Imperial College of Science, Technology and Medicine
Theory Group

Hydrodynamic as a method of Analysis for RHIC data and beyond

Arshia Tajlili Moghanjoghi

Supervisor: Prof. Toby Wiseman

Submitted in part fulfilment of the requirements for the degree of
Masters of Science in Quantum Fields and Fundamental Forces of the
Imperial College, September 2022

Abstract

Hot interacting quantum fields can be described by Hydrodynamics. In this review the overview of using kinetic theory and holography to expand on the hydrodynamics of the system is explored. It is explained that the region of applicability is larger than what was assumed and that is due to the possibility to track the fluid for longer as the result of fluid/gravity. Finally radii of convergences for the sound and sheer mode were calculated. It was shown that they converge around , $\omega = \frac{i}{2}$ in the small coupling limit for sheer viscosity and it was also shown that analytically extending from the hydrodynamics principal sheets gives the non hydrodynamics modes.

Contents

Chapter 1

Introduction

1.1 Motivation and Objectives

In recent years, activity surrounding the modelling different quantum systems has been capturing more attention. One of the models that has been proposed to tackle to complexity of physics at heavy ion collision is Hydrodynamics. Hydrodynamics are usually constructed using conservation of energy or stress energy tensor. Using these equations we can limit the d.o.f of the system to describe various systems. It is a highly useful tool when it comes to describing plasma and fermionic ultra cold gas. Recent activity in RHIC (Relativistic Heavy Ion Collider) shows that elliptic flows serve as a great model for heavy nucleon-nucleon collison and hydrodynamics can be very useful to describe the system in certain temperature regimes when hydrodynamics modes exist. For years, physicist have been trying to describe very complicated real time strongly correlated systems in quantum mechanics, which are almost close to impossible to study using lattice correlators. It turns out that their dynamics actually matches the systems that are described by hydrodynamics with gradient expansions. They are also a great tool to describe kinetic theory which are models, that use Boltzmann distribution to describe the concept of quasi particles. Using these models it has been possible to generalise to other non conformal strongly correlated systems and do calculations in heavy ion collision. Therefore it has become more vital than ever to find out the points when one could achieve hydrodynamics physics. To this point, the fluid gravity proved to be very useful. Through gauge/gravity correspondence one can establish a connection between hydrodynamics and gravity and fluid gravity correspondence has proven to be a very important tool for studying nearly perfect fluids with higher order diffusive coefficients. What has been really impressive is that the region where hydrodynamics seems to be applicable in terms of the temperature of the QCD plasma seems larger than what is should be. Using fluid gravity correspondence one can analytically continue the description to regions where the hydrodynamization might have naively failed. It can also expand the description through non conformal fluids which are a better description of what is occurring at RHIC and CERN. The question to ask then, is at what points are the descriptions valid. This review wishes to explore the regimes of the existence of hydrodynamic expansion, their convergence regimes and theories that are possibly described by it to give a holistic overview of the applications of hydrodynamics in quantum field theory and relativistic and non relativistic quantum collisions.

During the review the order of the topics covered will be the following. In the chapters 2.1 and 2.2 basic principles of fluid dynamics such as the Navier stokes equations and the conservation of the momentum and energy will be covered in both the relativistic and the non relativistic limit. Following that the introduction of CFT hydrodynamics is explained following by a brief description of the world of CFTs and their correlation functions is presented in 2.3.1. As the aim of the paper is to probe different limits of hydrodynamics a description of the kinetic theory is provided as an alternative to weak coupling limits of quantum field theory.

Continuing upon that, the string theory background of gauge/gravity duality will be explained and finally gauge gravity duality and fluid gravity duality will be explored and the regions of hydrodynamics convergence will be analysed.

Chapter 2

Background Theory

2.1 Non relativistic Hydrodynamics

The governing conservation equation in non relativistic hydrodynamics is the Navier Stokes equation for the conservation of energy and momentum.

The variables that describe the flow of a viscous liquid are the pressure, temperature and shear and bulk viscosity. As pressure can be described by the temperature field, the overall dynamical fields to describe the non relativistic dynamics of fluids would be the temperature and velocity fields. To be able to describe the equations, it is best to introduce a material derivative which measures the time rate of change of F as seen by an observer moving with the fluid at position x[1].

The material derivative is then formulated as

$$\frac{DF}{DT} = \frac{\partial F}{\partial t} + v_j \frac{\partial F}{\partial x_j} \tag{2.1}$$

Using the material derivative above, it is possible to describe the fluid starting with the overall conservation equation for the ρV. ρ denoting mass density and V the volume. As the mass of the fluid remains unchanged, We can write the overall conservation of mass as [2]

$\mathrm{D}(\rho V)_{\overline{Dt=0}}$

And using

$$div\mathbf{v} = \lim_{V \to 0} \frac{1}{V} \oint_S v.d\mathbf{S}$$

(2.2)

Leading to the equations of continuity listed as

$$\frac{1}{\rho}\frac{D\rho}{Dt} + div\mathbf{v} = 0$$

(2.3)

$$\frac{\partial \rho}{\partial t} + div(\rho\ \mathbf{v}) = 0$$

(2.4)

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho \mathbf{v}_j) = 0$$

(2.5)

As explained in the introduction, the aim of hydrodynamics is to construct certain limitation to the dynamic and by solving the dynamics with regards to the restriction one can eventually be able to describe the fluid. It is therefore imperative to describe the particle momentum as it has a direct correlation to the velocity field that is to explored, as one of the descriptive variables of the fluid. For the momentum conservation one has to consider the effect of normally applied pressure p on the boundary surface S and the conservation equation is related to rate of the change of the momenutum of the fluid particle subjected to the applied pressure.[1] Here it is necessary to introduce viscous stress tensor σ which is the traction pressure normal to the surface. Using these elements one can describe the rate of change of momentum as

$$\rho V \frac{Dv_i}{Dt} = \oint (-p\delta_{ij} + \sigma_{ij}) n_j d\mathbf{S} + V F_{i_S} \tag{2.6}$$

Which using Gauss's theorem can be turned into

$$\rho \frac{Dv_i}{Dt} = -\frac{\partial p}{\partial x_i} + \frac{\partial \sigma_{ij}}{\partial x_j} + F_i \tag{2.7}$$

Where F is the applied force.

Viscous stress tensor is effectively related to the relative motion of the particles with respect to each other and their diffusion. Denoting the difference of the velocity between different particles as δv and expanding in first order one can represent the velocity gradient as anti symmetric and symmetric sectors. Anti symmetric part relates to a rigid body rotation at angular velocity $\frac{1}{2}\omega$ with 3 independent quantities $\omega = \nabla \times v$ [2]. The symmetric module is called the rate of strain tensor and corresponds to the other distortions can be written as the sum of elements. As the relative motion of the particles next to each other has to be rotation invariant due to Galilean transformations, whilst studying the relative motion the solid body rotation drops out and the first order of the viscous stress tensor must be just from the straining part of it. Then in order to introduce the viscosity terms the symmetric tensor is split into two terms.

$$e_{ij} = (e_{ij} - \frac{1}{3}e_{kk}\delta_{ij}) + \frac{1}{3}e_{kk}\delta_{ij} \tag{2.8}$$

This way the first term corresponds to the amount of straining motion while the total volume is unchanged, and the second term corresponds to an increase in volume completely homogeneous in all directions.

Then the shear and bulk coefficients correspond to the amount each of the straining motion, such that the viscous tensor coefficients respectively with symbols η and ζ are defined as

$$\sigma_{ij} = 2\eta(e_{ij} - \frac{1}{3}e_{kk}\delta_{ij}) + \eta'\frac{1}{3}e_{kk}\delta_{ij} \tag{2.9}$$

Up to first order variations then the momentum relation previously defined would provide the Navier Stokes equation.

$$\rho\frac{D\mathbf{v}}{Dt} = -\nabla p + \eta\nabla^2\mathbf{v} + (\eta' + \frac{1}{3}\eta)\nabla div\mathbf{v} + F \tag{2.10}$$

with p being the pressure density.

Using the thermodynamics in terms of densities, i.e

$$de = Tds - pdV \tag{2.11}$$

One can recover the energy equation

$$\rho T\frac{Ds}{Dt} = 2\eta(e_{ij} - \frac{1}{3}e_{kk}\delta_{ij})^2 + \eta'(div\mathbf{v})^2 + div(\kappa\nabla T) \tag{2.12}$$

Using the Hydrodynamics set up before, It is possible to define partition functions corresponding to the average behaviour of the fluid. The partition functions would essentially become the dynamical tool that is necessary to describe the system.

Partition functions of the sound modes and the diffusive modes are calculated to be at [3]

$$S_{ij}^{vv}(\omega,k) = \langle \delta v_i^T \delta v_j^T \rangle_{\omega,k} = \frac{2T}{\rho}(\delta_{ij} - \hat{k}_i\hat{k}_j)\frac{\nu k^2}{\omega^2 + \nu^2 k^4}$$

(2.13)

$\nu = \eta/\rho$ the kinematic viscosity.

Pressure correlation function gives the sound modes

$$S^{pp}(\omega,k) = \langle \delta p \delta p \rangle_{\omega,k} = 4\rho T c_s^3 \frac{\gamma c_s^2 k^2 + \gamma_T(\omega^2 - c_s^2 k^2}{\left(\omega^2 - c_s^2 \mathbf{k}^2\right)^2 + 4\gamma^2 c_s^2 \omega^2} \quad (2.14)$$

where c_s is the speed of sound and γ is the inverse sound attenuation length.

There are two contributions to it based on the change of fields, one from viscosity and one from thermal conductivity.

Hence the region of validity of the hydrodynamics can be observed using the correlation functions. The highest contribution is given at the region where $\omega \sim c_s k >> k^2\eta/\rho$.

2.2 Relativistic Hydrodynamics

Relativistic hydrodynamics is the relativistic extension of the previous section. In order for the hydrodynamics to be applicable on the prescription of the previous section when curvature is introduced, velocity fields are introduced. The starting point of the relativistic hydrodynamics will be the fluid stress energy tensor. The continuation relations from the previous part will take the form [4]

$$\nabla_\mu T^\mu = 0 \tag{2.15}$$

The least complex of the types of fluid described in the relativistic hydrodynamics is the perfect fluid.

Perfect fluids are assumed to have isotropic pressure and for that reason they are simply defined by their stress energy tensor taking the form [4]

$$\mathbf{T} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}$$

(2.16)

To study the dissipation in different cases one can introduce a split of longitudinal and transverse velocity using projectors

$$\Delta^{\parallel}_{\mu\nu} = u_\mu u_\nu \Delta^{\mu\nu = u_\mu u_\nu}(2.17)$$

Then the conservation equations assuming flat space split into to entropy conservation

$$\Delta_\mu(su^\mu) \tag{2.18}$$

Where s is the entropy density.

The transverse conservation equation is the relativistic Euler equation showing the inertia of the relativistic fluid being governed by $\epsilon + P$[5]

$$Du_\mu = -\frac{1}{\epsilon + P}\Delta_{\mu\nu}\partial^\nu P \tag{2.19}$$

Then there comes the concept of near fluidity , as the perfect fluid described only has matter in terms of the pressure density for nearly perfect fluid there would be added perturbations to stress energy tensor. In nearly perfect fluidity the concept of dissipation along longitudinal and transverse modes becomes important. However There is a distinct problem when it comes to introducing the perturbation terms. As there is 4 degrees of freedom coming to the choice of 4-velocity there is a liberty of a choice being made as the choice of frame would affect how the next order equations would look like.[5] A very widely used choice of frame is the Landau frame which leaves the dissipation corrections to the energy momentum tensor left in the same form as the non relativistic case discussed above. The landau frame imposes that in the local rest frame the stress tensor has the form

$$T^{00} = \epsilon T^{0i} = 0 \tag{2.20}$$

Now it is possible to talk about the perturbative variation δT. The first order viscous correction is defined in terms of sheer and bulk viscosity as discussed above along with the viscous stress tensor now defined relativistically as

$$\sigma^{\mu\nu} = \Delta^{\mu\alpha} \tag{2.21}$$

Using this and the fact that we are in the Landau frame, the first perturbative correction will yield, [6]

$$\delta T^{ab}_{(1)} = -\eta\sigma^{ab} - \zeta\theta P^{ab} \tag{2.22}$$

As $u_b \delta T^{ab}_{(1)} = 0$, In the landau frame, using the Euler relation and Gibbs-Duhem, along with the equations of the motion, and using the fact that our first order corrections to the hydrodynamics are divergence free we are left with a divergence term, this sets restriction on the values of the shear and bulk viscosity to be divergence free.

As the first order terms lead to a diverging theory, it is necessary to include the second order for it to be renormalisable. [3]

The second order corrections are [6]

$$\delta^{(2)}T^{\mu\nu} = \eta\tau_I I[{}^{\langle}D\sigma^{\mu\nu\rangle} + \frac{1}{3}\sigma^{\mu\nu}(\partial \cdot u)] + \lambda_1\sigma_\lambda^{\langle\mu}\sigma^{\nu\rangle\lambda} + \lambda_2\sigma_\lambda^{\langle\mu}\Omega^{\nu\rangle\lambda} + \lambda_1\Omega_\lambda^{\langle\mu}\Omega^{\nu\rangle\lambda} \tag{2.23}$$

Vorticity $\Omega^{\mu\nu}$ is written as

$$\Omega^{\mu\nu} = \frac{1}{2}\delta^{\mu\alpha}\delta^{\nu\beta}(\partial_\alpha u_\beta - \partial_\beta u_\alpha) \tag{2.24}$$

Which is the transverse projection of the antisymmetric changes in the velocity field.

Each of the coefficients can be determined using the various approaches discussed in this paper for different orders of coupling.

A subgroup of the equations explained above apply to conformal fluids. In the next chapter it is shown what restrictions conformal invariance implies for the hydrodynamic description of itself.

2.2.1 Conformal Fluid

Conformal fluid has certain restrictions as a result of conformal invariance. Firstly and the most obvious is that the stress tensor is needed to be traceless. It also requires the bulk viscosity to be zero, due to the symmetry restrictions of conformal invariance which will be discussed below. [7]

To study the hydrodynamic structure of a conformally invariant fluid one has to impose the conformal invariance of the metric on to the manifold. This is achieved through the weyl connection. Which transports the tensors in a conformally covariant fashion. [5]

The Weyl connection is defined through the covariant derivative as

$$\nabla_a^{Weyl}\gamma_{bc} = 2\omega A_a \gamma_{bc} \tag{2.25}$$

Hence the Weyl covariant derivative which preserves the weight of any tensor and transforms with the same weight is then written as

$$D_a = \nabla_a^{Weyl} + \omega A_a \tag{2.26}$$

The Weyl connection should be related to the affine connection on the fluid velocity field such that the action of the Weyl covariant derivative would result in the fluid velocity being transverse and traceless.

$$u^a D_a u^b = 0 D_a u^a = 0 \tag{2.27}$$

This then allows the connection one form to be uniquely defined based on the vector field.

$$A = u^c \nabla_c u_a - \frac{1}{d-1} u_a \nabla_c u^c \tag{2.28}$$

As the shear stress tensor is defined as the symmetric distortion of the stress energy tensor it can easily be defined through the weyl covariant derivative as $\sigma^{ab} = D^(a u^{b)}$ *It seems convenient now to introduc*

2.3.1 CFT

Conformal field theory is a field theory which its symmetry group is bigger than the symmetries of the Poincare group. For a more detailed review of the constructions of the CFT you can look at [7]. Conformal invariance implies a local scale invariance on the metric. I.e it is possible to scale the metric as $g_{\mu\nu}$=$g_{\mu\nu}\Gamma(x)$.

The conformal transformation on the metric results in

$$\eta_{\rho\sigma}\frac{\partial x'^{\rho}}{\partial x'^{\sigma}}\frac{\partial x'^{\mu}}{\partial x'^{\nu}} = \Lambda(x)\eta_{\nu\mu} \tag{2.43}$$

Focus momentarily on the algebra of the symmetry group for a field living in this space time it is possible to see that an infinitesimal coordinate transformation would lead the the constraint that

$$x'^{\mu} = x^{\mu} + \epsilon^{\mu} \tag{2.44}$$

The infinitesimal version of the succeeding equation is then

$$\eta_{\rho\sigma}\frac{\partial x'^{\rho}}{\partial x'^{\sigma}}\frac{\partial x'^{\mu}}{\partial x'^{\nu}} = \eta_{\rho\sigma}(\delta^{\rho}_{\mu} + \frac{\partial \epsilon^{\rho}}{\partial x^{\mu}} + \mathcal{O}(\epsilon^2))(\delta^{\sigma}_{\nu}\frac{\partial \epsilon^{\sigma}}{\partial x^{\nu}}) + \mathcal{O}(\epsilon^2)) = \eta_{\mu\nu} + (\frac{\partial \epsilon_{\mu}}{\partial x^{\nu}} + \frac{\partial \epsilon_{\mu}}{\partial x^{\nu}}) + \mathcal{O}(\epsilon^2) \tag{2.45}$$

Thanks to this one can derive the relation

$$\partial_{\mu}\epsilon_{\nu} + \partial_{\nu}\epsilon_{\mu} = \frac{2}{d}(\partial_{\rho}\epsilon^{\rho})\eta_{\nu}\mu \tag{2.46}$$

One sees that this restricts ϵ to be at most quadratic in x, hence the only terms that can exist are

$$\epsilon_{\mu} = a_{\mu} + b_{\mu\nu}x^{\nu} + c_{\mu\rho\nu}x^{\nu}x^{\rho} \tag{2.47}$$

As it is clear the first term corresponds to infinitesimal translation with the generator being the usual momentum operator.

Second term can have both anti symmetric and symmetric modules and hence based on the main relation it is possible to see that the symmetric part is constrained by the equation to be proportional to the metric as one has

$$2b_{(\mu\nu)} = \frac{2}{d}(\eta^{\rho\sigma}b_{\rho\sigma}\eta_{\nu}\mu \quad (2.48)$$

The transformation for which, $x'^{\mu} = (1+\alpha)x^{\mu}$ leads to the generator $D = -ix^{\mu}\partial_{n}u$. This is called the scale transformation.

For the third part of the infinitesimal transformation one can use the equation () to derive that

$$\partial_{\rho}\partial_{\mu}\epsilon_{\nu} + \partial_{\rho}\partial_{\nu}\epsilon_{\mu} = \eta_{\nu}\mu\frac{2}{d}(\partial\cdot\epsilon) \quad (2.49)$$

At which point it is possible to view the permutations of the indices and realise that one can write

$$c_{\mu\rho\nu} = c_{\mu\rho\nu} + c_{\mu\nu\rho} - c_{\nu\rho\mu} = \eta_{\mu\rho}b_{\nu} + \eta_{\mu\nu}b_{\rho} - \eta_{\nu\rho}b_{\mu}, b_{\mu} = \frac{1}{d} \quad (2.50)$$

Hence

$$x'\mu = x^{\mu} + 2(x.b)x^{\mu} - (x^{2})b^{\mu} \quad (2.51)$$

Which clearly leads to

$$K_{\mu} = -i(2x_{\nu}x^{\mu}\partial_{\mu} - (x^{2})\partial_{\nu}) \quad (2.52)$$

The infinitesimal dialtion transformation leads to a conserved Noether's current which in turn confirms the tracelessness of the stress energy momentum tensor. In the scope of conformal fields the requirement on the stress energy tensor is that it is necessarily traceless. Using this algebra it is possible to represent the field in So(2,d) which corresponds to the transformation groups od Ads space. The Stress energy tensor's restriction then by turn defines the liquid corresponding to the CFT.

Superconformal algebra follows similiar lines, in addition to the usual supersymmetry generators Q and the conformal algebra there are additional algebra to describe the representation of the fermionic parts. Clearly this means that one of them has to be the commutation between K and Q, called S. The anticommmutation of S and Q gives rise to the R charges that depending on the theory may exist. For cases of theories without spin 2, i.e gravitationally free field theories the maximum number of supersymmetries is 16 (as observed in the SYM), which leads to 32 generators for the fermionic sector, (combinations of the supersymmetric algebra and the spins). For d=4 which would be the main focus of this paper, the R charge is SU(4) and spinor and conformal representation of the fermionic generators will be $(4,4)+(\bar{4},\bar{4})$ in $SO(4,2)\times SU(4)$. With regards to correlators as the a result of the conformal invariance and scale invariance the two point function of an operator on the scalar field is related

$$\frac{1}{|x_1 - x_2|^{2\Delta}} \tag{2.53}$$

2.3.2 Heavy Ion Collision

Heavy iron collision reveals a very interesting story about the achievements of RHIC and their discovery. It was shown that the collision of two nuclei led to a very big asymmetry in the transverse momentum on different sides. I.e the distribution of the angle on the plane was uneven. What was then discovered is that the collisions led to a fluid model which was almost completely in agreements with perfect relativistic hydrodynamics. Perfect relativistic hydrodynamics as mentioned above has zero viscosity as it does not diffuse. Using this and the connection of the viscosity with the mean free path it was concluded that the result of the collision which is believed to be a quark gluon plasma is described as a strongly correlated system since the momentum transfer is very small and the model cannot be described using quasi particles of kinetic theory. The data also showed that the system reaches the local equilibrium and fluid flow very early on after the collision. The problem persists on the fact that the lattice calculations on the quark gluon plasma would be very complicated and as the fluid is strongly coupled the perturbative analysis is invalid. Hence the only possible outcome seems to be the study of other strongly coupled plasmas similiar enough to the QGP. [3] A fairly comprehensive description of the process experienced in the RHIC has been done in reference 115,116. The summary follows that important cross over regimes are indicated as the initialisation time τ_0 wehre the hydrodynamics characterstics begin to show before the local equilibrium is reached. There is also the freeze out tempreature where the system behaves non hydrodynamically. Analysis has pointed out that the freezeout regime has to do with the confinement/deconfinement phase transition and it is related to the point when particles begin to be confined and there exist a rapid decoupling. The measure of the validity of the hydrodynamics mode might be the calculations of the shear viscosity and the upper bound of which experimentally seems to be set at around $(3-5)/4\pi$.

2.3.3 QCD, quark and gluon plasma (flavourful theories)

To calculate the states of QCD, that is in the deconfined state there exists a more complicated aspect to it. Firstly the breaking of the central symmetry of the group as a result of a non zero polyakov loop. In the quark introduction the situation becomes much more complicated as there exists regimes of chiral synmmetry breaking at low temperature and their symmetry being left unbroken on the higher temperatures. One of the most puzzling bits of this is that lattice calculations above the 1.5 T_c tends to support the weakly coupled description of QCD where the perturbative expansion works and there seems to be an existence of quasi particles. However the the elliptic flow description shows that the plasma constituents must have severe interaction at above the the critical temperature. [3] This issue is resolved by comparing QCD to the N=4 super Yang mills. Experimental evidence shows that at high enough temperatures the susceptibility measured by the holographic methods tends to be in support of the calculations of susceptibility from the heavy iron collisions. Hence in a way dynamically this supports the fact that the quark gluon plasma acts as a strongly coupled system and therefore provides more evidence that the hydrodynamic description would have a higher region of validity with a dual of gravity which is later explored.

As the work will mostly work with the supersymmetric version of QCD, An introduction to how it schematically will look like is necessary.

N=4 Susy QCD is the theory with the highest amount of supersymmetry. In YM theory one has that the coupling of the N=4 gauge theory is $\lambda \equiv g_{YM}^2 N_c$. It includes a gluino, a Weyl fermion in adjoint representation of the gauge group and in the fundamental representation of the global $SU(4)_R$ symmetry consistent with the dual symmetries of the 5 sphere in the supergravity description. It also has a coloured higgs field which is also in the adjoint rep of the gauge group and in the anti symmetric tensor (dim=6) of the SU(4) R symmetry. []

$$L = -\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} - i\bar{\lambda}^a_i \sigma^\mu D_\mu i\bar{\lambda}^a_i + D^\mu \phi^{\dagger\, a}_{ij} D_\mu \phi^a_{ij} \tag{2.54}$$

With the fermionic spin indices suppressed.

For the theory it was found that the entropy in infinite coupling as a ratio of the free theory's entropy is obtained in an expansion of the coupling as

$$\frac{s_{\lambda=}}{s_0} = \frac{3}{4} + \frac{1.69}{\lambda^{3/2}} \tag{2.55}$$

As explained above the QCD plasma will undergo a phase transition from confinement to deconfinement. When it comes to phase transition it is important to study the critical fluctuations near the liquid gas phase transition. Evidence indicates that Near this point the sound modes are higher in energy than the diffusive modes and therefore are dominant. As a result one can see that the QCD is not behaving conformally and their non conformaty will be explored in chapter 2.5.5.

Reason for the conformal symmetry breaking is that the QCD is expected to have an extra phase transition corresponding the chiral critical point.

Although there is evidence that viscoisty minimum is located at the end point of the liquid-gas phase transition both the shear and bulk viscosities diverge near that end point. Data from [8].

As the QCD also experiences phases of quasi particle descritpion at low enough temperature the next chapter is dedicated to providing an alternative analysis for the weakly interacting quantum field theories through kinetic theory.

2.4 Weak interacting systems and hydrodynamics limit

2.4.1 kinetic theory

Kinetic theory is only valid when the system is weakly interacting. The theory consists of quasi particles with a mean free path much larger than the duration of their interaction. This coincides with the YM theory at weak coupling due to $\lambda_{mfp}1/(g^4T)$. Therefore they are described by a distribution function of particles $f(x, \mathbf{p})$. Which is the number of particles at a position x with the momentum $\mathbf{p}$. The Fourier transform of this must be smaller than the the momentum of the particles. At long distances these particles follow classical field theory and they are on shell. [3]

The stress tensor of this is given by

$$T^{\mu\nu}(x) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{p^\mu p^\nu}{E} f(x, \mathbf{p}) \tag{2.56}$$

The dynamic of such system is described by boltzmann equation

$$E\frac{d}{dt}f(x, \mathbf{p}) = p^\mu \partial_\mu f(x, \mathbf{p}) + E_p \frac{d\mathbf{p}}{dt} \frac{\partial}{\partial_p} f(x, \mathbf{p}) = \mathcal{C}[f] \tag{2.57}$$

Where $\mathcal{C}[f]$ is the contribution for all collisions [9]. In the presence of curvature one has to impose the changes that the curvature connection will have on the path of the particles and hence in a background metric that is not flat one would alter the boltzmann equation to

$$p^\mu \partial_m f(x, \mathbf{p}) - \Gamma^\lambda_{\mu\nu} p^\mu p^\nu \partial_{p_\lambda} f(x, \mathbf{p}) \tag{2.58}$$

Now it is possible to study metric fluctuation parallel to the linear response analysis that was done for the CFT in Kabo analysis. The response of the system then determines the transport coefficients for the Kinetic theory and allows a pathway for Hydrodynamics construction. The starting point then would be to introduce the metric fluctuations perturbively writing our general space metric two form as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ and then chosing the transverse components of the metric to the direction of the wave vector travel, z.

If one has the deviation from the equilibrium written as

$$f(x, \mathbf{p}) = f_{eq}(E_p) + \delta f(x, \mathbf{p}) \tag{2.59}$$

As the result of the disturbance and assuming rotational invariance the equilibrium position is altered from the perturbation as $E_p \rightarrow \sqrt{|(\eta_{\mu\nu} + h_{\mu\nu})p^\mu p\nu|}$ to

$$f_{eq} = f_0 + f_0' p^x p^y \frac{|v_p}{p} h_{xy} \approx_{v=E/p} f_0 + f_0' \frac{p^x p^y}{E} h_{xy} \tag{2.60}$$

Collision terms are very complicated to calculate, one would have for an elastic scattering the very complicated. One can then instead use the relaxation time approximation which assumes a local equilibrium and knowing that oscillations from the equilibrium are driven back to the equilibrium the relaxation time τ_R should be of the same order as the λ_{mfp} [9].

The equilibrium is chosen to be

$$f_0 = \frac{1}{exp[\sqrt{p^i(\eta_{ij} + h_{ij})p^j}/T \mp 1} \tag{2.61}$$

Inputting the δf into the boltzman equation one then has

$$(\partial_t + v_p \partial_x)\delta f + n_p(1 + n_p)\frac{p^i p^j}{2E_p}\partial_t h_{ij} = 0 \tag{2.62}$$

the solution of which is going to be

$$\delta f(\omega, k) = -i\omega h_{xy} \frac{n_p(1+n_p)}{-i\omega + iv_p q + 1/\tau_R E} \tag{2.63}$$

Leading to the retarded correlator [3]

$$G_R^{xy,xy}(K) = -\int \frac{d^3p}{(2\pi)^3 v^x v^y \frac{\omega p^x p^y (n_p)(1+n_p)}{\omega - qv_p + \frac{i}{\tau_R}}} \tag{2.64}$$

Now Oen can work in the limit of q to zero which is possible by imposing that all momenta is smaller than the internal scale, which is the limit for the kinetic theory as well.

Using Green-Kubo relations then one recovers the form for shear viscosity as

$$\eta = -\tau_R \int \frac{d^3p}{(2\pi^)3} \frac{(p^x p^y)^2}{E^2} (n_p)(1+n_p) \tag{2.65}$$

One can also recover sound modes by relaxing the metric restriction set at the start and allowing longitudinal modes [9]

$$T^z z = -\frac{1}{2} G^{zzzz}(\omega, k) h_{zz}(\omega, k)$$

G$^{zzzz}(\omega, k) = (\epsilon_0 + P_0) \frac{c_s^2 \omega^2 - i\Gamma_s \omega^3}{\omega^2 - c_s^2 k^2 + i\Gamma_s k^2 \omega}$(2.66)Where Γ is the usual sound attenuation length.

2.5 Holography

2.5.1 Open vs closed strings

Starting with Type II string theory, p branes are the objects to look for when it comes to connecting Large N field theories and string theory as they modify the allowed boundary condition to include Dirichlet boundary conditions, i. e $x^p, ...x^d - 1 = 0$. These branes along with the closed strings introduce an open string with any value for the coordinates $x^0, ...x^p, and x^p + 1..x^d - 1 = 0$ which describe the excitation of the p branes [10]. The quantisation of them is at a 1-1 correspondence with the orientable open string which is regularily known. Massless modes are a vector and spinor superpartner with the gauge group U(1) in SYM. As the boundary condition implies the dissapearance of the 0 modes, the gauge vector bosons are only described by the 0,...,p coordinates. And scalars are described by p+1...d-1. In this sense the scalars can be viewed as the oscillations in the position of the p+1 brane. On the Dirichlet p brane more commenly known as the Dp brane, we can start to discuss the open string dynamics. Dp branes can be viewed as D branes with a defect, a fixed point where a closed string can split into two open strings being attached to the p dimensional subspace of the "hole". These open strings represent the excitation of the p-branes. As the strings reside on the boundary of the Dp brane, then if the subspace is p dimensional the world volume of the Dp brane is necessarily p+1 dimensional. Therefore taking the string action we will refer to this p+1 dimensional space. The open string represents the fluctuations of the d-brane. The fields only exist on the world volume, hence they can be used to describe the fluctuations of the D-brane in the transverse direction using their excitations. There exist 9-p of them in 10 dimensional construction. The existence of the U(1) gauge field is very essential and leads to some of the more key properties explored when treating open strings as a thoery for non abelian gauge theory. As multiple branes come close to each other the presense of non abelian gauge fields becomes important. As an example imagine two D-branes close to each other, There exist four different combinations of open strings that can be created using different end points. Therefore the symmetries that occure from that combination will be the U(2) symmetry group. As the establish connection of open strings viewed in terms of a gauge theory, one can also represents this as a gauged field theory.

The gauge field then can have two indices for each end point which shows that it is represented by the U(2) gauge group. Moreover the bosonic field superpartners of the gauge field are represented in the adjoint group of the SU(2). As the representation is Nc*Nc in the adjoint with Nc being the number of the D-branes stacked together we can see that as an example for D-3 branes the fields are going to be $\phi i = 1, 2, 3, 4, 5, 6$ and coupled to four fermions. In effective field theory at low energies where the massive modes can be integrated out, it is possible to see that the space of n p branes near by is the full maximally supersymmetric gauge theory of SYM, with the interesting property that the beta function which is the measure of the coupling change with scale is zero, meaning that the coupling doesnt run with scale and that the low energy gravitational prescription is dual to a conformally invairant field theory [11]. The Lagrangian is written as

$$tr[\frac{1}{2g^2}F_{\mu\nu}F^{\mu\nu}+\frac{\theta_I}{8\pi^2}F_{\mu\nu}\bar{F}^{\mu\nu}-i\overline{\lambda}^a\overline{\sigma}^\mu D_\mu\lambda_a-D_\mu X^i D^\mu X^i+gC_i^{ab}\lambda_a[X^i,\lambda_b]+g\overline{C}_{iab}\overline{\lambda}^a[X^i,\overline{\lambda}^b]+\frac{g^2}{2}[X^i,X^j]^2] \tag{2.67}$$

As closed strings can propagate through the full 10 dimensioanl space time the lagrangian written above would receive higher order corrections, called Dirac born Infield corrections. However at low energies they can be neglected and give rise to the correct correspondance [3].

As one would like to work only in AdS space, the compact directions need to manifest themselves in another way. To be able to preform this reduction, a process called Kaluza-Klein (KK) compactification is used [12]. This compactification on S^5 allows the theory to include the SU(4) gauge group on the supergravity. Meaning one gauges the supergravity in order to work in a smaller subspace. The full SYM can be described by solely the AdS_5 as long as the spherical harmonics are included in the field. To draw a map of this, it is possible to say that the supergravity theory in 10 dimensions can be theorized as the one explained above by taking the gauge theory to be the fiber bundle over a compact manifold. Since the Gauge group in question is SU(4) which is a cover of SO(6) it is possible to represent this as a fiber bundle over S^5. As the discussion now includes an infinite volume compact curvature, the gauge field strength should be related to a volume form on S^5 and that exist stationary wave solutions which are quantised. The quantisation of them leads to frequencies

$$\omega|R| = \lambda_{\pm} + l + 2n \tag{2.68}$$

.

Using the fact that the masses of states in the kaluza-klein have the infinite tower construction, the mass restriction is set to be $(mR)^2 = l(l+4)$. Which would lead to the the definition of λ being related to l. Hence bounding the frequency from below and creating conformal time periodic dependency for all of the scalar fields in the supergravity multiplet (As described above compactifying the time dimension for the theory into an S1). The importance of the quantisation comes to importance when there is a comparison of the fields in the SYM and the spectrum. Going back to formulating the gauged supergravity, The spectrum of the N=8 supergravity is the practically the same as the gauged supergravity with the only difference being that 12 of the vector fields take the form of anti symmetric two forms. That is to say that as the result of the kaluza-klein compactification the representation of the vector supermultiplets now includes indices for its representation in the gauge theory. To be able to write the action, the scalar fields now will transform covariantly on the gauged global symmetry as usual.

The region of stability allows tachyonic modes as the exponets above would converge for the values of

$$m^2R^2 \geq -\frac{d^2}{4} \tag{2.69}$$

It turns out as the kaluza-klein excitiations are of order one, as the radius of S^5 is comparable to the AdS_5 Radius, The dimension one fields on the conformal field theory are exactly the mass excitation of the kaluza-klein theory which was thoroughly calculated in [].

2.5.2 Gauge gravity Correspondence

To show that the space time symmetries of AdS are related to the conformal symmetries of the minkowski space, We can take use of the conformal compactification used by carter penrose when describing the spacetime of classical blackholes. Starting with the minkowski space with the lorentzian signiture,we change coordinates as usual and analytically continue to the boundary of the space time [11].

The conformal minkowski space time then has the metric

$$ds^2 = \frac{1}{cos^u_+ cos^2 u(du_+ du_- + \frac{1}{4} sin^2(\theta d\Omega^2 p - 1(2.70)}$$

where $u_\pm = \tau \pm \theta/2$

absorbing the scale factor $\frac{1}{cos^u_+ cos^2 u}$ into the metric one can see this corresponds to Einstein's static universe and has the geometry of $R \times S^p$ with the generators of the global time translation taking the form

$$H = \frac{1}{2}(P_0 + K_0) = J_{0,p+2} P_0 : \frac{1}{2}(\frac{\partial}{\partial u_+} \frac{\partial}{\partial u_-}) \tag{2.71}$$

$$K_0 : \frac{1}{2}(u_+^2 \frac{\partial}{\partial u_+} + u_-^2 \frac{\partial}{\partial u_-}) \tag{2.72}$$

Therefore it is possible to show that using the killing vectors symmetries, that the time like killing vector is going to be related to the combination of translation and and special conformal group, which then is going to be the SO(2) subgroup of the maximally compact subgroup of SO(2,p+1). It is clear that that SO(p+1) represents the symmetries of the Sp sphere and hence it is possible to say that the maximally compact subgroup discussed above is isomorphic to $R * S^p$. On the other hand looking at the AdS spacetime it is possible to see that the universal cover of it is isomorphic to half of the static universe. The boundary of this space is exactly isomorphic to the boundary of the p+1 minkowski which can be seen as the Einstein static universe is covered half way through. The clear definition of H also allows the CFTs on the minkwoksi to be extended onto the geometry described above.

Now we can move on to discuss the symmetries of the AdS. To be able to embed the AdS space, it should be constructed as a hyperboloid. As it is often found in general relativity there exist some inconsistencies in regards to arbitrary choices such as coordinates. In this paper most of the choices as such are consistent with the original paper on the corespondance. Following this it is possible to write the hyperbola as

$$X_0^2 + X_p + 2^2 - \tag{2.73}$$

obviously the symmetry due to the conservation of the hyperbola would lead to the action of SO(2,p+1) on a module. The embedding of the anti-desitter in the flat space then takes the form [11]

$$ds^2 = -dX_0^2 - dX_p + 2^2 + \sum_{i=1}^{p+1} \tag{2.74}$$

The full AdS metric will be then

$$ds^2 = R^2(-cosh^2\rho d\tau^2 + d\rho^2 + sinh^2 d\Sigma^2) \tag{2.75}$$

The spherical coordinates have a symmetry group of SO(p+1). Near the boundary $\rho- > 0$ the metric could be mapped to $S^1 * R^{p+1}$. Hence near boundary, The maximally compact subgroup of what was of this mentioned above the SO(2)*SO(p+1) and the universal cover is the symmetry group. To preserve casual structure which would be broken by having the time like coordinate mapped on the S^1 one performs another coordinate transformation to unwrap the circle, $tan\theta = sinh\rho$ and the metric full causal metric is

$$ds^2 = \frac{R^2}{cos^2\theta}(-d\tau^2 + d\theta^2 + sin^2\theta d\Omega^2) \tag{2.76}$$

The causal structure of AdS written in this format is related to the Einstein's static universe although it is apparent that $\tau >= \frac{\pi}{2}$ instead of the full π, meaning that the boundary of AdS only covers half of the Einstein's static universe, any metric that has this behaviour on the boundary is called assymptotically AdS, which is the larger class of metrics that can be studied in this correspondance, the same way that the field theory is applicable in any asymptotically Minkowski space [11].

As the KK reduction has been explained one can expand the fields as towers of fields in AdS by including harmonics of S^5 such that

$$\phi(x,\Omega) = \sum_{\ell} \phi_\ell(x) Y_\ell(\Omega) \tag{2.77}$$

And then one can work on the AdS reduction and its equivilance to N=4, SYM in 4d.

In string theory the coupling constant of the string is given by the value of dilaton at infinity. $g_s = e^{\Phi}$. Here the infinity chosen will be the boundary (∂AdS) [13]. If there exist a new local operator with a source term $\phi(x)$ the generalisation of the argument above imposes that there must exist a field in the bulk which has the value corresponding to ϕ at the boundary.

$$\phi(x) = \Phi|_{AdS}(x) = \lim_{z \to 0} \Phi(x,z). \tag{2.78}$$

With the addition of mass into the theory, there would be the need for scaling dimension as there exist a scale symmetry in need of conservation. Imagine the theory expanded around the quadratic order of Φ, After solving this one finds out the solutions are in the asymptotic form of [3]

$$\Phi(z,k) \approx A(k) z^{d-\Delta} + B(k) z^{\Delta} \tag{2.79}$$

The mass dimensions are

$$\Delta = \frac{d}{2}\nu \ \nu = \sqrt{m^2 R^2 + \frac{d^2}{4}}$$

(2.80)

As the scale isometry would restrict a transformation of scale to

$$\Phi(z,x) \rightarrow \Phi(\Lambda z, \Lambda x)A(\Lambda x) = \Lambda^{\Delta-d}A(x) \tag{2.81}$$

Now to the argument before about the proptionality of the boundary source to the bulk field at the boundary one has to impose dimensional restrictions, i.e

$$\phi(x) = \Phi_{\partial AdS} \equiv lim_{z\rightarrow 0}z^{\Delta-d}\Phi(z,x) \tag{2.82}$$

Which restricts the source to have the dimensions $d-\Delta$ and Hence the field operator would have to have dimensions Δ.

For Spin-2 fields, (graviatons), the scaling dimensionality leads to a bulk p-form having the scale of the highest root of

$$m^2R^2 = (\Delta - p)(\Delta + p - d) \tag{2.83}$$

One of the amazing outcomes of this formulation is that in terms of the correlates if one imagines that the boundary field is identified with A(x) and that the expectation value of the operator

$$\langle \mathcal{O} \rangle_{\phi} = 2\nu B(x) \tag{2.84}$$

which using linear response theorem would result in the proportionality

$$G_E(\omega_E, \vec{k}) = \frac{\langle \mathcal{O}(\omega_E, \vec{k}) \rangle}{\phi(\omega_E, \vec{k})} = 2\nu \frac{B(\omega_E, \vec{k})}{A(\omega_E, \vec{k})} \tag{2.85}$$

This relation is crucial as it shows that the poles of the two point function are bijective with the solutions of the equations of motion for the bulk field at the boundary.

Hence A(x) has a mass scaling dimension of d-Δ, and B(x), of Δ. As energy-momentum tensor is a conserved operator for transitionally invariant theory, (Since they are the generators of it), it becomes a very important operator that has the external metric changes as its source, $g_\mu\nu(x)$. This metric also corresponds to the boundary value of the bulk metric and shows that the gravity theory is dynamical if the energy-momentum tensor is a conserved operator.

Generalising to any source field on the gravity side with the dual operator partner in CFT side, The field operator correspondence can be written in terms of its generating functional

$$\langle e^{\int d^4x\phi_0(\vec{x})\mathcal{O}(\vec{x})}\rangle = \mathcal{Z}_{String}[\phi(\vec{x}), z)|_{z=0} = \phi_0(\vec{x}] \tag{2.86}$$

Now to calculate explicit correlation function for a theory that has a scalar as a source, one takes a look at the connected diagrams, their contribution can be written as

$$W_{gauge}[\phi_0] = -log\,\langle e^{\int d^4x\phi_0(\vec{x})\mathcal{O}(\vec{x})}\rangle_{CFT} =\approx extremum I_{SUGRA}[\phi_0] \tag{2.87}$$

Now considering the scalar in the supergravity,

using the besel functions described before one would have in 4 dimensions

$$\langle\mathcal{O}(\vec{x})\mathcal{O}(\vec{y})\rangle = \epsilon^{2(\Delta-4)}\frac{2\Delta-4}{\Delta}\frac{\Gamma(\Delta+1)}{\pi^2\Gamma(\Delta-2)}\frac{1}{|\vec{x}-\vec{y}|^{2\Delta}} \tag{2.88}$$

Which is in the agreement with the CFT correlation function [11]. And the process can be continued for higher order functions.

As the usage of hydrodynamics will require dependence on thermodynamics it would be worth while to study the correspondence for field theories at finite temperature.

2.5.3 CFT and blackholes, blackbrane

We now turn our focus to study some systems using holography. After the construction it could be easily conjectured that the macroscopic properties of the blackhole would have a dual in the effective field theory in question. This thermodynamic properties then in turn give us a way of describing various hydrodynmamic properties in the fluid gravity correspondance. Therefore it is crucial to study the thermodynamical properties of the blackholes in the dual field theory more closely. In the paper [input number]. One of the first analysis of the correpondance in such way was done on reisnner nordstrom charged black holes [14]. Obviously their action along with the gravitational part has the field strength tensor from the maxwell action and can be written as

$$I = -\frac{1}{16\pi G}\int d^{n+1}x\sqrt{-g}[R - F^2 + \frac{n(n-1)}{l^2}_M \tag{2.89}$$

With $\Lambda = n(n-1)l^2$ with respect to the length scale l.

The metric takes the form of

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Sigma^2_{n-1} \tag{2.90}$$

with $f(r) = 1 - \frac{m}{r^{n-2}} + \frac{1^2}{r^{2n-4} + \frac{r^2}{l^2}}$ using the parameter q one then can determine the pure gauge potential and can fix the gauge by demanding euclidean regularity of the one form at the horizon. In order to discuss thermodynamics We look at the euclidean sector of the metric which comes with the idea that we find the period of euclidean time (imaginary time) in terms of the area and etc. The period of the imaginary time arises from the periods of the killing vector which arises from wanting the solutions to have a regular forms and can be written in terms of potential $\Phi = \frac{1}{c}\frac{q}{r_+^{n-2}}$

$$\beta = \frac{4\pi l^2 r_+^{2n-3}}{(n-2)l^2(1-\Phi^2) + nr_+^2} \tag{2.91}$$

This in turn can be understood as the euqilibrium of blackholes. Hence The period can be rewritten as a thermodynamical equation of state. For example for n=3, the equations of state lead to the Blackhole temperature of

$$T = \frac{\Phi^2(1-\Phi^2)+Q^2}{2Q\Phi} \tag{2.92}$$

To calculate different energies, it is necessary to regularize the calculation since the gravitational action and hence the energy is infinte [14]. Therefore one method that usually gets carried out is to introduce a background metric which matches the solutions and subtracting a sort of action contribuition. In the RN case introdced above it is possible to use metrics that are assymptotic to the AdS. This method such as some of the dimension regularisation methods in quantum field theory regulaises the action by adding boundary counterterms to it. The Gibbs potential contribution is calculated to be

$$W[\Phi,T] = \frac{1}{12}[3\frac{Q}{\Phi}(1-\Phi^2(-(\frac{Q}{\Phi})^3] \tag{2.93}$$

The helmolz free energy takes into account the relationship with the normallisation factor so depending on the normalisation, the change of boundary conditions would affect it.

$$F[Q,T] = \frac{1}{12}[3\frac{Q}{\Phi}(1-\Phi^2(-(\frac{Q}{\Phi})^3+9Q\Phi] \tag{2.94}$$

While plotting this the writers of the paper found out that there exist a shape as they called it resembling a swallow tail for values of T¿T_crit or Q lowwr than the critical point. Obviously the critical points have been calcualted to be the points where the F[Q,T] has a zero derivative with respect to each. The swallow tales physical significance comes in to the play for the phase transition condition. The condition can be translated in terms of the areas inclosed by the isotherm curves, where the phase transition is viewed as an equal area law. The equations of state show that for the intrinsic variable Φ and extrinsic variable Q there exist three branches of solutions for sub critical charge. For the thermodynamical stability it follows that there exist two different bracnehs. The branches of the solutions have different phase changes and it can be shown that the second branch extending between the two slopes of dQ$/d\phi$. This is electrically stable as it has positive slope and it can be computed exactly where the electrical stability starts.

The thermodynamical properties discussed above can then be translated into different models of blackholes. The fact that there is swallow tail and there exist phase transitions at different temperature could shed some light on phase transitions in dual field theories.

In turn there is the correpondance between blackhole dynamics and thermodynamics. It was argued that generally that the boundary theory of a gauged gravity theory with a gauge group of rank c is described effectively by a conformal fluid with a set of c U(1) charges. Using the thermodynamical properties of the charged static blackholes it is possible to determine the partition function. However, there are conditions to be satisfied in order to use the equations of fluid dynamics. The first, is that fluctuations around correlation functions has to be minimal, which is the case in the large N dynamics of the field theory. Secondly a fluid has to be in a local equilibrium, hence it is only useful when the length scale of of the variations of thermodynamical variables and the curvature of the space is larger in comparison to the mean free path which is an effective description of the equilibration length scale. This is dual to a condition that the horizon radius of the blackhole is large compared to the AdS radius [15].

It is also possible to study the blackhole through extrinsic curvature calculations.

2.5.4 fluid gravity correspondance

Imagine setting up the field theory an describing the fields in the adjoint rep as a combination of anti fundamental and fundamental, except for a mixing term which is going to have 1/N contribution there exist an equivilance between the SU(N) and the U(N) of the anti/fund. If this is imagined as a surface then there is a way to topologically discuss the feynman diagrams produced by this prescription.

The perturbation theory one is at a 1-1 with closed orientable strings with the coupling constant related to 1/N. Fields expectation value, analogous to the string theory vertext operators in the world sheet. D brane construction for an extremal black p brane solution. The world duality relates d branes not only as the source of closed strings but as the boundary of the open strings. Therefore the same gauge theory construction built from the supergravity theory applicable in the low energy regime is applicable by imagining N dbranes stacking up together. The prescription related to the D branes is more useful when it comes to string perturbation as it uses to string world sheet. Then using the duality between open and closed strings on the D-branes it was possible to develope the AdS/CFT description. Some of the physical phonemena that influenced the gauge/duality prescription include the Greybody factors and blackholes, which showed that two open strings collision on the D brane sourcing the closed string in the bulk is a process mimicking the hawking radiation [11]. The calculations for the cross section of the particle coming from infinity absorbed by blackholes was shown to have agreements in both calculations made by two point correlators and the supergravity solutions. This in turn sparked the idea that the two are interconnected and related to each other in regions where t'hooft coupling limits are opposite. For more information check out (the references). Similiar processes using D1-D5 brane which has the 1+1 quantum theory it can be shown that the far region describes the near region much like how the throat of the minkowski region in the Ads-Scharwarzschild can be used to match conditions between the black hole thermodynamics and quantum thermodyanmics in the far outside of the throat region.

In AdS/CFT there exist an isomorphism of the particle states in the classical hilbert space of string thoery and single trace operators of the gauge theory. As the space of single trace operators is infinite there is a infinite class of gauge theories which have dynamics described by einstein gravity with negative cosmological constant. As explained above at sufficiently high temperatures, near equilibrium it is possible to describe the dynamics of the quantum field theory using hydrodynmaics. What can be derived from this is that in the long wave length regime the Einstein's equations in the bulk should reduce to the equations of hydrodyanmics in the boundary. As derived above if the equatiosn of hydrodynamics have been determined to second order gradient expansion then one expects the fluid dynamical equations to correspond to einstein's gravity in second order perturbation. i.e inhomogeniousk, time dependent black holes with slowly varying but otherwise generic horizon profiles. Boundary stress tensor is related to to normalisable metric perturbations about a state. Holographic renormalisation as an approach to recursively perturb the metric in higher dimensions. The best way to start with the duality would be the global equilibrium, then that allows us to observe when the equations of state cannot describe the boundary away from that equilibrium. Hawking tempeature has already been established as a dual of the temperture field where as the fluid dymamical velocity is dual to the horizon boost velocity of the black hole [6]. As the blackhole has assymptotically Ads Behaviour and the temperature of Schwarzchild-Ads Blackhole grows linearly with horizon we are ensured thermodynamic stability in the long Ads Radius Regime. To create a gradient expansion what is being done on the hydrodynamics level is slightly varying the temperature and velocity field from position to position, the dual of this would be similar to smoothly patching together different configurations of blackholes varying in temperature and boost. find a good source to get the symmetries and etc out of it to be able to discus the first order expansion. As T(x) and u(x) are invariant under redefinition, which implies that there is a freedom to choose the frame for u away from the equilibrium. upon choosing the liquid rest frame called the landau frame which has the from (input form) and it fixes the ambiguity of the fields by relating the velocity and the temperature to the stress tensor. Remember that the construction of the stress tensor is related to the velocity fields. Naturally as the stress tensor is related to the entropy by the relationship (describe the relationship) you would expect there to be an entropy current whose divergence is point wise non negative, and these constraints lead to the entropy current taking the form

$$J_s^a = su^a - \frac{1}{T} u_b \Pi^{ab}_{(1)} \tag{2.95}$$

To construct the solutions the starting point would be the solution that corresponds to a global thermal equilibrium in Shwarzschild-AdS

$$ds^2 = -r^2 f(r/T) dt^2 + \frac{dr^2}{r^2 f(r/T)} + r^2 \delta_{ij} dy^i dy^j$$

(2.96)

Where $f(r) \equiv 1 - \frac{(4\pi dr)}{d}$ The solution has been adapted from the usual AdS-Shwarzschild

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_{d-1}^2 \tag{2.97}$$

with the identification that the relation between the blackhole temperature and the horizon radius is $r_+ \equiv \frac{(4\pi Tr)}{d}$. [15] It is possible to use a boosted frame along the spatial directions to get a solution that is written in terms of the velocity field which along with the temperature gives the hydrodynamic properties of the bulk.

To generalise this point it is possible to adapt general AdS seed geometry. One can study the away from equilibrium properties of the system by creating perturbations to the seed geometry.

$$ds^2 = -2u_a(x)dx^a dr - r^2 f(r/T)u_a(x)u_b(x)dx^a dx^b + r^2 P_{ab} dx^a dx^b \tag{2.98}$$

Where $P_{ab} = u_a u_b + h_{ab}$, the metric is written In ingoing coordinates. With a gauge fixing $g_{rr} = 0$ and $g_{ra} = -u_a$.

As perturbations are assumed to involve a perturbation series expansion, the metric is expanded into

$$g_\mu \nu = \sum_{k=0} g_{\mu\nu}^{(k)}(T(x), u^(\epsilon x) \tag{2.99}$$

Hence to each order of perturbation what is needed to be done is to solve the Einstein equation too that order.

The corrections of the seed metric can be written out as

$$H[g^{(0)(T^{(0)}}, u^{a(0)}]g^{(n)} = s_n \tag{2.100}$$

where the operator H is constructed from the information in the equilibrium.

The general understanding of what the equation entails is to look at the Einstein equations and the meaning of each part. The E_{ra} would be a conservation equation for the momentum in the radial direction. The $E_r r$ part is the Hamiltonian (energy) constraints for the radial evolution. The E_{ra} can be solved for each slice since the direction is parallel to the radial direction and the E_{ab} would be the dynamical equations that can move the solutions between slices. Hence as it is possible to study the E_{ra} equations at each slice it makes sense to bring the dynamical solutions to the boundary and study them there and then it can be solved for the whole system. It turns out that what is studied at the boundary is just the conservation of the stress tensor at that point. Now it is possible to re express the situation in terms of expression from hydrodynamics for the generic metric derived above.

$$ds^2 = -2u_a(x)dx^a(dr + \mathcal{D}_b(r,x)dx^b) + \mathcal{G}_{ab}(r,x)dx^a dx^b \tag{2.101}$$

To be able to parametrise these values it is possible to taylor expand the stress tensor. Doing such would lead the second order perturbation to take the form

$$\partial_\lambda \partial_\mu {T_0}^{\lambda\mu} = 0 \tag{2.102}$$

In landau frame.

As the seed metric preserves the SO(3) rotation one can apply constraints specifically to the scalar vector and tensor channels. A summary of these constraints is found in []. For second order the number of channels increases and as the fluid is supposed to be conformal the Weyl covaraince from chapter 2.3 also plays a role. The values for the parameters above can be summarised as

$$\mathcal{D}_a = r\mathcal{A}_a - \mathcal{S}_a c - \lfloor_1(r/t) P_a^b D_c \sigma_b^c + u_a[\frac{1}{2}r^2 f(r/T) + \frac{1}{4}(1 - f(r/T)\omega_{cd}\omega^{cd} + \lfloor_2(r/T)\frac{\omega_{cd}\omega^{cd}}{d-1}] \tag{2.103}$$

$\mathrm{G}_{ab} = r^2 P_{ab} - \omega_a^c \omega_{cb} + 2(r/T)^2 \}_1(br)[\frac{4\pi T}{d}\sigma_{ab} + \}_1(r/T)\sigma_a^c \sigma_{cb}] -$
$\}_2(br)\frac{\sigma_{cd}\sigma^{cd}}{d-1}P_{ab} - \}_3(r/T)[\mathcal{I}_{1ab}\frac{1}{2}\mathcal{I}_{3ab} + 2\mathcal{I}_{2ab}] + \}_4(r/T)[\mathcal{I}_{1ab} + \mathcal{I}_{4ab}]$

$S_{ab} = \frac{1}{d-2}(\mathcal{R}_{ab-\frac{\mathcal{R}}{2(d-1)}_{ab}}$ and $\mathcal{R}$ is the Weyl covariant curvature tensor. The tensors not introduced previously are to produce a weyl covariant basis and their details along with the coefficients can be found in [5] and [15].

To finish the construction it is possible to simply write the stress energy tensor found (2.24) by making the replacement infront of the coefficents τ_I, λ_i to their respective weyl covariant partners $\mathcal{I}_i^{ab}$. The coefficients themselves are calculated in [15].

In order to introduce a physical attribute to the convergence modes described later, it is possible to use small fluctuation of the first order dissapative coefficients above in an oscillatory fashion. Such that In first order the infinetesimal shifts to coefficents would be related to $e^i wv + kx$. By solving for the hydrodynamic equations one can derive a dispersion relationship for them. The convergence of these modes provides the realm in which the hydrodynamic description are valid. [5]

Another natural way to consider what has been done is the fact that the hydrodynamics description derived from gravity has strong connections with the membrane paradigm discussed in [16].

In the r limit of the blackhole horizon classical Einstein gravity tells us there lives a fictitious fluid on the horizon, membrane paradigm. membrane paradigm and the boundary theory fluid.

In some prescriptions it has been observed that the equations are exactly precise at low frequency regimes and correspond exactly to that of the mebrane paradigm. What this comparison creates is a clearer description of the connections of the thermodynamics in the gravitational point of view to ones of the statistical physics. The paper shows how one can describe the transport coefficients of the boundary liquid in terms of the sole near horizon of the blackhole geometry. However away from the lowest order of the expansion of frequency, the full geometry affects the relationship. The procedure to relate the membrane to the boundary fluid then at the non low frequency limit relies on introducing constant radius hypersurfaces and introuding a dynamical flow equation that moves across the radius to the boundary. This in a way is an intuitive analogue of the dynamical equations introduce in the fluid gravity section. To this point It is important to have an overview of this included in the paper.

The membrane paradigm can be summarised by starting of outside of the horizon. If one demands that causality is respected then the effective action on the observer is the outside the horizon action plus the effective action of the horizon on the observer. This is not the same as the complete action at the boundary of horizon. The dynamics of the boundary is determined by demanding that the observer is stationary with respect to it.

There are certain physical properties that can be derived from this. Firstly the membrane conductivity can be found by demanding that there exist a U(1) gauge field in the bulk.

The bulk action then will have the term

$$S_{bulk} = -\int d^{d+1}x\sqrt{-g}\frac{1}{4g_{d+1}^2(r)F_{MN}F^{MN}} \tag{2.104}$$

Where g(r) is a radius dependent gauge coupling.

The boundary term at the horizon is determined by varying the action and that determines the boundary action that cancels it out to be

$$S_{Surf} = \int d^d\sqrt{-\gamma}\frac{j^\mu}{\sqrt{-\gamma}}A_\mu{}_{\Sigma} \tag{2.105}$$

Where the Legendre conjugate variable of A_μ is set to be j^μ.

Hence the current induced due to the boundary on the horizon would be

$$J^\nu_{mb} \equiv \frac{(j^v(r_0))}{\sqrt{-\gamma}} = -\frac{1}{g_{d+1}^2}\sqrt{grr}F^{r\nu}(r_0) \tag{2.106}$$

Applying the near boundary coordinates and the appropriate gauge ($A_r = 0$) leads to the interpretation that the current is the response of the horizon membrane to the electric field with membrane conductivity

$$\sigma_{mb} = \frac{1}{g^2_{d+1}(r_0)} \tag{2.107}$$

Same structure can be used to calculate the shear viscosity of the membrane. As shear viscosity is the response of the stress-energy tensor when there exist massless scalar modes, The relevant action will be

$$S_out = -\frac{1}{2}\int d^{d+1}x\sqrt{-g}\frac{1}{q(r)}(\nabla\phi)^2\bigg|_{r>r_0} \tag{2.108}$$

Boundary term is then

$$\S_s urf = \int d^d\sqrt{-\gamma}\frac{\Pi}{\sqrt{-\gamma}}\phi(r,x)\bigg|_{\Sigma} \tag{2.109}$$

For which then the membrane scalar charge to be calculated as

$$\Pi_m b \equiv \frac{\Pi_{r_0}}{\sqrt{-\gamma}} = -\frac{\sqrt{g^{rr}}\partial r\phi(r_0)}{q(r_0)} \tag{2.110}$$

assuming that the field is the off diagonal component of the graviton, one then has the coupling as $q = 16\pi G_N$ and the shear viscosity can be measured as

$$\eta_{mb} = \frac{1}{16\pi G_N} \tag{2.111}$$

and using the fact that the entropy density for a black hole is going to be $s = 1/4G_N$ one deduces the famous bound of $\eta_{mb}s_{mb} = 1/4\pi$ which strengthens the argument of a connection between the two regions.

The bulk evolution is simply in zero in the low frequency regime as one has

$$\Pi = \frac{-\sqrt{-g}}{q(r)}g^{rr}\partial_r\phi\partial_r\Pi = \frac{-\sqrt{-g}}{q(r)}g^{rr}g^{\mu\nu}k_\mu k_\nu\phi \tag{2.112}$$

Second equation drops out due to the killing vector properties of black holes and the symmetries, however these one forms are to approach zero at the low frequency regime which means that the radial evolution doesn't happen.

The more interesting regime is the high/infinite frequency regime where there exist complicated structures that cannot be described classically by the membrane paradigm. This regime shows a hydrodynamics flow [16].

Starting with constant r membrane hypersurfaces and defining the usual transport coefficient for each of them.

$$\bar{\chi}(r, k_\mu) = \frac{\Pi(r, k_\mu)}{i\omega\phi(r, k_\mu)}$$

(2.113)

From the previous two equations it is possible to derive the evolution of it in terms of a flow equation.

$$\partial_r \bar{\chi}(r) = i\omega \frac{\sqrt{g^{rr})}}{\sqrt{g_t t}} [\frac{\bar{\chi}^2}{\phi(r)} - \Sigma_\phi(r)(1 - \frac{k^2}{\omega^2} \frac{g^{zz}}{g^{tt}}] \tag{2.114}$$

$$\Sigma = \frac{1}{q(r)} \sqrt{\frac{-g}{g_{rr} g_{tt}}} \tag{2.115}$$

The flow equation then can be integrated to derive the response of AdS/CFT to all different freuquencies. In a way analytically continuing the method into regimes where it should not have any applications.

As an example of this flow one can take the momentum diffusion. Small perturbation in momentum density away from the equilibrium in the z direction is taken. In this case the graviton fields in the gauge $h_{ar} = 0$ where a is the normal directions to z, are decoupled from the rest and the remaining components are h_{at} and h_{az}. As diffusion would only involve non diagonal components the momenta is $T^{a\mu}$, mapping the plane spanned by a using a Kaluza Klein reduction to a gauge field, the graviton modes are mapped to a gauge field and what is found is that the correlator in the retarded green's function would take the form

$$h^a_\mu = A_\mu, T^\mu_a = j^\mu$$

$\mathrm{G}^{az,az}_R = \frac{s}{4\pi}\frac{\omega^2}{i\omega - D_s k^2}$

(2.116)

Poles of which are found out by determining the diffusion constant

$$D_s = \frac{\eta}{\epsilon + p} = 4G_N s \int dr' \frac{g_{rr} g_{tt}}{\sqrt{-g} g_x x}^{r_0}$$

(2.117)

And overall one can calculate the evolution of momenta density using the correlator. The overall importance of this method is that now there exist a map of classical gravity to hydrodynamics.

What it could provide is that using this method the low energy and high energy regimes are now well connected and calculations based on AdS/CFT now can be analytically extended and vice versa. It also provides a flow equation from the non linear hydrodynamic prescription between relativistic and non relativistic limits [1].

Another point to added to the description is the universality of the shear stress tensor value.

This can be analysed using a general blackbrane in any dimension.

A general black brane in D-dimensions has a metric of the form

$$ds^2 = f(\xi)(dx^2 + dy^2_+ g_{\mu\nu} d\xi^\mu d\xi\nu) \tag{2.118}$$

Using the area law the entropy is porpotional to

$$S = \frac{A}{4G} \tag{2.119}$$

The absorption cross section of the gravitons polarised in the xy direction that propagates normal to the brane is calculated by the greens function of T_{xy} which is the operator coupled to the boundary metric h_{xy} [17].

$$\sigma_{abs} = -2\frac{\kappa^2}{\omega} ImG^R(\omega) = \frac{\kappa^2}{\omega} \int dtd\mathbf{x} \left\langle T_{xy}(t, \mathbf{x}) T_{xy}(0, \mathbf{0}) \right\rangle \tag{2.120}$$

Which then comparing to the Kubo Relationship previously discussed would lead us to the clear relationship between the absorption cross section and the shear viscosity.

$$\eta = \frac{\sigma_{abs}}{2\kappa^2} \tag{2.121}$$

What is interesting is that it can be shown that the cross section is calculable by solving a wave equation for the h^x_y. It then leads to the observation that by considering the low energy limit where the perturbations to the geometry are very small and assuming that as the gravitons are polarised and the only non vanishing first order of the perturbation is h_{xy}, It could be shown that by solving the Einstein equations that the behaviour of the perturbation is the same as a minimally coupled scalar. Hence the absorption cross section of the graviton is the same as a massless scalar.

The theory follows from [17]

$$R_{MN} = T_{MN} - \frac{T}{D-2} g_{MN} \tag{2.122}$$

The stress-energy tensor depends on various matter fields such as the dilaton. It can be then shown that as the result of the O(2) rotational symmetry for xy that all the matter fields are perturbation free as they would be coupled to the perturbed metric h_{xy} which would break the symmetry.

Hence the Einstein Equation can be rewritten as

$$-(L + \frac{T^{(0)}}{D-2})\delta_{\alpha\beta f + h_{\alpha}\beta}$$

(2.123)

The Lagrangian is that of the matter fields. By rewriting the LHS of Einsteins equation in linear order of h and comparing it to the unperturbed results deduced from the blackbrane metric one can have.

$$R_{xy} = -\frac{1}{2}\Box h_{xy} + \frac{1}{f}\partial^{\mu}\partial_{\mu}h_{xy} - \frac{(\partial f)^2}{f^2}h_{xy} = -\frac{1}{2}[\frac{\Box f}{f} - \frac{(\partial f)^2}{f^2}]h_{xy}$$

(2.124)

Which then in turn by changing variables of h_{xy} to fh^x_y It is possible to see that the equation that this 1-1 tensor satisfies is exactly that of the massless non interacting scalar, i.e $h^x_y = 0$. Based on the work of [18], then it can be said imperically that as the bound of the scalar absorption cross section of a scalar in the blackbrane metric is equal to the area of the horizon and since the entropy is related to the horizon by the relation $s = a/4G$ by re introducing the plank constant scales and the rest one retrieves the minimum value of the shear viscosity has to be equal to the

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B} \tag{2.125}$$

The universality of this theory was tested by the work of [19], which provided evidence for and against the theory ad showed that including matter fields in the fundamental representation violate this bound.

The paper discussed how the different Quantum corrections affect the theory. For this purpose one can start by writing the action schematically. As the dual field theory is conformal one can expect the gravity action in AdS to take the look of

$$S = \int d^5\sqrt{-g}\frac{12}{L^2} + R + L^2\lambda_1 W^2 + L^4\lambda_2 W^3 + L_3^6 W^4 \tag{2.126}$$

W is the weyl tensor and the specific inputs of the parameters for the weyl tensor and the λ_n can be calculated using perturbative string theory. In zero temperature when the gauge theory is supersymmetric the second term vanishes and without matter in the fundamental representation (usually quarks) the first term also drops out. Considering these simplifications it was shown that

$$\frac{\eta}{s} = \frac{1}{4\pi}(1 + \frac{15\zeta(3)}{\lambda^{3/2}}) \tag{2.127}$$

Which respects the bound discussed before for the infinite coupling t'hooft limit.

However a 4-d conformal theory can be described by two central charges which then using the Gauge/Gravity duality yield the anomaly calculation that sets $_1 \cong (c-a)/8c$. Calculating the gravity action and hence calculating the $\frac{\eta}{s}$, using the fluid gravity method previously discussed, it can be seen that the leading order which is the first term of the gravtiy action would have the effect of

$$\frac{\eta}{s} = \frac{1}{4\pi}(1 - 8\lambda_1 + O(\lambda_3, \lambda_1^2)) \tag{2.128}$$

Hence the bound is only respect for the negative values of the coefficient. In theories where c-a is non negative one has the violation of the bound. It was found that in the large class of the CFTs and $SU(N_c)$ gauge most of the theories violate this bound and the situation is only exacerbated once there exist a gauge field in the action. However as the leading corrections are of the order $1/N_c$ they will be very small so although the bound is not quite universal It could be used as a point of vicinity.

2.5.5 Plasma Balls

Plasma balls are droplets of deconfined plasma surrounded by the confining vaccum. Conjectured by [20], These plasma balls map to localised blackholes. As the dual gauge theory of interest is expected to be at a finite temperature one needs to restrict the theory to only background gravity solutions which can have blackbranes of finite energy density higher than a critical value exxamined by them. A study based on this later confirmed the findings by showing that blackholes in 6 dimensions that assymptote to the Scherk-Shwarz (anti periodic fermion boundary conditions on the KK compactification) compactifcation of AdS_6 [21] are dual to the plasma balls described above. Specifically as the boundary of these blackholes will be dual to the Scherk-Shwarz compacification the 4+1 dimensional CFT, the long wave length effective description is dual to the 3+1 Navier stokes equation. The viscosity parameters of which can be derived from the fluid gravity correspondance.

The topology of these gravitational solutions is and $R^d \times afilledtorus$. As there exist two cycles in a filled torus as such, there can exist two distinct solutions based on which cycle is filled in. When the torus is a square one can have a phase transition between the two solutions as there is no prefered direction and hence the thermodynamical porperties are described by the phase transition temperature and the changes between the solutons. The identification of the phase transition in the large coupling limit when one has large anamolous dimension that allow the existence of gluballs with masses much higher than the graviton leading to their dual theory simply being the classical string theory oscillations. The configuration is set up such that the blackhole decay is dual to a gluon plasma ball decaying using hadronization. There are a multitude of papers discussing the gravity dual of the plasma balls. One of the main reasons of interest in the field of fluid gravity is that fluid gravity correspondent is only valid for plasma without a boundary surface. Reason for it being that at the surface the denisty and pressure have a large variance between points and the hydrodynamics conditions break down. As the plasma balls and discs possess a boundary one requires the full gravitational solutions in order to produce a dual to the boundary fluid. The construction in [22] showed that at regions with infrared cutoff which is controlled by breaking the conformal symmetry, the infrared localised black holes in an assymptotic AdS_4 backrgound are in fact dual to the plasma balls.

The construction begins at the metric

$\mathrm{ds}^2 = \frac{\ell}{(x-y)^2}[-\ell^{-2}H(y)dt^2 + \frac{dx^2}{G(x)} + G(x)d\phi^2]$
$where H(y) = y^2(1+2\mu y), G(x) = 1 - x^2 - 2\mu x^3$

$\ell = \sqrt{-3/\Gamma}$ is the cosmological radius.

G(x) contains a dimensionless parameter and one can restrict the parameter so that the G(x) contains three roots that are

$$-\frac{1}{2\mu} < x_0 < x_1 < 0 < x_2 \tag{2.129}$$

To simplify the solution one can take the limit $\mu \to 0$ which leads to the solution of x_1 asymptoting to -1.

One recovers the patch with the subgroup symmetry of poincare in the empty AdS_4.

$$ds^2 = \frac{\ell^2}{z^2}(dz^2 - dt^2 + dr^2 + r^2 d\phi^2) \tag{2.130}$$

Where one has -ℓ/y and arccos(x) as polar coordinates of the (r,z) plane. Using the fact that one wants implement an infrared cutoff, It is possible to implement a wall with vacuum energy momentum tensor as its souurce.

The gemoetry of the cutoff works by restricitng that $x_1 < x0$. As the conditions of the wall are satisfied at x=0 and one could have the wall extend to infinity as the horizon will become essentially infinite at the critical point μ_c where x has the value of $-\sqrt{3}$, With infinite horzion temperature and area.

The area can be calculated using kumar intergration and is set to be

$$A_H = 4\pi\ell^2 \frac{(x_1^2 - 1)^2}{x_1^2(3 - x_1^2} \tag{2.131}$$

At this limit the horizon exhibits an interesting effect called low-wetting, it loses its spherical profile and the horizon grows larger alonmg the directions parallel to the infrared wall and creates a water droplet like shape. At this limit there exist regimes of negative curvature around the center of the horizon. This is surprising as what was expected is that at around the center one would recover the geometry of the black brane. The solutions of which is found in the paper. But the most important property is that the mass is of the order

$$m = \frac{\ell}{3\sqrt{3}}$$

(2.132)

Which gives the sense that the blackhole will not reach planar geometry in the large horizon radius limit. What this then indicates is that eventhough the horizon diverges, the blackhole does not extend infinetly and in fact it is exhibiting a warped hyperbolic geometry.

Following the paper it is suitable to give the plasma ball CFT some formulations as well [**?**].

Imagine the metric written in Fefferman-Graham coordinates (z, x^i),

$$ds^2 = \frac{\ell^2}{z^2}(dz^2 + g_{z,x}dx^i dx^j) \tag{2.133}$$

Such that the metric can be expended in terms of a power series in orders of z.

the third order gives the stress tesnor of the dual theory

$$\langle T_{ij}(x)\rangle = \frac{3\ell^2}{16\pi G} g_{ij}^{(3)}$$

(2.134)

The calcualations are once again preseneted there. There are certain conclusions one can derive from this, first of all is that the metric on which the CFT is the boundary theory is non flat and is dependent on x_1. Hence one gets different geometries for different values of the parameter.

One can then see the duality by observing the stress energy tensor as a function of radial distance and it was found that in fact in the centre there exist a perfect fluid and as the critical limit is approached the pressure and energy density remains the same until the boundary of the plasma where it drops off rapidly. THis proves that the dual object has a boundary that is non hydrodynamical and that it is in fact describing a plsama ball. This regions as said before coincides with the ricci scalar for the boundary metric of $R^0 - 6\ell^2$

and to check it is possible to even see that the stress energy tensor is exactly the same witout a coformal factor. The external curvature is the controlling limit of how much the geometry of the boundary is warped to allow the formation of the plasma ball and in the critical limit

$$K|_{r_p=r_{ball}} \rightarrow 1.5\ell^-1 \tag{2.135}$$

which is finite and the edge remains the same shape. Through this analysis certain charactersitics of the wall can be studied. Firstly the characteristics of the spontenously broken conformal invariance, which should lead to massless modes could be studied through the massless radion produced and its effect on the confinement deconfinement phase transition.

It was found that through this that the deconfinement temperature is vanishingly small , hence the formation of plasmas is not first order as the plasma phase is related to the temperature. As this idea is not stable it seems like that the only way to stabilise the plasma ball is to give the raidon degrees of freedom mass and allow the modifications of the curvature of.

The aim of the inclusion of this is to view fireballs along with the necessary identification of dynamical gravity in the boundary theory, Which recovers the first order phase transition. Hence there exist interpolations of fluid dynamics and gravity in a deconfined plasma ball. This Theory and the conjecture inspired the work of calculating the dynamics of plasma balls from holography in numerical simulations. For the simulation. As described before AdS solitons are found to be the dual to the confining phase of the N=4 Superyang mills and the metric is described as a metric with a flat conformal boundary and a compact S^1 directon. The numerical scheme works by solving einstein's field equations in terms of solutions that include the soliton as the background with some sort of non small metric deformation [**?**].

The soliton has a metric

$$\hat{g} = \frac{1}{(1-\rho^2)^2}(-dt^2 + \frac{4^2}{f(\rho)}d\rho^2 + dx_1^2 + dx_2^2 + f(\rho)d\theta^2)$$

with f(ρ) $= 1-(1-\rho^2)^4$(2.136)IR bottom is at $\rho = 0$. Assuming no symmetries except for the compactified circle

one can have a general form of the metric with all of the cross terms. Confinement scale and the deconfinement temperature are both related to the period of the circle by $T_c = 1/\Delta\theta = \Lambda$. If the AdS-soliton has a vanshing stress-tensor, i.e the counter term for the renormalisation scheme is taken such that the lhs of the background einstein equation is set to zero, the black branes on this background have the thermodynamical quantities

$$(\epsilon, P_1 = P_2, p\theta) = \frac{\pi^2}{4}(3T^4 + \Lambda, T^4 - \Lambda^4, T^4 + 3\Lambda^4)$$

(2.137)

The analysis takes into account of matter formation by coupling massless scalar field ϕ to the gravity. The dual of it is going to have the value of the scalar at the boundary as the source term and a scalar operator dual to it as its momentum conjugate. The CFT hasnt had the stress tensor of it modified and hence by construction it is still trasless.

The result of the simulation indicated that for the anystropic distribution of the scalars there is a black hole formation at the IR bottom with the blackhole being extended in one direction more than the other, resembling the almond shape interaction viewed at the RHIC.

The results explain an unusual behaviour. As the scalar field is supposed to be described by a set of gapped and quantum like modes, in late time it should have been expected that they will couple and the frequencies of each mode reaches its assymptotic stage, however shockingly it was discovered that fluctuations and mode mixing is still present although at a very small amount even in late times. As this is supposed to be dual to the Quark Gluon plasma reaching its final freezout stage, what has been observed is dual to the freezeout not being present at the times previously expected. These are the results of the modes bouncing back and fourth between the IR bottom and the AdS boundary, as they cause periodic fluctuations. The study points out this strongly interacting regime exist when the wavelength of the modes are comparable to the size of the blackhole. This phase is described in QCD as a transfer of energy between IR and UV modes coupled to the dynamical oscillations caused (dynamical) by the dynamical boundary metric.

For more realistic model of QCD where there is an epxectation for the existence of quark which are matter fields in the fundamental representation, there would be an additional existence of

D7 branes and their respective orientfold planes.

2.5.6 confinement-deconfinement phase transition

At finite temperature as the ground state energy is changed the supersymmetry and conformal invariance is broken. Computation of entropy for the finite temperature U(N) Yang-Mills is quite arduous. However using a free field approximation it could be shown that the results of the identification were quite successful. In this construction then it was shown that for the identification to apply one should set the hawking temperature in the supergravity which is once again related to the area of the horizon equal to the field theory temperature. As a result it was shown that [3]

$$F_SYM = \frac{4}{3}F_SUGRA \tag{2.138}$$

In the strong coupling limit the supergravity relation would be equivilant. In the weak coupling regime the calculation was done from the expansion to two loop in perturbation theory. The strong results came from the leading corrections to the supergravity action. Both of these corrections become the subject of interest in the hydrodynamic prescription. In order to achieve the t'Hooft large N limit It is known that the loop stringy corrections are not counted in, hence there is an extermination similiar to the process of stationary point approximation applied to the generating functional of the supergravity fields. The biggest problem is that as gravity as at all points divergent it is non renormalisable, hence they are only valid using particular asymptotic boundary solutions. This leads to certain multiple solutions to the theory, which should be summed over to obtain the singular solution to the gauge theory side. The importance of this choice of solution is that it leads to phase transitions between the multiple solutions leading to the deconfinement-confinement transition which would be discussed in the extent of certain confinfing-deconfining quantum solutions such as the plasma balls in an AdS background.

The choice of the solution leads to distinct and different topological spaces. First one with $S^4 \times S^1$ with the boundary being $S^3 \times S^1$. Discussed in [12] it is possible to have a completely different topologically distinct structure $R^2 \times S^3$. In regards to spin structure the second manifold has a distinct structure as it is simply connected, However as the S^1 is not simply connected, there is a non uniqueness to its spin since it will be possible to have different non continuous patches. Hence the representation of the spin can take two different forms, thermal and supersymmetric. The second topology restricts the first space dubbed X1 to have the thermal structure. Leaving the supersymmetric patch to X2. Since both of them have a saddle point contribuition, using a cut off radius as a regularisation scheme to reach definite values and defining a consistent time for both (the circumfrance of S1 for X1 is the same as the geodesic length of the time killing vector of the other) It is possible to write the action difference as

$$I(X_2) - I(X_1) = \frac{\pi^2 r_+^3 (R^2 - r_+^2)}{4G_5(2r_+^2 + R^2} \tag{2.139}$$

Whichever space's contribution is bigger it leads to a different theory. This leads to two regions of validity. One where this value is positive, where it is the blackhole in the AdS or if the X1 contribution is bigger and the sign is negative then thee thermal gas of particles in AdS is favoured. Using the coupling limits of QCD it was found that these in field theory are the phase transitions from confinement to deconfinement. Hence One expects to work with quasi particles in AdS when describing a deconfined field theory. This idea of the confinement deconfinement is of importance while studying gauge theories that exhibit assymptotic confinement. QCD an assymptotically free and confining theory could benefit a study from this approach. The starting point is to compactify time on a circle and use anti periodic boundary condition. The metric produced is the euclidean black hole. The effective dimensionless gauge coupling of QCD is determined by the fact that the periodic boundary condition on the circle will lead to the radius of the compactifiying circle to provide the uv cutoff. The dimensionless gauge coupling at the cutoff distance is determined by g_{SN}.

This prescription can then gave rise to the analysis through shockwave solutions. In shockwave soluions one uses a soliton background which is the dual X1 described before and by applying gravitational shockwaves that produce blackholes study a deconfined phase in a confined background [23].

The metric of this analysis is

$$ds2 = 2dx^+dx^{+(dx^+)2\Phi(x^i)\delta(x^+)+d\vec{x}^2}(2.140)$$

With Φ being a harmonic 0-form of the manifold and satisfying the Poisson equation

$$\Delta_{D-2}\Phi(x^i) = -16\pi Gp\delta^{D-2}(x^i) \tag{2.141}$$

This becomes the background in the curved space. The gravitational scatterings was shown to produce blackholes. Now turning the attention to the pion scattering, It was conjectured that the position of the IR brane is the pion dual in the gravity theory.

The scalar as a product of the pion field shockwaves collision, the soliton, should be dual to the blackhole description. Some scale prescription shows that that the blackhole creation begins at the scale M_P which in the gauge theory relates to $M_p = N^{1/4}\lambda_{QCD}$ and the gauge theories maximal "Froissart behaviour" is observed when the size of the blackhole is its AdS radius.

However before the saturation of the band and before the AdS reaching the IR brane, the scattering leads to a gauge theory with the cross section related to $s^1/11$. And the maximal Froissart behaviour is reached before the expected energy bound. Hence showing the description works before the bound is saturated. So there exist at a lower energy band a dual description of the blackhole for the lightest glueball(the lightest pion excitation where there exist no other pions in the system except for it). As the KK modes are the lightest of the gravity modes, a prescription was implied to check whether these two are dual.

The prediction led to the conjecture that these pion fields exactly describe the Color Glass Condensate (CGC), that eventually expands into the Quark Gluon Plasma, which eventually decays into the free pions. Hence one can track the evolution of the plasma as reasonably high energies in the gravity dual. The first phase would be dual to the blackhole Formation. The decay of the soliton into pion is then the gravitons radiating away from the blackhole and the temperature for the freeze out is the hawking's temperature of the blackhole. This led to the realization that this may be the temperature of the phase transition from confinement into deconfinement. To check the prediction the used the spherically symmetric nature of the black hole at the IR brane to derive the temperature of the blackhole as

$$T = \frac{M_1}{4\pi} \tag{2.142}$$

Where M_1 is the mass of the KK graviton. As the conjecture implies that these modes correspond to the lightest pion modes, once the pion average mass is inserted, the results should indicate the temperature for the freeze out regime. The value was calcualted to be 175.76 MeV which is fantastically close to the 176 MeV calculated for the freezout regime at the RHIC.

This section shows a familiar cross between the field theory and gravity theory through thermodynamics. To expand on the effect of hydrodynamics on shockwave solution the next chapter is dedicated to the study of the hydrodynamics simulations and efficacy of the gradient expansion on these solutions.

2.5.7 shockwave solutions

Using holography it has been possible to derive a multitude of dual theories that have various uses. As mentioned in the section 2.3.2, the data from RHIC support hydrodynamics viscous flow for the system. Therefore it is logical to present gravity models that could be dual to this collision and study the fluid gravity correspondence to evaluate the hydrodynamics of such systems. It is also a great model for the response of at what limits the collision can be described by hydrodynamics. As the process of QGP formation can be dual to the gravitational collapse and blackhole formation and hadronisation and the relaxation of QGP to the confined state mapped to the blackhole evaporation, all stages of the evolution can be mapped to gravity models that can be numerically studied in an iterative fashion.

One of the dual theories that describes a simple model of QGP production is the gravitation shockwaves collision which form blackholes. Nucleus being dual to the planar shockwaves and proton the transversely localised shockwaves. One then can numerically solve the dynamical einstein equations and track the evolution of the SYM stress tensor. The gradient expansion of the hydrodynamics stress tensor coupled to the SYM has orders of power $1/\ell T_{eff}$ where ℓ is he scale over which $T^{\mu\nu}$ varies and T_{eff} is the effective temperature. It is not a surprise then that the effective temperature acts as the representation of mean free path. The model presented in this section focuses on the small mean free path which is the more strongly coupled regime of the scale. The model [24]has the shockwaves moving in the z direction at the speed of light. In Fefferman-Graham coordinates the metric of these shockwaves is

$$ds^2 = r^2[-dt^2 + d\mathbf{x}^2 + \frac{dr^2}{r^4}] + h_\pm(x_\perp, z_\mp, r)dz_\mp^2 \tag{2.143}$$

$$h_\pm(x_\perp, z_\mp, r) \equiv \int \frac{d^2k}{(2\pi)^2} e^{i\mathbf{k}\cdot x_\perp} H^{\pm(\tilde{\mathbf{k}}, z_\pm \frac{8I_2(k/r)}{k^2r^2}} \tag{2.144}$$

from holography it is possible to view the boundary theory stress tensors in terms of the transverse Fourier transform (i.e fourier transform in the transverse directions) of the $H^{\tilde{\pm}}$ such that

$$T^{00} = T^{zz} = \pm T^{0z} = H_\pm(x_\perp, z_\mp) \tag{2.145}$$

The numerical simulation then apporoximates a simple choice of the shock profiles such that it consists of a longitudinal δ function in terms of a gaussian $\delta_\omega(z) = \frac{1}{\sqrt{2\pi\omega^2}} e^{-\frac{1}{2}z^2/\omega^2}$ and a normalised longitudinally integrated energy density per area which then helps with the localisation. It could be represented by $\mu_+(x_\perp)^3 = e^{-\frac{1}{2}x2_\perp/\sigma^2}$. The evolution of the geometry is tracted using a near boundary approximation and gradient expansion described above in an assymptotically AdS space time. If one has the near the boundary metric as $g_{\mu\nu} = \eta_{\mu\nu} + g_{\mu\nu}^{(4)}/r^4 + O(1/r^5)$ which evolves the stress energy tensor as

$$T^{\mu\nu} = g_\mu \nu^{(4)} + \frac{1}{4}\eta_{\mu\nu} g_{00}^{(4)} \tag{2.146}$$

The simulations seems to show that the region with hydrodynamics validity is when $|x|\tilde{\sigma}$ and depends on the transverse size of the proton. One can use the temporal eigen value which is the energy and the eigen vector of it which is the fluid velocity calculated from the extact stress tensor and approximate the stress tensor for hydrodynamicas using the mechanism outlined in the previous sections. The plot shows the rapid decay between the values of the stress tensor and the hydrodynamic expression for it. Using a measure of anistropy it was shown that the ideal hydrodynamics case is not a good approximation and the results pointed out that the first order gradient is almost as large as the ideal stress. Which shows the extreme capability of hydrodynamics description even in a region with large gradient. As the large gradients are accompanied by large initial viscosity parameters which drive the rapid development of the transverse flow. Leading to a higher region of validity of hydrodynamics prescription.

Phys. Rev. Lett. 115 (2015) 241602, [1506.02209] Colliding Shockwaves

2.5.8 non conformal gauge/gravity duality models

Until very recently most of the simulations provided by the shockwave solutions were done for theories dual to conformal field theories, however as QCD is a theory which carries a scale factor for most of the temperatures, It raises questions about the quality of the models that use Holography and as a result mostly conformal field theories as an approxiamtion to Heavy Ion collisions. The procedure for producing these models is the following [25].

Models that are dual to the CFT are deformed by a source which breaks scale in variance and produces an Renormalisation Group flow [26]. The procedure is quite similar to the explanation of adding quarks as sources to the theory explained above. One of the intriguing aspects of what is shown is that as the relation between the energy and density which used to be fixed by symmetry (The stress tensor energy tensor being traceless) is not anymore under that constraint. Therefore the freeze out and relaxation includes and extra channel relating to the asymptotic behaviour of both called EoSization.

As realised before to deform the theory one can always couple a scalar field to the gravity model.

The gravity action then is changed to

$$S = \frac{2}{\kappa_5^2} = \int d^5x\sqrt{-g}[\frac{1}{4}R - \frac{1}{2}(\nabla\phi)^2 - V(\phi)]$$

(2.147)

The potential is then chosen to satisfy some properties. As one would like to keep the vacuum solution asymptotically AdS at UV, Secondly the second scalar derivative at $\phi = \phi_{max}$ would constrict the mass of the field into be $-3/L^2$, allowing the dual operator to have dimension 3 at the UV regime.

The potential then accordingly is chosen to be

$$L^2V(\phi) = -3 - \frac{3}{2}\phi^2 - \frac{1}{3}\phi^4 +$$

$$(1\frac{}{2\phi_M^4 + \frac{1}{3\phi_M^2})\phi^6 - \frac{1}{12\phi_M^4}\phi^8}$$

(2.148)

Which has a maximum at $\phi = 0$ and a minimum at $\phi = \phi_M$.

The dimensions for the IR region (at $\phi = \phi_M$) are presented in the paper [26]. Using (FG) coordinates the solution following the assumption that it will be asymptotically AdS will be be

$$ds^2 = \frac{L^2}{u_{FG}^2}du_{FG}^2 + e^{2a_{FG}u(FG)}\eta_{\mu\nu}dx^\mu dx^\nu \tag{2.149}$$

Using principles of super symmetry one can then obtain an arbitrary addition of super potentials related to the potential and as there exist auxiliary fields related to the metric coefficients one can derive the solution necessary (based on the choice of super potential in the paper), The equations are the following and their analytical solution is presented in the paper

$$u_{FG}\frac{da_{FG}}{du_{FG}} = \frac{2}{3}W, u_{FG}\frac{d\phi}{du_{FG}} = -\frac{\partial W}{\partial \phi} \tag{2.150}$$

To apply gradient expansion then it is possible to take the 5 dimensional AdS assymptoting metric in the FG form

and power expansion of the holographic coordinate

such that

$$g_{\mu\nu} = \eta_{\mu\nu} + g^{(2)}_{\mu\nu}u^2_{FG} + g^{(4)}_{\mu\nu}u^4_{FG} + ...\phi = \phi_0 u_{FG} + \phi^{(2)}u^3_{FG}$$

(2.151)

Producing up to u^4_{FG} the 1 point correlators dual to the metric and the field ϕ for the field theory respectively as

$$\langle T_{\mu\ nu}\rangle = \frac{2L^3}{\kappa_5^2}[g^{(4)}_{\mu\nu} + \Lambda\phi^{(2)} - \frac{\Lambda^4}{18}\frac{\Lambda^4}{4\phi_M^2}\eta_{\mu\nu}]$$

(2.152)

with the UV limit of ϕ being the source of the dual operator.

$$\langle \mathcal{O} \rangle = -\frac{2L^3}{\kappa_5^2}(2\phi^{(2)} + \frac{\Lambda^3}{\phi_M^2}$$

(2.153)

Taking the trace of the penultimate equation one can derivate the equation

$$\langle T_\mu^\mu \rangle = -\Lambda \langle \mathcal{O} \rangle \tag{2.154}$$

Which was to be expected for the breaking of the conformal invariance.

The numerical simulation follows a very similiar procedure to the shockwave solutions described before for the conformal plasma balls, hence stating the results of the simulation seems sufficient enough.

As $\eta/s = 1/4\pi$ in all the Einstein gravity models, it becomes unimportant to measure it. However the ratio of the bulk viscosity to the shear viscosity becomes an important index of the measure of the conformality of the fluid.

The ratio is calculated using

$$\frac{\zeta}{\eta} = 4(\frac{dlogs}{d\phi_H})^{-2} \tag{2.155}$$

Once again the results show that hyrdoynamisation comes before isotropisation (where the fluid has isotropic pressure) as the transverse pressure is still about 70% larger than the longitudinal modes. The EoSization time which is defined as the time taken for the average pressure to agree with the equilibrium by 10% accuracy is controlled by the expectation value of operator dual to the scalar and its peak time seems to be decrease at high energies. As the conformal effects become more strong and it assymptotes to the hydrodynamisation time. This is explained through the fact dynamics of the condesate decouple from the stress tensor. What is interesting is that hydrodynamisation time is smaller than the Eosization time in more non conformal systems, and it indicated a value for the degree of non conformality necessary for this to happen in terms of the bulk viscosity. THe bulk viscosity has be around 0.025 for this effect to occure, which shows the response of the fluid to increased bulk viscosity.

2.6 convergence of hydrodynamic modes

The rise of interest in finding the radius of convergence is described by all of the proceeding sections before hand. As the green's functions of the stress tensor in equilibrium has infinitely many poles in the fluid gravity prescription it allows numerical results to be manifested. The trust in these numerical results is based on how good the hydrodynamic prescription is. Amongst these poles the most relevant ones as the work presented mainly focuses on CFTs are the shear and sound channel described above [27].

The modes can be simply written as

$$\omega_\perp = i\frac{\eta}{sT}k^2 + \mathcal{O}(k^4) \tag{2.156}$$

for the sheer mode,

$$\omega_{\pm=\pm\frac{1}{\sqrt{3}}k-i\frac{2}{3}\frac{\eta}{sT}k^2+\mathcal{O}(k^4)}(2.157)$$

2.6.1 Large coupling limit

Why do we care about the convergence of the hydrodynamics mode. It is infact because if the gradient expansion mentioned in section 2.2 is actually viable, by having a convergent series to the expansion we can have away from equilibrium microscopic description of whatever theory is described by the stress energy tensor. Recent studies shows hydrodynamic gradient expansion is divergent. The work of [28] points out to the fact that there exist a finite radius of convergence. This section will follow with studying the convergent series and a general way to gain analytical control over the divergence of the series for general values. The holographic theory that was presented in this paper is the Einstein Maxwell theory for AdS_4. Which ofcourse would contain RN blackbrane in the bulk for the equilibrium. Paralllel to what you will see for the kinetic theory one observes the structure of the complex k plane to determine the radius of convergence. This leads to a multi sheeted modes of $(\omega(k))$ with the principal sheet being related to the hydrodynamics properties of the theory and contains branch points

$$k = \pm ik^* where k^* = \frac{1}{2\mu}\sqrt{\frac{\epsilon + p}{2\mu\sqrt{\eta}}} \tag{2.158}$$

This was accredited physically to the collision of the branch point and the hydrodynmaic mode on the k axis. What has been interesting is that if one analytically continues the quasi normal modes past the branch point they will move on to a secondary shorter lived sheet which corresponds to the non hydrodynamic modes hence drawing a connection between the different modes through this prescription. One can then use the analytical continuation to determine the breakdown of hydrodynamics in the complex plane.

Starting with AdS_4 RN blackbrane with horizon at (z=1) can be written as the metric

$$ds^2 = \frac{1}{z^2}(-f(z)dt^2 + \frac{dz^2}{f(z)} + dx^2 + dy^2 \tag{2.159}$$

And

$$f(z) = 1 - (1 + \frac{\mu^2}{4})z^3 + \frac{\mu^2 z^4}{4} \tag{2.160}$$

As one is dealing with the convergence radius of the shear modes, one perturbs the metric and gauge filed transversely

$$\delta g(t, x, y, z) = \frac{e^{-i+ikx}}{z^2}(h_{ty}(z)2dtdy + h_{xy}(z)2dxdy) \tag{2.161}$$

$$\delta A(t,x,y,z) = e^{-i+ikx} h_y \tag{2.162}$$

One can form a master equation satisfying both ODEs.

$$-f(f\phi'_{\pm})' + f(q^2 + \mu^2 z^2 - \mu z c_{\pm})\phi_{\pm} - \omega^2 \phi_{\pm} = 0 \tag{2.163}$$

On the boundary ($z \to 0$), the boundary condition is the absence of sources hence producing quasi normal modes for the bulk.

One then can expand both the shear mode and the solution to the perturbation in terms of a series

$$\phi_{+}(z) = \sum_{n=0}^{\inf} \psi_n(z) k^{2n} \tag{2.164}$$

$$\omega(k) = \sum_{n=0}^{\inf} \omega_b k^{2n} \tag{2.165}$$

Expanding ϕ in terms of (ω) then links the two series to each other.

For calculating the radius of convergence it is convenient to define Pade's approximant which defines a ratio between two polynomials as

$$\mathcal{P}_k(k) = \frac{\sum_{n=0}^{\inf} a_n k^n}{1 + \sum_{j=1}^{\inf} b_j k^j} \tag{2.166}$$

Which then gives the closest pole to k=0 to be $k = \approx 0.753i$

Which corresponds to a pair of modes

$$\omega(0)/\mu = \pm 0,7493 - 0.5128i$$

(2.167)

The large order behaviour of the system is dominated by a factorial form

$$J^n \tilde{A} \frac{\Gamma(n+\alpha)}{\chi^{n+\alpha}} \tag{2.168}$$

χ and α are some constants. the factorial form is controlled by the parameter χ which was coined as singulants.

The article[27] devised a way of using Dingle's singulants to achieve control over its away from equilibrium behaviour for non linear flows. Singulants are (define singulants and their behaviour briefly). They applied the large order behaviour and the singulant theory to the gradient expression. Assuming patches of space would have different flows and different velocities it is possible to have the parameters defined above as scalar fields.

The ansatz can be summarised as

$$\Pi^n_{\nu\mu}(t,\vec{x}) = A^n_{\nu\mu}(t,\vec{x}) \frac{\Gamma(n+\alpha(t,\vec{x}))}{\chi(t,\vec{x})^{n+\alpha(t,\vec{x})}} \tag{2.169}$$

There are discussions on why the main focus is longitudenal flow. The reason for that is that you can always go into the liquids rest frame and it adds a degree of spherical symmetry and translational invariance. The paper focuses on different regions of validity and their overlap of domain. The study finds that the singulants have a duality to a particular domain of gradient expansion. Moreover there is a region of absolute overlap between linear response theory,hydrodynamics and the singulants which is the most resolved region.

We can quickly summarise the singulants in the longitidunal flow as follows. As the flow is parralel to the motion it is possible to pick out a time and spatial direction. The velocity then has to be constrained to the plane. This would mean that the velocity vector has a definition

$$U^\mu \partial_m u = coshu\partial tsinhu\partial x \tag{2.170}$$

And any two tensor that is transverse and traceless and symmetric should be projected on the hyperplane This could be achieved using

$$A^{\mu\nu} = (2-d)(\eta^{\mu\nu} + U^{\mu\nu} - \frac{d-1}{d-2} P_T^{\nu\mu})A(t,x) \tag{2.171}$$

Summarise this the gradient expansion becomes only related to the ... In the holography case which is obviosuly coupled to the strong interactions the gradient expansion is the dual to the gradient expansion of the metric such that one can write the metric as

$$g_{AB} = \sum g^n_{AB}(X)\epsilon^n \tag{2.172}$$

obtained by the holographic renormalisation obtained above.

As discussed one would like to match the two gradient expansions together hence the orders of the metric expansion will behave the same as the perturbations of the non linear fluid. In the limit of large n it is expected that the recursion relations become linear and the u dependent terms drop out. For the holographic case then the equations of motion at the large n simplify to the solutions related to the solutions of the singulant. As in the action of $\partial_{\mu_1}...\partial_{\mu p}$ on the gravitational term corresponds to $\partial_{\mu_1}...\partial_{\mu p}\chi$

Another way to realise this which does help with the understandings of the model is to introduce plane wave fluctuations of pi. Then the same as the WKB approximation and mapping the zeroth order thermodynamics equations to the equations of singulant dynamics.

To describe the longitudinal flow it is possible to create a geometry dual to it, the metric of such space assuming that the boundary is described by x^μ and the coordinate r takes us away from the boundary,

$$ds^2 = -2U_\mu(x)dx^\mu(dr + V_\nu(r,x)dx^\nu) + G_\mu\nu(r,x)dx^\mu dx^\nu \tag{2.173}$$

$G_\mu\nu$ is a transverse tensor to the liquid velocity. expressing the coordinates as

$$x^\mu = (\tau, \sigma, x_\perp^{(1)}, x_\perp^{(2)})$$

(2.174)

where the first coordinate is time like and second is spacelike and together they form the coordiantes of the longtidunal plane. There is a possibility to diagonalise the boundary metric by the choice of coordiantes such that

$$dh^2 = -e^{2a(\tau,\sigma)}d\tau^2 + e^{2b(\tau,\sigma)}d\sigma^2 + d\vec{x}_\perp^2 \tag{2.175}$$

the fluid velocity is the time like killing vector on the metric.

The holography conjecture drew a very clear connection between the stress energy tensors from each theory such that

$$\langle T_\mu\nu \rangle = t_\mu\nu \tag{2.176}$$

The calculations of each of the components of the stress energy tensor of the gravity i.e the right hand side can be found in the reference [6]. The conclusion is that in the landau frame where the fluid velocity defines the time orientation of the liquid stress tensor (the lhs). Then using the process described above it is possible to write the zeroth order solutions taking into acoount the bulk spacetime being assymptotically AdS. Then then by using the series relations it is possible to obtain the large order behaviour of the gradient expansion. The only thing to note is that there exist two regimes of solutions, the first is the infrared regime found at the horizon and the high energy regime found at the assymptotes of infinity.

For different sections of the Einsteins equation there exist the different solutons. You can also find the dynamical equations of motions summarised in the paper. The significane of what is found is that if you imagine infinetisemal hydrodyanmiocs fluctuations around a thermal state of energy ϵ_0. And expecting the other result from previous chapters to be applicable it is possible to write them in terms of transport coefficients. And what can then be defined is a clear understanding of the dispersion relations for the sound channels using the calculations of the momentum dependendent sound attenuation length. The result of this is that at the poles of the sound attentuation lenth it is possible for the corrections to the stress energy tensor to be finite. Therefore it can be shown that by linearising the recursive relationships described above and using the gradient expansion approach the dynamical equations of motion have analytical sense in Dingle's description of the singulants.

Small coupling limit Kinetic theory has gained interest as the result of its effective description of QCD in high temperatures. Therefore it is convenient to find out what momentum radius produces a converging hydrodynamic prescription. In second order of the momentum expansion (i.e one higher order of perturbation than what was described above one would have the retarded greens function for the shear and sound channel as

$$\frac{G^{xy,xy}_{R,\perp}(\omega,k)}{\epsilon+P} = \frac{2k\tau(2k^2\tau^2+3(1-i\tau\omega)^2+3i(1-i\tau\omega)(k^2\tau^2+(1-i\tau\omega)^2L}{2k\tau(3+2k^2\tau^2-3i\tau)+3i(k^2\tau^2+(1-i\tau\omega)^2L} \tag{2.177}$$

and the sound channel written as

$$\frac{G^{xz,xz}_{R,\parallel}}{-3(\epsilon+P)} = \frac{1}{3}+\omega^2\tau\frac{2k\tau+i(1-i\tau\omega)L}{2k\tau(k^2\tau+3i\omega)+i(k^2\tau+3\omega(i+\tau\omega))L} \tag{2.178}$$

With L being

$$L = log(\frac{\omega-k+\frac{i}{\tau}}{\omega+k+\frac{i}{\tau}} \tag{2.179}$$

The correlation functions have produced a branch cut at $\omega = -\frac{i}{\tau}+k$ due to the value of the logarithmic being singular. This has been described as the result of the stress tensor obtaining contributions from particles coming in from various angles that lead to a logarithmic distribution.

To further analyse this [29] has presented a series expansion and has made use of the implicit function theorem.

Firstly the shear channel has poles of the two point function at (τ is set to 1 for now)

$$2k(3+2k^2-3i\omega)+3i(k^{22}+(1-i\omega)^2L=0 \tag{2.180}$$

Away from the poles its leading contribution is

$$\omega_\perp(k) = -\frac{i}{5}k^2+... \tag{2.181}$$

As one needs to test the convergence, it is possible to write the series expansion as

$$\omega_\perp = \sum_{q=1}^{\inf c_q k^{2q}(2.182)}$$

and then replace it as the solution for values near k=0, one can then find that since the coefficients have the behaviour

$$lim_{q\to\inf}|\frac{c_{q+1}}{c_q} = |k_\perp^*|^{-2} \tag{2.183}$$

and numerical simulation set the value of

$$|k_\perp^*| = \frac{3}{2} \tag{2.184}$$

They then confirmed the results using symmetric Pade approximants where which maps the real values to the complex plane. Poles in this method become lines of pole condensastion and it was shown that there existed to of these condensations starting at

$$k = k_{\pm} = \pm 1.50020004i \tag{2.185}$$

One can then turn to realise that the branch point singularities exist at $k = \frac{3}{2}i$, $\omega = \frac{i}{2}$. One can check that these are valid solutions of (num) but also that at the coordinates the hydrodynamic pole collides with the non hydrodynamic branch point. However for the case of real k it can be observed that $\omega_{\perp}$ can in fact cross the branch cuts, meaning that it has moved from the principal sheet to a different sheet by analytical continuation. And the same procedure can be carried out to find the collision for gapless modes. Therefore the situation can be generalised for them, and the point k=$3\frac{}{2idoescorresondtoalograithmicbranchpointof}\omega_{\perp}$.

One can bring the same analysis to the sound channel.

for the sound channel the pole is at

$$2k(k^2 + 3i\omega) + i(k^2 + 3\omega(i + \omega))L \tag{2.186}$$

Then the series expansion ansatz around k=0 bring s to

$$\omega_{\parallel}^{\pm} = \pm\frac{k}{\sqrt{3}} + \sum^{\inf_{q+2} c_q^{\pm} k^q (2.187)}$$

Once again plugging the series expansion into the pole and to find the location of the singularities one can carry on with the Pade approximation. The first lines that coincide with the k=0 starts at $k_0 = 0.7513375i$ and goes along the positive imagiary axis. The other two are symmetric on the imaginary axis and start at the point $k_{\pm} = \pm 0.0102799 + 0.7409764$. Since their imaginary value is less than k_0 it is expected that they are the ones setting the radius of convergence.

To further the understanding of these symmetric points the pole is turned into an ODE for $\omega_p ar$

$C(\omega_{\|}(k), k)\omega_{\|}'(k) - D(\omega_{\|}(k), k) = 0$

(2.188)

$$C(\omega, k) = ik^2 + 6k^3\omega + 3ik\omega^2 - 6k\omega^3$$

(2.189)

$$D(\omega, k) = k^4 + 6ik^2\omega + 8k^2\omega^2 - 9i\omega^3 - 9\omega^4 \quad (2.190)$$

solving along a one parameter θ such that $k = \xi e^{i\theta}$ the results is indicative that at $\xi = |k_+|$, the first derivative i.e

$$\frac{d}{d\xi}\omega_{\|}^{-}(\xi e^{i\theta_+} \quad (2.191)$$

diverges and that the k_+ point is associated to the point where the $\omega_{\|}$ diverges. One can expand the arguments to the infinite poles expanded around the non principle sheet.

The conclusion is that to describe the rich background of the theory one has to deal with is the analytically continued Green's function which is defined on a multi sheeted Riemannian surface.

Chapter 3

Discussion

The point of this review is to simply bring in together the elements of hydrodynamics explored as an effective theory for dealing with collisions in the RHIC. Throughout this work it was shown that hydrodynamics gradient expansion is achievable through an interative process with gravity, allowing high order of precision with the simulations using fluid gravity. It was also shown that the fluid can be probed using quasi particle description of kinetic theory. Hence being able to describe QCD at both applied coupling. Moreover it was shown that there exist a clear and correct prediction method for the RHIC collisions in plasma balls and shockwave simulations. Using the experiments and the simulations described in the paper it was argued that hydrodynamics correctly predicts the hydrodynamisation time occuring before equilibration time and the fact that it is possible to simulate the models much earlier after the collision than expected effectively with hydrodynamics. The radii of convergence were calculated for the different methods of holography and kinetic theory. Finally it was shown that although the modes in the hydrodynamics prescritpion are factorally divergent there can be analytic control gained using solitons and the method was described.

Bibliography

[1] R.-G. Cai, L. Li, and Y.-L. Zhang, "Non-relativistic fluid dual to asymptotically ads gravity at finite cutoff surface," 2011.

[2] "Equations of motion," in *Hydrodynamics and Sound*, pp. 1–13, Cambridge University Press, Oct. 2006.

[3] J. Casalderrey-Solana, H. Liu, D. Mateos, K. Rajagopal, and U. A. Wiedemann, "Gauge/string duality, hot qcd and heavy ion collisions," 2011.

[4] E. Gourgoulhon, "An introduction to relativistic hydrodynamics," *EAS Publications Series*, vol. 21, pp. 43–79, 2006.

[5] S. Bhattacharyya, S. Minwalla, V. E. Hubeny, and M. Rangamani, "Nonlinear fluid dynamics from gravity," *Journal of High Energy Physics*, vol. 2008, pp. 045–045, feb 2008.

[6] T. Schäfer and D. Teaney, "Nearly perfect fluidity: from cold atomic gases to hot quark gluon plasmas," *Reports on Progress in Physics*, vol. 72, p. 126001, Nov. 2009.

[7] J. D. Qualls, "Lectures on conformal field theory," 2015.

[8] H. B. Meyer, "Calculation of the shear viscosity in SU(3) gluodynamics," *Physical Review D*, vol. 76, nov 2007.

[9] P. B. Arnold, G. D. Moore, and L. G. Yaffe, "Effective kinetic theory for high temperature gauge theories," *Journal of High Energy Physics*, vol. 2003, pp. 030–030, jan 2003.

[10] E. Witten, "Bound states of strings and p-branes," *Nuclear Physics B*, vol. 460, pp. 335–350, feb 1996.

[11] O. Aharony, S. S. Gubser, J. Maldacena, H. Ooguri, and Y. Oz, "Large n field theories, string theory and gravity," *Physics Reports*, vol. 323, pp. 183–386, jan 2000.

[12] E. Witten, "Anti de sitter space and holography," 1998.

[13] J. Maldacena *International Journal of Theoretical Physics*, vol. 38, no. 4, pp. 1113–1133, 1999.

[14] A. Chamblin, R. Emparan, C. V. Johnson, and R. C. Myers, "Holography, thermodynamics, and fluctuations of charged AdS black holes," *Physical Review D*, vol. 60, oct 1999.

[15] V. E. Hubeny, S. Minwalla, and M. Rangamani, "The fluid/gravity correspondence," 2011.

[16] N. Iqbal and H. Liu, "Universality of the hydrodynamic limit in AdS/CFT and the membrane paradigm," *Physical Review D*, vol. 79, Jan. 2009.

[17] P. K. Kovtun, D. T. Son, and A. O. Starinets, "Viscosity in strongly interacting quantum field theories from black hole physics," *Phys. Rev. Lett.*, vol. 94, p. 111601, Mar 2005.

[18] R. Emparan, "Absorption of scalars by extended objects," *Nuclear Physics B*, vol. 516, pp. 297–314, apr 1998.

[19] A. Sinha and R. C. Myers, "The viscosity bound in string theory," *Nuclear Physics A*, vol. 830, pp. 295c–298c, nov 2009.

[20] O. Aharony, S. Minwalla, and T. Wiseman, "Plasma balls in large- in/i gauge theories and localized black holes," *Classical and Quantum Gravity*, vol. 23, pp. 2171–2210, mar 2006.

[21] H. Bantilan, P. Figueras, and D. Mateos, "Real-time dynamics of plasma balls from holography," *Physical Review Letters*, vol. 124, May 2020.

[22] R. Emparan and G. Milanesi, "Exact gravitational dual of a plasma ball," *Journal of High Energy Physics*, vol. 2009, pp. 012–012, aug 2009.

[23] H. Nastase, "The rhic fireball as a dual black hole," 2005.

[24] P. M. Chesler, “Colliding shock waves and hydrodynamics in small systems,” *Physical Review Letters*, vol. 115, Dec. 2015.

[25] M. Attems, J. Casalderrey-Solana, D. Mateos, D. Santos-Oliván, C. F. Sopuerta, M. Triana, and M. Zilhão, “Paths to equilibrium in non-conformal collisions,” *Journal of High Energy Physics*, vol. 2017, jun 2017.

[26] M. Bianchi, D. Z. Freedman, and K. Skenderis, “Holographic renormalization,” *Nuclear Physics B*, vol. 631, pp. 159–194, jun 2002.

[27] M. P. Heller, A. Serantes, M. Spaliński, V. Svensson, and B. Withers, “Relativistic hydrodynamics: a singulant perspective,” 2021.

[28] B. Withers, “Short-lived modes from hydrodynamic dispersion relations,” *Journal of High Energy Physics*, vol. 2018, June 2018.

[29] M. P. Heller, A. Serantes, M. Spaliński, V. Svensson, and B. Withers, “Convergence of hydrodynamic modes: insights from kinetic theory and holography,” *SciPost Phys.*, vol. 10, p. 123, 2021.