Imperial College of Science, Technology and Medicine
Department of Physics

Shift on W boson mass

I Cheong Hong

Submitted in part fulfilment of the requirements for the degree of
Msc QFFF
September 2022

Abstract

In April 2022, the CDFII experiment that measures the mass of w boson shows anomalous among previous experiments and the Standard model prediction. In this master thesis, we have gone through the background knowledge which is needed to understand modern W boson analysis. This thesis gave an introduction to both experimental aspects and theoretical calculations. The experimental side includes the setup of the CDFII experiment and methodology, while the theoretical side mentioned loop correction calculation, and the most important – SMEFT(Standard model effective field theory). SMEFT is used in [4] to analyse the shift in the W boson mass. The aim of this thesis is to give the reader knowledge of the W boson analysis so the reader is capable to understand the academic papers that is on this topic.

Acknowledgements

I would like to express my best thanks to:

- My supervisor Prof.Kelly Kellogg Stelle
- All of my colleagues in the QFFF
- My family and friends

Contents

List of Figures

Chapter 1

Introduction

1.1 Motivation and Background

In order to examine the Standard Model, physicists have been working on the precise measurement of fundamental particles for many years. For example, the observation of the Higgs boson in 2012 by LHC(Large Hardon Collider) is a piece of strong evidence for the standard model. However, a recent report [1] from CDF (Collider Detector at Fermilab) Collaboration proposed that the mass of W boson measured in CDF II is $M_w = 80433.5 \pm 9.4$ MeV, which is different compared to the Standard model predicted value, 80354 ± 7MeV. This is interesting because the data is taken at the most precise measurement ever. There are many possible reasons for this mass shift in both theoretical and experimental aspects. In this thesis, I provided an overview of the CDF II experiment which **hugely** based on the CDF II report [1]as well as an introduction to theoretical calculation for the mass of W boson at the level of Master students. Steps for some equation are inlcuded.

Chapter 2

Experimental Aspect— CDF II result

2.1 Introduction

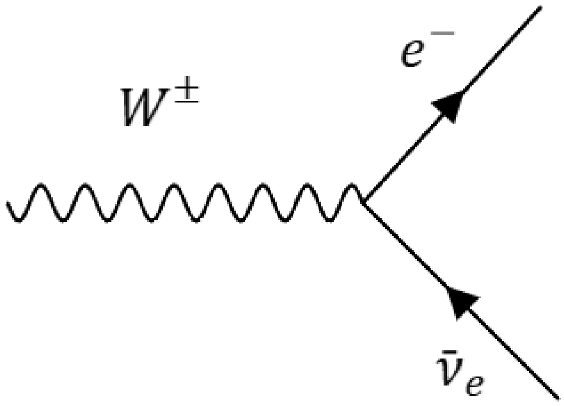
CDF(Collider Detector at Fermilab) [15] is one of the detectors located at the Tevatron accelerator ring at Fermilab, Chicago. When proton and antiproton that accelerate to the centre of mass energy 1.96 Tev [1] by the Tevatron collide, they produce many subatomic particles during the collision. The CDF tracks the product particle using 7 layers of silicon at the innermost layer in order to measure the momentum. The second layer act as a calorimeter, which is used to measure the energy of the resulting hardon. The third layer is a muon detector which measures the particles that did not absorb by the calorimeter. These three parts combine and give the data which can be used to analyse what happens during the collision. In the CDFII experiment, detectors are calibrated by comparing the mass of the J/ψ meson to the experimental value.

2.2 Measurement of W boson

2.2.1 W boson decay

However, the experiment does not directly take the measurement of the W boson. In the CDFII measurement, they analyse a high purity sample of lepton decay data, which is the process that W boson decay to lepton (Figure 2.1) $W \to e + \bar{\nu_e}$ and $W \to \mu + \bar{\nu_\mu}$. The mass of the w boson is reconstructed using the kinematic data of electrons and muons. The branching ratios [12]for these two process are $0.1046 \pm 0.0042 \pm 0.0014$ for electron and $0.1050 \pm 0.0041 \pm 0.0021$for muon.

Figure 2.1: W boson decay to leptons – electron example



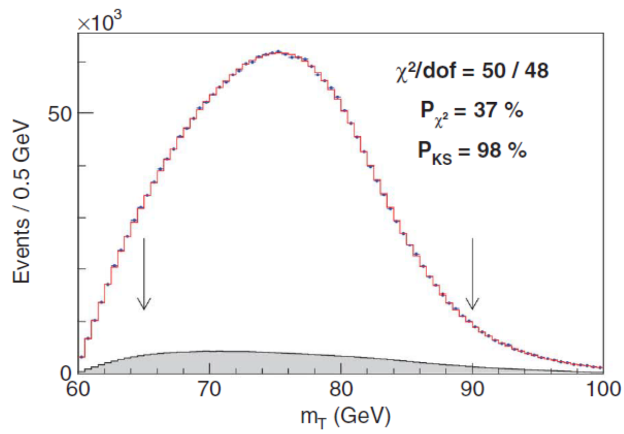
2.2.2 Trigger system

In order to obtain useful information and a pure sample for $W \to e_\nu$ and $W \to m\nu$, a trigger system is applied to filter the data. Only Muon with p_T (Transverse momentum) > 18 GeV or electron $E_T > 18$ GeV will be recorded by the online trigger system. Those data are downloaded for offline analysis. In the offline selection part, electron must have $p_T > 18$ GeV and energy $E_T > 30$ GeV. Muon must have $p_T > 30$ GeV, and both electron and muon's COT (central outer tracking drift chamber) track must meet requirements for quality. The cosmic-ray muons and Z boson event data are being rejected from the data.

2.2.3 W boson mass reconstruction

The w boson mass is reconstructed using kinematics data of the product leptons. Because the longitude momentum of neutrino is not measurable, so what we reconstruct here is the transverse momentum and transverse mass.(The transverse momentum of the neutrino p_T^{ν} can be inferred by the conservation of transverse momentum.) This reconstruction uses one called the 'Transverse momentum method' [19]. The transverse mass of the W boson is given by $m_T = \sqrt{2(p_T^e p_T^{\nu} - \vec{p_T^e} \cdot \vec{p_T^{\nu}})}$. The steps are given in the appendices A. The resulting reconstructed m_T is a distribution, which will peak at m_w (so-called Jacobian Peak). (For details see appendices B) The figure 2.2shows the m_T distribution measured in the CDF II experiment.

Figure 2.2: m_T distribution at CDF II experiment [1]



2.2.4 Monte Carlo – RESBOS

In order to infer the w boson rest mass from transverse mass distribution, a custom Monte Carlo simulation(RESBOS, Resummation for Bosons) [5] is used. It takes boson mass, and transverse momentum as input(require coupling constant a_s as external input) and performs next to leading order QCD calculation. The output is the differential cross-section of the process. Here is the abstract of the Monte Carlo:

1. take a mass value as the starting point of the program

2. use the code to generate m_T and p_T distribution.

3. Compare to the experimental data, repeat to get the best fit

This Monte Carlo uses NNPDF3.1 [6] as the Parton distribution function (PDF), which denotes a 3.9 MeV PDF uncertainty to the inferred W boson mass.

2.2.5 Result

In order to make sure the measurement is correct, CDF II also measures the mass of the Z boson. The result for the Z boson is $m_z = 91194.3$ MeV, which is consistent with the world average. Fig.2.2 shows the resulting data. The blue points are experimental data and the red indicates the best fit for Monte Carlo. The resulted W boson mass is $m_w = 80433.5 \pm 9.4$MeV, which is the most precise measurement ever(Fig 2.4) , shows a 7 σ difference compare to the standard model prediction. This experiment also shows a large deviation from the previous experiment.(Fig.2.3) Thus, it might be a hint to theory beyond the standard model.

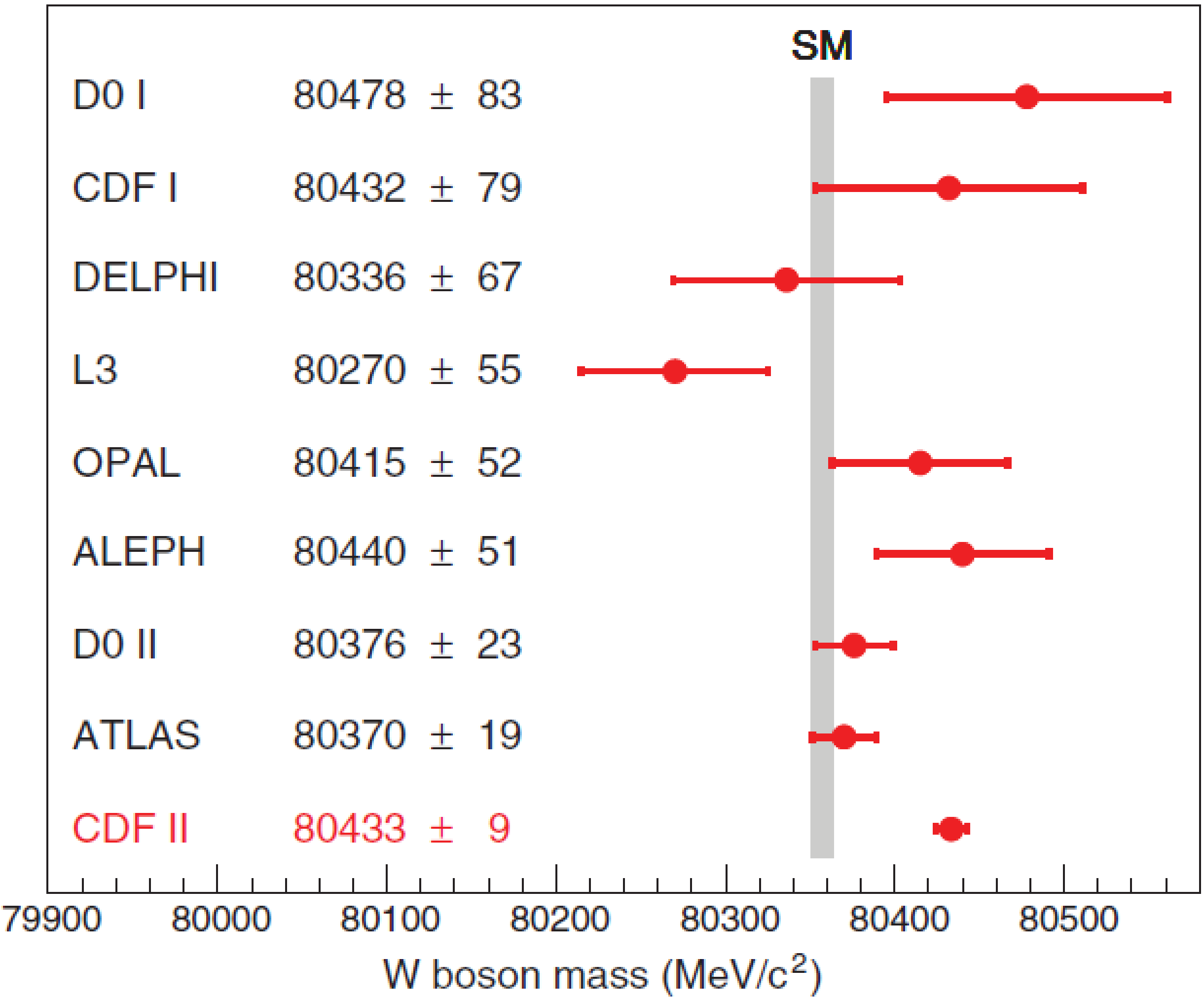


Figure 2.3: Comparison between CDF II data and previous experiment [1]

Source	Uncertainty (MeV)
Lepton energy scale	3.0
Lepton energy resolution	1.2
Recoil energy scale	1.2
Recoil energy resolution	1.8
Lepton efficiency	0.4
Lepton removal	1.2
Backgrounds	3.3
p_T^Z model	1.8
p_T^W/p_T^Z model	1.3
Parton distributions	3.9
QED radiation	2.7
W boson statistics	6.4
Total	9.4

Figure 2.4: Summary of uncertainty [1]

Chapter 3

Introduction to the Standard model

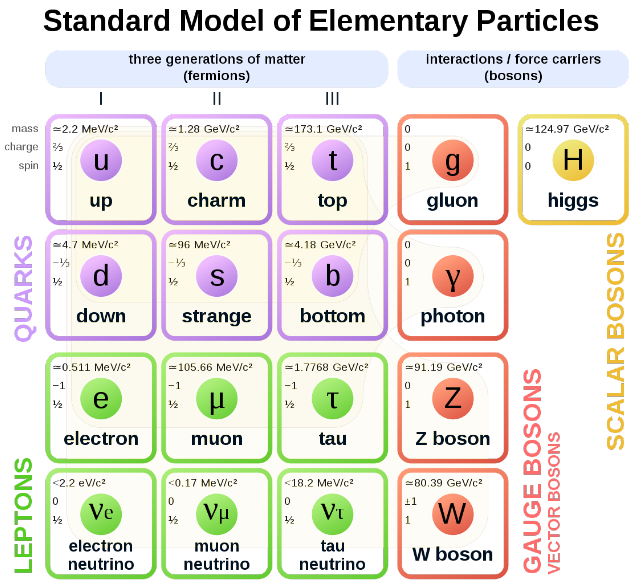
Before going to cutting-edge theory describing the shift of the W boson mass, I re-introduce the 'standard' standard model here for readers unfamiliar with particle physics or needing a recap on it. Also, I will define notation and convention in this chapter.

3.1 The Standard Model

The standard model is the most successful model in physics. It states that fundamental particles make up everything in the world, and particles are classified into two classes: boson and fermion. Fermion constructs matter and the boson is the force carrier. Fig3.1 shows all the fundamental particles in the standard model.

There are four fundamental forces in nature. Gravity, strong force, weak force, electromagnetic force. The standard model describes all of them well except gravity, and the rest of the fundamental forces are described by quantum fields. The standard $SU(3) \times SU(2)_L \times U(1)$ Lagrangian describe all of these quantum fields, which are:

Figure 3.1: Particle zoo



$$
\begin{aligned}
\mathcal{L} = & -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - -\frac{1}{4}W^{l}_{\mu\nu}W^{l\mu\nu} - \frac{1}{4}G^{a}_{\mu\nu}G^{a\mu\nu} \\
& + (D_\mu\phi)^\dagger(D_\mu\phi) + m^2\phi^\dagger\phi - \frac{1}{2}\lambda^2(\phi^\dagger\phi)^2 \\
& + \sum_{f=1}^{3}(\bar{l}^f_L i\not{D}l^f_L + \bar{l}^f_R i\not{D}\bar{l}^f_R + \bar{q}^f_L i\not{D}\bar{q}^f_L + \bar{d}^f_R i\not{D}\bar{d}^f_R + \bar{u}^f_R i\not{D}\bar{u}^f_R \\
& - \sum_{f=1}^{3} y^f_l(\bar{l}^f_L\phi l^f_R + h.c.) \\
& - \sum_{f,g=1}^{3}(y^{fg}_d\bar{q}^f_L\phi d^g_R + y^{fg}_u\bar{q}^f_L\tilde{\phi}u^g_R + h.c.)
\end{aligned}
\tag{3.1}
$$

3.1.1 Definitions and notations

And the definition of notations above are :

Gauge fields

U(1) gauge field :

$$
\begin{aligned}
& B_\nu, \\
& B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu
\end{aligned}
\tag{3.2}
$$

SU(2) gauge field :

$$
\begin{aligned}
& W^l_\nu, l = 1 \ldots 3 \\
& W^l_{\mu\nu} = \partial_\mu W^l_\nu - \partial_\nu W^l_\mu + g\epsilon_{ljk} W^j_\mu W^k_\nu
\end{aligned}
\tag{3.3}
$$

SU(3) gauge field :

$$
\begin{aligned}
& G^a_\nu, a = 1 \ldots 8 \\
& G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f_{abc} G^b_\mu G^c_\nu
\end{aligned}
\tag{3.4}
$$

$\mu\nu$ are Lorentz indices, a is the SU(3) indices, l is the SU(2) indices. f_{abc} is the SU(3) generator, g and g_s is the coupling constant for SU(2) and SU(3) gauge field. ϵ_{ljk} is the anti-symmetric tensor. Dual gauge field is defined as $\tilde{X} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}X^{\rho\sigma}$, where X is the gauge field.

matter fields

In the Standard Model, fermion has three-generation, and I labelled them as f and g in the summation. The colour indices are ignored for quark fields.L/R means the handedness of the particle. The lepton fields is the following:

$$l^1_L = \left(\begin{smallmatrix} \nu_{eL} \\ e_L \end{smallmatrix} \right) \tag{3.5}$$

$$l^2_L = \left(\begin{smallmatrix} \nu_{\mu L} \\ \mu L \end{smallmatrix} \right) \tag{3.6}$$

$$l^3_L = \left(\begin{smallmatrix} \nu_{\tau L} \\ \tau L \end{smallmatrix} \right) \tag{3.7}$$

$$l^1_R = e_R \tag{3.8}$$

$$l^2_R = \mu_R \tag{3.9}$$

$$l^2_R = \tau_R \tag{3.10}$$

and the quark field :

$$q_L^f = \begin{pmatrix} u_L^f \\ d_L^f \end{pmatrix} \tag{3.11}$$

The right-handed up/down type quark is written as u_R^f and d_R^f. The Yukawa couplings is y_e^f

Higgs field and covariant derivative

Define the covariant derivative : $D_\mu = \partial_u - ig_s\frac{1}{2}\lambda^a G_\mu^a - ig\frac{1}{2}\tau^l W_\mu^l - ig^{'} Y B_\mu$

The Higgs doublet ϕ

The Higgs vacuum expectation value is $\phi_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$

where $v^2 = \frac{2\mu^2}{\lambda} = \frac{1}{\sqrt{2}G_F} =$. G_F is the Fermi constant. Experimentally, $v = 246.22$GeV.

3.2 Masses of gauge boson and Higgs mechanism

In the Lagrangian, the mass term is the coefficient that appears with the field's quadratic (For example, the mass term of Higgs field $m^2\phi^\dagger\phi$). It is clear that in Eq.3.1 does not have an explicit mass term(as well as field mixing) for the gauge boson. In order to get the masses for physical bosons, the Higgs mechanism and electroweak symmetry breaking is needed. The breaking electroweak symmetry pattern is SU(2)×U(1)⟶U(1). Below is an abstract of the process:

Consider SM Lagrangian with only $B_{\mu\nu}, W_{\mu\nu}$ gauge boson and Higgs field (other is not relevant yet):

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W_{\mu\nu}^l W^{l\mu\nu} \\ & + (D_\mu\phi)^\dagger(D_\mu\phi) + m^2\phi^\dagger\phi - \frac{1}{2}\lambda^2(\phi^\dagger\phi)^2 \end{aligned} \tag{3.12}$$

where

$$D_\mu = \partial_u - ig\frac{1}{2}\tau^l W^l_\mu - ig^{'}YB_\mu \tag{3.13}$$

Choose the vev $\phi_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$

, so that the $m^2\phi^\dagger\phi - \frac{1}{2}\lambda^2(\phi^\dagger\phi)^2$ vanishes.

Expand the $(D_\mu\phi)^\dagger(D_\mu\phi)$

$$(D_\mu\phi)^\dagger(D_\mu\phi) = |(\partial_u - ig\frac{1}{2}\tau^l W^l_\mu - ig^{'}YB_\mu)\frac{1}{\sqrt{2}}\left(\begin{smallmatrix} 0 \\ v \end{smallmatrix}\right)|^2 \tag{3.14}$$

The hypercharge $Y(\phi) = \frac{1}{2}$

$$\begin{aligned}
&= \frac{1}{8}|(-ig\tau^l W^l_\mu - ig^{'}B_\mu)\left(\begin{smallmatrix} 0 \\ v \end{smallmatrix}\right)|^2 \\
&= \frac{v^2}{8}|(-ig\tau^l W^l_\mu - ig^{'}B_\mu)\left(\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right)|^2 \\
&= \frac{v^2}{8}|(-ig(\left(\begin{smallmatrix} 0 & W^1 \\ W^1 & 0 \end{smallmatrix}\right) + \left(\begin{smallmatrix} 0 & -iW^2 \\ iW^2 & 0 \end{smallmatrix}\right) + \left(\begin{smallmatrix} W^3 & 0 \\ 0 & -W^3 \end{smallmatrix}\right)) - ig^{'}\left(\begin{smallmatrix} B & 0 \\ 0 & B \end{smallmatrix}\right))\left(\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right)|^2 \\
&= \frac{v^2}{8}|\left(\begin{smallmatrix} gW^1 - igW^2 \\ g^{'}B - gW^3 \end{smallmatrix}\right)|^2 \\
&= \frac{v^2}{8}(g^2((W^1_\mu)^2 + (W^2_\mu)^2) + (gW^3_\mu - g'B_\mu)^2)
\end{aligned}$$

This term can be written in a mass matrix form $\frac{v^2}{8}\begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & -gg^{'} \\ 0 & 0 & -gg^{'} & g^{'2} \end{pmatrix}\begin{pmatrix} W^1_\mu \\ W^2_\mu \\ W^3_\mu \\ B_\mu \end{pmatrix}$

Because there is a mix between W^3 and B, diagonalization of the mass matrix is needed to obtain a physical boson.

If diagonalize directly,

$$\lambda_1 = 0 \qquad c^1 = \begin{pmatrix} 0 \\ 0 \\ sin\theta \\ cos\theta \end{pmatrix}$$

$$\lambda_2 = \frac{g^2v^2}{8} \qquad c^2 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\lambda_3 = \frac{g^2v^2}{8} \qquad c^3 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\lambda_4 = \frac{(g^2+g^{'2})v^2}{8} \qquad c^3 = \begin{pmatrix} 0 \\ 0 \\ cos\theta \\ -sin\theta \end{pmatrix}$$

where define the Weinberg angle $cos\theta = \frac{g}{\sqrt{g^{'2}+g^2}}$

Define the following physical fields :

$W^{\pm} = \frac{1}{\sqrt{2}}(W^1\mu \mp iW^2_\mu)$

$A_\mu = \frac{1}{\sqrt{g^2+g^{'2}}}(gW^3_\mu + g'B_\mu)$

$Z_\mu = \frac{1}{\sqrt{g^2+g^{'2}}}(gW^3_\mu - g'B_\mu)$

where $W^{\pm}$ is the W bosons, Z_μ is the Z boson and A_μ is the photon field.

Substitute the fields defined above back to the original mass matrix

$W_\mu^+ W^{-\mu} = \frac{1}{2}((W_\mu^1)^2 + (W_\mu^2)^2)$,

the mass term $\frac{v^2 g^2}{8}((W_\mu^1)^2 + (W_\mu^2)^2) = \frac{v^2 g^2}{4} W_\mu^\pm W^{\mp\mu} = m_w^2 W_\mu^\pm W^{\mp\mu}$, so $m_w = \frac{vg}{2}$

For the Z boson:

$\frac{(g^2+g^{'2})v^2}{8}(cos\theta W_\mu^3 - sin\theta B_\mu)^2 = \frac{(g^2+g^{'2})v^2}{8} Z_\mu Z^\mu = \frac{m_z^2}{2} Z_\mu Z^\mu$

$m_z = \frac{\sqrt{(g^2+g^{'2})}v}{2}$

The $\mathcal{L}$ after electroweak symmetry breaking and in terms of physical fields is:

$$\begin{aligned} \mathcal{L}_0 = & -\frac{1}{4} A_{\mu\nu} A^{\mu\nu} - \frac{1}{2} W_{\mu\nu}^+ W^{-\mu\nu} - \frac{1}{4} Z_\mu Z^\mu \\ & + m_w^2 W_\mu^+ W^{-\mu} + \frac{m_z^2}{2} Z_\mu Z^\mu \end{aligned} \tag{3.15}$$

Chapter 4

Loop correction and accurate prediction

However, the mass of the W boson calculated above is the bare mass(or the mass in Lagrangian). It is not the physical mass. In order to get an accurate prediction, loop correction is needed. For example, the tree-level prediction of $(m_w)_{tree} = 80.939GeV$ and the averaged experimental value$(m_{wexp}) = 80.3$ (From PDG) is different by many S.D. For the purpose of introduction, a scalar field example here is introduced. The example below is based on the lecture notes from the QED course at Imperial College London.

4.1 Scalar field example

Consider two point function for scalar field ϕ in ϕ^4 theory (ignore the wave function renormalization here, which set $Z_0 = 1$):

$$F(p) = \int d^4xe^{ipx} \langle\Omega| \mathcal{T}\phi(x)\hat{\phi(0)} |\Omega\rangle \tag{4.1}$$

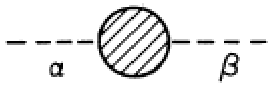


Figure 4.1: For example 1 PI, from [18]

Assume it can be written in propagator form

$$F(p) \sim \frac{i}{p^2 - m^2 + i\epsilon} \tag{4.2}$$

The pole here is $p^2 \sim m^2$, where defined the physical mass m. The two-point function can also be written in the form:

$$F(p) = F_0(p) + F_0(p)(-i\Pi(p))F_0(p)$$

where $-i\Pi(p)$ is the sum of all amputated diagrams (removed all external propagators) and $F_0(P)$ is

$$F_0(p) = \frac{i}{p^2 - m_0^2 + i\epsilon}$$

m_0 is the bare mass.

The $-i\Pi(p)$ can also be seen as the sum of all amputated 1-PI (1 particle irreducible diagram) (Fig.4.1)

$$-i\Pi(p) = -i\Sigma(p) + (-i\Sigma(p))F_0(p)(-i\Sigma(p)) + (-i\Sigma(p))F_0(p)(-i\Sigma(p))F_0(p)(-i\Sigma(p)) + ... \tag{4.3}$$

So that

$$
\begin{aligned}
F(p) &= F_0(p) + F_0(p)(-i\Pi(p))F_0(p) \\
&= F_0(p)(1 + (-i\Pi(p))F_0(p)) \\
&= F_0(p)(1 + (-i\Sigma(p)F_0(p) + ((-i\Sigma(p))F_0(p))^2 + ...) \\
&= F_0(p)\frac{1}{1 + i\Sigma(p)F_0(p)} \\
&= \frac{i}{F_0(p)^{-1} - \Sigma(p)} \\
&= \frac{i}{p^2 - m_0^2 + i\epsilon - \Sigma(p)}
\end{aligned}
\tag{4.4}
$$

Compare to 4.2, we can identify the physical mass:

$$p^2 - m_0^2 + i\epsilon - \Sigma(p) = 0_{|p^2=m^2}$$

The $\Sigma(p)$ is also called self-energy. This method is also called on-shell renormalization.

4.2 W boson 1 loop correction

However, when we try to renormalize the W boson observable, on-shell renormalization is not the best choice. It is because a usual renormalization always alter the experimental observable $e, g, g^{'}, m_w, m_z, \theta_w$. For the purpose of simplicity and applicability, a new renormalization scheme is used.[18]. Many physicists choose the input parameters as G_F, m_z, α because these three can be measured in experiments very well. Below is a short introduction to this scheme. Note that quantity with a lower indices 0 is renormalized quantity, or called bare quantity. Begin with the Standard model Lagrangian after electroweak SSB:

$$\mathcal{L}_m^{VB} = \frac{v_0^2}{2}(\frac{g_0^2}{2}W_\mu^+ W^{-\mu} + \frac{1}{4}(g_0 W_\mu^3 - g_0' B_\mu)^2) \tag{4.5}$$

where all the fields and coupling constant are renormalized. To generate counterterms, one alters the bare coupling constant:

$$g_0 = g - \delta g$$
$$v_0^2 = v^2 - \delta v^2$$

And the Lagrangian 4.5 can be written in :

$$L_m^{VB} = \frac{v^2 - \delta v^2}{8}[(A_\mu(g'c - gs) - Z_\mu(gc + g's) - \delta g'(cA_\mu - sZ_\mu) + \delta g(cZ_\mu + sA_\mu)]^2 \tag{4.6}$$
$$+ \frac{(v^2 - \delta v^2)(g - \delta g)^2}{4} W_\mu^+ W^{-\mu} \tag{4.7}$$

where the c=cosθ_w, s=sinθ_w. θ_w is the Weinberg angle that $tan(\theta) \equiv \frac{g'}{g}$.
Ignore higher order terms $O(((\delta g)^2, (\delta g')^2, (\delta g)(\delta g'))$:

$$\mathcal{L}_m^{VB} = \frac{m_z^2 - \delta m_z^2}{2} Z_\mu Z^\mu + \delta m_{za}^2 Z_{m\mu} A^\mu + (m_w^2 - \delta m_w^2) W_\mu^+ W^{-\mu} \tag{4.8}$$
$$\delta m_z^2 = (g^2 + g'^2)\frac{\delta v^2}{4} + \frac{v^2}{4}\delta(g^2 + g'^2) \tag{4.9}$$
$$\delta m_w^2 = \frac{v^2 \delta g^2 + g^2 \delta v^2}{4} \tag{4.10}$$
$$\delta m_{za}^2 = \frac{m_z^2}{(g^2 + g'^2)^{(\frac{1}{2})}}(c\delta g' - s\delta g) \tag{4.11}$$

The field rescaling and renormalisation are ignored here due to the reason of keeping Largangian simple. For the purpose of studying a physical mass matrix, it is sufficient to use such a scheme. In order to identify the physical masses of Z and W bosons, need to introduce the unrenormalize boson self-energy:

$$\Pi_{zz}^{\mu\nu}(q^2) = A_{zz}(q^2)g^{\mu\nu} + B_{zz}(q^2)q^\mu q^\nu \tag{4.12}$$
$$\Pi_{zz}^{\mu\nu}(q^2) = A_{ww}(q^2)g^{\mu\nu} + B_{ww}(q^2)q^\mu q^\nu \tag{4.13}$$

Figure 4.2: t_{zz} and t_{ww} term [18]

where the $\Pi_{zz}^{\mu\nu}$ is the same as defined 4.3.(Also refer to Fig4.1) One can write the amputated propagator in 4.12 form because the boson propagator is written in the form

$$D^{\mu\nu}(k) = -\frac{g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}}{k^2 - m^2 + i\epsilon} \tag{4.14}$$

To obtain the physical mass m_w and m_z, choose

$$\delta m_z^2 = ReA_{zz}(m_z^2) + t_{zz} \tag{4.15}$$

$$\delta m_w^2 = ReA_{ww}(m_w^2) + t_{ww} \tag{4.16}$$

The t_{zz} and t_{ww} are the contribution from a tadpole and tadpole counter-term of the Higgs field (Fig 4.2). One can choose the vacuum expectation value so the t_{zz} and $t_{ww} = 0$

Combing the equations above, we can find

$$\delta m_{zA}^2 = \frac{m_w^2}{2sc} Re(\frac{A_{zz}(m_z^2)}{m_z^2} - \frac{A_{ww}(m_w^2)}{m_w^2} + O(\alpha^2)) \tag{4.17}$$

This counterterm is very important in analysing hadronic contribution to $Z\gamma$ mixing self-energy. (since it comes with the A and Z fields in the Lagrangian).

The above renormalization does not compete. We still need to consider boson-quark interaction and renormalization of electric charge. They are all done in [18]. In the next section, we jump to the most important conclusion and prediction of this scheme, loop correction for muon decay.

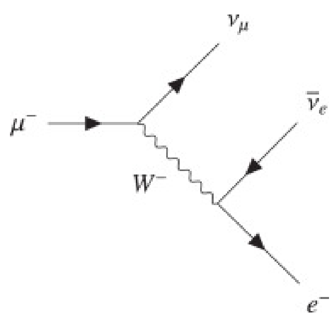


Figure 4.3: Example of Muon decay [16]

4.3 Loop correction for muon decay

Muon decay is important because this process gives important hints and constraints to the W boson and Fermi constant mass. The Feynman diagram for tree level muon decay is 4.3 From this process one can deduce an important and famous relation (note the g here is the physical g):

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_w^2} \tag{4.18}$$

To proceed with the process of radiation correction, we need to sum over all possible Feynman diagrams in 1-loop order. Fig.4.4 shows the self-energy, tadpole and counterterm contribution to the muon decay. The sum in this section is:

$$M_4 = M^0[\frac{A_{ww}(q^2) - ReA_{ww}(m_w^2)}{q^2 - m_w^2} - \frac{2\delta e}{e} + \frac{c^2}{s^2}Re(\frac{A_{zz}(m_z^2)}{m_z^2} - \frac{A_{ww}(m_w^2)}{m_w^2})] \tag{4.19}$$

where

$$M^0 = -\frac{g^2}{2}(\bar{u}_{v_\mu}\gamma_\mu a_- u_\mu)(\bar{u}_e\gamma_\mu a_- \nu_{\nu_e})\frac{-i}{q^2 - m_w^2} \tag{4.20}$$

is the tree-level propagator. e is the EM coupling,

$$e = gs$$

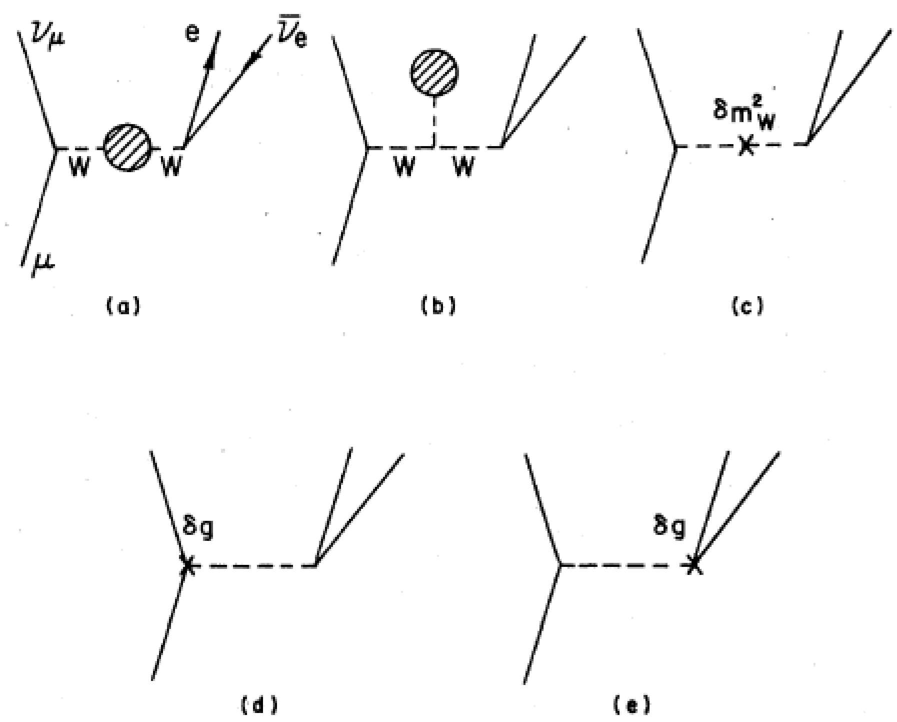


Figure 4.4: Self-energy part [18]

and the counter term

$$\delta e = c^3 \delta g' + s^3 \delta g \tag{4.21}$$

.

This counterterm is adjusted to cancel correction to photon emission. Fig.4.5 shows all diagrams that have 1-loop correction at the interaction vertex. The contribution of those diagrams are:

$$M_5 = M^0 \frac{g^2}{16\pi^2}[\frac{c^2}{s^2}(1+c^2)lnc^2 + 2 \tag{4.22}$$

$$- 64\pi^2 ic^2 \int_n \frac{1}{(k^2 - m_z^2)(k^2 - m_w^2)} \tag{4.23}$$

$$- 64\pi^2 is^2 \int_n \frac{1}{(k^2)(k^2 - m_w^2)} \tag{4.24}$$

The circle in the vertex means all possible ways virtual bosons are attached to the fermion line. In addition, 2 more Feynman diagrams called boxed diagrams that showed in Fig.4.6 needed to be considered. The contributions are:

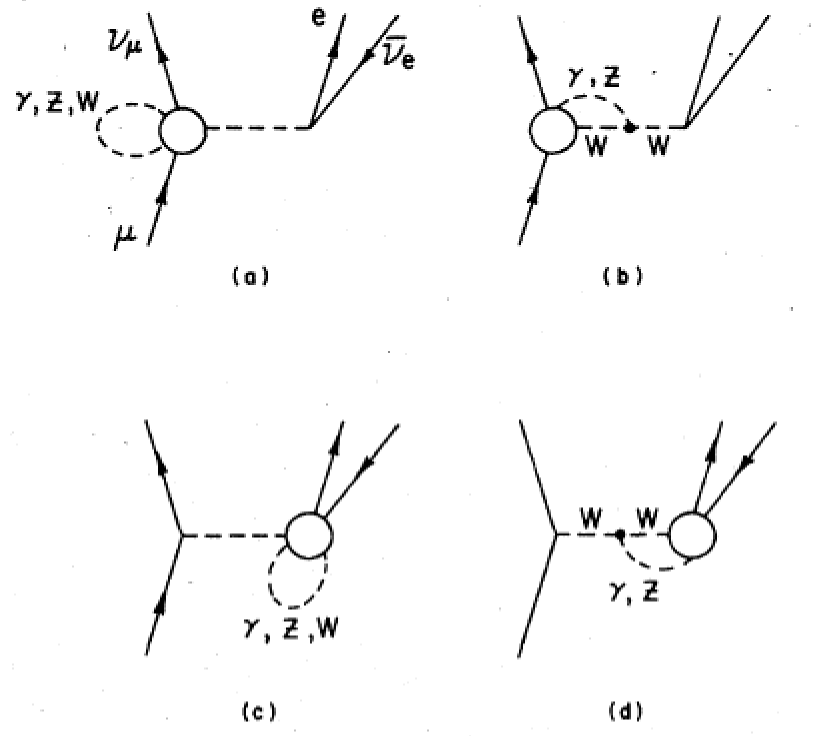


Figure 4.5: Vertex diagram at 1loop order [18]

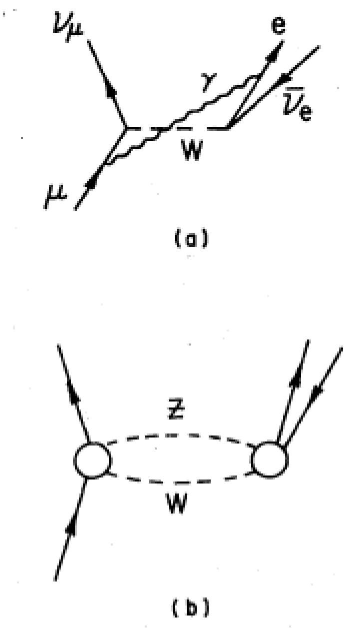


Figure 4.6: In addition Feynman diagram (Box diagram)[18]

$$M_{box} = M^0 \frac{g^2}{32\pi^2} c^2 ln(\frac{1}{c^2})(5\frac{c^2}{s^2} - 3\frac{s^2}{c^2}) \tag{4.25}$$

Adding Eq.4.22 4.19 4.25,

$$M = M^0 \Delta r$$

And use a redefinition of coupling g to absorb the correction(Note that in this step the usual definition of G_F changes due to a different choice of renormalization scheme):

$$\hat{g}^2 = g^2[1 + \Delta r] \tag{4.26}$$

Combine it to 4.18

$$\begin{aligned} m_w^2 &= \frac{\sqrt{2}}{8} \frac{\hat{g^2}}{G_F} \\ &= \frac{\sqrt{2}}{8} \frac{g^2(1+\Delta r)}{G_F} \\ &= \frac{\sqrt{2}}{8} \frac{e^2(1+\Delta r)}{s^2 G_F} \\ &= \frac{\sqrt{2}}{8} \frac{4\pi\alpha(1+\Delta r)}{s^2 G_F} \\ &= \frac{\pi\alpha(1+\Delta r)}{s^2 G_F \sqrt{2}} \\ &= \frac{\pi\alpha(1+\Delta r)}{(1-c^2) G_F \sqrt{2}} \\ &= \frac{\pi\alpha(1+\Delta r)}{(1-\frac{m_w^2}{m_z^2}) G_F \sqrt{2}} \end{aligned}$$

And we arrived at the famous 1-loop correction equation:

$$m_w^2(1 - \frac{m_w^2}{m_z^2}) = \frac{\pi\alpha(1+\Delta r)}{\sqrt{2} G_F} \tag{4.27}$$

where Δr can be written as [2] :

$$\Delta r^\alpha = \Delta\alpha - \frac{c^2}{s^2}\Delta\rho + \Delta r_{rem}(M_H) \tag{4.28}$$

$\Delta\alpha$ describes the effect from fermionic correction to the fine structure constant. $\Delta\alpha \propto log(m_f)$. $\Delta\rho$ is the shift to the ρ parameter, which defined as $\rho \equiv \frac{m_w^2}{m_z^2 c^2}$. $\Delta\rho = \Delta\rho(m_t^2)$ where m_t is mass of top quark. Δr_{rem} are classified as the remaining part which contains the dependence on the Higgs mass. For details and steps of Eq.4.28, see [13]

For predicting W boson mass, the equation 4.29 is employed

$$m_w^2 = m_z^2(\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\pi\alpha(1+\Delta r)}{\sqrt{2}m_z^2 G_F}}) \tag{4.29}$$

It is easy to show that 4.29 is equivalence to 4.27 (See appendix C)

4.4 Oblique parameters

In order to parametrize new physics contributions to electroweak radiative correction, a set of parameters proposed by Peskin and Takeuchi [17], called Oblique parameters (S T U parameters) is used. These parameters are only affected by self-energy correction from new physics (Example of self-energy type diagram Fig 4.19). There are three assumptions to using STU parameters:

[13]

1.No additional electroweak gauge boson other than γ Z W (Electroweak group is $SU(2)_L \times U(1)$

2. Only oblique correction (self-energy part) needs to be considered

3. The energy scale of new physics is higher compared to the electroweak scale

QED Ward identity implies the self-energy of photon and Z-γ mixing is 0 ($\Pi_{\gamma\gamma}(0) = 0, \Pi_{z\gamma}(0) =$

0), and perform a Taylor expansion :

$$\Pi_{\gamma\gamma}(q^2) = q^2\Pi'_{\gamma\gamma}(0) + \dots \tag{4.30}$$

$$\Pi_{z\gamma}(q^2) = q^2\Pi'_{z\gamma}(0) + \dots \tag{4.31}$$

$$\Pi_{zz}(q^2) = \Pi_{zz}(0) + q^2\Pi'_{zz}(0) + \dots \tag{4.32}$$

$$\Pi_{ww}(q^2) = \Pi_{ww}(0) + q^2\Pi'_{ww}(0) + \dots \tag{4.33}$$

where the Π' means the derivatives of the vacuum polarization function. This form of setup leaves 6 undetermined parameters. Input experiment data G_F, α, m_z, one can reduce to 3 undetermined parameters. Here define these 3 parameters as:

$$\alpha S = 4s^2c^2[\Pi'_{zz}(0) - \frac{c^2 - s^2}{sc}\Pi'_{Z\gamma}(0) - \Pi'\gamma\gamma(0)] \tag{4.34}$$

$$\alpha T = \frac{\Pi_{ww}(0)}{m_w^2} - \frac{\Pi_{zz}(0)}{m_z^2} \tag{4.35}$$

$$\alpha U = 4s^2[\Pi'_{ww}(0) - c^2\Pi'_{zz}(0) - 2sc\Pi'_z\gamma(0) - s^2\Pi'_{\gamma\gamma}(0)] \tag{4.36}$$

The definition of T can also represent a shift in the ρ parameter:

$$\rho = 1 + \delta\rho_{sm} + \alpha T \tag{4.37}$$

Many of the predictions predict U ¡¡T, thus in much research people assume U = 0. If custodial symmetry is a real symmetry, it will cause T = U = 0 When no new physics is presented, S=T=U=0. These sets of parameters are useful when we try to see how the new physics is presented and affects the experience observable. Also, it has a strong linkage to SMEFT (which will be discussed in a later chapter). The determination of STU parameters via fitting experimental data are [3]:

S =0.03±0.10,T=0.05±0.12,U=0.03±0.10

Using m_H=126GeV m_t = 173GeV

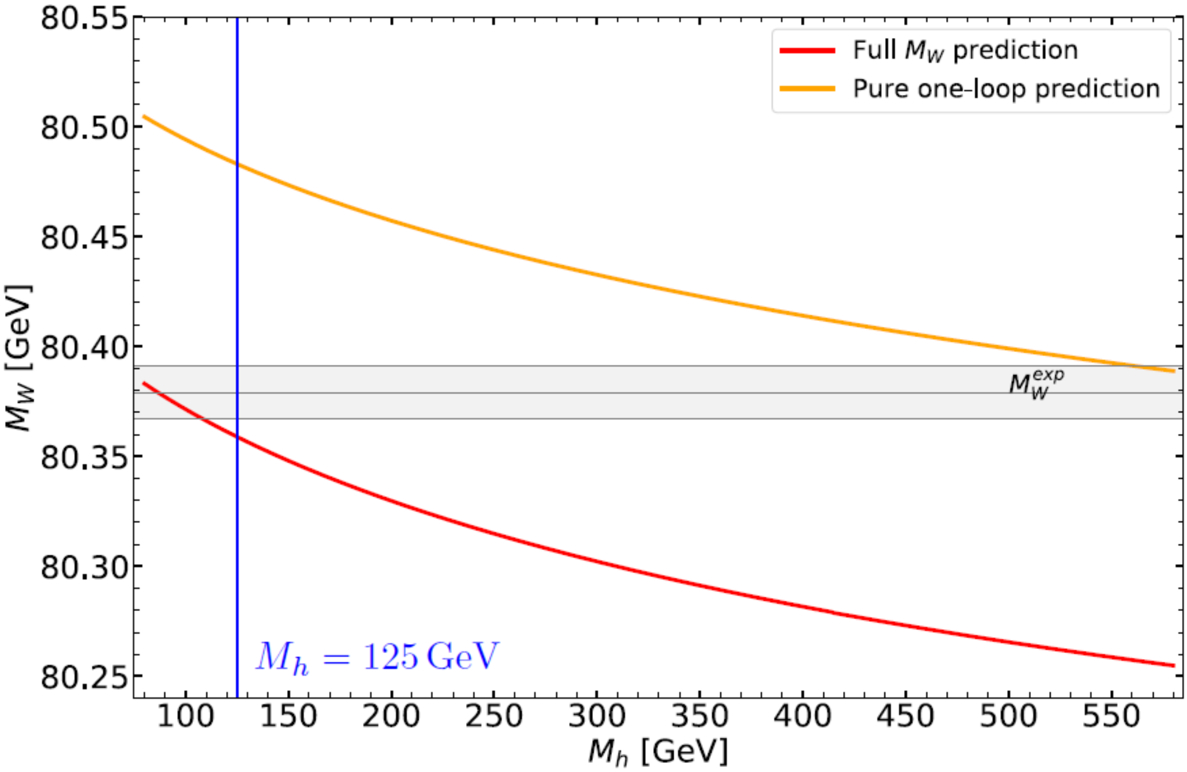


Figure 4.7: State of the art prediction, from Possible hints for new physics from EWPO and Higgs, searches, Imperial College Seminar, [M. Berger, S. Heinemeyer, G. Moortgat-Pick, G. W. '22]

4.5 State-of-the-art prediction

Unfortunately, the 1-loop result is not enough for precise measurement. Fig.4.7 shows the difference between prediction from higher loop order and one loop order. If we just use a 1-loop result it will indicate a heavy Higgs mass, which is incompatible with the experimental result. All the loop corrections are absorbed to Δr as one loop result previously. A accurate prediction of W boson mass [10] m_w = 80354 MeV using full higher loop result : [2].

$$\Delta r_{2loop} = \Delta r^{(\alpha)} + \Delta r^{(\alpha\alpha_s)} + \Delta r^{(\alpha\alpha_s^2)} + \Delta r^{(\alpha\alpha_s^3 m_t^2)} + \Delta r_{bos}^{(\alpha^2)} + \Delta r^{G_F^2\alpha_s m_t^4} + \Delta^{(G_F^3 m_t^6)} \tag{4.38}$$

where Δr^{α} is the one loop result. Other terms include two loops, three loop and approximate four loop QCD result, 2loop pure fermionic and bosonic electroweak correction.

A numerical approximation is used to predict the W boson mass[2]:

$$m_w = m_w^0 + c_t\Delta_t + c_t'\Delta_t^2 + c_z\Delta_z + c_\alpha\Delta_\alpha + c_{\alpha s}\Delta_{\alpha s}MeV \tag{4.39}$$

$$\Delta_t = (\frac{m_t}{173GeV})^2 - 1 \tag{4.40}$$

$$\Delta_z = \frac{m_z}{91.1876GeV} - 1 \tag{4.41}$$

$$\Delta_\alpha = \frac{\Delta\alpha_{had}^5(m_z^2)}{0.0276} - 1 \tag{4.42}$$

$$\Delta_\alpha s = \frac{\alpha_s(m_z^2)}{0.119} - 1 \tag{4.43}$$

$$m_w^0 = 80.359.5 \tag{4.44}$$

$$c_t = 520.5 \tag{4.45}$$

$$c_t' = -67.7 \tag{4.46}$$

$$c_z = 115000 \tag{4.47}$$

$$c_\alpha = -503 \tag{4.48}$$

$$c_{\alpha s} = -71.6 \tag{4.49}$$

m_t is the mass of the top quark. The $\alpha_s(m_z^2)$ is the hadronic contribution, and $\alpha_s(m_z^2)$ represent the running of strong coupling,where $\alpha_s = \frac{g_s}{4\pi}$.

Chapter 5

SMEFT — the Standard model effective field theory

The prediction above worked well and shows a good match to several experiments2.3, except for the very precise result from CDFII. At this point, we finally get to the main problem — the anomalous w boson mass. After the publication of the CDF group, theorists suggested many models explain the difference between prediction and CDFII results. One of them is SMEFT (Standard model effective field theory)[8] . It was originally proposed by W. BUCHMLJLLER and D. WYLER in 1985. Emanuele Bagnaschi, John Ellis, Maeve Madigan, Ken Mimasu, Veronica Sanze, and Tevong You use this model to analyse the shift in W boson mass [4] after the CDFII result was out. In this chapter, an introduction and overview of SEMFT are presented below.

5.1 Effective field theory

If we assumed that, the Standard model is actually an effective field theory, which only describes physics well at an energy scale less than m_w(Since it works very well at the present experiment), and the heavy fields are being integrated out in the standard model. It is natural to think that adding terms that contain the power of $\frac{1}{\Lambda}$ will "compete" the standard model at energy scale

Λ. We can propose the SMEFT lagrangian as the following:

$$\mathcal{L}_{eff} = \mathcal{L}_0 + \frac{1}{\Lambda}\mathcal{L}_1 + \frac{1}{\Lambda^2}\mathcal{L}_2 + higherdimensions... \tag{5.1}$$

$\mathcal{L}_0$ is the standard model lagrangian, the $\mathcal{L}_1$ are the dimension five terms and $\mathcal{L}_2$ are the dimension six terms. It is easy to see the dimension of those operators since the $\mathcal{L}_{eff}$ is dimension four, and $\frac{1}{\Lambda}$ denote a dimension of -1. For sure we can always construct terms that are higher than 6 dimensions, but as a starting point (also One can also impose a condition that the lagrangian is $SU(3)XSU(2)XU(1)$ invariance.This is a very helpful and useful condition when constructing operators later. Besides $SU(3)XSU(2)XU(1)$ symmetry, one also can impose baryon number conservation and lepton number conservation in order to construct higher dimension operators.(It is not necessarily) In conclusion, to construct a higher dimension standard model operator, one needs to impose the following conditions:

1. $SU(3)XSU(2)XU(1)$ invariance
2. Lorentz invariance
3. Baryon number and lepton number conservation(Optional)

5.2 Dimension five operators

The goal of this chapter is to construct dimension five operators.

Full fermionic / bosonic/ scalar operators is not possible (fermion field dimension is $\frac{3}{2}$ and bosonic/scalar field is 2). For it to be Lorentz invariance, the lepton field must couple with its charge conjugate field. In order to have SU(2) invariance, the scalar field must also be coupled with itself. The only operator that satisfies condition (1) and (2) is: (However this field violates the lepton number)

$$\mathcal{L}_1 = \epsilon_{ij}\bar{l}_R^{Ci}\phi^j\epsilon_{kl}l_L^k\phi^l + c.c. \tag{5.2}$$

where $\bar{l}_R^{Ci} = \begin{pmatrix} \bar{\nu_{eL}}^C \\ \bar{e_L}^C \end{pmatrix}$ is the charge conjuate of SU(2) doublet lepton field,ϵ_{ij} is the anti-symmetric tensor $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. This operator will give a Majorana mass term for neutrino, which leads to a huge problem for this operator. After solving the Majorana mass (appendix D), the mass term is in the order of $G_F^{-1}\Lambda^{-1}$. Since the known neutrino mass from the experiment is in the order of 0.1 eV, the energy scale Λ will be in the order of 10^{14} GeV. This result indicated that this dim-five operator is super heavy, so it is not useful in the current analysis. We then move to the next field, the dim-six operator.

5.3 Dimension Six operators

The $\mathcal{L}_2$ can be written in the form:

$$\mathcal{L}_2 = \sum_i \alpha_i O_i \tag{5.3}$$

α_i here is the Wilson coefficients, a dimensionless coupling constant. The original work from Buchmüller and Wyler [8] has constructed 80 operators in dimension six. For the purpose of analysing W boson mass shift, the operators that will affect W boson mass at the tree level are (The dual field is included here but it will not enter the calculation below):

$$O_{\phi W} = \frac{1}{2}(\phi^\dagger\phi)W^l_{\mu\nu}W^{l\mu\nu} \tag{5.4}$$

$$O_{\phi\tilde{W}} = \frac{1}{2}(\phi^\dagger\phi)\tilde{W}^l_{\mu\nu}W^{l\mu\nu} \tag{5.5}$$

$$O_{\phi B} = \frac{1}{2}(\phi^\dagger\phi)B_{\mu\nu}B^{\mu\nu} \tag{5.6}$$

$$O_{\phi\tilde{B}} = \frac{1}{2}(\phi^\dagger\phi)\tilde{B}_{\mu\nu}B^{\mu\nu} \tag{5.7}$$

$$O_{WB} = (\phi^\dagger\tau^l\phi)W^l_{\mu\nu}B^{\mu\nu} \tag{5.8}$$

$$O_{\tilde{W}B} = (\phi^\dagger\tau^l\phi)W^l_{\mu\nu}B^{\mu\nu} \tag{5.9}$$

$$O_\phi^{(1)} = (\phi^\dagger\phi)(D_\mu\phi^\dagger D^\mu\phi) \tag{5.10}$$

$$O_\phi^{(3)} = (\phi^\dagger D^\mu\phi)(D_\mu\phi\phi^\dagger) \tag{5.11}$$

To see the effect of these operators, we start from the lagrangian and perform an SSB to determine the mass of those bosons. The overall procedure is the same but with an additional dimension of six terms. The gauge boson lagrangian with dimension six terms (ignore G field here) :

$$\begin{aligned}\mathcal{L}_{WB} =& \mathcal{L}_{0WB} + \mathcal{L}_{2WB}\frac{1}{\Lambda^2} \\ =& -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^l_{\mu\nu}W^{l\mu\nu} \\ &+ \frac{1}{\Lambda^2}(\alpha_{\phi w}\frac{1}{2}(\phi^\dagger\phi)W^l_{\mu\nu}W^{l\mu\nu} + \alpha_{\phi B}\frac{1}{2}(\phi^\dagger\phi)B_{\mu\nu}B^{\mu\nu} + \alpha_{WB}(\phi^\dagger\tau^l\phi)W^l_{\mu\nu}B^{\mu\nu})+ \\ &(D_\mu\phi)^\dagger(D_\mu\phi) + \frac{1}{\Lambda^2}(\alpha^{(1)}_\phi(\phi^\dagger\phi)(D_\mu\phi^\dagger D^\mu\phi) + \alpha^{(3)}_\phi(\phi^\dagger D^\mu\phi)(D_\mu\phi\phi^\dagger)\end{aligned} \tag{5.12}$$

It looks long but we can do it step by step.Substituting the vev to ϕ and $\phi^\dagger$ and $(D_\mu\phi)^\dagger(D_\mu\phi)$ which is calculated in the Chapter3 , the equations above becomes:

$$\begin{aligned}\mathcal{L}_{WB} =& -\frac{1}{4}B_{\mu\nu}B^{\mu\nu}(1-\alpha_{\phi w}\frac{v^2}{\Lambda^2}) - \frac{1}{4}W^l_{\mu\nu}W^{l\mu\nu}(1-\alpha_{\phi B}\frac{v^2}{\Lambda^2}) \\ &+ \frac{1}{\Lambda^2}\alpha_{WB}(\phi^\dagger\tau^l\phi)W^l_{\mu\nu}B^{\mu\nu}+ \\ &\frac{v^2}{8}(g^2((W^1_\mu)^2+(W^2_\mu)^2)+(gW^3_\mu - g'B_\mu)^2)(1+\frac{v^2}{\Lambda^2}\alpha^{(1)}_\phi) + \alpha^{(3)}_\phi(\phi^\dagger D^\mu\phi)(D_\mu\phi\phi^\dagger)\end{aligned} \tag{5.13}$$

The only non-zero indices for α_{WB} term is l=3, where $\tau^3 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$. This term becomes a current mixing term, which increases the difficulty of our work. The $\alpha^{(3)}_\phi$ term contains the mixing mass term. The lagrangian will look like this:

$$\begin{aligned}\mathcal{L}_{WB} =& -\frac{1}{4}B_{\mu\nu}B^{\mu\nu}(1-\alpha_{\phi w}\frac{v^2}{\Lambda^2}) - \frac{1}{4}W^l_{\mu\nu}W^{l\mu\nu}(1-\alpha_{\phi w}\frac{v^2}{\Lambda^2}) \\ &+ \frac{v^2}{2\Lambda^2}\alpha_{WB}W^3_{\mu\nu}B^{\mu\nu}+ \\ &\frac{v^2}{8}(g^2((W^1_\mu)^2+(W^2_\mu)^2)+(gW^3_\mu - g'B_\mu)^2)(1+\frac{v^2}{2\Lambda^2}\alpha^{(1)}_\phi) + \alpha^{(3)}_\phi\frac{v^4}{16}(gW^3_\mu - g'B_\mu)(gW^{3\mu} - g'B^\mu)\end{aligned} \tag{5.14}$$

In order to determine the mass of the physical boson field, one needs to perform a diagonalization. However, compare to the standard model lagrangian, the equations above contain mass mixing and current mixing terms are mentioned above. To deal with this problem, we first diagonalize the mass mixing term with others first. The new mass matrix and be written as: $\frac{v^2}{8}\begin{pmatrix} g^2(1+\frac{\alpha_\phi^{(1)}}{2\Lambda^2}) & 0 & 0 & 0 \\ 0 & g^2(1+\frac{\alpha_\phi^{(1)}}{2\Lambda^2}) & 0 & 0 \\ 0 & 0 & g^2(1+\frac{\alpha_\phi^{(1)}}{2\Lambda^2})+g^2v^2\frac{\alpha_\phi^{(3)}}{2\Lambda^2} & -gg^{'}(1+\frac{\alpha_\phi^{(1)}}{2\Lambda^2})-gg'v^2\frac{\alpha_\phi^{(3)}}{2\Lambda^2} \\ 0 & 0 & -gg^{'}(1+\frac{\alpha_\phi^{(1)}}{2\Lambda^2})-gg'v^2\frac{\alpha_\phi^{(3)}}{2\Lambda^2} & g'^2(1+\frac{\alpha_\phi^{(3)}}{2\Lambda^2})+g'^2v^2\frac{\alpha_\phi^{(3)}}{2\Lambda^2} \end{pmatrix}$

Diagonalize this matrix directly (with the help of python code, see appendix E), and we get

$$\lambda_1 = 0 \qquad c^1 = \begin{pmatrix} 0 \\ 0 \\ sin\theta \\ cos\theta \end{pmatrix}$$

$$\lambda_2 = \frac{g^2v^2}{8}(1+\frac{v^2}{2\Lambda^2}\alpha_\phi^{(1)}) \qquad c^2 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\lambda_3 = \frac{g^2v^2}{8}(1+\frac{v^2}{2\Lambda^2}\alpha_\phi^{(1)}) \qquad c^3 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\lambda_4 = \frac{(g^2+g^{'2})v^2}{8}(1+\frac{v^2}{2\Lambda^2}(\alpha_\phi^{(1)}+\alpha_\phi^3)) \qquad c^3 = \begin{pmatrix} 0 \\ 0 \\ cos\theta \\ -sin\theta \end{pmatrix}$$

The good news is the eigenvector is not changed with the additional terms. We can still use the previous definition for the physical boson field. However, the eigenvalues for this matrix are not the mass term in our usual definition. Beware that there is a scale in front of the field kinetic terms, one needs to perform a rescaling of the field in order to reach our preferred lagrangian form. If there is a factor of d^2 in front of the field strength tensor:

$$\mathcal{L} = \frac{-1}{4} d^2 W_{\mu\nu} W^{\mu\nu} + \ldots$$

Using a rescaling of field

$$W \to \frac{1}{d} W \tag{5.15}$$

$$m_w^2 \to \frac{1}{d^2} m_w^2 \tag{5.16}$$

Apply the rescaling to the w boson mass, and $d^2 = (1 - \alpha_{\phi B} \frac{v^2}{\Lambda^2})$. Taking the order that is up to v^2, we get

$$m_{weft}^2 = \frac{g^2 v^2}{4}(1 + \frac{v^2}{2\Lambda^2}\alpha_\phi^{(1)}) \to \frac{g^2 v^2}{4}(1 + \frac{v^2}{2\Lambda^2}\alpha_\phi^{(1)}) \frac{1}{(1 - \alpha_{\phi B} \frac{v^2}{\Lambda^2})} \tag{5.17}$$

$$= \frac{g^2 v^2}{4}(1 + \frac{v^2}{2\Lambda^2}\alpha_\phi^{(1)})(1 + \alpha_{\phi B} \frac{v^2}{\Lambda^2} + \ldots) \tag{5.18}$$

$$= \frac{g^2 v^2}{4}(1 + \frac{v^2}{2\Lambda^2}(\alpha_\phi^{(1)} + 2\alpha_{\phi B})) \tag{5.19}$$

The final lagrangian is:

$$\mathcal{L}^2 = -\frac{1}{4} A_{\mu\nu} A^{\mu\nu} - \frac{1}{2} W_{\mu\nu}^+ W^{-\mu\nu} - \frac{1}{4} Z_\mu Z^\mu + m_w^2 W_\mu^+ W^{-\mu} + \frac{m_z^2}{2} Z_\mu Z^\mu \tag{5.20}$$

with

$$m_{weft}^2 = m_{w0}^2 (1 + \frac{v^2}{2\Lambda^2}(\alpha_\phi^{(1)} + 2\alpha_{\phi B})) \tag{5.21}$$

$$m_{zeft}^2 = m_z (1 + (\frac{1}{2}\alpha_{zz} + \frac{1}{4}\alpha_\phi^{(1)} + \frac{1}{4}\alpha_\phi^{(3)}) \frac{v^2}{\Lambda^2}) \tag{5.22}$$

where $m_{w0}^2 = \frac{1}{4} g^2 v^2$. Note that after field redefinition the electroweak field is also redefined. For example, the W^3 and B field definition is changed. Our main goal is to analyse W boson, so this part is skipped. The equations above is the origin formalisation of w boson mass shift that proposed in [8]. For a modern formalisation, one need to implent a new basis for operators – the Warsaw basis.

Chapter 6

Mordern SMEFT analysis

As mentioned in the previous chapter, there are 80 operators in the original paper[8]. In fact, some of those operators can be written using another operator – a paper by B. Grzadkowski, M. Iskrzyński, M. Misiak, and J. Rosiek[9] shows that using the conservation of baryon number and equation of motion, the number of operators can be reduced to 59 independent operators, and this is called Warsaw basis. Using this new basis, a different approach to analysis w boson mass shift is presented. In this chapter we will not discuss deeply how to construct a Warsaw basis as the complete list of operators is already given in [9]. Instead, this chapter will focus on how the SMEFT analysis of the W boson shift work in Warsaw basis. Also, this chapter competed for some ambiguity in Chapter 5, such a redefinition of SM parameters. Though working on a warsaw basis, the notation for the Wilson coefficient sticks with [8]. Every operator that comes with higher order than $\frac{v^2}{\Lambda^2}$ will be ignored.

6.1 Reintroduce the electroweak observable

In the SMEFT, the definition of parameters is different to the parameters that we defined in the SM Largagian(which is ignored in the previous section for simplicity) in SMEFT. It is because additional terms in SMEFT cause the shift of definition. To perform a modern SMEFT analysis[7], a clear definition of parameters is needed. In the following, the letter with a hat is the experimentally measured parameter. Note that for convenience, all the Wilson coefficient (α_i) comes with a hidden factor $\frac{1}{\Lambda^2}$ in this chapter.

6.2 Effective measured mixing angle

The usual definition of the mixing angle is $s^2 = \frac{g^2}{g^2+g'^2}$

Writing this in terms of observable, we define the measured effective mixing angle :

$$\hat{s} = s_{eff} = \frac{1}{2} - \frac{1}{2}(\sqrt{1 - 4\frac{\pi\hat{\alpha}}{\sqrt{2}\hat{G}_F\hat{m}_z^2}}) \tag{6.1}$$

6.3 Fermi constant

The definition of the Fermi constant also changed. In the Fermi's local effective lagrangian for muon decay,

$$\mathcal{L}_{Fermi} = -\frac{4\hat{G}_F}{\sqrt{2}}(\bar{\nu}_\mu\gamma^\mu P_L\mu)(\bar{e}\gamma_\mu P_L\nu_e) \tag{6.2}$$

Due to the presence of the new operators, the origin relation between the Fermi constant and vacuum expectation value no longer holds. SMEFT contribute additional Fenynam diagrams to this process(Fig.6.1,6.2) . If assuming flavour symmetry, two operators affecting directly this process. They are:

$$O_{\phi l}^{(3)} = (\phi^\dagger i\hat{D}_\mu^I\phi)(\bar{l}_p\tau^I\gamma^\mu l_r) \tag{6.3}$$

$$O_{ll} = (\hat{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t) \tag{6.4}$$

where $\hat{D}$ is defined as

$$\phi^\dagger i\hat{D}\phi = i\phi^\dagger(D_\mu - \overleftarrow{D}_\mu)\phi$$

$$\phi^\dagger i\overleftarrow{D}_\mu\phi = (D_\mu\phi)^\dagger\phi$$

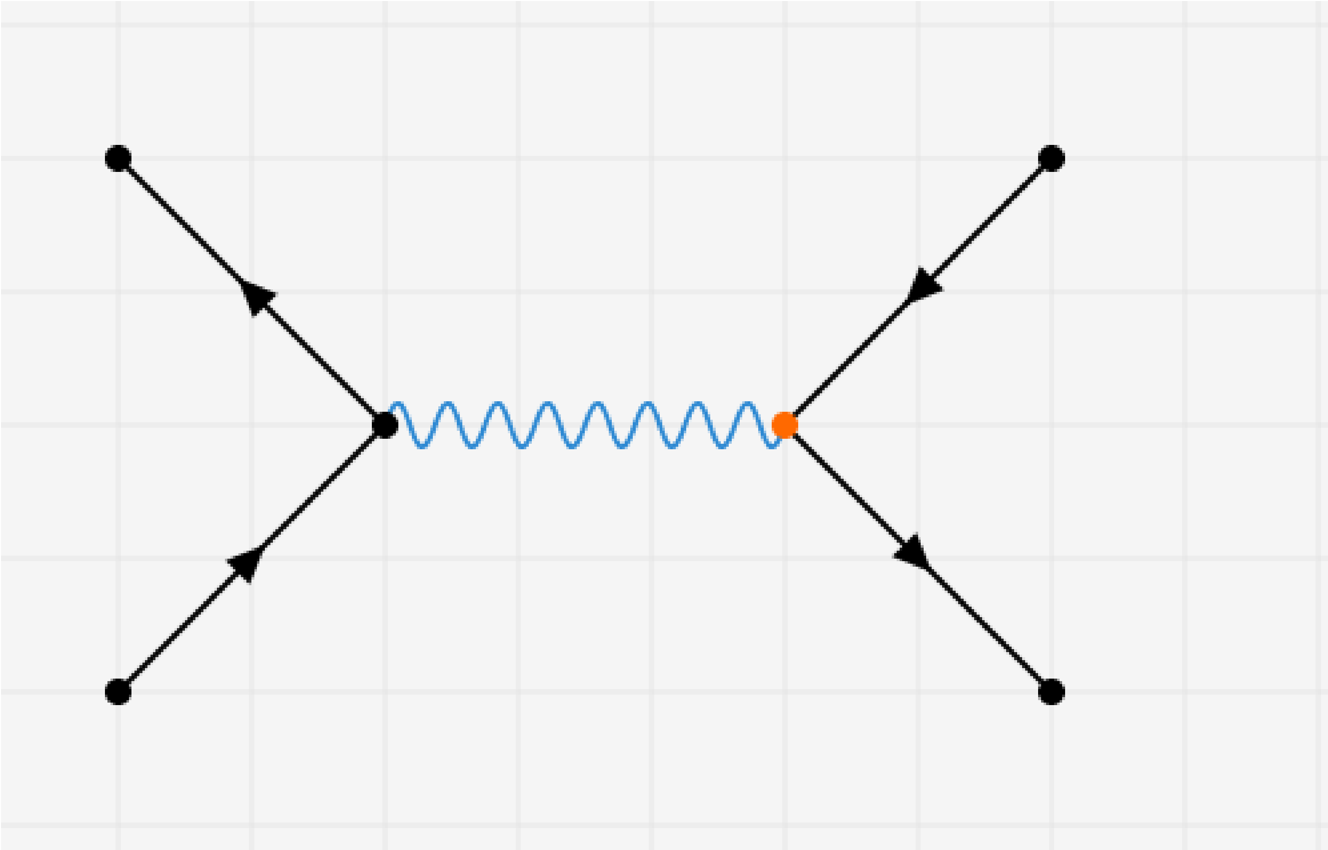


Figure 6.1: Feynman for $O_{\phi l}$ in muon decay. Orange dot means the SMEFT operator coupling

These two operators contribute directly to the muon decay lagrangian, and so affect the relationship between Fermi constant and vacuum expectation value. The SMEFT relation is:

$$\hat{G_F} = \frac{1}{\sqrt{2}\bar{v}^2} - \frac{1}{\sqrt{2}}\alpha_{ll} + \sqrt{2}\alpha_{\phi}^{(3)} \tag{6.5}$$

And of course, the definition with respect to the vacuum expectation value changed. The vacuum expectation value in SMEFT is represented by $\bar{v}$ and the relationship between the SM vev and SMEFT is:

$$\bar{v} = (1 + \frac{3\alpha_{\phi} v^2}{8\lambda})v \tag{6.6}$$

λ is the Higgs coupling. The corresponding operators:

$$O_{\phi} = (\phi^{\dagger}\phi)^3 \tag{6.7}$$

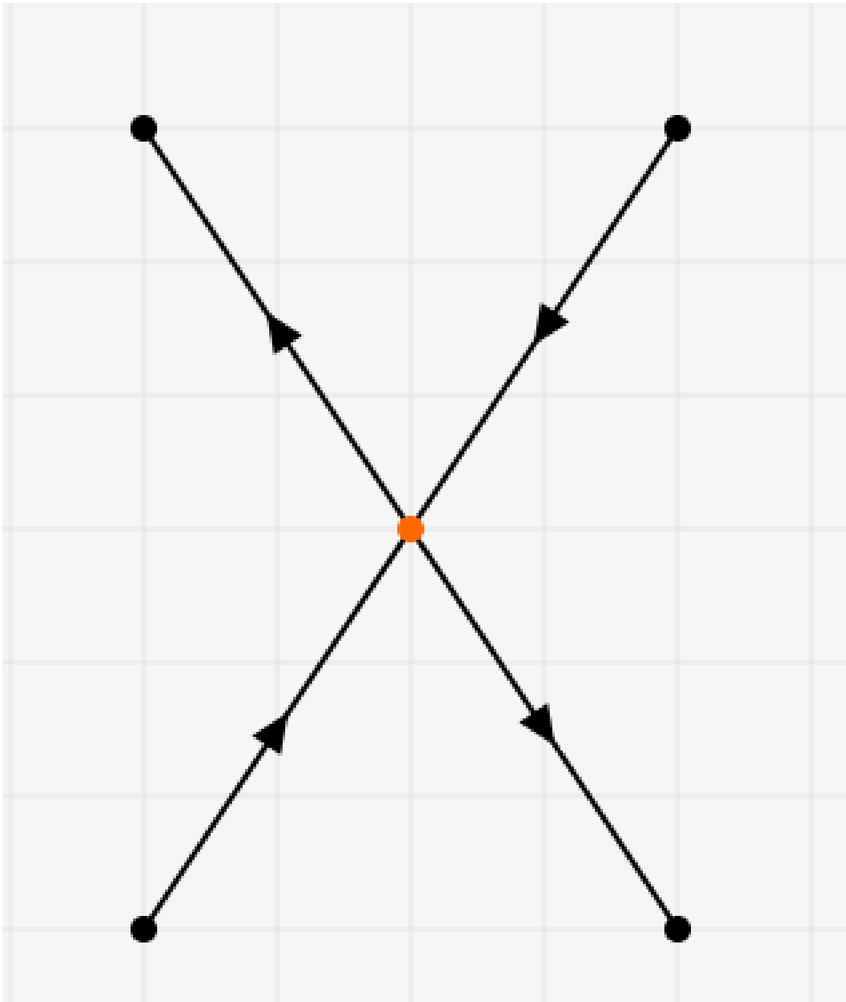


Figure 6.2: Feynman for O_{ll} in muon decay. Orange dot means the SMEFT operator coupling

It is natural to define δG_F as:

$$\bar{v}^2 = \frac{1}{\sqrt{2}\hat{G}_F} + \frac{\delta G_F}{\hat{G}_F} \tag{6.8}$$

so

$$\delta G_F = \frac{1}{\sqrt{2}\hat{G}_F}(\sqrt{2}\alpha_{\phi l} - \frac{\alpha_{ll}}{\sqrt{2}}) \tag{6.9}$$

6.4 Z boson mass

The mass eigenstate of Z boson is (from the SSB and diagonalisation procedure in Chapter5, but redefined in Warsaw basis) :

$$\bar{m}_z^2 = \frac{v^2}{4}(\bar{g}^2 + \bar{g}'^2) + \frac{1}{8}\bar{v}^4\alpha_\phi^{(3)}(\bar{g}^2 + \bar{g}'^2) + \frac{1}{2}\bar{v}^4\bar{g}\bar{g}'\alpha_{WB} \tag{6.10}$$

and δm_z^2

$$\delta m_z^2 \equiv \hat{m_z^2} - \frac{\bar{v}^2}{4}(\bar{g}^2 + \bar{g}'^2) = -\frac{1}{2\sqrt{2}}\frac{m^2}{\hat{G_F}}\alpha_\phi^{(3)} - \frac{22^{\frac{1}{4}}\sqrt{\pi}\sqrt{\hat{\alpha}}\hat{m}_z}{\hat{G_F}^{\frac{3}{2}}}\alpha_{WB} \tag{6.11}$$

6.5 Coupling constant

The SMEFT coupling constant is renormalised as:

$$g = \bar{g}(1 + \alpha_{\phi W}\bar{v}^2) \tag{6.12}$$

$$g' = \bar{g}'(1 + \alpha_{\phi B}\bar{v}^2) \tag{6.13}$$

Working at tree level, express the coupling constant using the input parameters:

$$\bar{g}^2 + \bar{g}'^2 = 4\sqrt{2}G_F\hat{m}_z^2(1 - \sqrt{2}\delta G_F - \frac{\delta\hat{m}_z^2}{\hat{m}_z^2}) \tag{6.14}$$

$$\bar{g}^2 = \frac{4\pi\hat{\alpha}}{\hat{s}^2}[1 + \frac{\delta s^2}{\hat{s}^2} + \frac{\hat{c}}{\hat{s}}\frac{1}{\sqrt{2}G_F}\alpha_{WB}] \tag{6.15}$$

6.6 Mixing angle in SMEFT

A kinetic term mixing (being introduced in the previous chapter(5.13) cause the definition of mixing angle to change. The SMEFT definition of mixing angle is :

$$\bar{s}^2 = \frac{\bar{g}^2}{\bar{g}^2 + \bar{g}'^2} + \frac{\bar{g}\bar{g}'(\bar{g}^2 - \bar{g}'^2)}{\bar{g}^2 + \bar{g}'^2}\bar{v}^2\alpha_{WB} \tag{6.16}$$

Define δs:

$$\delta s^2 = \hat{s}^2 - \bar{s}^2 = \frac{\hat{s}\hat{c}}{2\sqrt{2}\hat{G}_F(1 - 2\hat{s}^2)}[\hat{s}\hat{c}(\alpha_\phi^{(3)} + 4\alpha_{\phi l} - 2\alpha_{ll}) + 2\alpha_{WB}] \tag{6.17}$$

6.7 W boson

The definition of W boson mass is SMEFT is:

$$\bar{m}_w^2 = \frac{\bar{g}^2\bar{v}^2}{4} \tag{6.18}$$

Substituting result and definition in the previous section, and express it in terms of observable:

$$\bar{m_w}^2 = \frac{\pi\hat{\alpha}}{\hat{s}^2}[1 + \frac{\delta s^2}{\hat{s}^2} + \frac{\hat{c}}{\hat{s}}\frac{1}{\sqrt{2}G_F}\alpha_{WB}](\frac{1}{\sqrt{2}\hat{G}_F} + \frac{\delta G_F}{\hat{G}_F}) \tag{6.19}$$

Take m_w^2(with simple algebra) factor out and take only linear order :

$$\bar{m}_w^2 = \frac{1}{\sqrt{2}\hat{G}_F}\frac{\pi\hat{\alpha}}{\hat{s}^2}[1+\frac{\delta s^2}{\hat{s}^2}+\frac{\hat{c}}{\hat{s}}\frac{1}{\sqrt{2}G_F}\alpha_{WB}](1+\sqrt{2}\delta G_F) \tag{6.20}$$

$$\bar{m}_w^2 = {m_w}^2\frac{\pi\hat{\alpha}}{\hat{s}^2}[1+\frac{\delta s^2}{\hat{s}^2}+\frac{\hat{c}}{\hat{s}}\frac{1}{\sqrt{2}G_F}\alpha_{WB}+\sqrt{2}\delta G_F) \tag{6.21}$$

Define the shift as:

$$m_w^2 - \delta m_w^2 = \bar{m}_w^2 \tag{6.22}$$

$$\delta m_w^2 = -m_w^2(\frac{\delta s^2}{\hat{s}^2}+\frac{\hat{c}}{\hat{s}}\frac{1}{\sqrt{2}G_F}\alpha_{WB}+\sqrt{2}\delta G_F) \tag{6.23}$$

$$\frac{\delta m_w^2}{m_w^2} = \Delta(4\alpha_{WB}+\frac{c}{s}\alpha_\phi^{(3)}+4\frac{s}{c}\alpha_{\phi l}-2\frac{s}{c}\alpha_{ll}) \tag{6.24}$$

where

$$\Delta \equiv \frac{cs}{(c^2-s^2)2\sqrt{2}\hat{G}_F} \tag{6.25}$$

The 6.24is simplified using python code in Appendix(F). Another way to write this in terms of vacuum expectation value (the fashion that is presented in [4] is simply replaced $G_F$and release the$\frac{1}{\Lambda^2}$. We get:

$$\frac{\delta m_w^2}{m_w^2} = -\frac{s2\theta}{c2\theta}\frac{v^2}{4\Lambda}(4\alpha_{WB}+\frac{c}{s}\alpha_\phi^{(3)}+4\frac{s}{c}\alpha_{\phi l}-2\frac{s}{c}\alpha_{ll}) \tag{6.26}$$

which is the disered result. A gloal fit [4] based on the this equation analysis the CDFII result for W boson, and give values for Wilson cofficient.

Chapter 7

Conclusion

Due to the limitation of time and effort here is the end of the thesis. There are a few possible reasons for the mass shift. It can be caused by SMEFT, higher loop correction, experimental uncertainty from Monte Carlo, also the Higgs triplet, which did not mention in this thesis, can contribution to this process. With future detector in a higher energy the W boson mass problem might be a hints to new physics. In summary, this thesis had a brief introduction to the recent W boson mass shift from experimental methodology to SMEFT analysis. Some derivation and steps for equations and the use of computation techniques are also presented in this thesis. After finishing reading this thesis one should have a basic understanding of the background of W boson mass shift and some approaches to it. For those who are interested in this topic, higher loop correction and Higgs triplet are good topics to study further.

Appendix A

Transverse mass of W boson

The steps are base on [14] and [11]

Define transverse mass $m_T^2 = m_{rest}^2 + p_T^2 = E^2 - p_z^2$, where p_T^2 is the transverse momentum, $p_T^2 = p_x^2 + p_y^2$. It is invariant under Lorentz boost at z direction.

Here we ignore the p_z for the whole system first, and set the frame to W rest frame, so

$P_W^\mu = (m_T, 0)$, and $P_e^\mu = (E_{Te}, p_{Te}^\mu), P_\nu^\mu = (E_{T\nu}, p_{T\nu}^\mu)$

$m_T^2 = (E_{Te} + E_{T\nu}, p_{Te}^\mu + p_{T\nu}^\mu)^2$

$m_T^2 = E_{Te}^2 + E_{T\nu}^2 + 2E_{Te}E_{T\nu} - ({p_{Te}^\mu}^2 + {p_{T\nu}^\mu}^2 + 2p_{Te}^\mu \cdot p_{T\nu}^\mu)$

Here set electron and neutrino as a massless particle, so $E_T = p_T$

We get

$m_T = \sqrt{2(p_T^e p_T^\nu - \vec{p_T^e} \cdot \vec{p_T^\nu})} = \sqrt{2p_T^e p_T^\nu(1 - cos\phi)}$ as required.

Appendix B

Transverse mass distribution and Jacobian Peak

This base on
https://physics.stackexchange.com/questions/609727/particle-physics-understanding-the-jacobian-peak
and
[19]

Here let's discuss about the transverse mass method and a bit history. Historically, because the observable is the lepton transverse momentum p_T^l, people try to infer the W boson mass directly from p_T^l distribution.

Consider $\frac{d\sigma}{dp_T^l}$, which is $d\sigma$ the differential cross section .

We define ϕ is the angle respect to the w boson beam direction, so that $p_T^l = \vec{p^l} sin\phi$ and $p_L^l = \vec{p^l} cos\phi$ (longitude momentum)

$\frac{d\sigma}{dp_T^l} = \frac{d\sigma}{d\phi}\frac{d\phi}{dp_T^l}$

and $\frac{d\phi}{dp_T^l} = (\frac{dp_T^l}{d\phi})^{-1} = \frac{1}{\vec{p^l} cos\phi} = \frac{1}{p_L^l}$

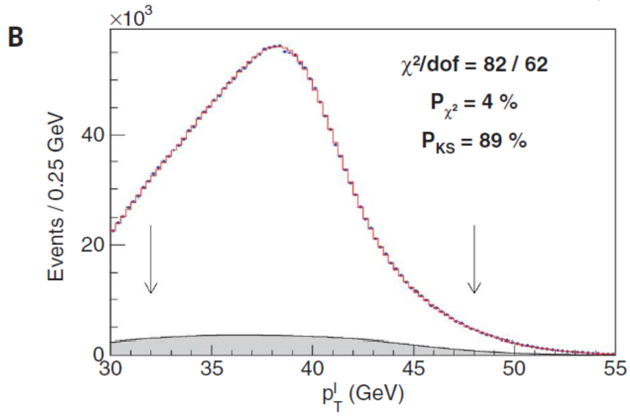
The lepton has energy $E = \frac{m_w}{2}$, so $p_L^l = \sqrt{(\frac{m_w}{2})^2 - p_T^2}$

which leads to

$\frac{d\sigma}{dp_T^l} = \frac{d\sigma}{d\phi}\frac{1}{\sqrt{(\frac{m_w}{2})^2 - p_T^2}}$

We can see a sudden drop when $p_T = \frac{M_W}{2}$, this sudden drop is called Jacobian Peak. However, due to QCD correction which affect W boson transverse momentum and detector's limit, the peak is smeared. An example of p_T distribution is given at B.1

Figure B.1: A typical p_T transverse momentum distribution (from CDFII report)



In order to get a better result, rather than using the lepton transverse momentum, one can use the transverse mass (See Appendix A) because it is more stable against QCD correction. [19]. If we do a substitution , $p_T = E_T = \frac{m_T}{2}$

$\frac{d\sigma}{dm_T} = \frac{d\sigma}{d\phi}\frac{2}{\sqrt{(\frac{m_w}{2})^2 - \frac{m_T}{2}^2}}$

The m_T shows a similar jacobian peak at $m_T = m_w$, see Fig2.2.

Appendix C

Equation for predicting W boson mass

$$m_w^2 = m_z^2\left(\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\pi\alpha(1+\Delta r)}{\sqrt{2}m_z^2 G_F}}\right) \quad \text{(C.1)}$$

$$\frac{m_w^2}{m_z^2} - \frac{1}{2} = \sqrt{\frac{1}{4} - \frac{\pi\alpha(1+\Delta r)}{\sqrt{2}m_z^2 G_F}} \quad \text{(C.2)}$$

$$\left(\frac{m_w^2}{m_z^2}\right)^2 + \frac{1}{4} - \frac{m_w^2}{m_z^2} = \frac{1}{4} - \frac{\pi\alpha(1+\Delta r)}{\sqrt{2}m_z^2 G_F} \quad \text{(C.3)}$$

$$\frac{m_w^2}{m_z^2}\left(\frac{m_w^2}{m_z^2} - 1\right) = \frac{-\pi\alpha(1+\Delta r)}{\sqrt{2}m_z^2 G_F} \quad \text{(C.4)}$$

$$m_w^2\left(1 - \frac{m_w^2}{m_z^2}\right) = \frac{\pi\alpha(1+\Delta r)}{\sqrt{2}G_F} \quad \text{(C.5)}$$

$$\quad \text{(C.6)}$$

which is same as the equation 4.27

Appendix D

Neutrnio Majorana mass for Dimension five operator

The dim 5 term is :

$$\mathcal{L}_1 = \epsilon_{ij}\bar{l}_R^{Ci}\phi^j\epsilon_{kl}l_L^k\phi^l + c.c. \tag{D.1}$$

To infer majorana mass term for neutrino, we set $\phi^j = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$ and because $\epsilon_i j$ is anti-symmetric tensor, the indicies that give non zero result is the neutrino mass term, when i=1 ,j=2, k=1,l=2. The $\mathcal{L}_{1mass} = \frac{v^2}{2}\bar{\nu}_L^C\nu_L$, so the $m_{majorana} \approx \frac{1}{G_F\Lambda}$ (Do not forget the $\frac{1}{\Lambda}$ in the original expression 5.1). To match with the current experiemntal data for neutrino mass, the energy scaleΛ for this operator will leads to 10^14 GeV.

Appendix E

Python code for diagonalization mass matrix

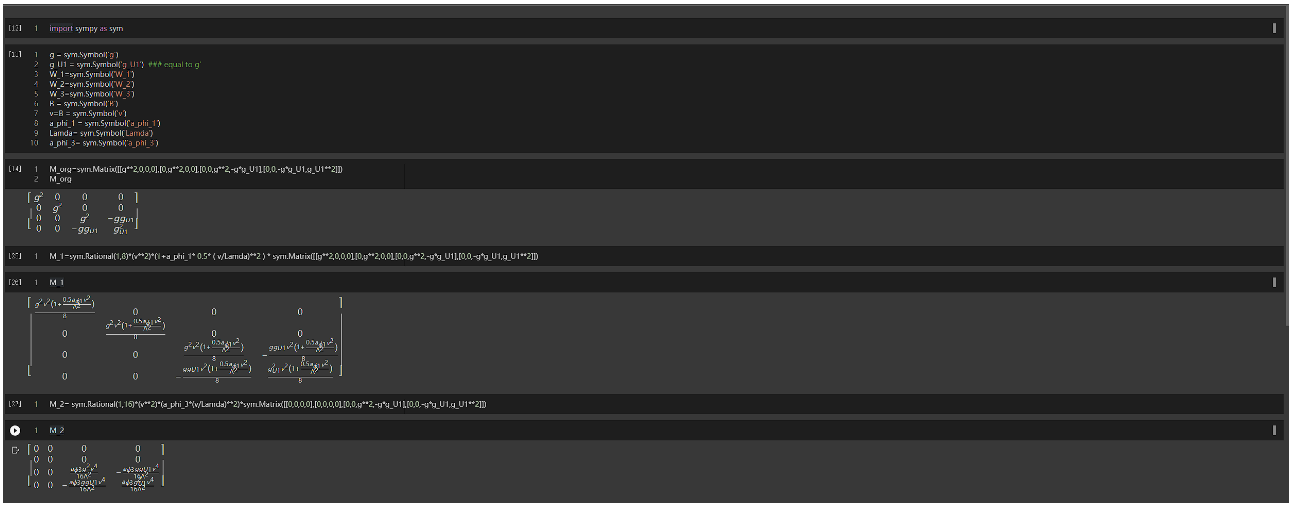


Figure E.1: Part 1 of the code

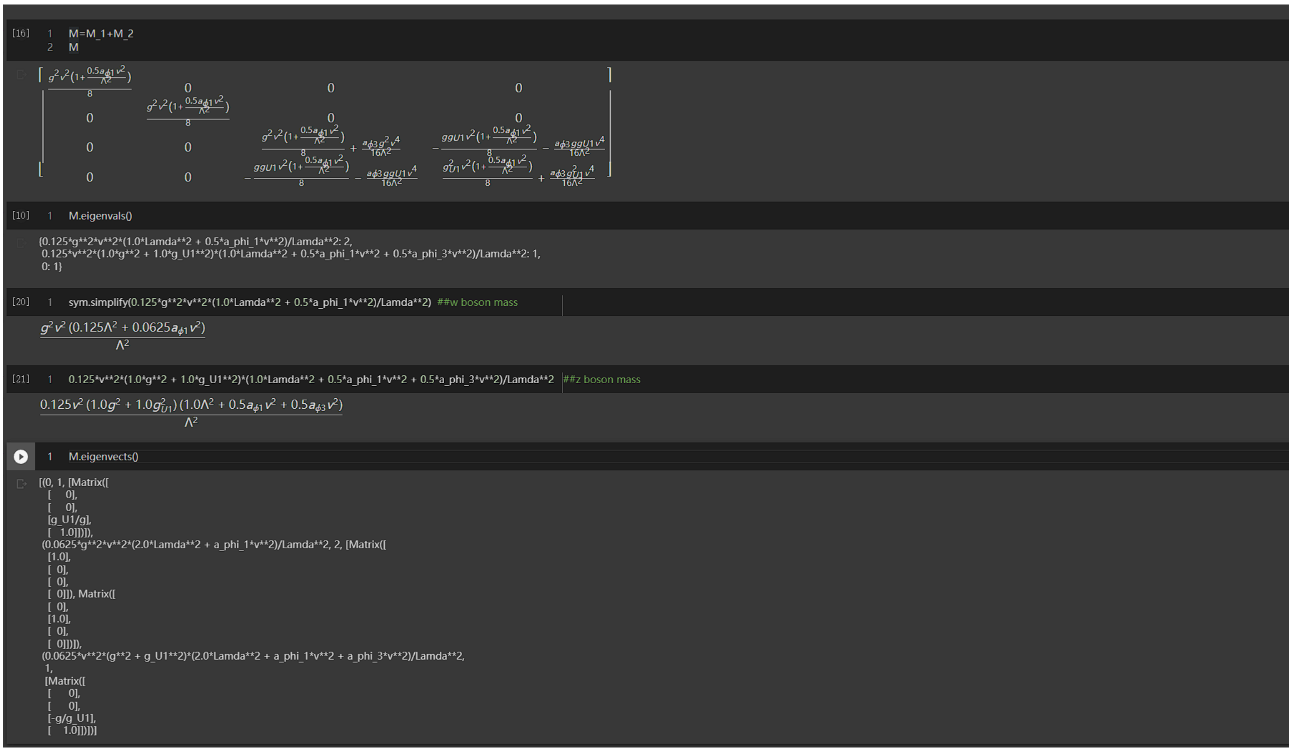


Figure E.2: Part 2 of the code

Appendix F

Python code for simpification

```python
theta=sym.Symbol('Theta')
s = sym.sin(theta)
c = sym.cos(theta) ### equal to g'
G=sym.Symbol('G')
c_hd=sym.Symbol('c_hd')
c_hl=sym.Symbol('c_hl')
c_ll= sym.Symbol('c_ll')
c_hwb= sym.Symbol('c_hwb')

```

```python
delta_G = (1/(sym.sqrt(2)*G))*(sym.sqrt(2)*c_hl- c_ll/sym.sqrt(2))
```

```python
delta_s = - s*c/(2*sym.sqrt(2)*G*(1-2*s**2))*(s*c*(c_hd +4*c_hl-2*c_ll  )+2*c_hwb)
```

```python
solve= delta_s/(s**2) + c/(s*sym.sqrt(2)*G)* c_hwb + sym.sqrt(2) *delta_G
```

```python
from sympy.integrals.transforms import simplify
simplify(solve)
```

$$-\frac{\sqrt{2}\left(\frac{c_{hd}\cos(2\Theta)}{2} + \frac{c_{hd}}{2} - 2c_{hl}\cos(2\Theta) + 2c_{hl} + 2c_{hwb}\sin(2\Theta) + c_{ll}\cos(2\Theta) - c_{ll}\right)}{4G\cos(2\Theta)}$$

```python
Delta = c*s/((c**2-s**2)*2*sym.sqrt(2)*G)
```

```python
simplify(Delta)

```

$$\frac{\sqrt{2}\tan(2\Theta)}{8G}$$

```python
simplify(solve/Delta)
```

$$-\frac{c_{hd}}{\tan(2\Theta)} - \frac{c_{hd}}{\sin(2\Theta)} + \frac{4c_{hl}}{\tan(2\Theta)} - \frac{4c_{hl}}{\sin(2\Theta)} - 4c_{hwb} - \frac{2c_{ll}}{\tan(2\Theta)} + \frac{2c_{ll}}{\sin(2\Theta)}$$

Figure F.1: Part 1 of the code

Bibliography

[1] T. Aaltonen et al. High-precision measurement of the W boson mass with the CDF II detector. *Science*, 376(6589):170–176, 2022.

[2] M. Awramik, M. Czakon, A. Freitas, and G. Weiglein. Precise prediction for the W boson mass in the standard model. *Phys. Rev. D*, 69:053006, 2004.

[3] M. Baak, M. Goebel, J. Haller, A. Hoecker, D. Kennedy, R. Kogler, K. Moenig, M. Schott, and J. Stelzer. The Electroweak Fit of the Standard Model after the Discovery of a New Boson at the LHC. *Eur. Phys. J. C*, 72:2205, 2012.

[4] Emanuele Bagnaschi, John Ellis, Maeve Madigan, Ken Mimasu, Veronica Sanz, and Tevong You. SMEFT analysis of m_W. *JHEP*, 08:308, 2022.

[5] C. Balazs and C. P. Yuan. Soft gluon effects on lepton pairs at hadron colliders. *Phys. Rev. D*, 56:5558–5583, 1997.

[6] Richard D. Ball et al. Parton distributions from high-precision collider data. *Eur. Phys. J. C*, 77(10):663, 2017.

[7] Laure Berthier and Michael Trott. Towards consistent Electroweak Precision Data constraints in the SMEFT. *JHEP*, 05:024, 2015.

[8] W. Buchmuller and D. Wyler. Effective Lagrangian Analysis of New Interactions and Flavor Conservation. *Nucl. Phys. B*, 268:621–653, 1986.

[9] B. Grzadkowski, M. Iskrzynski, M. Misiak, and J. Rosiek. Dimension-Six Terms in the Standard Model Lagrangian. *JHEP*, 10:085, 2010.

[10] Johannes Haller, Andreas Hoecker, Roman Kogler, Klaus Mönig, Thomas Peiffer, and Jörg Stelzer. Update of the global electroweak fit and constraints on two-Higgs-doublet models. *Eur. Phys. J. C*, 78(8):675, 2018.

[11] Tao Han. Collider phenomenology: Basic knowledge and techniques. In *Theoretical Advanced Study Institute in Elementary Particle Physics*, pages 407–454, 8 2005.

[12] UCL HEP. W boson decays https://www.hep.ucl.ac.uk/ jpc/all/ulthesis/node45.html.

[13] J. L. Hewett. The Standard model and why we believe it. In *Theoretical Advanced Study Institute in Elementary Particle Physics (TASI 97): Supersymmetry, Supergravity and Supercolliders*, pages 3–83, 6 1997.

[14] J.D. Jackson. 38. Kinematics (Particle data group), https://pdg.lbl.gov/2006/reviews/kinemarpp.pdf.

[15] Fermi lab. Energy frontier - cdf https://www.fnal.gov/pub/tevatron/experiments/cdf.html.

[16] Gleb Lukicov. Alignment of the straw tracking detectors for the fermilab muon $g-2$ experiment and systematic studies for a muon electric dipole moment measurement. 12 2020.

[17] Michael E. Peskin and Tatsu Takeuchi. A New constraint on a strongly interacting Higgs sector. *Phys. Rev. Lett.*, 65:964–967, 1990.

[18] A. Sirlin. Radiative Corrections in the SU(2)-L x U(1) Theory: A Simple Renormalization Framework. *Phys. Rev. D*, 22:971–981, 1980.

[19] J. Smith, W. L. van Neerven, and J. A. M. Vermaseren. The Transverse Mass and Width of the W Boson. *Phys. Rev. Lett.*, 50:1738, 1983.