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Table-Top Proposals for Witnessing the Quantum Nature of Gravity

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Abstract

Understanding quantum gravity has posed a significant challenge within the field of physics. While widely accepted in the physics community, there is still no experimental evidence to substantiate the existence of quantum gravity. Recently, there has been an emergence of table-top proposals aimed at investigating these non-classical properties in gravity. Most notably, by observing the gravitationally induced entanglement of two massive nanoparticles. We conduct a comprehensive examination of three such proposals and evaluate their experimental feasibility. We assess the advantages and disadvantages of each and, by responding to criticism directed towards these table-top experiments, we discuss what their results would tell us about the quantum nature of gravity.

Acknowledgements and Dedication

I would like to firstly thank my supervisor Prof. Halliwell. He not only introduced me to the majority of topics discussed in this paper but also willingly dedicated his time to patiently address every question I had, no matter how trivial. Prof. Halliwell's guidance and support have been invaluable throughout this journey, and I am greatly appreciative. I'd also like to extend my gratitude to my friends, Talia Rahall and Ben Gregory, who taught me to create aesthetically pleasing graphs and to correctly use commas, respectively.

I dedicate this paper to my soon to be nephew, who I am sure will share my passion for the beauty of the quantum world.

Contents

Chapter 1

Introduction

Quantum gravity

Since the early 20th century, the foundations of our knowledge of the universe have rested upon the pillars of two guiding fundamental theories of physics; quantum theory and general relativity (GR). Both theories have a plethora of empirical evidence confirming them to the highest degree of accuracy. Nevertheless, when trying to merge the two, by promoting the classical field of gravity described by GR into a quantum field, difficulties have arisen; with contemporary physics not providing a consensus on how these theories should be unified [1–3]. Among these theories, the most notable examples are string theory [4] and loop quantum gravity [5]. However, none has yet to provide a complete and consistent quantum theory of gravity or made predictions that are experimentally testable at the length scales currently attainable with existing technologies.

Alternative models of gravity

This lack of empirical evidence for quantum gravity (QG) has led to the emergence of a group of classical gravity (CG) theories [6–12]. Most significantly, semi-classical gravity models, wherein matter is treated quantum mechanically, but spacetime is described as fundamentally classical. Here, we provide a concise description of two CG models.

Attempting to establish a direct coupling between classical gravity and quantum matter through the standard Einstein equations is not logically sound, since it involves equating a c-number ($G_{\mu\nu}$) to an operator ($\hat{T}_{\mu\nu}$) [8]. To remedy this, an alternative approach would be to consider coupling the Einstein tensor to the expectation value of the stress-energy tensor, such that

$$G_{\mu\nu} = \frac{8\pi G}{c^4} \langle\Psi|\hat{T}_{\mu\nu}|\Psi\rangle = \frac{8\pi G}{c^4} \langle\hat{T}_{\mu\nu}\rangle. \tag{1.1}$$

In these **mean-field models** gravity is described by a potential Φ sourced from the expectation value of the mass density. In the non-relativistic limit of these models massive particles are governed by the Schrödinger-Newton equation [13]:

$$i\hbar\partial_t\Psi(t,\mathbf{r}) = \left(-\frac{\hbar^2}{2m}\nabla^2 - Gm^2 \int d^3\mathbf{r}' \, \frac{|\Psi(t,\mathbf{r}')|^2}{|\mathbf{r}-\mathbf{r}'|} \right)\Psi(t,\mathbf{r}). \tag{1.2}$$

Another such CG theory is the **stochastic collapse model**, introduced originally as an endeavor to address the *measurement problem*[1] in quantum mechanics. Here (in the non-relativistic limit) a stochastic 'noise' term is added to the Schrödinger equation, effectively transforming it into a diffusive differential equation. The primary proposition of this model postulates that the 'noise' leads to decoherence in position space [14]. In other words, it causes the suppression of space-time superpositions such that large enough masses 'collapse' into localised position states.

Gravitational induced entanglement of masses

The contrasting views on the quantum nature of gravity, coupled with advances in quantum technologies [15], have fueled growing interest in ascertaining empirical evidence of quantum effects (or lack thereof) in gravity. Given the extremely weak nature of gravitational interactions, detecting gravity's quanta (gravitons) through momentum transfer in a detector is an exceedingly challenging task, if not fundamentally impossible [16]. Therefore, we are prompted to inquire whether there exists an alternative experiment that can be conducted to examine the quantum characteristics of gravity in a low energy laboratory setting. The possibility of carrying out such an experiment was originally discussed by Richard Feynman in 1957 [17] and recently a series of table-top experiments utilizing quantum information (QI) theory have been proposed.

Most notably an experiment proposed by Bose et al. [18] and, separately, by Marletto and Vedral [19] in 2017. The central claim is that by measuring the entanglement generated between two masses, each in a quantum superposition of spacetime geometries, we can deduce the quantum nature of gravity. This is due to the information theoretic argument that any physical entity mediating the quantum entanglement of two objects must itself be quantum. Ergo, if the two masses only interact through gravitational forces, we can conclude any entanglement in the evolved state of the system is evidence of the gravitational field being quantum.

Since the release of this seminal paper, numerous other table-top experiments in order to witness the non-classical nature of gravity have been proposed. Here we will highlight two such experiments. The first, as suggested by Matsumura et al. [20], utilizes a Leggett-

[1]The measurement problem in quantum mechanics pertains to explaining how the wave function describing a system collapses when 'measurements' are taken.

Garg inequalities approach, while the second, introduced by Howl et al. [21], uses the fact that only quantum fields, not classical ones, can generate non-Gaussianity in the quantum field state of matter. It is important to note there have been other non-laboratory proposed tests for quantum gravity, such as cosmological observation [22], but these have yet to provide any conclusive empirical evidence and are not subject of discussion in this paper. The interested reader is directed towards [23] for descriptions of such phenomenological models.

A road map of this paper

The remainder of this paper is organised as follows.

In chapter 2, we begin by introducing the low-energy limit of linearized gravity and the central principle that local operations and classical communications (LOCC) cannot generate entanglement within a system. We then describe the experimental setup proposed by both Bose et al. [18] and Marletto and Vedral [19] to measure quantum gravity; showing that by assuming a quantum gravitational field that contains superpositions of semi-classical states, entanglement is generated in this system. Following this, we outline the techniques used to measure entanglement in a system. Next, we will derive the phase associated with the degree of entanglement generated in the Newtonian limit of QG, highlighting any assumptions made. Finally, we conclude by summarizing the experimental feasibility of detecting the results of this BMV proposal.

Since its publication, the BMV experiment has received some questions regarding its validity as a test for quantum gravity. In Chapter 3, we address and attempt resolve these concerns. First, we examine issues regarding the claim that the BMV experiment is a 'witness for quantum gravity' and conclude that it is more accurately described as a test for the 'non-classical nature of gravity', measuring whether spacetime geometries can exist in a quantum superposition of states. Next, we derive a Lorentz covariant and gauge invariant expression for the phase induced by linearized gravity, in an attempt to alleviate any concerns regarding locality. Our analysis demonstrates that the Newtonian limit, which is used in the original derivation of the BMV effect, is a valid approximation of the true gravitational action of the system in the non-relativistic, static regime.

In chapter 4, we explore another table-top proposal to test for quantum gravity, that being the Leggett-Garg inequalities approach suggested by Matsumura et al. [20]. First outlining the assumptions of macrorealism that lead to the Leggett-Garg inequalities, we then define a two-time quasi-probability. Explaining that measuring a negative quasi-probability serves as evidence that a system adheres to the superposition principle of quantum mechanics. We proceed by analysing the setup designed to measure this quasi-probability in an attempt to observe the quantum nature of gravity. Furthermore, we provide an assessment of the experiment's strengths and weaknesses, drawing comparisons with the aforementioned BMV experiment.

We do the same in chapter 5, but this time for the markedly distinct proposal put forward by Howl et al. [21]. Here we first define non-Gaussian states and explain that a Hamiltonian that involves terms higher order than quadratic in operators induces non-Gaussianity. We show explicitly that quantized gravity can create non-Gaussianity in a system, but the classical picture of gravity cannot. Following this, we summarize the authors proposal to measure non-Gaussianity in a single Bose-Einstein condensate, highlighting the advantages and disadvantages of this experiment. Once again, evaluating in reference to the previously discussed table-top experiments.

Chapter 2

Quantum Entanglement of Two Massive Particles as a Witness for Quantum Gravity

Here we derive the Bose-Marletto-Vedral (BMV) effect [18, 19], outlining and justifying the assumptions necessary for this derivation and discussing the experimental feasibility of detection.

2.1 Linearized Gravity as an Effective Field Theory

The gravitational field is a manifestation of the geometry of curved spacetime. When deriving the BMV effect we are dealing with gravitational forces involving small masses that induce minimal curvature of spacetime. Thus, to a very good approximation, we can a use first order (in $\hbar$) perturbation around Minkowski spacetime, such that

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{2.1}$$

where $|h_{\mu\nu}| \ll 1$ to maintain linearity and $\eta_{\mu\nu}$ is the Minkowski metric. We can write a weak field approximation for static masses in terms of the Newtonian potential $\phi(\mathbf{x})$ as

$$ds^2 = (1 + \frac{2\phi(\mathbf{x})}{c^2})(c^2dt^2) - (1 - \frac{2\phi(\mathbf{x})}{c^2})d\mathbf{x}^2, \tag{2.2}$$

where we are working with the $(+, -, -, -)$ signature.

Earlier, we stated the difficulties in merging general relativity and quantum mechanics into a single, consistent theory of everything, leading to disputes over the correct mathematical modeling of quantum gravity. However, the unification of these two fundamental theories is only problematic at high energy scales, where divergences occur which cannot be addressed using the usual quantum field theory (QFT) renormalization methodologies [14]. Nevertheless, by performing linear perturbations around flat spacetime as above (but with the small fluctuations $\hat{h}_{\mu\nu}(\mathbf{x}, t)$ now a quantized field) we can define an effective quantum field theory (EFT), which is valid at energies below the order of the Planck scale $(M_p) \sim 10^{19}\, GeV$. The quantization of linearized gravity closely follows the theoretical framework of other QFT's of fundamental forces. In this context, the graviton, a massless spin-2 boson, is analogous to the photon, while the Newtonian potential corresponds to the gravitational counterpart of the Coulomb potential in quantum electrodynamics. We then consider the Newtonian interaction between masses to have originated from tree-level diagrams of the exchange of these virtual gravitons [24]. Here, we refer to 'virtual' in the sense that quanta are off the mass shell (off-shell), meaning they do not conform to the Einstein energy-momentum relations of particles. According to this definition, a graviton is considered a non-classical entity. It is precisely this linearized quantum description of gravity that these table-top experiments of quantum gravity induced entanglement of masses (QGEM) propose to empirically validate.

It is important to note this EFT is the low energy limit for all QG theories which contain massless spin-2 bosons. As the BMV experiment tests the quantum nature of linearized gravity, observing such effect will not distinguish between different main QG models such as string theory and quantum loop gravity; it is the high energy behaviour which will give such an insight. However, if BMV effects aren't measured it would disprove these aforementioned quantum gravity theories and we would have to look elsewhere, such as gravitational collapse models suggested by Roger Penrose [7].

2.2 A Classical Field Cannot Cause Entanglement

A central principle of information theory is the statement that entanglement between two quantum states cannot occur if the states are mediated by a classical channel. In QI this is usually framed in terms of local operations and classical communications (LOCC), where any sensible measurement of the entanglement of a system must be non-increasing under the action of LOCC [25]. This is shown explicitly for a system containing two matter states (as in the BMV experiment) in Appendix A.

Throughout this experiment, we presuppose local interactions within both classical and quantum contexts. Our primary objective is not the verification of locality itself; instead, we aim to establish the quantum properties of the gravitational field following the assump-

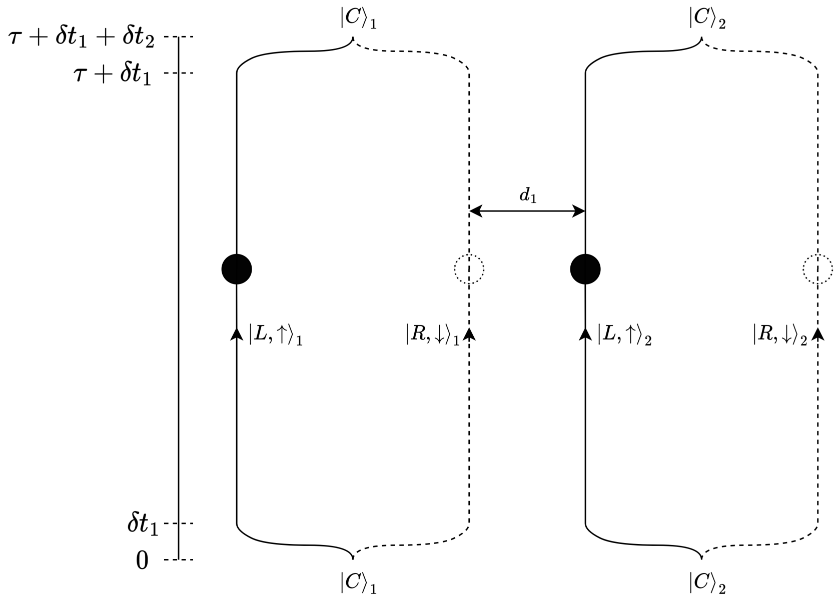

Figure 2.1: Experimental setup proposed to witness the quantum nature of gravity using two adjacent interferometers. Here each test mass is initially in a localized state $|C\rangle_i$, where a Stern-Gerlach interferometer splits the masses into a quantum superposition of two spacial separated states $|L,\uparrow\rangle_i$ and $|L,\uparrow\rangle_i$. Masses are then held in superposition for time τ, until being refocused to the localized state $|C\rangle_i$. The distance between the two closest arms is d_1.

tion of locality. Locality, in this context, functions as a constraint preventing entanglement generation over a distance without communication through a quantum mediator.

2.3 Experimental Setup

We now focus our attention to the BMV experimental procedure that was proposed to measure the quantum nature of gravity in a laboratory setting. The experimental setup is shown in Figure 2.1 and is composed of two Stern-Gerlach (SG) interferometers situated a short distance from one another.

The first step consists of both interferometers (labelled $i = 1, 2$) splitting each of the identical ($m_1 = m_2$) test masses into a superposition of spatially separated states. In a Stern-Gerlach setting this is usually implemented using masses which are initially in a superposition of spin states. Such that, under the action of an inhomogeneous magnetic field, the states unitarily evolve into

$$\frac{1}{\sqrt{2}} |C\rangle_i \otimes (|\uparrow\rangle + |\downarrow\rangle) \rightarrow \frac{1}{\sqrt{2}}(|L,\uparrow\rangle_i + |R,\downarrow\rangle_i) \tag{2.3}$$

where $|L\rangle$ and $|R\rangle$ represent the spacial states of each arm of the interferometer. After

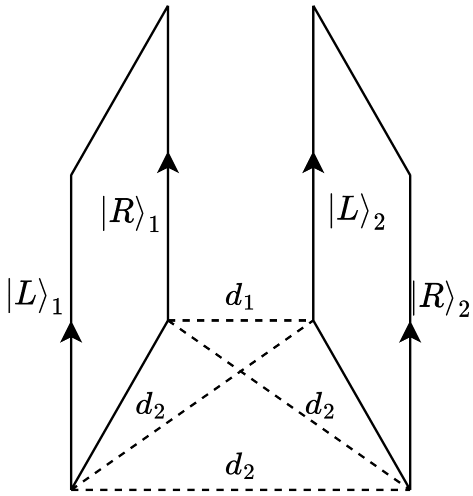

Figure 2.2: Illustration showing the arrangement of the two Stern-Gerlach interferometers such that distance between the masses in three of the spatially superposed states is equivalent. Later we pick the distance between the interferometer arms such that $d_1 \ll d_2$ for simplicity.

such an operation the total state of the system becomes

$$|\psi(t=\delta t_1)\rangle = \frac{1}{\sqrt{2}}(|L,\uparrow\rangle_1 + |R,\downarrow\rangle_1) \otimes \frac{1}{\sqrt{2}}(|L,\uparrow\rangle_2 + |R,\downarrow\rangle_2) \tag{2.4}$$

$$= \frac{1}{2}(|LL\rangle + |LR\rangle + |RL\rangle + |RR\rangle), \tag{2.5}$$

where we have used more compact notation of representing $|L,\uparrow\rangle_1 \otimes |L,\uparrow\rangle_2$ as $|LL\rangle$.

Assuming that the masses on different paths have different gravitational interactions energies depending on the relative distance between them in each quantum state and that they are only interacting through said gravitational field. After time τ the joint state will become

$$|\psi(t=\tau+\delta t_1)\rangle = \frac{1}{2}(e^{i\phi_{LL}}|LL\rangle + e^{i\phi_{LR}}|LR\rangle + e^{i\phi_{RL}}|RL\rangle + e^{i\phi_{RR}}|RR\rangle) \tag{2.6}$$

where ϕ represents the relative phase acquired by each component. Arranging the geometry of the two interferometers in such a way as shown in Figure 2.2, where the distances between three of the arms is equal, we can factor out $e^{i\phi_{LL}} = e^{i\phi_{LR}} = e^{i\phi_{RR}} = e^{i\phi_0}$, rewriting Eq.(2.6) as

$$|\psi(t=\tau+\delta t_1)\rangle = \frac{e^{i\phi_0}}{2}(|LL\rangle + |LR\rangle + e^{i\Delta\phi}|RL\rangle + |RR\rangle). \tag{2.7}$$

Finally the SG interferometers map the orbital entanglement back to spin entanglement by recombining the interferometer paths, such that

$$\begin{aligned}|\psi(t=\delta t_2+\tau+\delta t_1)\rangle &= \frac{1}{2}(|\uparrow\rangle_1|\uparrow\rangle_2+|\uparrow\rangle_1|\downarrow\rangle_2+e^{i\Delta\phi}|\downarrow\rangle_1|\uparrow\rangle_2+|\downarrow\rangle_1|\downarrow\rangle_2)|C\rangle_1|C\rangle_2\\ &= \frac{1}{2}(|\uparrow\uparrow\rangle+|\uparrow\downarrow\rangle+e^{i\Delta\phi}|\downarrow\uparrow\rangle+|\downarrow\downarrow\rangle\,, \end{aligned}\tag{2.8}$$

where, for simplicity, the overall phase has been dropped and we have used the more concise notation $|\uparrow\rangle_1|C\rangle_1\otimes|\uparrow\rangle_2|C\rangle_2=|\uparrow\uparrow\rangle$. This last step is done because it is easier to measure entanglement when the states are represented in a spin basis. It is important to note that, for ease of calculation, we have made the assumption $\delta t_1, \delta t_2 \ll \tau$ so that any phase acquired by the states in these periods is negligible and can be ignored. This can be corrected with more in-depth analysis.

2.4 Measuring Entanglement

As explained in subsection 2.2, LOCC cannot cause the increase of entanglement in a system. Thus the claim is that detecting the BMV effect by measuring entanglement in the evolved state of the system is direct evidence that the gravitational field mediating the interaction of the two masses must exhibit quantum properties.

An entangled system is one that cannot be separated. Specifically, a pure bipartite state $|\psi_{12}\rangle$ associated with the Hilbert space $\mathcal{H}_1\otimes\mathcal{H}_2$ is said to be separable if it can be written as a tensor product state of the form

$$|\psi_{12}\rangle=|\psi_1\rangle\otimes|\psi_2\rangle\,,\tag{2.9}$$

where $|\psi_1\rangle\in\mathcal{H}_1$ and $|\psi_2\rangle\in\mathcal{H}_2$. It is clear to see in Eq.(2.8) that the total state is not separable unless $\Delta\phi=2\pi n$ for some integer n, where in this particular instance the system could be factorised to

$$|\psi\rangle=\frac{1}{\sqrt{2}}(|\uparrow\rangle+|\downarrow\rangle)_1|C\rangle_1\otimes\frac{1}{\sqrt{2}}(|\uparrow\rangle+|\downarrow\rangle)_2|C\rangle_2\,,\tag{2.10}$$

this represents the extreme regime in which no entanglement occurs. We now need to devise a methodology for quantifying the entanglement within our evolved state.

2.4.1 Von Neumann entropy

For bipartite states, such as ours, a standard way to quantify entanglement is through the Von Neumann entropy of the density operator

$$S_i=-Tr[\hat{\rho}_i\log_2(\hat{\rho}_i)],\tag{2.11}$$

with the reduced density matrix defined as

$$\hat{\rho}_i = Tr_j[\hat{\rho}], \tag{2.12}$$

where $i \neq j$. Calculating $\hat{\rho}_1$ for our evolved state Eq.(2.8) we get

$$\hat{\rho}_1 = \frac{1}{4}\Big(2\left|\uparrow\right\rangle\left\langle\uparrow\right| + (1+e^{-i\Delta\phi})\left|\uparrow\right\rangle\left\langle\downarrow\right| + (1+e^{i\Delta\phi})\left|\downarrow\right\rangle\left\langle\uparrow\right| + 2\left|\downarrow\right\rangle\left\langle\downarrow\right|\Big), \tag{2.13}$$

which can be expressed in matrix form as

$$\hat{\rho}_1 = \frac{1}{4}\begin{bmatrix} 2 & 1+e^{-i\Delta\phi} \\ 1+e^{i\Delta\phi} & 2 \end{bmatrix}. \tag{2.14}$$

Now solving using the standard eigenvalue equation we obtain

$$\lambda_{\pm} = \frac{1}{2} \pm \frac{\sqrt{1+\cos\Delta\phi}}{2\sqrt{2}}. \tag{2.15}$$

It is then trivial to diagonalize $\hat{\rho}_1$, giving the Von Neumann entropy for general $\Delta\phi$

$$S_1 = -(\lambda_+ \log_2 \lambda_+ + \lambda_- \log_2 \lambda_-). \tag{2.16}$$

When the state is separable, as in the case of Eq.(2.10), $\lambda_+ = 1$ and $\lambda_- = 0$, resulting in zero entropy (here we have defined $0\log(0) = 0$). The system is maximally entangled when $\Delta\phi = \pi n$ for odd n, for which $\lambda_{\pm} = \frac{1}{2}$, giving a Von Neumann entropy of $S_1 = 1$.

2.4.2 Spin Witness Protocol

In realistic experiments, decoherence, through entanglement with the environment, can occur [24]. This makes it difficult to attribute entanglement in the system solely to the gravitational field. Furthermore, assessing the entropy requires quantum state tomography to measure a complete set of observables in order to reconstruct the density matrix ρ of the system [26]. This measurement process demands a substantial amount of effort.

To avoid these issues, we can implement an entanglement witnesses $\mathcal{W}(\hat{\rho})$, which measures correlations between the two masses and searches for violations of Bell inequalities, whilst ignoring any entanglement between the states and their environment. These witnesses are unique to the specific entangled state being measured. For our setup the appropriate witness is:

$$\mathcal{W} = |\left\langle(\hat{\sigma}_x)_1 \otimes (\hat{\sigma}_z)_2 | (\hat{\sigma}_x)_1 \otimes (\hat{\sigma}_z)_2\right\rangle - \left\langle(\hat{\sigma}_y)_1 \otimes (\hat{\sigma}_y)_2 | (\hat{\sigma}_y)_1 \otimes (\hat{\sigma}_y)_2\right\rangle|, \tag{2.17}$$

where $\hat{\sigma}_i$ are the Pauli spin matrices [24]. If $\mathcal{W}(\hat{\rho}) > 1$ then the state is proven to be entangled. Conversely, measuring $\mathcal{W}(\hat{\rho}) \leq 1$ gives us no information about the evolved system. Unlike the Von-Neumann entropy, witnesses do not provide an entanglement measure, meaning they do not quantify the amount of entanglement in the evolved state.

2.5 Gravitational Phase Induced

In deriving the BMV effect we have assumed that the gravitational field contains superpositions of semiclassical states. The unitary time evolution of each of these bipartite states can be calculated from the proper time along the test masses world lines. Here we follow the generally covariant calculation given in [27] to evaluate this gravitational phase.

2.5.1 Assumptions

In order to use the Newtonian limit of our weak field approximation of linearized gravity Eq.(2.2), we first need to make a few assumptions. Later (cf. section 3.2) we will validate these presuppositions, deriving a Lorentz covariant and gauge invariant expression for the gravitational induced phase from first principles. Demonstrating this simple model is an accurate approximation of the on-shell action of linearized gravity. These assumptions are as follows:

1. **General Relativity holds for mesoscopic masses** $\sim 10^{-13}kg$ used in the experiment. These are the largest feasible masses we can use to create the spacial superposition needed (cf. section 2.6). Which, although currently not experimentally verified, is a reasonably conservative assumption.

2. We are **operating in the non-relativistic regime**. This means that the speeds $|\boldsymbol{v}|$ of the test masses are a lot less than the speed of light ($|\boldsymbol{v}| \ll c$).

3. The velocities are equal ($\boldsymbol{v}_1 = \boldsymbol{v}_2$), such that the **distance between the masses is constant** ($\dot{d}_{12} = 0$). This also encompasses the supposition that acceleration due to their gravitational attraction is negligible, a condition valid for masses of this magnitude.

4. The near-field approximation, in which the **gravitational interactions are modeled as being instantaneous**. While General Relativity clearly adheres to principles of locality, if the time the masses are held in spatial superposition, denoted as τ, is significantly longer than the time during which the system is not static $\frac{d_2}{c}$, we can approximate the field as static.

2.5.2 Derivation

The Newtonian potential for masses of radius R is given by[1]

$$\phi(\mathbf{r}) = \begin{cases} -\frac{(3R^2-r^2)Gm}{2R^3}, & \text{if } r < R \\ -\frac{Gm}{r}, & \text{otherwise.} \end{cases} \tag{2.18}$$

Thus, using Eq.(2.2), we can calculate the proper time elapsing for each particle as

$$\begin{aligned} s &= \int_{\delta t_1}^{\delta t_1+\tau} ds = \int_{\delta t_1}^{\delta t_1+\tau} dt\sqrt{1-\frac{3Gm}{Rc^2}-\frac{2Gm}{dc^2}} \\ &= \tau\sqrt{1-\frac{3Gm}{Rc^2}-\frac{2Gm}{dc^2}} \approx \tau(1-\frac{3Gm}{2Rc^2}-\frac{Gm}{dc^2}), \end{aligned} \tag{2.19}$$

where d is the distance between the two masses in each superposition state and, in the last step we dropped the higher order terms in the binomial expansion, as $\frac{Gm}{dc^2} \ll R \ll d$. The first two terms are the same for all spacial states and so they only contribute to the overall phase in Eq.(2.7), such that the only meaningful difference in proper time between the entangled particle states is

$$\delta s = -\frac{Gm\tau}{dc^2}. \tag{2.20}$$

Now, using the fact the time evolution of the quantum state of a massive particle is given by $e^{-\frac{imc^2 s}{\hbar}}$, we can conclude

$$\Delta\phi = -\frac{mc^2\delta\tilde{s}}{\hbar} = \frac{Gm^2\tau}{\hbar\tilde{d}}, \tag{2.21}$$

with $\tilde{d} = \frac{d_1 d_2}{d_2 - d_1}$. Arranging our interferometers such that $d_1 \ll d_2$ we can further simplify Eq.(2.21) to

$$\Delta\phi \approx \frac{c\tau}{d_1}\Big(\frac{m}{m_{\text{Planck}}}\Big)^2, \tag{2.22}$$

where $m_{\text{Planck}} = \sqrt{\frac{\hbar c}{G}}$ is a constant $\sim 2 \times 10^{-8} kg$.

2.6 Experimental Feasibility

When measuring the BMV effect, if the phase shift $\Delta\phi$ is too small then the induced entanglement will be insignificant and not measurable, but if it is too large relative to the resolution of the measuring equipment it also will not be detectable. The system is first maximally entangled when $\Delta\phi = \pi$ so we should aim to adjust our experimental parameters in order to produce a phase shift of comparable magnitude $\Delta\phi \sim 1$.

[1]Here $r_s \ll R \ll d$, where $r_s = \frac{2Gm}{c^2}$ is the Schwarzschild radius of the mass so we don't have issues using spherical coordinates.

The distance between the two closest arms of the interferometers d_1 has be large enough so that there are no significant electromagnetic forces between the test masses. As, in order to validate the BMV experiment, gravitational interactions must be the source of the majority of the entanglement between the states. Since our masses are neutrally charged the most notable source of electromagnetic noise is due to the Casimir effect, which is the relativistic van der Waals force due to quantum fluctuations. The Casimir-Polder potential when the masses are in state $|LR\rangle$ is

$$V(d) \approx k \frac{R^6}{(d_1)^7} \left(\frac{\epsilon - 1}{\epsilon + 2} \right)^2, \tag{2.23}$$

where $k = \frac{23\hbar c}{(4\pi)^3 \epsilon_0^2}$. We assume this effect is negligible for all other spatially separated states as $d_1 \ll d_2$.

Using cryogenically cooled micro-diamonds $\epsilon \sim 5.7$, as suggested in [18], at a separation of $d_1 \sim 200\mu m$ the Casimir-Polder potential would be a tenth the strength of the gravitational potential. However, it has been suggested that at the experimental parameters proposed by Bose et al. [18], gravitationally-induced quantum state reduction (GQSR) would occur, on average, at $0.01s$, ergo disentangling the state [28].

Thus, using parameters to nullify GQSR effects $\tau \sim 1ms$ and Casimir-Polder forces $d_1 \sim 200\mu m$, Eq.(2.22) dictates that we would require the superposition of test masses of the order $\sim 10^{-13}kg$. Although setting masses of this scale in spacial superpositions is challenging to implement, advances in quantum technologies make it seem feasible in the near future. For example, systems have been proposed using NOON states of Bose-Einstein condensates (BEC) [28] and opto-mechanical oscillators [29, 30] as the masses in question.

Chapter 3

Resolving Issues with the Bose-Marletto-Vedral Experiment of Gravity Induced Entanglement

Since being proposed [18, 19] as a witness of quantum effects in gravity, the BMV experiment has been questioned in many papers, with some authors raising concerns with its relevance. Here we highlight some of these issues and the reasons why they do not diminish the interest in measuring this effect.

3.1 Semantics of Non-Classical Fields

3.1.1 Reginatto-Hall Argument

In [31], it is argued that it is not strictly correct to label the BMV experiment a *witness for quantum gravity*, but that it is in fact more accurately described as a *test for the non-classicality, under suitable constraints, of gravity.* The subtle distinction here is that measuring the BMV effect does not preclude all other descriptions of spacetime situated between standard classical general relativity and a fully quantum representation. Here, our description of a classical field is one in which each point of spacetime has fixed values depending on the probability of the field configuration, that is, the field can only be in one state available to it at every instant [24]. Conversely a quantum field is described using amplitudes allowing for the possibility of a field to exist in a superposition of multiple states. It is precisely this definition of non-classicality that this experiment is testing for in gravity.

However, by extending this definition of a classical field, one can show it is possible to

Model	Condition 1	Condition 2	Condition 3
Mean-field	✓	×	✓
Stochastic collapse	×	✓	✓
String Theory	✓	✓	×

Table 3.1: The classification of the two semi-classical models given in section 1 of this paper, using Galley, Giacomini and Selby's no-go theorem on the nature of gravitational fields [32]. Here we have also included a quantum gravity model for comparison.

create a hybrid description, which is not fully quantum, that can create entanglement between quantum particles. This has been done by Hall and Reginatto using a configuration ensemble model [31]. Consequently, it is imperative to make clear that this experiment proposes not to exclude all conceivable explanations of these interference measurements generated by entanglement. But, instead it is designed to make an observation which is predicted by all established quantum gravity models and not by conventional classical gravity [18]. The BMV experiment can accordingly be more accurately described as an investigation aimed at observing that spacetime geometries obey the superposition principle of quantum mechanics.

3.1.2 No-Go Theorem on the Nature of the Gravitational Field

Recently, Galley, Giacomini and Selby developed a no-go theorem providing the structure in which to analyze all conceivable gravitational theories [32], without presupposing any particular gravitational description. This is done through the use of Generalised Probabilistic Theories (GPTs), which utilize classical probability theory as a framework to describe non-classical phenomena.

The derivation of their results is very mathematically rigorous; however, the conclusion is as follows. By creating a system, such as in the BMV experiment, where two masses **A** and **B** interact gravitationally through a field **G**, they assert that the following statements are incompatible [32]:

1. **G** is able to generate entanglement;
2. **A** and **B** do not interact directly but only through the mediator **G**;
3. **G** is classical.

Note here that neither **A**, **B** nor **G** is assumed to be quantum, allowing for any number of hybrid descriptions of this system.

These conditions can be used to systematically assess whether a gravitational theory is consistent with the BMV effect without the need to reconcile the question of whether it is a theory that is either conventionally classical or quantum in nature. For instance, in the

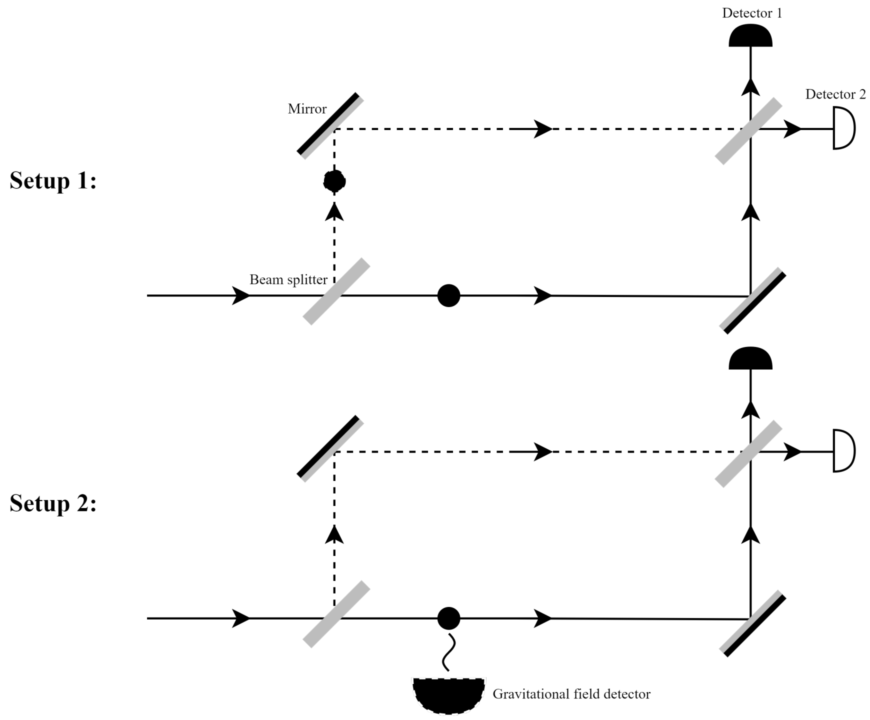

Figure 3.1: Schematics of an experiment proposed to test the measurement postulate of quantum mechanics for gravity [33]. Here, identical masses are subjected to Mach-Zehnder interferometers. In setup 2, an intermediate measurement of the gravitational field sourced by the mass is made while it is in a superposition of spacial states. If gravity is described quantum mechanically, the particle will collapse to one spacial state altering the probabilistic outcome, such that interference measurements differ from setup 1.

stochastic collapse model, decoherence in position space due to the gravitational 'noise', results in the spontaneous localisation of the states. Consequently, the absence of state superposition within each interferometer prevents the generation of entanglement, thereby resulting in a violation of condition 1 (see Table 3.1). Unfortunately, this no-go theorem does not apply to the Reginatto-Hall model proposed in [31], as in its current state it is not a GPT (for the interested reader more on this can be found in [32]).

3.1.3 Measuring other Quantum Postulates in Gravity.

The question of what conclusions can be drawn about the quantum nature of gravity through the BMV experiment is further explored by Hanif et al. [33]. The authors suggest that measurements of the BMV effect imply that spacetime geometries can exist in quantum superpositions. However, they emphasize that the quantum nature of an entity is not solely defined by its adherence to the superposition principle. They propose for a complimentary experiment to be conducted that tests for the measurement postulate, which concerns the intrinsically invasive nature of quantum fields, where the act of measuring a quantum system leads to an instantaneous update of the system.

This is done through another multi-interferometer test (refer to Figure 3.1). In this setup, two separate identical masses undergo Mach-Zehnder interferometry. In one of these interfereometers a probe performs an intermediate measurement of the gravitational potential of the test mass. It is imperative that the mass probe exclusively interacts with the test mass through its gravitational field and not through any other means. The measurement outcomes of these two scenarios are subsequently compared.

A quantum theory of gravity would predict the collapse of the wavefunction when the measurement takes place in setup 2, leading to the absence of interference effects, thereby altering the probability of the massive particle arriving at either detector. However, in a classical theory of gravity this collapse would not occur, thus the probabilistic results of detection wouldn't change from setup 1 to setup 2.

For instance, in the case of the *mean-field model*, the detector records a gravitational potential $\phi_0(\bar{x})$, where $\bar{x}$ represents the distance between the midpoint of the two possible particle trajectories and the probe. This reading gives no information about which spatial state the particle is in; therefore, no measurement collapse would occur.

3.2 Lorentz Covariant and Gauge Invariant Linearized Gravity

When deriving the BMV effect, we employed an approximation that relies on an instantaneous, non-relativistic depiction of gravitational interaction between masses (see subsection 2.5.1), rather than a dynamic field-based approach. Some authors [34, 35] have argued that, by using this Newtonian limit of gravity, we cannot conclude that gravity is non-classical. Moreover, these approximations confuse the notion of locality of interactions, a central point in claiming that the mediating gravitational field is non-classical. Since we have used the information theoretic argument, **local** operations and classical communications (LOCC) cannot entangle two quantum states.

Here, we follow [36], deriving a Lorentz covariant and gauge invariant expression for the gravitational induced phases from first principles, showing that at the slow-moving near-field limits of our experiment we recover Eq.(2.21). We, once again, employ a linearized gravity effective field theory (EFT), which remains a valid approximation since we are operating within a low-energy regime, given the small spacetime curvature induced by the masses ($|\hat{h}_{\mu\nu}| \ll 1$). For clarity we have dropped all the hats from operators in the following derivations.

3.2.1 Path Integral Approach to Entanglement Generation

As shown earlier, in section 2.3, the two test masses are placed in motion $\boldsymbol{x}_a^{s_a}(t)$ dependent on their intrinsic spin $s_a \in \{\uparrow, \downarrow\}$, where $a = 1, 2$ denotes each mass. We can represent each of the internal spin configuration between two states as $|\sigma\rangle = \otimes_a |s_a\rangle$, such as in Eq.(2.8). Furthermore, we denote the gravitational perturbation due to the masses as $\mathcal{G}$, which couples to system and mediates the entanglement.

Starting with initial states

$$\left|\Psi^i\right\rangle = \left|\psi^i\right\rangle \otimes \sum_\sigma A_\sigma \left|\sigma\right\rangle, \tag{3.1}$$

such that A_σ are complex amplitudes, $\left|\psi^i\right\rangle = \left|\mathcal{G}^i[x_a^i]\right\rangle \otimes \left|\boldsymbol{x}_a^i\right\rangle$ and $\boldsymbol{x}_a^i$ is the initial position of the masses, where the spatial states in different interferometer branches are taken to be orthogonal. The final state can be found by the unitary time evolution operator that can be separated as

$$U_{i\to f} = U_{i\to f}^\sigma \otimes \sum_\sigma \left|\sigma\right\rangle \left\langle\sigma\right|, \tag{3.2}$$

since the particle spins do not change along the interferometer paths. Our objective is to determine $U_{i\to f}^\sigma$, which is responsible for inducing the gravitational phase and generating entanglement.

Using path integral formalism, the partition function of our system can be written as

$$\mathcal{Z} = \int \mathcal{D}\Psi e^{\frac{iS}{\hbar}} = \prod_a \int \mathcal{D}\mathcal{G}\mathcal{D}x_a \exp\Big(\frac{iS[x_a, \mathcal{G}[x_a]]}{\hbar}\Big). \tag{3.3}$$

Utilizing a stationary phase approximation and retaining only the field configuration $\mathcal{G}$ that solves the classical gravitational field equations of masses following paths $x_a(t)$ [36], then

$$U_{i\to f}^\sigma \propto \prod_a \int_i^f \mathcal{D}x_a \exp\Big(\frac{iS[x_a, \mathcal{G}[x_a]]}{\hbar}\Big)|\psi^f\rangle\langle\psi^i|. \tag{3.4}$$

This can be further simplified by a second stationary phase approximation. Keeping only the contribution from the classical path $x_a^{s_a}$ we get

$$U_{i\to f}^\sigma \propto \exp\left(\frac{i(S_0^\sigma[x_a^{s_a}] + S_\mathcal{G}^\sigma[x_a^{s_a}, \mathcal{G}[x_a^{s_a}]])}{\hbar}\right)|\psi^f\rangle\langle\psi^i|, \tag{3.5}$$

where we have split the action ($S = S_0 + S_\mathcal{G}$) into a term S_0 independent of $\mathcal{G}$ and an interacting term $S_\mathcal{G}$ coupling the masses to the mediating gravitational field [36]. For the sake of ease, we assume S_0 is the same for all states $|\sigma\rangle$. Ergo, by factorising it out, it can be rendered a global phase that can be ignored. Giving us a final state:

$$|\Psi^f\rangle = U_{i\to f}|\Psi^i\rangle \propto |\psi^f\rangle \otimes \sum_\sigma A_\sigma e^{i\theta_\sigma}|\sigma\rangle, \tag{3.6}$$

with

$$\theta_\sigma = \frac{S^\sigma_\mathcal{G}[x_a^{s_a}, \mathcal{G}[x_a^{s_a}]]}{\hbar} \tag{3.7}$$

the gravitational phase induced, entangling the spin degrees of freedom.

3.2.2 Action in de Donder Gauge

In appendix B.1, we start from the gauge and Lorentz invariant Fierz-Pauli action, which is the Einstein-Hilbert action to quadratic order in the metric perturbation around Minkowski spacetime $h_{\mu\nu}$ (see Eq.(2.1) [37]. Showing that the on-shell action of the linearized gravitational field is:

$$S_\mathcal{G} = \frac{1}{4}\int d^4x\, h_{\mu\nu}T^{\mu\nu}. \tag{3.8}$$

In this calculation, we fixed the action in de Donder gauge

$$\partial^\nu \bar{h}_{\mu\nu} = \partial^\nu (h_{\mu\nu} - \frac{1}{2}h\eta_{\mu\nu}) = 0, \tag{3.9}$$

where $T^{\mu\nu}$ is the energy-momentum tensor and $h = \eta^{\mu\nu}h_{\mu\nu}$. The de Donder gauge is the gravitational analogue of the Lorenz gauge in electromagnetism.

The Euler-Lagrange equations for the metric perturbation are

$$\Box h_{\mu\nu} = -\frac{16\pi G}{c^4}\bar{T}_{\mu\nu}. \tag{3.10}$$

This can be solved using a retarded Green function for the d'Alembertian operator $\Box$ (see Appendix B.2) to get the wave equation

$$h_{\mu\nu}(t, \mathbf{x}) = \frac{4G}{c^4}\int d^3y\, \frac{\bar{T}_{\mu\nu}(t_r, \mathbf{y})}{|\mathbf{x}-\mathbf{y}|}, \tag{3.11}$$

where

$$t_r = t - \frac{|\mathbf{y}-\mathbf{x}|}{c}, \tag{3.12}$$

is the retarded time. The retarded time is significant, with Eq.(3.11) showing that disturbances in the gravitational field at spacetime coordinates $(t, \mathbf{x})$ are due to the sum of all energy and momentum sources on the past light cone [38] (see Figure 3.2). The

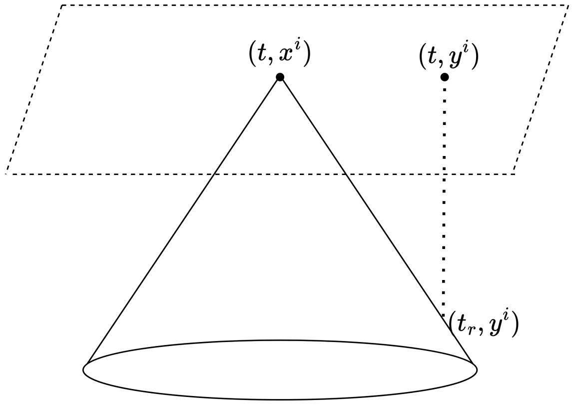

Figure 3.2: Perturbations in the gravitational field at (t, x^i) are determined by spacetime disturbances on the past light cone.

energy-momentum tensor for N point-like[1] masses is

$$T^{\mu\nu}(t, \mathbf{x}) = \sum_{a=1}^{N} T_a^{\mu\nu}(t)\delta(\mathbf{x} - \mathbf{x}_a(t)), \tag{3.13}$$

such that

$$T_a^{\mu\nu}(t) = m_a \gamma_a(t) v_a^\mu(t) v_a^\nu(t), \tag{3.14}$$

$$\bar{T}^{\mu\nu} = T^{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\eta_{\alpha\beta}T^{\alpha\beta}, \tag{3.15}$$

where $v_a^\mu(t) = (c, \mathbf{v}_a)$, with $\mathbf{v}_a$ the three-velocity and $\gamma_a(t) = (1 - |\mathbf{v}_a|^2/c^2)^{-\frac{1}{2}}$ the Lorentz factor [39]. Upon substituting Eq.(3.11) into Eq.(3.10), we perform some delta function manipulation [36] and integrate over space, obtaining the on-shell action:

$$S_{\mathcal{G}}^{\sigma} = \frac{G}{c^4} \sum_{a,b}^{a \neq b} \int dt \, \frac{\bar{T}_a^{\mu\nu}(t_{ab}) T_{b\mu\nu}(t)}{|\mathbf{d}_{ab}(t)| - \mathbf{d}_{ab}(t) \cdot v_a(t_{ab})/c} \tag{3.16}$$

with t_{ab} the retarded time between the two particles and $\mathbf{d}_{ab}(t) = \mathbf{x}_b(t) - \mathbf{x}_a(t_{ab})$ the retarded displacement. This is shown in more detail in Appendix B.2. Evidently $a = b$ terms are ignored, as they pertain to the scenario in which both particles are in the same interferometer, resulting in a purely infinite phase contribution to the state.

This action of linearized gravity is Lorentz covariant. Thus, through Eq.(3.7), we have shown that θ_σ, which the BMV experiment is designed to observe, also possesses these properties. This addresses any locality-related concerns in the process of gravitationally induced entanglement. We now show that our earlier calculation of θ is just an approximation of this Lorentz covariant expression.

[1] A valid approximation as here the distance between the particles is a lot greater than the their size. In more technical terms this means the spacial states of the particles are orthogonal.

3.2.3 Retrieving the Newtonian Limit from our Assumptions

In deriving the BMV effect we assumed, in subsection 2.5.1, that our masses moved at (equal $|\mathbf{v}_a| = |\mathbf{v}_b| = |\mathbf{v}|$) *non-relativistic speeds* and that gravitational field interactions were *instantaneous*. Together these two approximations yield the *Newtonian limit*.

Slow-moving approximation

For the slow moving test masses $|\mathbf{v}| \ll c$ we have

$$\gamma = (1 - |\mathbf{v}|^2/c^2)^{-\frac{1}{2}}) = 1 + \mathcal{O}(|\mathbf{v}|^2/c^2) \approx 1, \tag{3.17}$$

such that expanding Eq.(3.16) in factors of the speed of light c and cancelling terms of the order $\mathcal{O}(|\mathbf{v}|/c)$ and above we get

$$\frac{\bar{T}_a^{\mu\nu}(t_{ab})T_{b\mu\nu}(t)}{c^4} = m^2(1 + \mathcal{O}(|\mathbf{v}|/c)) \approx m^2, \tag{3.18}$$

for the numerator and

$$|\mathbf{d}_{ab}(t)| - \mathbf{d}_{ab}(t) \cdot v_a(t_{ab})/c \approx |\mathbf{d}_{ab}(t)|, \tag{3.19}$$

for the denominator, where we have taken the masses to be equal. Therefore, the action for linearized gravity in this *slow-moving approximation* is

$$S_{\mathcal{G}}^{\sigma} \approx \frac{1}{2} G \sum_{a,b}^{a \neq b} \int dt \frac{m^2}{d_{ab}^{\sigma}(t)}. \tag{3.20}$$

We observe that, in this context, the action is not entirely independent of the speed of light. There remains a factor of c in the implicit equation for the retarded displacement, which is dependent upon t_r. Thus, the interaction, resulting from this non-relative approximation of action, is still local.

Near-field approximation

In the *near-field approximation* the time taken for information to travel between the masses is very small compared to the time the particles are held in spacial superposition $d_{ab}/c \ll \tau$. This amounts to the retarded time functions closely approximating to the coordinate time $t_r \approx t$ for the majority of time we are integrating over. This approximation leads to the following expression for the action:

$$S^{\sigma}_{\mathcal{G}} \approx \frac{G}{c^4} \sum_{a,b}^{a\neq b} \int dt \, \frac{\bar{T}^{\mu\nu}_a(t) T_{b\mu\nu}(t)}{|\mathscr{D}_{ab}(t)| - \mathscr{D}_{ab}(t) \cdot v_a(t)/c}, \tag{3.21}$$

where $\mathscr{D}_{ab}(t) = \mathbf{x}_b(t) - \mathbf{x}_a(t)$, in which we have replaced the retarded time function with the coordinate time.

3.2.4 Newtonian limit

Both of these approximations do not necessarily overlap; physical systems can be structured in a way that validates one while invalidating the other. Taking both the *near-field* and the *slow-moving* approximations amounts to replacing the retarded displacement in Eq.(3.19) with $\mathscr{D}_{ab}(t)$. In our setup, we treat $\mathscr{D}_{ab}(t)$ as a constant displacement d, ignoring the tiny acceleration effect between the particles due to the gravitational interaction. Finally, by replacing the retarded displacement with d in Eq.(3.7), we arrive at the *Newtonian limit* for the induced gravitational phase, expressed as:

$$\phi_{\sigma} \approx \frac{Gm^2}{\hbar d} \int_0^{\tau} dt = \frac{Gm^2 \tau}{\hbar d}. \tag{3.22}$$

This perfectly corresponds to Eq.(2.22). This demonstrates that our naive calculation of the phase induced by gravity, responsible for entanglement, is merely an approximation of an on-shell, Lorentz covariant and gauge invariant action. So although in our derivation, we employed a simplified limit of the field, it does not imply that the BMV effect is the result of an unphysical, non-dynamical gravitational field, as some have suggested. These arguments are further supported in the papers presented by Belenchia et al. [40] and separately, Danielson et al. [41].

Chapter 4

A Leggett-Garg Inequalities Approach for Testing the Non-Classical Nature of Gravity

In this chapter and the subsequent one, we explore two other recent table-top proposals to test for quantum gravity. Here, we begin by examining an approach introduced by Matsumura et al. [20] that utilizes Leggett-Garg inequalities, which are known to be violated in quantum systems. It is proposed that by measuring violations in these inequalities by a system interacting solely through gravitational forces, we can draw conclusions about the quantum nature of the gravitational field.

4.1 Leggett-Garg Inequalities

4.1.1 Assumptions of macrorealism

In their seminal 1985 paper, Leggett and Garg [42] introduced what are often superficially referred to as the temporal counterparts to the well-known spacial CHSH/Bell's inequalities [43, 44]. Initially proposed to test for the potential presence of macroscopic[1] quantum coherence in a laboratory environment [45, 46], they were derived by outlining a set of assumptions that you would expect a macroscopic system to adhere to. Quoting directly from their paper [42], these assumptions are:

1. *Macroscopic realism*: A macroscopic system with two or more macroscopically distinct states available to it will at all times be in one or the other of these states.

[1]Generally, the systems explored using LG inequalities are seldom macroscopic; nevertheless, the inequalities remain valid.

2. *Noninvasive measurability* (NIM) at the macroscopic level: It is possible, in principle, to determine the state of the system with arbitrarily small perturbation on its subsequent dynamics.

Subsequently, a third assumption was incorporated for completeness [47, 48]:

3. *Induction*: future measurements cannot affect the outcome of measurements on the present state.

Together these constitute the *assumptions for macrorealism*. To prevent any ambiguity, assumption 1 is frequently referred to as 'macroscopic realism per se' (MRps). These assumptions ensure the existence of an 'underlying joint probability distribution' [49] and lead to a set of Leggett-Garg (LG) inequalities.

4.1.2 Two and Three Time Leggett-Garg Inequalities

We begin by defining a simple system with a single dichotomic observable Q with possible values $s = \pm 1$. By measuring this observable at multiple times t_i, we can construct a dataset and subsequently compute single time averages $\langle Q(t_i) \rangle = \langle Q_i \rangle$ as well as the temporal correlation function between sequential measurements C_{ij}. The correlation function is defined as

$$C_{ij} = C(t_i, t_j) = \sum_{s_i, s_j \pm 1} s_i s_j p(s_i, s_j), \tag{4.1}$$

where $p(s_i, s_j)$ represents the joint probability of measuring outcomes s_i and s_j at times t_i and t_j, respectively [46]. Following from the *assumptions for macrorealism*, one can derive the Leggett-Garg inequalities for any arbitrary number of time measurements, denoted as LGn's. This won't be displayed here, but I direct the interested reader to [42, 45, 50], for these derivations. For the case where three time measurements are taken the LG3 inequalities are as follows:

$$1 + C_{12} + C_{23} + C_{13} \geq 0, \tag{4.2}$$

$$1 - C_{12} - C_{23} + C_{13} \geq 0, \tag{4.3}$$

$$1 + C_{12} - C_{23} - C_{13} \geq 0, \tag{4.4}$$

$$1 - C_{12} + C_{23} - C_{13} \geq 0. \tag{4.5}$$

When measuring a system, LG inequalities are violated if any of the three assumptions for macrorealism fail.

For the remainder of this section we are most interested in the two-time Leggett-Garg inequalities (LG2) . These can be derived by defining the positive expression

$$(1 + s_1 Q_1)(1 + s_2 Q_2) \geq 0. \tag{4.6}$$

Next, assuming the existence of an 'underlying joint probability distribution' in a macrorealistic system, we can take the average of Eq.(4.6), leading to the following LG2 inequalities [50–53]:

$$1 + s_1 \langle Q_1 \rangle + s_2 \langle Q_2 \rangle + s_1 s_2 C_{12} \ \geq \ 0. \tag{4.7}$$

4.1.3 Properties of the Quasi-Probability

To better understand how the LG2 inequalities can be used to differentiate between classical and quantum systems, we first define the *two-time quasi-probability* by promoting the LHS of Eq.(4.7) to operators:

$$q(s_1, s_2) = \frac{1}{4}(1 + s_1 \left\langle \hat{Q}_1 \right\rangle + s_2 \left\langle \hat{Q}_2 \right\rangle + s_1 s_2 C_{12}), \tag{4.8}$$

where the added factor of $\frac{1}{4}$ ensures that $q(s_1, s_2)$ sums to one. The term 'quasi' is used because Eq.(4.8) can take negative values, analogous to the behaviour of Wigner distributions in phase space (cf. section 5.1). Lüders bound states that the minimum value for the two-time quasi-probability is $-\frac{1}{8}$.

The LG2 inequalities assert that the observation of a negative quasi-probability for a system serves as an indicator for the violation of macrorealism. To see what this tells us about the quantum nature of the system, we begin by introducing the statistical version of the NIM requirement for macrorealism, represented as follows:

$$\sum_{s_1} p(s_1, s_2) = p(s_2). \tag{4.9}$$

Here $p(s_2)$ is probability of measuring s_2 at time t_2 without a prior measurement having been performed. This was originally proposed by Kofler and Brukner [54] and termed the *no signalling in time* condition (NSIT). The NSIT condition holds for classical systems but generally does not hold for quantum systems due to the measurement postulate of quantum mechanics, described earlier in subsection 3.1.3. This is shown explicitly for our system in Appendix C. However, the same does not hold true for the two-time quasi-probability, which possesses the following properties:

$$\sum_{s_1} q(s_1, s_2) = p(s_2) \tag{4.10}$$

$$\sum_{s_2} q(s_1, s_2) = p(s_1) \tag{4.11}$$

for both classical and quantum systems. This implies that the quasi-probability satisfies a condition analogous to the NSIT condition [52].

Another salient property of the quasi-probability is its equivalence to the temporal correlation function defined by the measurement probability $p(s_1, s_2)$ [52], that is

$$(t_1, t_2) = C_{12} = \sum_{s_1, s_2 \pm 1} s_1 s_2 p(s_1, s_2) = \sum_{s_1, s_2 \pm 1} s_1 s_2 q(s_1, s_2). \tag{4.12}$$

It is noted that the quantum mechanical form of the correlation function is[46]:

$$C_{12} = \frac{1}{2} \left\langle \hat{Q}_1 \hat{Q}_2 + \hat{Q}_2 \hat{Q}_1 \right\rangle. \tag{4.13}$$

To measure the quasi-probability of our system, we conduct separate experiments to measure $\langle Q_1 \rangle$, $\langle Q_2 \rangle$ and $C(t_1, t_2)$. To ensure the correlation function of a quantum system cannot be simulated using an invasive classical measurement model, it is imperative we employ non-invasive measurement protocols. These protocols, such as ideal-negative measurements [55], are designed to mitigate classical disturbances arising from "clumsiness", thus ensuring a classical system satisfies the NIM condition of macrorealism. While implemented them in practice can be challenging, Joarder et al. [56] have provided a comprehensive approach to address and close various "clumsiness loopholes".

When measuring the quasi-probability using these non-invasive measurement techniques and following from the property described in Eq.(4.10), we can conclude that the quasi-probability satisfies NIM, characterizing the 'non invaded' aspect of the system at two distinct point in time [52]. Consequently, if the value of the quasi-probability is negative, it indicates a violation of MRps (as *induction* is assumed for all realistic physical models). A violation of MRps effectively signifies that the system exists in a superposition of states. Therefore, measuring a negative quasi-probability for a system amounts to demonstrating it adheres to the superposition principle of quantum mechanics.

4.2 Experimental Proposition

4.2.1 Measurement operator

In the proposed experiment, the dichotomic variable we are interested in measuring is denoted $\hat{Q} = \mathbf{n} \cdot \boldsymbol{\sigma}$, corresponding to the spin value ($s = \pm 1$) along a direction vector $\mathbf{n}$. To simplify the subsequent derivation, we will adopt the specific choice of $\mathbf{n} = (1, 0, 0)$, such that $\hat{Q} = \hat{\sigma}^x$. Following from this, the associated projective measurement operator of our quantum variable $\hat{Q}$ is defined as follows:

$$\hat{P}_s = \frac{1}{2}(\mathbb{1} + s\hat{\sigma}^x), \tag{4.14}$$

satisfying $\hat{P}_s = \hat{P}_s^\dagger = \hat{P}_s^2$ and $\sum_s \hat{P}_s = \mathbb{1}$. Starting with an initial system ρ_0 the single time average of measuring s at a time t is

$$\begin{aligned} \left\langle \hat{Q}(t) \right\rangle &= \sum_{s=\pm 1} s p_s(t) = \sum_{s=\pm 1} sTr[\hat{P}_s(t)\rho_0] \\ &= \sum_{s=\pm 1} \frac{1}{2}\Big(sTr[\rho_0] + Tr[s^2\hat{\sigma}^x(t)\rho_0]\Big) \\ &= Tr[\hat{\sigma}^x(t)\rho_0], \end{aligned} \tag{4.15}$$

where we are in the Heisenberg picture, such that

$$\hat{\sigma}^x(t) = \hat{U}^\dagger(t)\hat{\sigma}^x(0)\hat{U}(t) = e^{-\frac{i\hat{H}t}{\hbar}}\hat{P}_s(0)e^{\frac{i\hat{H}t}{\hbar}}. \tag{4.16}$$

Similarly, from Eq.(4.13), the temporal correlation function for sequential measurements at times t_1 and then t_2 is given as:

$$C(t_1, t_2) = \frac{1}{2}Tr[\{\hat{\sigma}^x(t_1), \hat{\sigma}^x(t_2)\}\rho_0], \tag{4.17}$$

with the curly brackets representing the anti-commutator of the two operators. Therefore, to establish the quasi-probability we must define the Hamiltonian of our system in order to identify the unitary operators responsible for governing the temporal evolution of the system.

4.2.2 Setup

To use the two-time quasi-probability function to test for the non-classical nature of gravity, Matsumura et al. [20] propose an experiment consisting of a hybrid system setup, which is based on the setup presented by Carney et al. [57]. Here, a low frequency oscillator is coupled to a particle in a superposition of two spacial states. Depicted in Figure 4.1,

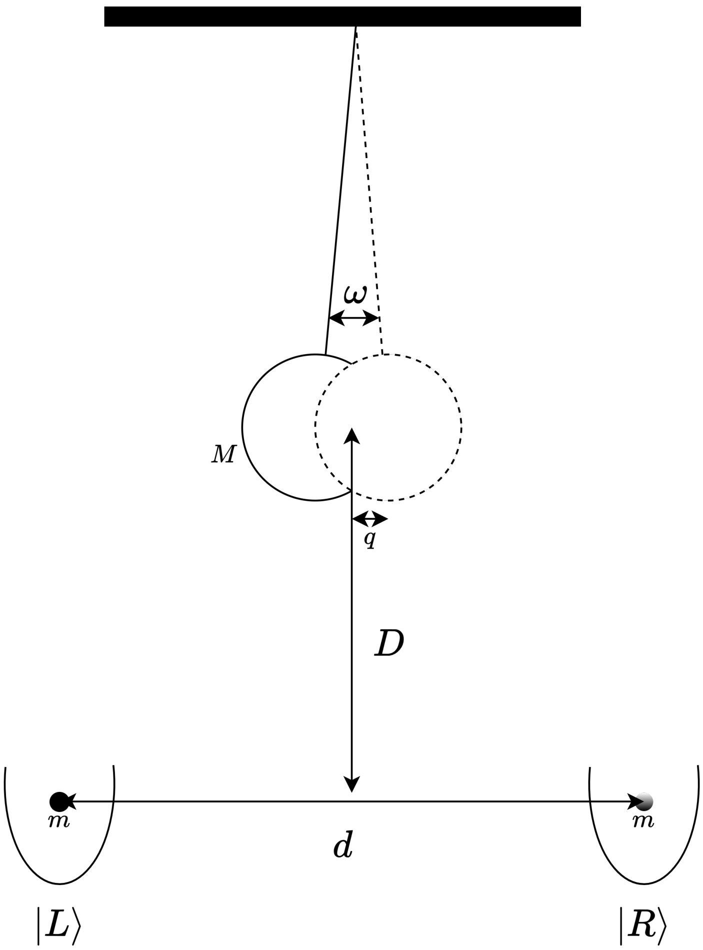

Figure 4.1: A hybrid system composed of a particle with mass m existing in a spacial superposition of states represented as $|L\rangle$ and $|R\rangle$ coupled to a mechanical oscillator of mass M, position variable q and angular frequency ω. This coupling is facilitated through a mediating gravitational field and no other interactions take place. By performing measurements on the evolved state of the particle, it is proposed [20] that detecting the violation of Leggett-Garg inequalities is evidence for the non-classical nature of gravity.

the equilibrium point of the mechanical oscillator, characterized by an angular frequency ω and mass M, is situated a distance D from a particle with mass m in a superposition state, where the separation between the two interferometer arms is d. For clarity we will be dropping hats on operators and setting $\hbar = 1$ for the rest of this section.

As with the BMV experiment, we are assuming that the oscillator and the particle are only interacting through a gravitationally mediated field, such that the Hamiltonian is:

$$\hat{H} = \underbrace{\omega a^\dagger a}_{\textbf{free Hamiltonian of the oscillator}} + \underbrace{\mathcal{G}}_{\textbf{gravitational interaction term}}, \tag{4.18}$$

where $\hat{a}^\dagger$ and $\hat{a}$ are the creation and annihilation operators respectively. In this context, we have deliberately omitted the Larmour procession term $\Omega\hat{\sigma}^z$ included in the derivation by Matsumura et al. [20], as we are only focusing on effects of gravity.

Again implementing the valid low-energy assumption of the Newtonian limit of gravity (see subsection 2.5.1 and 3.2.3), we have [20]

$$\hat{\mathcal{G}} = -\frac{GMm}{\sqrt{D^2 + (q + d\sigma^z/2)^2}}. \tag{4.19}$$

Under the assumption that the maximum position from equilibrium (q_{max}) of the oscillator is a lot smaller than both d and D we can make the approximation

$$\hat{\mathcal{G}} \approx \frac{GMmd}{(D^2 + d^2/4)^{3/2}}\sigma^z q = \frac{g}{\sqrt{2}}\,\sigma^z\,q', \tag{4.20}$$

with

$$g = \frac{1}{\sqrt{2M\omega}}\frac{GMmd}{(D^2 + d^2/4)^{3/2}}, \tag{4.21}$$

$$q' = \sqrt{M\omega}\,q. \tag{4.22}$$

From the Hamiltonian of the system, an expression for the unitary time evolution operator can be derived. Utilizing principles of quantum field theory, it can be shown that the time evolution operator is:

$$U(t) = e^{-iHt} = e^{-i\omega a^\dagger a t}\,e^{\sigma^z(\alpha(t)a - \alpha^*(t)a^\dagger)}, \tag{4.23}$$

where

$$\alpha(t) = \lambda(e^{-i\omega t} - 1) = -2\lambda i\,e^{\frac{-i\omega t}{2}}\sin\left(\frac{\omega t}{2}\right), \tag{4.24}$$

here $\lambda = g/\omega$. This quantity is significant as it sets the scale for all observables being measured in this experiment. For the interested reader please refer to [20] for a derivation of this last step.

We prepare our experiment as in Figure 4.1, such that the initial state of the hybrid system is

$$|\Psi(0)\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)_a \otimes |0\rangle_b$$
$$= \frac{1}{\sqrt{2}}(|0\rangle |0\rangle + |1\rangle |0\rangle). \tag{4.25}$$

By acting the unitary operator defined in Eq.(4.23) on $|\Psi(0)\rangle$ and ignoring overall phase, the state evolves as[20, 57]:

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}}(|0\rangle |\alpha(t)\rangle + |1\rangle |-\alpha(t)\rangle), \tag{4.26}$$

where

$$|\alpha(t)\rangle = |\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle = e^{-\frac{|\alpha|^2}{2}} e^{\alpha a^\dagger} e^{-\alpha^* a} |0\rangle \tag{4.27}$$

are coherent states. Unlike Fock states, two different coherent states $(|\alpha\rangle, |\beta\rangle)$ are not orthogonal and have the inner product property

$$\langle\beta|\alpha\rangle = e^{-\frac{1}{2}(|\beta|^2 + |\alpha|^2 - 2\beta^*\alpha)}. \tag{4.28}$$

4.2.3 Calculating the two-time quasi-probability

We now calculate two-time quasi-probability for our system. To do this we need to determine the single time averages $\langle \hat{Q}_1 \rangle$ and $\langle \hat{Q}_2 \rangle$, as well as the two time correlation function $C(t_1, t_2)$ and insert these values into Eq.(4.8). Please note that when calculating these values, the interferometer measures the spin states. More technically, this implies that the projective measurement operator only operates on the Hilbert space in which the particle in spatial superposition lives.

Using the cyclic properties of the trace we can rewrite Eq.(4.15) as:

$$\left\langle \hat{Q}(t) \right\rangle = Tr[\hat{U}^\dagger(t)\hat{\sigma}^x(0)\hat{U}(t)\rho_0] = Tr[\langle\Psi(0)| \hat{U}^\dagger(t)\hat{\sigma}^x(0)\hat{U}(t) |\Psi(0)\rangle]$$
$$= \langle\Psi(t)| \hat{\sigma}^x |\Psi(t)\rangle . \tag{4.29}$$

Now substituting in Eq.(4.26) we have

$$\begin{aligned}\left\langle \hat{Q}(t) \right\rangle &= \frac{1}{2}(\langle 0| \langle \alpha(t)| + \langle 1| \langle -\alpha(t)|)\, \hat{\sigma}^x \,(|0\rangle |\alpha(t)\rangle + |1\rangle |-\alpha(t)\rangle) \\ &= \frac{1}{2}(\langle 0| \langle \alpha(t)| + \langle 1| \langle -\alpha(t)|)(|1\rangle |\alpha(t)\rangle + |0\rangle |-\alpha(t)\rangle) \\ &= \frac{1}{2}(\langle \alpha(t)| - \alpha(t)\rangle + \langle -\alpha(t)|\alpha(t)\rangle) = \langle \alpha(t)| - \alpha(t)\rangle, \end{aligned} \tag{4.30}$$

where we have used the orthogonality property of the spacial states. This can easily be solved by utilizing the inner product of coherent states given in Eq.(4.28) and our definition of $\alpha(t)$ in Eq.(4.24), such that

$$\left\langle \hat{Q}(t) \right\rangle = \langle \alpha(t)| - \alpha(t)\rangle = e^{-2|\alpha^2|} = e^{-8\lambda^2 \sin^2(\frac{\omega t}{2})}. \tag{4.31}$$

A similar calculation for the temporal correlation function yields the slight more complex result [20]:

$$C(t_1, t_2) = \cos\Theta(t_1, t_2)\, e^{-8\lambda^2 \sin^2(\frac{\omega\tau}{2})}, \tag{4.32}$$

where $\tau = t_2 - t_1$ and

$$\Theta(t_1, t_2) = 8\lambda^2 \sin\left(\frac{\omega\tau}{2}\right)\left[(\cos\left(\frac{\omega\tau}{2}\right) - \cos\left(\frac{\omega(t_1+t_2)}{2}\right)\right]. \tag{4.33}$$

Substituting Eq.(4.31) and Eq.(4.32) into Eq.(4.8) we get our expression for the two-time quasi-probability function:

$$q(s_1, s_2) = \frac{1}{4}\Big(1 + s_1 e^{-8\lambda^2 \sin^2(\frac{\omega t_1}{2})} + s_2 e^{-8\lambda^2 \sin^2(\frac{\omega t_2}{2})} + s_1 s_2 \cos\Theta(t_1, t_2)\, e^{-8\lambda^2 \sin^2(\frac{\omega\tau}{2})}\Big). \tag{4.34}$$

4.3 Analysis

4.3.1 Results

After disabling the gravitational interaction between the two systems by taking $\lambda \to 0$, we find that Eq.(4.34) simplifies to

$$\begin{aligned} q(s_1, s_2) &= \frac{1}{4}(1 + s_1 + s_2 + s_1 s_2) \\ &= \frac{1}{4}(1 + s_1)(1 + s_2) \geq 0. \end{aligned} \tag{4.35}$$

This observation demonstrates, that in the absence of gravitational interactions between the two hybrid systems, the LG2 inequalities are satisfied.

By reinstating the gravitational interactions, Figure 4.2 evidences that at a pair of time

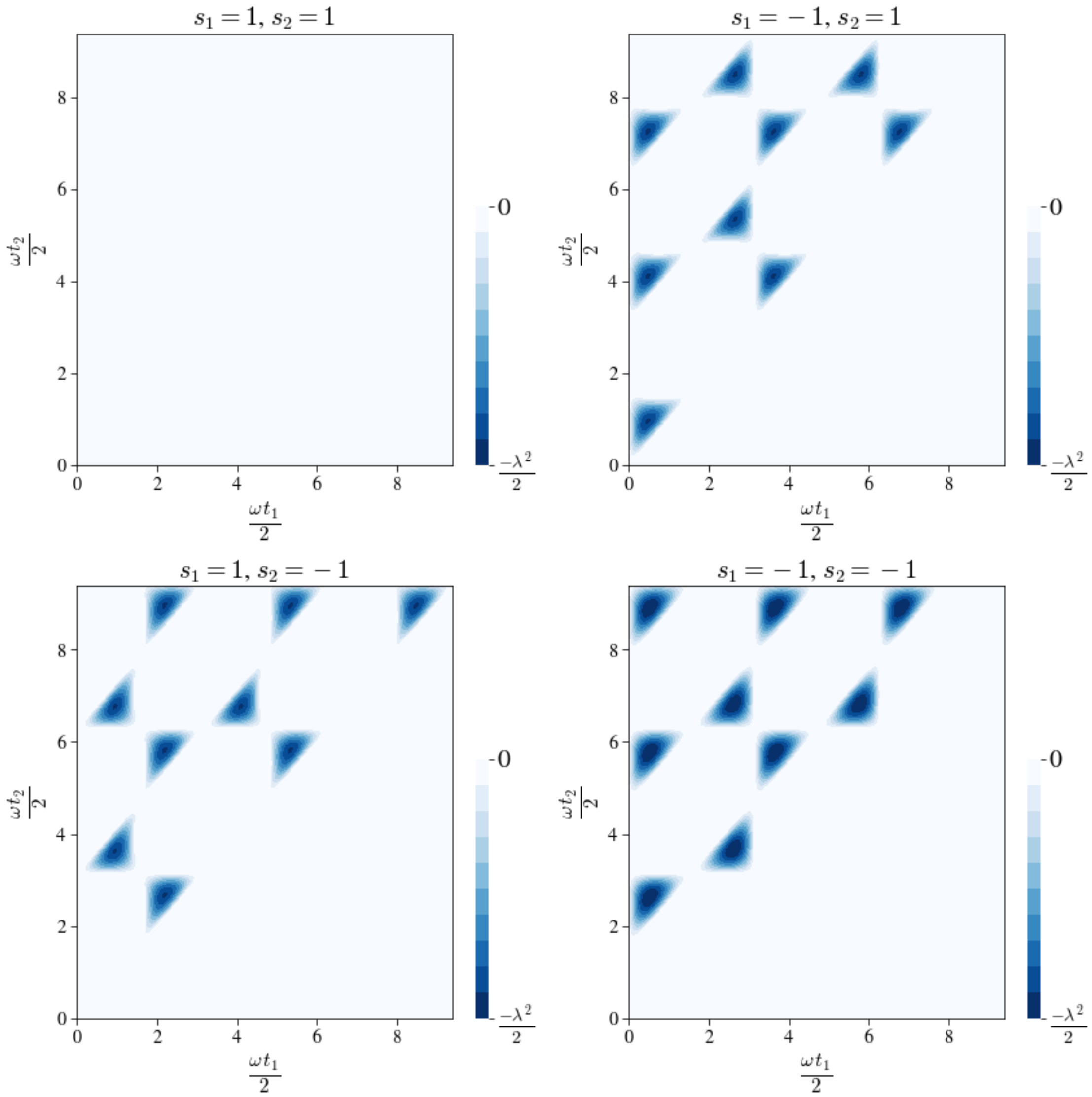

Figure 4.2: The blue regions show the negative values of the two-time quasi-probability defined in Eq.(4.34). We have shown this for all iterations of spin measurement outcomes (s_1, s_2). Here you can see that in the case of $s_1 = 1$ and $s_2 = 1$ the quasi-probability is always positive.

correlated measurements t_1, t_2, the quasi-probability can take negative values. This negativity depends on both the spin measurement outcomes (s_1, s_2) and times of measurements, with no negativity occurring in the case $(s_1, s_2) = (1, 1)$. This demonstrates that, if negative quasi-probabilities were measured, the LG2 inequalities, and therefore macrorealism, would be violated due to gravitational interactions mediating the two systems.

4.3.2 Feasibility

The measurability of the quasi-probability of our system is contingent upon the parameter λ. Both Matsumura et al. [20] and Carney et al. [57] propose that, by employing highly pragmatic experimental parameters, we would estimate λ to be on the order of approximately 10^{-14}. To assess the impact of this estimation on feasibility, we now expand the quasi-probability to the order of $\mathcal{O}(\lambda^2)$ in λ, such that

$$\begin{aligned} q(s_1, s_2) = \frac{1}{4}\Big(&1 + (s_1(1 - 8\lambda^2 \sin^2(\frac{\omega t_1}{2}) + s_2(1 - 8\lambda^2 \sin^2(\frac{\omega t_2}{2})) + \mathcal{O}(\lambda^4)) \\ &+ s_1 s_2(1 + \mathcal{O}(\lambda^4))(1 - 8\lambda^2 \sin^2(\frac{\omega\tau}{2}) + \mathcal{O}(\lambda^4))\Big) \\ &\approx \frac{1}{4}(1 + s_1 + s_2 + s_1 s_2) \\ &- 2\lambda^2\Big(s_1 \sin^2(\frac{\omega t_1}{2}) + s_2 \sin^2(\frac{\omega t_2}{2}) + s_1 s_2 \sin^2(\frac{\omega\tau}{2})\Big). \end{aligned} \tag{4.36}$$

From Eq.(4.36) the minimum value for the two-time quasi-probability of our system when $(s_1, s_2) = (-1, -1)$, $(-1, 1)$, or $(1, -1)$ can be calculated as [20]:

$$q_{min}(s_1, s_2) = -\frac{\lambda^2}{2}. \tag{4.37}$$

For instance, a $q_{min}(-1, -1)$ occurs when $\omega t_1 = \frac{\pi}{3} + 2\pi n$ and $\omega t_2 = \frac{5\pi}{3} + 2\pi m$ for any integers n, m. In the case that $\lambda \sim 10^{-14}$, this quasiprobability would be minuscule, rendering its detection practically impossible.

Thus, in order to make this proposed experiment viable, we need to make q_{min} more negative. A solution to this is adopting a different initial state of the oscillator to that of the single particle state in Eq.(4.25). For example, choosing our initial state to be a thermal state with density operator

$$\rho = \frac{1}{\pi\bar{n}} \int d^2\alpha\, e^{-\frac{|\alpha|^2}{\bar{n}}} |\alpha\rangle \langle\alpha|, \tag{4.38}$$

where $\bar{n}$ is the mean occupation of thermal phonons and $|\alpha\rangle$ is some arbitrary coherent state. It can be shown [20, 57] that the minimum value for the quasi-probability becomes

$$q_{min}(s_1, s_2) = -(2\bar{n} + 1)\frac{\lambda^2}{2}. \tag{4.39}$$

Reintroducing $\hbar$'s, the mean occupation number is associated to the temperature T of the system via the expression [20]:

$$\bar{n} = \frac{k_B T}{2\hbar\omega}. \tag{4.40}$$

For our system this is roughly of the order $\sim 10^{14}$.

Finally, Carney et al. [57] have suggested that linear sensitivity in λ can be achieved through a 'boosted' preparation method, where the oscillator is initially prepared in an entangled state. Alternatively, Matsumura et al. [20] argue that 'squeezing' the initial state of the oscillator can significantly increase the degree to which the LG inequalities are violated. Further details can be found in their respective papers cited above.

4.3.3 Entanglement generation

We now quantify the entanglement generated in our hybrid system to demonstrate the direct parallels between measuring the quasi-probability and employing an entanglement witness. Implementing the same procedure outlined in subsection 2.4.1, we calculate the Von Neumann entropy for our hybrid system. From Eq.(4.26), the reduced density matrix of the particle undergoing interferometry is

$$\hat{\rho}_a = Tr_b[\hat{\rho}] = \frac{1}{2}\begin{bmatrix} 1 & e^{-8\lambda^2 \sin^2(\frac{\omega t}{2})} \\ e^{-8\lambda^2 \sin^2(\frac{\omega t}{2})} & 1 \end{bmatrix}, \tag{4.41}$$

such that the corresponding eigenvalues are

$$\lambda_{\pm}^{eig} = \frac{1}{2} \pm \frac{1}{2} e^{-8\lambda^2 \sin^2(\frac{\omega t}{2})}. \tag{4.42}$$

Using Eq.(2.16) we again observe that the entropy is zero at times $t = 2n\pi$, as $\lambda_+ = 1$ and $\lambda_- = 0$, showing the state is initially separable. Additionally, it is apparent that if we turn gravitational interactions off (by setting $\lambda = 0$), the entropy vanishes for all times. This observation demonstrates that the presence of a gravitational field mediating the two hybrid systems is a prerequisite for the emergence of entanglement.

By investigating the entanglement of the system at times $t = n\pi$, where n is an odd integer, and expanding in powers of λ, we find

$$\lambda_{\pm}^{max} = \frac{1}{2} \pm \frac{1}{2} e^{-8\lambda^2} = \frac{1}{2} \pm \frac{1}{2}(1 - 8\lambda^2 + \mathcal{O}(\lambda^4)) \tag{4.43}$$

leading to a Von Neumann entropy in the limit $\lambda \ll 1$:

$$\begin{aligned}
S_{max} &= -(4\lambda^2 + \mathcal{O}(\lambda^4))\ \log\big(4\lambda^2 + \mathcal{O}(\lambda^4)\big) - (1 - 4\lambda^2 + \mathcal{O}(\lambda^4))\ \log\big(1 - 4\lambda^2 + \mathcal{O}(\lambda^4)\big) \\
&= -(4\lambda^2 + ...)\log\big(4\lambda^2 + ...\big) - (1 - 4\lambda^2 + ...)(-4\lambda^2 + ...) \\
&\approx -8\lambda^2 \log(2\lambda) + 4\lambda^2. \qquad (4.44)
\end{aligned}$$

This demonstrates that the amount of entanglement generated by the hybrid system is contingent on λ in a manner akin to the negativity of the quasi-probability (see Eq.(4.37)). This alludes to the fact that both are a consequences of the same quantum phenomenon: the superposition of spacetime geometries.

4.4 Closing remarks

Due to the negligible amount of entanglement generated in a system defined by our approximate parameters, obtaining a statistically significant result for the two-time quasi-probability will require conducting numerous measurements of $\langle Q(t_1)\rangle$, $\langle Q(t_2)\rangle$, and the correlation function $C(t_1, t_2)$. The correlation function involves measurements at two different times, necessitating a significantly larger data set compared to the single-time expectation values to maintain accuracy. This demanding measurement process is one of the disadvantages of this experiment. Additionally, there is a concern that the low strength of these signals could pose challenges when attempting to mitigate decoherence effects and eliminate any entanglement generated by non-gravitational quantum fields. For example, to nullify van der Waal forces it would be necessary to increase the distance between the oscillator and the particle; however, this would further weaken the detection of a negative quasi-probability. However this test only requires the observation of a single subsystem, as opposed to the multi-partite quantum system proposed in the BMV approach, which is a potentially more manageable feat for the experimentalist.

Leggett-Garg inequalities cannot be violated in a classical system, so much like the BMV experiment, this test would be a witness for the non-classical nature of gravity. This proposal can be regarded as a variation on the earlier BMV experiment, in the sense that it is just a measure of whether two systems locally interacting exclusively with the gravitational field can generate entanglement. Therefore, it suffers from similar issues to those raised in chapter 3. Nevertheless, this experiment offers a view on the quantumness of gravity through the lens of the more rigorous confines of the assumptions for macrorealism. With future advances in quantum measurement technologies, the experiment offers the scope to extend these Leggett-Garg inequality tests to three time LG3s (as long as it is ensured that non-invasive measurements are implemented). This would give a more complete picture of gravity as a quantum entity.

Chapter 5

Non-Gaussianity as a signature of Quantum Gravity

Now, we shift our focus to a fundamentally different approach for detecting the quantum characteristics of gravity in a laboratory setting. In which Howl et al. [21] utilize the fact that only quantum, not classical, fields can generate non-Gaussianity in the quantum field state of matter.

5.1 The Wigner Function and Gaussian States

A quantum field can be viewed as a collection of quantum simple harmonic oscillators (QSHO). Each of these oscillators is described by the Hamiltonian:

$$\hat{H} = \hbar\omega_k(\hat{a}_k^\dagger \hat{a}_k + \frac{1}{2}), \tag{5.1}$$

where, again, $\hat{a}_k^\dagger$ and $\hat{a}_k$ are the creation and annihilation operators, and ω_k represents the angular frequency of the oscillator. These QHSO collectively contribute to the field's dynamics [19, 21]. Momentum and position-like operators for each of these modes, referred to as 'quadrature operators', are defined as follows [58]:

$$\hat{x}_k = \frac{1}{\sqrt{2}}(\hat{a}_k + \hat{a}_k^\dagger), \tag{5.2}$$

$$\hat{p}_k = \frac{1}{i\sqrt{2}}(\hat{a}_k - \hat{a}_k^\dagger). \tag{5.3}$$

Denoting $|x_k\rangle \, and \, |p_k\rangle$ as eigenvectors of these quadrature operators with eigenvalues x_k and p_k respectively, we can define a 'phase-space' parameterized by these continuous variables.

Using this phase-space formalism it is now possible to describe our system using a probability distribution known as the Wigner function. The Wigner function corresponding to the the state described by the density operator ρ is

$$W(x,p) = \frac{1}{2\pi\hbar}\int \langle x+y|\hat{\rho}|x-y\rangle e^{-ipy/\hbar}\,dy. \tag{5.4}$$

This function is real valued and, as it represents a probability density, it is normalized to one. However, similar to Eq.(4.8) in subsection 4.1.3 the Wigner function is a quasi-probability distribution. States for which Eq.(5.4) takes negative values vanish in the classical limit ($\hbar \to 0$) and are therefore recognized as highly non-classical states [59].

Gaussian states are described by Wigner functions that take the form of a Gaussian function [58] and Gaussian transformations are operations that map Gaussian states to other Gaussian states [60]. For a scalar field $\hat{\phi}$, only non-Gaussian states have a negative Wigner function [57]. Moreover, Non-gaussian states are necessary for the violation of bell inequalities, drawing further parallels between the Wigner function and the two-time quasiprobability defined in the previous chapter, where a negative two-time quasi-probability indicates a violation of Leggett-Garg inequalities.

The unitary time evolution operator of a state with Hamiltonian $\hat{H}$, which consists of terms that are at most quadratic in the quadrature operators, is a Gaussian transformation [57, 61]. As gravity couples to all particles that have energy and momentum, quantizing gravity leads to mass terms in a system's Hamiltonian that involves terms higher order than quadratic in operators, inducing non-Gaussianity. Assuming all other non-gravitational quantum interactions can be neglected, and thus no self-interaction terms are present in the Hamiltonian of the free scalar field, any indication of non-Gaussianity emerging in a quantum field state of matter that was initially Gaussian would be evidence of a quantum theory of gravity [21]. This holds true for both the high and low curvature regimes of quantum gravity. In the following subsection 5.2.1 we will show this explicitly for our low-energy experimental setup where we are working with the perturbative weak-field perspective of gravity.

5.2 Experimental Proposition

Here we demonstrate how this quantum witness for gravity can be implemented using a table-top test on a single, localized Bose-Einstein condensate (BEC).

5.2.1 Classical Gravity and Quantum Gravity in the Newtonian Limit

A BEC is a collection of N bosons described by the field operators $\hat{\Psi}(\mathbf{r})$ and $\hat{\Psi}(\mathbf{r})^\dagger$, that annihilate and create an atom at the position r, respectively [62]. For a BEC in which the bosons have mass m the mass density $(\hat{\rho})$ is given by:

$$\hat{\rho}(\mathbf{r}) = m\hat{N} = m\hat{\Psi}^\dagger(\mathbf{r})\hat{\Psi}(\mathbf{r}) \tag{5.5}$$

In the low-energy system of a BEC, we can once again employ the non-relativistic Newtonian approximation of gravity. In this approximation, the classical interaction Hamiltonian is [21]:

$$H^{int}_{classical} = \frac{1}{2}\int d^3\mathbf{r}\rho(\mathbf{r})\phi(\mathbf{r}), \tag{5.6}$$

where $\rho(\mathbf{r})$ is the classical mass density and $\phi(\mathbf{r})$ the classical Newtonian potential. By solving the Poisson equation

$$\nabla^2\phi(\mathbf{r}) = 4\pi Gm\rho(\mathbf{r}) \tag{5.7}$$

we get the the gravitational self-potential [63]:

$$\phi(\mathbf{r}) = -Gm\int d^3\mathbf{r}'\frac{\Psi^*(\mathbf{r})\Psi(\mathbf{r})}{|\mathbf{r}-\mathbf{r}'|}. \tag{5.8}$$

If gravity adheres to quantum theory, it necessitates the quantization of both the classical mass density and the gravitational field $\phi(\mathbf{r})$. However, in a semi-classical description, only the former is quantized. Resulting in the respective interaction Hamiltonian's for a semi-classical system in which gravity is classical $\hat{H}_{CG}$ and a fully quantum picture of the Newtonian limit of gravity $\hat{H}_{QG}$[21]:

$$\hat{H}^{int}_{CG} = \frac{m}{2}\int d^3\mathbf{r}\ \hat{\Psi}^\dagger(\mathbf{r})\hat{\Psi}(\mathbf{r})\phi(\mathbf{r}), \tag{5.9}$$

$$\begin{aligned}\hat{H}^{int}_{QG} &= \frac{m}{2}\int d^3\mathbf{r} : \hat{\Psi}^\dagger(\mathbf{r})\hat{\Psi}(\mathbf{r})\hat{\phi}(\mathbf{r}) : \\ &= -\frac{Gm^2}{2}\int d^3\mathbf{r}d^3\mathbf{r}'\frac{: \hat{\Psi}^\dagger(\mathbf{r})\hat{\Psi}(\mathbf{r})\hat{\Psi}^\dagger(\mathbf{r}')\hat{\Psi}(\mathbf{r}') :}{|\mathbf{r}-\mathbf{r}'|}.\end{aligned} \tag{5.10}$$

In Eq.(5.10) the colons represent the Wick ordering operation used in quantum field theory (QFT) to eliminate infinite vacuum energy terms and in the last step we substituted in the quantized version of Eq.(5.8).

As previously outlined, only a Hamiltonian that is not linear or quadratic in quantum operators can generate non-Gaussianity. Consequently, we can see that only the interaction Hamiltonian $\hat{H}_{QG}$, associated to the quantization of spacetime curvature, not the classical gravity Hamiltonian $\hat{H}_{CG}$, can induce non-Gaussianity in our BEC system. It is imperative to emphasize that Eq.(5.10) represents the non-relativistic, low-energy, static regime for all Quantum Gravity theories, while Eq.(5.9) serves the same role in classical gravity theories. By observing the generation of Non-Gaussianity in the quantum state of the Bose-Einstein condensate, we confirm a phenomenon predicted by all quantum gravity models and unsupported by classical gravity models.

5.2.2 Measuring Non-Gaussianity

Cumulants κ_n are a commonly used tool in testing for non-Gaussianity in a quantum system. Defined by the formula [64]:

$$\kappa_n = \langle \hat{q}^n \rangle - \sum_{k=1}^{n-1} \binom{n-1}{k-1} \langle \hat{q}^{n-k} \rangle \, \kappa_k, \tag{5.11}$$

where $\binom{n-1}{k-1}$ is the binomial coefficient and $\hat{q}$ is some generalised quadrature [21]

$$\hat{q}(\theta) = \hat{a}e^{-i\theta} + \hat{a}^\dagger e^{i\theta}. \tag{5.12}$$

With the expectation values of powers of this generalised quadrature calculated using the Wigner function, given by Eq.(5.4), as follows:

$$\langle \hat{q}^n \rangle = \int dxdp \, W(x,p) q^n. \tag{5.13}$$

The forth-order cumulant κ_4 is the lowest-order indicator of non-Gaussianity in systems defined by symmetric Wigner distributions like ours [64]. From Eq.(5.11) it is straightfoward to derive

$$\kappa_4 = \langle \hat{q}^4 \rangle - 4\langle \hat{q} \rangle \langle \hat{q}^3 \rangle - 3\langle \hat{q}^2 \rangle^2 + 12\langle \hat{q}^2 \rangle \langle \hat{q} \rangle^2 - 6\langle \hat{q} \rangle^4. \tag{5.14}$$

It can be shown [21] that the signal-to-noise ratio (SNR) of our calculated value of κ_4 is:

$$\text{SNR} = \frac{|\kappa_4|}{\sqrt{Var(\kappa_4)}} \propto \frac{|\kappa_4|}{\mathcal{M}}, \tag{5.15}$$

with $\mathcal{M}$ the number of independent measurements of our system. In making the initial state, of the weakly interacting BEC we are measuring, a *squeezed* Gaussian state, Howl et al. [21] demonstrate that:

$$\text{SNR} \sim \sqrt{\frac{2\mathcal{M}}{\pi}} \, \frac{GM^2 t}{\hbar R}, \tag{5.16}$$

where M and r are the mass and effective radius of the BEC, respectively and t is the time the BEC is left to gravitationally self-interact. We have chosen to omit the detailed derivation of the values of the signal-to-noise ratio given above, as in reviewing this proposal our primary focus is on the theoretical approach rather than the practical calculations of measurements involving Bose-Einstein condensates. For a comprehensive understanding of how both Eq.(5.15) and Eq.(5.16) were calculated please refer to the original paper by Howl et al. [21].

Notably, Eq.(5.16) is analogous to our expression for the phase induced in the BMV experiment. Specifically, by performing the substitutions $M \to m$, $t \to \tau$ and $R \to \tilde{d}$, we recover Eq.(2.21) multiplied by a factor $\sqrt{\frac{2\mathcal{M}}{\pi}}$. This highlights the significance of this experiment, as here we can achieve more feasible experimental parameters, than proposed in subsection 2.6 for the BMV experiment, by increasing the number of measurements $\mathcal{M}$. In the next section, we will discuss additional advantages offered by this proposal.

5.3 Advantages of the Proposal

With all table-top tests designed to measure the quantum aspects of gravity, it is essential to be able to neglect other non-gravitational quantum interactions. Or alternatively establish a mathematically and experimentally rigorous methodology to differentiate these non-gravitational quantum interactions from gravitational ones.

For the BMV experiment, we proposed in section 2.6, that in order to reduce the effects of Van der Waal forces the distance between the interacting quantum states needed to be increased. This had the detrimental effect of reducing the amount of entanglement generated in the state by gravitational forces, making the BMV effect harder to detect. A similar situation arises in the Leggett-Garg case, necessitating an increased distance between the oscillator and the particle, thus decreasing λ and rendering the detection of a negative quasi-probability a lot more challenging (refer to Eq.(4.37)).

However, unlike these other proposals a phenomenon of Feshbach resonances in ultra cold quantum gases allows you to control the interactions between the bosons in the BEC [65]. It is possible these unwanted electromagnetic (EM) effects [1] may be mitigated by applying external EM fields [57]. This could be achieved without affecting the strength of the gravitational interactions, resulting in an increase in SNR. A distinct advantage in comparison to the previous models.

Furthermore, the masses proposed in section 2.6 in order get a meaningful measurement of the BMV effect have yet to be achieved in opto-mechanical oscillators. However, this experiment implements squeezed state BECs that have already been created in the laboratory. We should note that this is only true in the quantum regime and no squeezed

[1] As Bose-Einstein condensates have a neutral overall charge, these electromagnetic forces tend to be a result of both van der Waal forces and magnetic dipole-dipole interactions [57].

state BECs have been generated at the scales of the parameters proposed by Howl et al. [21], with all indications suggesting that it will be very challenging.

Finally, much like the Leggett-Garg witness for QG, here we are only required to conduct tests on a single system rather than the multi-partite system proposed in the BMV experiment.

Chapter 6

Conclusion

6.1 Summary

In this study, we delved into table-top experimental approaches designed to investigate the quantum aspects of gravity. Our exploration revealed both the strengths and weaknesses inherent in all the proposed experiments. We also derived the expected outcomes for a low-energy system in which gravity can be approximated by linearized quantum gravity. In our findings we demonstrated that employing the Newtonian limit of the quantized gravitational field does not imply any derived effects are a result of an unphysical, non-dynamical gravitational field.

For any such proposals, there invariably emerge the contrarian physicists capable of devising creative, non-quantum explanations for the results. Explanations that often offer no useful application in describing the physical world in which we live, except to pander to the quantum skeptic. It is thus crucial to bear in mind that the role of an experimental proposals such as these is to design experimentally achievable observations predicted by models of quantum gravity, but not by all conventional theories of classical gravity.

While we have shown that, in their current state, these proposals push the limits of what may seem experimentally feasible at this time. I believe that, with the continued advances in quantum information technologies and the substantial ongoing work aimed at improving on all the prior experimental proposals, the future holds promise for observing quantum properties of gravity in the laboratory.

Furthermore, in my view, there is no need for the various table-top experimental proposals to be considered in isolation. Observing the results of multiple tests can offer a more comprehensive perspective on the quantum nature of gravity.

6.2 Further Considerations

An area of particular interest for future work involves extending the Leggett-Garg inequalities test to measure the three-time LG3 equations. This is a significantly more measurement-intensive task as it involves calculating three different correlation functions. However, there are physical scenarios in which LG2 violations have been measured while LG3 violations are not observed, and vice versa. Exploring these scenarios from a gravitational perspective would be particularly interesting, even purely from a theoretical standpoint. Furthermore, by exploring the different possibilities for the initial state of the oscillator, it would be of great significance to discover an optimally prepared state that maximizes the signal of Leggett-Garg inequality violations.

Regarding the non-Gaussian test for quantum gravity, given the challenges associated with creating macroscopic Bose-Einstein condensates (BECs), an alternative proposal suggests the use of classical coherent initial states [21]. Investigating the feasibility of detection when using these initial states would be an significant endeavor.

These proposals have generated significant interest in relation to the Planck mass [21, 66]. Unlike the other Planck units, the Planck mass lacks a well-defined physical interpretation. Christodoulou and Rovelli suggest that these experiments shed light on the significance of the Planck mass scale: the scale at which the quantum superposition principle of curved spacetime becomes observable [27]. It would be intriguing to explore the recurring presence of this unit in the context of quantum gravity witnesses.

Lastly, with more time, it would have been enlightening to demonstrate where the common semi-classical descriptions of gravity break down in each of the proposals.

Appendix A

Entanglement Generation in a Bipartite State

Here, using an example of a system comprised of two massive states, we show explicitly that LOCC cannot produce entanglement [67].

We can write an initial system composed of two separable matter states as

$$\psi_{initial} = |0\rangle_A \otimes |0\rangle_B \,, \tag{A.1}$$

where $|0\rangle_A$ and $|0\rangle_b$ are two spacial separated ground states of a harmonic oscillator. By introducing an interaction potential $\hat{H}_{AB}$ of a field that mediates the two matter systems we can show the perturbed state is

$$\psi_{final} = \frac{1}{\mathcal{N}} \sum_{n,N} C_{nN} |n\rangle_A \otimes |N\rangle_B \,, \tag{A.2}$$

where $\mathcal{N}$ is the normalisation factor. Here $C_{00} = 1$ and

$$C_{nN} = \frac{\langle n|_A \otimes \langle N|_B \hat{H}_{AB} |0\rangle_A \otimes |0\rangle_B}{2E_0 - E_n - E_N}, \tag{A.3}$$

are the other state coefficients. Equation (A.2) can then be separated into the form

$$\psi_{final} = \underbrace{(|0\rangle + \sum_{n>0} C_{n0} |n\rangle)(|0\rangle + \sum_{N>0} C_{0N} |N\rangle)}_{\text{separable state}} + \underbrace{\sum_{n,N>0} (C_{nN} - C_{n0}C_{0N}) |n\rangle |N\rangle}_{\text{entangled state}}, \tag{A.4}$$

with all of the entanglement encoded into the second term of (A.4). It is easy to see that for a classical mediating field, where $\hat{H}_{AB} |0\rangle_A \otimes |0\rangle_B = \lambda |0\rangle_A \otimes |0\rangle_B$ for some complex number λ, only $c_{00} \neq 0$ due to the orthogonality between the ground $(|0\rangle_A \otimes |0\rangle_B)$ and the excited states $(|n\rangle_A \otimes |N\rangle_B)$. Hence, the entanglement term in (A.4) is equal to zero. Likewise, the same can be said for an interaction operator acting on only one of the two states. For example; $\hat{H}_{AB} |0\rangle_A \otimes |0\rangle_B = k |n\rangle_A \otimes |0\rangle_B$ where only C_{00} and $C_{n0} \neq 0$. This is the type of system used in the BMV experiment, where two separable states are used.

Appendix B

Action for Linearized Gravity

Here we derive the on-shell action for the linearized massive gravity of N point-like masses in de Donder gauge. We will be working in the with the $(-,+,+,+)$ metric signature.

B.1 Deriving the Simplified Action for Linearized Gravity

Starting from the Lorentz invariant Fierz-Pauli action:

$$S_{FP} = \frac{c^4}{8\pi G}\int d^4x\Big(-\frac{1}{4}\partial_\rho h_{\mu\nu}\partial^\rho h^{\mu\nu} + \frac{1}{2}\partial_\rho h_{\mu\nu}\partial^\nu h^{\mu\rho} + \frac{1}{4}\partial_\mu h\partial^\mu h - \frac{1}{2}\partial_\nu h^{\mu\nu}\partial^\mu h\Big), \quad \text{(B.1)}$$

where $|h_{\mu\nu}| \ll 1$ is the perturbation around the flat Minkowski metric $\eta_{\mu\nu}$. To couple to matter we add a $h^{\mu\nu}T_{\mu\nu}$ term to this action such that

$$S_{\mathcal{G}} = \frac{c^4}{16\pi G}\int d^4x\Big(-\frac{1}{2}\partial_\rho h_{\mu\nu}\partial^\rho h^{\mu\nu} + \partial_\rho h_{\mu\nu}\partial^\nu h^{\mu\rho} + \frac{1}{2}\partial_\mu h\partial^\mu h - \partial_\nu h^{\mu\nu}\partial^\mu h\Big) + \frac{1}{2}\int d^4x h^{\mu\nu}T_{\mu\nu}, \quad \text{(B.2)}$$

here $T_{\mu\nu}$ is the energy-momentum tensor for the matter. This action is invariant under gauge transformations, where under any **infinitesimal** change of spacetime coordinates $x^\mu \to x^\mu - \xi^\mu(x)$, the metric changes by

$$h_{\mu\nu} \to \nabla_\mu\xi_\nu + \nabla_\nu\xi_\mu = \partial_\mu\xi_\nu + \partial_\nu\xi_\mu. \quad \text{(B.3)}$$

Choosing the de Donder gauge (equivalent to the Lorenz gauge in electromagnetism) defined by picking the coordinate change

$$\Box\xi_\mu = \partial^\nu(h_{\mu\nu} - \frac{1}{2}h\eta_{\mu\nu}) = \partial^\nu\bar{h}_{\mu\nu} = 0, \quad \text{(B.4)}$$

the action simplifies to

$$S_{\mathcal{G}} = \frac{c^4}{64\pi G}\int d^4x\Big(-\partial_\rho h_{\mu\nu}\partial^\rho h^{\mu\nu} + \frac{1}{2}\partial_\mu h\partial^\mu h\Big) + \frac{1}{2}\int d^4x h^{\mu\nu}T_{\mu\nu}. \tag{B.5}$$

By varying the action with respect to $h_{\mu\nu}$ and integrating by parts, ignoring the boundary terms which are taken to vanish at infinity for this on-shell action, we invoke the principle of least action to get the wave equations:

$$\Box h_{\mu\nu} = -\frac{16\pi G}{c^4}\bar{T}_{\mu\nu}. \tag{B.6}$$

Substituting this back into Eq.(B.5), we get a concise form of the action

$$S_{\mathcal{G}} = \frac{1}{4}\int d^4x\, h_{\mu\nu}T^{\mu\nu}. \tag{B.7}$$

B.2 The Linearized Gravitational Action for N Point Like Masses

To solve the wave equation we implement the use of a Green function $G(x^\rho - y^\rho)$, which is the solution to the wave equation sourced by a delta function:

$$\Box_x\, G(x^\rho - y^\rho) = \delta^{(4)}(x^\rho - y^\rho). \tag{B.8}$$

By inserting a delta function into Eq.(B.6) and replacing with the Green function we find

$$h_{\mu\nu}(x^\rho) = -\frac{16\pi G}{c^4}\int d^4y\, G(x^\rho - y^\rho)\bar{T}_{\mu\nu}(y^\rho). \tag{B.9}$$

As we are interested in the gravitational disturbance due to the influences of energy and momentum sources in the past, the Green function in question is the retarded (causal) Green function

$$G(x^\rho - y^\rho) = \frac{\delta\big(|\mathbf{x}-\mathbf{y}| - (x^0 - y^0)\big)}{4\pi|\mathbf{x}-\mathbf{y}|}\theta(x^0 - y^0), \tag{B.10}$$

where $\theta(x^0 - y^0)$ is the Heaviside step function [38]. Plugging into Eq.(B.9) and integrating over y^0 we arrive at the wave equation:

$$h_{\mu\nu}(t,\mathbf{x}) = \frac{4G}{c^4}\sum_{a=1}^{N}\int d^3y\,\delta(\mathbf{y}-\mathbf{x}_a(t_r))\frac{\bar{T}_{a\mu\nu}(t_r)}{|\mathbf{x}-\mathbf{y}|}, \tag{B.11}$$

here $t_r = t - |\mathbf{y}-\mathbf{x}|/c$ is the retarded time. We have also substituted in our expression for the energy-momentum tensor for N point like masses Eq.(3.13). By employing integration techniques using dummy variables and utilizing the identity [36]

$$\delta(f(x)) = \frac{\delta(x - x')}{|\delta_x f(x')|}, \tag{B.12}$$

we can integrating out the Dirac delta functions [36]. We arrive at

$$h_{\mu\nu}(t, \mathbf{x}) = \frac{4G}{c^4} \sum_{a=1}^{N} \frac{\bar{T}_{a\mu\nu}(t_r)}{|\mathbf{d}_a| - \mathbf{d}_a \cdot \mathbf{v}_a(t_r)/c}, \tag{B.13}$$

with $\mathbf{v}_a = d\mathbf{x_a}/dt$ and $\mathbf{d}_a = \mathbf{x} - \mathbf{x}_a(t_r)$. Substituting Eq.(B.13) and Eq.(3.13) into Eq.(B.7) and performing the spacial integration we get our final solution for the action

$$S_{\mathcal{G}}^{\sigma} = \frac{G}{c^4} \sum_{a,b}^{a \neq b} \int dt \, \frac{\bar{T}_a^{\mu\nu}(t_{ab}) T_{b\mu\nu}(t)}{|\mathbf{d}_{ab}(t)| - \mathbf{d}_{ab}(t) \cdot v_a(t_{ab})/c}, \tag{B.14}$$

where the retarded displacements and times are implicitly defined [36]:

$$\mathbf{d}_{ab}(t) = \mathbf{x}_b(t) - \mathbf{x}_a(t_{ab}), \tag{B.15}$$

$$t_{ab} = t - \frac{|\mathbf{d}_{ab}(t)|}{c}. \tag{B.16}$$

Appendix C

Showing the NSIT Condition Does Not Hold for Quantum Systems

Defining a projective measurement operator

$$\hat{P}_s = \frac{1}{2}(\mathbb{1} + s\hat{Q}), \tag{C.1}$$

such that $\hat{P}_s = \hat{P}_s^\dagger = \hat{P}_s^2$ and $\sum_s \hat{P}_s = \mathbb{1}$. Starting with an initial system ρ_0 the probability of measuring s at a time t_1 is

$$p(s_2) = Tr[\hat{P}_s(t_2)\rho_0], \tag{C.2}$$

where we are in the Heisenberg picture, therefore

$$\hat{P}_s(t) = \hat{U}^\dagger(t)\hat{P}_s(0)\hat{U}(t) = e^{-i\hat{H}t}\hat{P}_s(0)e^{i\hat{H}t}. \tag{C.3}$$

Similarly the probability of two sequential measurements at times t_1 and $t_2(\geq t_1)$ is

$$\begin{aligned} p(s_1, s_2) &= Tr[\hat{P}_{s_2}(t_2)\hat{P}_{s_1}(t_1)\rho_0\hat{P}_{s_1}(t_1)^\dagger\hat{P}_{s_2}(t_2)^\dagger] \\ &= Tr[\hat{P}_{s_2}(t_2)\hat{P}_{s_1}(t_1)\rho_0\hat{P}_{s_1}(t_1)]. \end{aligned} \tag{C.4}$$

It then easy to see that

$$\sum_{s_1} p(s_1, s_2) \neq p(s_2) \tag{C.5}$$

unless

$$\sum_{s_1} \hat{P}_{s_1}(t_1)\rho_0\hat{P}_{s_1}(t_1) = \rho_0. \tag{C.6}$$

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