Energy flux to subgrid scales as obtained from particle tracking

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&

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Motivation

$\tilde{A}_{ij}$ properties

Borue V. and Orszag S.A. (1998) Local energy flux and subgrid-scale statistics in three-dimensional turbulence, J. Fluid Mech., 366, 1–31.

Chertkov M., Pumir A. and Shraiman B. I. (1999) Lagrangian tetrad dynamics and the phenomenology of turbulence, Phys. Fluids, 11(8), 2394-2410.

Tao B., Katz J. and Meneveau C. (2002) Statistical geometry of subgrid-scale stresses determined from holographic particle image velocimetry measurements, J. Fluid Mech. 457, 35–78.

van der Bos F., Tao B., Meneveau C. and, Katz J. (2002) Effects of small-scale turbulent motions on the filtered velocity gradient tensor as deduced from holographic particle image velocimetry measurements, Phys. Fluids, 14(7), 2456–2474.

LES context

Pope S. B. (2004) Ten questions concerning the large-eddy simulation of turbulent flows, New J. Phys., 6, 35.

Wang B. C and Bergstrom D. J. (2005) A dynamic nonlinear subgrid-scale stress model, Phys. Fluids, 17, 035109.

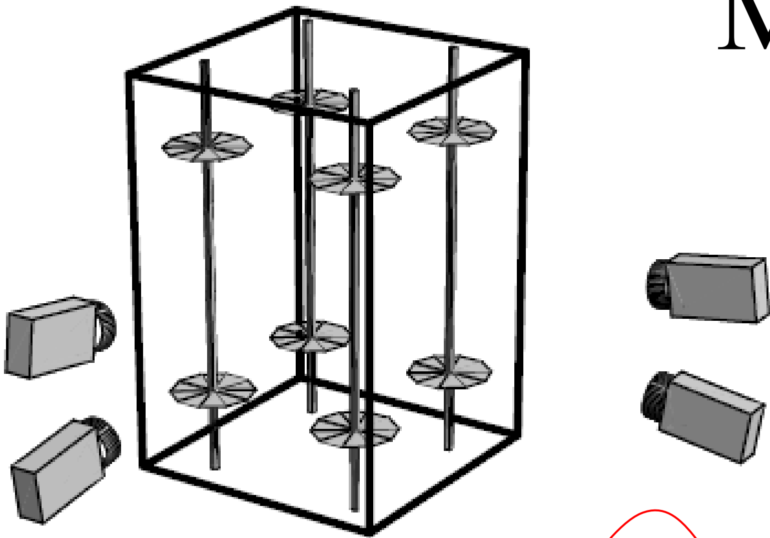
What can 3D-PTV contribute?

so far: HPIV, 2D PIV, DNS

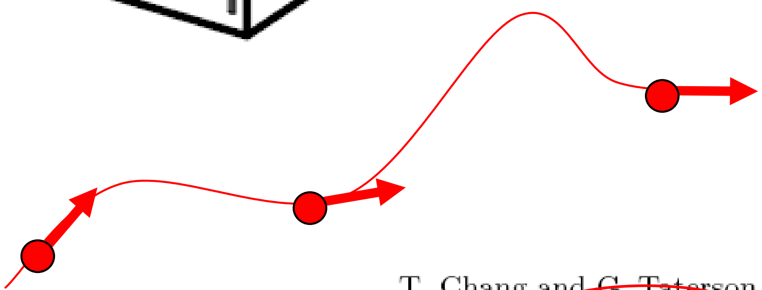
Content of this presentation

- 3D particle tracking
- filtered derivatives
- energy flux how to decompose/represent it
- energy flux from PTV & nonlinear model
- correction for nonlinear model
- conclusions

Main idea of 3D-PTV



to follow a 3D (!) particle position as opposed to 2D PIV!



started 1983

T. Chang and G. Taterson. Application of image processing to the analysis of three-dimensional flow fields. *Opt. Engng*, 23:283–287, 1983.

R. Racca and J. Dewey. A method for automatic particle tracking in a three-dimensional flow field. *Experiments in Fluids*, 6:25–32, 1988.

Marko Virant and Themistocles Dracos. 3D PTV and its application on Lagrangian motion. *Meas. Sci. Technol.*, 8:1529–1552, 1997.

Søren Ott and Jakob Mann. An experimental investigation of the relative diffusion of particle pairs in three dimensional turbulent flow. *J. Fluid Mech.*, 422:207–223, 2000.

A. La Porta, Greg A. Voth, Alice M. Crawford, Jim Alexander, and Eberhard Bodenschatz. Fluid particle accelerations in fully developed turbulence. *Nature*, 409:1016–1017, Feb. 2001.

Beat Lüthi, Arkady Tsinober, and Wolfgang Kinzelbach. Lagrangian measurement of vorticity dynamics in turbulent flow. *J. Fluid Mech.*, 528:87–118, 2005.

Mickaël Bourgoin, Nicholas T. Ouellette, Haitao Xu, Jacob Berg, and Eberhard Bodenschatz. Pair Dispersion in Turbulence. *Science*, 311:835–838, 2006.

Jacob Berg, Beat Lüthi, Jakob Mann, and Søren Ott. An experimental investigation: backwards and forwards relative dispersion in turbulent flow. *Phys. Rev. E*, 74(1):016304, 2006.

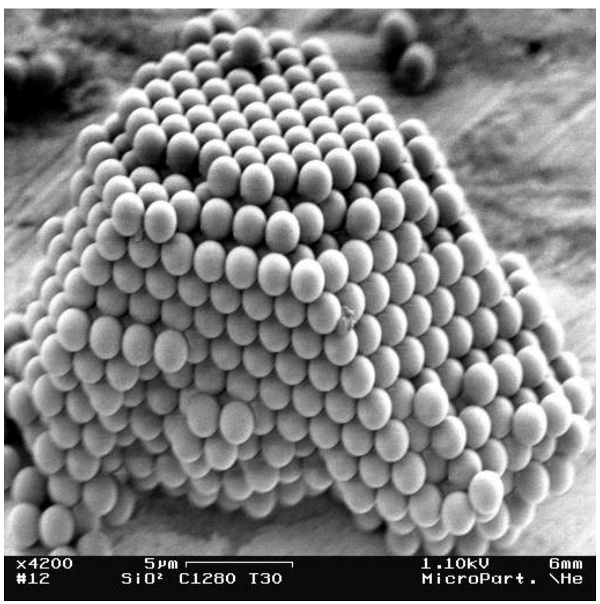
List of technical aspects

•flow tracers
•illumination
•cameras
•observation volume
•camera callibration
•particle detection
•from 2D to 3D positions
•particle tracking

Flow tracers

high tech, accurate, expensive:

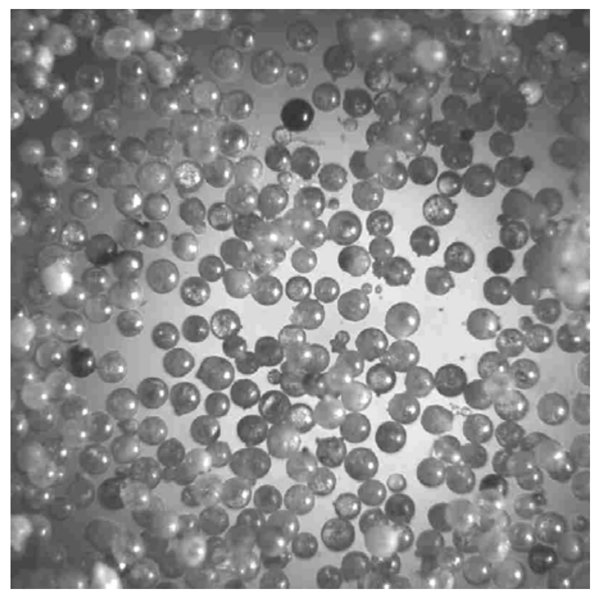
micro particles GmbH



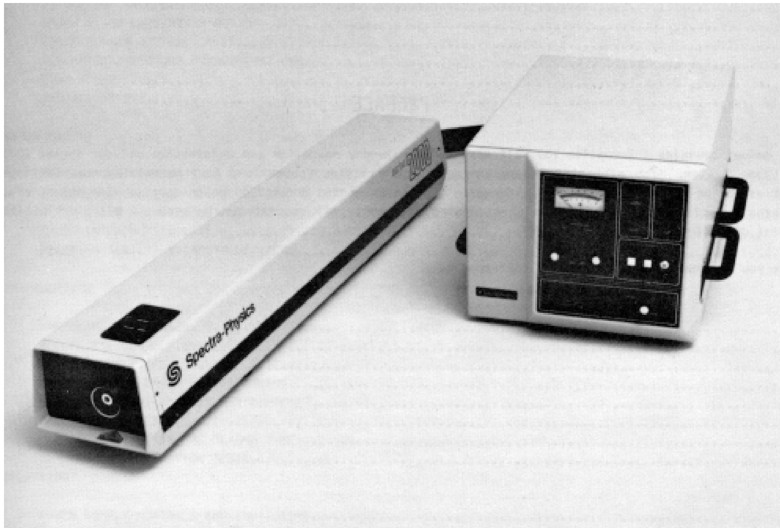
low tech, accurate, cheap:

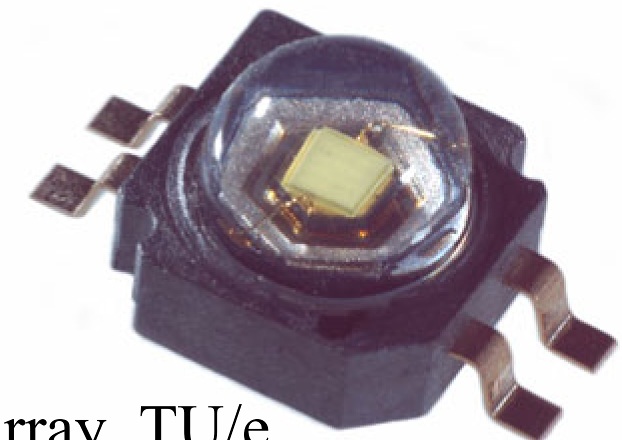
Idea: Søren Ott & Jakob Mann, Risø, Denmark

fly ash→sieving→∅50-60µm



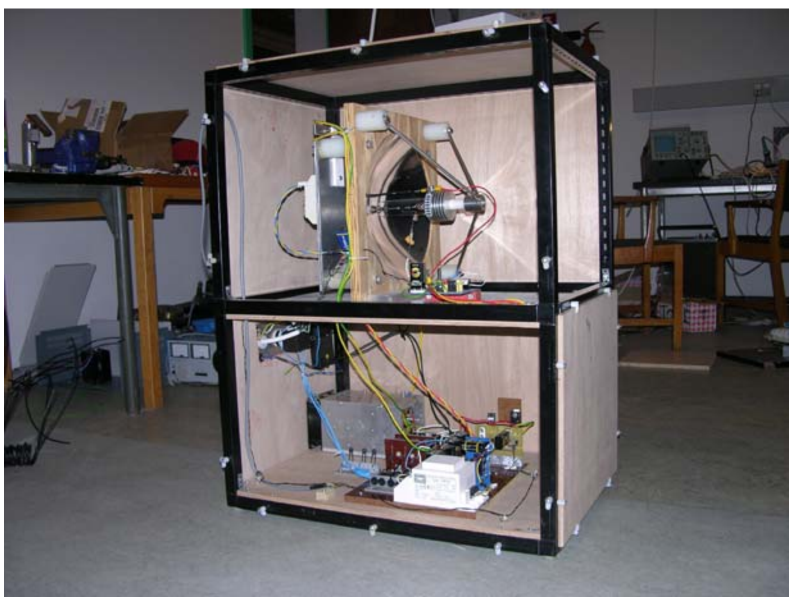
Illumination





LED array, TU/e

Lorenzo del Castello, Herman Clercx



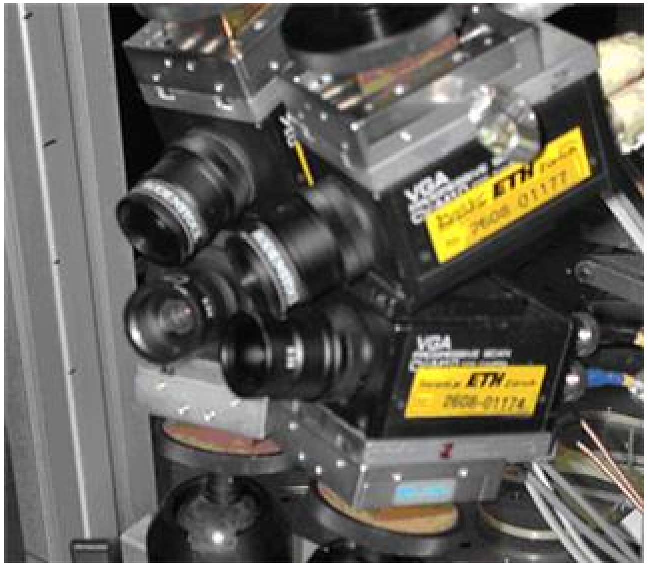
trend towards smarter solutions

Fast digital cameras

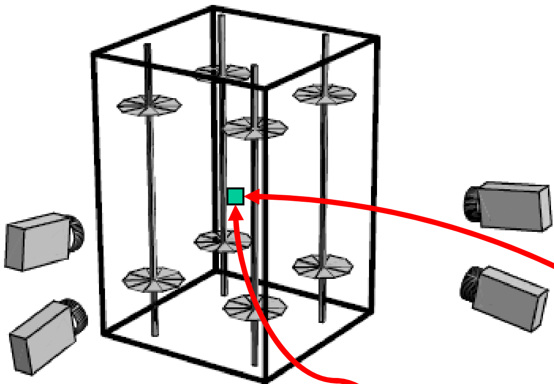
pixel: 500x500
frame rate: 50Hz

pixel: 1000x1000
frame rate: 5000Hz
or
pixel: 250x250
frame rate: 80’000Hz

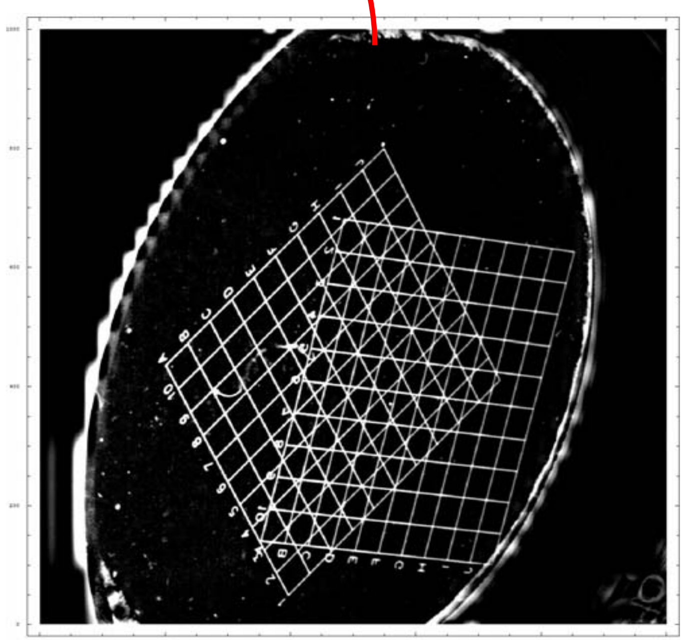
data storage is main bottelneck

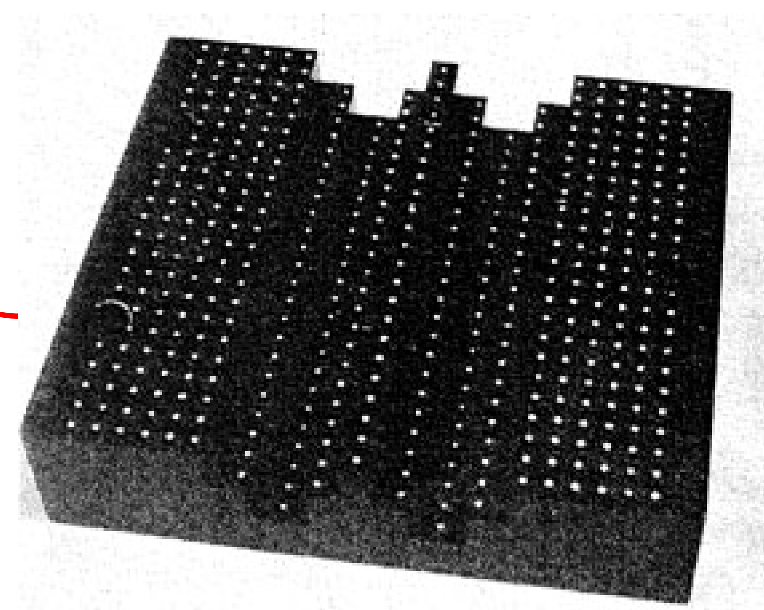


Camera callibration



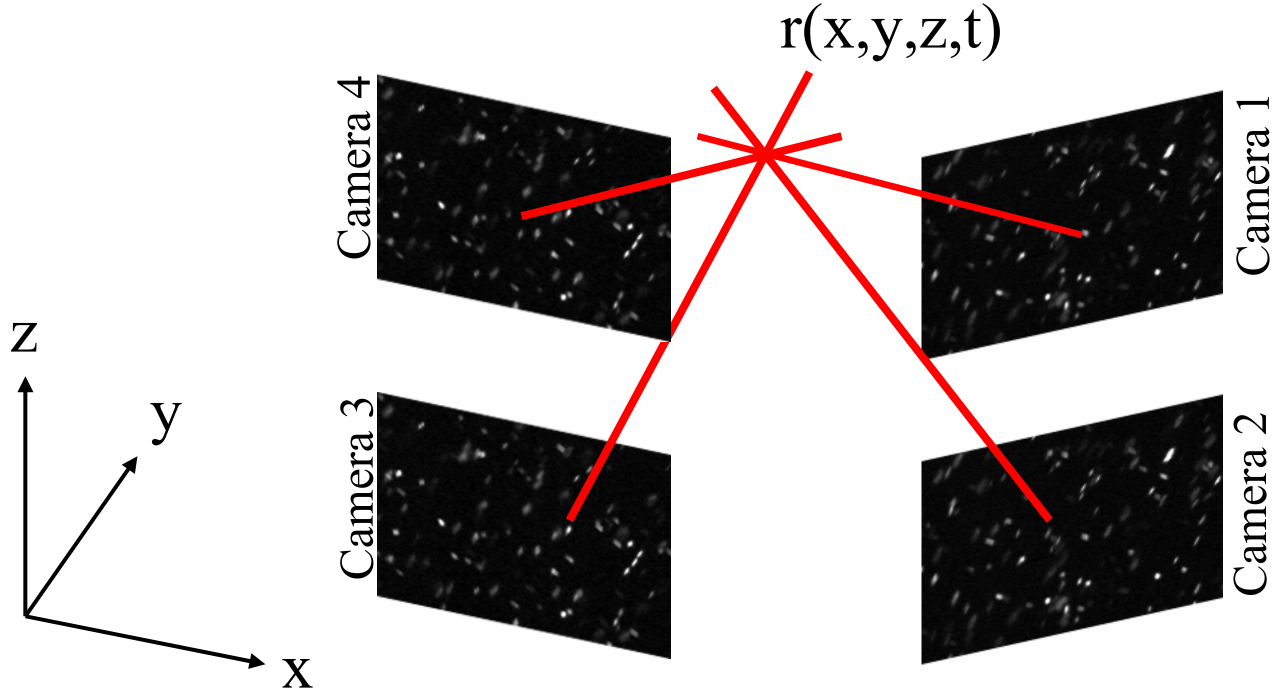
- teach the cameras with know grid points
- problem: how to have space filling target?
- solution in part: callibration on flow tracers



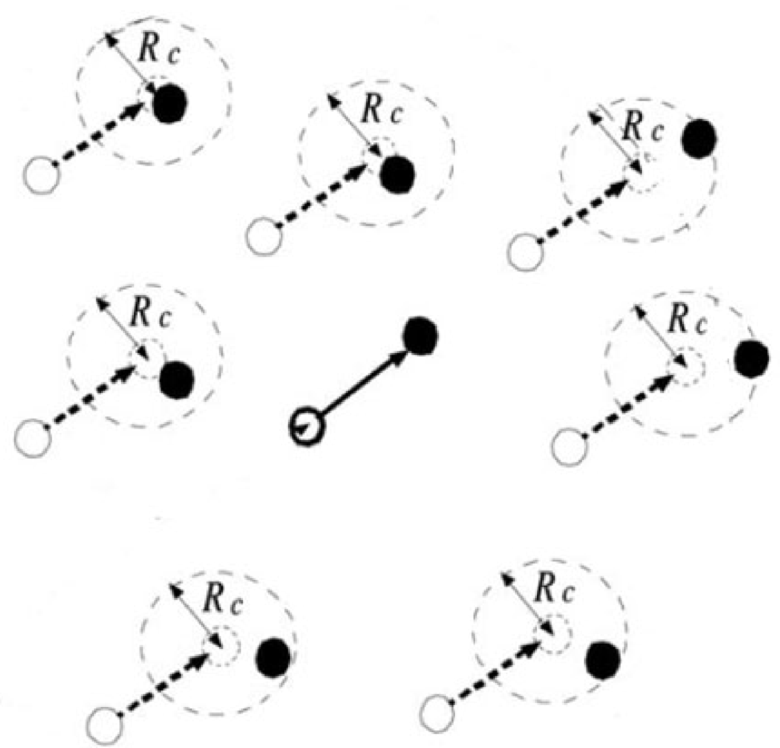


From 2D to 3D position

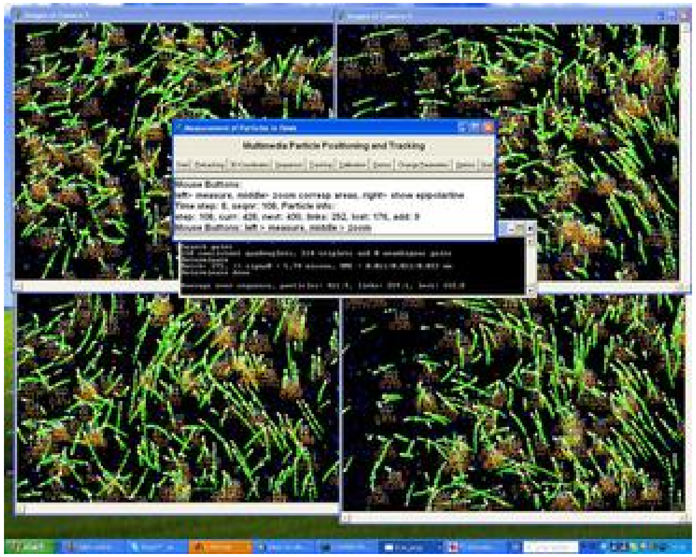
callibration and 2D position accuracy, seeding density, etc.



Tracking through consequtive images

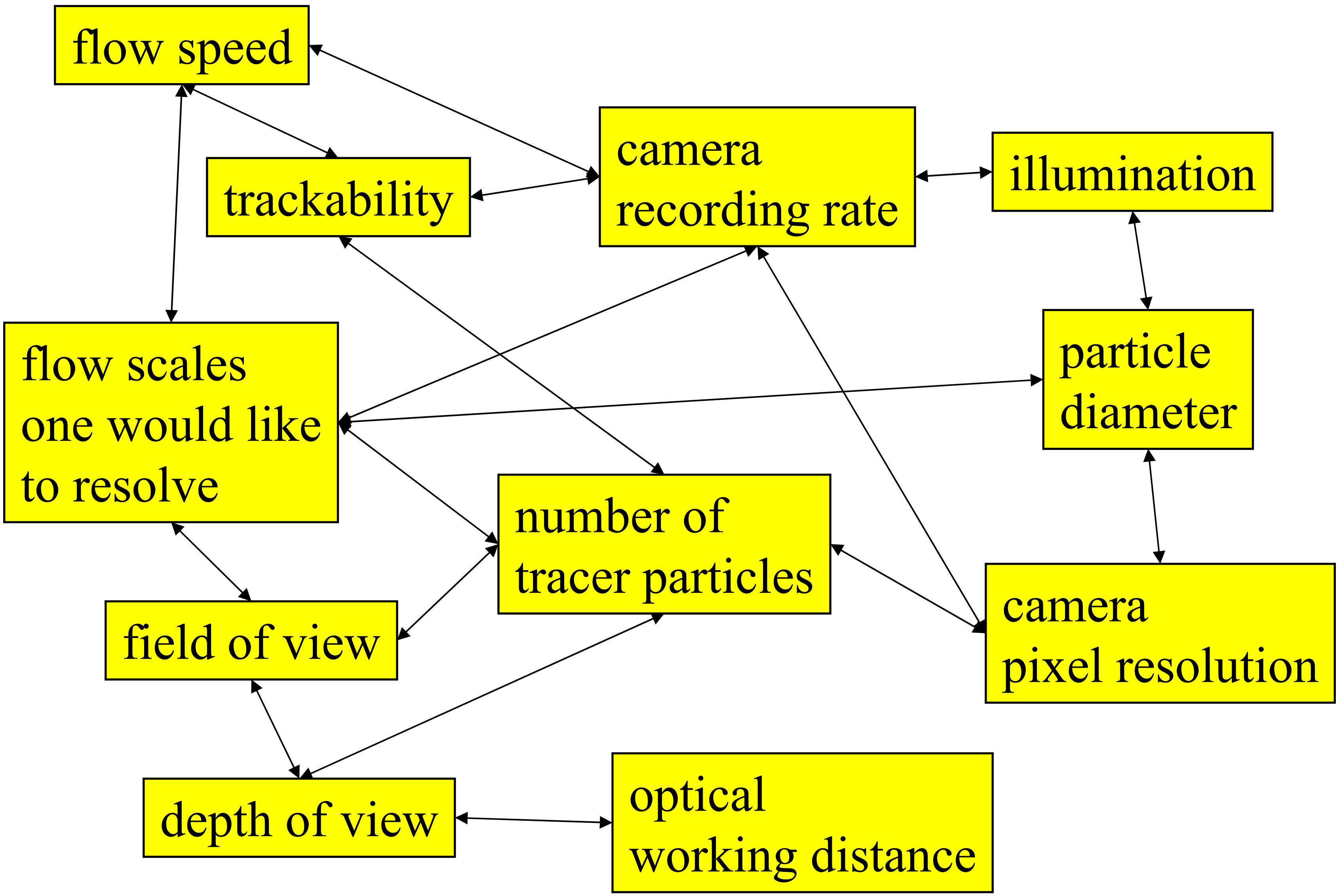


tracking criteria: particle must not travel further than their typical spacing



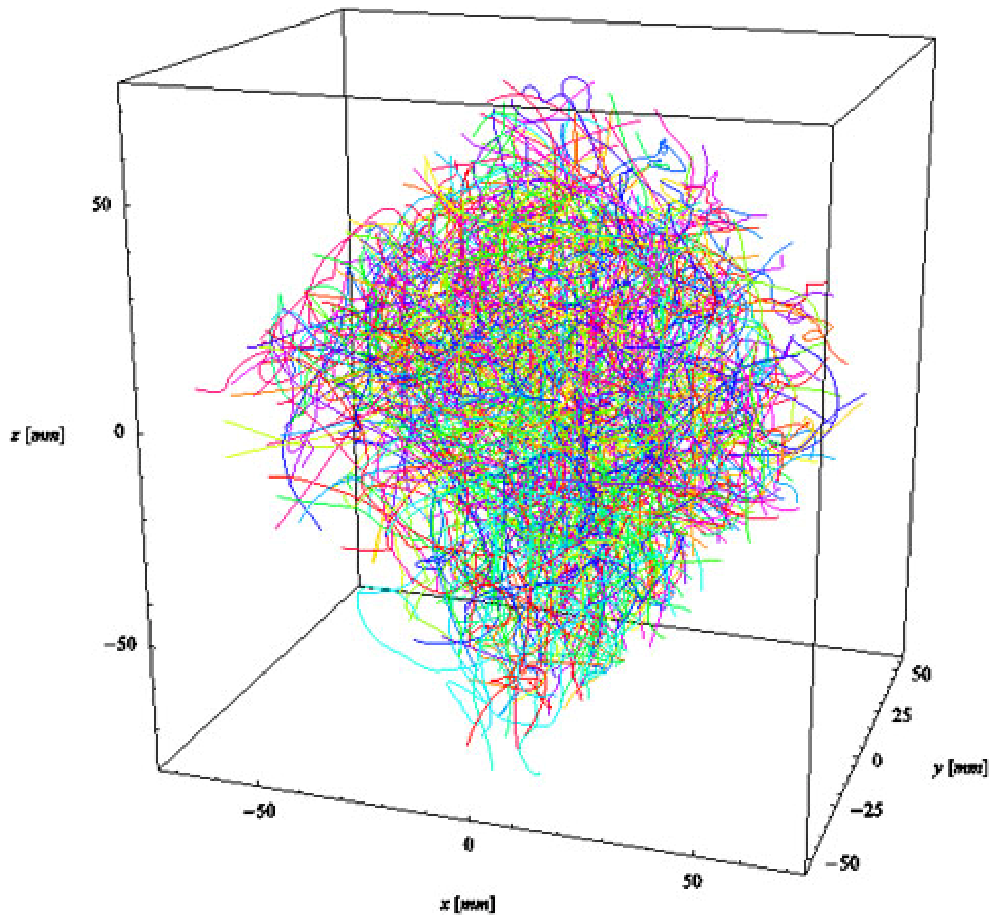
codes available at www.3dptv.schtuff.com

Many dependencies, many choices…

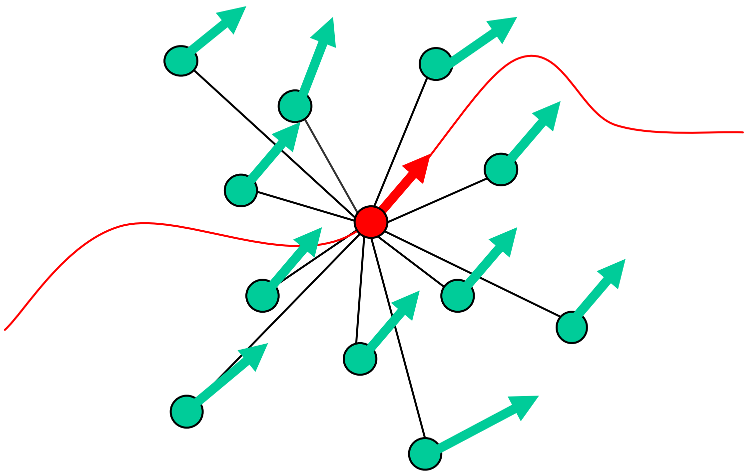


Final output is the start for analysis

if all goes well, one can finally start 'learning' about the flow



Velocity derivatives



differentiate
convoluted velocity field
to get
velocity derivatives

challenge to get
HIGH SEEDING DENSITY

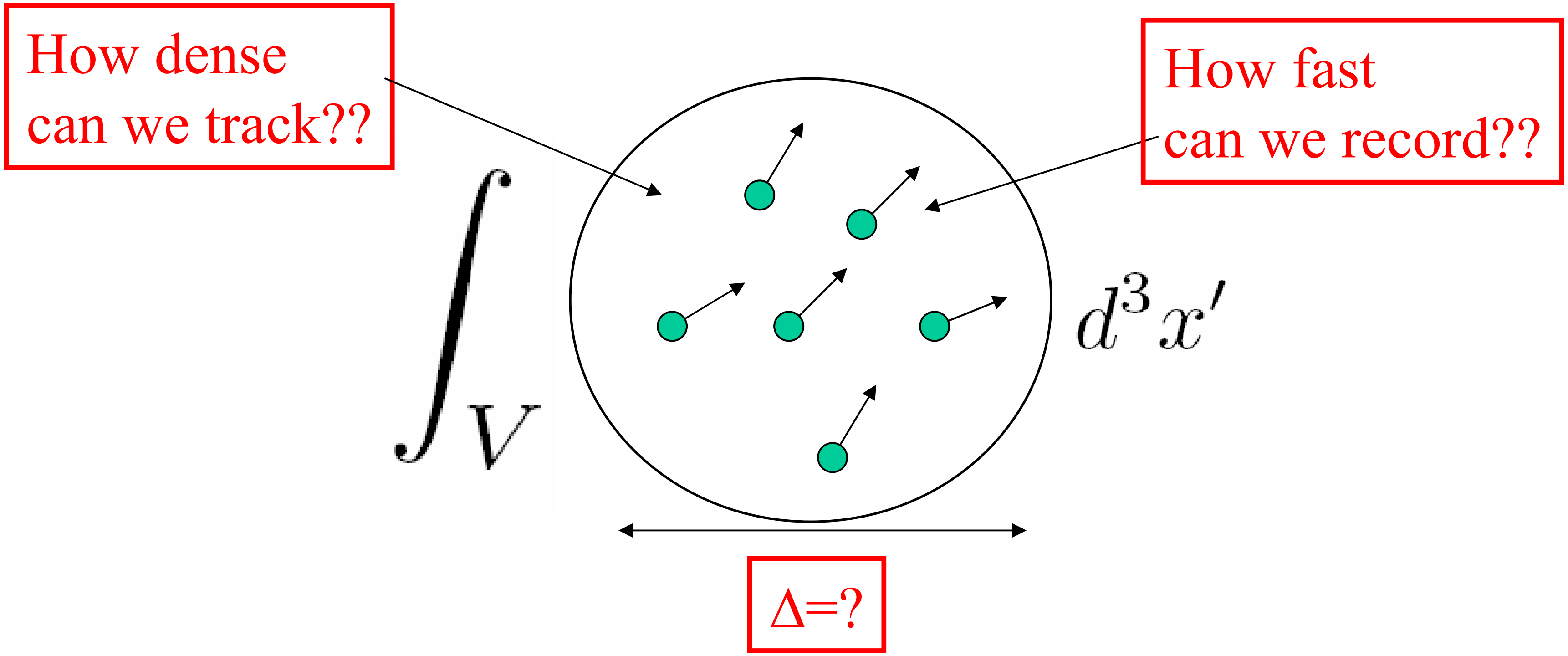
$$\tilde{\boldsymbol{u}}(\boldsymbol{x}) \approx \frac{4\pi(\Delta/2)^3}{3n} \sum_{\boldsymbol{x}' \in B_\Delta(\boldsymbol{x})} \rho_\Delta(\boldsymbol{x}-\boldsymbol{x}')\, \boldsymbol{u}(\boldsymbol{x}')$$

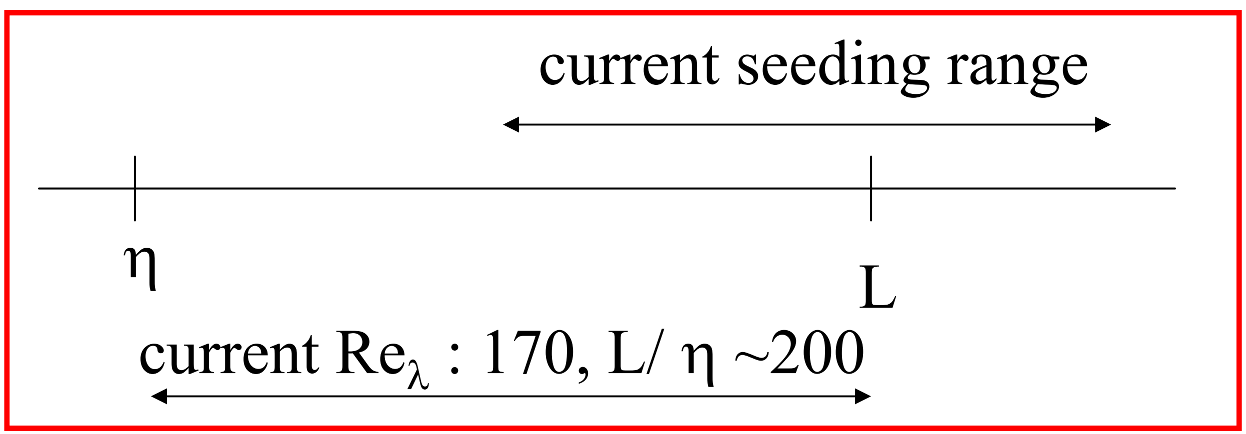
$$\rho_\Delta(r) = \begin{cases} \frac{15((\Delta/2)^2 - r^2)}{8\pi(\Delta/2)^5} & \text{for } r \le \Delta/2 \\ 0 & \text{for } r > \Delta/2 \end{cases}$$

$$\tilde{A}_{ij}(\boldsymbol{x}) \approx \frac{20}{(n-1)\Delta^2} \sum_{\boldsymbol{x}' \in B_\Delta(\boldsymbol{x})} (x'_j - x_j)\, u_i(\boldsymbol{x}').$$

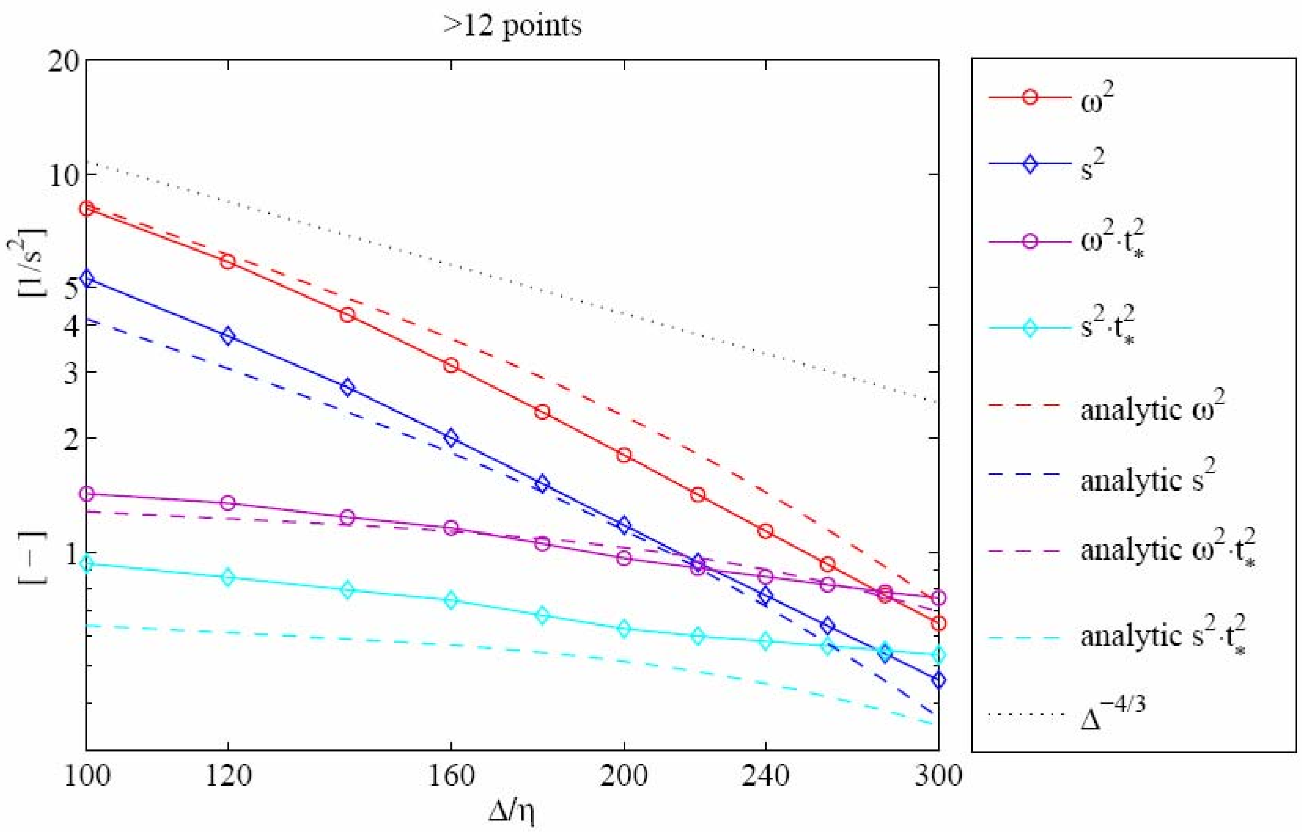
B. Lüthi ETH, Søren Ott Risø, Jacob Berg, Jakob Mann

Particle seeding, scales?

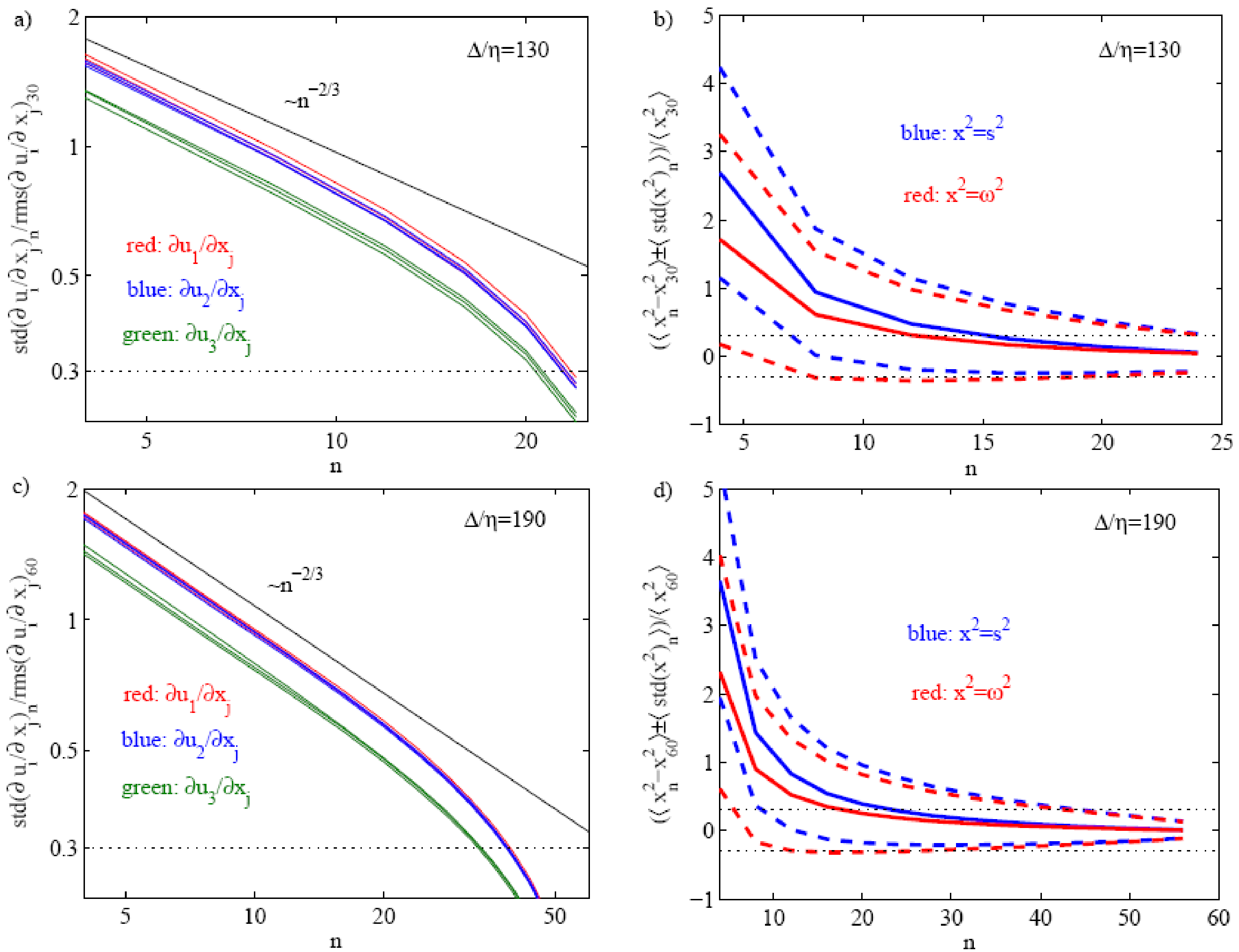




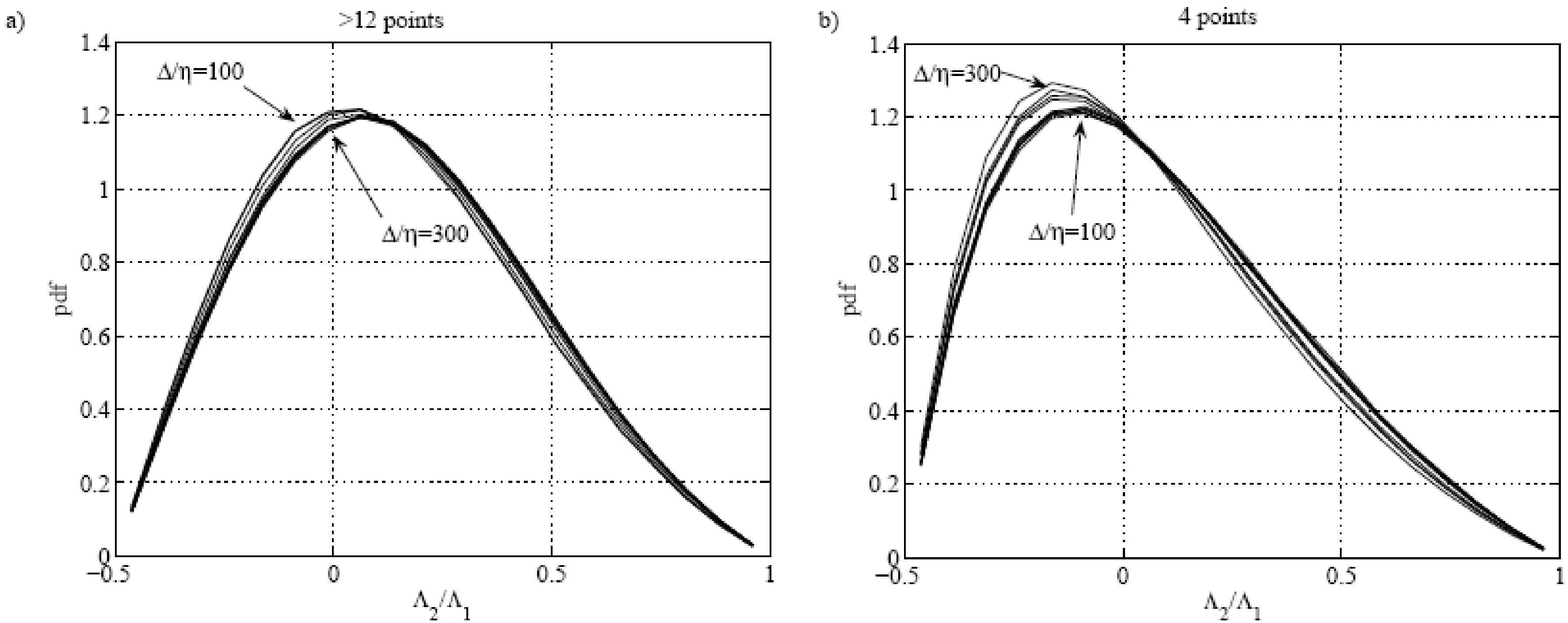
Velocity gradients



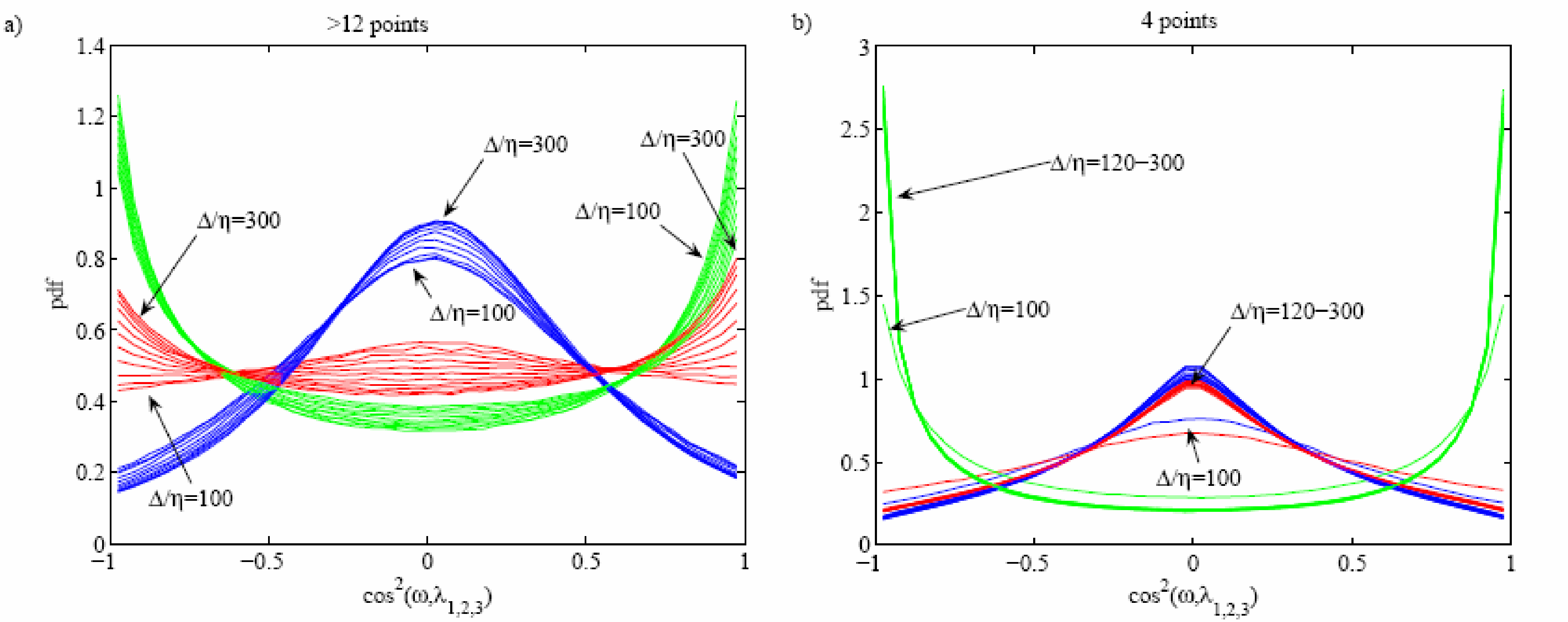
Velocity gradients



Self amplification

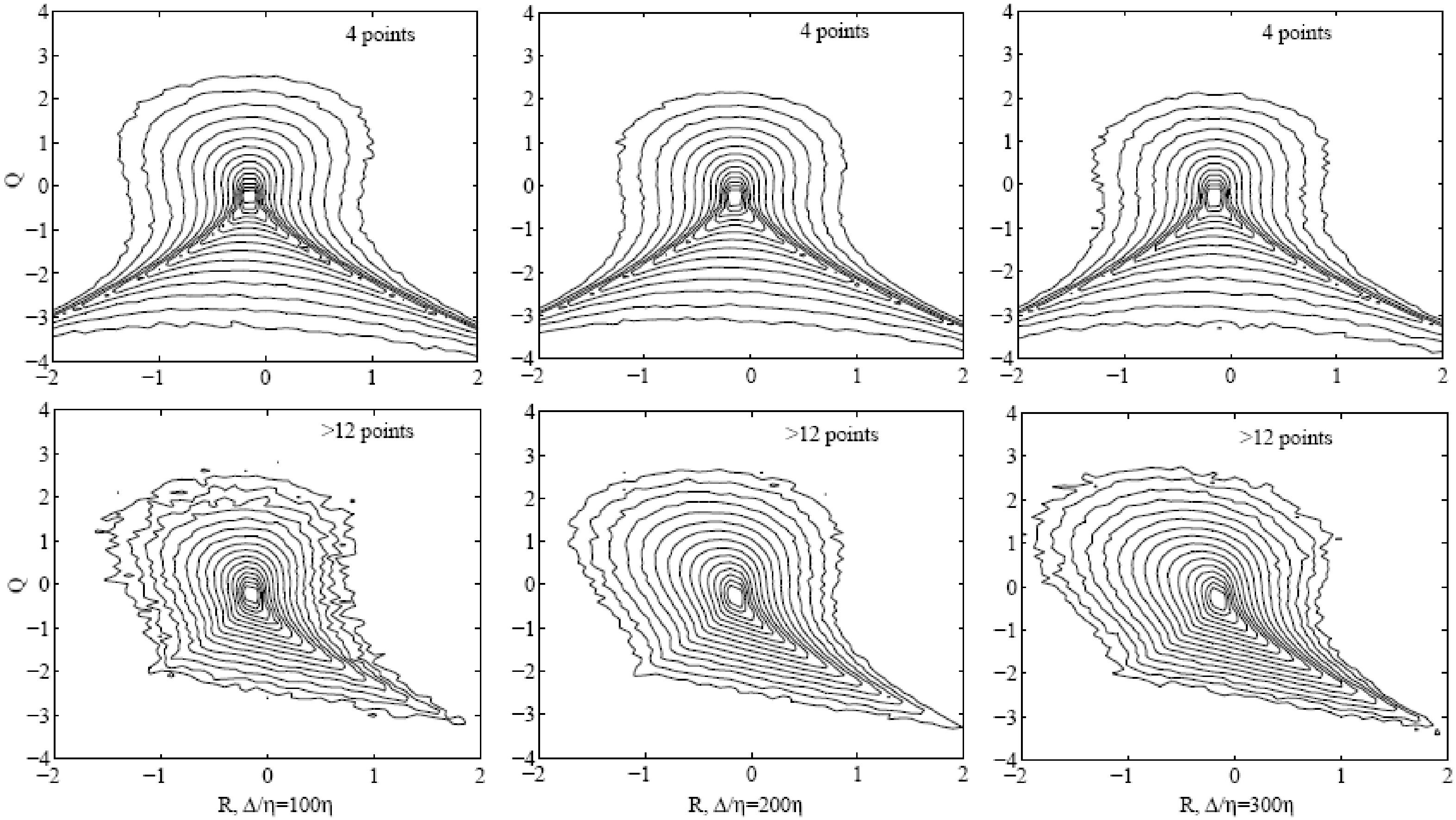


Structure



a)
>12 points
1.4
1.2
1
0.8
0.6
0.4
0.2
0
pdf
−1
−0.5
0
0.5
1
cos2(ω,λ1,2,3)
Δ/η=300
Δ/η=300
Δ/η=300
Δ/η=100
Δ/η=100
Δ/η=100
b)
4 points
3
2.5
2
1.5
1
0.5
0
pdf
−1
−0.5
0
0.5
1
cos2(ω,λ1,2,3)
Δ/η=120−300
Δ/η=100
Δ/η=120−300
Δ/η=100

RQ invariant maps



LES context

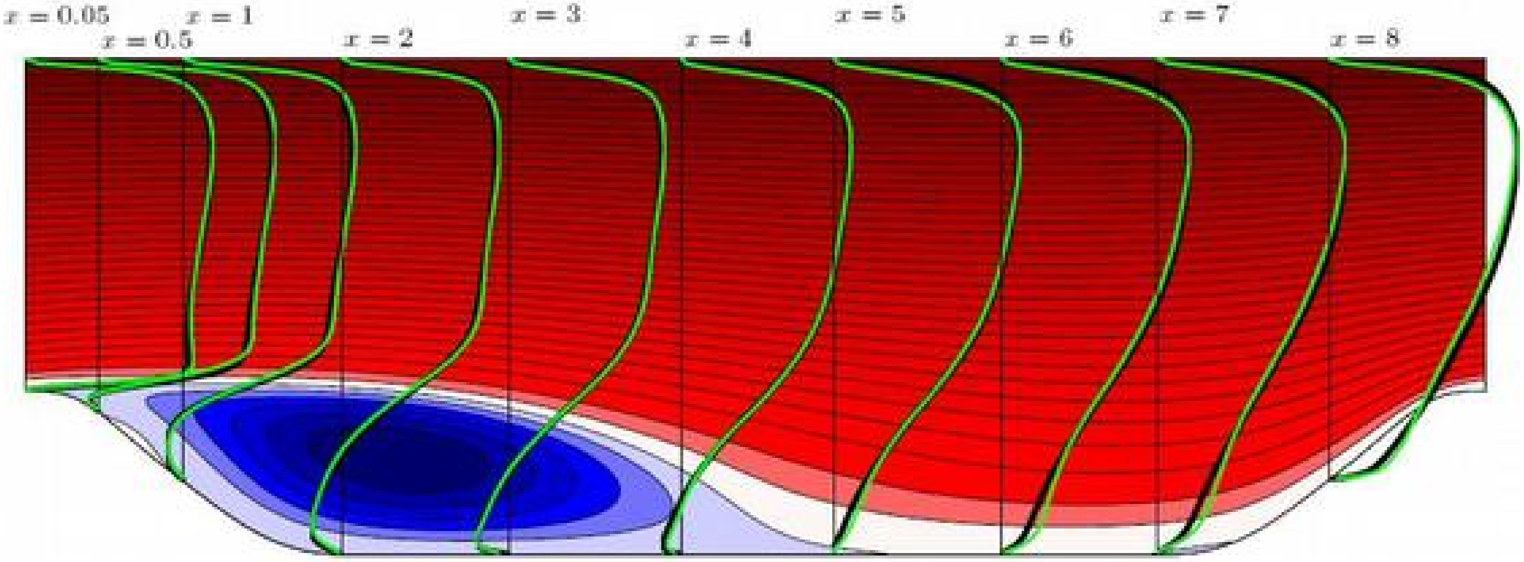


Figure 3. Streamfunction contours of the average velocity field and mean streamwise velocity profiles at ten downstream positions ($\langle \mathrm{Re}_{S_h b} \rangle_t \approx 2800$). Positive streamfunction values appear in red, negative values in blue. ········ Edges of mean flow recirculation zones. —— Present LES, —— DNS by Peller & Manhart.[1]

Jörg Ziefle, Kleiser LES group ETH

Definition of SGS TKE production rate[1] or 'energy flux'

$$\bar{u}_{i,i} = 0\,,$$

$$\dot{\bar{u}}_i + (\bar{u}_i \bar{u}_j)_{,j} = -\bar{p}_{,i}/\rho - \tau_{ij,j} + \nu \bar{u}_{i,jj}$$

$$\tau_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j$$

$$\tau^*_{ij} = \tau_{ij} - \frac{\delta_{ij}}{3}\tau_{kk}$$

$$\mathcal{P}_r = -\,\tau^*_{ij}\bar{S}_{ij}$$

also referred to as:

- energy flux
- SGS dissipation

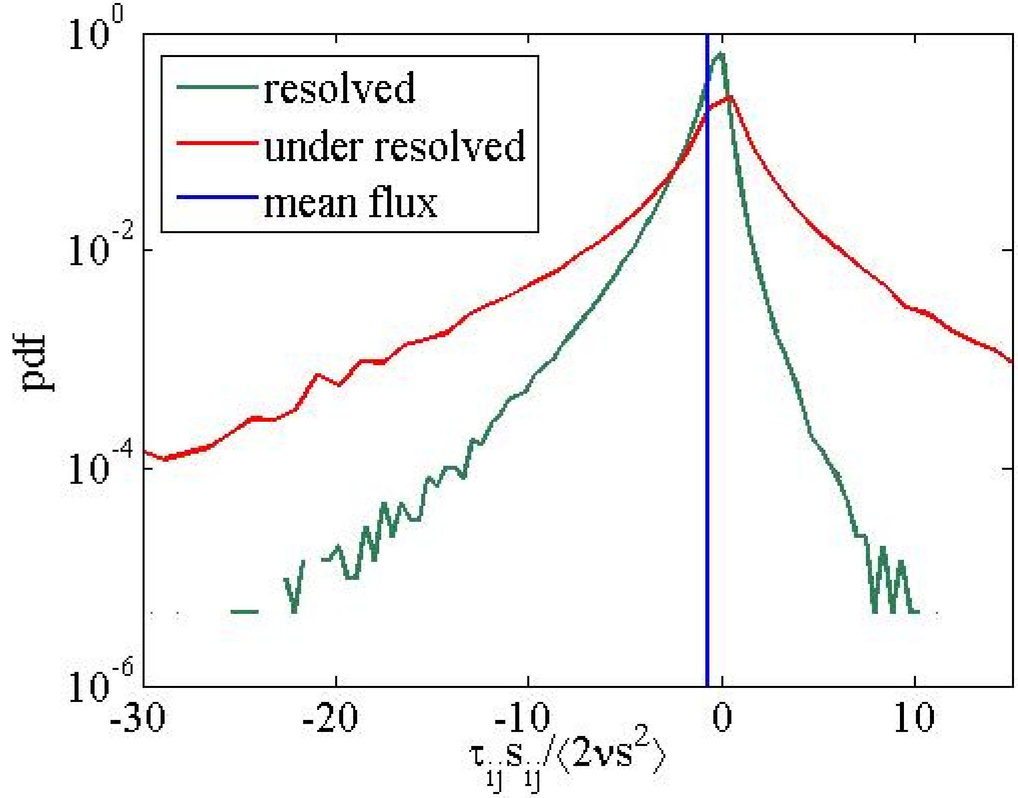
[1] S. B. Pope, *Turbulent Flows* (Cambridge University Press, Cambridge, 2000).

Role of PTV for LES?

•holographic PIV, Tao et al. 2002, van der Bos et al. 2002
•sonic anemometer array, Higgins et al. 2003
•3D-PTV

•check SGS modeling assumptions
•check is possible in real flows

Energy flux from 3D-PTV

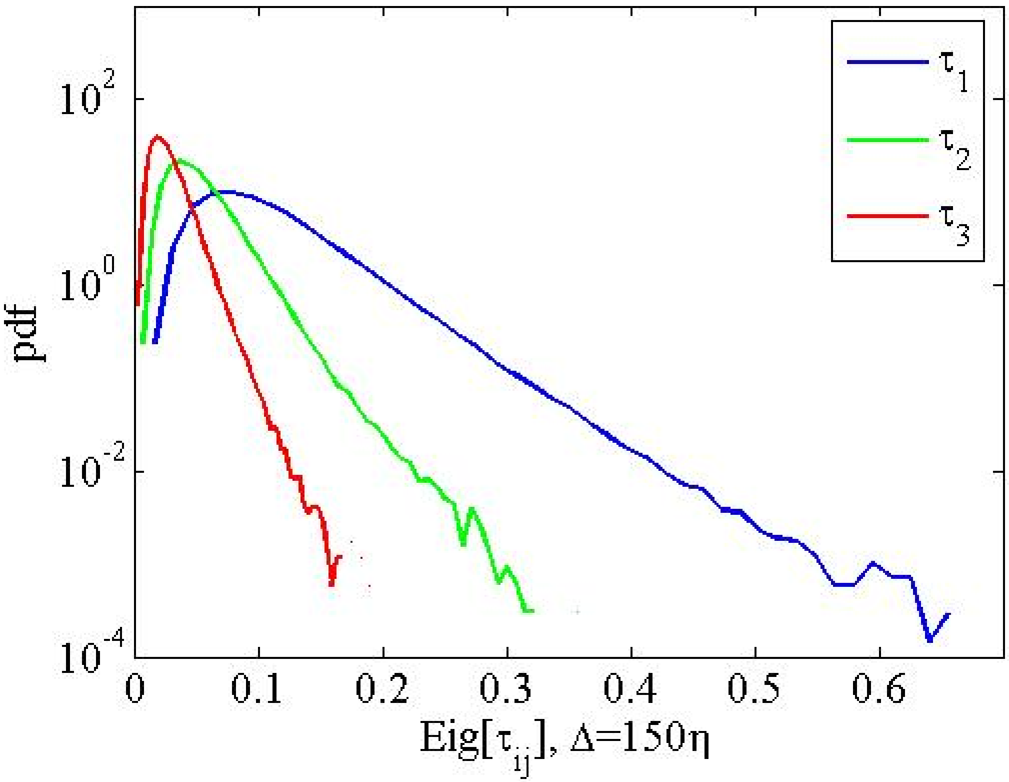


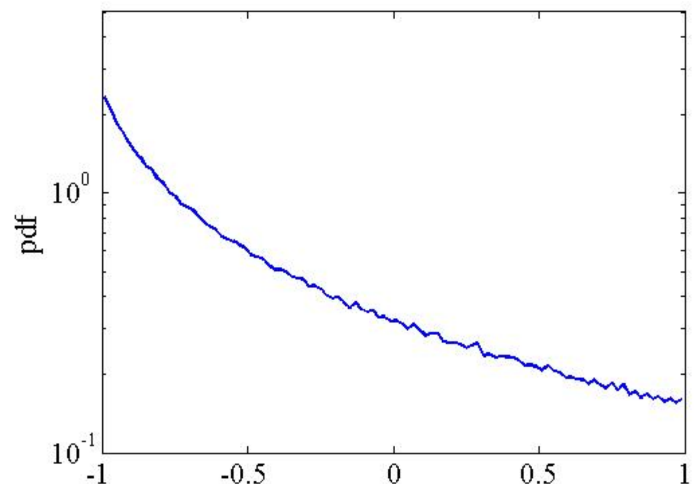
mean flux ~ 0.7 ε

Tao et al. 2002:
mean flux > 2 ε

how to trust flux?
analysis and sources of error?

Eigenvalues of τ_{ij}

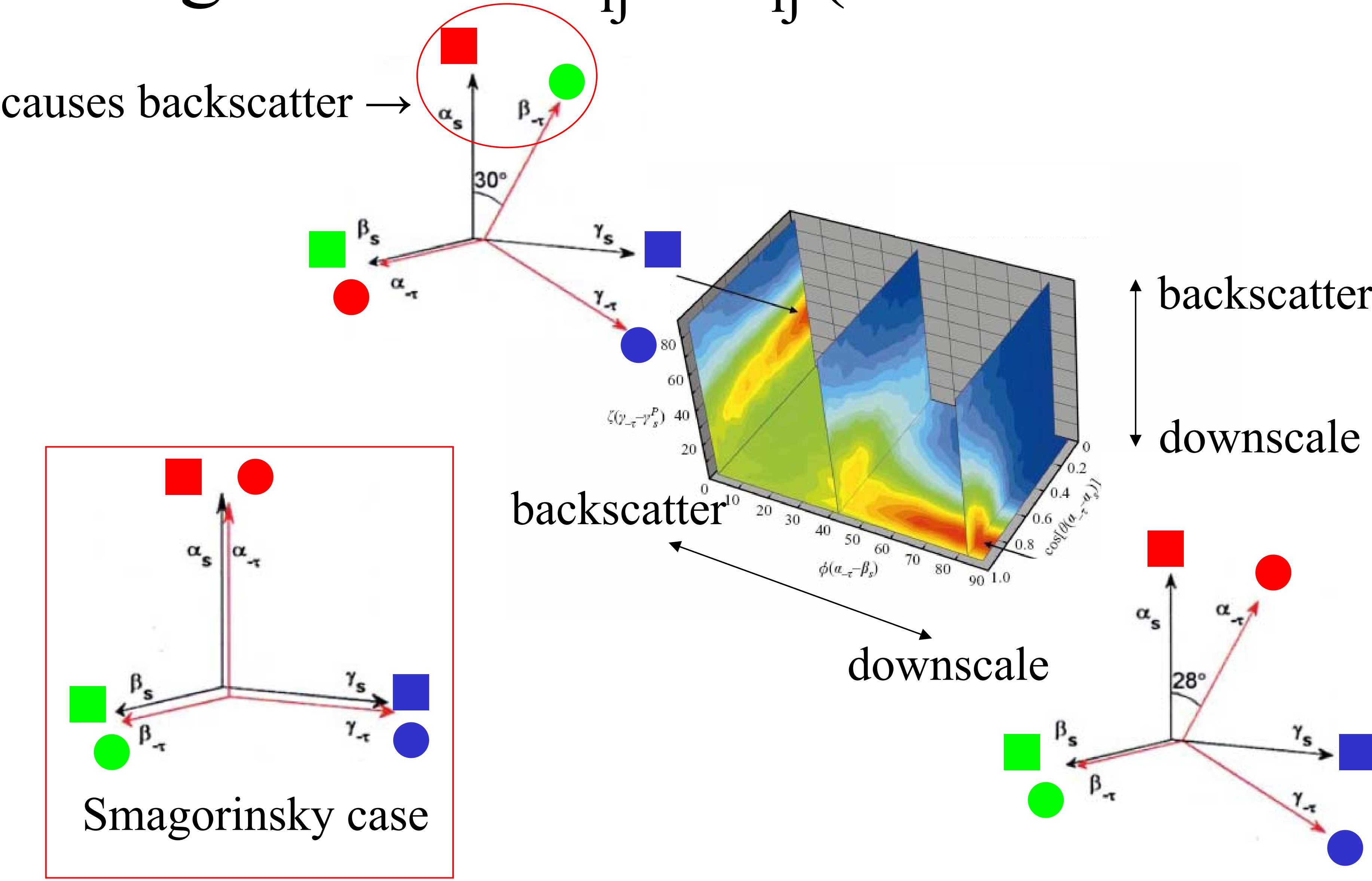




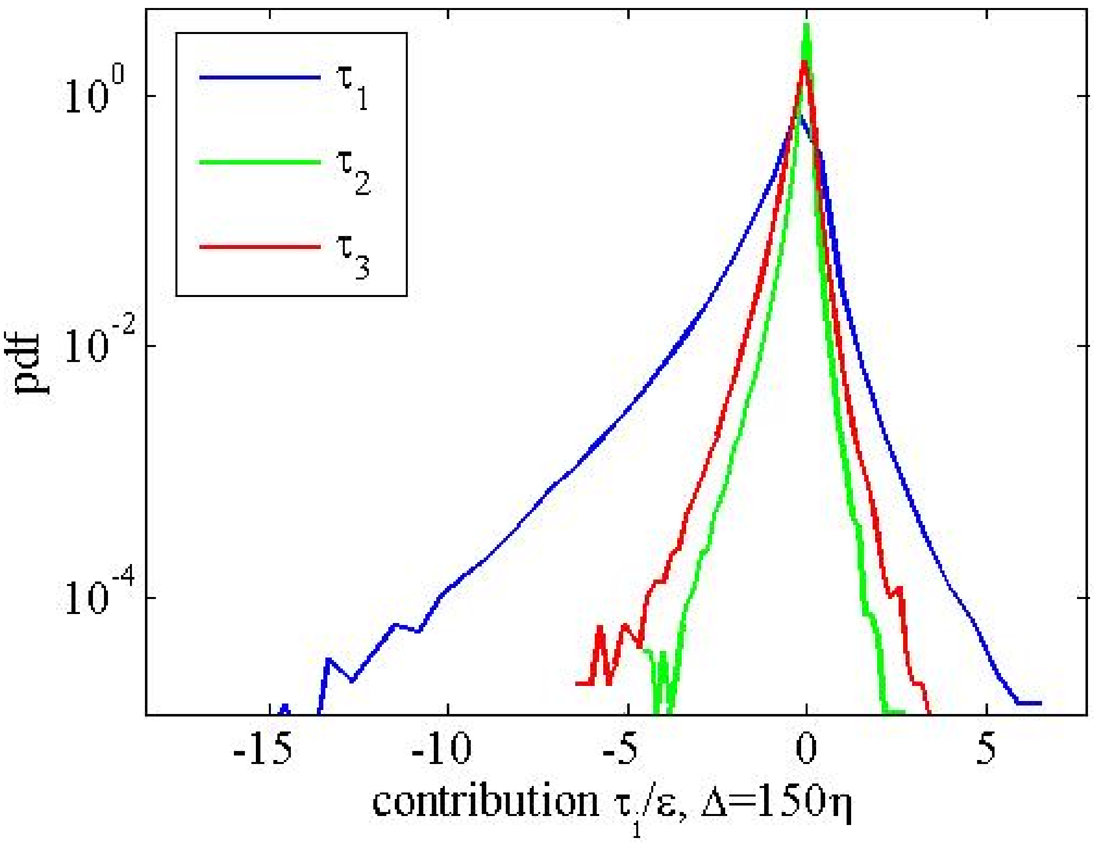
$$s^* = -3\sqrt{6}\alpha_s\beta_s\gamma_s/(\alpha_s^2+\beta_s^2+\gamma_s^2)^{3/2}$$

Chumakov 2006 JFM also introduces q*

Alignment of τ_{ij} to s_{ij} (Tao et al. 2002)

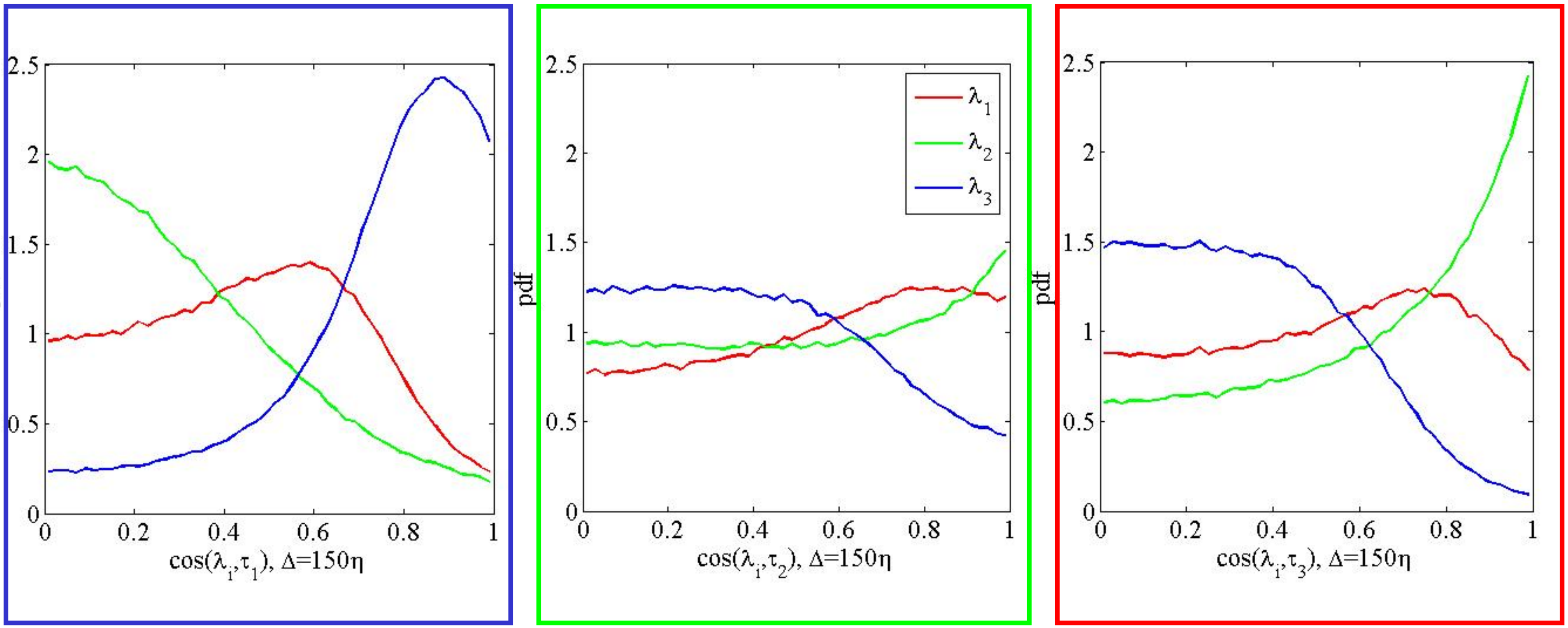


Contributions to flux



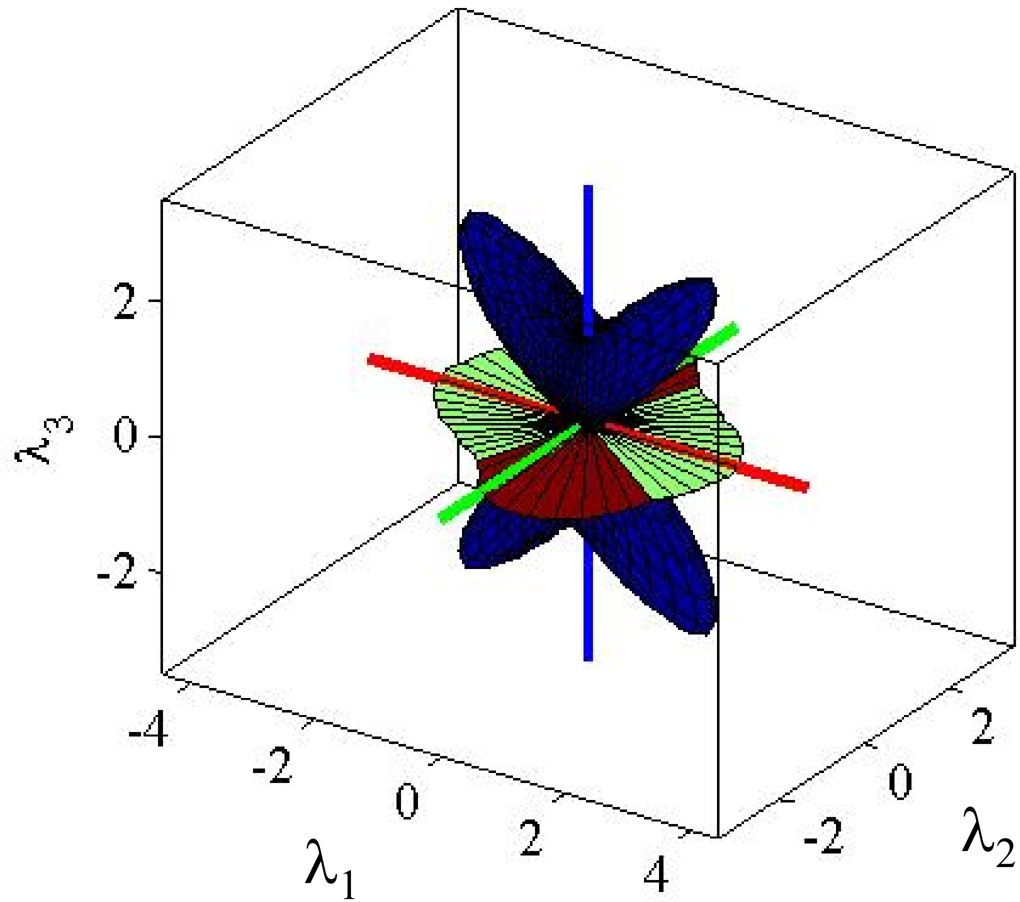
most important τ_1 least important τ_2

Alignment of τ_{ij} to s_{ij}



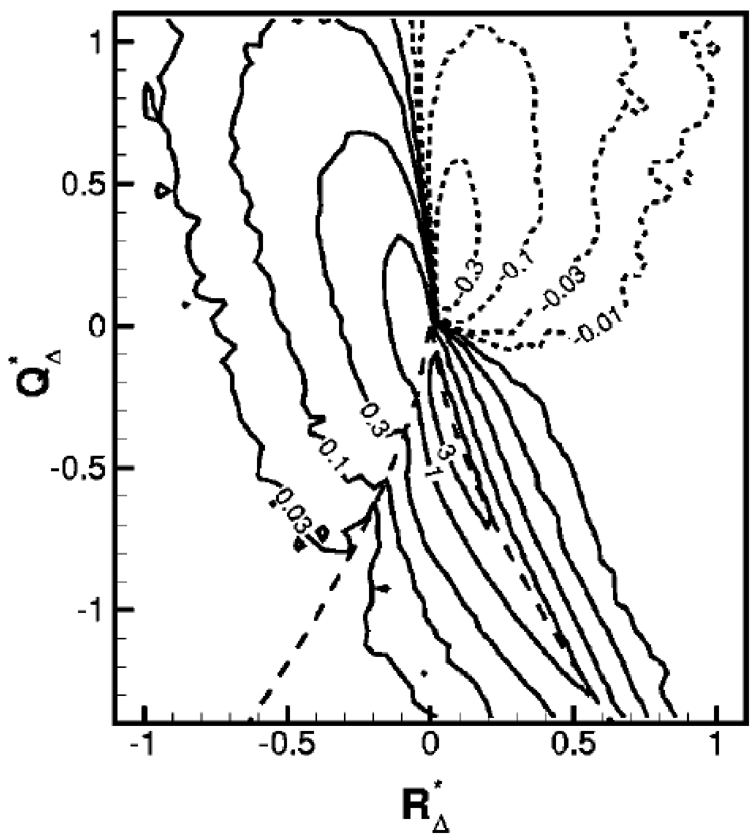
- τ_1, τ_3 most relevant
- τ_1 aligned with λ_3
- τ_2,τ_3 perpendicular to λ_3

Alignment of τ_{ij} to s_{ij}



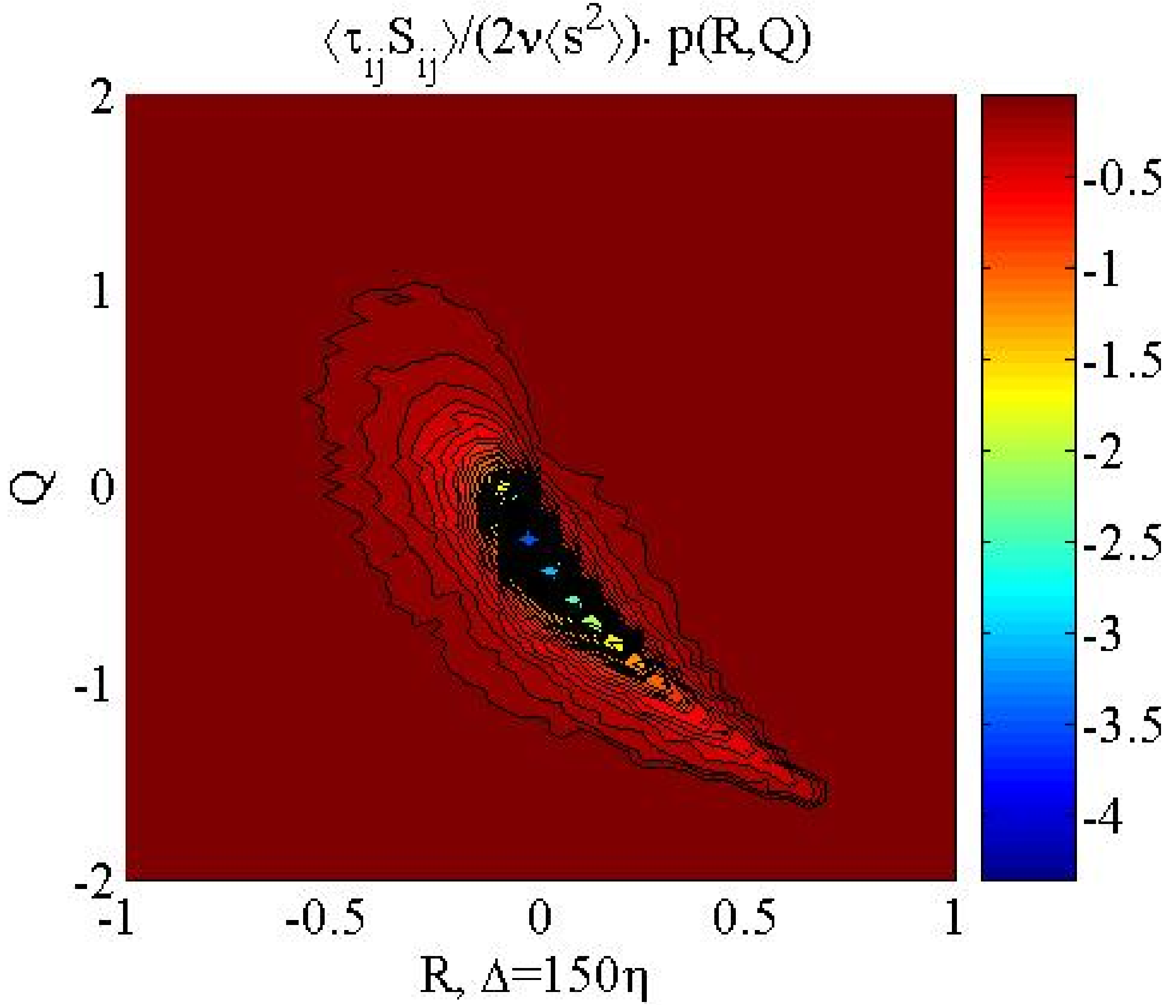
Flux in terms of RQ

(d)

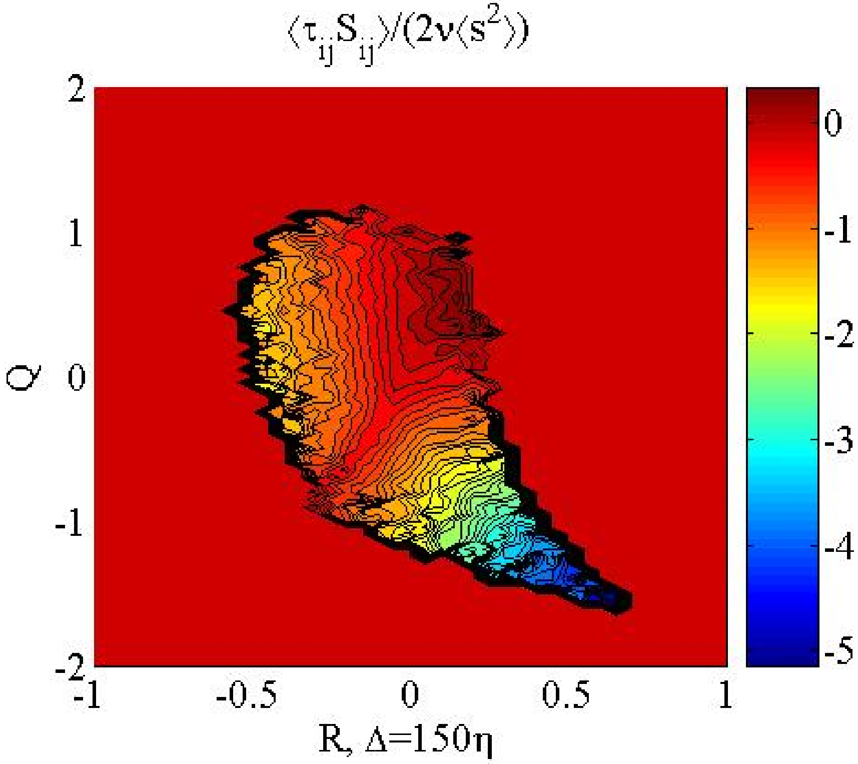


van der Bos et al. 2002,
Physics of Fluids 14(7)
holographic PIV

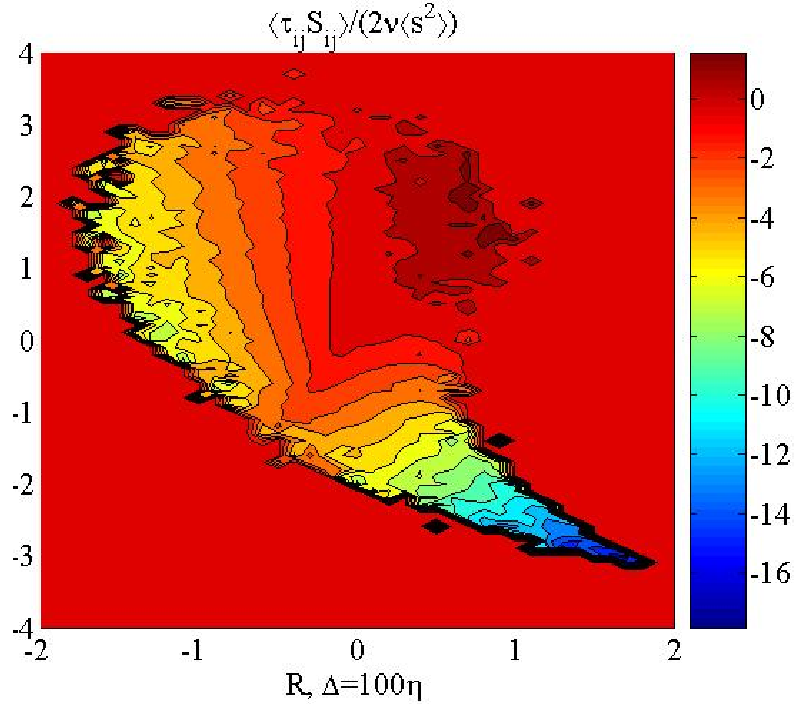
Flux in terms of RQ



Flux in terms of RQ



PTV



DNS

public data from
Biferale, Boffetta, Toschi etc.

Smagorinsky, nonlinear, mixed, …

scalar eddy viscosity:
- related to strain
- no backscatter possible
- stable

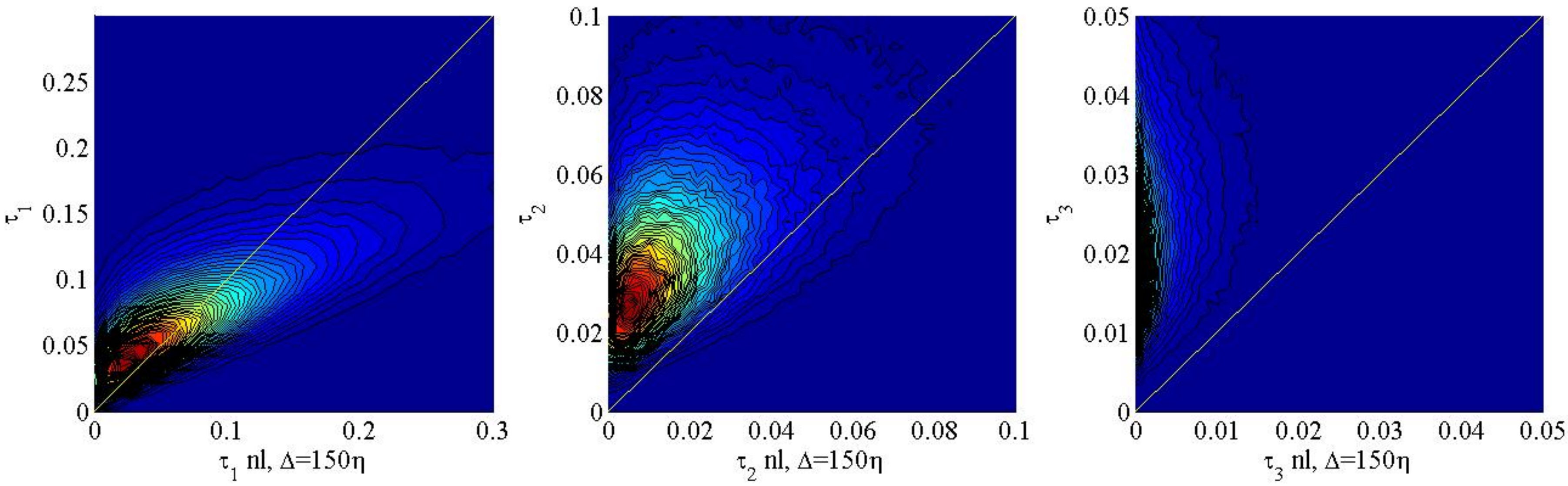
tensor eddy viscosity:
- related to strain and vorticity production
- allows for 'backscatter'
- is unstable

$$\tau_{ij}^{\mathrm{smag},d} = -2c_s^2\Delta^2|\widetilde{S}|\widetilde{S}_{ij} ,$$

$$\tau_{ij}^{nl} = c_{nl}\Delta^2\widetilde{A}_{ki}\widetilde{A}_{kj} ,$$

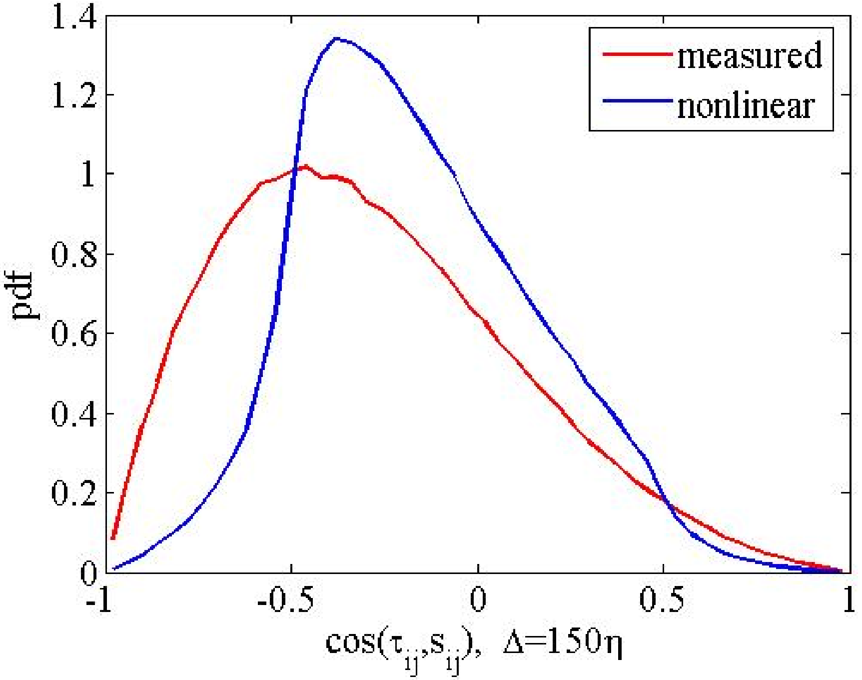
$$\tau_{ij}^{\mathrm{mix}} = c_{nl-m}\Delta^2\widetilde{A}_{ki}\widetilde{A}_{kj} - 2c_{s-m}^2\Delta^2|\widetilde{S}|\widetilde{S}_{ij}$$

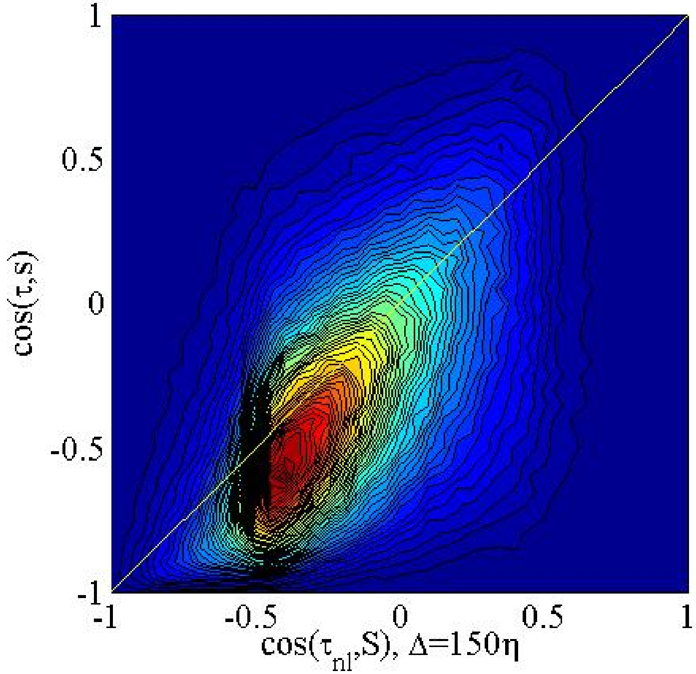
Validation : Eigenvalues of τ_{ij}



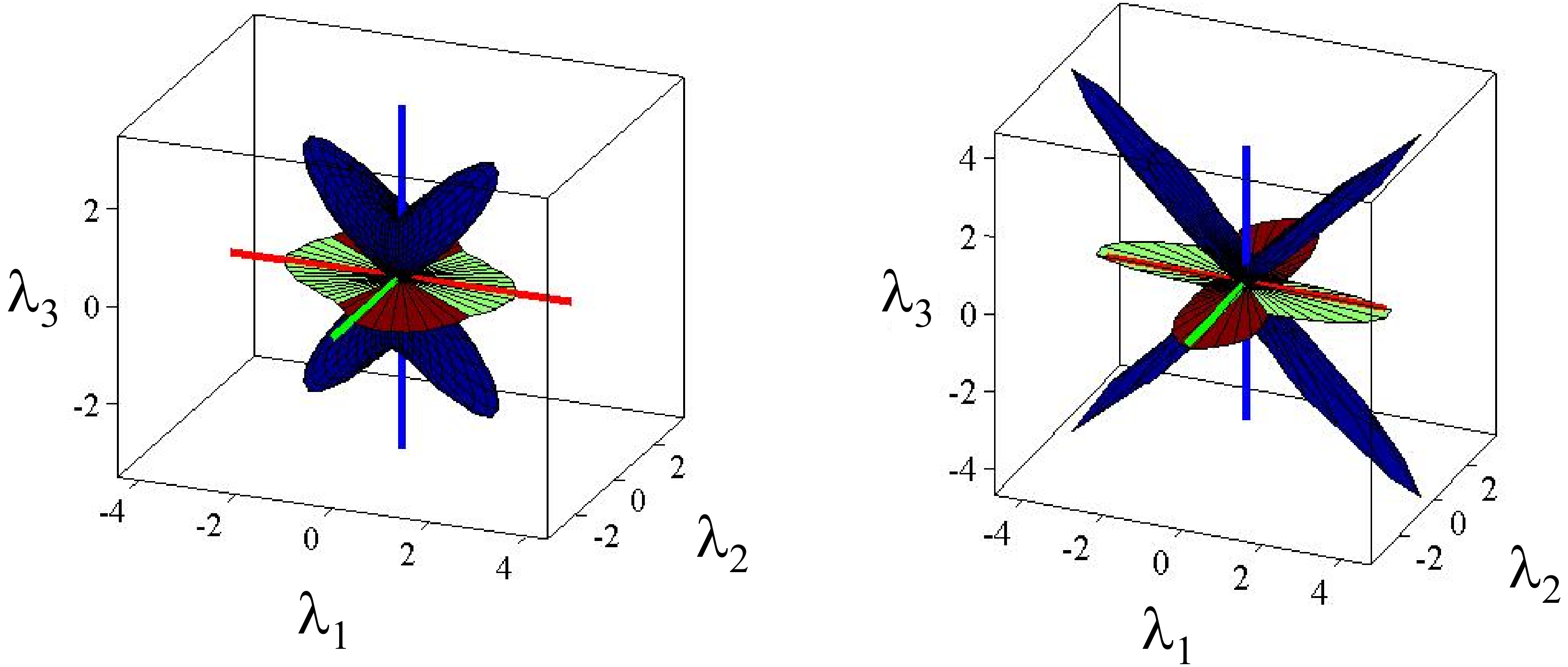
nonlinear model:
- overestimates large τ_1
- underestimates τ_2 and τ_3

Validation : Alignment of τ_{ij} to s_{ij}





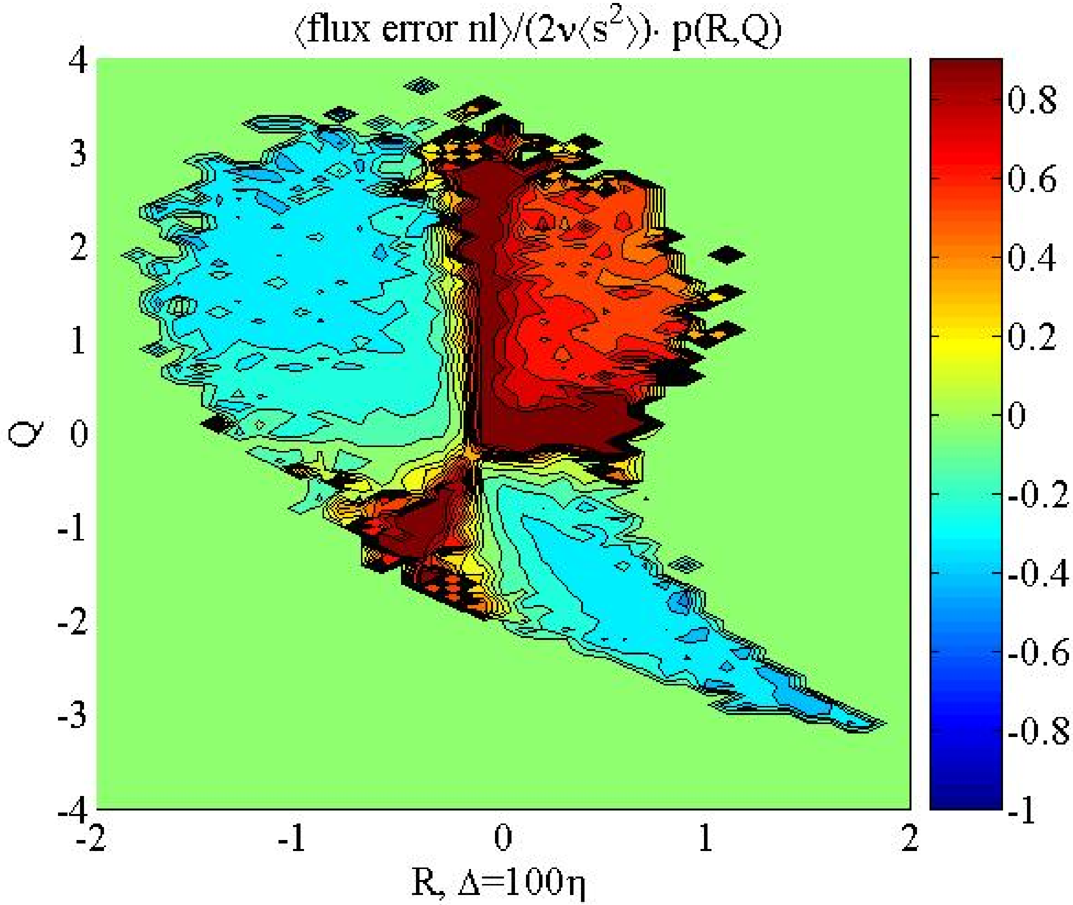
Validation: Alignment of τ_{ij} to s_{ij}



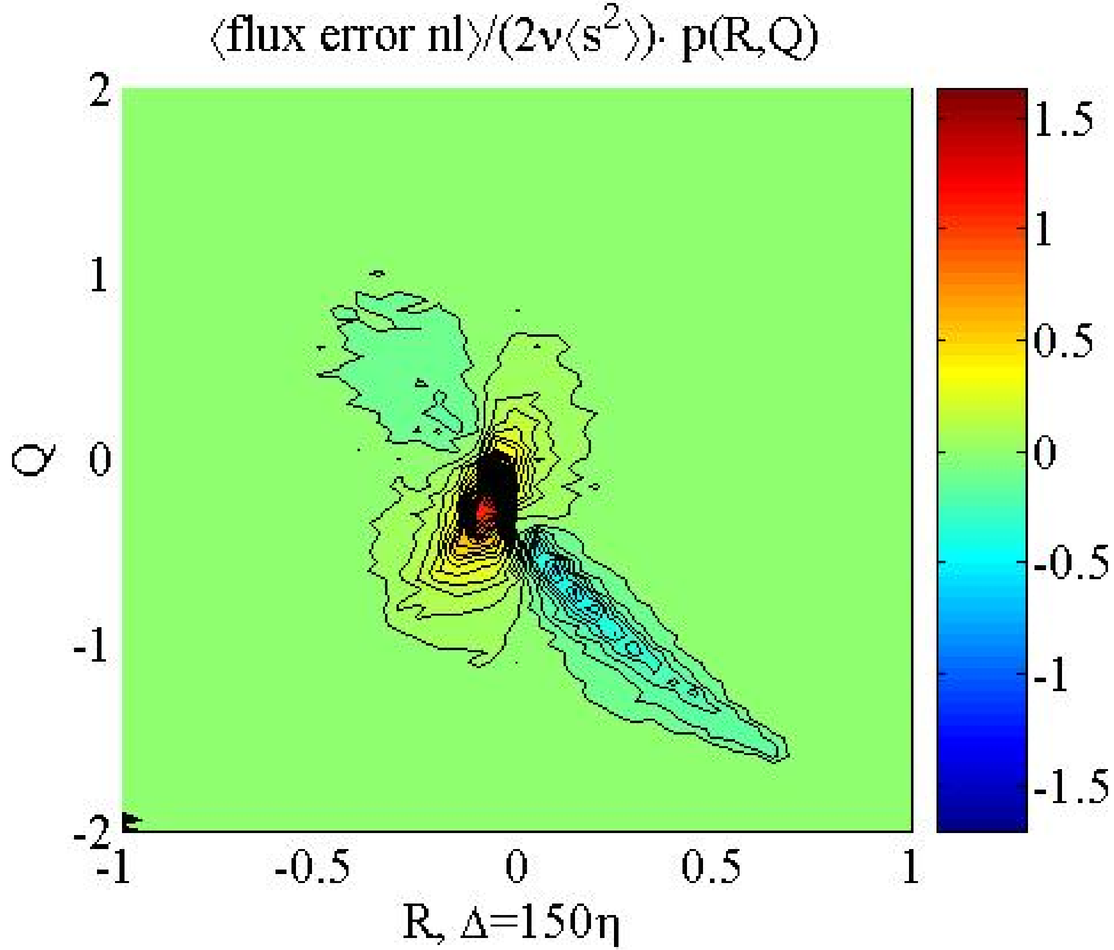
nonlinear model:

- too deterministic
- too little τ_1 λ_3 alignment
- too much τ_2 λ_1 alignment

Flux error in terms of RQ

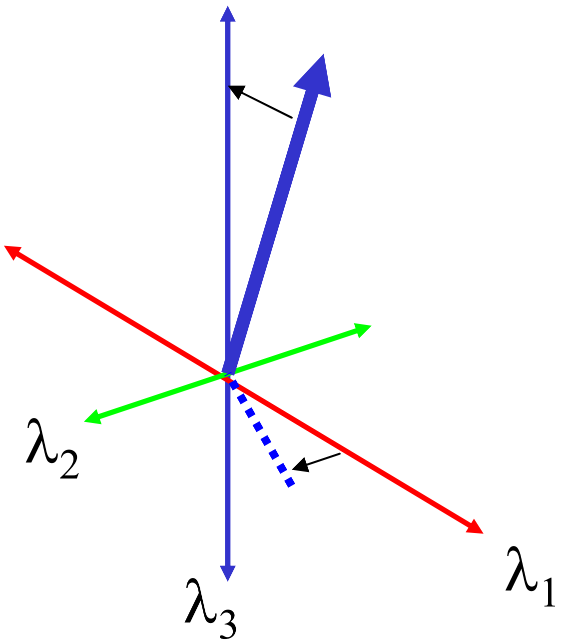


Flux error in terms of RQ



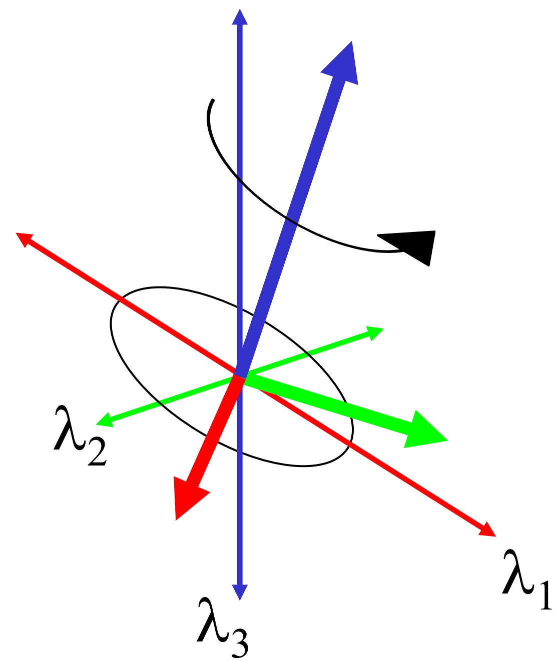
Possible correction for nonlin. model

2 rotations to get τ_1 'right'



blue on red → more backscatter
blue on blue → more energy flux to small scales

1 rotation around τ_1



green on red → more backscatter
green on green → more Smagorinsky → more energy flux to small scales

Possible correction

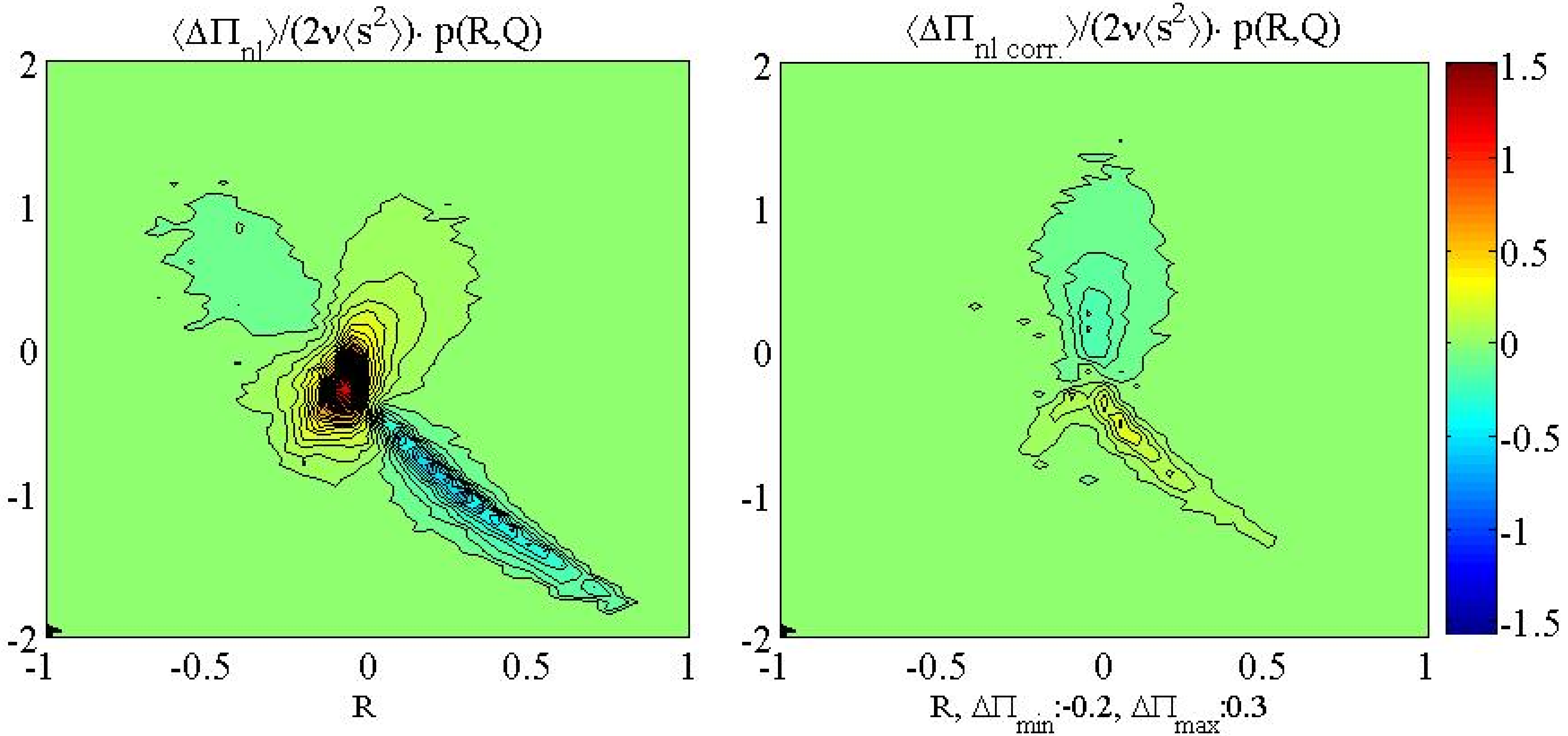


Fig. 3. Normalized prediction error density $\Delta\Pi / \langle 2\nu s^2 \rangle \cdot p(R, Q)$ for a) the nonlinear model and b) for the corrected nonlinear model. Yellow to red colours denote over prediction of back-scattering or to weak energy flux from large to small scales, and light to dark blue colors show where energy flux from large to small scales is too strong.

Conclusion

•particle tracking can access (LES) energy flux
•can be used to study SGS models
•e.g. the ’nonlinear model’
•we find systematic misalignment
•’corrected model’ has less flux error

•need&possibility for more specific experiments