A thesis submitted for the degree of MSc Quantum Fields and Fundamental Forces

B-anomalies beyond the Pati-Salam Model

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1 Introduction

The Standard Model (SM) now has the final missing component due in large part to LHC's discovery of a 125 GeV Higgs boson. However, we have no direct hints of any new physics so far. There are many different models that have been tested already at the LHC which puts bounds on the mass scale beyond the TeV scale in some cases even closer to the 10 TeV scale. In this situation, it is almost clear that we might face a mass gap between the Standard model and the new physics that we would like to understand. To cross this gap, we need to construct the bridge from the Standard model (low-energy physics) footprints and effective tools used to probe the way to arrive the UV-complete theory. If new physics has a heavy mass gap where effective field theories are playing important roles, we can actually replace the Standard model with the analogue with the Fermi theory for the Standard model which is the SM effective theory (SMEFT). However, there are too many possible interactions that we can have. Even in the presence of hints in the new physics, with the number of new possible couplings, it seems to be a very hard situation.

The SM lagrangian contains two unnatural features pointing towards new physics associated with the Higgs potential and the Yukawa sector. The first one is the Higgs hierarchy problem essentially telling us that there needs to be something that stabilizes the mass which typically should be around the TeV scale. The other one is a standard model flavour puzzle where the Yukawa couplings have a very hierarchical structure for each family of SM fermions. In the absence of the Yukawa interactions and the Higgs interactions to the SM lepton fields, the kinetic terms of the leptons are invariant under the global $U(3)_{L_L} \times U(3)_{E_R}$ symmetry telling us that we cannot distinguish among all three families of the lepton. We called this property as **the Lepton Flavour Universality (LFU)**. Although there are many observations verifying the existence of the LFU, with an increasing set of experimental anomalies in semileptonic B-meson decay, they show us the hint of **Lepton Flavour Universality Violation (LFUV)**. There are two types of B-anomalies: 1. the neutral current transition in the SM $b \to sll$ and 2. the charged current counterpart $b \to cl\nu$. As studied in [26],[27],[28],[29] the recent experiments show the deviation from the SM at the average discrepancies at the level of 4σ.

The B-anomalies can be explained by a single U_1 leptoquark mediator which couple the SM quark and lepton together constituting the source for the violation of lepton universality. With the effective field theory low-energy fits of the simplified model of leptoquark mediator [3],[4],[5] it suggests that U_1 leptoquark is the best single mediator to explain the LFUV. The gauge structure of this leptoquark implies that it would be embedded in a UV-complete model. The quark-lepton unification model of Pati and Salam [17] views the lepton as the fourth colour of the quark based on $SU(4) \times SU(2)_L \times SU(2)_R$ gauge symmetry. The model predicts the exotic particle, a gauge vector leptoquark in the same representation as in the simplified model. However, the Pati-Salam model predicts the bound on leptoquark mass more than 200 TeV ballpark resulting from the semi-leptonic meson decay $K_L \to \mu e$ [15],[18] telling us that the energy scale of the new physics is beyond our interest since we would like to stabilize the Higgs mass at around TeV scale.

The search for a renormalizable model with a TeV-scale leptoquark has led ta way to consider the 4321 model [1],[2],[7] based on the extension of the Pati-Salam gauge group to the $G_{4321} = SU(4) \times SU(3)' \times SU(2)_L \times U(1)'$. For the fermion sector, we consider a flavour universal model in which the would-be SM fermions are singlets under $SU(4)$ structure. The model predicts the right quantum numbers of U_1 leptoquark and also predicts the flavour non-universal structure given the CKM procedure.

In this work, we organise the paper as follows. In section 2 we present the basic introduction to flavour physics view through symmetry glasses. We will see a large global symmetry in various parts of the SM. In section 3, we introduce the idea of Lepton flavour Universality, summarize the current bounds of B-anomalies experiments and present a way to construct theory through U_1 leptoquark. In section 4 we will see the anatomy of the Pati-Salam model and its two dangerous problems. Finally, in section 5 we present the current model for this field of study the 4321 model and digest its anatomy to see the consistent leptoquark gauge boson.

2 The Standard Model and its Flavour

The Standard Model (SM) of elementary particles is a renormalizable theory based on the gauge group and the particle field content which can be defined as follows:
(i) It is completely specified by the gauge (local) symmetry

$$\mathcal{G}_{\mathrm{SM}} = SU(3)_c \times SU(2)_L \times U(1)_Y$$

(ii) The fermion content for each generation are arranged in the five representations of $\mathcal{G}_{\mathrm{SM}}$:

$$Q_{Li}(3,2)_{+1/6}, \quad U_{Ri}(3,1)_{+2/3}, \quad D_{Ri}(3,1)_{-1/3}, \quad L_{Li}(1,2)_{-1/2}, \quad E_{Ri}(1,1)_{-1}$$

Each of the fermion field comes in three generations or **"flavours"** (i = 1,2,3). Where

$$Q_{Li} = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix} \quad \text{for left-handed quarks}$$

$$\text{and} \quad U_{Ri} = u_R, c_R, t_R, \quad D_{Ri} = d_R, s_R, b_R \quad \text{for right-handed quarks}$$

$$L_{Li} = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix} \quad \text{for left-handed leptons}$$

$$\text{and} \quad E_{Ri} = e_R, \mu_R, \tau_R \quad \text{for right-handed leptons}$$

A right-handed neutrino is something we do not include in the SM since it would be completely "neutral" under the gauge symmetry. The Standard Model lagrangian can be divided into three main parts

$$\mathcal{L}_{\mathrm{SM}} = \mathcal{L}_{\mathrm{kin}} + \mathcal{L}_{\mathrm{Yukawa}} + \mathcal{L}_{\mathrm{Higgs}} \tag{2.1}$$

Where the Higgs potential is spontaneously broken into $\mathcal{G}_{\mathrm{SM}} \to SU(3) \times U(1)_{em}$ by the vacuum expectation value of a single Higgs doublet, $H(1,2)_{1/2}$ $\left(\langle H^0 \rangle = v/\sqrt{2}\right)$. However, the Higgs field is not based on the symmetry principle like the mediators of the SM forces since all the interactions associated with Higgs; we have to assign the couplings by hand (i.e. Higgs Mechanism cannot predict particle masses).

In order to have the kinetic terms which are gauge invariance, one has to introduce the covariant derivatives:

$$\begin{aligned}
D_{Q,\mu} &= \partial_\mu + ig_s T^a G^a_\mu + ig\tau^a W^a_\mu + ig' Y(Q) B_\mu \\
D_{U,\mu} &= \partial_\mu + ig_s T^a G^a_\mu + ig' Y(U) B_\mu \\
D_{D,\mu} &= \partial_\mu + ig_s T^a G^a_\mu + ig' Y(D) B_\mu \\
D_{L,\mu} &= \partial_\mu + ig\tau^a W^a_\mu + ig' Y(L) B_\mu \\
D_{E,\mu} &= \partial_\mu + ig\tau^a W^a_\mu + ig' Y(E) B_\mu
\end{aligned} \tag{2.2}$$

Where G^a_μ are the eight gluon fields, W^a_μ are weak gauge bosons and B_μ is the hypercharge boson. T^a (a = 1,2,...,8) and τ^a (a=1,2,3) are the generators of $SU(3)_c$ and $SU(2)_L$ respectively. All gauge couplings are the same for each generation (gauge couplings are universal).

2.1 Global Symmetries

However, for the kinetic part of the fermion lagrangian, there are a large global symmetry satisfied by $\mathcal{L}^{\mathrm{f}}_{\text{kinetic}}$,

$$\mathcal{L}^{\mathrm{f}}_{\text{kinetic}} = i\overline{Q}_{Li}\not{D}Q_{Li} + i\overline{U}_{Ri}\not{D}U_{Ri} + i\overline{D}_{Ri}\not{D}D_{Ri} + i\overline{L}_{Li}\not{D}L_{Li} + i\overline{E}_{Ri}\not{D}E_{Ri} \tag{2.3}$$

In this part, it is evident that the interaction lagrangian is flavour universal and CP conserving. One can transform each term above by a unitary transformation.

$$Q_L \to V_Q Q_L, \quad U_R \to V_U U_R, \quad D_R \to V_D D_R, \quad L_L \to V_L L_L, \quad E_R \to V_E E_R \tag{2.4}$$

where V_Q, V_U, V_D, V_L and V_E are $U(3)$ independent rotations in flavour space. Thus, the lagrangian is invariant under

$$U(3)^5 = U(3)_{Q_L} \times U(3)_{U_R} \times U(3)_{D_R} \times U(3)_{L_L} \times U(3)_{E_R} \tag{2.5}$$

This can be decomposed as

$$\mathcal{G}_{\text{flavor}} = U(1)^5 \times SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R} \times SU(3)_{L_L} \times SU(3)_{E_R} \tag{2.6}$$

$$U(1)^5 = U(1)_B \times U(1)_L \times U(1)_Y \times U(1)_{\mathrm{PQ}} \times U(1)_E \tag{2.7}$$

The groups apart from $U(1)^5$ correspond to non-trivial flavour mixing. However, when we consider the Yukawa terms in SM,

$$-\mathcal{L}_{\text{Yukawa}} = Y^d_{ij}\overline{Q}_{Li}\phi D_{Rj} + Y^u_{ij}\overline{Q}_{Li}\tilde{\phi}U_{Rj} + Y^e_{ij}\overline{L}_{Li}\phi E_{Rj} + \text{ h.c.} \tag{2.8}$$

where the dual field $\tilde{\phi}$ is given as $\tilde{\phi} = i\tau_2\phi^\dagger$ and Yukawa matrices Y^d, Y^u and Y^e are complex 3×3 matrices. Since each Yukawa matrix is flavour dependent ($Y^f \not\propto 1$),this will be source of flavour and CP violating. If we turn on this Yukawa interactions, the global flavour symmetry $\mathcal{G}_{\text{flavor}}$ is broken because we know that masses of the fermions are different from the observations. Thus, the remained symmetry is

$$\mathcal{G}_{\text{flavour}} \to U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau \times U(1)_Y \tag{2.9}$$

where $U(1)_B$, $U(1)_e$, $U(1)_\mu$, $U(1)_e$ and $U(1)_\tau$ are the baryon number, electron number, muon number and tau number respectively. These are accidental symmetries (i.e. not required when we construct SM). They depend on how we set the model from the choice of gauge symmetry, field content and renormalisability. Within the SM, flavour physics is completely controlled by the Yukawa couplings that are determined by the fermion masses. Moreover, the unbroken global symmetry gives us strong consequences upon the Standard Model listed below:

1. Proton decay ($p \to e^+\pi$) is forbidden since proton is the lightest particle that has a baryon number.

2. There are no flavour changing neutral current in the charged leptons. Lepton flavour violation is forbidden by $U(1)_e \times U(1)_\mu \times U(1)_\tau$ (e.g. $\mu \to e\gamma$ is forbidden.)

2.2 CKM Structure

In order to diagonalise the mass matrices derived from Yukawa interactions, Yukawa couplings requires two independent unitary matrices (bi-unitary transformation). We can write the Yukawa terms (2.8) as

$$\overline{Q}_L \left(V_D \lambda^d U_D^\dagger\right) D_R \phi + \overline{Q}_L \left(V_U \lambda^u U_U^\dagger\right) U_R \tilde{\phi} + \overline{L}_L \left(V_E \lambda^e U_E^\dagger\right) E_R \phi + \text{ h.c.} \tag{2.10}$$

where $\lambda^d = \mathrm{diag}(y_d, y_s, y_b)$, $\lambda^u = \mathrm{diag}(y_u, y_c, y_t)$ and $\lambda^e = \mathrm{diag}(y_e, y_\mu, y_\tau)$
Before electroweak symmetry breaking, we can use the global symmetry to rotate the basis without changing the fermion kinetic part given as

$$Q_L \to V_D Q_L, \quad U_R \to U_U U_R, \quad D_R \to U_D D_R, \quad L_L \to V_E L_L, \quad E_R \to U_E E_R \tag{2.11}$$

We have

$$\overline{Q}_L \lambda^d D_R \phi + \overline{Q}_L \left(V^\dagger \lambda^u\right) U_R \tilde{\phi} + \overline{L}_L \lambda^e E_R \phi + \text{ h.c.} \tag{2.12}$$

where $V^\dagger = V_D^\dagger V_U$ is physical and unitary called the CKM matrix. So, we pick a basis where down quark's mass matrix is diagonal ($Y^d = \lambda^d$), up quark's mass matrix is a product of CKM matrix and diagonal matrix ($Y^u = V^\dagger \lambda^u$) and charged lepton's mass matrix is diagonal ($Y^e = \lambda^e$).

In this basis, after the Higgs gets VEV, we have

$$\bar{d}_L^i M_D^{ik} d_R^k + \bar{u}_L^i M_U^{ik} u_R^k + \bar{e}_L^i M_E^{ik} e_R^k + \text{ h.c.} \tag{2.13}$$

where $M_D = v\lambda^d/\sqrt{2}$, $M_U = V^\dagger \left(v\lambda^u\right)/\sqrt{2}$ and $M_E = v\lambda^e/\sqrt{2}$. To diagonalise the M_U, we have to rotate u_L and d_L separately to non gauge invariant basis.
The mass eigenstates fields are

$$Q_L = \begin{pmatrix} V^\dagger u_L \\ d_L \end{pmatrix} \quad L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \tag{2.14}$$

This draws consequences on the SM which are
1. Charged-current weak interactions become flavour non-diagonal(flavour violating) at tree-level for the interactions among quarks with the size governed by the off-diagonal element of the CKM matrix.

$$\mathcal{L}_{c.c.} = \frac{g}{\sqrt{2}} V_{ik} \bar{u}_{Li} \gamma_\mu W^{\mu+} d_{Lk} + \frac{g}{\sqrt{2}} V_{ik}^* \bar{d}_{Lk} \gamma_\mu W^{\mu-} u_{Li} \tag{2.15}$$

2. Higgs interactions with fermion are flavour-diagonal but the coupling strength are proportional to its mass which is different for each generation.(i.e.flavour diagonal but non-universal)

$$\mathcal{L}_{\text{Higgs}} = m_{u_i} \bar{u}_L^i u_R^i \left(1 + \frac{h}{v}\right) + m_{d_i} \bar{d}_L^i d_R^i \left(1 + \frac{h}{v}\right) + m_{e_i} \bar{e}_L^i e_R^i \left(1 + \frac{h}{v}\right) \tag{2.16}$$

3. Interactions of the photon, Z-boson and gluon are flavour universal and flavour diagonal. So, there are no Flavour Changing Neutral Currents (FCNCs) at tree-level. However, it is possible to have FCNCs process at loop-level with internal $W^{\pm}$ bosons.

$$\mathcal{L}_{\text{N.C.}} = g_Z\left(d_L\right)\bar{d}^m_{Li}\gamma_\mu Z^\mu \delta_{ik} d^m_{Lk} + g_Z\left(u_L\right)\bar{u}^m_{Li}\gamma_\mu Z^\mu \delta_{ik} u^m_{Lk} + g_Z\left(e_L\right)\bar{e}^m_{Li}\gamma_\mu Z^\mu \delta_{ik} e^m_{Lk} \tag{2.17}$$

these are true for the other neutral gauge bosons as well.

2.3 Counting Parameters

The Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix is the 3×3 unitary matrix so it consists of $3^2 = 9$ parameters (3 angles + 6 phases). However, not all parameters are physical as they can be absorbed as unobservable parameters into the up-type and down-type quarks, respectively. We can make pure phase transformation on the 6 quark flavours. There is one global phase which all quarks transform with the same phase. This phase is a symmetry transformation and corresponds to the baryon number conservation. Consequently, we are then left with three mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$ and one complex phase δ called the Kobayashi-Maskawa phase which is the only source of CP violation in the quark sector of SM.
The CKM matrix is very hierarchical and its form is not unique.

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \sim \begin{pmatrix} 1 & 0.2 & 0.004 \\ 0.2 & 1 & 0.04 \\ 0.008 & 0.04 & 1 \end{pmatrix} \tag{2.18}$$

The standard parametrization recommended by the Particle Data Group (PDG) is

$$\begin{aligned} V_{\text{CKM}} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \end{aligned} \tag{2.19}$$

where $c_{ij} \equiv \cos\theta_{ij}, s_{ij} \equiv \sin\theta_{ij}$ $(i, j = 1, 2, 3)$
As suggested by the hierarchical form given above, it is convenient to express the CKM matrix in the Wolfenstein parametrization where four mixing parameters are (λ, A, ρ, η)

$$V_{\text{CKM}} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}\left(\lambda^4\right) \tag{2.20}$$

Let us again try to explain the global structure in more explicit way. The charge lepton Yukawa lagragian is given by

$$\mathcal{L}_{\text{Yukawa}} \supset -Y^e_{ij}\bar{L}_{Li}\phi E_{Rj} + \text{ h.c. } \rightarrow -\bar{L}_L\left(V_E\lambda^e U_E^\dagger\right)E_R\phi + \text{ h.c.} \tag{2.21}$$

Y^e is 3×3 complex matrix. It consists of 9 real and 9 imaginary numbers. The two unitary matrices have 6 angles and 12 phases in total. However, when we rotate L_{Li} and E_{Ri} with the same phase, the λ^e stays the same corresponding to $U(1)_e \times U(1)_\mu \times U(1)_\tau$ global symmetry. Finally, in total we are left with $9 - 2 \times 3 = 3$ real and $9 - (2 \times 6 - 3) = 0$ parameters. The rest of physical parameters are nothing but lepton masses. In short, we can say that the $U(3)_{L_L} \times U(3)_{E_R}$ is broken to $U(1)_e \times U(1)_\mu \times U(1)_\tau$ by $Y^e \not\propto 1$.

With the same explaination in the lepton case, in quark section after we choose the basis given above (2.14), we can also say that $U(3)_{Q_L} \times U(3)_{U_R} \to U(1)_u \times U(1)_c \times U(1)_t$ when $Y^u \not\propto 1$ and $U(3)_{Q_L} \times U(3)_{D_R} \to U(1)_d \times U(1)_s \times U(1)_b$ when $Y^d \not\propto 1$. However, since $[Y_d, Y_u] \neq 0$ (we cannot rotate the same phase for both up and down quark cases), then $U(1)_u \times U(1)_c \times U(1)_t \times U(1)_d \times U(1)_s \times U(1)_b \to U(1)_B$ (baryon number).

2.4 Custodial Symmetry

By specifying the Higgs sector in the SM, there is an accidental global symmetry called "custodial symmetry". To see this more explicitly, we introduce another representation for the same Higgs field by re-writing it as a "bidoublet".

$$\Phi(x) = \begin{pmatrix} \Phi^0(x)^* & \Phi^+(x) \\ -\Phi^+(x)^* & \Phi^0(x) \end{pmatrix} \tag{2.22}$$

where first column is the conjugate doublet $\tilde{\Phi} \equiv i\sigma^2 \Phi^*$ and the second column is the original Φ. Both of them transform as a doublet under $SU(2)$.
The lagrangian for the Higgs sector takes the form

$$\mathcal{L}(\Phi, \partial_\mu \Phi) = \frac{1}{4} \operatorname{Tr}\left[\partial_\mu \Phi^\dagger \partial_\mu \Phi\right] + \frac{m^2}{4} \operatorname{Tr}\left[\Phi^\dagger \Phi\right] + \frac{\lambda}{4!} \left(\frac{1}{2} \operatorname{Tr}\left[\Phi^\dagger \Phi\right]\right)^2 \tag{2.23}$$

Which is invariant under the global $SU(2)_L \times SU(2)_R$ transformations:

$$\Phi \to \Phi(x)' = L\Phi(x)R^\dagger = \exp\left(i\theta_L^a \tau^a\right) \Phi \exp(-i\theta_R^b \tau^b), \quad L \in SU(2)_{\mathrm{L}}, \quad R \in SU(2)_{\mathrm{R}} \tag{2.24}$$

The vacuum expectation value takes the form:

$$\langle \Phi \rangle_{\mathrm{vev}} = \frac{1}{\sqrt{2}} \begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix} \propto I_{2\times 2} \tag{2.25}$$

which breaks $SU(2)_L \times SU(2)_R \to SU(2)_D$ (i.e. preserves diagonal subgroup where $\theta_L^a = \theta_R^a$). We call this $SU(2)_D$ symmetry the custodial symmetry. However, we can match these global symmetries $SU(2)_L \times SU(2)_R$ to the gauge symmetries as 1. Global $SU(2)_L$ is the gauged $SU(2)_L$, 2. The T_3 generator of global $SU(2)_R$ is the hypercharge $U(1)_Y$ generator, 3. The T_3 generator of the custodial $SU(2)_D$ is

the electric charge operator (unbroken generator), and 4. When the hypercharge generator of $SU(2)_R$ is gauged, the global symmetry is broken without promoting it to the $SU(2)$ gauge symmetry.
Let us consider the gauge boson mass matrix of electroweak sector in SM:

$$M^2 = \frac{v^2}{4} \begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & -gg' \\ 0 & 0 & -gg' & g^2 \end{pmatrix} \tag{2.26}$$

where we write in the basis (W^1, W^2, W^3, B)

In the limit where $g' \to 0$, the massive vectors are degenerate and $\rho_0 \equiv M_W^2/M_Z^2 \cos^2\theta_W = 1$ because of the $SU(2)_D$ custodial symmetry in rotations among (W^1, W^2, W^3).

3 Lepton Flavour Universality

Leptons appear in two parts of the Standard Model lageangian, the gauge sector and the Yukawa sector. For the gauge sector, we have

$$\mathcal{L}_{\rm SM} \supset i\left(\bar{L}_{Li}\gamma^{\mu}D_{\mu}L_{Li} + \bar{E}_{Ri}\gamma^{\mu}D_{\mu}E_{Ri}\right) \tag{3.1}$$

which is invariant under the global symmetry $U(3)_{L_L} \times U(3)_{E_R}$ telling us that there is no way to distinguish among electron, muon, and tau. Gauge interactions containing the covariant derivatives couple with the same strength and are proportional to $\propto g\delta_{ij}$ for all the leptons which are to say that gauge interactions are **Lepton Flavour Universal** (LFU).

If we switch on the Yukawa terms, the Yukawa sector starts to distinguish electron, muon, and tau and the universality will be broken in two ways. At the level of the lepton masses, the lepton masses are different ($m_e \neq m_\mu \neq m_\tau$) or by differentiating them by considering the Higgs interactions, the strength of the interactions are proportional to the lepton masses and thus are different. However, for all flavour observables, the Higgs interactions are irrelevant so the only effect deviating from the lepton universality is from mass terms. So, it is important to look at the observables that can test the lepton flavour universality of the gauge interactions. If we start to see some deviations from the SM, there should be another mediator that distinguishes among the leptons. In other words, three families of leptons have the same charge under the SM gauge bosons (γ, g ,W and Z) which could be the accidental low energy property. The new physics may have different behavior at high energies which we do not see by this low energy gauge boson observables.

In short, Lepton Flavour Universality (LFU) stands for identical behavior of the charged leptons in the limit where we neglect their masses which is a consequence of the accidental flavour symmetry in SM gauge sector when turning off the Yukawa couplings. LFU has been verified with extremely high accuracy in several systems: $Z \to ll \quad [\sim 0.1\%], \tau \to l\nu\nu \quad [\sim 0.1\%], K \to (\pi)l\nu \quad [\sim 0.1\%]$ and $\pi \to l\nu \quad [\sim 0.01\%]$

In the last few years, LHCb, Babar and Belle reported some deviations or anomalies from SM predictions in B-meson decays which indicate a non-universal behavior of different lepton species in semi-leptonic decay of b quark which is the third generation quark into the second generation quarks (charm and strange), $b \to c, s$. There are two main different set of measurements:

1. Flavour Changing Charged Current ($b \to c\ell\nu_\ell$)
2. Flavour Changing Neutral Current ($b \to s\ell\ell$).

3.1 Flavour Changing Charged Current

3.1.1 $b \to c\tau\nu$

the first observable is the ratio:

$$R(X) = \frac{\mathcal{B}(\bar{B} \to X\tau\bar{\nu})}{\mathcal{B}(\bar{B} \to Xl\bar{\nu})} \tag{3.2}$$

where $X = D, D^*$ and $l = \mu, e$. We take the ratio since we cannot compute numerators and denominators separately precisely because of the hadronic uncertainties of the decay processes. So, the ratio we get is very clean without QCD uncertainty. However, from this observable, the effect of SM is at the size $\frac{G_F}{\sqrt{2}}V_{bc}^* = \frac{1}{(1.7\text{TeV})^2}$. If we would like to match the experimental value, the effect of the new physics has to be huge which is difficult to build a model if we want the light degree of freedoms.

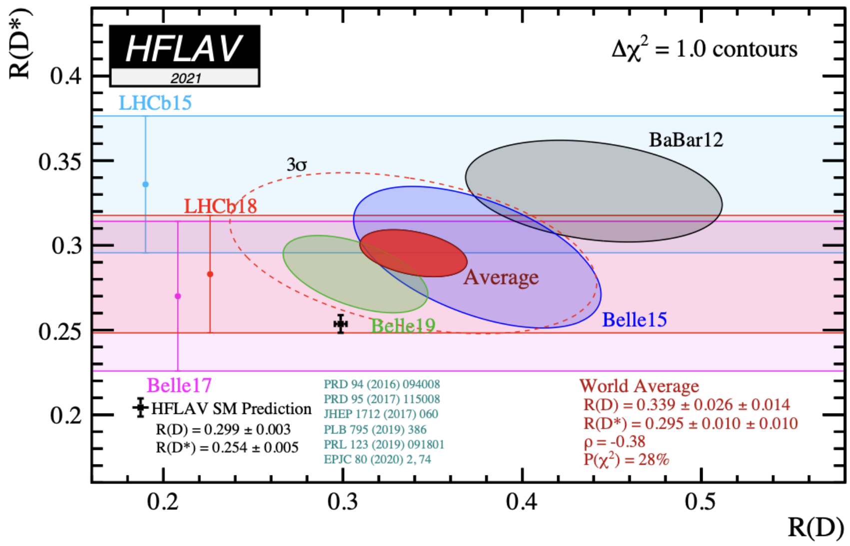

Figure 1: The measurements are reported in the plane $R(D^*)$ and $R(D)$.The black point is the SM prediction with error bars. An average of this SM predictions and the experimental average deviate from each other by about 3.4σ.(Plot taken from the [29])

3.2 Flavour Changing Neutral Current

The important vertex for this type is the interaction between neutral gauge bosons and two charged leptons ($V_{(\gamma/Z^0)l^+l^-}$) which is basically universal for all lepton species. Flavour Changing Neutral Current (FCNCs) are suppressed in the SM since they start at loop-level and are thus sensitive to new heavy particles through virtual corrections.

3.2.1 $B \to K^* \mu^+ \mu^-$ angular distributions

K^* will promply decay into kaon and pion. So, the decay is actually four-body decay problem which is described by four kinematic variables q^2 (dilepton invariant mass squared), $\theta_\ell, \theta_{K^*}$,and ϕ. We can construct the observable as

$$\frac{d^4\Gamma\left[B \to K^*(\to K\pi)\ell\ell\right]}{dq^2 d\cos\theta_\ell d\cos\theta_{K^*} d\phi} \tag{3.3}$$

We can then define P_5' when we integrate all the three angles $\theta_\ell, \theta_{K^*}$,and ϕ.

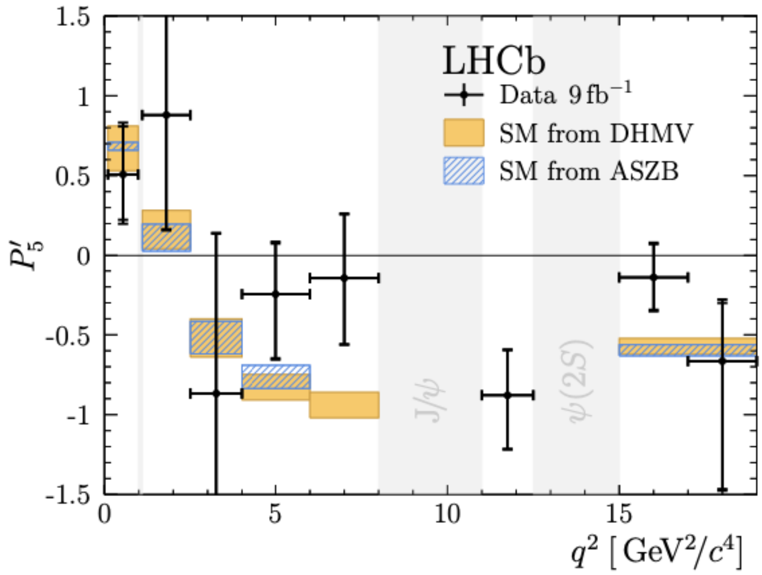

Figure 2: The discrepancies of P_5' against energy. The orange boxes and the blue boxes are the SM predictions which deviate from the black points, the experimental data. (Plot taken from the [28])

3.2.2 LFU ratios R_K and R_{K^*}

We can define an observable ratio in the dilepton mass-squared range as

$$R_K = \int_{q^2_{\min}}^{q^2_{\max}} \frac{\mathrm{d}\Gamma\left[B^+ \to K^+\mu^+\mu^-\right]}{\mathrm{d}q^2}\,\mathrm{d}q^2 \Big/ \int_{q^2_{\min}}^{q^2_{\max}} \frac{\mathrm{d}\Gamma\left[B^+ \to K^+e^+e^-\right]}{\mathrm{d}q^2}\,\mathrm{d}q^2 \tag{3.4}$$

Theoretically, the ratio is very clean because QCD uncertainty is cancelling in the ratio leaving an error that is completely dominated by QED and lepton mass effect where the masses of the muon and electron are small compared to the mass of the bottom quark. SM prediction is $R_K^{SM} = 1 + \delta_{R_K}$ with $|\delta_{R_K}| < 1\%$ coming from bremsstrahlung photon is slightly different between electron and muon which is very small effect. Instead of having $R_K = 1$, we have $R_K\left(1.1 < q^2 < 6.0\mathrm{GeV}^2/c^4\right) =$

$0.846^{+0.042}_{-0.039}(\text{stat})^{+0.013}_{-0.012}(\text{syst})$.This is the most precise measurement to date and is consistent with the SM expectation at the level of 0.10% (3.1 standard deviations).

For the R_{K^*}, the branching fractions is defined as a double ratio of $B^0 \to K^{*0}\ell^+\ell^-$

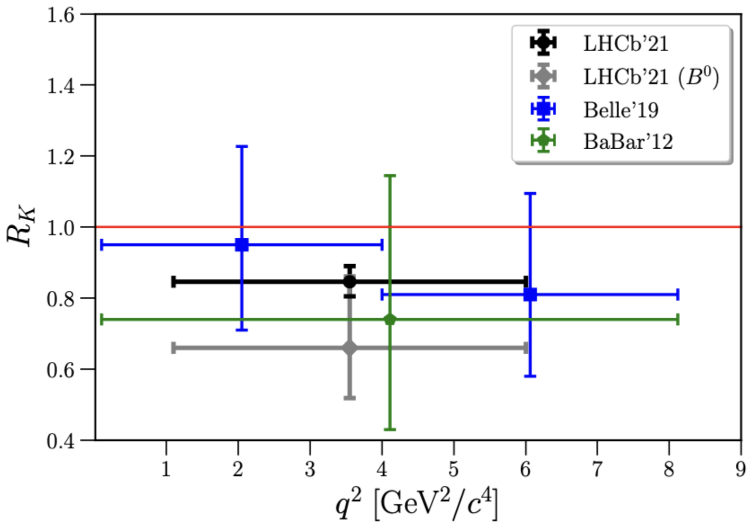

Figure 3: Measurements of R_K in the low-q^2 range (Plot taken from the [29])

and $B^0 \to K^{*0}J/\psi\,(\to \ell^+\ell^-)$:

$$\mathcal{R}_{K^{*0}} = \frac{\mathcal{B}\left(B^0 \to K^{*0}\mu^+\mu^-\right)}{\mathcal{B}\left(B^0 \to K^{*0}J/\psi\left(\to \mu^+\mu^-\right)\right)} \Bigg/ \frac{\mathcal{B}\left(B^0 \to K^{*0}e^+e^-\right)}{\mathcal{B}\left(B^0 \to K^{*0}J/\psi\left(\to e^+e^-\right)\right)} \tag{3.5}$$

The ratio is measured in two region of the dilepton invariant mass squared, [27]

$$R_{K^{*0}} = \begin{cases} 0.666^{+0.11}_{-0.07}(\text{stat}) \pm 0.03(\text{syst}) & \text{for } 0.045 < q^2 < 1.1\text{GeV}^2/c^4 \\ 0.69^{+0.11}_{-0.07}(\text{stat}) \pm 0.05\ (\text{syst}) & \text{for } 1.1 \quad < q^2 < 6.0\text{GeV}^2/c^4 \end{cases} \tag{3.6}$$

compatible with the SM predictions at the level of 2.1–2.3 and 2.4–2.5 standard deviations in the two q^2 regions, respectively.

Remarkably, all these data taken together points to a very coherent new physics effect beyond the Standard Model of the B-anomalies.

3.3 Single-Mediator Simplified Models

If we look for the mediator that can explain the B-anomalies ,in particular $R(D^{(*)})$, there are two types of mediators that we can consider leptoquarks mediator that connect the quark and lepton currents and colorless (W', B') mediators. The leptoquark mediator is interesting since it will generate the effect on $\Delta F = 2$ process and $\tau \to \mu\nu\bar{\nu}$ (lepton transition) at loop-level whereas colourless mediators are at tree-level. Also, LQs give rise to deviations from SM in the semileptonic process at tree-level and deviation in the pure quark or pure lepton process at loop-level.

There are many leptoquarks that we can consider for example scalar LQs: $S_1 = (3,1)_{-1/3}$, $R_2 = (3,2)_{7/6}$, $R_2 = (3,2)_{1/6}$, $S_3 = (3,3)_{-1/3}$ and vector LQs: $U_1 = (3,1)_{2/3}$, $U_3 = (3,3)_{2/3}$. However, from the table below taken from [3], there is only one vector leptoquark $U_1 = (3,1)_{2/3}$ that can explain both anomalies in $R_{K^{(*)}}$ and $R_{D^{(*)}}$. Furthermore, U_1 leptoquark predicts no tree-level effect in $b \to S\nu_{(\tau)}\nu_{(\tau)}$. We will focus on this U_1 leptoquark.

Model	$R_{K^{(*)}}$	$R_{D^{(*)}}$	$R_{K^{(*)}}$&$R_{D^{(*)}}$
$S_1 = (3,1)_{-1/3}$	$\times$	$\checkmark$	$\times$
$R_2 = (3,2)_{7/6}$	$\times$	$\checkmark$	$\times$
$\widetilde{R}_2 = (3,2)_{1/6}$	$\times$	$\times$	$\times$
$S_3 = (3,3)_{-1/3}$	$\checkmark$	$\times$	$\times$
$U_1 = (3,1)_{2/3}$	$\checkmark$	$\checkmark$	$\checkmark$
$U_3 = (3,3)_{2/3}$	$\checkmark$	$\times$	$\times$

(3.7)

3.3.1 The U_1 vector leptoquark

We consider the effective simplified model of vector leptoquark $U_1^\mu \sim (\mathbf{3},\mathbf{1})_{2/3}$ coupled to both left-handed and right-handed SM fields of leptoquark current proposed studied in [3],[4] and [5].

$$\mathcal{L} \supset \frac{g_U}{\sqrt{2}} U_1^\mu \left[\beta_L^{i\alpha}\left(\bar{q}_L^i \gamma_\mu \ell_L^\alpha\right) + \beta_R^{i\alpha}\left(\bar{d}_R^i \gamma_\mu e_R^\alpha\right)\right] + \text{h.c.} \tag{3.8}$$

where the couplings β_L and β_R are complex 3×3 matrices which are given by

$$\beta_L = \begin{pmatrix} 0 & 0 & \beta_L^{d\tau} \\ 0 & \beta_L^{s\mu} & \beta_L^{s\tau} \\ 0 & \beta_L^{b\mu} & 1 \end{pmatrix}, \qquad \beta_R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \beta_R^{b\tau} \end{pmatrix} \tag{3.9}$$

It provides a good description of all low-energy data as suggested in Fig.5 and Fig.4 . With its gauge structure, it seems to be the promising mediator that may point to quark-lepton unification.

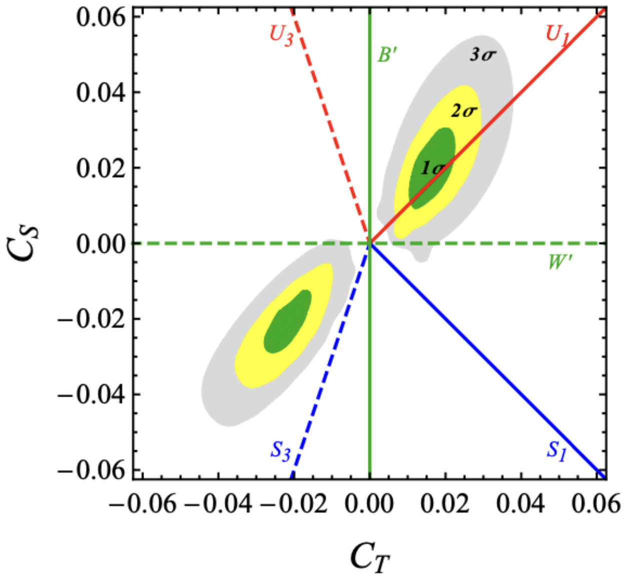

Figure 4: The correlation between triplet (C_T) and singlet (C_S) operators in single-mediator models compared to the EFT fit resulting from low-energy observables which suggests that vector LQ U_1 is the best single-mediator case.(Plot taken from the [4])

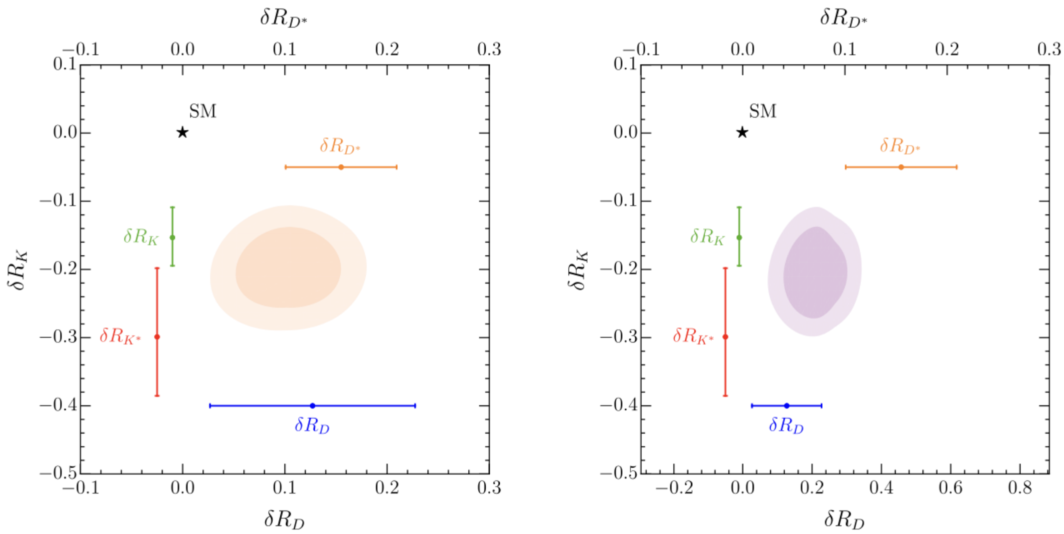

Figure 5: 1σ and 2σ regions for the ratios $\delta R_{K^{(*)}} = (R_{K^{(*)}} - R^{\rm SM}_{K^{(*)}})/R^{\rm SM}_{K^{(*)}}$ and $\delta R_{D^{(*)}} = (R_{D^{(*)}} - R^{\rm SM}_{D^{(*)}})/R^{\rm SM}_{D^{(*)}}$ for the LFU-violating observables resulting from the low-energy fit for $\beta_R^{b\tau} = 0$ (orange) and $\beta_R^{b\tau} = -1$ (purple) (Plot taken from the [5])

4 Pati-Salam Model

As suggested by the U_1 leptoquark gauge structure, it is natural to consider a gauge symmetry and the smallest UV completion model that predicts the right quantum numbers of this leptoquark is constructed from $SU(4)$ gauge group which was first proposed by Pati and Salam [17] in 1974 under $G_{\text{PS}} = SU(4) \times SU(2)_L \times SU(2)_R$ gauge theory.

In this chapter, we will analyse the properties and features of the Pati-Salam model proposed by M.P. Worah [20] as the first model or Model 1 Pati-Salam followed closely by [30]. Pati-Salam model is the model which incorporates quark-lepton unification or views the lepton as the fourth colour extended from the quark three colours. The Pati-Salam gauge group G_{PS} extends the SM by identifying the $SU(3)$ colour group as a subgroup of an $SU(4)$ gauge group and extending the electroweak sector to be left-right symmetric: $G_{\text{PS}} = SU(4) \times SU(2)_L \times SU(2)_R$.

4.1 Higgs Sector

We introduce two scalars bosons, the right-handed bosons which are in the representation (4,1,2) of G_{PS} and the left-handed bosons which are in the representation (4,2,1) of G_{PS}. We can write in the 4×2 matrix form as

$$R^{\alpha i} = \begin{pmatrix} R_{u1} & R_{d1} \\ R_{u2} & R_{d2} \\ R_{u3} & R_{d3} \\ R_\nu & R_e \end{pmatrix} \quad \text{and} \quad L^{\alpha i} = \begin{pmatrix} L_{u1} & L_{d1} \\ L_{u2} & L_{d2} \\ L_{u3} & L_{d3} \\ L_\nu & L_e \end{pmatrix} \tag{4.1}$$

where $i = 1, 2$ is the index for $SU(2)_{L(R)}$ and $\alpha = 1, 2, 3, 4$ are the index for $SU(4)$. The transpose conjugate is given by the notation $R_{\alpha i} = (R^{\alpha i})^*$ and $L_{\alpha i} = (L^{\alpha i})^*$.

The two Higgs bosons transform under G_{PS}, written as matrix multiplication, as

$$L \to U_4(L)U_{2L}^T \qquad \text{and} \qquad R \to U_4(R)U_{2R}^T \tag{4.2}$$

where $U_4 \in SU(4)$, $U_{2L} \in SU(2)_L$ and $U_{2R} \in SU(2)_R$.
The most general Higgs potential constructed from these two Higgs bosons that is invariant under the transformation above is given in [30] as:

$$\begin{aligned} V(L,R) = &- 2\mu_L^2 L_{i\alpha}L^{i\alpha} + \lambda_{L1}\left(L_{i\alpha}L^{i\alpha}\right)^2 + \lambda_{L2}L_{i\alpha}L^{j\alpha}L^{i\beta}L_{j\beta} - \\ &- 2\mu_R^2 R_{i\alpha}R^{i\alpha} + \lambda_{R1}\left(R_{i\alpha}R^{i\alpha}\right)^2 + \lambda_{R2}R_{i\alpha}R^{j\alpha}R^{i\beta}R_{j\beta} + \\ &+ \lambda_{LR1}L_{i\alpha}L^{i\alpha}R_{j\beta}R^{j\beta} + \lambda_{LR2}L_{i\alpha}R^{j\alpha}L^{i\beta}R_{j\beta} + \\ &+ \lambda_{LR3}\left(L_{i\alpha}R^{j\alpha}L^i_\beta R^\beta_j + \text{ h.c. }\right) \end{aligned} \tag{4.3}$$

The breaking pattern of the model 1 is specified by minimising the potential above and the vacuum expectation value (vev) structure of both left and right-handed higgs bosons which leads to the following symmetry breaking chain:

$$\begin{array}{c} SU(4)\times SU(2)_L\times SU(2)_R \\ \Big\downarrow{\scriptstyle \langle R\rangle} \\ SU(3)_C\times SU(2)_L\times U(1)_Y \\ \Big\downarrow{\scriptstyle \langle L\rangle} \\ SU(3)_C\times U(1)_Q \end{array} \tag{4.4}$$

where the first right-handed boson is responsible for breaking $G_{\rm PS}$ to the SM group and the second left-handed boson is responsible for breaking the SM group down to the electroweak group. With these breaking structures, we can define the charge as

$$Q=\frac{\sigma_{3L}}{2}+\frac{Y}{2}\quad\text{where}\quad\frac{Y}{2}=\left(\frac{\sigma_{3R}}{2}+\frac{B-L}{2}\right)=\left(\frac{\sigma_{3R}}{2}+\sqrt{\frac{2}{3}}\frac{\lambda_{15}}{2}\right) \tag{4.5}$$

The hypercharge Y is a linear combination between the diagonal generator of $SU(2)_R$ and the generator of B-L.

The vev of the right-handed Higgs $\langle R\rangle$ has to preserve both $SU(3)_c$ and $U(1)_Y$ or $\lambda^a\langle R\rangle=0$ and $Y\langle R\rangle=0$ where λ^a are the $SU(3)$ generators which are the subgroup generators of the $SU(4)$ (the $T^1,T^2,...,T^8$). Also, the left-handed Higgs has to satisfy $\lambda^a\langle L\rangle=0$ and $Q\langle L\rangle=0$. We have:

$$\langle L\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}0&0\\0&0\\0&0\\v_L&0\end{pmatrix}\qquad\text{and}\qquad\langle R\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix}0&0\\0&0\\0&0\\v_R&0\end{pmatrix} \tag{4.6}$$

We start with $G_{\rm PS}=SU(4)\times SU(2)_L\times SU(2)_R$ which has $21=15+3+3$ massless gauge bosons, coming from 15 $SU(4)$ generators, 2×3 from each $SU(2)_L$ and $SU(2)_R$ generator, and end up with residual $SU(3)_C\times U(1)_Q$ symmetry group which has $9=8+1$ massless gauge bosons, coming from 8 $SU(3)$ generators and single $U(1)$ generator.So, we expect to see $22-9=13$ massless goldstone bosons corresponding to 12 broken generators or 12 massive gauge bosons after the Higgs Mechanism.

Higgs masses can be derived by considering the second derivative of the Higgs potential and imposing the vevs structure:

$$\left[\frac{\partial^2V(L,R)}{\partial L^{y\nu}\partial L_{x\mu}}\right]_{VEV},\quad\left[\frac{\partial^2V(L,R)}{\partial R^{y\nu}\partial L_{x\mu}}\right]_{VEV},\quad\left[\frac{\partial^2V(L,R)}{\partial R^{y\nu}\partial R_{x\mu}}\right]_{VEV},\quad\left[\frac{\partial^2V(L,R)}{\partial L^{y\nu}\partial R_{x\mu}}\right]_{VEV} \tag{4.7}$$

and put these in the mass-matrix form in the basis $((L_u,R_u,L_\nu,R_\nu,L_d,R_d,L_e,R_e))$:

$$M^2_{LR}=\begin{pmatrix}M^2_{LR_u}&0&0&0\\0&M^2_{LR_\nu}&0&0\\0&0&M^2_{LR_d}&0\\0&0&0&0\end{pmatrix} \tag{4.8}$$

where the diagonal elements are given by 2×2 matrices:

$$M^2_{LR_u} = \frac{v_R^2}{2}\lambda_{LR2}\begin{pmatrix} -1 & \frac{v_L}{v_R} \\ \frac{v_L}{v_R} & -\frac{v_L^2}{v_R^2} \end{pmatrix} \tag{4.9}$$

$$M^2_{LR_\nu} = v_R^2\begin{pmatrix} (\lambda_{L1}+\lambda_{L2})\frac{v_L^2}{v_R^2} & \frac{(\lambda_{LR1}+\lambda_{LR2})}{2}\frac{v_L}{v_R} \\ \frac{(\lambda_{LR1}+\lambda_{LR2})}{2}\frac{v_L}{v_R} & (\lambda_{R1}+\lambda_{R2}) \end{pmatrix} \tag{4.10}$$

$$M^2_{LR_d} = v_R^2\begin{pmatrix} -\left(\lambda_{L2}\frac{v_L^2}{v_R^2}+\frac{\lambda_{LR2}}{2}\right) & \lambda_{LR3}\frac{v_L}{v_R} \\ \lambda_{LR3}\frac{v_L}{v_R} & -\left(\frac{\lambda_{LR2}\frac{v_L^2}{2}+\lambda_R}{v_R^2}\right) \end{pmatrix} \tag{4.11}$$

After diagonalising them and putting them in the physical bases, we can count the goldstone bosons by looking at the massless scalar spectrum. Since L_e and R_e do not exist in any mass term and they are complex scalar fields corresponding to two real scalar fields, in this case, we have four goldstone bosons. For the L_u and R_u, we can rotate them with an orthogonal matrix:

$$\begin{pmatrix} L_u^a & R_u^a \end{pmatrix}^* \begin{pmatrix} -A^2 & AB \\ AB & -B^2 \end{pmatrix}\begin{pmatrix} L_u^a \\ R_u^a \end{pmatrix} = \begin{pmatrix} H_{1u}^a & H_{2u}^a \end{pmatrix}^* \begin{pmatrix} 0 & 0 \\ 0 & C^2 \end{pmatrix}\begin{pmatrix} H_{1u}^a \\ H_{2u}^a \end{pmatrix} \tag{4.12}$$

where $a = 1, 2, 3$ the colour number. In this case, we have $2 \times 3 = 6$ goldstone bosons. The last two goldstone bosons are the combination of L_ν and R_ν. So, in total, we have 12 goldstone bosons consistent with our breaking structure from G_{PS} to $SU(3)_c \times U(1)_Q$.

4.2 Gauge Sector

As usual, the gauge bosons have to be in the adjoint representations of $SU(2)_{L(R)}$ and $SU(4)$ defined as follow:
For $SU(2)_L$ and $SU(2)_R$,

$$W^\mu_{L,R} = \sum_{a=1}^{3}\frac{\sigma^a_{L,R}}{2}W^{\mu a}_{L,R} = \frac{1}{2}\begin{pmatrix} W^{\mu 3} & W^{\mu 1}_{L,R} - iW^{\mu 2}_{L,R} \\ W^{\mu 1}_{L,R} + iW^{\mu 2}_{L,R} & -W^{\mu 3} \end{pmatrix}_{L,R} \tag{4.13}$$

We can define the $W_L(R)$ gauge bosons as in the SM:

$$W^{\mu\pm}_{L(R)} = \frac{W^{\mu 1}_{L(R)} \mp iW^{\mu 2}_{L(R)}}{\sqrt{2}} \quad \text{and} \quad W^{\mu 0}_{L(R)} = W^{\mu 3}_{L(R)} \tag{4.14}$$

or explicitly

$$W_{L\mu} \equiv \frac{1}{2}\begin{pmatrix} W^0_{L\mu} & \sqrt{2}W^+_{L\mu} \\ \sqrt{2}W^-_{L\mu} & -W^0_{L\mu} \end{pmatrix}, \quad W_{R\mu} \equiv \frac{1}{2}\begin{pmatrix} W^0_{R\mu} & \sqrt{2}W^+_{R\mu} \\ \sqrt{2}W^-_{R\mu} & -W^0_{R\mu} \end{pmatrix} \tag{4.15}$$

For $SU(4)$, we have

$$G^\mu = \sum_{a=1}^{15} \frac{\lambda^a}{2} G^{\mu a} = \\ = \frac{1}{2} \begin{pmatrix} \frac{B^\mu}{\sqrt{6}} + \frac{G^{\mu 8}}{\sqrt{3}} + G^{\mu 3} & G^{\mu 1} - iG^{\mu 2} & G^{\mu 4} - iG^{\mu 5} & G^{\mu 9} - iG^{\mu 10} \\ G^{\mu 1} + iG^{\mu 2} & \frac{B^\mu}{\sqrt{6}} + \frac{G^{\mu 8}}{\sqrt{3}} - G^{\mu 3} & G^{\mu 6} - iG^{\mu 7} & G^{\mu 11} - iG^{\mu 12} \\ G^{\mu 4} + iG^{\mu 5} & G^{\mu 6} + iG^{\mu 7} & \frac{B^\mu}{\sqrt{6}} - \frac{2G^{\mu 8}}{\sqrt{3}} & G^{\mu 13} - iG^{\mu 14} \\ G^{\mu 9} + iG^{\mu 10} & G^{\mu 11} + iG^{\mu 12} & G^{\mu 13} + iG^{\mu 14} & -\sqrt{\frac{3}{2}} B^\mu \end{pmatrix} \tag{4.16}$$

where $B^\mu = G^{\mu 15}$ is the $B-L$ gauge field that couples to the hypercharge Y and $G^{\mu a}$ with $a = 1, \ldots 8$ are the gluons for $SU(3)_C$. Again, we can put them in the forms:

$$G_{12}^{\mu\pm} = \frac{G^{\mu 1} \mp iG^{\mu 2}}{\sqrt{2}}, \quad G_{13}^{\mu\pm} = \frac{G^{\mu 4} \mp iG^{\mu 5}}{\sqrt{2}} \quad \text{and} \quad G_{23}^{\mu\pm} = \frac{G^{\mu 6} \mp iG^{\mu 7}}{\sqrt{2}} \tag{4.17}$$

$$X_1^{\mu\pm} = \frac{G^{\mu 9} \mp iG^{\mu 10}}{\sqrt{2}}, \quad X_2^{\mu\pm} = \frac{G^{\mu 11} \mp iG^{\mu 12}}{\sqrt{2}} \quad \text{and} \quad X_3^{\mu\pm} = \frac{G^{\mu 13} \mp iG^{\mu 14}}{\sqrt{2}} \tag{4.18}$$

$$G_\mu \equiv \frac{1}{2} \begin{pmatrix} G_{3\mu} + \frac{G_{8\mu}}{\sqrt{3}} + \frac{B_\mu}{\sqrt{6}} & \sqrt{2} G_{12\mu}^+ & \sqrt{2} G_{13\mu}^+ & \sqrt{2} X_{1\mu}^+ \\ \sqrt{2} G_{12\mu}^- & -G_{3\mu} + \frac{G_{8\mu}}{\sqrt{3}} + \frac{B_\mu}{\sqrt{6}} & \sqrt{2} G_{23\mu}^+ & \sqrt{2} X_{2\mu}^+ \\ \sqrt{2} G_{13\mu}^- & \sqrt{2} G_{23\mu}^- & -\frac{2G_{8\mu}}{\sqrt{3}} + \frac{B_\mu}{\sqrt{6}} & \sqrt{2} X_{3\mu}^+ \\ \sqrt{2} X_{1\mu}^- & \sqrt{2} X_{2\mu}^- & \sqrt{2} X_{3\mu}^- & -\frac{3B_\mu}{\sqrt{6}} \end{pmatrix} \tag{4.19}$$

where $X_1^{\mu\pm}$, $X_2^{\mu\pm}$ and $X_3^{\mu\pm}$ are in $(3, 1, 2/3)$ representation of the SM group which are consistent with the U_1 leptoquaks that we want.The mass terms of gauge bosons can be derived by considering the kinetic term of the two Higgs fields where the covariant derivative is defined as $D^\mu \equiv \partial^\mu + ig_L W_L^\mu + ig_R W_R^\mu + ig_4 G^\mu$.

$$\begin{aligned} \mathcal{L} &= D^\mu L^{i\alpha} D_\mu L_{i\alpha} + D^\mu R^{i\alpha} D_\mu R_{i\alpha} \\ &\subset \frac{g_L^2 v_L^2}{8} \left[W_{\mu L}^0 W_L^{\mu 0} + 2 W_{\mu L}^+ W_L^{\mu -} \right] - \frac{3 g_L g_4 v_L^2}{4\sqrt{6}} B^\mu W_{\mu L}^0 \\ &+ \frac{g_R^2 v_R^2}{8} \left[W_{\mu R}^0 W_R^{\mu 0} + 2 W_{\mu R}^+ W_R^{\mu -} \right] - \frac{3 g_R g_4 v_L^2}{4\sqrt{6}} B^\mu W_{\mu R}^0 \\ &+ \frac{g_4^2 (v_L^2 + v_R^2)}{4} \left[X_{1\mu}^- X_1^{\mu +} + X_{2\mu}^- X_2^{\mu +} + X_{3\mu}^- X_3^{\mu +} + \frac{3}{4} B_\mu B^\mu \right] \end{aligned} \tag{4.20}$$

The gauge boson masses read $M_{W_L}^2 = g_L^2 v_L^2/4, M_{W_R}^2 = g_R^2 v_R^2/4$ and $M_X^2 = g_4^2(v_R^2 + v_L^2)/4$ for the charged particles. For the neutral gauge bosons, we need to diagonalise the mass matrix because of the mixing structure above. After diagonalising the neutral gauge boson mass matrix, we have

$$M_0^2 = \frac{1}{8} \begin{pmatrix} 0 & 0 & 0 \\ 0 & m_-^2 & 0 \\ 0 & 0 & m_+^2 \end{pmatrix} \tag{4.21}$$

where we rotate the basis from $\left(W^0_{\mu L}, W^0_{\mu R}, B_\mu\right)$ to $\left(A_\mu, Z_\mu, Z'_\mu\right)$.

$$\begin{aligned}\frac{m_\pm^2}{8} = \frac{1}{16}\left(\frac{3}{2}g_4^2\left(v_R^2+v_L^2\right)+g_R^2v_R^2+g_L^2v_L^2\right) \pm \\ \pm\frac{1}{16}\sqrt{\left(\frac{3}{2}g_4^2\left(v_R^2-v_L^2\right)+g_R^2v_R^2-g_L^2v_L^2\right)^2+9g_4^4v_R^2v_L^2}\end{aligned} \tag{4.22}$$

In total, we have 12 massive gauge bosons consisting of 6 leptoquarks ($X^\pm_{a\mu}$), 4 charged bosons ($W^\pm_{\mu L(R)}$) and 2 neutral gauge bosons (Z_μ, Z'_μ) consistent with the number of 12 goldstone bosons after Higgs mechanism.

For the fermion sector of the model 1, the SM fermions get mass from radiative loop corrections not at tree-level (except only Beyond SM neutrinos which attain mass at tree-level). In order to have the Yukawa structure exist at tree-level, we will consider the model 2 developed by [19] which differs from model 1 by introducing Higgs bidoublet as in the section 2.4 instead of the left-handed Higgs to break the electroweak symmetry down to $U(1)_Q$.

The Higgs bidoublets are in the representation (1,2,2) of the SM group denoted as Φ_i^I where $i = 1, 2$ and $I = 1, 2$ are the $SU(2)_R$ and $SU(2)_L$ indices, respectively transforming as matrix multiplication:

$$\Phi \to U_{2L}\Phi U_{2R}^\dagger \tag{4.23}$$

where $U_{2L} \in SU(2)_L$ and $U_{2R} \in SU(2)_R$ with the conjugate transpose notation defined as $(\Phi_I^i)^* = \Phi_i^I$. Together with the transformation in equation (4.2) for the right-handed Higgs fields, we can construct the general invariant Higgs potential as

$$\begin{aligned}\mathbf{V}(\Phi) = &-\mu_2^2\,\mathrm{Tr}\left[\Phi^\dagger\Phi\right]+\eta_3\left(\mathrm{Tr}\left[\Phi^\dagger\Phi\right]\right)^2+\eta_4\,\mathrm{Tr}\left[\Phi^\dagger\Phi\Phi^\dagger\Phi\right]\\ &-\mu_3^2\left[\mathrm{Tr}\left[\Phi^\dagger\tilde{\Phi}\right]+\text{ h.c. }\right]+\eta_5\left[\left(\mathrm{Tr}\left[\Phi^\dagger\tilde{\Phi}\right]\right)+h.c.\right]\\ &+\eta_6\left[\mathrm{Tr}\left[\Phi^\dagger\Phi\Phi^\dagger\tilde{\Phi}\right]+\text{ h.c. }\right]\end{aligned} \tag{4.24}$$

The refined breaking chain is as follow:

$$\begin{aligned} &SU(4)\times SU(2)_L\times SU(2)_R\\ &\quad\big\downarrow \langle R\rangle\\ &SU(3)_C\times SU(2)_L\times U(1)_Y\\ &\quad\big\downarrow \langle \Phi\rangle\\ &SU(3)_C\times U(1)_Q\end{aligned} \tag{4.25}$$

Since it preserves the path of the breaking chain, the number of the broken generator is still the same. Hence, the number of goldstone bosons is unchanged also for the massive gauge bosons. However, the VEV structure has to change because of introducing the Higgs bidoublet Φ. This will affect the Higgs spectrum, gauge boson spectrum and some definitions of physical fields. With this breaking chain and minimising the potential above, the VEV has to be in the form:

$$\langle\Phi\rangle = \begin{pmatrix} u_1 & 0 \\ 0 & u_2 \end{pmatrix} \tag{4.26}$$

4.3 Yukawa Structure

The Pati-Salam symmetry unifies quarks and leptons into a single representation (the lepton number as a fourth colour of $SU(4)_C$). The SM fermions are unified in the same representation as we have for the Higgs fields in model 1, in representation (4,2,1) for left-handed fermions and (4,1,2) for right-handed fermions under G_{PS} defined in the matrix form as

$$\Psi^{\alpha i(\mathbf{f})}_{L,R} = \begin{pmatrix} u_1 & d_1 \\ u_2 & d_2 \\ u_3 & d_3 \\ \nu_e & e^- \end{pmatrix}^{(\mathbf{f})}_{L,R} \tag{4.27}$$

where $i = 1, 2$ is the index for $SU(2)_{L(R)}$, $\alpha = 1, 2, 3, 4$ for $SU(4)$ and $f = 1, 2, 3$ corresponds to the three fermion generations. Therefore the gauge transformation rules for the fields, written as matrix multiplication, are

$$\Psi_L \to U_4(\Psi_L)U^T_{2L} \quad \text{and} \quad \Psi_R \to U_4(\Psi_R)U^T_{2R} \tag{4.28}$$

where $U_4 \in SU(4)$, $U_{2L} \in SU(2)_L$ and $U_{2R} \in SU(2)_R$
With this minimal content, the Yukawa lagrangian is given by

$$-\mathcal{L}^{\text{min}}_Y = \bar{\Psi}_L Y_1 \Phi \Psi_R + \bar{\Psi}_L Y_2 \epsilon^T \Phi^* \epsilon \Psi_R + \text{ h.c.} \tag{4.29}$$

or in the form:

$$-\mathcal{L}^{\text{min}}_Y = \left[Y^{(\mathbf{f})}_1 \Psi^{(\mathbf{f})}_{L\alpha I} \Phi^I_i + Y^{(\mathbf{f})}_2 \Psi^{(\mathbf{f})}_{L\alpha I} \tilde{\Phi}^I_i \right] \Psi^{\alpha i(\mathbf{f})}_R + \text{ h.c.} \tag{4.30}$$

where $\epsilon := i\sigma_2$ acts on the $SU(2)_L$ and $SU(2)_R$ indices and Y_1 and Y_2 are 3×3 Yukawa matrices in the flavor space. In the case without inter-family mixing, the Y_1 and Y_2 are diagonal and proportional to the mass of the fermions: $Y_1 = \text{diag}(y^{(1)}_1, y^{(2)}_1, y^{(3)}_1)$ and $Y_2 = \text{diag}(y^{(1)}_2, y^{(2)}_2, y^{(3)}_2)$. Once the Higgs fields get VEVs, we can expand above as:

$$\begin{aligned} -\mathcal{L}^{\text{min}}_Y = &(u_1 y_1 + u_2 y_2)^{(\mathbf{f})} \left(\bar{u}^k_L u^k_R + \bar{\nu}_{eL} \nu_{eR}\right)^{(\mathbf{f})} \\ &+ (u_1 y_2 + u_2 y_1)^{(\mathbf{f})} \left(\bar{d}^k_L d^k_R + \bar{e}_L e_R\right)^{(\mathbf{f})} + \text{ h.c.} \end{aligned} \tag{4.31}$$

where $(\mathbf{f})$ is the fermion families and $k = 1, 2, 3$ is the colour index for quarks. We have $m_u = m_\nu$ and $m_d = m_e$: the up-quark mass is the same as the neutrino and the down-quark mass is the same as the electron. Therefore, this model 2 predicts the inconsistent fermion masses compared to the SM fermions which is the first problem of the Pati-Salam minimal model.

4.4 $\mathrm{U_1}$ Leptoquark in PS model

The second dangerous problem of the Pati-Salam model lies in the interactions of fermions and gauge bosons, in particular the leptoquarks $X_1^{\mu\pm}, X_2^{\mu\pm}$ and $X_3^{\mu\pm}$. We consider the fermion kinetic terms and look closely at the interactions relating to leptoquarks of $SU(4)$ gauge fields.

$$\begin{aligned}\mathcal{L}^f_{\text{kinetic}} &= i\bar{\Psi}_L\gamma^\mu D_\mu \Psi_L + i\bar{\Psi}_R\gamma^\mu D_\mu\Psi_R \\ &\subset g_4\Psi_{L\alpha i}\gamma^\mu (G_\mu)^\alpha_\beta \Psi_L^{\beta i} + g_4\Psi_{R\alpha i}\gamma^\mu (G_\mu)^\alpha_\beta\Psi_R^{\beta i}\end{aligned} \tag{4.32}$$

where the G_μ is given in the equation (4.19). We can expand it in the form (only left-handed part) as

$$\begin{aligned}&= \frac{g_4}{\sqrt{2}}\Psi_{L1i}\gamma^\mu(X^+_{1\mu})\Psi_L^{4i} + \frac{g_4}{\sqrt{2}}\Psi_{L2i}\gamma^\mu(X^+_{2\mu})\Psi_L^{4i} + \frac{g_4}{\sqrt{2}}\Psi_{L3i}\gamma^\mu(X^+_{3\mu})\Psi_L^{4i} \\ &= \frac{g_4}{\sqrt{2}}X_\mu\left(\bar{d}_L\gamma^\mu e_L + \bar{u}_L\gamma^\mu\nu_L\right)\end{aligned} \tag{4.33}$$

where the colour index is absorbed in both for leptoquarks and for quarks. The full leptoquark interaction is in the form of

$$\mathcal{L}_X = \frac{g_4}{\sqrt{2}}X_\mu\left(\bar{d}_L\gamma^\mu e^i_L + \bar{u}_L\gamma^\mu\nu^i_L + \bar{d}_R\gamma^\mu e^i_R + \bar{u}_R\gamma^\mu\nu^i_R\right)^{(\mathbf{f})} + \text{h.c.} \tag{4.34}$$

where $(\mathbf{f})$ is the fermion families index. The above lagrangian is written in the flavour basis and thus diagonal and universal. Without the assumption given in the Yukawa sector that there is no mixing between generations and Yukawa matrices are diagonal, we can revive the complex 3×3 structure of the Yukawa matrix and hence the CKM structure will appear in the left-handed fermion interactions once we are in the mass basis after diagonalising the fermion mass matrix.

Obviously, the leptoquark interaction is the source of the violation of lepton universality as we discussed in section 3.3. This massive leptoquark fits well in the $SU(4)$ gauge group of the Pati-Salam model arising from the breaking $\mathrm{SU(4)} \to \mathrm{SU(3)_C \times U(1)_{B-L}}$. However, the problem arising in the minimal Pati-Salam model is the strong bounds on the leptoquark (LQ) couplings to the first and the second generation fermions. Especially, the FCNC semi-leptonic meson decays $K_L(sd) \to \mu e$ put the bounds on the mass of the LQ beyond 200 TeV [15],[18] which gives not interesting physics since we would like to talk about the TeV scale physics to keep the naturalness of the Higgs mass around it.

The minimal Pati-Salam group, in particular model 2, explicitly show two problems. One is that the fermion spectrum does not match the standard model $m_u = m_\nu$ and $m_d = m_e$. The other is the processes at tree-level predict the rapid lepton flavour violation process.

5 4321 Models

As suggested by the phenomenology of the $U_1(3,1,2/3)$ leptoquark in B-anomalies and the strong bounds from the semileptonic meson decay into lepton pairs, we can construct a UV-complete model based on the PS group which is flavour universal model. The way to protect the light families from the new physics problems in the minimal Pati-Salam model is to de-correlate the $SU(4)$ group from the Standard model colour group by taking flavour universal $G_{\text{PS}} \subset SU(4) \times SU(2)_L \times U(1)_R$ and enlarge the colour part from $SU(4)$ into $SU(4) \times SU(3)'$ in kind of close analogy of what we have in QED and electroweak theory. That is when we break $SU(4)\times SU(3)'$ to $SU(3)_C$ of the SM group the $SU(3)_C$ is a residual subgroup which is similar to the $U(1)_Q$ the diagonal subgroup of the electroweak $SU(2)_L \times U(1)_Y$ group. Without loss of generality, in electroweak part, instead of using $SU(2)_L \times SU(2)_R$ we can use $SU(2)_L \times U(1)'$ to give the electroweak structure as in the SM after symmetry breaking as well.

The resulting model is based on $G = SU(4) \times SU(3)' \times SU(2)_L \times U(1)'$ gauge group called "**the 4321 model**". The scalar content of the 4321 model is summarized in the table below:

	$SU(4)$	$SU(3)'$	$SU(2)_L$	$U(1)'$
H	1	1	2	1/2
ξ	$\overline{4}$	1	1	$-1/2$
Φ	$\overline{4}$	3	1	1/6
Ω_{15}	15	1	1	0

(5.1)

Therefore the gauge transformation rules for the fields, written as matrix multiplication, are

$$H \to U_2 H \quad , \quad \xi \to U_4^* \xi \quad , \quad \Phi \to U_4^* \Phi U_{3'}^T \quad \text{and} \quad \Omega_{15} \to U_4^* \Omega_{15} U_4^\dagger \tag{5.2}$$

where $U_4 \in SU(4), U_{3'} \in SU(3)'$ and $U_{2L} \in SU(2)_L$.
The breaking chain of the 4321 model is followed as:

$$\begin{gathered} G_{4321} = SU(4) \times SU(3)' \times SU(2)_L \times U(1)' \\ \big\downarrow {\scriptstyle \langle \Phi \rangle} \\ SU(3)_C \times SU(2)_L \times U(1)_Y \\ \big\downarrow {\scriptstyle \langle H \rangle} \\ SU(3)_C \times U(1)_Q \end{gathered} \tag{5.3}$$

From the breaking chain, the only $\Phi = (\bar{4},3,1,1/6)$ suffice for the breaking G_{4321} down to G_{SM} where the spontaneous breaking proceeds such that $SU(3)_C$ is the diagonal subgroup of $SU(3)_4 \times SU(3)'$ and $U(1)_Y$ is the diagonal subgroup of $U(1)_4 \times U(1)'$. Also, the Higgs doublet $\Phi = (1,1,2,1/2)$ is responsible for the breaking of the SM group G_{SM} down to $SU(3)_C \times U(1)_Q$ as usual like the SM Higgs. It will give mass to the fermions. For the other fields, ξ will play a role in the fermion sector as induce the mixing of the leptons and vector-like leptons and Ω_{15} will distinguish the mass for the vector-like fermions that we will discuss later.

With the structure of symmetry given above, we can define the hypercharge as

$$Y = \sqrt{\frac{2}{3}} T^{15} + Y' \tag{5.4}$$

analog to the one we have in equation (4.5) of the PS model but without $SU(2)_R$ structure.

5.1 Higgs Potential

Let us construct the singlets out of the representations of Φ and ξ in order to find the general Higgs potential term with at most dimension-4 operators to keep the renormalisable structure of the theory.

We adopt the notation of the fields as $\xi^\alpha \sim (\bar{4}, 1, 1)$ and $\Phi^\alpha_i \sim (\bar{4}, 3, 1)$ with $\alpha = 1, 2, 3, 4$ is the $SU(4)$ index and $i = 1, 2, 3$ is the $SU(3)'$ index and define the conjugate transpose as $\xi_\alpha = (\xi^\alpha)^*$ and $\Phi^i_\alpha = (\Phi^\alpha_i)^*$.

5.1.1 Quadratic terms

Consider the group multiplications

$$\begin{aligned}
3 \otimes 3 &= \overline{3} \oplus 6 \\
\overline{3} \otimes 3 &= 1 \oplus 8 \\
\overline{3} \otimes \overline{3} &= 3 \oplus \overline{6} \\
4 \otimes 4 &= 10 \oplus 6 \\
\overline{4} \otimes 4 &= 1 \oplus 15 \\
\overline{4} \otimes \overline{4} &= \overline{10} \oplus 6
\end{aligned} \tag{5.5}$$

therefore the only gauge invariant quadratic terms we can write have the form:

$$(\overline{4}, 3, 1) \otimes (4, \overline{3}, 1) \to \Phi^\alpha_i \Phi^i_\alpha \quad \text{and} \quad (\overline{4}, 1, 1) \otimes (4, 1, 1) \to \xi_\alpha \xi^\alpha \tag{5.6}$$

5.1.2 Cubic terms

Consider the group multiplications

$$\begin{aligned}
3 \otimes 3 \otimes 3 &= 1 \oplus 8 \oplus 8 \oplus 10 \\
\overline{3} \otimes 3 \otimes 3 &= 3 \oplus 3 \oplus \overline{6} \oplus 15 \\
\overline{3} \otimes \overline{3} \otimes 3 &= \overline{3} \oplus \overline{3} \oplus 6 \oplus \overline{15} \\
\overline{3} \otimes \overline{3} \otimes \overline{3} &= 1 \oplus 8 \oplus 8 \oplus \overline{10} \\
4 \otimes 4 \otimes 4 &= \overline{4} \oplus \overline{20} \oplus \overline{20} \oplus \overline{20}'' \\
\overline{4} \otimes 4 \otimes 4 &= 4 \oplus 4 \oplus 20 \oplus 36 \\
\overline{4} \otimes \overline{4} \otimes 4 &= \overline{4} \oplus \overline{4} \oplus \overline{20} \oplus \overline{36} \\
\overline{4} \otimes \overline{4} \otimes \overline{4} &= 4 \oplus 20 \oplus 20 \oplus 20''
\end{aligned} \tag{5.7}$$

Although $(\overline{4},3,1)\otimes(\overline{4},3,1)\otimes(\overline{4},3,1)$ can form a singlet under $SU(3)$, it is not possible to form the singlets under $SU(4)$. So, there are no cubic terms in the potential.

5.1.3 Quartic terms

Consider the group multiplications

$$\begin{aligned} 3\otimes 3\otimes 3\otimes 3 &= 3\oplus 3\oplus 3\oplus \overline{6}\oplus \overline{6}\oplus 15\oplus 15\oplus 15\oplus \overline{15} \\ \overline{3}\otimes 3\otimes 3\otimes 3 &= \overline{3}\oplus \overline{3}\oplus \overline{3}\oplus 6\oplus 6\oplus 6\oplus \overline{15}\oplus \overline{15}\oplus \overline{24} \\ 3\otimes \overline{3}\otimes 3\otimes \overline{3} &= 1\oplus 1\oplus 8\oplus 8\oplus 8\oplus 8\oplus 10\oplus \overline{10}\oplus 27 \end{aligned} \tag{5.8}$$

$$\begin{aligned} 4\otimes 4\otimes 4\otimes 4 &= 1\oplus 15\oplus 15\oplus 15\oplus 20'\oplus 20'\oplus 35\oplus 45\oplus 45\oplus 45 \\ \overline{4}\otimes 4\otimes 4\otimes 4 &= 6\oplus 6\oplus 6\oplus 10\oplus 10\oplus 10\oplus \overline{10}\oplus 64\oplus 64\oplus 70 \\ 4\otimes \overline{4}\otimes 4\otimes \overline{4} &= 1\oplus 1\oplus 15\oplus 15\oplus 15\oplus 15\oplus 20'\oplus 45\oplus \overline{45}\oplus 84 \end{aligned} \tag{5.9}$$

The possible terms are

$$(\overline{4},3,1)\otimes(4,\overline{3},1)\otimes(\overline{4},3,1)\otimes(4,\overline{3},1)\rightarrow \Phi_i^\alpha\Phi_\alpha^i\Phi_\beta^j\Phi_j^\beta \quad \text{and} \quad \Phi_i^\alpha\Phi_\alpha^j\Phi_\beta^i\Phi_j^\beta \tag{5.10}$$

$$(\overline{4},1,1)\otimes(4,1,1)\otimes(\overline{4},1,1)\otimes(4,1,1)\rightarrow \xi^\alpha\xi_\alpha\xi^\beta\xi_\beta \tag{5.11}$$

The mixing between the two Higgs fields Φ and ξ has two possible terms which can form the singlets under the gauge transformation. One is

$$(\overline{4},3,1)\otimes(4,\overline{3},1)\otimes(\overline{4},1,1)\otimes(4,1,1)\rightarrow \Phi_i^\alpha\Phi_\alpha^i\xi^\beta\xi_\beta \tag{5.12}$$

As discussed in the Cubic terms, we can form the singlet under $SU(3)'$ out of three Φ fields but the $SU(4)$ part is not singlet. We have to compensate the $SU(4)$ structure by one ξ field in order to have gauge invariant term.

$$(\overline{4},3,1)\otimes(\overline{4},3,1)\otimes(\overline{4},3,1)\otimes(\overline{4},1,1)\rightarrow \epsilon_{\alpha\beta\gamma\delta}\epsilon^{ijk}\left(\Phi\right)_i^\alpha\left(\Phi\right)_j^\beta\left(\Phi\right)_k^\gamma\left(\xi\right)^\delta \tag{5.13}$$

The most general Higgs potential constructed from these two Higgs bosons that is gauge invariant is given in matrix multiplication form as

$$\begin{aligned} V(\Phi,\xi) &= \mu_3^2\,\mathrm{Tr}\left(\Phi^\dagger\Phi\right)+\lambda_1\left(\mathrm{Tr}\left(\Phi^\dagger\Phi\right)-\frac{3}{2}v_3^2\right)^2+\lambda_2\,\mathrm{Tr}\left(\Phi^\dagger\Phi-\frac{1}{2}v_3^2\mathbb{1}_3\right)^2 \\ &+\mu_1^2\left|\xi\right|^2+\lambda_3\left(\left|\xi\right|^2-\frac{1}{2}v_1^2\right)^2+\lambda_4\left(\mathrm{Tr}\left(\Phi^\dagger\Phi\right)-\frac{3}{2}v_3^2\right)\left(\left|\xi\right|^2-\frac{1}{2}v_1^2\right) \\ &+\lambda_5\xi^\dagger\Phi\Phi^\dagger\xi+\lambda_6\left(\left[\Phi\Phi\Phi\xi\right]_1+\text{ h.c. }\right) \end{aligned} \tag{5.14}$$

or in form of Φ_i^α and ξ^α with $\Phi_\alpha^i=(\Phi_i^\alpha)^*$ and $\xi_\alpha=(\xi^\alpha)^*$ where $\alpha=1,..,4$ and $i=1,2,3$

$$\begin{aligned} V(\Phi,\xi) &= \mu_3^2\Phi_i^\alpha\Phi_\alpha^i+\lambda_1\left(\Phi_i^\alpha\Phi_\alpha^i-\frac{3}{2}v_3^2\right)^2+\lambda_2\left(\Phi_i^\alpha\Phi_\alpha^j-\frac{1}{2}v_3^2\delta_i^j\right)\left(\Phi_\beta^i\Phi_j^\beta-\frac{1}{2}v_3^2\delta_j^i\right) \\ &+\mu_1^2\xi^\alpha\xi_\alpha+\lambda_3\left(\xi^\alpha\xi_\alpha-\frac{1}{2}v_1^2\right)^2+\lambda_4\left(\Phi_i^\alpha\Phi_\alpha^i-\frac{3}{2}v_3^2\right)\left(\xi^\beta\xi_\beta-\frac{1}{2}v_1^2\right) \\ &+\lambda_5\xi_\alpha\Phi_i^\alpha\Phi_\beta^i\xi^\beta+\lambda_6\left(\epsilon_{\alpha\beta\gamma\delta}\epsilon^{ijk}\left(\Phi\right)_i^\alpha\left(\Phi\right)_j^\beta\left(\Phi\right)_k^\gamma\left(\xi\right)^\delta+\text{ h.c. }\right) \end{aligned} \tag{5.15}$$

where $[\Phi\Phi\Phi\xi]_1 = \epsilon_{\alpha\beta\gamma\delta}\epsilon^{ijk}\,(\Phi)_i^\alpha\,(\Phi)_j^\beta\,(\Phi)_k^\gamma\,(\xi)^\delta$ is the term that breaks explicitly the global $U(1)$ symmetry of the scalar potential in order to prevent the appearance of unwanted massless Goldstone bosons (GBs) after spontaneous symmetry breaking.

5.2 Minimizing the potential

We consider first only Φ field without mixing terms, the first derivative of Higgs potential (in the limit $\mu_3 = 0$) is

$$\frac{\partial V(\Phi)}{\partial \Phi_k^\sigma} = 2\lambda_1\left(\Phi_i^\alpha\Phi_\alpha^i - \frac{3}{2}v_3^2\right)\Phi_\sigma^k + \lambda_2\left(\Phi_i^\alpha\Phi_\alpha^k - \frac{1}{2}v_3^2\delta_i^k\right)\Phi_\sigma^i + \lambda_2\left(\Phi_\beta^k\Phi_j^\beta - \frac{1}{2}v_3^2\delta_j^k\right)\Phi_\sigma^j \tag{5.16}$$

We define: $X_i^j = \Phi_\alpha^i\Phi_j^\alpha$ which is hermitian since $(X^\dagger)_j^i = (X_i^j)^* = (\Phi_j^\alpha\Phi_\alpha^i)^* = \Phi_\alpha^j\Phi_i^\alpha = X_j^i$ then

$$\frac{\partial V(\Phi)}{\partial \Phi_k^\sigma} = 2\lambda_1\left(\mathrm{Tr}\,(X) - \frac{3}{2}v_3^2\right)\Phi_\sigma^k + 2\lambda_2\left(X_i^k - \frac{1}{2}v_3^2\delta_i^k\right)\Phi_\sigma^i \tag{5.17}$$

Since X is hermitian, we can diagonalize it by a unitary transformation

$$X \to X' = U_3^* X (U_3^*)^\dagger = U_3^*\Phi^*\Phi(U_3^*)^\dagger = U_3^*\Phi^*(U_4)^T U_4^*\Phi(U_3^*)^\dagger = (U_4^*\Phi(U_3)^T)^* U_4^*\Phi(U_3)^T = \tilde{\Phi}^*\tilde{\Phi}$$

So, X' has the form

$$X' = \begin{pmatrix} x_1' & & \\ & x_2' & \\ & & x_3' \end{pmatrix} \tag{5.18}$$

Since $\mathrm{Tr}\,[X] = \mathrm{Tr}\left[U^\dagger\Sigma' U\right] = \mathrm{Tr}\,[X'] = \sum_{i=1}^3 x_i'$, we can rewrite (5.17) as

$$\frac{\partial V(\Phi)}{\partial \Phi_k^\sigma} = \left[2\lambda_1\left(\sum_{i=1}^3 x_i - \frac{3}{2}v_3^2\right) + 2\lambda_2\left(x_k - \frac{1}{2}v_3^2\right)\right]\Phi_\sigma^k = 0 \tag{5.19}$$

We consider only non-trivial solution that can make SSB. Then, the set of equations defining vevs is

$$2\lambda_1\left(\sum_{i=1}^3 x_i - \frac{3}{2}v_3^2\right) + 2\lambda_2\left(x_k - \frac{1}{2}v_3^2\right) = 0, \quad k = 1,\cdots,3 \tag{5.20}$$

We have

$$6\lambda_1\left(\sum_{i=1}^3 x_i - \frac{3}{2}v_3^2\right) + 2\lambda_2\left(\sum_{i=1}^3 x_k - \frac{3}{2}v_3^2\right) \quad = 0 \tag{5.21}$$

$$\sum x_{i=1}^3 = 3v^3/2 \tag{5.22}$$

and $x_1 - x_2 = x_1 - x_3 = x_2 - x_3 = 0$, from which we obtain $x_1 = x_2 = x_3 = v^3/2$.

$$\langle X \rangle = \frac{1}{2} \begin{pmatrix} v_3^2 & 0 & 0 \\ 0 & v_3^2 & 0 \\ 0 & 0 & v_3^2 \end{pmatrix} \tag{5.23}$$

Since we want to break $SU(4)\times SU(3)'\times SU(2)_L\times U(1)' \to SU(3)_C\times SU(2)_L\times U(1)_Y$, we can put the VEV in the form which is given in Georgi's paper [11] where they consider the breaking of a partial unification model $SU(N+3)_H\times SU(3)_{C'}\times SU(2)_L\times U(1)_{Y'}$ down to $SU(N)_H \times SU(3)_C \times SU(2)_L \times U(1)_Y$ which is analog to our case for $N = 1$.

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_3 & 0 & 0 \\ 0 & v_3 & 0 \\ 0 & 0 & v_3 \\ 0 & 0 & 0 \end{pmatrix} = \frac{v_3}{\sqrt{2}} \delta_{\alpha i} \tag{5.24}$$

For the other scalar fields, the breaking pattern is proceeded by acquiring the VEVs in the form:

$$\langle \xi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ v_1 \end{pmatrix} = \frac{v_1}{\sqrt{2}} \delta_{\alpha 4}, \quad \langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \text{and} \quad \langle \Omega_{15} \rangle = \frac{v_{15}}{\sqrt{2}} \delta_{A15} \tag{5.25}$$

If we consider together the μ_1, μ_3 and also the mixing terms with ($\lambda_6 \neq 0$), we can have conditions on μ_1 and μ_3 by minimising the first derivative of the full Higgs potential as:

$$\begin{aligned} \frac{\partial V(\Phi)}{\partial \Phi_l^\sigma} &= \mu_3^2 \Phi_\sigma^l + 2\lambda_1 \left(\Phi_i^\alpha \Phi_\alpha^i - \frac{3}{2} v_3^2 \right) \Phi_\sigma^l + 2\lambda_2 \left(\Phi_i^\alpha \Phi_\alpha^l - \frac{1}{2} v_3^2 \delta_i^l \right) \Phi_\sigma^i \\ &+ \lambda_4 \left(\xi_\beta \xi^\beta - \frac{1}{2} v_1^2 \right) \Phi_\sigma^l + \lambda_5 \xi_\sigma \Phi_\beta^l \xi^\beta \\ &+ \lambda_6 \left[\epsilon_{\alpha\beta\sigma\delta} \epsilon^{ijl} \Phi_i^\alpha \Phi_j^\beta + \epsilon_{\alpha\sigma\alpha\delta} \epsilon^{ilk} \Phi_i^\alpha \Phi_k^\gamma + \epsilon_{\sigma\beta\gamma\delta} \epsilon^{ljk} \Phi_j^\beta \Phi_k^\gamma \right] \end{aligned} \tag{5.26}$$

and

$$\begin{aligned} \frac{\partial V(\Phi)}{\partial \xi^\sigma} &= \mu_1^2 \xi_\sigma + 2\lambda_3 \left(\xi_\alpha \xi^\alpha - \frac{1}{2} v_1^2 \right) \xi_\sigma + \lambda_4 \left(\mathrm{Tr}(X) - \frac{3}{2} v_3^2 \right) \xi_\sigma \\ &+ + \lambda_5 \xi_\alpha \Phi_i^\alpha \Phi_\sigma^i + \lambda_6 \epsilon_{\alpha\beta\gamma\sigma} \epsilon^{ijk} (\Phi)_i^\alpha (\Phi)_j^\beta (\Phi)_k^\gamma \end{aligned} \tag{5.27}$$

After Φ and ξ getting VEVs, where $\langle \Phi_1^1 \rangle = \langle \Phi_2^2 \rangle = \langle \Phi_3^3 \rangle = v_3/\sqrt{2}$ and $\langle \xi^4 \rangle = \langle \xi_4 \rangle = v_1/\sqrt{2}$, we have
for $l = 1$ and $\sigma = 1$ in the first derivative with respect to Φ fields:

$$\left[\frac{\partial V(\Phi)}{\partial \Phi_1^1} \right]_{\mathrm{VEV}} = \frac{\mu_3^2}{\sqrt{2}} v_3 + 3\lambda_6 \epsilon_{\alpha\beta 14} \epsilon^{ij1} \Phi_i^\alpha \Phi_j^\beta = 0 \quad \to \quad \mu_3^2 = -3\lambda_6 v_1 v_3 \tag{5.28}$$

and for $\sigma = 4$ in the first derivative with respect to ξ fields:

$$\left[\frac{\partial V(\Phi)}{\partial \xi^4}\right]_{\text{VEV}} = \frac{\mu_1^2}{\sqrt{2}} v_1 + \lambda_6 \epsilon_{\alpha\beta\gamma 4} \epsilon^{ijk} \Phi_i^\alpha \Phi_j^\beta \Phi_k^\gamma = 0 \quad \rightarrow \quad \mu_1^2 = -3\lambda_6 \frac{v_3^3}{v_1} \tag{5.29}$$

Thus, the stationary equations are satisfied by

$$\mu_3^2 = -3\lambda_6 v_1 v_3, \quad \text{and} \quad \mu_1^2 = -3\lambda_6 \frac{v_3^3}{v_1} \tag{5.30}$$

5.3 Higgs Masses

We start with $G_{4321} = SU(4) \times SU(3)' \times SU(2)_L \times U(1)'$ which has $27 = 15 + 8 + 3 + 1$ massless gauge bosons, coming from 15 $SU(4)$ generators, 8 from each $SU(3)'$, 3 from $SU(2)_L$ and single $U(1)'$ generator, and we end up with the $SU(3)_C \times SU(2)_L \times U(1)_Y$ symmetry group which has $12 = 8 + 3 + 1$ massless gauge bosons, coming from 8 $SU(3)$ generators, from 3 $SU(2)_L$ and single $U(1)_Y$ generator. So, we expect to see $27 - 12 = 15$ massless goldstone bosons corresponding to 15 broken generators or 15 massive gauge bosons after the Higgs Mechanism.

For the Higgs spectrum, we can derive from evaluating the second derivative of the Higgs potential with respect to all possible combinations of Φ, ξ, Φ^*,and ξ^* fields and impose the VEVs structure.

$$\begin{aligned}\frac{\partial^2 V(\Phi,\xi)}{\partial \Phi_\rho^m \partial \Phi_l^\sigma} &= \left[\mu_3^2 + 2\lambda_1 \left(\Phi_\alpha^i \Phi_\alpha^i - \frac{3}{2} v_3^2\right) + \lambda_4 \left(\xi_\beta \xi^\beta - \frac{1}{2} v_1^2\right)\right] \delta_m^l \delta_\sigma^\rho \\ &+ 2\lambda_1 \Phi_\sigma^l \Phi_m^\rho + 2\lambda_2 \left(\Phi_\alpha^l \Phi_m^\alpha - \frac{1}{2} v_3^2 \delta_m^l\right) \delta_\sigma^\rho + 2\lambda_2 \Phi_\sigma^i \Phi_i^\rho \delta_m^l \\ &+ \lambda_5 \xi_\sigma \xi^\beta \delta_m^l \delta_\beta^\rho + 3\lambda_6 \left(\epsilon_{\alpha\beta\sigma\rho} \epsilon^{ijk} \Phi_i^\alpha \Phi_j^\beta\right)\end{aligned} \tag{5.31}$$

$$\frac{\partial^2 V(\Phi,\xi)}{\partial \Phi_m^\rho \partial \Phi_l^\sigma} = 2\lambda_1 \Phi_\sigma^l \Phi_\rho^m + 2\lambda_2 \Phi_\sigma^m \Phi_\rho^l + 3\lambda_6 \left[\epsilon_{\alpha\rho\sigma\delta} \epsilon^{iml} \Phi_i^\alpha + \epsilon_{\rho\beta\sigma\delta} \epsilon^{mjl} \Phi_j^\beta\right] \xi^\delta \tag{5.32}$$

$$\begin{aligned}\frac{\partial^2 V(\Phi,\xi)}{\partial \xi_\rho \partial \xi^\sigma} &= \mu_1^2 \delta_\sigma^\rho + 2\lambda_3 \left(\xi_\alpha \xi^\alpha - \frac{1}{2} v_1^2\right) \delta_\sigma^\rho + 2\lambda_3 \xi_\sigma \xi^\rho \\ &\qquad + \lambda_4 \left(\text{Tr}(X) - \frac{3}{2} v_3^2\right) \delta_\sigma^\rho + \lambda_5 \Phi_i^\rho \Phi_\sigma^i\end{aligned} \tag{5.33}$$

$$\frac{\partial^2 V(\Phi,\xi)}{\partial \xi^\rho \partial \xi^\sigma} = 2\lambda_3 \xi_\sigma \xi_\rho \tag{5.34}$$

$$\frac{\partial^2 V(\Phi,\xi)}{\partial \xi^\rho \partial \Phi_l^\sigma} = \lambda_4 \Phi_\sigma^l \xi_\rho + \lambda_5 \xi_\sigma \Phi_\rho^l + 3\lambda_6 \left[\epsilon_{\alpha\beta\sigma\rho} \epsilon^{ijl} \Phi_i^\alpha \Phi_j^\beta\right] \tag{5.35}$$

$$\frac{\partial^2 V(\Phi,\xi)}{\partial \xi_\rho \partial \Phi_l^\sigma} = \lambda_4 \Phi_\sigma^l \xi^\rho + \lambda_5 \xi^\sigma \Phi_\rho^l \tag{5.36}$$

The scalar Φ and ξ can be decomposed into the SM fragments as

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}} v_3 \mathbb{1}_3 + \chi \\ T_3 \end{pmatrix}, \quad \text{and} \quad \xi = \begin{pmatrix} T_1^* \\ \frac{1}{\sqrt{2}} v_1 + S_1 \end{pmatrix} \tag{5.37}$$

where $\mathbb{1}_3$ is the 3×3 identity matrix, t^a are the $SU(3)$ generators and $\chi = \frac{1}{\sqrt{6}}S_3 I_3 + O^a t^a$. These complex Higgs fields are decomposed under the unbroken SM gauge group as

$$\begin{array}{l} \Phi \to S_3 \sim (\mathbf{1},\mathbf{1},0) \oplus T_3 \sim (\mathbf{3},\mathbf{1},2/3) \oplus O_3 \sim (\mathbf{8},\mathbf{1},0) \\ \xi \to S_1 \sim (\mathbf{1},\mathbf{1},0) \oplus T_1^* \sim (\overline{\mathbf{3}},\mathbf{1},-2/3) \end{array} \tag{5.38}$$

More explicitly, we can write

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}v_3 + \frac{1}{\sqrt{6}}S_3 + \frac{1}{2}\left(O_3 + \frac{1}{\sqrt{3}}O_8\right) & \frac{1}{2}\left(O_1 - iO_2\right) & \frac{1}{2}\left(O_4 - iO_5\right) \\ \frac{1}{2}\left(O_1 + iO_2\right) & \frac{1}{\sqrt{2}}v_3 + \frac{1}{\sqrt{6}}S_3 + \frac{1}{2}\left(-O_3 + \frac{1}{\sqrt{3}}O_8\right) & \frac{1}{2}\left(O_6 - iO_7\right) \\ \frac{1}{2}\left(O_4 + iO_5\right) & \frac{1}{2}\left(O_6 + iO_7\right) & \frac{1}{\sqrt{2}}v_3 + \frac{1}{\sqrt{6}}S_3 - \frac{1}{\sqrt{3}}O_8 \\ (T_3)_1 & (T_3)_2 & (T_3)_3 \end{pmatrix}$$

The scalar spectrum is not trivial to derive since we have to collect all the masses from the above second derivatives of the Higgs potential and use the definition of the decomposition above. By sorting the fields according to the SM quantum numbers, the scalar spectrum is as follows:

For octet fields O^a, we collect all the possible fields corresponding to the octet fields. For example, in the diagonal elements of the Φ field, we collect all the combinations of Φ_1^1, Φ_2^2, Φ_3^3, Φ_1^1, $(\Phi_1^1)^*$, $(\Phi_2^2)^*$ and $(\Phi_3^3)^*$ at dimension 2 associated with the combinations of O_3 and O_8 and use the definition $(\mathrm{Re}(O))^2 = (OO + 2O^*O + O^*O^*)/4$ and $(\mathrm{Im}(O))^2 = (OO - 2O^*O + O^*O^*)/4$ together with the masses derived from (5.31)-(5.36). We obtain:

$$\begin{array}{l} \mathcal{M}^2_{\mathrm{Re}\,O} = 2\left(\lambda_2 v_3^2 - 3\lambda_6 v_1 v_3\right) \\ \mathcal{M}^2_{\mathrm{Im}\,O} = 0 \end{array} \tag{5.39}$$

In this case, we have 8 goldstone bosons associated to the $\mathrm{Im}\,O$ fields that will be eaten to give mass to the coloron (g') in gauge sector. The positivity of the mass eigenvalues yields the condition $\lambda_2 v_3 > 3\lambda_6 v_1$

For triplet sector, we repeat the same procedure by collecting Φ_1^4, Φ_2^4, Φ_3^4, Φ_4^1, Φ_4^2 and Φ_4^3 associated with the T_3 fields and ξ_1, ξ_2, ξ_3, ξ^1, ξ^2 and ξ^3 associated with the T_1 fields. Therefore, we have the mass matrix in the basis (T_3,T_1):

$$\mathcal{M}_T^2 = \begin{pmatrix} \frac{1}{2}\lambda_5 v_1^2 - 3\lambda_6 v_1 v_3 & \frac{1}{2}\lambda_5 v_1 v_3 - 3\lambda_6 v_3^2 \\ \frac{1}{2}\lambda_5 v_1 v_3 - 3\lambda_6 v_3^2 & \frac{1}{2}\lambda_5 v_3^2 - 3\lambda_6 v_3^3/v_1 \end{pmatrix} \tag{5.40}$$

We can rotate the basis:

$$\begin{pmatrix} T_R \\ T_{\mathrm{GB}} \end{pmatrix} = \frac{1}{\sqrt{v_3^2 + v_1^2}} \begin{pmatrix} v_1 & v_3 \\ -v_3 & v_1 \end{pmatrix} \begin{pmatrix} T_3 \\ T_1 \end{pmatrix} \tag{5.41}$$

to diagonalise the matrix above. We obtain the mass eigenvalues:

$$\begin{array}{l} \mathcal{M}^2_{T_R} = \left(\frac{1}{2}\lambda_5 - 3\lambda_6 \frac{v_3}{v_1}\right)\left(v_1^2 + v_3^2\right) \\ \mathcal{M}^2_{T_{\mathrm{GB}}} = 0 \end{array} \tag{5.42}$$

In this case, we have $2 \times 3 = 6$ goldstone bosons associated to the T_{GB} fields that will be eaten to give mass to the leptoquark (U_1) in gauge sector. The positivity of the mass eigenvalues yields the condition $\lambda_5 v_1 > 6\lambda_6 v_3$.

For the singlet sector, we collect the same fields as in the octet sector with two additional fields ξ_4 and ξ^4 associated with the S_3, S_3^*, S_1 and S_1^*. We obtain the mass matrix in the basis (S_3, S_3^*, S_1, S_1^*) as

$$\begin{pmatrix} \frac{1}{2}(3\lambda_1+\lambda_2)v_3^2+3\lambda_6 v_1 v_3 & \frac{1}{2}(3\lambda_1+\lambda_2)v_3^2-\frac{3}{2}\lambda_6 v_1 v_3 & \sqrt{\frac{3}{2}}\left(3\lambda_6 v_3^2+\frac{1}{2}\lambda_4 v_1 v_3\right) & \frac{1}{2}\sqrt{\frac{3}{2}}\lambda_4 v_1 v_3 \\ \frac{1}{2}(3\lambda_1+\lambda_2)v_3^2-\frac{3}{2}\lambda_6 v_1 v_3 & \frac{1}{2}(3\lambda_1+\lambda_2)v_3^2+3\lambda_6 v_1 v_3 & \frac{1}{2}\sqrt{\frac{3}{2}}\lambda_4 v_1 v_3 & \sqrt{\frac{3}{2}}\left(3\lambda_6 v_3^2+\frac{1}{2}\lambda_4 v_1 v_3\right) \\ \sqrt{\frac{3}{2}}\left(3\lambda_6 v_3^2+\frac{1}{2}\lambda_4 v_1 v_3\right) & \frac{1}{2}\sqrt{\frac{3}{2}}\lambda_4 v_1 v_3 & \lambda_3 v_1^2 & \lambda_3 v_1^2-3\lambda_6\frac{v_3^3}{v_1} \\ \frac{1}{2}\sqrt{\frac{3}{2}}\lambda_4 v_1 v_3 & \sqrt{\frac{3}{2}}\left(3\lambda_6 v_3^2+\frac{1}{2}\lambda_4 v_1 v_3\right) & \lambda_3 v_1^2-3\lambda_6\frac{v_3^3}{v_1} & \lambda_3 v_1^2 \end{pmatrix} \tag{5.43}$$

Upon diagonalization,

$$\mathcal{M}_S^2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 3\lambda_6 v_3\left(\frac{3}{2}v_1+v_3^2/v_1\right) & 0 & 0 \\ 0 & 0 & M_{S_1}^2 & 0 \\ 0 & 0 & 0 & M_{S_2}^2 \end{pmatrix} \tag{5.44}$$

in the basis ($S_{\rm GB}$,S_0,S_1',S_2) where $M_{S_1}^2$ and $M_{S_2}^2$ have a complicated expression given in [14]. The zero eigenvalue corresponds to the eigenvector:

$$S_{\rm GB} = \frac{1}{\sqrt{v_1^2+\frac{2}{3}v_3^2}}\left(\frac{v_3}{\sqrt{3}}S_3-\frac{v_3}{\sqrt{3}}S_3^*-\frac{v_1}{\sqrt{2}}S_1+\frac{v_1}{\sqrt{2}}S_1^*\right) \tag{5.45}$$

and the $\mathcal{M}_{S_0}^2 = 3\lambda_6 v_3\left(\frac{3}{2}v_1+\frac{v_3^2}{v_1}\right)$ eigenvalue corresponds to the eigenvector

$$S_0 = \frac{1}{\sqrt{v_3^2+\frac{3}{2}v_1^2}}\left(-\frac{v_1}{\sqrt{2}}S_3+\frac{v_1}{\sqrt{2}}S_3^*-\frac{v_3}{\sqrt{3}}S_1+\frac{v_3}{\sqrt{3}}S_1^*\right) \tag{5.46}$$

and thus gives the condition on mass positivity as $\lambda_6 > 0$. In this case, we have a single goldstone boson associated to the S_{GB} field that will be eaten to give mass to the Z' colorless gauge boson.

In analogy with the breaking chain in the equation (5.3), we have total $15 = 8+6+1$ goldstone bosons, coming from 8 GBs in the octet sector, $3 \times 2 = 6$ GBs in triplet sector and single GB in singlet sector which will give mass to 15 gauge bosons after Higgs mechanism.

5.4 Gauge Sector

The gauge fields corresponding to the $G_{4321} = SU(4) \times SU(3)' \times SU(2)_L \times U(1)'$ gauge group are denoted respectively as $H_\mu^\alpha, G_\mu', W_\mu^i,\ B_\mu'$, the gauge couplings by

g_4, g_3, g_2, g_1 and the generators by T^α, T^a, T^i, Y' with indices $\alpha = 1, \ldots, 15, a = 1, \ldots, 8, i = 1, 2, 3$. The gauge spectrum can be derived from the kinetic part of the Higgs boson after we introduce the covariant derivative on each scalar field Φ and ξ where $A = 9, 10, ..., 14$ spans over the $SU(4)/(SU(3)_4 \times U(1)_4)$ coset.

The spontaneous symmetry breaking $G_{4321} \to G_{SM}$ will give rise to the 15 massive gauge bosons: a leptoquark, coloron and a colorless boson. Here, we neglect the effects of Ω_{15} and the electroweak symmetry breaking. The covariant derivatives are defined (recall that the scalar field are in $\Phi \sim (\overline{4}, 3, 1, 1/6)$ and $\xi \sim (\overline{4}, 1, 1, -1/2)$) as

$$
\begin{aligned}
D_\mu \Phi &= \partial_\mu \Phi + i g_4 H^a_\mu T^a \Phi + i g_4 H^A_\mu T^A \Phi + i g_4 H^{15}_\mu T^{15} \Phi - i g_3 G^a_\mu T^a \Phi - \frac{1}{6} i g_1 B'_\mu \Phi \\
D_\mu \xi &= \partial_\mu \xi + i g_4 H^a_\mu T^a \xi + i g_4 H^A_\mu T^A \xi + i g_4 H^{15}_\mu T^{15} \xi + \frac{1}{2} i g_1 B'_\mu \xi
\end{aligned}
\tag{5.47}
$$

which transform in matrix multiplications as

$$
D_\mu \xi \to U_4^* (D_\mu \xi) \quad \text{and} \quad D_\mu \Phi \to U_4^* (D_\mu \Phi) U_{3'}^T \tag{5.48}
$$

The gauge invariant terms for the Higgs kinetic part are written as

$$
\mathcal{L}^{\text{h}}_{\text{kinectic}} = \text{Tr} \left(D_\mu \langle \Phi \rangle \right)^\dagger D^\mu \langle \Phi \rangle + \left(D_\mu \langle \xi \rangle \right)^\dagger D^\mu \langle \xi \rangle \tag{5.49}
$$

The generators are normalized in such a way that $\text{Tr}\left[T^\alpha T^\beta\right] = \frac{1}{2}\delta^{\alpha\beta}$ in the fundamental representation. We can derive for example the diagonal terms:

$$
g_4^2 \,\text{Tr}\left[H^a_\mu \Phi^\dagger_{\text{vev}} T^a H^{\mu b} T^b \Phi_{\text{vev}}\right] = g_4^2 H^a_\mu H^{\mu b} \,\text{Tr}\left[(\Phi^\dagger_{\text{vev}})_{ij} (T^a)_{jk} (T^b)_{kl} (\Phi_{\text{vev}})_{lm}\right] \tag{5.50}
$$

where $\Phi^{\text{vev}}_{ij} = \dfrac{v_3}{\sqrt{2}} \delta_{ij}$, $\Phi_{i4} \sim \delta_{i4} = 0$ and given the normalization above. Thus,

$$
= g_4^2 H^a_\mu H^{\mu b} \,\text{Tr}\left[\delta_{ij} (T^a)_{jk} (T^b)_{kl} \delta_{lm}\right] = \frac{g_4^2 v^2 3}{2} H^a_\mu H^{\mu, b} \,\text{Tr}\left[T^a T^b\right] = \frac{g_4^2 v^2}{4} H^a_\mu H^{\mu, a} \tag{5.51}
$$

for the mixing $H^a_\mu H^{\mu, A}$ term:

$$
M^2 \sim \text{Tr}\left[(T^a)_{ij} \, \delta_{jk} \, (T^a)_{kl} \, \delta_{lm} \right] = \text{Tr}\left[T^a T^A\right] \sim \delta^{aA} = 0 \quad (a \neq A) \tag{5.52}
$$

for the mixing with B_μ, we have

$$
M^2 \sim B_\mu H^{\mu, a} \,\text{Tr}\left[T^a\right] = 0 \tag{5.53}
$$

Repeating the same procedure for the other fields, we obtain:

$$
\begin{aligned}
\mathcal{L}^{\text{gauge}}_{\text{mass}} = &\frac{1}{2} \left(H^a_\mu G'^a_\mu \right) \begin{pmatrix} g_4^2 & -g_4 g_3 \\ -g_4 g_3 & g_3^2 \end{pmatrix} \frac{v_3^2}{2} \begin{pmatrix} H^{b\mu} \\ G'^{b\mu} \end{pmatrix} + \frac{1}{8} g_4^2 \left(v_1^2 + v_3^2 \right) H^A_\mu H^{\mu A} \\
&+ \begin{pmatrix} H^{15}_\mu & B'_\mu \end{pmatrix} \begin{pmatrix} \dfrac{1}{16}\left(3 g_4^2 v_1^2 + g_4^2 v_3^2\right) & \dfrac{g_1 g_4 \left(-3 v_1^2 - v_3^2\right)}{8\sqrt{6}} \\ \dfrac{g_1 g_4 \left(-3 v_1^2 - v_3^2\right)}{8\sqrt{6}} & \dfrac{1}{4} g_1^2 \left(\dfrac{v_1^2}{2} + \dfrac{v_3^2}{6} \right) \end{pmatrix} \begin{pmatrix} H^{15\mu} \\ B'^\mu \end{pmatrix}
\end{aligned}
\tag{5.54}
$$

diagonalize the two mass matrices with the new basis given by

$$U_\mu^{1,2,3(\pm)} = \frac{1}{\sqrt{2}}\left(H_\mu^{9,11,13} \mp iH_\mu^{10,12,14}\right) \tag{5.55}$$

as we have in the Pati-Salam model for leptoquarks. and

$$\begin{aligned} g_\mu'^a &= \frac{g_4 H_\mu^a - g_3 G_\mu'^a}{\sqrt{g_4^2 + g_3^2}} \\ Z_\mu' &= \frac{g_4 H_\mu^{15} - \sqrt{\frac{2}{3}} g_1 B_\mu'}{\sqrt{g_4^2 + \frac{2}{3} g_1^2}} \end{aligned} \tag{5.56}$$

We obtain the gauge boson spectrum as

$$\begin{aligned} M_U^2 &= \frac{1}{4} g_4^2 \left(v_1^2 + v_3^2\right) \\ M_{g'}^2 &= \frac{1}{2}\left(g_4^2 + g_3^2\right) v_3^2 \\ M_{Z'}^2 &= \frac{1}{4}\left(\frac{3}{2} g_4^2 + g_1^2\right)\left(v_1^2 + \frac{1}{3} v_3^2\right) \end{aligned} \tag{5.57}$$

for leptoquark, coloron and colorless gauge bosons respectively. The massless orthogonal counterparts of (5.56) corresponding to $SU(3)_c \times U(1)_Y$ are given by

$$\begin{aligned} g_\mu^a &= \frac{g_3 H_\mu^a + g_4 G_\mu'^a}{\sqrt{g_4^2 + g_3^2}} \\ B_\mu &= \frac{\sqrt{\frac{2}{3}} g_1 H_\mu^{15} + g_4 B_\mu'}{\sqrt{g_4^2 + \frac{2}{3} g_1^2}} \end{aligned} \tag{5.58}$$

Totally, after the breaking $G_{4321} \to G_{SM}$, we have 15 massivw gauge bosons: 6 leptoquarks in the representation $U_1 \sim (3, 1, 2/3)$, 8 colorons $g' \sim (8, 1, 0)$ and a single colerless boson $Z' \sim (1, 1, 0)$.

If $v_1 = v_3 = v$ and in the limit $g_{1,3} \ll g_4$, the massive gauge bosons have degenerate masses

$$M_U = M_{g'} = M_{Z'} = \frac{1}{\sqrt{2}} g_4 v$$

which shows us that there should be a global symmetry associated to this in order to protect this structure. The underlying symmetry is analogous to the one we have in the electroweak part of the SM in the absence of the gauge coupling g' and the Yukawa couplings. In this case the Higgs potential has an accidental symmetry, the $SU(2)_L \times SU(2)_R$ global symmetry which is broken by the VEV structure of the Higgs bidoublet down to $SU(2)_D$, the custodial symmetry, as we discussed in the section 2.4. In the same spirit that we have in the SM, the 4321 models have the accidental global symmetry $SU(4) \times SU(4)'$ which is broken by the VEV structure of a bi-fundamental scalar down to the $SU(4)_D$. The subgroup $SU(3)' \times U(1)'$ of the $SU(4)'$ group and the full $SU(4)$ group are gauged. Hence, the global symmetry breaking leads to the breaking of the $SU(4) \times SU(3)' \times U(1)'$ down to the $SU(3)_D \times U(1)_D$. This $SU(4)_D$ symmetry cannot distinguish the spectrum of the gauge bosons since it imposes the symmetry of rotating the gauge fields in the mass matrix.

Let us consider $u'_R = (1, 3, 1, 2/3)$, the would-be SM right-handed down quark, under the 4321 group with the assumption that it is singlet under $SU(4)$ group. Then, writing out the part of the covariant derivative belonging to this representation:

$$D_\mu u'_R = \partial_\mu u'_R - i g_3 G'^a_\mu T^a u'_R - \frac{2}{3} i g_1 B'_\mu u'_R \tag{5.59}$$

We can rewrite the expressions (5.58) as

$$\begin{aligned} G'^a_\mu &= \frac{g_4 g^a_\mu - g_3 g'^a_\mu}{\sqrt{g_3^2 + g_4^2}} \\ B'_\mu &= \frac{g_4 B_\mu - g_1 \sqrt{\frac{2}{3}} Z'_\mu}{\sqrt{g_4^2 + \frac{2}{3} g_1^2}} \end{aligned} \tag{5.60}$$

then

$$\begin{aligned} D_\mu u'_R = \partial_\mu u'_R - i g_3 \left(\frac{g_4 g^a_\mu - g_3 g'^a_\mu}{\sqrt{g_3^2 + g_4^2}} \right) T^a u'_R - 2/3 g_1 \left(\frac{g_4 B_\mu - g_1 \sqrt{\frac{2}{3}} Z'_\mu}{\sqrt{g_4^2 + \frac{2}{3} g_1^2}} \right) u'_R \\ \subset \partial_\mu u'_R - i \frac{g_4 g_3}{\sqrt{g_4^2 + g_3^2}} g^a_\mu T^a u'_R - \frac{2}{3} i \frac{g_4 g_1}{\sqrt{g_4^2 + \frac{2}{3} g_1^2}} B_\mu u'_R \end{aligned} \tag{5.61}$$

Comparing with $D_\mu u'_R = \partial_\mu u'_R - i g_s g^a_\mu T^a u'_R - \frac{2}{3} i g_Y B_\mu u'_R$, we conclude that the matching condition with the SM gauge couplings is

$$\begin{aligned} g_s &= \frac{g_4 g_3}{\sqrt{g_4^2 + g_3^2}}, \quad g_Y = \frac{g_1 g_4}{\sqrt{g_4^2 + \frac{2}{3} g_1^2}} \\ g_U &= g_4, \quad g_{g'} = \sqrt{g_4^2 - g_s^2}, \quad g_{Z'} = \frac{1}{2\sqrt{6}} \sqrt{g_4^2 - \frac{2}{3} g_Y^2} \end{aligned} \tag{5.62}$$

5.5 Yukawa Sector

In this section, we consider the flavour universal model developed in [1],[7] where all the would-be SM fermions denoted with prime are singlets under $SU(4)$ group and are charged under the $SU(3)' \times SU(2)_L \times U(1)'$ with the quantum numbers as in the SM (the hypercharge $U(1)_Y$ as their $U(1)'$ charge and $SU(3)_C$ as their $SU(3)'$). The would-be Sm fermions transform trivially under $SU(4)$ and hence we cannot construct the leptoquark interactions among them which are the source of

LFV. In order to have the leptoquark interaction with the would-be SM fermions, we introduce three vector-like heavy fermions $\Psi^i_{L,R}$ with $i = 1, 2, 3$ and assign the $SU(4) \times SU(2)_L$ structure to them. The fermion content is summarized in the table below:

Field	$SU(4)$	$SU(3)'$	$SU(2)_L$	$U(1)'$
q'^i_L	1	3	2	1/6
u'^i_R	1	3	1	2/3
d'^i_R	1	3	1	$-1/3$
ℓ'^i_L	1	1	2	$-1/2$
e'^i_R	1	1	1	-1
Ψ^i_L	4	1	2	0
Ψ^i_R	4	1	2	0

(5.63)

where $i = 1, 2, 3$ denotes the flavour index and $\Psi^i_{L,R}$ decomposed in the SM as $\Psi_{L,R} = \left(Q'_{L,R}, L'_{L,R}\right)^T$ in the representations: $Q'_{L,R} \sim (\mathbf{3}, \mathbf{2}, 1/6)$ and $L'_{L,R} \sim (\mathbf{1}, \mathbf{2}, -1/2)$.

The invariant Yukawa Lagrangian is

$$\begin{aligned}\mathcal{L}_Y = &-\bar{q}'_L Y_d H d'_R - \bar{q}'_L Y_u \tilde{H} u'_R - \bar{\ell}'_L Y_e H e'_R + \text{ h.c.} \\ &- \bar{q}'_L \lambda_q \Phi^T \Psi_R - \bar{\ell}'_L \lambda_\ell \xi^T \Psi_R - \overline{\Psi}_L \left(M + \lambda_{15}\Omega_{15}\right) \Psi_R + \text{ h.c.}\end{aligned} \tag{5.64}$$

The first line is the SM-like fermion lagrangian and the second line associates to the mixing among the SM-like and the vector-like fermions where $\tilde{H} = i\sigma_2 H^*$ and $Y_{u,d,e}, \lambda_{q,\ell,15}, M$ are 3×3 complex matrices.

As implied by the large global symmetries $U(3)^5$ of the SM-like fermion kinetic terms, one can transform each field by a unitary transformation in the basis that $Y_d = \lambda_d$, $Y_u = V^\dagger \lambda_u$, and $Y_e = \lambda_e$ where where $\lambda_d = \text{diag}\left(y_d, y_s, y_b\right), \lambda_u = \text{diag}\left(y_u, y_c, y_t\right)$ and $\lambda_e = \text{diag}\left(y_e, y_\mu, y_\tau\right)$ are diagonal matrices. The SM-like lagrangian becomes

$$\mathcal{L}_{\text{SM- like}} = -\bar{q}'_L V^\dagger \lambda_u u'_R \tilde{H} - \bar{q}'_L \lambda_d d'_R H - \bar{\ell}'_L \lambda_e e'_R H + \text{ h.c.} \tag{5.65}$$

We can write the mixing term in (5.64) as

$$\begin{aligned}\mathcal{L}_{\text{mix}} &= -\bar{q}'_L \lambda_q \Psi_R \Phi - \bar{\ell}'_L \lambda_\ell \Psi_R \xi - \overline{\Psi}_L \left(\hat{M} + \lambda_{15}\Omega_{15}\right) \Psi_R + \text{ h.c.} \\ &= -\bar{q}'_L V_q \lambda_q^{\text{diag}} U_q^\dagger \Psi_R \Phi - \bar{\ell}'_L V_l \lambda_l^{\text{diag}} U_l^\dagger \Psi_R \xi - \overline{\Psi}_L \left(\hat{M} + \lambda_{15}\Omega_{15}\right) \Psi_R + \text{ h.c.}\end{aligned} \tag{5.66}$$

We can redefine the V_q matrix to V_D and absorb the changes to $U_q^\dagger \to U_{q'}^\dagger$ and also redefine the V_l matrix to V_e and absorb the changes to $U_l^\dagger \to U_{l'}^\dagger$. We obtain

$$\mathcal{L}_{\text{mix}} = -\bar{q}'_L V_D \lambda_q^{\text{diag}} U_{q'}^\dagger \Psi_R \Phi - \bar{\ell}'_L V_e \lambda_l^{\text{diag}} U_{l'}^\dagger \Psi_R \xi - \overline{\Psi}_L \left(\hat{M} + \lambda_{15}\Omega_{15}\right) \Psi_R + \text{ h.c.} \tag{5.67}$$

One can transform:

$$q'_L \to V_D q'_L, \quad l'_L \to V_e l'_L, \quad \text{and} \quad \Psi_{L,R} \to U_{q'} \Psi_{L,R} \tag{5.68}$$

Finally, we have

$$\mathcal{L}_{\rm mix} = -\bar{q}'_L \lambda_q^{\rm diag} \Psi_R \Phi - \bar{\ell}'_L \lambda_l^{\rm diag} W^\dagger \Psi_R \xi - \overline{\Psi}_L \left(\hat{M} + \lambda_{15} \Omega_{15} \right) \Psi_R + \text{ h.c.} \tag{5.69}$$

where $W^\dagger = U_{l'}^\dagger U_{q'}$ and with assumption that $\lambda_{15} \propto \hat{M} \propto \mathbb{1}$. Following the assumptions given in [7], we have

$$\begin{aligned} \lambda_q^{\rm diag} &\equiv {\rm diag}\,(\lambda_1^q, \lambda_2^q, \lambda_3^q) \\ \lambda_l^{\rm diag} W^\dagger &\equiv {\rm diag}\left(\lambda_1^\ell, \lambda_2^\ell, \lambda_3^\ell\right) \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{LQ} & -\sin\theta_{LQ} \\ 0 & \sin\theta_{LQ} & \cos\theta_{LQ} \end{pmatrix} \end{aligned} \tag{5.70}$$

As in the PS model, we can represent the $\Psi_{L,R}$ in the matrix form as

$$\Psi_{L,R}^{\alpha i (\mathbf{f})} = \begin{pmatrix} U_1 & D_1 \\ U_2 & D_2 \\ U_3 & D_3 \\ V & E \end{pmatrix}_{L,R}^{(\mathbf{f})} \tag{5.71}$$

We consider the first term in (5.69), after getting VEV:

$$\begin{aligned} -\bar{q}'_L \lambda_q^{\rm diag} \Psi_R \Phi_{\rm VEV} + \text{ h.c.} &= \frac{v_3}{\sqrt{2}} \lambda_q^{\rm diag} (\bar{q}'_L)_i^c \Psi_R^{\alpha i} \delta_{\alpha c} + \text{ h.c.} = \frac{v_3}{\sqrt{2}} \lambda_q^{\rm diag} (\bar{q}'_L)_i^c (\Psi_R)_c^i + \text{ h.c.} \\ &= \frac{v_3}{\sqrt{2}} \lambda_q^{\rm diag} \left[\bar{u}_L U_R + \bar{d}_L D_R + \overline{U}_L u_R + \overline{D}_L d_R \right]^c \end{aligned} \tag{5.72}$$

where $c = 1,2,3$ is the colour index, $i = 1,2$ is $SU(2)_L$ index and $\alpha = 1,2,3,4$ is the $SU(4)$ index. Repeating the same procedure for the other terms, we obtain the 6×6 mass matrices:

$$\begin{aligned} \mathcal{M}_u &= \begin{pmatrix} V^\dagger \lambda_u \frac{v}{\sqrt{2}} & \lambda_q^{\rm diag} \frac{v_3}{\sqrt{2}} \\ 0 & \hat{M}_Q \end{pmatrix}, \quad & \mathcal{M}_d &= \begin{pmatrix} \lambda_d \frac{v}{\sqrt{2}} & \lambda_q^{\rm diag} \frac{v_3}{\sqrt{2}} \\ 0 & \hat{M}_Q \end{pmatrix} \\ \mathcal{M}_N &= \begin{pmatrix} 0 & \lambda_\ell^{\rm diag} \frac{v_1}{\sqrt{2}} \\ 0 & \hat{M}_L \end{pmatrix}, & \mathcal{M}_e &= \begin{pmatrix} \lambda_e \frac{v}{\sqrt{2}} & \hat{\lambda}_\ell W^\dagger \frac{v_1}{\sqrt{2}} \\ 0 & \hat{M}_L \end{pmatrix} \end{aligned} \tag{5.73}$$

For example, the $\mathcal{M}_e$ is in the basis $(e, \mu, \tau, E^1, E^2, E^3)$ where

$$\hat{M}_Q = \hat{M} + \frac{\lambda_{15} v_{15}}{2\sqrt{6}}, \quad \text{and} \quad \hat{M}_L = \hat{M} - \frac{3\lambda_{15} v_{15}}{2\sqrt{6}} \tag{5.74}$$

The interaction of the vector like fermions with leptoquark U_μ can be obtained by considering the kinetic term of the fermion lagrangian together with covariant derivative as we have in the PS model:

$$i\overline{\Psi}_L\gamma^\mu D_\mu \Psi_L \supset \frac{g_4}{\sqrt{2}} U_\mu \overline{Q}_L \gamma^\mu W L_L = \frac{g_4}{\sqrt{2}} U_\mu \overline{Q}_L \gamma^\mu \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{LQ} & \sin\theta_{LQ} \\ 0 & -\sin\theta_{LQ} & \cos\theta_{LQ} \end{pmatrix} L_L \tag{5.75}$$

Upon diagonalization of the fermion mass matrices (5.73), the mass eigenstates are the combinations of $(e, \mu, \tau, E^1, E^2, E^3)$. So, the SM quark and lepton interact with the leptoquark as

$$\frac{g_4}{\sqrt{2}} \beta_{ij} U_\mu \bar{q}^i_L \gamma^\mu \ell^j_L \tag{5.76}$$

where

$$\beta = \mathrm{diag}\,(s_{q_{12}}, s_{q_{12}}, s_{q_3})\, W\, \mathrm{diag}\,(0, s_{\ell_2}, s_{\ell_3}) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & c_{\theta_{LQ}} s_{q_{12}} s_{\ell_2} & s_{\theta_{LQ}} s_{q_{12}} s_{\ell_3} \\ 0 & -s_{\theta_{LQ}} s_{q_3} s_{\ell_2} & c_{\theta_{LQ}} s_{q_3} s_{\ell_3} \end{pmatrix} \tag{5.77}$$

6 Conclusions

The lepton flavour universality (LFU) is not only the global symmetries of the kinetic lagrangian of the SM but also the physics probing a way to new physics beyond SM as well. The violation of the lepton flavour universality enlarges knowledge on how we understand the flavour puzzle (the hierarchical structure of Yukawa) together with the stabilization of the Higgs mass. U_1 leptoquark is the component that we need to construct the UV-complete theory which predicts the existence of this gauge particle suggested from the low-energy data of the effective single mediator simplified model. We identify it as the best single mediator for B-anomalies which couple quark and lepton directly.

The Pati-Salam model is the first model that contains the right quantum numbers of the leptoquark and gives mass to its after Higgs gets VEVs. Since we unify the quark and lepton together in a single representation under $SU(4)$ group, the fermions content will inherit the structure of the $SU(4)$ as well and hence the fermions can couple to the leptoquark directly. However, two serious problems of the PS model are 1. It predicts the wrong fermion spectrum and 2.The interaction between quark and lepton via leptoquark when we consider the semileptonic meson decay will give the constraint on the mass of leptoquark which is beyond 200 TeV scale that is not of our interest.

We extend the colour part of the PS model to get the 4321 model. Since the 4321 model contain $SU(4)$ gauge group, it can also have the exotic leptoquark with the right QM numbers as we have in the PS model. With the appearance of the mixing term which breaks the global $U(1)$ symmetry, we have a consistent number of goldstone bosons and gauge bosons corresponding to the broken generators. The Yukawa structure gives rise to the definitions of the coupling mixing matrix for the leptoquark interactions in our flavour universal model since by introducing the vector-like fermion we have more freedom to define the CKM structure than in the PS model in order to match the phenomenology of the low-energy physics.

Another model that we can consider related closely to this flavour universal 4321 model is [2] where they give the $SU(4)$ charge to the third generation fermion and hence flavour non-universal from the beginning. Also, in the three-site model where we give PS groups belonging to the different generations and break them with the different scales we can have a non-universal structure from the starting point.

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