Imperial College London

The Speed of Gravity in the Low-Energy EFT of GR

Department of Theoretical Physics

Author: Zakary Buras-Stubbs

Supervisor: Claudia de Rham

Submitted in partial fulfillment of the requirements for the MSc degree QFFF

of Imperial College London

September 2021

Abstract

We consider the effects of higher order curvature corrections to General Relativity, focusing on those produce by the inclusion of terms of order $R^4_{\alpha\beta\gamma\delta}$, with regards to the speed of propagating gravitational waves in Schwarzschild spacetime.

Focusing on the presence of superluminal speeds in odd modes, we consider whether any time advanced produced is not in violation of causality due to its unresolvable nature at the scale of the EFT, as well as checking for self consistency, and observing that any propagating modes remain luminal at the Event Horizon of a Black Hole.

Acknowledgements

I would like to first thank Claudia de Rham for the tremendous support and guidance she has provided throughout. Thank you also to Jun Zhang for extremely useful discussions and assistance.

I would also like to thank my family and Inês Ferreira for putting up with me.

Contents

Notation and Conventions

The following conventions shall be used throughout this work unless otherwise stated:

The signature of 4-dimensional spacetime is taken to be {-,+,+,+}.

Units are defined such that the speed of light in a vacuum is given by $c = 1$.

The reduced Planck mass is defined as $M_{pl}^2 = \frac{1}{8\pi G}$.

We use standard Einstein summation convention for repeated indices.

Partial differentiation is indicated by $\frac{\partial}{\partial x^\mu} = \partial_\mu$, and covariant differentiation given by ∇_μ.

1 Introduction

The recent successes and progress of Gravitational Wave (GW) astronomy, via the LIGO-Virgo collaborations Refs.[1-4], have clearly opened up new pathways to understanding compact objects, General Relativity (GR), and cosmology as a whole.
The ability to observe interactions at high energies and curvatures provides a previously unavailable laboratory to test fundamental physics, and our current Standard Model, in exotic and otherwise unattainable environments within the confines of our solar system. Namely, any unknown process or large numbers of new particles generated may impact the dynamics of the system in question, for example a binary black hole merger, leaving an observable imprint on the Gravitational Waves sent to us.

While these kinds of energetic conditions may seem most suited to searches for these finger prints to provide a foundation to explore beyond SM physics, and indeed in the search for light particles that only couple weakly to the standard model like Axions, it is equally useful in providing a greater understanding of GR, the latter being our main interest.

Within the classical regime of GR, with minimal coupling to matter, behaviour is governed by the Einstein-Hilbert action. Due to its form, any modifications stemming from background curvature will induce the Lagrangian relating to tensor fluctuations of the form

$$L = (\dot{h})^2 - (\boldsymbol{\nabla} h)^2 - m_{eff}^2 h^2, \tag{1}$$

where the modifications have resulted in the presence of an effective mass term, m_{eff}. This however does not cause the waves to deviate from luminal speeds, as the sound speed, c_s, is determined by

$$L = (\dot{h})^2 - c_s^2(\boldsymbol{\nabla} h)^2 - m_{eff}^2 h^2. \tag{2}$$

Since the modifications induce no kinetic or gradient terms, they have no impact on the sound speed of the wave, and hence the effective mass does not manifest itself as an actual mass.

This is an inherent property of GR due to the second order nature of the action, and as such any deviations of GW speeds must result from higher order corrections to the Einstein-Hilbert action, [5].

General Relativity in and of itself is, however, an effective theory, and as such is open to corrections in low and high energy regimes. Extensions to this in the context of Effective Field Theory will primarily take the form of introducing new massive particles, necessarily at an energy scale that have yet to be explored. Furthermore, at low energies, the effects of massive particles above the energy scale of interest result in the inclusion of new higher order terms constructed from the Riemann Curvature tensor. This follows by the process of integrating out the massive particles that cannot exist at this energy, with these higher order terms suppressed by a mass scale beyond which we expect the theory to lose validity.

It is the latter of these that we concern ourselves with, in particular we consider the effects of these higher order curvature corrections on the speed of GWs in the Low Energy EFT of Gravity.

The basic framework of how we use Effective Field Theory to provide a modification to our current knowledge, and hopefully produce new physics, is that we attempt to make the most general extension to the existing theory within the regime of interest. This may appear to be a rather large task, with the obvious benefit being that we are not required to fully specify our new action and from there compute measurable differences in underlying physics.

However this method is not without a guiding philosophy, namely we must respect symmetries or principles that readily reveal themselves within current experiment, and ensure that measurable modifications, or new particles, are outside of the domain in which they should have already been observed.

An important element in maintaining this balance, and remaining at energy level at which our EFT would be valid, is the introduction of an energy scale or cutoff, in this instance we choose Λ. This defines what is meant by low energies, and at energy scales below Λ we construct our theory such that extensions are perturbative, and hence under control.

With this in mind, it is reasonable to wonder whether there is any new physics to be tested at low energies, as surely any low energy effects should be visible in tests made within our solar system. This, however, neglects to acknowledge General Relativity's unique dependence on space-time curvature independent of the energies being considered, with observable curvature scales resulting from black hole mergers being on the order of inverse kilometres, as compared to the $10^{-8}km^{-1}$ scales typically found on earth and in the solar system.

A key attribute in understanding GR is the manner in which spacetime geometry effects the propagation of particles through it and the effect it has on the causal structure. A well understood example of this is the Shapiro Time Delay, the time delay induced by propagation through a curved background space-time, [6]. However, in using an EFT we necessarily must consider the further effects of using a modified form of GR, giving us and effective time delay of the form

$$\Delta T = \Delta T_g + \Delta T^{EFT}, \tag{3}$$

where ΔT_g is the time delay generated by standard propagation through curved background spacetime, and ΔT^{EFT} is the time delay generated by the EFT corrections. This has typically been used as a test for whether a choice of EFT is causal, as if ΔT^{EFT} is negative then there will be a time advance, and hence superluminal travel.

This however may be too strict of a constraint, as it pays no mind to whether, within the context of the EFT, ΔT^{EFT} is large enough to be resolvable, and is hence relevant on the length scales that the EFT is sensitive to.

More specifically for spherically symmetric asymptotically flat spacetimes, we may make use of a generalised form of the Eisenbud-Wigner scattering time delay, as in Ref.[16], and moreover may specify this to specific fixed choices of angular momentum due to spherical symmetry, providing a time delay ΔT_l.

The paper shall be organised as follows: In Section 2, we consider the possible forms of our EFT, and decide on the specific construction to analyse. In Section 3, we find the effects modifications made by the EFT have on the equations of motion, and hence on the form of the metric that solves them. In Section 4, we introduce the metric perturbations, and consider the equations of motion generated by the odd modes, as well as finding the governing master equations for the degrees of freedom for a modified and unmodified Schwarzschild metric, and from these deriving the sound speed of the propagating degrees of freedom. In Section 5, we discuss superluminal propagation in a wider context and consider how the problem of causality was solved in the context of QED. In Section 6, we consider the effective metric as seen by low energy gravitational waves, and compare the event horizons experienced with the modified Schwarzschild metric derived in Section 3. In Section 7, we discuss the validity of the theory, and check whether any time advances generated may be mitigated by being unresolvable by the EFT. We end with a summary and discussion in Section 8. Appendix A contains the exact equations of motion generated by the odd modes, and the exact Master Equation produced.

2 The Construction of the Effective Field Theory

We now move on to discuss in further detail the precise form that our EFT will take, and the overall manner in which it is constructed. This will involve considering the associated shortcomings and properties that arise in the wider literature, and reviewing why a given choice may be ignored.

It is well established that the form of classical GR does not reflect nature, and indeed one would at the very least expect 1-loop interactions of already well understood phenomena to induce alterations to the form of Gravity.

On the level of GR however there is only a single term that we could consider including in our EFT by diffeomorphism invariance, this is of course the standard Einstein-Hilbert term,

$$L_{GR} = \sqrt{-g} M_{pl}^2 \frac{R}{2}, \tag{4}$$

where $g = det(g_{\mu\nu})$ for a metric $g_{\mu\nu}$, M_{pl} is the Planck mass, and R is the Ricci Scalar. This provides the well known Einstein's Field Equations in a vacuum,

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0 \tag{5}$$

for a Ricci Tensor $R_{\mu\nu}$.

As stated, the process with which we construct our higher order extensions from this foundation hinges on setting an upper limit on the energy our system can exist in using a cutoff, Λ, integrating out any particles with masses higher than this cutoff.

In our case we seek to ensure that no massive particles are present, by setting this cutoff below the mass of the lightest known particle. This cutoff also serves the purpose of restraining higher order terms that we may introduce, preventing their effects from being comparable to those seen in unmodified GR.

Quadratic Curvature Corrections

Having seen that the allowable linear curvature terms are already embodied by GR, we now must move on to consider possible terms stemming from the next lowest order in curvature.

At the level of quadratic, or dimension 4, terms there is again only a single unique choice,

$$R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}. \tag{6}$$

However, this may be rewritten to provide terms of a form following that in Ref.[7]

$$L_{D4} = \sqrt{-g}(c_{R^2}R^2 + c_{W^2}W^2 + c_{GB}R_{GB}^2), \tag{7}$$

where $W^2 = W_{\alpha\beta\mu\nu}W^{\alpha\beta\mu\nu}$ for Weyl tensor $W_{\alpha\beta\mu\nu}$, and $R_{GB} = R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} - 4R_{\mu\nu}R^{\mu\nu} + R^2$ is the Gauss-Bonnet term.

In four dimensions however, the Gauss-Bonnet term happens to be the Euler density, which is topological and as such will be ignored. Hence we may rephrase our expression as

$$L_{D4} = \sqrt{-g}(c_1R^2 + c_2R_{\mu\nu}R^{\mu\nu}), \tag{8}$$

where $c_1 = c_{R^2} - \frac{2}{3}c_{W^2}$, and $c_2 = 2c_{W^2}$.

Clearly in a Ricci flat vacuum, such as with Schwarzschild, it is not possible for this to give first order corrections to the theory as all first order terms vanish trivially.

Cubic Curvature Corrections

Having ruled out dimension 4 terms, we now move on to consider dimension 6, or cubic curvature, terms. Again following Ref.[7], the corrections to GR are

$$
\begin{aligned}
L_{D6} = \frac{\sqrt{-g}}{\Lambda^2}(&d_1 R\Box R + d_2 R_{\mu\nu}\Box R^{\mu\nu} + d_3 R^3 + d_4 R R_{\mu\nu}^2 \\
&+ d_5 R R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} + d_6 R_{\mu\nu}^3 + d_7 R^{\mu\nu}R^{\alpha\beta}R_{\alpha\beta\mu\nu} \\
&+ d_8 R^{\mu\nu}R_{\mu\alpha\beta\gamma}R_{\nu}{}^{\alpha\beta\gamma} + d_9 R_{\mu\nu}{}^{\alpha\beta}R_{\alpha\beta}{}^{\gamma\delta}R_{\gamma\delta}{}^{\mu\nu} \\
&+ d_{10} R_{\mu\alpha\nu\beta}R^{\alpha\gamma\beta\delta}R_{\gamma}{}^{\mu}{}_{\delta}{}^{\nu}).
\end{aligned} \tag{9}
$$

This is the first higher curvature extension to GR that we introduce which produces first order corrections in Ricci flat space-times, however it is not without avowed problems. The arguments laid out in Ref.[8] are highlighted as sources of dissonance between this particular construction and its validity in parts of the literature, for example see Refs.[9], [10], and [11].

In Ref.[8], it was argued that the low energy effective theory induced superluminal speeds and time advances, and as such allegedly fails to be a causal theory.

In that context, in order to mitigate this in the UV completion, causality would require an infinite tower of higher spin particles coupled to standard model fields with gravitational strength, an effect that surely should have already been observed. However, we must proceed with caution so as to not rule out valid low energy theories.

We start by considering the appearance of an infinite tower of higher spin particles as a resolution to the effects in the UV completion of the theory. From the perspective of our low energy EFT, these higher spin particles will simply take the form of more, higher order, corrections to the theory, and as such be suppressed by larger and larger amounts,

making their effects questionable at this level.
Moreover, within the low energy EFT context, the manifestation of these superluminal speeds would still be a symptom of causality breaking over large distances even if the problem is corrected as we move into the high energy regime.

We are therefore left to consider possible resolutions to time advances that occur, indeed we choose to take a more subtle approach as to why these apparently causality violating theories may still remain salvageable.
By introducing a cutoff, we have in essence also introduced a minimum length scale that our theory concerns itself with. That is distances, and time scales, much smaller than those attributed to any Gravitational Waves in our theory should be unresolvable.
As such we don't necessarily require that these waves cannot stray into superluminal speeds, but merely that if they do they must (1) be suppressed in such a way that any advance may not be integrated over an arbitrarily long time to become macroscopically, or indeed infinitely, large, and (2) that any advance made is much smaller than the scale that the theory is sensitive to, that is we expect a time advance to be causal if

$$\Delta T \ll \omega^{-1}, \tag{10}$$

for a gravitational wave of frequency ω.

In spite of this, it is for the aforementioned difficulties that the cubic order terms are currently ignored in favour of quartic order terms as the same issues with regards to UV completion are not a concern at this order.
However, as we will see, the possibility of superluminal speeds still very much exists for

dimension 8 terms, for example in Ref.[12] this was used to limit the possible values of coefficients in the EFT. But, as stated, the goal of an EFT is to provide the most general extension to the theory, and as such we must be very careful not to exclude certain aspects of a theory too quickly, or naively ignore something appearing contradictory at first that may be reconciled when placed in the context of other elements of the theory.

With these concepts in mind, we follow these previous parts of the literature, leaning on the side of caution, and in the comfort of knowing that the behaviour of GW speeds and causality have been reasonably well explored at this order.

Therefore, we move on to consider the next lowest order in consistent higher curvature corrections and treat their cubic counterparts as an unphysical choice that may be ignored. This is done in essence to check the behaviour of this order, and see if the superluminal speeds it may generate are permissible within the context of an unresolvable time advance.

Quartic Curvature Corrections

Having ruled out, or chosen to ignore for our own purposes, all of the previously mentioned EFT corrections we are now left with quartic curvature, or dimension 8 corrections, of the form

$$L_{D8} = \sqrt{-g} M_{pl}^2 \left(\frac{c_1}{\Lambda^6} C^2 + \frac{c_2}{\Lambda^6} C\tilde{C} + \frac{c_3}{\Lambda^6} \tilde{C}^2 \right), \tag{11}$$

where $C = R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta}$, $\tilde{C} = R_{\alpha\beta\gamma\delta} \tilde{R}^{\alpha\beta\gamma\delta}$, and $\tilde{R}^{\alpha\beta\gamma\delta} = \epsilon^{\alpha\beta}{}_{\mu\nu} R^{\mu\nu\gamma\delta}$. As mentioned, this order of correction does not share the issues of UV completion with the cubic curvature terms, but can nevertheless produce superluminal gravitational wave modes.

In the name of consistency and providing a clear path to comparison, we define our Lagrangian in a similar fashion to Refs.[9],[10], and [11],

$$2\sqrt{-g} M_{pl}^2 \left(R + \frac{c_1}{\Lambda^6} C^2 + \frac{c_2}{\Lambda^6} C\tilde{C} + \frac{c_3}{\Lambda^6} \tilde{C}^2 \right). \tag{12}$$

This has the further benefit of allowing us to ignore the presence of M_{pl} when writing our equations of motion, as this is simply a overall constant factor.

From this we proceed to calculate the equations of motion, by taking the variation with respect to the metric, $g_{\mu\nu}$, providing

$$\begin{aligned}R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\frac{g_{\mu\nu}}{2}(\frac{c_1}{\Lambda^6}C^2 + \frac{c_2}{\Lambda^6}\tilde{C}^2 + \frac{c_3}{\Lambda^6}C\tilde{C}) \\ +\frac{c_1}{\Lambda^6}(4R_{\mu}{}^{\alpha\beta\gamma}R_{\nu\alpha\beta\gamma}C + 8\nabla^{\alpha}\nabla^{\beta}(R_{\mu\alpha\nu\beta}C)) \\ +\frac{c_2}{\Lambda^6}(2R_{\mu}{}^{\alpha\beta\gamma}\tilde{R}_{\nu\alpha\beta\gamma}C + 2R_{\mu}{}^{\alpha\beta\gamma}R_{\nu\alpha\beta\gamma}\tilde{C} \\ +4\nabla^{\alpha}\nabla^{\beta}(R_{\mu\alpha\nu\beta}\tilde{C}) + 4\nabla^{\alpha}\nabla^{\beta}(\tilde{R}_{\mu\alpha\nu\beta}C)) \\ \frac{c_3}{\Lambda^6}(4R_{\mu}{}^{\alpha\beta\gamma}\tilde{R}_{\nu\alpha\beta\gamma}\tilde{C} + 8\nabla^{\alpha}\nabla^{\beta}(\tilde{R}_{\mu\alpha\nu\beta}\tilde{C})).\end{aligned} \tag{13}$$

Notably, terms associated with c_2 and c_3 do not modify the background solution. Considering only c_1 terms, one should have,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{c_1}{\Lambda^6}(4R_{\mu}{}^{\alpha\beta\gamma}R_{\nu\alpha\beta\gamma}C + 8\nabla^{\alpha}\nabla^{\beta}(R_{\mu\alpha\nu\beta}C) - \frac{g_{\mu\nu}}{2}(C^2)) \tag{14}$$

This may still seem different from the equations of motion found in the aforementioned literature, however if we note that in a Ricci flat spacetime

$$(4R_a^{cde}R_{bcde} - g_{ab}C) = 0, \tag{15}$$

we recover the equations of motion of the same form found in Ref.[10]

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{c_1}{\Lambda^6}(8\nabla^{\alpha}\nabla^{\beta}(R_{\mu\alpha\nu\beta}C) + \frac{g_{\mu\nu}}{2}(C^2)). \tag{16}$$

In other parts of the literature these equations have been constrained by requiring $0 < c_i$ in order to preserve causality by preventing superluminal speeds, however in the vein of similar arguments we have made we shall pay no mind to this particular constraint and instead look at the theory through the lens of whether resolvable time advances are made. Having now considered the construction of our EFT, we are finally in a place to confidently continue on in analysing the behaviour it induces in Gravitational Waves propagating within it.

3 The Modified Schwarzschild Metric

Our higher curvature extensions to the Lagrangian, and hence to the equations of motion, will necessarily lead to modifications to the metric that solves them. This can possibly have consequences on the precise causal structure of our theory, as well as direct effects on Gravitational wave propagation.

As we are concerned with the static, spherically symmetric spacetime around a black hole, it is rational that we start with an ansatz in the vein of the Schwarzschild metric, with corrections to first order in ϵ, a small dimensionless variable defined as

$$\epsilon = \frac{1}{\Lambda^6 M^6}, \ \epsilon_i = c_i \epsilon, \tag{17}$$

where we again remind the reader that we will be ignoring the effects of c_2 and c_3.

We therefore start with the Schwarzschild ansatz

$$ds^2 = -f_t dt^2 + \frac{1}{f_r} dr^2 + C(r) r^2 (d\theta^2 + \sin^2(\theta) d\phi^2) \tag{18}$$

where,

$$\begin{aligned} f_t &= (1 - \frac{r_s}{r}) - \epsilon_1 A(r) \\ f_r &= (1 - \frac{r_s}{r}) + \epsilon_1 B(r), \end{aligned} \tag{19}$$

and we take $r_s = 2M$ to be the standard Schwarzschild radius, for a black hole of mass M.

This may then be substituted into our Lagrangian, which may then be varied with respect to $A(r)$, $B(r)$, and $C(r)$, or alternately $g^{\mu\nu}$ itself, to provide equations that may be used to specify our unknown variables.

At first it may seem somewhat alarming as our equations cannot be solved in their current state if we expand in full. However, if we notice that the terms in our equations of motion resulting from the higher order curvature corrections are small by construction we may therefore only need expand these higher order terms to order zero in ϵ. That is, we may treat the Lagrangian to be

$$2\sqrt{-g}M_{pl}^2(R+(\frac{c_1}{\Lambda^6}C^2+\frac{c_2}{\Lambda^6}C\tilde{C}+\frac{c_3}{\Lambda^6}\tilde{C}^2)|_{g_{\mu\nu}(\epsilon=0)}). \tag{20}$$

A further simplification may be made by recognising that we still have the freedom to choose a gauge in which we impose $C(r)=1$, however care must be taken so as to not lose information that could be gleaned by the equations produced via variation with respect to $C(r)$ by applying this gauge prematurely.

The equations generated by variation of our unknown functions in our metric, followed by enforcing the gauge $C(r)=1$ are:

$$\begin{gathered}
\mathscr{E}_A = 0 \\
\mathscr{E}_B = 0 \\
\mathscr{E}_C = \frac{c_1}{2\Lambda^6 r^{10}(r-r_s)^2}\sin(\theta)(\Lambda^6 r^{10}(2r-r_s)r_s A \\
+\Lambda^6 r^{10}(2r-r_s)r_s B + (r-r_s)(\Lambda^6 r^{11}(2r-3r_s)A' \\
+\Lambda^6 r^{11}(-2r+r_s)B' + 2(r-r_s)(-288(36r-41r_s)r_s^3+\Lambda^6 r^{12}A''))).
\end{gathered} \tag{21}$$

This is clearly insufficient to specify the forms of $A(r)$ and $B(r)$, as the equations stemming from variation with respect to A and B vanish trivially.

Accordingly we are required to take the equations of motion from our arsenal in order discern any further information. This too may seem somewhat of a daunting task, however we have more tools that may be used to simplify this procedure.

Notably, we are only interested in terms at order ϵ exactly, with order zero solutions simply being Einstein's Equations evaluated in a Schwarzschild vacuum,

$$G_{\mu\nu} = 0. \tag{22}$$

Hence, again identifying that our higher curvature corrections are small by construction, the relevant equation of motion is effectively of the form

$$\mathscr{E}_{\mu\nu} = G_{\mu\nu} - (\frac{c_1}{\Lambda^6}(4R_{\mu}{}^{\alpha\beta\gamma}R_{\nu\alpha\beta\gamma}C + 8\nabla^{\alpha}\nabla^{\beta}(R_{\mu\alpha\nu\beta}C) - \frac{g_{\mu\nu}}{2}(C^2))|_{g_{\mu\nu}(\epsilon=0)}) = 0, \tag{23}$$

from which we select

$$\mathscr{E}_{0,0} = -\epsilon_1(r - r_s)\frac{8r_s^3(-720r + 737r_s) + B'r^{11}}{r^{13}} = 0, \tag{24}$$

and

$$\mathscr{E}_{1,1} = \frac{\epsilon_1}{r^{11}(r - r_s)}(8(216r - 121r_s)r_s^3 + r^{11}A') = 0. \tag{25}$$

$\mathscr{E}_{0,0}$ solves for,

$$B = \frac{-1152rrs^3 + 1072rs^4 + r^9a_1}{r^{10}} \tag{26}$$

where we are free to select $a_1 = 0$. And using $\mathscr{E}_{1,1}$ we may then solve for

$$A = \frac{256 r_s^3}{r^9} - \frac{176 r_s^4}{r^{10}} + b_1 - \frac{b_1 r_s}{r}, \tag{27}$$

where we may then set $b_1 = 0$ for simplicity.

We are then provided with our modified Schwarzschild metric,

$$\begin{aligned} ds^2 = -(1 - \frac{r_s}{r} + \epsilon_1(-2\Big(\frac{r_s}{r}\Big)^9 + \frac{11}{8}\Big(\frac{r_s}{r}\Big)^{10}))dt^2 \\ +(1 - \frac{r_s}{r} + \epsilon_1(-9\Big(\frac{r_s}{r}\Big)^9 + \frac{67}{8}\Big(\frac{r_s}{r}\Big)^{10}))^{-1}dr^2 \\ +r^2(d\theta^2 + \sin^2(\theta)d\phi^2). \end{aligned} \tag{28}$$

With our new metric, specified to first order in ϵ, in hand, it is clear that the higher dimensional operators introduced will result in deviations from the event horizon radius that we are accustomed to in Schwarzschild. Furthermore, due to the nature of our EFT, physical singularities must not form within regions in which the theory is valid, the horizon being such a location for macroscopic black holes. It ensues that a modified horizon radius, r_H, must be defined, for which, to eliminate the prospect of physical singularities forming, we require f_t and f_r to vanish concurrently and at the same location,

$$f_t(r_H) = f_r(r_H) = 0.$$

This need only be satisfied to first order in ϵ, as we treat any higher order terms as negligible. By setting $f_t(r_H) = 0$, we find that

$$r_H = r_s - \epsilon_1 \frac{5}{8} r_s, \tag{29}$$

which also fortunately satisfies $f_r(r_H) = 0$, whereby we see that the inclusion of higher order terms has resulted in a different horizon radius to that of Schwarzschild and we have thankfully managed to avoid any physical singularities in the region where our theory should be valid.

4 The Perturbed Equations of Motion

4.1 Metric Perturbations

The heart and soul of Gravitational Wave propagation equations is understanding the behaviour of spacetime under perturbations about the background metric. Namely we start by defining our perturbed metric

$$\bar{g}_{\mu\nu} = g_{\mu\nu} + \varepsilon h_{\mu\nu}, \tag{30}$$

where ε is a small dimensionless parameter distinct from ϵ, and in the spirit of our previous derivations, we only consider perturbations to first order in ε, and where $g_{\mu\nu}$ is defined in Section 3.

Furthermore, we employ boundary conditions consistent with our background metric, namely that this perturbed spacetime is asymptotically flat,

$$\lim_{r\to\infty} h_{\mu\nu} = 0. \tag{31}$$

Where we use the notation that,

$$h^{\mu\nu} \equiv g^{\mu\alpha} g^{\nu\beta} h_{\alpha\beta}, \tag{32}$$

and the perturbation has the following properties,

$$(g_{\alpha\beta} + \varepsilon h_{\alpha\beta})(g^{\beta\gamma} - \varepsilon h^{\beta\gamma}) = \delta^{\gamma}_{\alpha}, \tag{33}$$

and hence

$$\bar{g}^{\mu\nu} = g^{\mu\nu} - \varepsilon h^{\mu\nu}. \tag{34}$$

4.2 The Master Equation

We are now in a position to analyse perturbations on our newly found background metric, and hence find the equations of motion governing gravitational wave propagation. In this light, we employ techniques used in Ref.[13] for analysing perturbations about a Schwarzschild background.

We are free to start by decomposing the metric perturbation into its odd and even components, that is $h_{\mu\nu} = h^o_{\mu\nu} + h^e_{\mu\nu}$, dependant on how they are effected by parity transformations, $(\theta, \phi) \to (\pi - \theta, \pi + \phi)$. Furthermore, due to the spherical symmetry present in the background, terms of different parity and degree, l, are not free to mix, where l is the angular momentum eigenvalue associated with the state.

Additionally, we are not forced to consider spherical harmonics for some arbitrary m, as for specified angular momentum eigenvalue l and wave number k we are led to the same radial equation. As such, it is most convenient to select $m = 0$ as this will dramatically simplify our calculations by removing ϕ dependence from the modes.

Working in the Regge-Wheeler gauge [13], the odd perturbations are given by

$$h^{o}_{\mu\nu} = e^{-i\omega t} \begin{pmatrix} 0 & 0 & 0 & h_0 \\ 0 & 0 & 0 & h_1 \\ 0 & 0 & 0 & 0 \\ h_0 & h_1 & 0 & 0 \end{pmatrix} \sin(\theta) Y_l'(\theta), \tag{35}$$

and even perturbations given by

$$h^{e}_{\mu\nu} = e^{-i\omega t} \begin{pmatrix} f_t H_0 & H_1 & 0 & 0 \\ H_1 & H_2/f_r & 0 & 0 \\ 0 & 0 & r^2 K & 0 \\ 0 & 0 & 0 & r^2 \sin^2(\theta) K \end{pmatrix} \sin(\theta) Y_l'(\theta), \tag{36}$$

where $Y_l(\theta) = Y_{l0}(\theta)$ are the spherical harmonics for $m = 0$, for which l is their degree, and the prime indicates differentiation with respect to θ. We note that the perturbation mode functions are h_0 and h_1 in the case of odd modes, and H_0, H_1, H_2, and K are the functions associated with even modes. It is important to stress that these functions do not represent individual degrees of freedom, and indeed that GR only has the capacity for 2 independent degrees of freedom, with a single being granted for each mode.

Despite our attempts at simplification, there is still a great deal of work involved in extracting information on GW propagation from these perturbations. Namely the odd modes will result in three distinct equations, and the even modes will produce seven. Therefore our perturbed equations of motion must be massaged in such a way that we are left with a single equation describing propagation for each mode.

To accomplish this we must use the perturbed equations of motion, which may be found in Appendix A, in conjunction with the Regge-Wheeler and Zerilli equations to determine a master equation for our theory, as well as the governing master variable, $\Psi^{o/e}$. These master variables represent the true odd (o) and even (e) degrees of freedom in the propagating gravitational wave, and the other perturbation mode functions are uniquely determined in terms of their respective master variable.
For simplicity, we shall focus on those generated by the odd mode only, and hence forgo the further use of the odd/even superscript, knowing implicitly we are discussing the odd mode, as in the current context the odd and even modes are fully decoupled.

In the same vein as with our metric corrections, we must start by finding our master variable and equation in the context of GR before finding corrections stemming from order ϵ terms.
Moreover, one might expect that the inclusion of leading order corrections of the quartic curvature modifications would lead to higher derivative equations of motion, as these terms have the capacity to provide fourth order derivative terms. This is all the more reason that we require these lower order equations to temper these terms, using the prescription laid out in Ref.[5], to ensure that the master equation remains second order.
For completeness we shall do this process in full for unmodified Schwarzschild metric, recognising that this method will also apply to the modified case.

We start by assuming that the master variable for Schwarzschild takes the form

$$\Psi_{GR} = f_{GR}(r)h_1, \tag{37}$$

as in Eq.(A.3) we see that h_0 is related to h_1 by a first order PDE and hence both are fully determined in terms of Ψ_{GR} by simply setting it as proportional to one of the two. The form of $f_{GR}(r)$ may then be determined by requiring Ψ satisfy the Regge-Wheeler and Zerilli equations, given in GR by

$$\frac{\mathrm{d}^2\Psi_{GR}}{\mathrm{d}r_*^2} + (\omega_0^2 - (1 - \frac{r_s}{r})V_{GR})\Psi_{GR} = 0, \tag{38}$$

where more generally the tortoise coordinate r_* is defined by $\frac{dr}{dr_*} = \sqrt{f_t(r)f_r(r)}$, however in the case of unmodified Schwarzschild this is simply

$$\frac{dr}{dr_*} = 1 - \frac{r_s}{r},$$

and

$$V_{GR} = \frac{J}{r^2} - \frac{3r_s}{r^3}, \tag{39}$$

where $J = l(l+1)$.

In isolation this is clearly insufficient to derive $f_{GR}(r)$, therefore we must call on the equations of motion for the unmodified metric found in Appendix A.1.

We start by eliminating h_0 from our equations by solving Eq.(A.3), providing the form of h_0 in terms of our other variables,

$$h_0 = \frac{i(2M-r)}{\omega r^3}(-2Mf_{GR}\Psi_{GR} + 2Mr\Psi_{GR}f'_{GR} - r^2\Psi f'_{GR} + 2Mrf_{GR}\Psi'_{GR} - r^2 f_{GR}\Psi'_{GR}), \tag{40}$$

for which primes indicate differentiation with respect to r.

Substituting this into Eq.(A.2), the equation may be rearranged to the form of Eq.(38), whereby we may solve for $f_{GR}(r)$ by matching the coefficients, rephrasing Eq.(A.2) such that the coefficient of Ψ''_{GR} is

$$(1-\frac{2M}{r})^2. \tag{41}$$

Once this is done, $f_{GR}(r)$ may be then found by matching the coefficients of Ψ'_{GR} to those specified in Eq.(38) providing us with

$$2(2M-r)((-5M+r)f_{GR}+(2M-r)rf'_{GR}) = 2Mr^3 f_{GR}(1-\frac{2M}{r}), \tag{42}$$

which may then be solved to yield

$$f_{GR}(r) = \frac{c(r-r_s)}{r^2}, \tag{43}$$

for an arbitrary choice of constant c, which we set $c = \frac{i}{\omega}$ for simplification purposes.
Furthermore, we also glean the form of Ψ''_{GR} and $\Psi^{(3)}_{GR}$ in Eq.(A.8) and Eq.(A.9) respectively. These relations have the benefit of allowing us to eliminate any terms containing third order derivatives or higher that would otherwise add unphysical degrees of freedom to our system.

With the master equation and variable known for the case of GR we are free to use this to find the prescription for our Low-Energy modification to GR defined by the metric in Eq.(28).
In the case of modifications made by our EFT, as we now know the form the master variable takes for Schwarzschild one can expect that any corrections to this will be of order ϵ.
We therefore anticipate the master variable associated with these corrections to be of the

form

$$\Psi = \frac{i\sqrt{f_t f_r}}{r\omega}(1-\epsilon_1 f(r))h_1, \tag{44}$$

where as previously stated

$$f_t = (1 - \frac{r_s}{r} + \epsilon_1(-2\Big(\frac{r_s}{r}\Big)^9 + \frac{11}{8}\Big(\frac{r_s}{r}\Big)^{10}))$$

and

$$f_r = (1 - \frac{r_s}{r} + \epsilon_1(-9\Big(\frac{r_s}{r}\Big)^9 + \frac{67}{8}\Big(\frac{r_s}{r}\Big)^{10})).$$

One would also expect that these order ϵ modifications should propagate into the expected form of the master equation itself, modifying Eq.(38), and indeed it is now recast as

$$\frac{\mathrm{d}^2\Psi}{\mathrm{d}r_*^2} + \frac{\omega^2}{c_s^2}\Psi - \sqrt{f_t f_r}(V_{GR} + \epsilon_1 V)\Psi = 0. \tag{45}$$

Analogously to the above, substituting our master variable into Eq.(A.12), we find that

$$\begin{aligned} h_0 = -\frac{(2M-r)(r\Psi' + \Psi)}{r} - \frac{128M^8\epsilon}{r^{10}}\Big(& r(231M^2 - 239Mr + 63r^2)\Psi' \\ & + (2661M^2 - 2462Mr + 567r^2)\Psi\Big). \end{aligned} \tag{46}$$

From here, we may substitute the above into Eq.(A.15), which may then be rearranged to provide the form of the full master equation, Eq.(A.16).

Using this we may once again solve for $f(r)$ by comparing the coefficients of Ψ', from where we find that

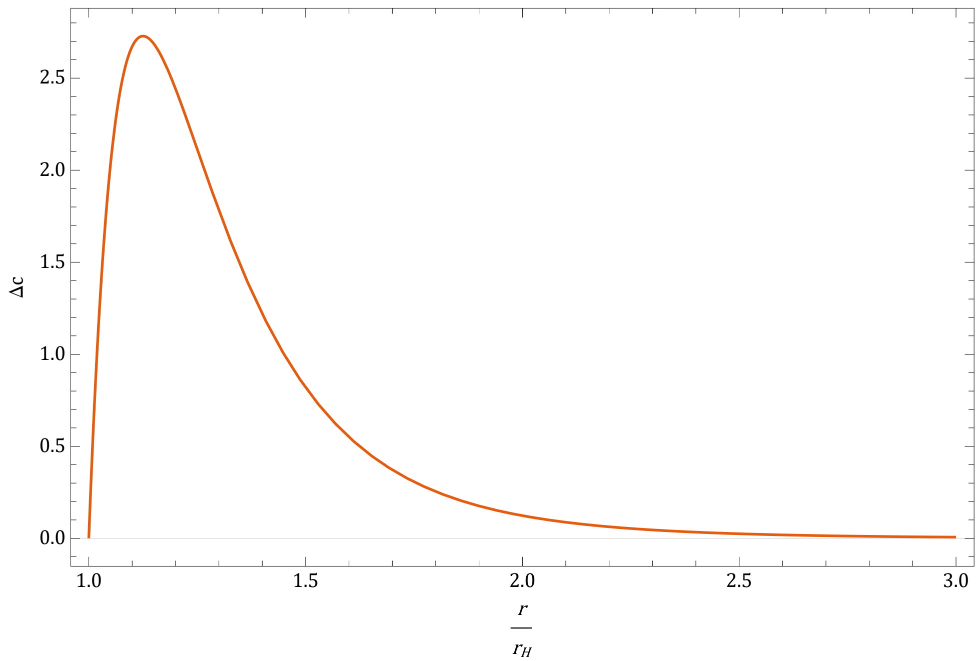

Figure 1: Deviation of low energy speed from unity. The deviation vanishes at the modified horizon radius, as well as asymptotically, $r \to \infty$. The maximum deviation occurs at a distance $r = \frac{9}{8} r_H$. Whether this deviation is superluminal or subluminal depends entirely on the sign of c_1, with negative corresponding to faster than light, and vice versa.

$$f(r) = \frac{1152 M^8 (13M - 7r)}{r^9}. \tag{47}$$

With this in hand we may now read off the sound speed,

$$\begin{aligned} c_s^2 &= 1 - \epsilon_1 \Delta c, \\ \Delta c &= \frac{63(r - r_s) r_s^8}{r^9}. \end{aligned} \tag{48}$$

The deviations from unity may be seen in Figure 1, where we note that the sound speed approaches unity for $r \to r_H$ as well as for $r \to \infty$.

The latter of these is relatively self explanatory, as our spacetime is asymptotically flat one would expect the sound speed to approach that of Minkowski, that is luminal. The former is more subtle and relates to the Horizon Theorem stated in Ref.[14], that will be discussed in further detail in the next section and Section 6, and how this relates to ensuring our theory is consistent and free of singularities in regions that are valid. Furthermore it is clear that the choice of $c_1 < 0$ will inevitably result in superluminal sound speeds.

5 Comments on Superluminal Propagation

It is at this point that we choose to address the elephant in the room. That is, as it stands we are discussing deviations from a speed of unity due to direct coupling to space-time curvature, and have not immediately struck down the notion of speeds deviating from unity in such a way that would lead to superluminal sound speeds, however small they may be.

As mentioned it is due to this exact behaviour that constraints were placed on coefficients on dimension 8 corrections in Ref.[12], and furthermore is one of the reasons that dimension 6 corrections were ignored in Ref.[9]. Up to this point however we have argued that superluminality is not necessarily the death knell of a theory if it is sufficiently restrained. Questions that may immediately come to mind in this instance are, what effect will these deviations have on the causal structure of our space-time, and will these deviations stand in stark contrast with the principles used to obtain them.

It is therefore possibly best to tackle this conundrum in a somewhat roundabout fashion, and drawing on similar effects previously studied in QED in the analogous circumstance of photon propagation in some background gravitational field considered in Ref.[15]. In this paper it was noted that corrections to the Einstein-Maxwell field equations induced tidal forces that would alter the behaviour of photon propagation, namely it was noted that superluminal photon speeds could be produced by these, and indeed it was argued that this behaviour was non-controversial and did not violate causality.

A reasonable concern is that superluminal propagation would quite readily set up a paradoxical situation in which information is sent backwards in time. Namely, if there is an observer who sees a secondary event happen before the initial one, we are left with a set up in which a path exists along which a return signal can be sent and received before the original signal has been emitted. This is as faster-than-light motion would imply that information is no longer constrained by the usual light cone, and hence may travel in a space-like fashion.

In the context of Ref.[15], the apparent paradox appeared to solve itself with the fortunate detail that superluminal travel in one direction was met with subluminal travel in the other, such that a returning signal could only arrive at its destination after the emission of the original.

A secondary but no less important concern is that of how this behaviour lines up with a foundational concept in Relativity, the Equivalence Principle. The key to understanding this relationship is in clarifying what the Equivalence Principle represents, and how it displays itself within General Relativity.

We therefore split the Equivalence Principle into its component parts, namely that, (1) for each point in space-time there exists a frame which is locally Minkowski,and (2) the laws of physics in all Local Inertial Frames (LIFs) are equivalent, and reduce to their Special Relativistic form at the origin of these LIFs.

The former effectively seeks to determining the behaviour of particles in a system where curvature is ignored, and amounts to the fundamental requirement that the space-time of General Relativity is Riemannian, or Pseudo-Riemannian. This requirement does not stand in contrast with any modifications we have made, as when curvature terms are ignored, we do indeed recover Minkowski space-time.

The latter is necessarily violated by the construction of our theory, as the presence of local higher order curvature terms will necessarily result in the inequivalence of LIFs, in which laws do not necessarily reduce to their Special Relativistic form. In this instance, the latter requirement only stands as a stronger constraint on our theory, there to exclude any direct curvature coupling, and hence is not necessarily fundamental to the Equivalence Principle as a whole.

A final issue worth highlighting is whether any superluminal propagation would result in a macroscopic, observable, time advance, or whether the level of deviation from luminal speeds is restrained in such a way that any advance made is never resolvable within the scales that the EFT is sensitive to, and indeed that no matter what timescale is integrated over any advance remains unresolvable.
It is with these convictions in mind, as well as the aim of an EFT to provide the most general, reasonable, extension to an existing theory, that we press on to consider whether it is sensible to rule out the superluminal effects of these low energy theories due to macroscopic causality violations that may occur.

6 The Effective Metric of Low Frequency GWs

In order to understand the causal structure associated with GW propagation, we must extract the Effective Metric, $Z_{\mu\nu}$, experienced by the Gravitational Waves at low energies. This is distinct from the background metric seen by other fields including those seen by high energy Gravitational Waves.

We may obtain this by considering a scalar, Φ, propagating on this effective background, and demanding it satisfy

$$Z_{\mu\nu} D^\mu D^\nu \Phi + U\Phi = 0 \tag{49}$$

For some effective potential U, where D^μ is the covariant derivative with respect to $Z_{\mu\nu}$, and where we further note that, in the coordinate system we work in,

$$Z_{\mu\nu} = \begin{pmatrix} -Z_t & 0 & 0 & 0 \\ 0 & Z_r^{-1} & 0 & 0 \\ 0 & 0 & r^2 Z_\Omega & 0 \\ 0 & 0 & 0 & r^2 \sin^2(\theta) Z_\Omega \end{pmatrix}. \tag{50}$$

Substituting $\Phi = e^{-i\omega t}\Psi Y_l(\theta)/r^2$ into Eq.(49) provides

$$\Psi'' + \Big(\frac{(Z_r Z_t)'}{2Z_r Z_t} + \frac{Z'_\Omega}{Z_\Omega}\Big)\Psi' + \Big(\frac{\omega^2}{Z_t Z_r} - \frac{J}{r^2 Z_\Omega Z_r} + \frac{U}{r^2 Z_r}\Big)\Psi = 0. \tag{51}$$

This clearly may be compared with our previous master equation, rephrased in the form

$$\Psi'' + \frac{(f_t f_r)'}{2 f_t f_r}\Psi' + \Big(\frac{\omega^2}{c_s^2 f_t f_r} - \frac{J}{r^2\sqrt{f_t f_r}} + V\Big)\Psi = 0. \tag{52}$$

Matching the coefficients, we may simply read off that

$$\begin{aligned}\frac{(f_t f_r)'}{2 f_t f_r} &= \frac{(Z_r Z_t)'}{2 Z_r Z_t} + \frac{Z'_\Omega}{Z_\Omega}, \\ c_s^2 f_t f_r &= Z_t Z_r \text{ and}, \\ \sqrt{f_t f_r} &= Z_\Omega Z_r.\end{aligned} \tag{53}$$

One may then find that

$$\begin{aligned}Z_t = Z_r &= \sqrt{f_t f_r}(1 - \frac{1}{2}\epsilon_1 \Delta c), \\ \text{and } Z_\Omega &= 1 + \frac{1}{2}\epsilon_1 \Delta c\end{aligned} \tag{54}$$

which to leading order gives,

$$\begin{aligned}Z_t = Z_r = 1 - \frac{r_s}{r} + (1 - \frac{r_s}{r})\epsilon \frac{128(213M^{10} - 230M^9 r + 63M^8 r^2)}{(2M - r)r^9}, \\ Z_\Omega = 1 + (1 - \frac{r_s}{r})\epsilon \frac{128(213M^{10} - 230M^9 r + 63M^8 r^2)}{(2M - r)r^9}.\end{aligned} \tag{55}$$

Now we have found the effective metric experienced by the low energy Gravitons, it is important that we compare this to the experience felt by photons on the background metric.

An obvious choice for comparison would be that of the Event Horizon seen in each scheme. We therefore check the horizon radius seen by GWs, where we see that $Z_t = Z_r = O(\epsilon^2)$, and $Z_\Omega = 1 + O(\epsilon^2)$, for $r = r_H$, just as with our modified Schwarzschild metric. Similarly, as $r \to r_H$ we find that $\Delta c \to 0$, meaning that the radial speed of the GWs is luminal on the shared Event Horizon, despite the fact that at every point near the black hole their speed, and therefore causal structures, differ.

This very neatly links to work done in Ref.[14] in the context of the EFT of QED below the electron mass, and the effect of space-time curvature on photon propagation speeds.

In this a Horizon Theorem was derived which shows that the effect of higher order, suppressed modifications to the theory have vanishing effects at the Event Horizon, forcing the speed of photons to be luminal. As an aside, this trick was also used to argue that in de Sitter space, as each observer has their own *cosmological* Event Horizon and by extension each point in space must be on the cosmological horizon of some observer, luminal speeds of photons must indeed then be enforced everywhere in space.

A similar theorem exists in the context of Gravity in Ref.[17], however due to both the increased complexity due to the presence of higher order equations of motion in considering EFTs of Gravity, and the fact that GR is inexorably coupled to any and all matter fields, this makes it tricky to consider anything other than a pure vacuum background.
In the setting of this EFT one need not go to these great lengths to check for consistency of the location of the horizon, we may instead attempt to show the necessity of this requirement by considering a situation in which our two metrics disagree on the location of the Black Hole horizon, reminding ourselves that our EFT should indeed still be valid at the Event Horizon.

We start by allowing our modified metric to see a horizon at $r = r_H$, and essentially consider our metric to have a form such that it is essentially that of Schwarzschild but where we have insisted $r_s \to r_H$. Using a similar approach with the effective metric seen by our Gravitational Waves, we assume they experience a slightly different horizon at $r = \bar{r}_H = r_H + \epsilon\delta r_H$, effectively setting $r_s \to \bar{r}_H$, meaning that our effective metric will be perfectly non-singular at $r = r_H$.
With these in hand we are free to construct a scalar invariant from functions of $g_{\mu\nu}$ and $Z_{\mu\nu}$. Defining $W_{\alpha\beta\gamma\delta}$ to be the Weyl tensor associated with our modified metric, and $\mathscr{W}_{\alpha\beta\gamma\delta}$

to be that associated with the effective metric viewed by GWs, we may then construct a scalar invariant, ξ, from the two. Up to first order in ϵ this takes the form

$$\xi = \mathscr{W}_{\alpha\beta\gamma\delta} W^{\alpha\beta\gamma\delta} = \frac{12 r_H^2}{r^6} + \frac{10 \delta r_H \epsilon}{r^5 (r - r_H)} - \frac{9 \delta r_H r_H^2 \epsilon}{r^6 (r - r_H)}. \tag{56}$$

This clearly leaves us in a situation in which we have a scalar invariant that is singular at the horizon at the order of our corrections, meaning that our ϵ order modifications have resulted in a physical singularity in a region in which the EFT should remain valid, and therefore not produce these kinds of effects. The only remaining course of action is to enforce $r_H = \bar{r}_H$, such that all of our particles agree on the location of the horizon, removing this issue.

7 Validity of the EFT

We now move on to consider the bounds within which this EFT remains valid, and hence when we can believe any aforementioned results, done following the method laid out in Ref.[16].

The most obvious first bound to note is that any corrections to propagation equations must remain perturbative.

Due to the spacetime of interest being Ricci flat, these corrections will be governed by the Weyl tensor. Hence we must construct an invariant from the Weyl tensor to extract any information about the corrections that is independent of field redefinitions one might make.

For an on-shell wave vector k^μ satisfying $k^2 \approx 0$, the highest order tensor we can can construct from k that is linear in curvature is $W_{\mu\nu\gamma\delta}k^\mu k^\nu k^\gamma k^\delta$, however due to the symmetries of the Weyl tensor this is zero. Therefore the next highest order tensor we may construct is

$$A_{\mu\nu} = W_{\mu\gamma\nu\delta}k^\gamma k^\delta \tag{57}$$

where by the aforementioned symmetries we find $A_{\mu\nu}k^\nu = A_{\nu\mu}k^\nu = A^\nu_\nu = 0$. We expect all scalar local operators to be suppressed by the cutoff scale, including A^α_α and $A^\alpha_\beta A^\beta_\alpha$. Hence one would expect at the very least that

$$Tr[A^n] \ll \Lambda^{4n}. \tag{58}$$

Considering a transverse wave vector $k_\mu = (-\omega, 0, 0, \pm\omega r^{1/2}\sin(\theta)/\sqrt{1 - r_s/r})$ we have the bound,

$$\omega^2 \ll \Lambda^2 \frac{r^3}{r_s}(1 - \frac{r_s}{r}). \tag{59}$$

However, using a radial travelling wave vector of the form $k_\mu = (-\omega, \pm\frac{\omega}{1-r_s/r}, 0, 0)$ we then have

$$A_{\alpha\beta}dx^\alpha dx^\beta = -\frac{r_s}{r^3}\omega^2(dt \mp \frac{dr}{1 - r_s/r})^2 \tag{60}$$

and therefore $A_{\alpha\beta} \propto k_\alpha k_\beta$, which satisfies the constraint in Eq.(58) trivially.

This compels us to consider the impact of higher derivative bounds such as,

$$A^{\alpha\beta}k^\mu\nabla_\mu A_{\alpha\beta} \ll \Lambda^8, \tag{61}$$

$$((k^\nu\nabla_\nu)^p A^{\alpha\beta})((k^\mu\nabla_\mu)^p A_{\alpha\beta}) \ll \Lambda^{8+4p}, \tag{62}$$

and

$$(k^\mu\nabla_\mu)^p(W^{\alpha\beta\gamma\delta}W_{\alpha\beta\gamma\delta}) \ll \Lambda^{8+2p}. \tag{63}$$

Considering the last two relations, we may take the limit $p \to \infty$, effectively providing $(k^\mu\nabla_\mu) \ll \Lambda^2$, which may be rephrased as

$$\omega \ll \Lambda^2 r, \tag{64}$$

if we assume our waves have a significant radial component for which $(k^\mu\nabla_\mu) \sim \omega\partial_r$ selects the radial dependence of background geometry.

This is clearly a stronger constraint than Eq.(59), and provides us a good basis to move on and check the validity of radial propagation of odd modes.

7.1 Radial Time Advance

This section considers the time advance generated by considering a photon and GW produced at a radius r_0 and projected radially to infinity. We define ΔT_{adv} as the amount of time by which the gravitational wave is advanced ahead of the photon. This provides a simple test for superluminality, and by which we can determine whether the advance made is resolvable before moving on to more complicated forms.

This time advance is given by

$$\Delta T_{adv} = \int_{r_0}^{\infty} dr \big(\frac{1}{1-2M/r} - \frac{1}{c_s(1-2M/r)} \big) \approx \int_{r_0}^{\infty} dr \frac{\Delta c}{2(1-2M/r)} \\ = 9\epsilon_1 \frac{r_s^8}{r_0^7} = \frac{576 c_1 r_s^2}{r_0^7 \Lambda^6}. \tag{65}$$

Using this notion of resolvability, we only require that any effects of superluminal propagation are unnoticeable on the energy scales that the EFT concerns itself with, as these unresolvable scales will be outside of the regime of validity that we a priori required our theory to be bound within. As such we for any time advance to be non-secular we at the very least require

$$\Delta T_{adv} \ll \omega^{-1}, \tag{66}$$

as this cannot be resolved by the gravitational waves by construction.

By reconsidering the form of Eq.(64), we can see that

$$\frac{r_s^2}{r_0^7} \ll \Lambda^2 \omega^{-1} \frac{r_s^2}{r_0^6}, \tag{67}$$

by which we then find

$$\Delta T_{adv} \sim \frac{r_s^2}{r_0^7 \Lambda^6} \ll \omega^{-1} \frac{r_s^2}{\Lambda^4 r_0^6} \ll \omega^{-1}, \tag{68}$$

as clearly $\frac{r_s^2}{\Lambda^4 r_0^6} < 1$. Therefore in this simple case, we see that the time advance made is sufficiently restrained to be unresolvable, and as such we argue it is valid with respect to causality.

7.2 Eisenbud-Wigner Time Delay

We now calculate corrections to the time delay in a more general context, that is we consider the corrections the EFT may have to the Shapiro Time Delay Ref.[6], modifications to propagation effected by the curvature of background spacetime. We do this by considering a generalisation to the Eisenbud-Wigner scattering time delay considered in Ref.[16]. This time delay is induced by considering the effect of scattering our Gravitational Waves on the background spacetime curvature, and measuring the impact of our EFT on the standard gravitational time delay.

We start by reconsidering the form of our master equation as

$$\frac{d^2\Psi}{dr_*^2} = -W\Psi, \tag{69}$$

where r_* is again the tortoise coordinate defined as $dr_* = \frac{dr}{\sqrt{f_t f_r}}$, and

$$W = \omega^2 - U_{GR} - \epsilon_1 U, \tag{70}$$

for which U is the potential associated with the higher order corrections. For a master equation of the form above, the phase shift is defined as,

$$\delta_l(\omega) = \int_{r_{*t}}^{\infty} dr_*(\sqrt{W} - \omega) - \omega r_{*t} + \frac{\pi}{2}(l + \frac{1}{2}) \tag{71}$$

where r_t is the turning point, and hence point of closest approach, in this scattering process. This turning point is defined by $\omega^2 - U_{GR}(r_t) = 0$, hence for this process to be valid we necessarily require that $\omega^2 < U_{GR}(r_{max})$, where r_{max} is the value of r maximising

$U_{GR}(r)$. Anything outside of this limit provides an undefined bound in the integral.

One can find that $U_{GR}(r_{max})$ is given by

$$U_{GR}(r_{max}) = -\frac{16J^3(-9+J-\sqrt{81-42J+9J^2})(-3+3J+\sqrt{81-42J+9J^2})}{(9+3J+\sqrt{81-42J+9J^2})^4 rg^2} \quad (72)$$

Following the procedure in Ref.[16] the time delay is given by,

$$\Delta T_l = 2\frac{d\delta_l}{d\omega} = 2\int_{r_{*t}}^{\infty} dr_* \Big(\frac{2\omega - \epsilon_1 \frac{\partial U}{\partial \omega}}{2\sqrt{W}}\Big) - 2r_{*t}, \quad (73)$$

and therefore the corrections on ΔT_l are given by

$$\Delta T_l^{EFT} = 2\frac{d\delta_l}{d\omega} = 2\int_{r_t+\delta r_t}^{\infty} dr \frac{1}{f+\delta f}\Big(\frac{2\omega - \epsilon_1 \frac{\partial U}{\partial \omega}}{2\sqrt{\omega^2 - U_{GR} - \epsilon_1 U}}\Big) - 2\int_{r_t}^{\infty} dr \frac{1}{f}\Big(\frac{\omega}{\sqrt{\omega^2 - U_{GR}}}\Big), \quad (74)$$

where $f = 1 - \frac{r_s}{r}$, and $\epsilon_i \delta f = \sqrt{f_t f_r} - f + O(\epsilon^2)$.

For convenience, and to avoid divergence at the turning point, we define,

$$\mathscr{A} = \frac{1}{f}\frac{\omega}{\sqrt{\omega^2 - U_{GR}}}, \quad (75)$$

as well as,

$$\delta\mathscr{A} = \epsilon_1 \frac{1}{f}\frac{\omega}{\sqrt{\omega^2 - U_{GR}}}\Big(\frac{U}{2(\omega^2 - U_{GR})} - \frac{1}{2\omega}\frac{\partial U}{\partial \omega} - \frac{\delta f}{f}\Big) \quad (76)$$

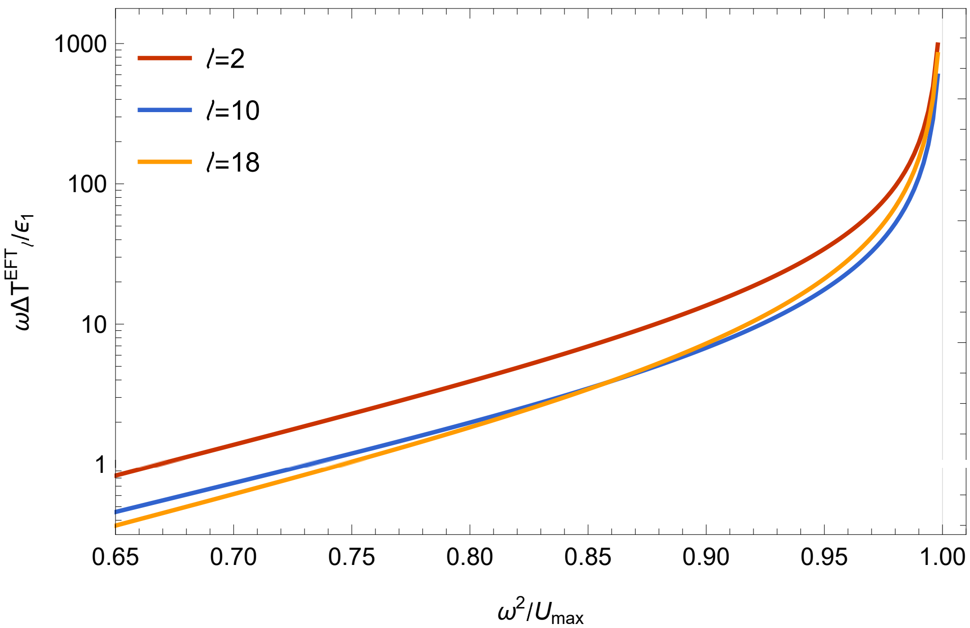

Figure 2: Results of numerical integration from produced from Eq.(77) for corrections of the EFT on the time delay, where we have considered the case of $c_1 < 0$. We know that superluminal propagation occurs if ΔT_l^{EFT} is negative, and indeed this becomes resolvable on the condition that $-\omega \Delta T_l^{EFT} > 1$. In order to prevent resolvable superluminal propagation for $\omega^2 < U_{GR}(r_{max}) = U_{max}$, we see that we approximately require $|\epsilon_1| < 10^{-3}$.

and hence the integral is,

$$\Delta T_l^{EFT} = -2 \int_{r_t}^{\infty} dr \mathscr{A} \left(\frac{\delta \mathscr{A}}{\mathscr{A}'} \right)'. \tag{77}$$

Unfortunately, ΔT_l^{EFT} does not have an analytic solutions, therefore we have no choice but to analyse this numerically. The results of this for a number of different choices of l may be found in Figure 2.

Necessarily we will have superluminal propagation if time delay corrections are negative. However, this is only problematic if the time advance is resolvable, that is, we have secular superluminality if,

$$-\omega \Delta T_l^{EFT} > 1 \tag{78}$$

From Figure 2 we see that our requirement for unresolvable superluminality for these choices of l is $|\epsilon_1| < 10^{-3}$, where we specify that $c_1 < 0$.

From this we observe that from the perspective of our arguments of permitting time advances, so long as they cannot be resolved by the theory, that an EFT of this form is reasonable based on the information explored.

8 Summary and Discussion

In this paper we have considered the effects of EFTs of gravity containing higher order curvature corrections, and investigated the effects of these on Gravitational Wave propagation as compared to standard GR. Namely, we have focussed on issues regarding superluminal GW speeds, and how this relates to whether the theory can be considered causal.

We started by considering, in Section 2, the rationale behind the construction of an EFT of gravity containing corrections in the form of higher order curvature terms constructed from the Riemann tensor in Ricci flat spherically symmetric spacetime. In this way, we noted that despite the issues of superluminality, and the appearance of an infinite tower of higher spin particles in the UV completion of the theory, terms associated with dimension 6 corrections had merit at the order of low energy EFTs of gravity and the superluminality could be overlooked if it can be considered unresolvable by the theory.

In spite of this, we pressed on to consider the effect of dimension 8 terms, the next choice after cubic curvature terms are ruled out. This was done to serve our own purposes of testing whether these terms may also have the capacity of unresolvable superluminality, and hence extend the range of theories that may be considered valid despite apparent issues with causality.

We then moved on, in Section 3, to consider the effects these quartic curvature terms may have on the form of the Schwarzschild metric, and what modifications that may result on the spacetime around Black Holes, for which the corrected metric was found to be Eq.(28). Furthermore these metric corrections induced an altered horizon radius,r_H given in Eq.(29), where it was found that $f_t(r_H)$ and $f_r(r_H)$ vanished at the same point in space and time thus protecting us from any singularities that may have been produced in the domain of validity of our perturbative corrections.

Next we examined the effects of tensor fluctuations on our altered metric in Section 4, from which the equations produced may be found in Appendix A. We started by stating the odd an even modes, Eqs.(35) and (36), before narrowing our scope to the odd modes only, due to the ease with which the relevant equations may be generated. From this we considered the master variable, a function containing the information pertaining to the single odd degree of freedom from which all the odd perturbation functions may be determined uniquely, as well as the master equation governing the behaviour of this. Starting with the standard unmodified Schwarzschild metric we found the master variable to be of the form in Eq.(43), from a master equation recast from Eq.(A.7) to be like that of Eq.(38). We then moved on to consider the effects produced by our order ϵ corrections, with a altered form of the Schwarzschild master equation as in Eq.(45). Corrections to the master variable were found to be as in Eq.47, with a modified sound speed given by Eq.48, where we noted that the speed became luminal at the event horizon, r_H, as well as at asymptotic infinity, $r \to \infty$, as can be seen in Figure 1.

In Section 5, we discussed the justification for considering superluminal speeds by contemplating them in the context of QED, as in Refs.[14] and [15] , and how the apparent paradoxes that may occur could be circumvented, as well as how these things contrast foundational principles in General Relativity.

Having explored the above, we pressed on to examine the metric as seen by the Gravitational Waves on this modified background in Section 6. We did this by considering a scalar field, Φ, propagating under the effective metric governed by Eq.(49), where it was seen that the metric itself would have the structure given in Eq.(50). Defining $\Phi = e^{-i\omega t}\Psi Y_l(\theta)/r^2$, we constructed a master equation, Eq.(51), that may be compared to that produced by the modified Schwarzschild metric, Eq.(52). Subsequently we were able to specify the effective metric to first order in ϵ as in Eq.55. From this we were in a position to check that the event horizon radius, r_H, was consistent for both metrics, and indeed went on to show using Eq.(56), to first order in ϵ, that this was necessary for physical singularities to not form within regions that the EFT should be valid.

Finally, in Section 7, we considered the regime of validity of the EFT, and what is therefore required for any time advances to not be resolvable in the theory. We started by constructing invariants from the Weyl tensor and an on-shell wave vector, finding that for radial propagating modes that the strongest constraint is set by Eq.(64).
This was then used to check whether the time advance of radially propagating Gravitational Waves over light was resolvable, requiring that we satisfy Eq.(66), from which we found in this instance that the time advance was sufficiently restrained to be considered unresolvable. Next we moved on to analyse the theory more generally by seeing the time advances generated by Eq.(77) stemming from a generalisation of the Eisenbud-Wigner scattering time delay [16]. From this we integrated numerically to generate Figure 2, which provides a basis for constraints on the form of ϵ_1 such that any time advances remain unresolvable to our theory, seeing that we require $|\epsilon_1| < 10^{-3}$.

The work done in this paper may be extended primarily by studying the behaviour of the even modes, as these will likely set further constraints on the possible values of ϵ_1, as well as by considering how the effects of keeping c_2 and c_3 manifest themselves in the behaviour already studied.

A The Perturbed Equations of Motion

In this Appendix we outline the exact form of the equations generated by metric perturbations on the equations of motion

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{c_1}{\Lambda^6}(8\nabla^\alpha\nabla^\beta(R_{\mu\alpha\nu\beta}C) + \frac{g_{\mu\nu}}{2}(C^2)), \tag{A.1}$$

both on a Schwarzschild and Modified Schwarzschild background metric, used in the derivation of the master equations and master variable.

A.1 Unmodified Perturbation Equations

We start by considering first order perturbations of our equations of motion for which metric corrections due to the modifications to GR in our Lagrangian are ignored.
This has the utility that these lower order equations of motion may be used to simplify higher order ones resulting from considering the equations governing the full modified theory, thus removing unphysical states that would otherwise prevent solutions from forming.
The distinct non-trivial unmodified odd equations of motion are defined as follows:

$$\mathscr{E}_{o03} = \frac{(-4M + Jr)h_0}{2r^3} + \frac{i\omega(2M - r)(2h_1 + rh_1'))}{2r^2} - \frac{1}{2}(1 + \frac{2M}{r})h_0'' = 0 \tag{A.2}$$

$$\mathscr{E}_{o13} = \frac{-i\omega h_0}{2(M - r)} + \frac{(2(-2 + J)M + r(2 - J + r^2\omega^2))h_1}{2(2M - r)r^2} - \frac{ir\omega h_0'}{4M - 2r} = 0 \tag{A.3}$$

$$\mathscr{E}_{o23} = \frac{ir\omega h_0}{4M-2r} + \frac{-2Mh_1 + (2M-r)rh_1'}{2r^2} = 0 \tag{A.4}$$

where $J = l(l+1)$.

Substituting h_0 and h_1 in terms of our master variable Ψ_{GR} as found in Section 3, that is,

$$h_1 = \frac{\omega r}{i(1-2M/r)}\Psi_{GR} \tag{A.5}$$

and

$$h_0 = -\frac{(2M-r)(\Psi_{GR} + r\Psi_{GR})}{r}, \tag{A.6}$$

the remaining non-trivial equations of motion become,

$$\begin{aligned}\mathscr{E}_{o03} = \frac{1}{2r^3(-2M+r)}\Big(&(J(-2M+r)^2 + (8M-3r)r^3\omega^2)\Psi_{GR} \\ &+(2M-r)((-16M^2+2(4+J)Mr - Jr^2 + r^4\omega^2)\Psi_{GR}' \\ &+(2M-r)r((2M-3r)(\Psi_{GR}'') + (2M-r)r\Psi_{GR}^{(3)}))\Big) = 0,\end{aligned} \tag{A.7}$$

and

$$\begin{aligned}\mathscr{E}_{o13} = \frac{i\omega}{2r(-2M+r)^2}&((-12M^2 + 2(3+J)Mr - Jr^2 + r^4\omega^2)\Psi_{GR} \\ &+(2M-r)r(-2M\Psi_{GR}' + (2M-r)r\Psi_{GR}'')) = 0.\end{aligned} \tag{A.8}$$

The latter of which may obviously be used to determine Ψ_{GR}'' in terms of lower order variables,

$$\Psi_{GR}'' = \frac{(12M^2 - 2(3+J)Mr + r^2(J - r^2\omega^2))\Psi_{GR}[r] + 2M(2M-r)r\Psi_{GR}')}{r^2(-2M+r)^2}, \tag{A.9}$$

which then may be used to find that

$$\Psi_{GR}^{(3)} = \frac{1}{r^3(-2M+r)^3}(2(12M^3 - 24M^2r - Jr^3 + Mr^2(9+2J+3r^2\omega^2))\Psi_{GR} \\ -(2M-r)r(12M^2 - 2(1+J)Mr + r^2(J - r^2\omega^2))\Psi'_{GR}). \tag{A.10}$$

A.2 Modified Perturbation Equations

Having extracted information from our theory in the influence of the unmodified Schwarzschild metric, we are now in a position to consider the modified perturbation equations to first order in epsilon.

The distinct non-trivial odd perturbation equations generated before the introduction of a master variable are as follows:

$$\begin{aligned} \mathscr{E}_{03} = (\frac{-4M+Jr}{2r^3} - \frac{128M^8(77M^2+18(-2+J)Mr-9Jr^2)\epsilon_1}{r^{12}})h_0 \\ +(\frac{i(2M-r)\omega}{r^2} + \frac{128iM^8(275M^2-243Mr+54r^2)\epsilon_1\omega}{r^{11}})h_1 \\ -\frac{1152M^8(22M^2-27Mr+8r^2)\epsilon_1}{r^{11}}h'_0 \\ +(\frac{i(2M-r)\omega}{2r} + \frac{64IM^8(473M^2-486Mr+126r^2)\epsilon_1\omega}{r^{10}})h'_1 \\ +(-\frac{1}{2}(1-\frac{2M}{r}) + \frac{64M^8(473M^2-486Mr+126r^2)\epsilon_1}{r^{10}})h''_0 = 0 \end{aligned} \tag{A.11}$$

$$
\begin{aligned}
\mathscr{E}_{13} = (\frac{i\omega}{2M-r} + \frac{128iM^9(47M-26r)\epsilon_1\omega}{r^9(-2M+r)^2})h_0 \\
+\Big(\frac{2(-2+J)M + r(2-J+r^2\omega^2)}{2(2M-r)r^2} \\
-\frac{64M^8\epsilon_1}{r^{11}(-2M+r)^2}(144(-2+J)M^3 - 18(-2+J)r^3 \\
+M^2r(432-216J-47r^2\omega^2) \\
+2Mr^2(-108+54J+13r^2\omega^2))\Big)h_1 \\
+(-\frac{ir\omega}{4M-2r} - \frac{64iM^9(47M-26r)\epsilon_1\omega}{r^8(-2M+r)^2})h_0' = 0
\end{aligned}
\tag{A.12}
$$

$$
\begin{aligned}
\mathscr{E}_{23} = (\frac{ir\omega}{4M-2r} + \frac{64IM^9(47M-26r)\epsilon_1\omega}{r^8(-2M+r)^2})h_0 \\
+(-\frac{M}{r^2} + \frac{128M^9(101M-54r)\epsilon_1}{r^{11}})h_1 \\
+(-\frac{1}{2}(1-\frac{2M}{r}) + \frac{64M^8(473M^2-486Mr+126r^2)\epsilon_1}{r^{10}})h_1' = 0
\end{aligned}
\tag{A.13}
$$

with the ansatz master variable of the form

$$
\Psi = \frac{i\sqrt{f_t f_r}}{r\omega}(1-\epsilon_1 f(r))h_1, \tag{A.14}
$$

for which

$$
f_t = (1 - \frac{r_s}{r} + \epsilon_1(-2\Big(\frac{r_s}{r}\Big)^9 + \frac{11}{8}\Big(\frac{r_s}{r}\Big)^{10}))
$$

and

$$
f_r = (1 - \frac{r_s}{r} + \epsilon_1(-9\Big(\frac{r_s}{r}\Big)^9 + \frac{67}{8}\Big(\frac{r_s}{r}\Big)^{10})).
$$

One might wonder at this point, why we were so interested in the perturbation equations defined by the unmodified metric and master variable. The key to this may be found in the fact that we are only interested in solutions that are continuously connected to those in the unmodified scheme. That is if we wished for $\epsilon \to 0$, we would expect our theory to oblige us by providing the solutions of its unmodified state, furthermore, clearly taking $\epsilon_1 = 0$ we are left with $\Psi = \Psi_{GR}$.

This is very convenient as it means that when considering terms of order ϵ in the equations of motion we will be able to assume that $\Psi = \Psi_{GR}$ as we regard terms of order ϵ^2 and higher as negligible. We will therefore have the capacity to apply constraints provided by the GR equations of motion to these terms in order to simplify the process.

Notably, the presence of h_0'' in Eq(A.10) likely guarantees the presence of a term of order $\Psi^{(3)}$ as Eq(A.12) implies that h_0 is linearly related to Ψ'. Furthermore, we note that even at the level of the equations for an unmodified metric $\Psi^{(3)}$ terms were present, and we only dealt with them by using another equation to rephrase them in terms of lower order components.

In this instance we have a final trick up our sleeve. It is unreasonable at this level for perturbations to result in new degrees of freedom within the regime of validity of the EFT, and hence these too should be removed using the results of lower order equations within the regime of GR.

After implementing the above, the remaining non trivial equations become,

$$\begin{aligned} -\frac{64iM^8\epsilon_1\omega}{(2M-r)^3r^{10}}\Big((124956M^4+2(-116529+545J)M^3r \\ +18r^4(324-8J+7r^2\omega^2) \\ -2Mr^3(25173-423J+230r^2\omega^2)+M^2r^2(162654-1661J+426r^2\omega^2))\Psi \\ +2Mr(-702M^3+1093M^2r-569Mr^2+99r^3)\Psi'\Big)=0, \end{aligned} \tag{A.15}$$

and

$$
\begin{aligned}
-\frac{64iM^8\epsilon_1\omega}{(2M-r)^3r^{10}}\Big((124956M^4+2(-116529+545J)M^3r \\
+18r^4(324-8J+7r^2\omega^2) \\
-2Mr^3(25173-423J+230r^2\omega^2)+M^2r^2(162654-1661J+426r^2\omega^2))\Psi \\
+2Mr(-702M^3+1093M^2r-569Mr^2+99r^3)\Psi'\Big) \\
+\frac{i\omega}{2(-2M+r)^2}\Big((2(3+J)M-(12M^2)/r-Jr+r^3\omega^2)\Psi \\
+(2M-r)(-2M\Psi'+(2M-r)r\Psi'')\Big)=0
\end{aligned}
\tag{A.16}
$$

where the second of these can be rearranged into the form of the master equation,

$$
\begin{aligned}
\frac{1}{r^{13}}\Big(-12M^2r^9+2(3+J)Mr^{10}-Jr^{11} \\
-7937280M^{11}\epsilon_1-128(-85026+623J)M^{10}r\epsilon_1+r^{13}\omega^2 \\
+2304M^8r^3\epsilon_1(324-8J+7r^2\omega^2)-1792M^9r^2\epsilon_1(2763-43J+18r^2\omega^2)\Big)\Psi \\
+\frac{1}{r^{12}}(2M(-2Mr^9+r^{10}+54912M^{10}\epsilon_1-53120M^9r\epsilon_1+12672M^8r^2\epsilon)\Psi') \\
+\frac{1}{r^{11}}((-2M+r)(-2Mr^9+r^{10}+9984M^{10}\epsilon_1-5632M^9r\epsilon_1)\Psi''=0.
\end{aligned}
\tag{A.17}
$$

As with GR in Section 4, one may now compare terms present to those expected in the Eq.(45), finding that,

$$
\Psi=\frac{i\sqrt{f_tf_r}}{r\omega}(1-\epsilon_1\frac{1152M^8(13M-7r)}{r^9})h_1. \tag{A.18}
$$

References and Notes

[1] B. P. Abbott et al. (LIGO Scientific and Virgo Collaborations), Observation of Gravitational Waves from a Binary Black Hole Merger, PhysRevLett.116.061102 (2016)

[2] B. P. Abbott et al. (LIGO Scientific and Virgo Collaborations), GW150914: First results from the search for binary black hole coalescence with Advanced LIGO, PhysRevD.93.122003 (2016)

[3] B. P. Abbott et al. (LIGO Scientific and Virgo Collaborations), GW151226: Observation of Gravitational Waves from a 22-Solar-Mass Binary Black Hole Coalescence, PhysRevLett.116.241103 (2016)

[4] B. P. Abbott et al. (LIGO Scientific and Virgo Collaborations), GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral, PhysRevLett.119.161101 (2017)

[5] C. de Rham, A. J. Tolley, Speed of gravity, PhysRevD.101.063518 (2020)

[6] I. I. Shapiro, Fourth Test of General Relativity, PhysRevLett.13.789 (1964)

[7] C. de Rham, J. Francfort, and J. Zhang, Black hole gravitational waves in the effective field theory of gravity, PhysRevD.102.024079 (2020)

[8] X. O. Camanho, J. D. Edelstein, J. Maldacena, and A. Zhiboedov, Causality Constraints on Corrections to the Graviton Three-Point Coupling, arXiv:1407.5597v1 (2014)

[9] S. Endlich, V. Gorbenko, J. Huang, L. Senatore, An effective formalism for testing extensions to General Relativity with gravitational waves, arXiv:1704.01590v3 (2017)

[10] N. Sennett, R. Brito, A. Buonanno, V. Gorbenko, L. Senatore, Gravitational-Wave Constraints on an Effective-Field-Theory Extension of General Relativity, PhysRevD.102.044056 (2020)

[11] V. Cardoso, M. Kimura, A. Maselli, and L. Senatore, Black Holes in an Effective Field Theory Extension of General Relativity, PhysRevD.121.251105

[12] A. Gruzinov, M. Kleban, Causality Constrains Higher Curvature Corrections to Gravity, Class.Quant.Grav.24.3521 (2007)

[13] T. Regge and J. A. Wheeler, Stability of a Schwarzschild singularity, PhysRev.108.1063 (1957)

[14] G. M. Shore, 'Faster than light' photons in gravitational fields: Causality, anomalies and horizons, Nucl.Phys.B460, 379 (1996).

[15]] I. T. Drummond, S. J. Hathrell, QED vacuum polarization in a background gravitational field and its effect on the velocity of photons, PhysRevD.22.343 (1980)

[16] C. de Rham and A. J. Tolley, Causality in Curved Spacetimes: The Speed of Light & Gravity, arXiv:2007.01847v2 (2020)

[17] T. Jacobson, G. Kang, R. C. Myers, On Black Hole Entropy, PhysRevD.49.6587 (1994)