Overview of Bohmian Mechanics and its extensions

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Abstract

It is well known that Copenhagen interpretation of quantum mechanics poses conceptual difficulties such as the measurement problem. Bohmian mechanics also known as de-Broglie Bohm theory is one of the simplest alternatives which explains quantum phenomena while avoiding these conceptual difficulties. The underlying idea of this theory is particles guided by the wave function. Although this idea seems natural, somehow this picture is ignored or even rejected by the majority of the physics community. In this dissertation, an overview of Bohmian mechanics and its extensions will be presented.

Contents

1 Introduction

In spite of the immense success of Newtonian physics on predicting the motion of macroscopic objects from falling of an apple to the orbit of the moon, experiments such as the double slit experiment, discovery of phenomena like black-body radiation and photoelectric effect between the late nineteenth and early twentieth century forced us to abandon the Newtonian description for the macroscopic world and opt for a more fundamental theory. At that time, new physics was needed.

In 1905, Einstein in his Nobel prize winning paper postulated that light, which was thought to be a wave, actually contains discrete energy quanta that we nowadays refer as photons in order to explain the photoelectric effect. This revolutionised the classical view on continuous entities like wave. The situation is similar to that on the first sight ocean waves seem to be continuous, but a deeper investigation shows that it is an effect of a enormous number of water molecules moving in a coherent way. Hence, the motion of the wave stems from motion of water molecules. Furthermore in 1926, de Broglie proposed the idea of matter wave stating that particles such as electrons and protons can display wave like behaviour. These ideas were then generalised by Max Born, Werner Heisenberg and Erwin Schrodinger separately to develop Matrix mechanics and wave mechanics respectively around 1925 which are the earliest version of nowadays quantum mechanics and proved their equivalence by Dirac in later years.

On the one hand, when a particle is observed, it always appears as a localised entity. On the other hand, it acts as if it is a wave when no observation is involved. An interpretation of quantum mechanics was needed in order to reconcile the properties. In the early development of quantum mechanics, a few interpretations such as pilot-wave by de Broglie and Copenhagen by Max Born and Werner Heisenberg were proposed. The most widely accepted interpretation is the Copenhagen interpretation due to its simplicity as one only needs to

solve the Schrodinger equation (17) and accept a few axioms to predict the results of an experiment. And to this day it is the most taught interpretation of quantum mechanics in universities and standard quantum mechanics textbooks.

Over the years, there are many objections of the Copenhagen interpretation. The main problem is usually referred as the measurement problem which will be explained in section (4). In short, Copenhagen interpretation not only has two dynamical rules which makes measurements somewhat special, but also to a certain degree implies subjective reality. Another minor problem is the superposition of states which was one of the problems that concerned Bohm. The superposition of states comes from the fact that the Schrodinger equation is linear. And based on that it is postulated that these states constitute a Hilbert space. If one assumes that the Schrodinger equation is only an approximation of a more fundamental equation which might involve a nonlinear term, then the construction of the Hilbert space is no longer valid and thus the operator description of measurements fails. Another less immediate problem is that the Copenhagen interpretation does not seem to give a clear ontology to quantum theory. This problem is usual ignored by majority of the scientists. As long as the predictions of the theory is consistent with experiments, why one should be bothered with problems like this. However, in my opinion physics is not just experiments. Experiments are just one of the many ways to 'test' the world. Therefore, this interpretation is closed to questions like what is objectively happening when measurement is not involved. The best answer one could get is that there is nothing since we cannot see or measure the wave function.

During the years, many other interpretations have been proposed as a solution to this problem. For example the many-worlds interpretation by Everett [19] which does not involve collapse of the wave function, but branching of realities, the Ghirardi–Rimini–Weber theory (GRW) [20] which involves spontaneous collapse and Bohmian Mechanics [6], [7] (a.k.a pilot-wave theory or de Broglie Bohm theory) which introduces particles position as additional

variable and thus maintains a realistic view on the quantum world.

The idea of particles guided by wave was first proposed by de Broglie in the early development of qunatum mechanics, but was abandoned due to the majority support of the Copenhagen interpretation. It is then rediscovered by Bohm in 1952. And in the paper [7], Bohm showed how the quantized electromagnetic field is described in this interpretation. This was the earliest extension of Bohmian mechanics to field theory. At that time the theory was not widely accepted by the scientific community because of political reasons. However, to Bell the pilot wave interpretation seems so natural as stated in his article [5]

> But why then had Born not told me of this 'pilot wave'? If only to point out what was wrong with it? ... Why is the pilot wave picture ignored in text books? Should it not be taught, not as the only way, but as an antidote to the prevailing complacency? To show that vagueness, subjectivity, and indeterminism, are not forced on us by experimental facts, but by deliberate theoretical choice?

It further inspired Bell to question the existence of local hidden-variable in quantum theory in later year which then led to one of the most fundamental theorem of quantum theory, the Bell's theorem [3]. Bell then extended Bohmian mechanics to include spin [4] and developed a version of Bohmian mechanics which contains creation and annihilation of particles [2].

In this dissertation, an overview of Bohmian mechanics and its extensions will be presented. The organisation of this dissetation is as follows. In section (2), classical mechanics in its different forms will be briefly discussed. In section (3), Bohmian mechanics will be intoduced with some examples. Then the measurement theory of Bohmian mechanics will be analysed in section (4). Then followed by a discussion of the criticisms of Bohmian mechanics. The two remaining sections (6) and (7) will focus on the extensions of Bohmian mechanics related to special relativity and quantum field theory respectively.

2 Classical Mechanics

It is believed that quantum mechanics is a more fundamental theory of reality. Therefore, classical mechanics should be from quantum mechanics in certain limit. Namely, when $\hbar$ tends to zero (see [1] for a more detail analysis of how classical mechanics is recovered in Bohmian mechanics). Thus, before going in to quantum mechanics, we first briefly explore different formalisms of classical mechanics. Also, some of the ideas will be needed when a Bohmian field theory is discussed in later sections. For simplicity, we only consider time-independent Lagrangian and Hamiltonian.

Consider a system of N particles under a potential V, instead of working with N points $[\vec{x}_1, ..., \vec{x}_N]$ in real space, $\mathbb{R}^3$, one introduces configuration space $\mathcal{Q} \equiv \mathbb{R}^{3N}$, whose coordinates are $q \equiv (\vec{x}_1, ..., \vec{x}_N)$ for mathematical convenience. Then given two points q_i and q_f in configuration space and travel time $t = t_f - t_i$. There is an infinite number of smooth paths connecting these two points. Then the action S, a map from the space of all possible smooth paths to the space of real number, is defined as follow.

$$S[q(t)] = \int_{t_i}^{t_f} L(q(t)), q\dot{(t)})dt \tag{1}$$

where $L(q(t)), q\dot{(t)}) = T - V$ is the Lagrangian of the system.

The actual path of the point in configuration space has to satisfies the principle of least action. Mathematically, it is

$$\delta S = 0 \tag{2}$$

In the following, three formulations of classical mechanics will be presented.

2.1 Lagrangian Mechanics

In Lagrangian mechanics, one derives the Euler-Lagrange equation by imposing the condition of principle of least action on the action as follow

$$\delta S = \Big(\frac{\partial L}{\partial \dot{q}}\delta q(t)\Big)_{t_i}^{t_f} - \int_{t_i}^{t_f} dt\Big[\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}}\Big]\delta q = 0 \tag{3}$$

Since the initial and end points are fixed (i.e $\delta q(t_i) = \delta q(t_f) = 0$), the first term vanishes. Then by the principle of least action, the integrand must be equal to zero. Hence, we have the Euler-Lagrange equation.

$$\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = 0 \tag{4}$$

Then one finds the trajectory of the system in the configuration space by solving the Euler-Lagrange equation. Hence, the motion of the system in real space.

2.2 Hamiltonian Mechanics

In Hamiltonian mechanics, one transforms the Lagrangian $L(q(t), \dot{q}(t))$ into the Hamiltonian $H(q(t), p(t))$ by Legendre transform, where $p(t) \equiv \frac{\partial L}{\partial \dot{q}}$ and $H = \dot{q}\frac{\partial L}{\partial \dot{q}} - L$. Then by comparing the two expressions of the total differential of H one expressing in terms of p and q another from the expression $H = \dot{q}\frac{\partial L}{\partial \dot{q}} - L$ and by the Euler-Lagrangian equation, one derives the following equation of motion for q and p.

$$\begin{aligned} \dot{p} &= -\frac{\partial H}{\partial q} \\ \dot{q} &= \frac{\partial H}{\partial p} \end{aligned} \tag{5}$$

The set of ordered pair (q, p) constitutes a phase space, $\mathbb{R}^{6N}$. A point say $(\bar{q}, \bar{p}) \in \mathbb{R}^{6N}$ represents the state of the system. Therefore, the system of first order differential equations can be solve either analytically or numerically when the Hamiltonian of the system and initial condition are given. Hence, the evolution of the system $(q(t))$ can be obtained.

2.3 Hamilton-Jacobi equation

Hamilton-Jacobi equation is considered the closet formulation of classical mechanics to quantum mechanics because it establishes the relation between wave and trajectory in classical mechanics. Unlike the two previous formulations where the action S plays a less significant role compared to equations (4),(5), one investigates what the action S tells us about the trajectory in Hamilton-Jacobi theory. Consider cases where the position (q_f) and time (t_f) of the end point are varied respectively. In the first case, the variation of S is the following

$$\delta S = \int_{t_i}^{t_f} dt \Big[\frac{\partial L}{\partial q}\delta q + \frac{\partial L}{\partial \dot{q}}\delta \dot{q}\Big] \tag{6}$$

By the Euler-Lagrangian equation (4),

$$\delta S = \int_{t_i}^{t_f} dt \Big[\frac{d}{dt}\frac{\partial L}{\partial \dot{q}}\delta q + \frac{\partial L}{\partial \dot{q}}\delta \dot{q}\Big] = \int_{t_i}^{t_f} \frac{d}{dt}\Big[\frac{\partial L}{\partial \dot{q}}\delta q\Big] \tag{7}$$

Since it is assumed that $\delta q(t_i) = 0$, and by the definition in Hamiltonian mechanics,

$$\delta S = \Big[\frac{\partial L}{\partial \dot{q}}\Big]_{t_f} \delta q(t_f) \tag{8}$$

Therefore, the variation of S due to the variation of the end point position is

$$\frac{\partial S}{\partial q} = p(t_f) \tag{9}$$

the variation of the action S due to change of t_f is

$$dS = Ldt = \frac{\partial S}{\partial q_f} dq_f + \frac{\partial S}{\partial t_f} dt_f \tag{10}$$

the second equality comes from the fact that when the initial time t_i and position q_i are fixed, the action S can be viewed as a function with two variables q_f and t_f. Then by equations (9) and (10),

$$\frac{\partial S}{\partial t} = L - \dot{q}p = -H\Big(q, \frac{\partial S}{\partial q_f}\Big) \tag{11}$$

This first order differential equation is the Hamilton-Jacobi equation.

2.3.1 Example - free particle

Consider a free particle. The Lagrangian is

$$L = \frac{1}{2} m\dot{q}^2 \tag{12}$$

and the Hamiltonian is

$$\frac{p^2}{2m} \tag{13}$$

Hence, the Hamilton-Jacobi equation is

$$\frac{\partial S}{\partial t} + \frac{1}{2m}\Big(\frac{\partial S}{\partial q}\Big) = 0 \tag{14}$$

One can check spherical wave of the form

$$S = \frac{m(q-q_0)^2}{2(t-t_0)} \tag{15}$$

where q_0 and t_0 are constants, is a solution to equation (14). One can also find a separable solution of the form $S = W(q) - Et$. This corresponds to the plane wave solution. Having found the solution to the Hamilton-Jacobi equation, the actual trajectory of the particle is found by using $p = \frac{\partial S}{\partial q}$ which is tangent to the trajectory. In the case of plane wave, the trajectory is trivial. By this example, one can see that the Hamilton-Jacobi equation establishes a relation between wave and particle in the context of classical mechanics. Therefore, Hamilton-Jacobi equation is usually regarded as the closet version of classical mechanics to quantum mechanics.

If we are dealing with a case in which initial position of a classical particle can only be described by probability distribution function $R^2(x,0)$ (i.e the probability of the initial position of the particle between x to dx at $t = 0$ is $R^2(x,0)dx$). Since the particle must take on a continuous path which evolves according to the Hamilton-Jacobi equation (11). As a result, there exists a continuity equation of the form

$$\frac{\partial R^2(x,t)}{\partial t} + \frac{\partial}{\partial x}\Big(R^2(x,t)v(x,t)\Big) = 0 \tag{16}$$

3 Bohmian Mechanics

3.1 Bohmian mechanics for spinless particles

In Bohmian Mechanics, the complete description of a system is given by the wave function ψ and its configuration Q. The wave function $\psi(q,t)$ evolves according to the Schrodinger equation in position representation (17), where $q = (\vec{x}_1, ..., \vec{x}_N)$. The evolution of the configuration is determined by the guiding equation (18) which can be regarded as a consequence of the Schrodinger equation. Bohmian mechanics is said to be deterministic because once the initial state of the system $(\psi(q,0), Q(0))$ is given, the dynamics of the state is uniquely determined for all future time by equations (17) and (18). Unlike orthodox quantum mechanics where there are two sets of dynamical rules, these two equations account for all the quantum phenomena involving spinless particles.

Consider a single spinless particles under the potential V, the Schrodinger equation is

$$i\hbar\frac{\partial\psi(\vec{x},t)}{\partial t} = \frac{-\hbar^2}{2m}\nabla^2\psi(\vec{x},t) + V(\vec{x},t)\psi(\vec{x},t) \tag{17}$$

and the guiding equation is

$$\frac{d\vec{X}(t)}{dt} = \frac{J(\vec{x},t)}{|\psi(\vec{x},t)|^2}\bigg|_{\vec{x}=\vec{X}(t)} = \frac{\hbar}{m}\,\mathrm{Im}\left(\frac{\psi^*\nabla\psi}{|\psi|^2}\right)\bigg|_{\vec{x}=\vec{X}(t)} \tag{18}$$

where $J(\vec{x},t)$ is the probability current density. Generalisation to many-particles is trivial in which $\vec{x}$ is replaced by $q = (\vec{x}_1, ..., \vec{x}_N)$ and the configuration X by Q. The second expression of equation (18) seems redundant as there is common factor of ψ^* in the numerator and denominator. However, the advantage of this expression is that it provides a easy transition to particles with spin.

The probability current density $J(\vec{x},t)$ comes from the associated continuity equation to the Schrodinger equation. By multiplying ψ and its complex conjugate ψ^* respectively on the Schrodinger equation and its complex conjugation. One has

$$\begin{aligned} \psi^* i\hbar\frac{\partial\psi}{\partial t} &= -\psi^*\frac{\hbar^2}{2m}\nabla^2\psi + \psi^* V\psi \\ -\psi i\hbar\frac{\partial\psi^*}{\partial t} &= -\psi\frac{\hbar^2}{2m}\nabla^2\psi^* + \psi^* V\psi \end{aligned} \tag{19}$$

Then subtracting one off the other, one gets

$$\frac{\partial|\psi|^2}{\partial t} = i\frac{\hbar}{2m}\nabla\Big(\psi^*\nabla\psi - \psi\nabla\psi^*\Big) \tag{20}$$

usually it is written in a more compact form of

$$\frac{\partial\rho}{\partial t} + \frac{\partial J}{\partial x} = 0 \tag{21}$$

where $\rho = \psi^*\psi = |\psi|^2$ and $J = \frac{\hbar}{2im}\Big(\psi^*\nabla\psi - \psi\nabla\psi^*\Big)$.

To see where the guiding equation comes from, we follow Bohm's paper [6] in 1952. Firstly, put the wave function ψ in polar form $\psi = Re^{i\frac{S}{\hbar}}$ then substituting it into the Schrodinger equation (17). The imaginary and real part then are,

$$\frac{\partial R}{\partial t} = -\frac{1}{2m}\Big[R\nabla^2 S + 2\nabla R\nabla S\Big] \tag{22}$$

$$\frac{\partial S}{\partial t} = -\Big[\frac{(\nabla S)^2}{2m} + V(x) - \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}\Big] \tag{23}$$

By comparing equation (23) and the Hamilton-Jacobi equation (11), then the natural interpretation of the phase S is to regard it as the quantum action. Unlike the classical Hamilton-Jacobi equation, there is an extra term $Q(x,t) \equiv -\frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}$ called the quantum potential. Equation (23) is commonly regarded as the quantum Hamilton-Jacobi equation. Rewriting $R = \sqrt{\rho}$, equation (22) then implies

$$\frac{\partial \rho}{\partial t} + \nabla\Big(\rho \frac{\nabla S}{m}\Big) = \frac{\partial R^2}{\partial t} + \nabla\Big(R^2 \frac{\nabla S}{m}\Big) = 0 \tag{24}$$

By comparing this to the continuity equation (16) of a particle with uncertain initial position in the classical case, it is natural to define the Bohmian velocity as

$$v = \frac{J(x,t)}{\rho} = \frac{\nabla S}{m} \tag{25}$$

it can be shown that the two expressions are equivalent to the previous expression. As a result, the guiding equation is merely a consequence of the Schrodinger equation. One then interprets $|\psi(x,t)|^2 dx$ as the probability of the particle at position x. Or equivalently, if a system has wave function $\psi(q,t)$, then its configuration Q is random with probability distribution $|\psi(q,t)|^2$. This is referred as the quantum equilibrium hypothesis. The justification of this is a delicate matter and is beyond the scope of this dissertation. A detailed argument of this is given in [11] by Durr, Goldstein and Zanghi in which typicality and the law of large number in probability theory are used for the argument. In addition, if at initial time the distribution is $|\psi(x,0)|^2$, then at later time the distribution is $|\psi(x,t)|^2$. This property is called equivariance. Since $|\psi(x,t)|^2$ acts as $R^2(x,t)$ in equation (16), the initial position of the particle cannot be known with certainty, but when the experiment is ran for many times $|\psi(x,t)|^2$ should be reproduced.

In Bohm's paper, he then formulated this theory in terms of force like Newton's equation.

Hence, one can simply treat a quantum system like a classical system but with an additional 'quantum' force coming from the quantum potential Q. Even though this way of looking at Bohmian Mechanics gives us some insights on how quantum phenomenons work when the word 'particle' is taken in the literal sense, it is like putting quantum mechanics in a classical mould.

After all Bohmian mechanics is a first order theory unlike Newton' equation which is second order. Hence, a more common view of equation (17) and (18) is to interpret and treat the wave function ψ like the Hamiltonian $H(q, p)$ in Hamiltonian formulation of classical mechanics in which the Hamiltonian generates a vector field in phase space and the evolution of the system is the integral curve of the field. In Bohmian mechanics, the wave function ψ generates a vector field in configuration space. Hence, the evolution of the system is the integral curve of the vector field.

3.2 Orthodox and Bohmian description of Double-slit experiment

In the double slit experiment, electrons are fired toward the slits one at a time and its position is registered by a screen in the opposite side of the source. After firing a large number of electrons, an interference pattern is observed. This suggests that the underlying theory determining the motion of electrons cannot be classical mechanics, but quantum mechanics. In standard quantum mechanics, by the principle of complementarity, a physical object displays its wave or particle property depending on the type of measurement. In the case of the double slit experiment, before going through the slits, an electron is represented by a plane wave. When the plane wave hits the slits, it creates two circular wave sources and hence an interference pattern. By Bohr's rule, the probability of finding the electron within x to $x + dx$ of the screen is $|\psi(x, t)|^2 dx$. When an electron is detected, it is said to be showing its particle-like property.

With this interpretation, questions such as, what is the position of the electron before it hits the screen, and more fundamentally what is an electron, can be very confusing or even cannot be answered. Therefore, it is unclear how this interpretation gives us an image of the world.

In contrast, Bohmian mechanics maintains the view that a particle is a particle. By quantum equilibrium hypothesis, the initial position of the electron cannot be known with certainty and can only be described by $|\psi(x,0)|^2$ due to the lack of knowledge. Then by the property of equivariance, at a later time the probability of the electron at x is $|\psi(x,t)|^2$. During the process the wave function $\psi(x,t)$ is only responsible for guiding the electron which has a continuous trajectory all along. In other word, the particle goes through one and only one of the slits while the wave function acts as ordinary wave which passes both slits and guides the particle. A figure of possible Bohmian trajectories (1) is shown at end of this section.

Problems that stated above regrading the Copenhagen interpretation do not exist in Bohmian mechanics. For example, the answer of the question of what is an electron is simple. An electron is a particle in the literal sense and it is represented by a point mathematically just like any other particles in classical mechanics. Since there is no electromagnetic field in the experiment, spin is not accounted for. Unlike Copenhagen interpretation, the image of a Bohmian universe is quite clear in which the fundamental physical entities are particles and their positions.

In [26], researchers reconstructed the trajectories of an ensemble of photons in a double slit experiment using weak measurement. The resulting trajectories of photons resemble that of the Bohmian trajectories.

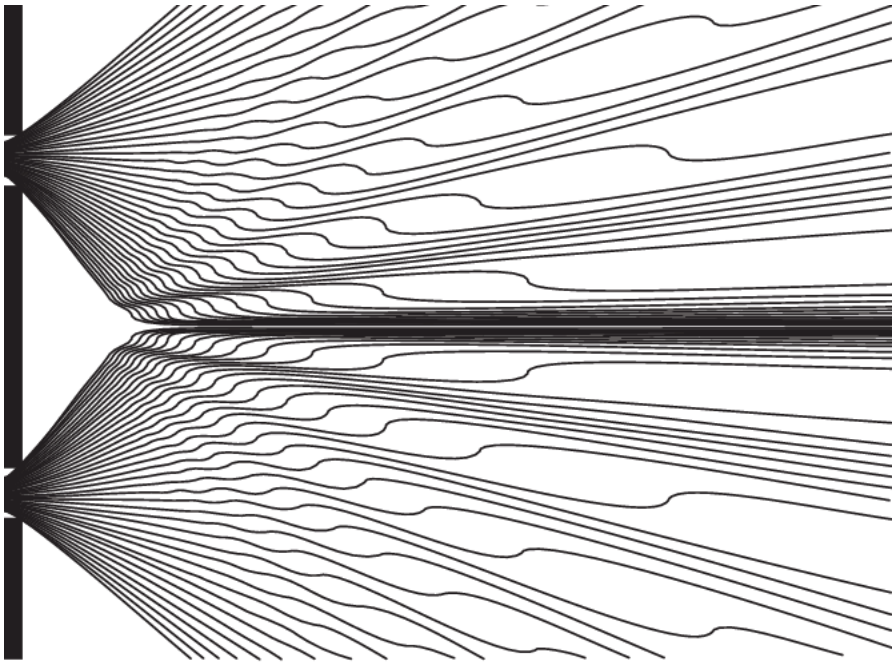

Figure 1: regions that have a high density of trajectories correspond to the bright fringes on the screen and vice versa for dark fringes. Figure from [17] based on [33]

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3.3 Spin in Bohmian mechanics

In standard quantum mechanics, spin is regarded as an intrinsic property of a particle in which there is no classical analogue. Therefore, it seems like additional variables are needed to account for every degree of spin freedom. In Bohmian mechanics, even though one can indeed introduce additional variables for spin, this tends to complicate the mathematics. However, Bell [4] showed us that spin can be treated as a property of the wave function instead of the particle's. For simplicity we will only discuss the case $s = \frac{1}{2}$. For a spin half particle, its wave function is a spinor wave function,

$$\vec{\psi}(\vec{x},t) = \begin{pmatrix} \psi_{\frac{1}{2}}(\vec{x},t) \\ \psi_{-\frac{1}{2}}(\vec{x},t) \end{pmatrix} \tag{26}$$

which is a map $\mathbb{R}^{3+1} \longmapsto \mathbb{C}^2$. And the evolution of this wave function given by the Pauli equation (27) instead of the Schrodinger equation (17),

$$i\hbar\frac{\partial\vec{\psi}(\vec{x},t)}{\partial t} = \Big[\frac{1}{2m}\vec{\sigma}(-i\hbar\nabla - q\vec{A}(\vec{x},t))^2 + V\Big]\vec{\psi}(\vec{x},t) \tag{27}$$

where $\vec{\sigma}$ are the Pauli matrices and $\vec{A}(\vec{x},t))$ is the vector potential. Then one can define the Bohmian velocity in a similar way as for spinless particles case.

$$\vec{v}(\vec{x},t) = \frac{\vec{J}(\vec{x},t)}{\rho(\vec{x},t)} \tag{28}$$

where $\rho = \vec{\psi}^\dagger \cdot \vec{\psi}$ and $\vec{J}$ is the probability current of the Pauli equation (27).
These two equations (27) and (28) together account for every phenomena involving spin. Unlike standard quantum mechanics, spin is not considered as an intrinsic property of a particle, but a property of the wave function and will affect the trajectory of a particle via the guiding equation (28). This is shown by the fact that the wave function is a function of only position and time. Also. there is no guiding equation for spin components of the particles. A detailed formulation of Bohmian mechanics involving spin can be found in [30].

3.4 Conditional and effective wave function

Before going into the how measurement is described in Bohmian mechanics. We discuss some physical implications of Bohmian mechanics. In a universe governed by Bohmian mechanics, because the theory is deterministic and the velocity of a particle depends on the instantaneous position of all other particles, there is a sense of 'wholeness' in which one is lead to the concept of wave function of the universe $\Psi^{(u)}$. An immediate question is that assuming the wave function of the universe $\Psi^{(u)}$ and its configuration $Q(t)$ evolves according to the

Schrodinger equation and the guiding equation respectively, why the subsystems which we deal with in a laboratory seem to follow the Schrodinger equation and guiding equation of the subsystem instead of being influenced by its environment. This then leads us to the concept of conditional wave function and effective wave function.

Consider the following, in a universe with N particles with configuration Q and wave function $\Psi^{(u)}(q,t)$. The wave function of the universe follows the many-particle Schrodinger equation

$$i\hbar\frac{\partial\Psi^{(u)}(q)}{\partial t} = \sum_{i=1}^{N} -\frac{\hbar^2}{2m_i}\frac{\partial^2\Psi^{(u)}(q,t)}{\partial {q_i}^2} + V(q,t)\Psi^{(u)}(q,t) \tag{29}$$

while its configuration evolves according to guiding equation

$$\frac{dQ}{dt} = \left.\frac{J(q,t)}{\left|\Psi^{(u)}(q,t)\right|^2}\right|_{q=Q(t)} \tag{30}$$

Say we are interested on a subset of the particles, a subsystem with $M < N$ particles whose configuration is X. Hence, the configuration of the universe is given by $Q = (X, Y)$ where Y is the configuration of the environment of the subsystem. Then we define the conditional wave function of the subsystem by

$$\phi(x) = \Psi^{(u)}(x, Y, t) \tag{31}$$

In general, the conditional wave function (31) is not governed by the Schrodinger equation but a more complicated equation. However, if the wave function of the universe $\Psi^{(u)}$ is of the form

$$\Psi^{(u)}(x,y,t) = \phi(x,t)\alpha(y,t) + \Psi^{\perp}(x,y,t) \tag{32}$$

where $\alpha(y,t)$ and $\Psi^{\perp}(x,y,t)$ do not overlap in y configuration space. In other word, they have disjoint macroscopic supports. If the configuration Y is in the support of $\alpha(y,t)$, then we define $\phi(x,t)$ as the effective wave function. And $\phi(x,t)$ will evolve according to the Schrodinger equation of the subsystem. This usually happens after a quantum measurement which will be discussed in section (4). A point to note is that effective wave function of a subsystem does not necessarily exist since the situation above does not always happen. But if it does exist, then it will be proportional to the conditional wave function which can be seen by comparing equation (31) and (32). In the rest of the paper, the wave function of the system will be the effective wave function.

3.5 Stationary states

In this interpretation, there is an interesting phenomena about a special type of stationary states. Just like standard quantum mechanics, stationary wave functions are the separable solutions to the Schrodinger equation (17) of the form $\phi(\vec{x})e^{-i\frac{Et}{\hbar}}$. Putting it into the following form

$$\psi(\vec{x},t) = R(\vec{x})exp\Big[i\frac{(\alpha(\vec{x}) - Et)}{\hbar}\Big] \tag{33}$$

the phase of the wave function is $S = \alpha(\vec{x}) - Et$. Then by $\vec{v} = \frac{1}{m}\nabla S$, one can see the if $\nabla S = \frac{\partial\alpha(\vec{x})}{\partial\vec{x}} = 0$, the position of the particle is constant. Hence, the particle appears to be stationary. However, this does not mean there is only one result when one measures the position of the particle. Because of the quantum equilibrium hypothesis, the distribution of position result will be $|\psi(\vec{x},t)|^2$. Electron in the $s = 0$ state in a hydrogen atom is one of the example that exhibits this behaviour. In fact, any system with a spherical symmetric wave function or real wave function will exhibit this behaviour.

Consider a free particle, the Schrodinger equation is

$$i\hbar\frac{\partial\psi(\vec{x},t)}{\partial t} = \frac{-\hbar^2}{2m}\nabla^2\psi(\vec{x},t) \tag{34}$$

then plane wave of the form $\psi(\vec{x},t) \propto exp[i\frac{(\vec{p}\cdot\vec{x}-Et)}{\hbar}]$. By $\nabla S = \frac{\partial\alpha(\vec{x})}{\partial\vec{x}} = \vec{p}$. Hence, the velocity of the particle is constant and the trajectory of it is a straight line as in the example in section (2). More examples and detail analysis of stationary states can be found in [6].

3.6 Choice of beables

In Bohmaian mechanics, we exclusively use the position representation of the Schrodinger equation. And the position of the particles as an addition 'hidden variable'or beable. However, in standard quantum mechanics one can use any representation of the Schrodinger equation. Therefore, one would question if it is possible to construct a Bohmian-type theory in which beable other than position, such as momentum, is used. A beable in Bell sense is an entity that exists in reality with or without observation. Also, it should give an image of the physical world. Therefore, it is a question of ontology of theory and whether it is consistent with the image of the world that we observe. In fact, additional to the position of particles, one can include other beable and this has been done by Holland [24]. See [18] for a detailed discussion regarding the ontology of Bohmian mechanics.

4 Measurement theory of Bohmian Mechanics

One of the most criticised issue of the orthodox quantum mechanics is its conceptual difficulty regarding measurements. It is usually referred as the measurement problem. In Bohmian mechanics, this problem does not exist since there is only one dynamical rule. Therefore, before analysing the measurement theory of Bohmian mechanics. A brief discussion of the measurement theory of Orthodox quantum mechanics is needed.

Firstly, in standard quantum mechanics the wave function ψ itself provides a complete description of the system. Secondly, the evolution of wave function is continuous and deterministic according to the Schrodinger equation in cases where measurement is not involved. Thirdly, upon measurement the wave function 'collapses' to one of the eigenstates of the corresponding operator. Mathematically, an observable A is associated with a Hermitian operator $\hat{A}$ whose eigenstates $|a_i\rangle$ span the Hilbert space in which the state $|\psi\rangle$ lives in. Therefore, $|\psi\rangle$ can be expressed in terms of those eigenstates as

$$|\psi\rangle = \sum_i^N \langle a_i|\psi\rangle \, |a_i\rangle = \sum_i^N c_i \, |a_i\rangle \tag{35}$$

where $\langle a_i|\psi\rangle$ is the inner product in Hilbert space, in position representation it is $\langle a_i|\psi\rangle = \int a_i^*(x)\psi(x)dx$. Also, non-degeneracy and discrete spectrum are assumed for simplicity. Then the act of measurement is represented by operator acting on the state as

$$\hat{A} \, |\psi\rangle = \sum_i^N \langle a_i|\psi\rangle \, a_i \, |a_i\rangle \tag{36}$$

where a_i is the eigenvalue of the eigenstate $|a_i\rangle$. The probability of getting the measurement result of a_i is $|\langle a_i|\psi\rangle|^2$ with $\sum_i^N |c_i|^2 = 1$. For example if one measured the value a_2, then the state $|\psi\rangle$ is said to collapse to the eigenstate $|a_2\rangle$. And for any subsequent measurement on

the system, one has to use the state $|a_2\rangle$. This quantum phenomena is called collapse of the wave function which is not accounted for by the Schrodinger equation in standard quantum mechanics, but by the measurement axiom.

Even though standard quantum mechanics has a great success on predicting the results of quantum measurements, it leaves some significant conceptual problems unanswered. First of all, this interpretation puts measurement in a special place in the theory. And hence unavoidably involves an observer. These leads to ambiguity on what a measurement is and what qualifies to be an observer. Even great physicists like Richard Feynman once said

> This is all very confusing, especially when we consider that even though we may consistently consider ourselves to be the outside observer when we look at the rest of the world, ... Does this then mean that my observations become real only when I observe an observer observing something as it happens? This is a horrible viewpoint. Do you seriously entertain the idea that without the observer there is no reality? Which observer? Any observer? Is a fly an observer? ...

And hence there is no specification on which dynamical law should be used in which circumstances. Secondly, the fact that one uses an operator to represent the environment of the system makes the separation of the system and its environment unclear.

In Bohmian Mechanics, the dynamics of a system is solely determined by the Schrodinger equation (17) and guiding equation (18). Instead of having another set of dynamical rule for measurement processes, one simply treats the interaction between the system and the measurement apparatus, such as a pointer, like any other interaction between particles to account for measurement processes. As a result, Bohmian mechanics is sometimes referred as the 'quantum mechanics without observers' since there is no involvement of an observer. Also, one can show that not only operators naturally emerge from the analysis of a Bohmian measurement, but also the measurement axiom in standard quantum mechanics is actually a consequence of it. However, due to computational burden it is impossible to practically

calculate the evolution of the coupled system because a classical describable measurement apparatus contains more than 10^{23} particles.

Even though the measurement theory of Bohmian mechanics gives the same results as the orthodox quantum mechanics, the mathematics behind is quite different. To analysis a measurement in Bohmian term, we follow a similar approach by [31] and [17].

Suppose the state of the system and pointer are $(\psi(x,t), X)$ and $(\phi(y,t), Y)$ respectively, where X and Y are the actual configurations of the system and pointer. Then the total wave function is $\Phi(x,y,t)$.

Assuming the pointer points at a discrete set of values so the value space is $\lambda = [\alpha_1, \alpha_2, ..., \alpha_N]$. If the wave function of the system is ψ_{α_1}, then the pointer should point at the value α_2 after the measurement. We further assume that this value space is completed, meaning that all possible wave functions are included. Also, since the pointer is a macroscopic object, ϕ_{α_i} and ϕ_{α_j} should not overlap for $i \neq j$. Thus, they represent different macrostates. A measurement apparatus that satisfies above conditions is referred as a good measurement device.

Before measurement by linearity, the wave function of the system is

$$\psi(x,t) = \sum_i^N c_i(t)\psi_{\alpha_i} \tag{37}$$

where we assume $\sum_i^N |c_i|^2 = 1$ for convenience. And the wave function of the pointer is ϕ_0 where it is the support of the pointer pointing null direction. During the measurement, the total wave function of the system plus pointer is in an entangled state

$$\Phi(x,y,t) = \sum_i^N c_i(t)\psi_{\alpha_i}(x)\phi_{\alpha_i}(y,t) \tag{38}$$

To get a unique result, the initial configuration of the system $(X(0))$ and pointer $(Y(0))$ are needed. During the measurement, the configuration of the total system $(X(t), Y(t))$ will

evolve under the effect of the total wave function (38). And the actual path of the total system will only lie in one of the non-overlapping wave packet of (38). Then by quantum equilibrium hypothesis, the configuration of the pointer (Y) after the measurement is given by one of the support of ϕ_{α_j}. Thus the probability of the pointer pointing at the value α_i is then given by the quantum equilibrium hypothesis,

$$\begin{aligned} P_{\alpha_j} &= \int_{support of \phi_{\alpha_j}} \int_x \left| \sum_i^N c_i(t)\psi_{\alpha_i}(x)\phi_{\alpha_i}(y,t) \right|^2 dxdy \\ P_{\alpha_j} &= |c_j(t)|^2 \int_{support of \phi_{\alpha_j}} \left|\phi_{\alpha_j}\right|^2 dy \int_x |\psi_{\alpha_i}(x)| dx \\ P_{\alpha_j} &= |c_j(t)|^2 \end{aligned} \tag{39}$$

Therefore, the probability of getting the value α_i is equal to the one from standard quantum mechanics.

Then for subsequent experiment, one has to use $\psi_{\alpha_j}(x)$ as the wave function of the system because the actual configuration of the system will only lie in the wave packet $\Phi_{\alpha_j} = \psi_{\alpha_j}\phi_{\alpha_j}$ and thus the other wave packets have no effect on the evolution of the system in future time. Then the wave function is said to be effectively collapsed. This is how the collapse of wave function in orthodox quantum mechanics described in Bohmian mechanics. And it is referred as effective collapse in Bohmian mechanics. The assumption of these non-overlapping wave packets to stay non-overlapping in later time is due to decoherence.

Furthermore, one can then show the orthogonality of the states of the system ψ_{α_i} and ϕ_{α_j}.

$$\int_y \int_x \left| \sum_i^N c_i \psi_{\alpha_i}(x)\phi_{\alpha_i}(y) \right|^2 dxdy = \sum_i^N |c_i| \int_x |\psi_{alpha_i}(x)|^2 dx \int_y |\phi_{\alpha_i}(y)|^2 dy \tag{40}$$

expressing this in another way gives

$$\int_y \int_x \left| \sum_i^N c_i \psi_{\alpha_i}(x) \phi_{\alpha_i}(y) \right|^2 dxdy = \sum_{i,j} c_i c_j^* \int_x \psi_{\alpha_i}(x) \psi_{\alpha_j}^*(x) dx \int_y \phi_{\alpha_i}(y) \phi_{\alpha_j}^*(y) dy \tag{41}$$

By comparing expressions (40) and (41), one gets for if $i \neq j$,

$$\int_x \psi_{\alpha_i}(x) \psi_{\alpha_j}^*(x) dx = 0 \tag{42}$$

and if $i = j$,

$$\int_x \psi_{\alpha_i}(x) \psi_{\alpha_i}^*(x) dx = 1 \tag{43}$$

Or in terms of Dirac notation,

$$\left\langle \psi_{\alpha_i} \middle| \psi_{\alpha_j} \right\rangle = \delta_{ij} \tag{44}$$

an orthogonality relation between the wave function of the system with different measurement values. Then define the operator

$$\hat{P}_i = |\psi_{\alpha_i}\rangle \langle \psi_{\alpha_i}| \tag{45}$$

Thus, the wave function of the system can be written as

$$\psi = \sum_i^N \hat{P}_i \psi = \sum_i^N c_i \psi_{\alpha_i} \tag{46}$$

where $c_i = \langle \psi_{\alpha_i} | \psi \rangle$. One can then verify that the operator $\hat{P}_i$ indeed forms a family of projector. Hence, the statistic of the experiment can be simply encoded using an operator $\hat{E} = \sum_i^N \alpha_i \hat{P}_i$.

As a result, analysing a measurement process just like any other interaction between particles with additional constraints on what a good measurement device in Bohmian mechanics not only induces a natural correspondence between experiments and operators in which the operator is merely a mathematical object allowing us to compute statistics of an experiment,

but also in fact defines a boarder class of allowed operators. This is called Positive-operator valued measure (POVM). And in standard quantum mechanics, one usually promotes a classical observable to operator. This defines a projection-valued measure (PVM) which is a special case of POVM. For a detail analysis of POVM in Bohmian see [17].

5 Criticisms of Bohmian Mechanics and some responds

5.1 Criticisms

Despite the fact that Bohmain mechanics gives the same predictions as standard quantum mechanics, there are a lot of oppositions on whether it should be the correct model for our universe. In this section, a few important ones will be discussed.

First of all, one of the most problematic issues of Bohmian mechanics is the explicit non-locality of the theory which can be seen in equation (18), where the velocity of a particle is under the influence of the instantaneous position of all other particles even they are spacelike separated. This strictly violates the spirit of special relativity in which an event outside the lightcone of another event cannot be causally related. This makes Bohmian mechanics exceptionally hard to combine with special relativity. Also, some might quote Bell's theorem stating that Bell's theorem ruled out all hidden-variable theory including Bohmian mechanics.

Another problem of Bohmian mechanics is that it seems to violate the action-reaction principle. In Bohmian mechanics, the wave function ψ acts as a guiding wave which generates a time-dependent vector field in the configuration space. Hence, the configuration of the system X is the integral curve of the vector field. However, there is no action from the configuration X on the wave function ψ. This can be seen simply in the Schrodinger equation (17) as the evolution of the wave function does not depend on the configuration.

Thirdly, as Bohm suggested one should interpret the wave function ψ like any other real physical fields such as electromagnetic field. When one thinks of a physical field, for example a scalar field, it defines a number at each point in space. However, the wave function defines a number at each point in configuration space. Therefore, if wave function is to be regarded as a physical field, then one must accept the fact that configuration space which was introduced for mathematical convenience in classical mechanics is actually physical.

Another concern of Bohmian mechanics is that it seems to be impossible to develop is version of Bohmian mechanics with particle creation and annihilation. Because position of particles is a beable of the theory. Hence, a particle is an entity that exists with or without observation.However, this is not true. A version of Bohmian mechanics that incorporates creation and annihilation of particle will be presented in section (7).

5.2 Responds

Regarding non-locality, Bell's theorem (See [28] for a proof and brief discussion of Bell's theorem and its implication of hidden variable theory) is telling us that locality in Einstein sense will always be violated by a theory that describes quantum phenomena in terms of some local hidden variables. As a result, one can say that non-locality is a feature of the universe. Therefore, Bohmian mechanics is not ruled out by Bell's theorem because the guiding equation is non-local. But Rather it brings out the explicit non-locality of the universe. As Bell stated in his book [5]

> That the guiding wave, in the general case, propagates not in ordinary three-dimensional space but in a multidimensional configuration space is the origin of the notorious "nonlocality" of quantum mechanics. It is a merit of the de Broglie–Bohm version to bring this out so explicitly that it cannot be ignored.

Even though this explicit non-locality poses difficulties on combining Bohmian mechanics with special relativity, there are a few versions of relativistic Bohmian mechanics that are consistent with experimental results.

On the violation of action-reaction principle in classical mechanics by Bohmain mechanics, since the action-reaction principle is essentially one of the consequences of the underlying Galilean symmetry of theory, in Bohmian mechanics Galilean symmetry can be achieved without imposing action from the configuration to the wave function. Therefore, action-

reaction principle is not violated, but is merely not a consequence of the theory for it to be Galilean invariant. In addition, if one considers the conditional wave function of the system (31), then the configuration of the environment of the subsystem does have an effect on the wave function.

In [16], the authors argued that these problems of Bohmian mechanics stems from a more fundamental question of what is the meaning of the wave function. A different view of the wave function ψ is suggested in which essentially the wave function is not considered as a real physical field as Bohm suggested. Instead, it should be regarded as an interpretation-less nonphysical object that is one of the element of the theory and not an entity described by the theory.

This is similar to the case of Hamiltonian in classical mechanics. Hamiltonian of a system is a function of phase space which is equally or even more unphysical than the configuration space. Since Hamiltonian is not considered as an physical entity described by the law, but part of the law. Therefore, there is no argument saying that phase space is physically real.

Furthermore, the authors hypothesise that instead of the time-dependent Schrodinger equation (17), the time-independent Schrodinger equation (5.2)

$$\hat{H}\Psi^{(u)} = E\Psi^{(u)} \tag{47}$$

is the fundamental equation that governs the universe at least in the non-relativistic regime. And the time-independent wave function of the universe $\Psi^{(u)}$ is responsible for generating a vector field in the configuration space and thus guides the motion of Q. In this case, Q is not the position of all the particles in the universe, but some abstract configurations. An immediate question of such hypothesis is where does time-dependent Schrodinger equation of the subsystem come from if the wave function of the system (i.e the universe) is time independent. An simple example is shown and argued that this should also happen in a

more general situation.

This interpretation of the wave function eliminates the problem of physical configuration space. However, if the wave function is no longer considered a physical entity, then the state of a system is only given by its configuration Q. For example, in double slit experiment, the wave function does not act as a guiding wave, a particle simply goes through one of slits and its trajectory is described by the law. Therefore, it is unclear how the interference pattern develops without invoking entity like a wave.

Actually, the problem of how the wave function should be interpreted and its meaning is not only a question on the mathematical level, but also on a philosophical level. A further discussion of the role of wave function in different interpretation of quantum mechanics and other related philosophical problems see [18] and [23].

6 Making Bohmian Mechanics relativistic

As mentioned in the previous section, non-locality in guiding equation poses difficulties on unifying Bohmian mechanics and special relativity. More precisely, since the velocity of one particle depends on the instantaneous position of all other particles, one has to specific a hypersurface or a reference frame to introduce simultaneity. It is believed that a relativistic version of Bohmian mechanics cannot be developed without invoking structures like these.
In relativistic setting, time and position are treated on an equal footing. Hence, a natural generalisation of the wave function of a N spinless particles system $\psi(\vec{x}_1, ..., \vec{x}_N, t)$, which is a map from the space $\mathbb{R}^{3N+1}$ to $\mathbb{C}$, is the multi-time wave function

$$\psi((t_1, \vec{x}_1), ..., (t_N, \vec{x}_N)) \tag{48}$$

where $((t_1, \vec{x}_1), ..., (t_N, \vec{x}_N))$ is referred as the spacetime configuration. Then the wave function we use in quantum mechanics is just the special case of the multi-time wave function (48) in which $t_1 = ... = t_N$. A detail analysis of the Multi-time wave function is given in [27]. In the following, we assume the consistency condition is fulfilled.
In the case of a single particle, the wave equation and guiding equation are

$$\hat{L}\psi(x) = 0 \tag{49}$$

$$\frac{dX^\mu}{d\lambda} \propto j^\mu(x)\Big|_{x=X(\lambda)} \tag{50}$$

where $\hat{L}$ is a linear combination of of some differential operators defining the wave equation, for example, for the Klein-Gordon equation $\hat{L} = \partial^2 + m^2$. λ is the parameter of the curve X^μ. These equations are Lorentz invariant. However, the guiding equation is no longer Lorentz

invariant if the system has N particles, then the guiding equation in such a case is,

$$\frac{dX_a^\mu(\lambda_a)}{d\lambda_a} \propto j^\mu(X_1(\lambda_1), ..., X_N(\lambda_N)) \tag{51}$$

Hence, one has to define at which point of X, j should be evaluated. The most naive way to do this is to use a preferred reference frame. However, this method is clearly not Lorentz invariant because different frame leads to different particle trajectories.

6.1 A few ways to define simultaneity

6.1.1 Past and Future lightcone

Since there is no issue on differentiating the temporal order of events that are timelike separated, all one has to do is to define simultaneity for spacelike separated events. Therefore, instead of using a preferred reference frame, there are a few different proposals. One could use either the future or past lightcone to define a simultaneous surface which was proposed by Squires and Goldstein in [35] and [22] respectively. However, both methods have their problems respectively. If one uses the past lightcone, then the resulting theory will be local. Therefore, this theory cannot be a satisfactory one as Bell's theorem demands non-locality. On the other hand, if one uses the future lightcome, then the resulting theory is in fact non-local, but it does not has an equivariant measure. Although as shown in next subsection, lack of statistical transparency might not be a problem, the theory suggests a microscopic arrow of time point to the past. Therefore, using future lightcone to define simultaneity is probably not a good choice.

6.1.2 Time foliation

A more general choice is to introduce a foliation which was proposed in [12] and an example is given based on [8]. Consider N Dirac particles, the wave function $\psi(x_1, ..., x_a, ..., x_N)$ satisfies N Dirac equation with electromagnetic 4-potential A^μ,

$$(i\gamma_a^\mu \partial_{a\mu} - e\gamma_a^\mu A_{a\mu} - m)\psi(x_1, ..., x_N) = 0 \tag{52}$$

where $a \in [1, ..., N]$ and $\gamma_a = I_1 \otimes ... \otimes \gamma_a \otimes ... I_N$. It is well known that the conserved 4-current for the Dirac equation is

$$j^\mu = \bar{\psi}\gamma^\mu\psi \tag{53}$$

where $\bar{\psi} = \psi^\dagger\gamma^0$ is the Dirac adjoint of ψ. Since $j^0 = \psi^\dagger\psi$ is positive definite. Thus, one can interpret it as probability density which is also equivariant in a particular Lorentz frame. In order to construct a pilot-wave model, an equation of motion for the addition beable in this case particle worldlines are introduced

$$\frac{dX_k^{\mu_k}}{d\lambda} = j^{\mu_1 ... \mu_N}(X_1(\Sigma), ..., X_N(\Sigma))\Pi_{i\neq k} n_{\mu_i}(X_i(\Sigma)) \tag{54}$$

where λ is a parameter of the curve X. Σ is called a time leaf belonging to the foliation $\mathcal{F}$ which is associated with the unit normal vector field n. $j^{\mu_1 ... \mu_N} = \bar{\psi}(\gamma^{\mu_1} \otimes ... \otimes \gamma^{\mu_N})\psi$ is the 4-current of the Dirac equation (52). At this point, there is no specification of what $n(x)$ needs to be, but future oriented. It can be shown that the theory has an equivariant measure and was done in the paper. Hence, one can recover the quantum equilibrium hypothesis as long as the hypersurfaces are time leaves, namely belong to the foilation. As a result, the results of this theory should be in principle agree with the orthodox quantum mechanics ones. It is argued that this will be also true even if the hypersurfaces do not belong to the

foliation. Then, the remaining question is what is $n(x)$, thus the foliation $\mathcal{F}$.

Regarding the first question, if the foliation $\mathcal{F}$ is dynamical, then it must be governed by some Lorentz covariant law for theory to be Lorentz covariant. An example of this is given by Tumulka in [39], in which the evolution of the foliation is

$$\nabla_\mu n_\nu - \nabla_\nu n_\mu = 0 \tag{55}$$

One of the solution to equation (55) is the absolute time of the universe. Therefore, the initial time leaf is a point corresponds to the Big Bang. A figure of the spacetime history of the universe is shown below. The suggestion of the foliation $\mathcal{F}$ being covariantly determined by the wave function is given in [12]. In this case, the foliation $\mathcal{F} = \mathcal{F}(\psi)$ is no longer an additional structure we have to introduce, but an object defined by the wave function.

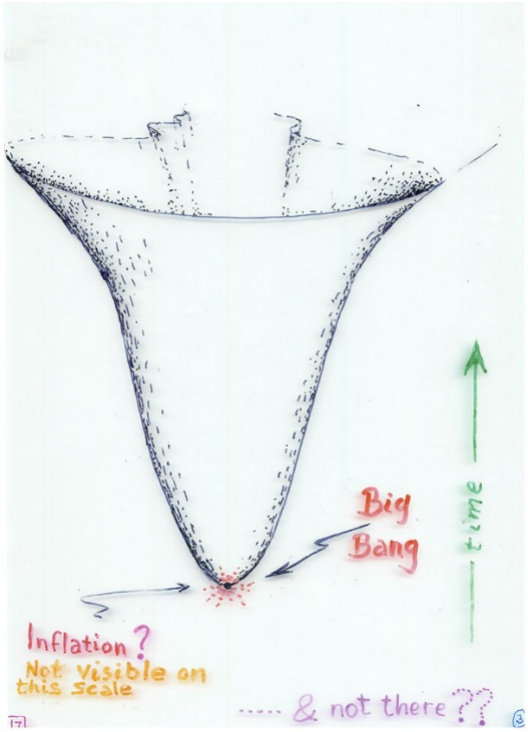

Figure 2: Every horizontal cut defines a simultaneous hypersurface. Figure from [32]

6.2 Synchronised trajectories

Synchronised trajectories is a method proposed in [29] and [25]. And this method still involves some foliation like structure. We will mainly follow [29] for the following discussion. Suppose we have a system of N spinless relativistic particles, the wave function satisfies N Klein-Gordon equation, and thus

$$(\sum_{a}^{N} \eta_{\mu\nu}\partial_a^\mu\partial_a^\nu + nm^2)\psi(x_1, ..., x_n) = 0 \tag{56}$$

where $\eta_{\mu\nu}$ has signature $(+,-,-,-)$ and $x_i = (t_i, \vec{x}_i)$. It is well known that there exists a conserved 4-current of the form $\psi^*\partial^\mu\psi - \psi\partial^{mu}\psi^*$ so we define

$$j_a^\mu(x_1, ..., x_n) = i(\psi^*\partial_a^\mu\psi - \psi\partial_a^\mu\psi^*) \tag{57}$$

thus $\partial_\mu^a j_a^\mu = 0$.

Then we put the wave function into polar form Re^{iS}, where R and S are the function of $x_a = (t_a, \vec{x}_a)$. Substituting it into the Klein-Gordon equation (56), then the real and imaginary respectively are

$$\eta_{\mu\nu}\partial_a^\mu\partial_a^\nu R - R\partial_{a\mu}S\partial_a^\mu S + nm^2R = 0 \tag{58}$$

$$\partial_{a\mu}R\partial_a^\mu S + \partial_a^\mu R\partial_{a\mu}S + R\eta_{\mu\nu}\partial_a^\mu\partial_a^\nu S = 0 \tag{59}$$

where the summation of the index a is accounted for by the repeated index. And they are usually put in the following form

$$-\frac{1}{2m}\partial_{a\mu}S\partial_a^\mu S+\frac{nm^2}{2}+Q=0 \tag{60}$$

where $Q=\frac{1}{2mR}\eta_{\mu\nu}\partial_a^\mu\partial_a^\nu R$.

$$\partial_a^\mu(R^2\partial_{a\mu}S)=0 \tag{61}$$

These are the relativistic version of equation (22) and (23). Then a pilot-wave model can be developed by introducing the equation

$$\frac{dX_a^\mu}{d\lambda}=-\frac{1}{m}\partial_a^\mu S(x_1,...,x_N)\Big|_{x_a=X_a(\lambda)} \tag{62}$$

where λ is a parametrization factor of the actual spacetime curve X_a^μ of the a-th particle in Minkowski spacetime. Similar to the non-relativistic case, the multi-time wave function is responsible for generating a time dependent vector field in the spacetime configuration space and $(X_1(\lambda),...,X_N(\lambda))$ is the integral curve of the vector field. Even though equation is non-local, it is Lorentz invariant. Note that all trajectories are parameterized in terms of the same factor λ. This is the reason this method is called synchronised trajectories. Actually, one can parameterizes each trajectory with a different parameter once the trajectory has been found.

A problem of this formulation is that since the Klein-Gordon equation is second-derivative in time, the quantity $|\psi|^2$ cannot be interpreted as probability density. In standard quantum mechanics, one circumvents this problem by second quantization in which ψ is no longer the probability density, but a field. However, the author suggested that the failure of interpreting $|\psi|^2$ as probability density does not mean probability cannot be calculated. More precisely, the author firstly argued that if fundamentally ψ is to be regarded as a field, then there is no clear connection on why the probabilistic interpretation of $|\psi|^2$ has such as a good agreement with experimental results in non-relativistic limit. Since relativistic quantum theory

lacks statistical transparency (i.e the probability of particle position can be calculated by only knowing the wave function), it is natural that $|\psi|^2$ cannot be interpreted as probability density. Yet if one assumes that particle trajectories exist, then statistical predictions can still be obtained. Therefore, because Bohmian mechanics assumes the existence of particle trajectories, it is not necessary to 'promote' to status of the wave function to field in relativistic Bohmian mechanics. However, there are a few problems such as superluminal velocities which cannot be observed. Nevertheless, it seems to contradict special relativity. There are in fact more ways to introduce simultaneity such as a covariant velocity field [21]. A figure of possible trajectory of a particle is shown below,

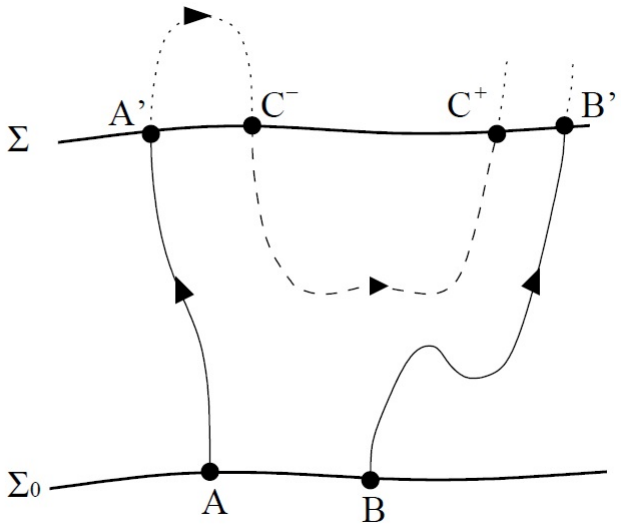

Figure 3: Dotted and dash lines indicate nonphysical trajectories. Solid lines represent physical trajectories of a particle. Measurement is taken on the hypersurface Σ. Figure from [29]

7 Bohmian Mechanics and QFT

As Bohmian mechanics accounts for every quantum phenomena like the orthodox quantum mechanics, it is then natural to extend Bohmian mechanics to quantum field theory which is considered as one of most fundamental theory of reality (See [38] for an overview of field beable Bohmian type field theory). In this section, two Bohmian type field theories based on field beable, which are examples in [38], will be presented. Then, followed by a brief dicussion of Bell-type theory based on particle beable. (See [39] and [40] for an overview) Quantum field theory is usually mathematically ill-defined without regularization and renormalization. In this section, it is assumed that the theory presented can be regularized and renormalized.

7.1 Schrodinger Field

Consider the following Lagrangian density of the Schrodinger field

$$\mathcal{L} = \left[\frac{i}{2}(\psi^*\dot{\psi} - \psi\dot{\psi}^*) + \frac{1}{2m}\psi^*\nabla^2\psi\right] \tag{63}$$

Separate ψ into real and imaginary part, $\psi = \psi_r + i\psi_i$. Then, the Lagrangian is

$$L = \int d^3x\mathcal{L} = \int d^3x\Big[(\psi_r\dot{\psi}_i + \psi_i\dot{\psi}_r) + \frac{1}{2m}(\psi_r\nabla^2\psi_r + \psi_i\nabla^2\psi_i + i\psi_r\nabla^2\psi_i + i\psi_i\nabla^2\psi_r)\Big] \tag{64}$$

the last two terms cancel each other by integration by parts and the assumption that field ψ goes to zero at spatial infinity. The conjugate momentum are the following,

$$\begin{aligned}\Pi_{\psi_r} &= \psi_i \\ \Pi_{\psi_i} &= -\psi_r\end{aligned} \tag{65}$$

Then, the Hamiltonian is

$$H = \int d^3x\mathcal{H} = \int d^3x\dot{\psi}_r\Pi_{\psi_r} + \dot{\psi}_i\Pi_{\psi_i} - \mathcal{L} = \frac{-1}{2m}\int d^3x\Big[\psi_r\nabla^2\psi_r + \psi_i\nabla^2\psi_i\Big] \tag{66}$$

By equation (65), since $\dot{\psi}_r$ and $\dot{\psi}_i$ cannot be expressed in terms of ψ_r, ψ_i, Π_{ψ_r} and Π_{ψ_i}, there are two constraints. Quantization of system with constraints are treated differently. Discussion and analysis of quantization of such type can be found in [38] and [10]. The unconstrained variables can be separated from the constrained ones by a canonical transformation.

$$\begin{aligned}
\psi_r &= \frac{1}{\sqrt{2}}(\phi + \tilde{\phi}) \\
\Pi_{\psi_r} &= \frac{1}{\sqrt{2}}(\Pi_\phi + \Pi_{\tilde{\phi}}) \\
\psi_i &= \frac{1}{\sqrt{2}}(\Pi_\phi - \Pi_{\tilde{\phi}}) \\
\Pi_{\psi_i} &= \frac{1}{\sqrt{2}}(\tilde{\phi} - \phi)
\end{aligned} \tag{67}$$

Then the physical Hamiltonian in which $\tilde{\phi} = \Pi_{\tilde{\phi}} = 0$ is

$$H = \int d^3x\mathcal{H} = \frac{-1}{4m}\int d^3x\Big[\phi\nabla^2\phi + \Pi_\phi\nabla^2\Pi_\phi\Big] \tag{68}$$

Then the system is quantized using canonical quantization in which ϕ and Π_ϕ are promoted to operators $\hat{\phi}$ and $\hat{\Pi}_\phi$. Also, by imposing the following commutation relation

$$[\hat{\phi}(\vec{x}), \hat{\Pi}_\phi(\vec{y})] = i\delta(\vec{x} - \vec{y}) \tag{69}$$

In Schrodinger picture, the time dependence lies in the state $|\Psi\rangle$ instead of the operators. The Hamiltonian in operator form is

$$\hat{H} = \frac{-1}{4m}\int d^3x\Big[\hat{\phi}\nabla^2\hat{\phi} + \hat{\Pi}_\phi\nabla^2\hat{\Pi}_\phi\Big] \tag{70}$$

In Schrodinger representation, one lets $\hat{\phi}\,|\phi\rangle = \phi(\vec{x})\,|\phi\rangle$, which is analogous to the position basis in usual quantum mechanics. In this basis, the conjugate momentum operator takes the form of $-i\frac{\delta}{\delta\phi}$. Then the evolution of the wave functional $\langle\phi|\Psi\rangle = \Psi[\phi(\vec{x}),t]$ is governed by the Schrodinger equation, in this case, a functional differential equation.

$$i\frac{\partial\Psi[\phi(\vec{x}),t]}{\partial t} = \frac{-1}{4m}\int d^3x\Big[\phi(\vec{x})\nabla^2\phi(\vec{x}) + (-i\frac{\delta}{\delta\phi(x)})\nabla^2(-i\frac{\delta}{\delta\phi(x)})\Big]\Psi[\phi(\vec{x}),t] \tag{71}$$

It can be rewritten as

$$i\frac{\partial\Psi[\phi(\vec{x}),t]}{\partial t} = \frac{-1}{2}\int\int d^3xd^3y\Big[\frac{\delta}{\delta\phi(\vec{x})}h(\vec{x},\vec{y})\frac{\delta}{\delta\phi(\vec{x})} - \phi(\vec{x})h(x,\vec{\vec{y}})\phi(\vec{y})\Big]\Psi[\phi(\vec{x}),t] \tag{72}$$

where $h(x,\vec{\vec{y}}) = \frac{-1}{2m}\nabla^2\delta(\vec{x}-\vec{y})$. In order to construct a Bohmian type field theory, one has to postulate a addition beable and its equation of motion. Therefore, one should first look for an equation analogous to equation (21) to get guiding equation of field configuration. The continuity equation associated to equation (72) can be derived in a similar way as described in section (3) for the continuity equation for the Schrodinger equation (17) in standard quantum mechanics. Since $\vec{x}$ and $\vec{y}$ are just the continuous versions of the discrete indices in classical mechanics. Therefore, one can go to discrete case to calculate the discrete version. Then replacing the sum by an integral and discrete indices by the continuous ones $\vec{x}$ and $\vec{y}$. One then can verify that the continuity equation is

$$\frac{\partial|\Psi[\phi(\vec{x}),t]|^2}{\partial t} + \int d^3x\frac{\delta j[\phi(\vec{x})]}{\delta\phi(\vec{x})} = 0 \tag{73}$$

where $j[\phi(\vec{x})] = \frac{-i}{2}\int d^3y h(\vec{x},\vec{y})(\Psi^*\frac{\delta\Psi}{\delta\phi(\vec{y})} - \Psi\frac{\delta\Psi^*}{\delta\phi(\vec{y})})$. Let $\Psi[\phi(\vec{x})] = |\Psi[\phi]|e^{iS[\phi]}$ and substitute it into the expression of j,

$$j[\phi(\vec{x})] = \int d^3y h(\vec{x},\vec{y})|\Psi|^2 \frac{\delta S[\phi]}{\delta(y)} \tag{74}$$

By comparing this the expression of Bohmian velocity (18). One then defines the following

$$\dot{\phi}(\vec{x}) = \frac{j[\phi(\vec{x})]}{|\Psi|^2} = \int d^3y h(\vec{x},\vec{y}) \frac{\delta S[\phi]}{\delta(y)} \tag{75}$$

Unlike the quantization of the Schrodinger field in standard quantum field theory, one has to single out the physical degree of freedom of the field in a Bohmian type field theory because the addition variable (beable) is related to the ontology of the theory which should give an image of the physical world. Therefore, if a unphysical field beable is treated equally as physical ones, it is unclear what physical entity this field corresponds to.

7.2 Electromagnetic Field

There are two ways to construct a Bohmian type theory for the electromagnetic field. The first was done by Bohm in his 1952 paper [7], another was done by Valentini in [41]. In this section, we follow a similar approach by Bohm.

Consider the following Lagrangian,

$$L = \int d^3x \mathcal{L} = \int d^3x - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \tag{76}$$

and the conjugate momentum field is

$$\Pi^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_0 A_\mu)} = \partial^\mu A^0 - \partial^0 A^\mu = F^{\mu 0} \tag{77}$$

By substituting $\mathcal{L}$ into Euler-Lagrange equation, one gets the equation of motion

$$\partial_\mu F^{\mu\nu} = 0 \tag{78}$$

this implies two constraints

$$\begin{aligned} \Pi^0 &= 0 \\ \partial_i \Pi^i &= 0 \end{aligned} \tag{79}$$

These two constraints implies there are two gauge variables and one of them is A_0. To identify, one can first preform a canonical transformation then express the transverse and longitudinal part in terms of them. Hence, showing that the second constraint implies the longitudinal part is a gauge variable. However, the same result can be obtained by simply imposing Coulomb gauge $\partial_i A_i = 0$ and $A_0 = 0$. By introducing the projection operator

$$P_{ij} = \frac{\partial_i \partial_j}{\nabla^2} \tag{80}$$

then the longitudinal part is

$$A_i^L(\vec{x}) = P_{ij} A_j(\vec{x}) = \frac{\partial_i \partial_j}{\nabla^2} A_j(\vec{x}) \tag{81}$$

by the Coulomb gauge condition $A_i^L(\vec{x}) = 0$. Hence,the gauge is completely fixed. And the physical variable is the transverse part

$$A_i^T(\vec{x}) = (\delta_{ij} - P_{ij}) A_j(\vec{x}) \tag{82}$$

and $A_i(\vec{x}) = A_i^L(\vec{x}) + A_i^T(\vec{x})$. By equation (76) and (77), the Hamiltonian in original canonical pair A_i and Π_i is

$$
\begin{aligned}
H &= \int d^3x \Pi^\mu \dot{A}_\mu - \mathcal{L} \\
H &= \int d^3x F^{\mu 0}(F_{0\mu} + \partial_\mu A_0) + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\
H &= \int d^3x E^2 + E^i \partial_i A_0 + \frac{1}{2}(B^2 - E^2) \\
H &= \int d^3x \frac{1}{2} \Pi_i \Pi_i + \frac{1}{4} F_{ij} F^{ij}
\end{aligned}
\tag{83}
$$

On the second line we used $\partial_o A_\mu = F_{0\mu} + \partial_\mu A_0$. From the second last to the last line, the term $E^i \partial_i A_0$ disappears because of integration by parts and the use of constraints. Then, E and B is expressed in terms of the conjugate momentum field Π_i and F_{ij}. This can be easily verified by writing F in its matrix form. Expanding the second term gives

$$
\begin{aligned}
\frac{1}{4}(\partial_i A_j - \partial_j A_i)^2 &= \frac{1}{2}(\partial_i A_j \partial_i A_j - \partial_i A_j \partial_j A_i) \\
&= -\frac{1}{2}(A_i \partial_j \partial_j A_i)
\end{aligned}
\tag{84}
$$

since partial derivatives commute, then by integration by parts, the gauge condition and the assumption that A_i goes to 0 at spatial infinity, $\partial_i A_j \partial_j A_i$ becomes 0. Then by $A_i(\vec{x}) = A_i^L(\vec{x}) + A_i^T(\vec{x})$ and the result of the gauge condition $A_i^L(\vec{x}) = 0$ the only physical field variables are $A_i^T(\vec{x})$, the physical Hamiltonian is

$$
H = \int d^3x \frac{1}{2} \Pi_i^T \Pi_i^T - \frac{1}{2}(A_i \nabla^2 A_i) \tag{85}
$$

where Π_i^T is the conjugate momentum field of A_i^T. In canonical quantization, the field A_i^T and its conjugate momentum Π_i^T become operators $\hat{A}_i^T$ and $\hat{\Pi}_i^T$ which take the following form in Schrodinger representation

$$
\begin{aligned}
\hat{A}_i^T(\vec{x}) &\longrightarrow A_i^T(\vec{x}) \\
\hat{\Pi}_i^T &\longrightarrow -i \frac{\delta}{\delta A_i^T(\vec{x})}
\end{aligned}
\tag{86}
$$

Therefore, the physical Hamiltonian in Schrodinger representation is

$$H = \frac{1}{2}\int d^3x\Big(- \frac{\delta^2}{\delta A_i^T(\vec{x})\delta A_i^T(\vec{x})} - (A_i^T\nabla^2 A_i^T)\Big) \tag{87}$$

Hence, the wave functional will $\Psi[A_i^T(\vec{x}), t]$ evolves according to the following Schrodinger equation

$$i\frac{\partial\Psi}{\partial t} = \frac{1}{2}\int d^3x\Big(- \frac{\delta^2}{\delta A_i^T(\vec{x})\delta A_i^T(\vec{x})} - (A_i^T\nabla^2 A_i^T)\Big)\Psi[A_i^T(\vec{x}), t] \tag{88}$$

The continuity equation can be found in a similar as in section (3).

$$\frac{\partial|\Psi|^2}{\partial t} + \int d^3x\frac{\delta}{\delta A_i^T}J[A_i^T] \tag{89}$$

where $J = \frac{i}{2}[\Psi^*\frac{\delta\Psi}{\delta A_i^T} - \Psi\frac{\delta\Psi^*}{\delta A_i^T}]$. Then by $\Psi = |\Psi|e^{iS}$ where $S = S[A_i^T, t]$, then $J = |\Psi|^2[\frac{\delta S}{\delta A_i^T}]$. Hence, postulating that A_i^T as beable whose evolution is determined by

$$\dot{A}_i^T(\vec{x}) = \frac{\delta S[A_i^T]}{\delta A_i^T(\vec{x})} \tag{90}$$

7.3 Problems of field ontology

There are a few problems with using field configuration as beable. The first is that different macrostates do not seem to be supported by some non-overlapping wave functionals. In usual Bohmian mechanics, we assume that a pointer pointing a two different directions corresponds to two actual configurations which are supported by two non-overlapping wave functions ψ_1 and ψ_2. Mathematically,

$$|\psi|^2 = |\psi_1|^2 + |\psi_2|^2 \tag{91}$$

where ψ is the total wave function. We use this assumption in section (4) in which they are wave packets. Therefore, in the field ontology case, one should expect that distinct mar-

costates or field configurations should be described by some non-overlapping wave functionals. Consider the one particle state of the Schrodinger field, whose functional form is

$$\Psi_1[\phi(\vec{x}),t] = \sqrt{2}\int d^3x\psi(\vec{x},t)\phi(\vec{x})\Psi_0[\phi(\vec{x}),t] \tag{92}$$

where $\Psi_0[\phi(\vec{x}),t]$ is the wave functional of the vacuum state, $\psi(\vec{x},t)$ is normalized solution of the Schrodinger equation. $\Psi_0[\phi(\vec{x}),t]$ can be obtained by first realising that $\hat{\psi} = b\int d^3k e^{-i\vec{k}\cdot\vec{x}}\hat{a}(\vec{k})$, where $\hat{a}(\vec{k})$ is the annihilation operator and b just a constant, is the field operator of the usual quantized version of Schrodinger field and can be written as $\frac{1}{\sqrt{2}}(\phi + \frac{\delta}{\delta\phi})$ in Schrodinger representation. Then by the condition $\hat{a}(\vec{k})\left|\Psi_0\right\rangle = 0$, one can get the functional form of the vacuum state. After that, the one particle state is obtained by the action of the conjugate field operator $\hat{\psi}^*$ on the vacuum state. Then consider the absolute value squared of $\Psi_1[\phi(\vec{x}),t]$,

$$|\Psi_1[\phi(\vec{x}),t]|^2 = 2\left|\int d^3x\psi(\vec{x})\phi(\vec{x})\right|^2 |\Psi_0[\phi(\vec{x}),t]|^2 \tag{93}$$

whose maximum can be found by requiring the functional derivative equal to zero. Let $\alpha[\phi(\vec{x})] = \int d^3x\psi(\vec{x})\phi(\vec{x})$,

$$\begin{aligned}\frac{\delta|\Psi_1[\phi(\vec{x}),t]|^2}{\delta\phi} &= 2\frac{\delta}{\delta\phi}\Big[|\alpha|^2|\Psi_0[\phi(\vec{x}),t]|^2\Big] \\ &= 2\Big[\frac{\delta|\alpha|^2}{\delta\phi}|\Psi_0|^2 + |\alpha|^2\frac{\delta|\Psi_0|^2}{\delta\phi}\Big]\end{aligned} \tag{94}$$

then use the product rule for $\frac{\delta|\alpha|^2}{\delta\phi}$ and $\frac{\delta|\Psi_0|^2}{\delta\phi}$ and the following

$$\begin{aligned}\frac{\alpha}{\delta\phi} &= \psi \\ \frac{\alpha^*}{\delta\phi} &= \psi^*\end{aligned} \tag{95}$$

$$\begin{aligned}\frac{\Psi_0}{\delta\phi} &= -\phi\Psi_0 \\ \frac{\Psi_0^*}{\delta\phi} &= -\phi\Psi_0\end{aligned} \tag{96}$$

then maximum condition, one finds that the maximum is at

$$\phi(\vec{x}) = \frac{1}{2}\Big(\frac{\psi^*}{\alpha^*} + \frac{\psi}{\alpha}\Big) \tag{97}$$

Consider a linear combination of one particle states $\Psi[\phi(\vec{x}),t] = N(\Psi_1^{(1)}[\phi(\vec{x}),t]+\Psi_1^{(2)}[\phi(\vec{x}),t])$. If $\Psi_1^{(1)}[\phi(\vec{x}),t]$ and $\Psi_1^{(2)}[\phi(\vec{x}),t]$ are non-overlapping wave functionals, then we would expect the analogue of equation (91),

$$|\Psi[\phi(\vec{x}),t]|^2 = N^2(\Big|\Psi_1^{(1)}[\phi(\vec{x}),t]\Big|^2 + \Big|\Psi_1^{(2)}[\phi(\vec{x}),t]\Big|^2) \tag{98}$$

which has to two maxima one at $\frac{1}{2}\Big(\frac{\psi^*_{(1)}}{\alpha^*_{(1)}} + \frac{\psi_{(1)}}{\alpha_{(1)}}\Big)$ and another at $\frac{1}{2}\Big(\frac{\psi^*_{(2)}}{\alpha^*_{(2)}} + \frac{\psi_{(2)}}{\alpha_{(2)}}\Big)$. However, the maximum of $|\Psi[\phi(\vec{x}),t]|^2$ is $\frac{1}{2}\Big(\frac{(\psi_{(1)}+\psi_{(2)})^*}{(\alpha_{(1)}+\alpha_{(2)})^*} + \frac{(\psi_{(1)}+\psi_{(2)})}{(\alpha_{(1)}+\alpha_{(2)})}\Big)$ Therefore, this indicates that $\Psi_1^{(1)}[\phi(\vec{x}),t]$ and $\Psi_1^{(2)}[\phi(\vec{x}),t]$ are overlapping. A detailed discussion and more examples are given in [38].

Another problem is related to regularization and renormalization. Even though equation (73) and (89) ensure the conservation of the quantity $|\Psi|^2$, the probability of the actual field configuration be $\phi(\vec{x})$ is $|\Psi[\phi(\vec{x}),t]|^2\Pi\mathcal{D}\phi$, which the analogue of the quantum equilibrium hypothesis in the usual Bohmian mechanics, is mathematically ill-defined as one is dealing with system with an infinity number of degree of freedom. Therefore, regularization and renormalization are needed. (See [38] for a detailed discussion) In addition, by Bell's theorem non-locality is unavoidable, thus incompatibility of Lorentz invariant remains as one of problems just like usual Bohmian mechanics. More examples can be found in [36] and [37].

7.4 Bell-type quantum field theories

Bell-type quantum field theories refer to the extension of Bohmian mechanics that incorporates creation and annihilation of particles. Unlike the two previous Bohmian type field theory, the beable is not field, but particle's worldline, an example is shown in figure (4) at the end of this section. This extension of Bohmian mechanics was first proposed by Bell [2] on a lattice and later extended and explored in [13], [9] and many other in the continuum limit. In this section, we will follow mainly [14].

In usual Bohmian mechanics presented in section (3), the number of particle in the system of interest (N) is fixed and thus the corresponding configuration space is of finite dimension, 3N. In order to incorporate particle creation and annihilation, a countably infinite dimensional configuration space is introduced

$$\mathcal{Q} = \bigcup_{i=0}^{\infty} \mathcal{Q}^{[i]} \tag{99}$$

where $\bigcup$ is the disjoint union, $\mathcal{Q}^{[i]}$ is the configuration space representing i number of particles. For example, $\mathcal{Q}^{[1]} = \mathbb{R}^3$. The evolution of the system is then represented by some curves in the configuration space $\mathcal{Q}$. If there is no creation or annihilation event, then the curve will be continuous and lives in $\mathcal{Q}^{[i]}$ and evolves according to the guiding equation. If there is a creation event, then the trajectory will have a discontinuous jump from the sub-configuration space $\mathcal{Q}^{[i]}$ to $\mathcal{Q}^{[i+1]}$ and if it is an annihilation event, then it jumps from $\mathcal{Q}^{[i]}$ to $\mathcal{Q}^{[i-1]}$. Just like usual Bohmian mechanics, the state of system is given by the wave function and its configuration (ψ, Q). The wave function evolves according to the Schrodinger equation with the Hamiltonian $H = H_{free} + H_{int}$. The dynamics of the configuration Q is determined by a more general guiding equation

$$\frac{dQ(t)}{dt} = Re\Big[\frac{\psi^*(Q(t))(\hat{\dot{q}}\psi)(Q(t))}{\psi^*(Q(t))\psi^(Q(t))}\Big] \tag{100}$$

where $\dot{\hat{q}} = \frac{i}{\hbar}[H_{free}, \hat{q}]$.

Unlike the usual Bohmian mechanics, there is an additional equation, which determines the probability of jump per time,

$$\sigma(dq|q^{'}) = \frac{2}{\hbar}\frac{[\mathrm{Im}\,\psi^*(q)\,\langle q|\,H_{int}\,|q^{'}\rangle\,\psi(q^{'})]^+}{\psi^*(q^{'})\psi(q^{'})} \tag{101}$$

where $x^+ = max(x, 0)$ is the positive part of $x \in \mathbb{R}$. These equations together defines a stochastic process called the Markov process (see [14] for further analysis). Therefore, this extension of Bohmian mechanics is no longer deterministic as the ususl Bohmian mechanics since the jump is described probabilistically. It is shown in [14] that one can associate a Bell-type QFT for most of the Hamiltonian. For example, for

$$H = -\hbar^2\frac{\partial^2}{\partial x^2} + V(x) \tag{102}$$

then the associated theory is the usual Bohmian mechanics. More examples of the Bell-type QFT is given in [15].

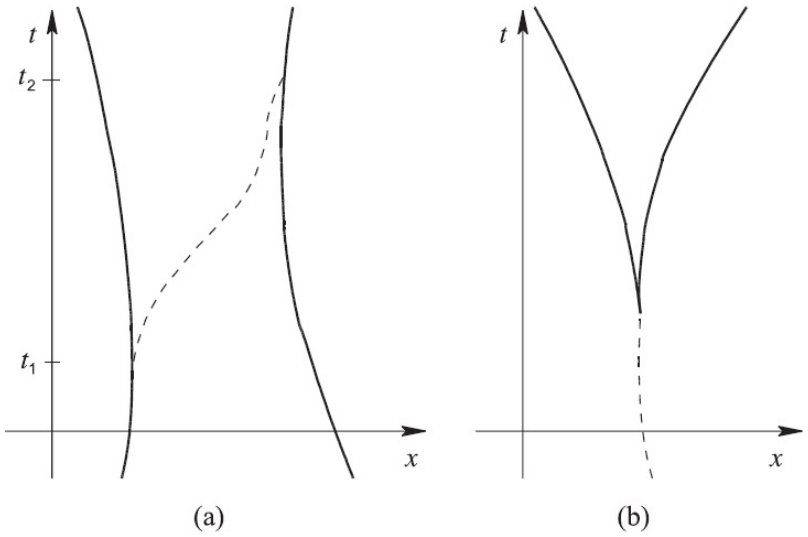

Figure 4: In figure (a), solid lines and the dashed line are the worldlines of electrons and the worldine of a photon respectively. The photon is emitted by an electron and absorbed by another electron. In figure (b), solid lines and the dashed line are the worldlines of electron, positron and the worldine of a photon respectively. The photon is annihilated and an electron-positron pair is created. Figure from [14]

8 Conclusion

Bohmian mechanics as one of the alternatives of standard quantum mechanics not only gives the same predictions for every expreiment as the Copenhagen interpretation, but also provides us a clear ontology and image of our world at least in non-relativistic regime. One of the main purpose of Bohmian Mechanics is to provide is realistic theory while avoiding conceptual problems. In turn, the cost is mathematical complexity. For example, in Bohmian mechanics, observers is no longer something special. The measurement and collapse axiom, that one has to accept without strong logical reason backing up in the Copenhagen interpretation of quantum mechanics are usually simply a consequence of Bohmian mechanics. Therefore, perhaps one should not regard Bohmian mechanics merely as an alternative to the orthodox quantum mechanics, but a more general theory that contains quantum mechanics.

Despite of its conceptional advantage, namely particles guided by wave, Bohmian mechanics remains widely unaccepted by the majority of the scientific community (see [34]). From the analysis of Bohmian mechanics and its extensions in this dissertation, I believe that the two main reasons of this mass unacceptance of the theory are

- The explicit non-locality in the guiding equation which seems to contradict the spirit of special relativity. Even though proposals have made, they usually involve some addition structures like a foliation in order to define simultaneity. Therefore, one might argue that the theory is not fundamentally Lorentz invariant.

- The lack of consistent choice of beable and mathematics rigour in its extension. Bosonic particles are better described by field ontology while Fermionic particles seems to be better described by particle ontology.

To overcome these problems more theoretical development of the theory are needed. Perhaps the pilot wave picture should be taught in a standard quantum mechanics class in universities in order to stimulate student to do more research related to the foundation of quantum mechanics.

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