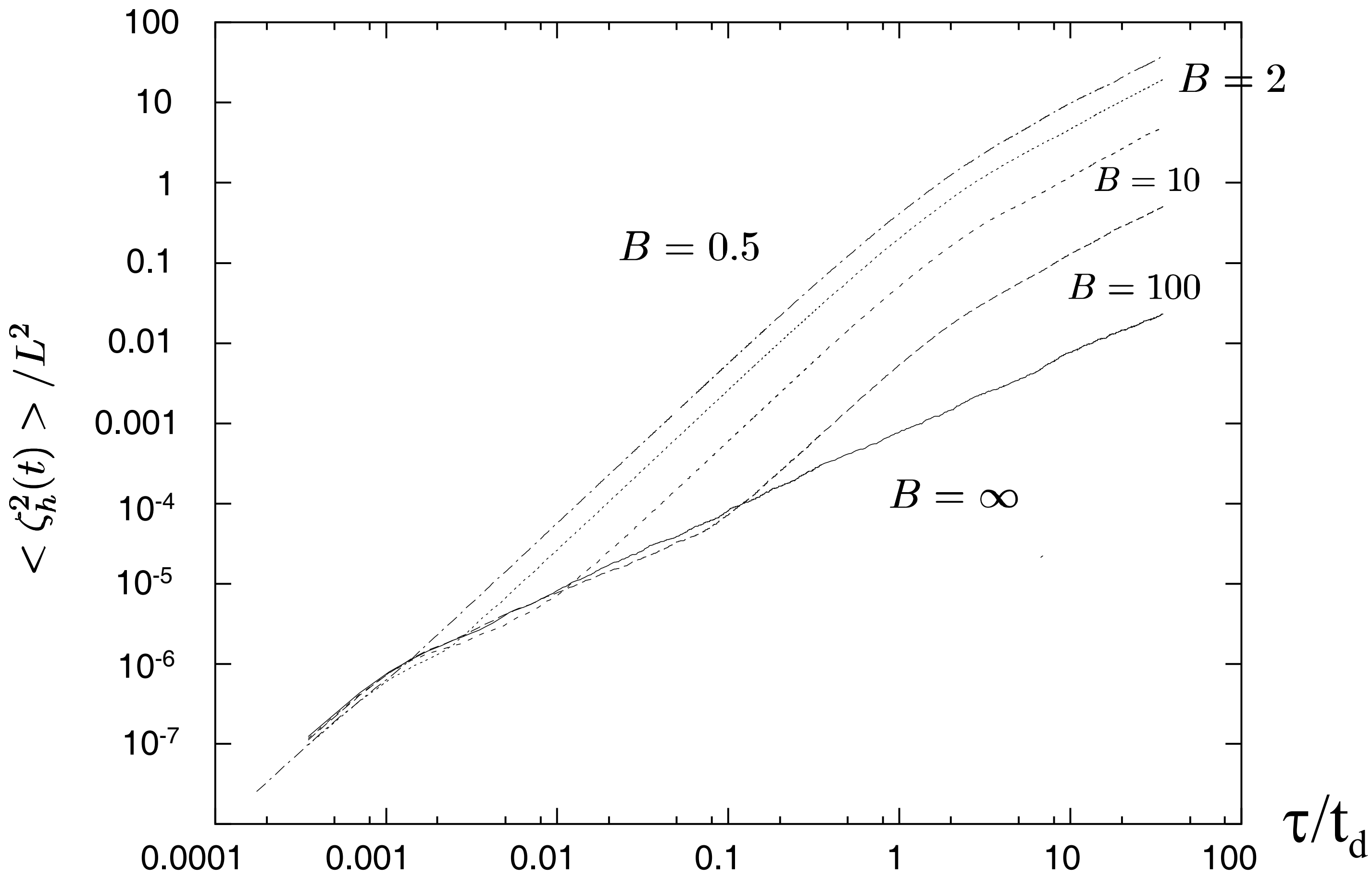


Effects of rotation on stratification with $N = 500$, $\frac{L}{\eta} = 12.5$.

From top to bottom $\Omega = 0$, 125, 500 and 2500.

KS rotation + stratification - horizontal 1-particle diffusion

(Nicolleau, Yu & Vassilicos 2004)



Effects of stratification on rotation with $\Omega = 500$.

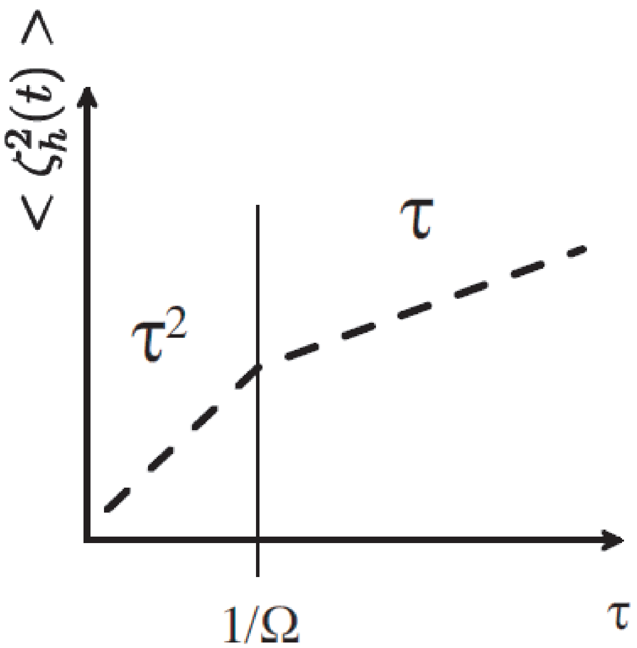
From top to bottom

$N = 2000$, 500, 100, 10,

and 0.

Case	Ω	N	L	u'	$\frac{1}{\eta}$	$\frac{\Delta_0}{\eta}$	B
A	0	500	1	0.35	12.5	1	0
B	100	1000	1	0.35	12.5	1	0.2
C	100	1000	2	0.35	12.5	1	0.2
D	100	1000	1	0.175	12.5	1	0.2
E	100	1000	1	0.35	6.5	1	0.2
F	200	2000	1	0.35	12.5	1	0.2
G	100	1000	1	0.35	12.5	0.02	0.2
H	100	1000	1	0.35	12.5	0.2	0.2
I	100	1000	1	0.35	12.5	5	0.2
J	500	2000	1	0.35	12.5	1	0.5
K	125	500	1	0.35	12.5	1	0.5
L	500	0	1	0.35	12.5	1	∞
M	500	10	1	0.35	12.5	1	100
N	500	100	1	0.35	12.5	1	10
O	2500	500	1	0.35	12.5	1	10
P	500	500	1	0.35	12.5	1	2
Q	500	500	2	0.35	12.5	1	2
R	500	500	1	0.175	12.5	1	2
S	500	500	1	0.35	6.5	1	2
T	1000	1000	1	0.35	12.5	1	2
U	500	500	1	0.35	12.5	0.2	2
V	500	500	1	0.35	12.5	5	2

simplified Corrsin hypothesis and rotating turbulence



Dash lines show
what can be predicted
by the simplified
Corrsin hypothesis.

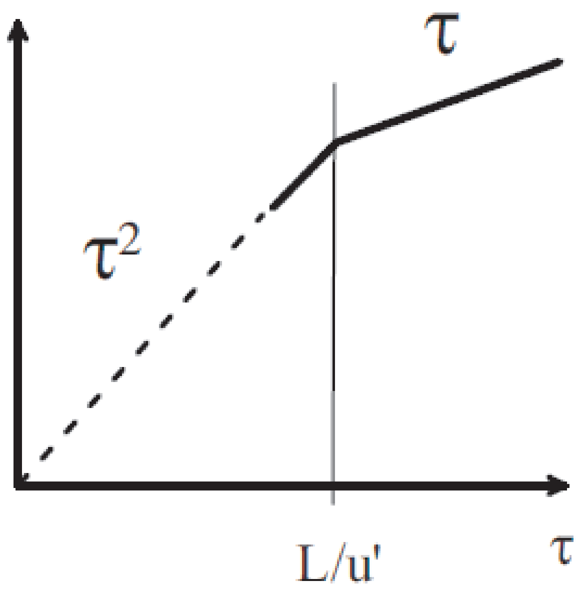
$$< \zeta_h^2(t) >= \frac{\frac{3}{2}u'^2}{8\Omega^2}\left[4\mathrm{Si}(2\Omega t)\Omega t + 2\cos 2\Omega t - \frac{\sin 2\Omega t}{\Omega t}\right]$$

where $\mathrm{Si}(s) = \int_0^s u^{-1}\sin u du$.

This equation derived from the simplified Corrsin hypothesis explains the two asymptotic behaviours observed for one-particle horizontal diffusion for pure rotation:

The ballistic régime when $\Omega t \to 0$ and the τ-régime when $\Omega\tau \to \infty$.

simplified Corrsin hypothesis and stratified turbulence



A similar equation can be derived for pure stratification:

$$< \zeta_h^2(t) >= \frac{3}{2}u'^2 \left\{ t^2 + \int_0^1 \left(\cos^2\theta \frac{\sin^2(Nt\sin\theta)}{N^2\sin^2\theta} \right) d\cos\theta \right\}$$

The equation does not contain any information of the turbulence or non-linear characteristic time L/u',

the τ^2-régime that is known to last up to times of the order of L/u' in the case of pure stratification cannot be related to this equation.

The simplified Corrsin hypothesis is not valid in cases without rotation.

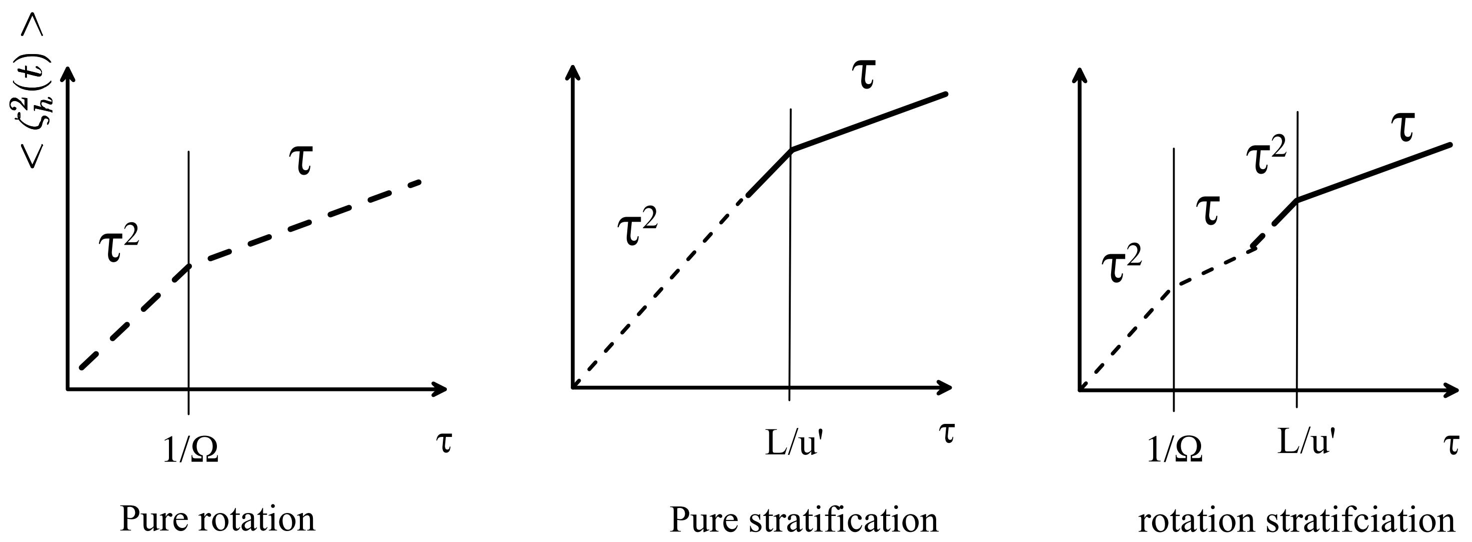
Sketch of superposition of rotation and stratification

τ-régimes are of very different natures in pure stratification and pure rotation:

i) In pure stratification the τ-régime is the well-known random walk that appears when the particle has been diffusing for longer than the turbulence characteristic time, it is independent of N and scales with L/u'.

ii) In pure rotation the τ-régime is not non-linear or random walk by nature, it is independent of the turbulence characteristic time and appears when the particle has been diffusing for longer than $1/\Omega$.

Sketch of superposition of rotation and stratification



- Ballistic régime ending when $\tau > \min\left(\frac{L}{u'}, \frac{1}{\Omega}\right)$
- A random walk after L/u'.

I.e. after a characteristic time ignored by the Simplified Corrsin Hypothesis (SCH), therefore this hypothesis is not valid whenever $N > 0$.

The main results of having rotation superimposed on stratified turbulence can be summarised as follows:

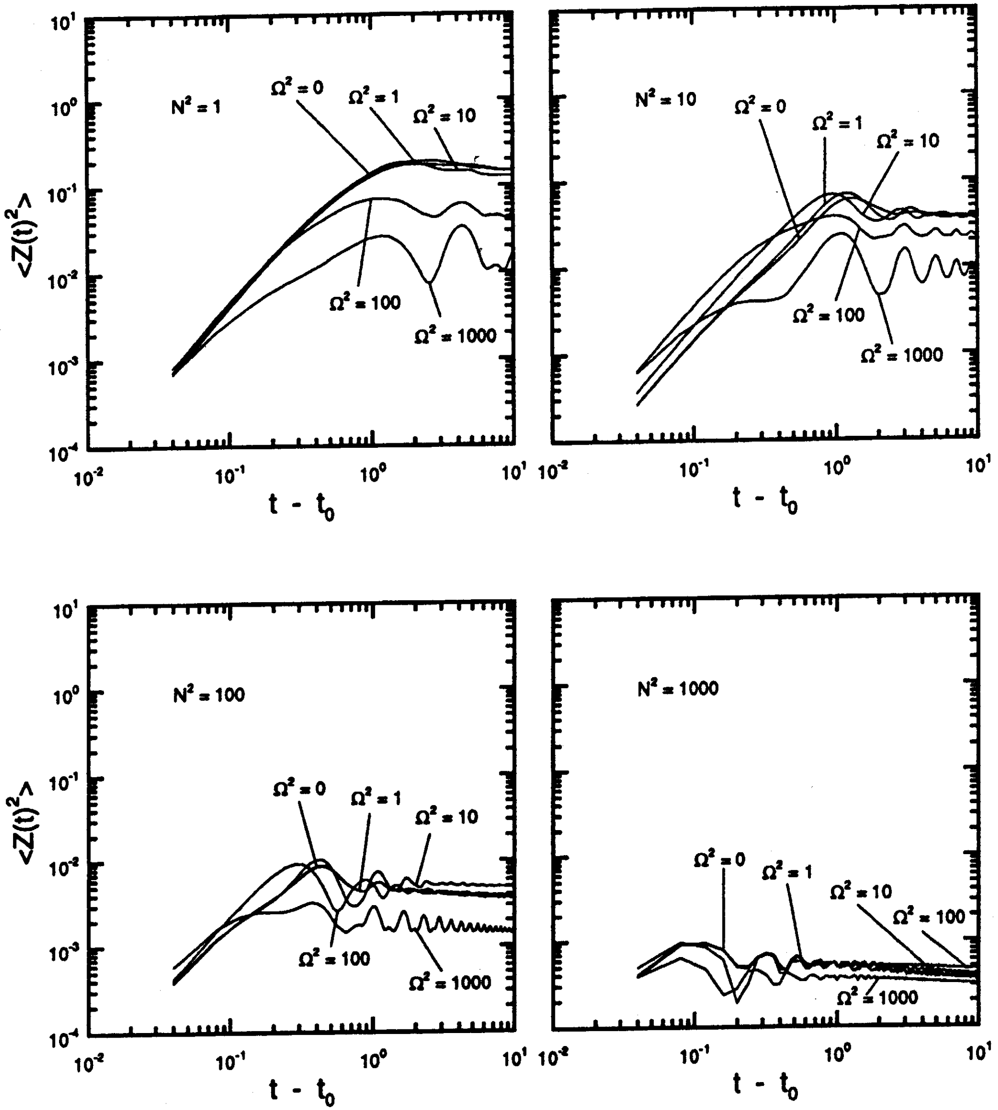
(1) The time when the random walk régime starts is only a function of L/u' regardless of the value of N or Ω.

(2) As N increases (or B decreases), the transition τ-régime is shortened until finally it disappears when $B < 1$.

(3) The diffusion after the ballistic τ^2-régime increases when B decreases.

(4) No effect of the superimposed stratification on the diffusion in the ballistic τ^2-régime is observed, and this régime finishes at a time related to Ω only.

DNS rotation + stratification - vertical 1-particle diffusion



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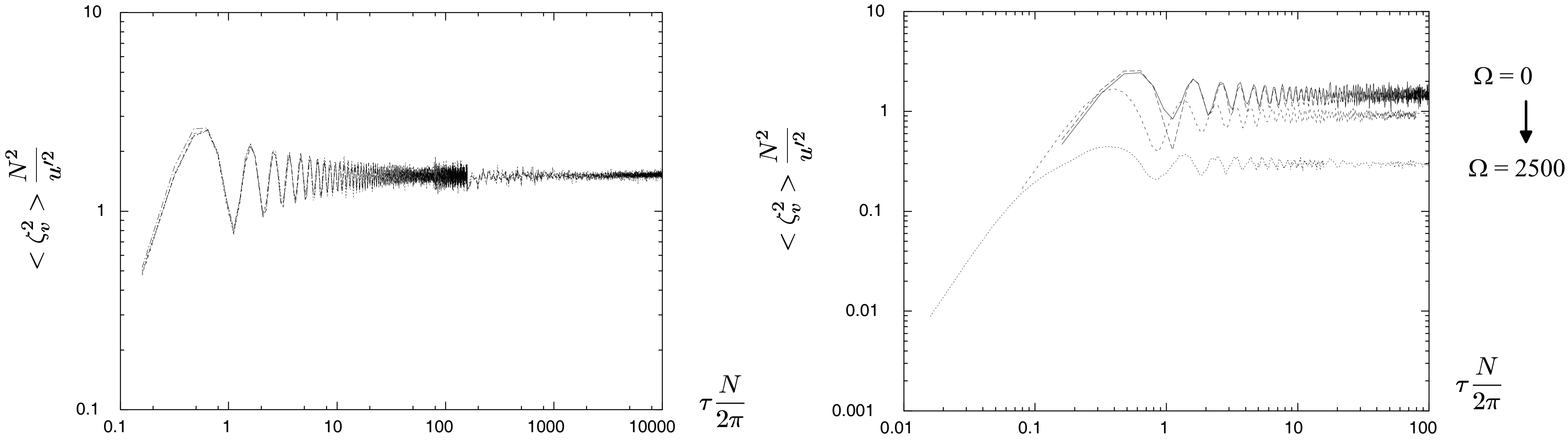
FEDSM99-7753 (1999)

Non-dimensional one-particle mean square vertical displacement

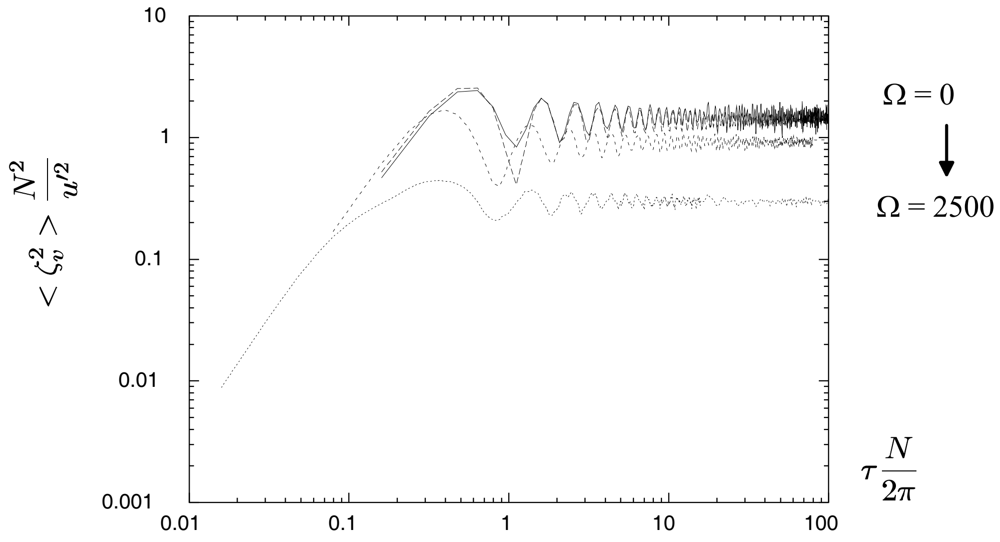


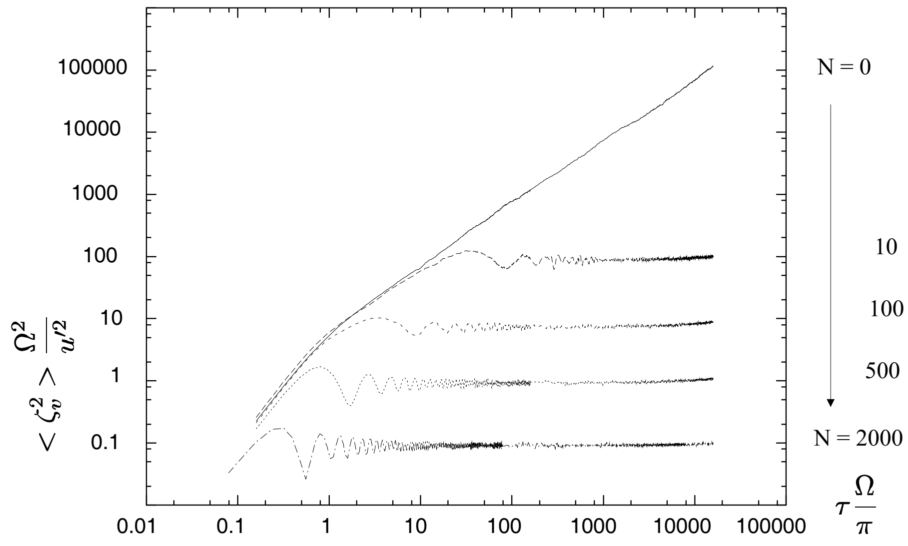
The diffusion pattern does not change very much, the difference is hardly discernable apart from the first loop of the oscillations where the diffusion sometimes shows higher or lower values than the case without rotation.

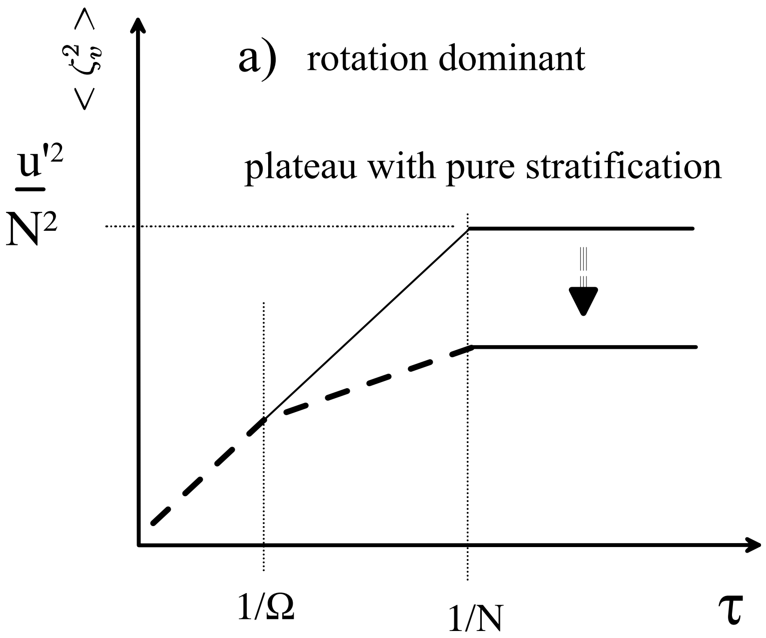
The superimposed rotation makes a distinctive difference only when the turbulence becomes rotation-dominated, $B > 1$.

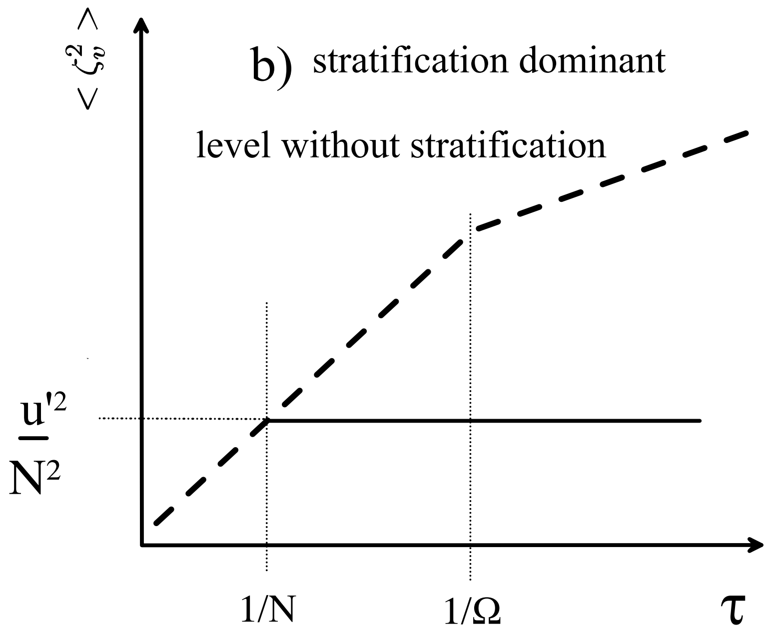
As Ω increases, the amplitude of the plateau is reduced, moreover, the starting time of the plateau is moved forwards. It is also observed that rotation has no significant influence on the diffusion in the ballistic τ^2-régime except that the régime is shortened as Ω increases.

one-particle vertical diffusion rotation & stratification









- In any case as already mentioned for $N\tau \gg 1$ a plateau is observed in accordance with the principle of energy conservation.

- A ballistic régime is observed when both stratification and rotation ballistic régimes co-exist that is up to $\tau = \min(\Omega^{-1}, N^{-1})$.

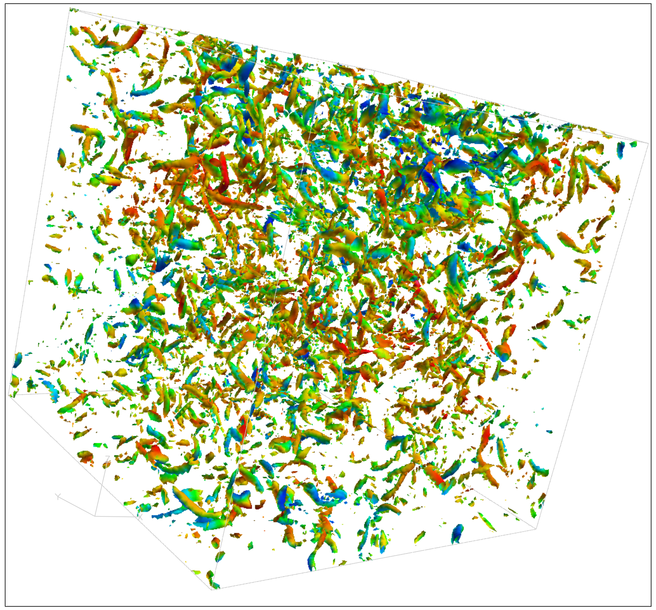
- When stratification is dominant ($B < 1$), the plateau is reached before the rotation waves can develop so that the rotation has no effect on the vertical diffusion.

- When rotation is dominant ($B > 1$), the rotation-τ-régime develops before stratification waves so that the plateau is lowered and a function of both Ω and N.

Role of spatial structures

KS for stratified turbulence does not contain any information about spatial structures such as layers or columns that are observed respectively for stratified turbulence and rotating turbulence

isotropic case, pure stratification, pure rotation



L. Liechtenstein & C. Cambon (2004)

So how important are these structures in the prediction of particles' diffusion?

- They are not sufficient: what matters in Lagrangian tracking is Lagrangian correlations not Eulerian's ones. So it is necessary to get accurate Lagrangian velocity correlation.

- They are not necessary to get a plateau: comparisons with DNS (Kimura & Herring 1999,Cambon-al 2004) show that the KS without the Eulerian structures predicts accurately the diffusion for one and two particles for stratified, rotating, and stratified and rotating turbulence.

spatial 'structure' stratification and rotation

$B = 0.1$, $B = 1$, $B = 10$

Energy argument: spatial 'structure' stratification

- It is worth noting that there is an energy argument that $N^2 < \zeta^2(\tau) >$ must be bounded, this argument is valid with or without rotation as the Coriolis force does not work.

- Therefore whatever B finite there exists a plateau irrespective of the Eulerian spatial structures predominantly layer-like or column-like or neither when $B = 1$.

- We can conclude that linear time-oscillations that are contained in KS are necessary and sufficient to predict accurately particle's diffusion in stratified and/or rotating turbulence,

- whereas Eulerian structures such as layers or columns are neither sufficient nor necessary.

Locality-in-scale hypothesis + spectrum

- in terms of diffusivity

$$\frac{d}{dt} < \Delta^2 >= \mathcal{F}[E(< \Delta^2 >), < \Delta^2 >]$$

- in terms of diffusion

$$< \Delta^2 >= \mathcal{G}[t, E(< \Delta^2 >), < \Delta^2 >]$$

Locality-in-scale hypothesis + spectrum

That is for $E(k) \sim k^{-\frac{5}{3}}$

- in terms of diffusivity

$$\frac{d}{dt} < \Delta^2 >= \beta < \Delta^2 >^{\frac{2}{3}}$$

- in terms of diffusion

$$< \Delta^2 >= G\epsilon t^3$$

Locality-in-scale hypothesis evaluation for $E(k) \sim k^{-\frac{5}{3}}$

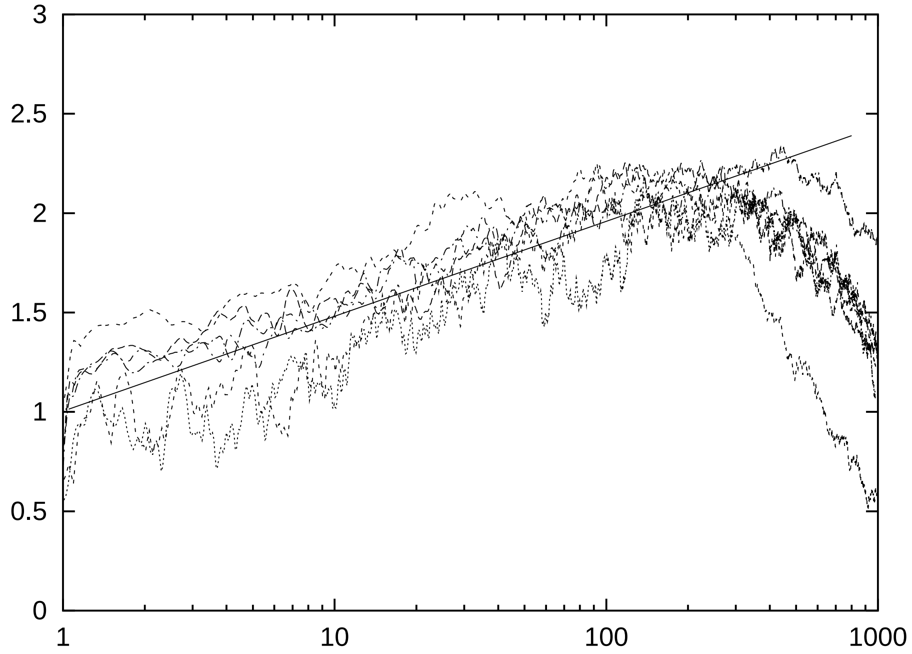
- in terms of diffusivity

$$\beta = \frac{\frac{d}{dt} < \Delta^2 >}{< \Delta^2 >^{\frac{2}{3}}}$$

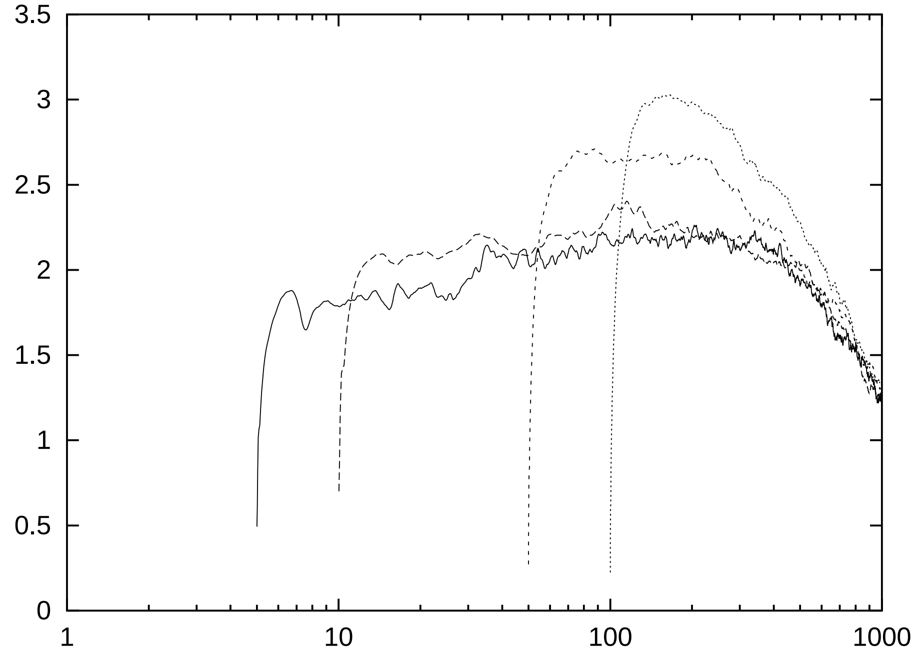
- in terms of diffusion

$$G = \frac{< \Delta^2 >}{\epsilon t^3}$$

β as a function of $\frac{\Delta(t)}{\eta}$ for isotropic turbulence



$\Delta_0 < \eta$



$\Delta_0 > \eta$

β as a function of $\frac{\Delta(t)}{\eta}$ for isotropic turbulence

$$\begin{cases} \beta = a \ln \frac{\Delta}{\eta} + b & \text{for } \frac{\Delta_0}{\eta} < 1 \\ \beta = b & \text{for } \frac{\Delta_0}{\eta} > 1 \end{cases}$$

where a and b are function of $\frac{\Delta_0}{\eta}$ that is

$$\begin{cases} \frac{d}{dt} < \Delta^2 >= (a \ln \frac{\Delta}{\eta} + b) < \Delta^2 >^{\frac{2}{3}} & \text{for } \frac{\Delta_0}{\eta} < 1 \\ \frac{d}{dt} < \Delta^2 >= b < \Delta^2 >^{\frac{2}{3}} & \text{for } \frac{\Delta_0}{\eta} > 1 \end{cases}$$

Conclusion

• We use KS coupled with Rapid Distortion Theory to model one and two-particle diffusion in turbulence with stratification and rotation. We show that the simplified Corrsin hypothesis introduced has to be restricted to pure rotation only.

• One-particle and two-particle diffusion is investigated in horizontal and vertical direction. For one-particle horizontal dispersion we observe four regimes:

- the ballistic regime when $\tau < 1/\Omega$

- a intermediary τ-regime which is not the random walk

- followed by a τ^2-regime up to $\tau \simeq L/u'$

- and finally the random walk τ-regime when $\tau > L/u'$

Conclusion

For one-particle vertical diffusion the effect of rotation is to lower the diffusion's plateau. A plateau is always observed as soon as there is stratification in accordance with the principle of energy conservation. We conclude that the Eulerian spatial structure that exists in real flows but not in KS plays a minor role in the vertical capping observed in stratified flows.

When considering two-particle diffusion, adding rotation to stratification has the same effect in the ballistic regime as the one observed for one-particle. I.e. it introduces an intermediary τ-regime which delays all the subsequent regimes.

We analyse the locality-in-scale hypothesis in the intermediary range of times and adopt [**?**]'s approach to study β_h as a function of $\frac{\delta_h}{\eta}$. We conclude that β_h does not depend on the turbulence's parameters but on B. Whereas, χ_h the range other which β_h is close to a constant is independent of B and a function of the turbulence's parameters.