Deformed Special Relativity: A possible modification of Relativity Theory close to the Planck scale

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1 Introduction

Deformed Special Relativity (DSR) is a proposal of how Einstein's theory of Relativity might experience changes when energies close to the Planck scale are considered. DSR therefore claims to be a theory which is valid in the semi-classical regime, which lies between a full theory of Quantum Gravity and the general relativistic regime. The idea of DSR has been around for almost exactly ten years now and was originally proposed by G. Amelino-Camelia in [1, 2]. Since then it has attracted quite some attention, not least because some of the qualitative predictions of DSR could be tested experimentally in the near future.

In this report we try to motivate the need for a DSR theory and introduce its basic concepts and notions. In particular we will introduce an invariant energy scale which is similar to the invariant speed of light in Sepcial Relativity (SR). This concept forms the heart of DSR, and as we will see, it has far-reaching consequences on the general form of the algebra of symmetry generators of the theory and the dispersion relation of particles. Based on recent papers we will also consider the problem of non-locality, which seems to be an inherent feature of some formulations of DSR. This discussion will also provide a direct link to experiments, which will turn out to place strong constraints on some of the theoretical considerations made.

In this report I hope to provide a general introduction to DSR theory, rather than a detailed discussion of only one particular aspect. Having said that, I have made a selection of certain topics which I found most interesting and I have invariably left out other aspects which would also deserve a detailed discussion. In writing this report I have drawn on various sources (see references), all of which use different notations, conventions and physical units. I have tried to use a common notation throughout this report and in particular for the commutation relations of the symmetry generators in sections 2.3 and 2.4. The units I am using are such that $c = 1 = \hbar$, however I maintain the right to use c's and $\hbar$'s explicitly, where it aids the understanding.[1] The reader should be warned that here and there factors of $i = \sqrt{-1}$ or constants of proportionality might be missing, which however should not cause problems for understanding the general concepts.

[1] In these units we have that $\ell_p = 1/E_p$ and $E_p = M_p$, where ℓ_p, E_p and M_p will be defined in the next section.

1.1 The need to change physics at the Planck scale

Many physicists today would agree that one of the most important unsolved problems in physics is to find a theory of Quantum Gravity (QG), i.e a theory which successfully describes the physics in certain regimes where both Quantum Mechanics (QM) and General Relativity (GR) have to be taken into account. Such regimes include, for example, the physics inside a black hole, and the physics close to the initial singularity of the universe. Several proposals for such a theory have been made in the last few decades, most notably String Theory and Loop Quantum Gravity (LQG). Both of these approaches are based on radically different ideas of what the fundamental physics is going to look like. However, they also share the common problem that so far no experiment has been carried out to rule in favour of either of the two theories or even to rule both of them out. This is partly due to a lack of testable predictions made by these theories and, partly, because present experiments are not achieving high enough energies to allow us to probe the physcially interesting regime. It is widely agreed, that the boundary of the regime at which a theory of Quantum Gravity will become the dominating theory is given by the Planck scale. As was already noted by Max Planck, when he introduced the constant which bears his name, the fundamental constants of physics, G (Newton's constant), $\hbar$ (Planck's constant) and c (speed of light), can be combined in such a way as to give constants of dimensions of length, time, energy and mass.[2] These four constants are the Planck length $\ell_p = \sqrt{\hbar G/c^3}$, Planck time $T_p = \sqrt{\hbar G/c^5}$, Planck energy $E_p = \sqrt{\hbar c^5/G}$ and the Planck mass $M_p = \sqrt{\hbar c/G}$ respectively. Disregarding what the correct theory QG might turn out to be, it is generally assumed that as one makes the transition from energies at or above E_p to much lower energies, one should find that in this limit the theory of QG reduces to the well-known theory of SR and GR. Therefore we expect a semiclassical regime which lies between SR/GR and QG, and this is precisely the regime where Deformed Special Relativity claims to be the dominating theory. Since at present we have no clear picture of what the final theory of QG will look like, it seems very reasonable to first concentrate on the semiclassical regime and hope that, with some inspiration from SR and the little we do know about QG, we might be able to contruct a theory which is valid in this regime. In this report we will concentrate on this semiclassical regime, for which DSR is considered the main candidate for the correct theory.

[2] It is of course possible to define other Planck quantities such as area, volume, temperature, etc. ...

1.2 The idea of an invariant energy scale E_p

As we have just seen, the boundary between the classical and quantum gravity regime is set by the Planck scale, or more precisely the Planck energy E_p. However the existence of such a scale immediately confronts one with a fundamental problem. This is summarized in the following question: The energy E_p is measured in the inertial frame of which observer? Clearly different observers could disagree about the exact value of E_p, since each one is measuring the energy in their respective rest frame. The relativity of frames is the central feature of SR, but just as SR seems to cause a problem here, it also gives us a clue about how to solve it. The solution will be to introduce E_p as an invariant scale, such that all obersvers agree about its value, when measuring it in their respective rest frame. This is in total analogy with the invariance of the speed of light c in SR.

The introduction of the invariant scale E_p lies at the heart the DSR proposal, for it has a drastic consequence on how we have to write the transformation laws between the frames of different observers. To phrase things in a slightly different way, we have to re-write the well known Lorentz transformations of SR in such a way that the energy E_p will be the same no matter what transformation we apply. We will explore the nature of the correct set of new transformation rules and other consquences in the following section.

Firstly, however let us point out that there is one significant difference between c and E_p as invariant scales. Namely c is a quantity in position/configuration space, whereas E_p lives in momentum space. This will cause some trouble as we will see later, since the theory of DSR is most naturally formulated in momentum space. We will see that the transition to position space is non-trivial and still open for debate.

2 The mathematical structure of DSR

As we have seen in the previous section, it is necessary to introduce E_p as an invariant energy scale, in order to define a clear border between the classical and the quantum gravity regime upon which all observers agree. If one assumes such an energy scale, this has an effect on the well-know structure of Special Relativity. We will present the changes in this section.

2.1 Is the Poincaré group still useful?

Now that we have given the motivation for introducing the invariant energy scale E_p, we want to ask the question of what effect it has on the mathematical structure of the theory which emerges from this assumption. In particular we want to know if the Poincaré algebra of the symmetry generators in SR is still valid. The answer is that this is not the case, since the Poincaré algebra does not support another observer-independent scale in addition to the speed of light. However there is an algebra called the κ-Poincaré algebra which does indeed have this property, see [3], and allows for a second invariant scale. The κ-Poincaré algebra is a deformation of the usual Poincaré algebra, and the right-hand sides of the commutation relations which make up the algebra contain the parameter κ, which encodes the second invariant scale. It should be said that in the limit of $\kappa \to \infty$ one recovers the usual Poincré algebra.

κ-Poincaré algebras have a very crucial property which distinguishes them from normal Lie algebras. To see this we need to look at things on a deeper level. Namely κ-Poincaré algebras are actually what are called quantum Hopf algebras. For a given Lie algebra there exists a corresponding quantum deformed algebra. For instance the quantum deformed group of the Lie group $SO(3,1)$ is denoted by $SO_q(3,1)$. The q is related to the aforementioned invariant scale κ and an appropriate limit of q will give us back the undeformed Lie group. We will discuss this further in section 4.2.1. For illustrational purposes let us present here the full algebra of $SO_q(3,1)$, which for instance can be found in [4]:

$$\begin{aligned}
[M_{2,3}, M_{1,3}] &= \frac{1}{z}\sinh\left(zM_{1,2}\right)\cosh\left(zM_{0.3}\right) \\
[M_{2,3}, M_{1,2}] &= M_{1,3} \\
[M_{2,3}, M_{0,3}] &= M_{0,2} \\
[M_{2,3}, M_{0,2}] &= \frac{1}{z}\sinh\left(zM_{0,3}\right)\cosh\left(zM_{1,2}\right) \\
[M_{1,3}, M_{1,2}] &= -M_{2,3} \\
[M_{1,3}, M_{0,3}] &= M_{0,1} \\
[M_{1,3}, M_{0.1}] &= \frac{1}{z}\sinh\left(zM_{0,3}\right)\cosh\left(zM_{1,2}\right) \\
[M_{1,2}, M_{0,2}] &= -M_{0,1} \\
[M_{1,2}, M_{0,1}] &= M_{0,2} \\
[M_{0,3}, M_{0,2}] &= M_{2,3} \\
[M_{0,3}, M_{0,1}] &= M_{1,3} \\
[M_{0,2}, M_{0,1}] &= \frac{1}{z}\sinh\left(zM_{1,2}\right)\cosh\left(zM_{0.3}\right),
\end{aligned} \tag{1}$$

where $z = \ln(q)$ and the $M_{\mu,\nu}$ are of course the generators of the algebra. A crucial point for the discussion that will follow is that the right-hand sides of the commutation relations for the generators of the quantum deformed algebra are not linear functions of the generators, but can be some analytic function of them. This can be seen in the above algebra where we have the analytic functions of hyperbolic sine and cosine on the right-hand sides. As an immediate consequence of this we can now use any analytic function of the generators to define a new basis for the quantum deformed algebra. This stands in contrast to the Lie algebra case, where only bases which are linear combinations of generators are allowed, see [4]. This freedom in the choice of a basis has led to several different proposals of bases for the κ-Poincaré algebra. The various can be found, explicitly written out, in [5]. All of these bases are equivalent however, and it is therefore more or less a matter of taste which of these bases one uses. Up to now there are mainly three bases which have been proposed in the literature and a . They are the bi-cross-product basis, the Magueijo-Smolin basis and the classical basis. Since the focus of the present article is not on the exact details of these bases, we are going to present only one of these bases explicitly here (see sections 2.3 and 2.4), namely the Magueijo-Smolin basis, and state its relation to the bi-cross-product basis. There is however a common feature which all of the bases that we have mentioned share. This is the fact that in all of these bases a choice has been made, such that the Lorentz subalgebra of the κ-Poincaré algebra is left unchanged. We will point this out once more further down in the text.[3] However, the commutators of the κ-Poincaré algebra, which involve the boosts and the momentum generators, differ from the the standard Poincaré algebra. One can also says that the Poincaré algebra is deformed in the Poincaré sector. This is also the origin of the word *Deformed* in the name Deformed Special Relativity. Subsequently people have suggested that the D in DSR could also stand for *Doubly*, with reference to the two observer independent quantities in DSR. The reason for leaving the Lorentz sector of the κ-Poincaré algebra unmodified and only modifiying the remaining commutators, is that if the Lorentz sector was modified as well, then upon integration, one would not obtain a group but a quasigroup, see [6]. It should be said that although mathematically we have the freedom to choose an arbitrary basis for the κ-Poincaré algebra, one might expect that there should ultimately be a physical argument which rules in favour of one particular basis and against any other basis.

We will now proceed with developing the mathematical structure of one possible formulation of DSR theory. This is the formulation developed by Magueijo and Smolin. First we have to state a set of principles from which we can develop the theory.

2.2 The principles of Relativity re-written

In order to derive the transformation rules of DSR it is necessary to re-write the principles on which SR is based and to add two additional principles. The first two principles are familiar from SR, and the origin of the last two should be clear from the discussion contained in sections 1.1 and 1.2. In our discussion we will follow [7]. The principles of DSR read:

1. *Relativity of inertial frames*: Observers in free, inertial motion are equivalent. It follows that there is no preferred state of motion and velocity is a purely relative quantity.

2. *Equivalence principle*: In a gravitational field, freely falling observers are inertial observers and are all equaivalent to each other.

3. *Observer independence of* E_p: All observers agree on the Planck energy as invariant energy scale and therefore also agree on its exact value.

4. *Correspondence principle*: At energies much smaller than E_p, conventional SR and GR are valid.

[3] There is also a basis, called the standard basis, in which the Lorentz sector is not of the form of the common Lorentz algebra.

2.3 Non-linear nature of transformation laws

Let us now derive the algebra of symmetry generators for the theory of DSR, by starting with the Lorentz sector. From principle 1 we see that a transformation group which relates measurements made by different observers must exist. According to principle 4, this transformation group must reduce to the usual Lorentz group of SR at energies much smaller than E_p. We expect that just like the Lorentz group in SR, the transformation group we are looking for also has six parameters, namely 3 rotations and 3 boosts. However the only group with this structure is the Lorentz group itself. This leaves us with the obvious problem that the Lorentz group does not satisfy principle 3, the invariance of E_p. The solution to this problem is to assume that our symmetry group is indeed the Lorentz group, but that it acts non-linearly on momentum space. So we must write down an explicit form for the action on momentum space, which is energy dependent and which leaves E_p invariant. It turns out that this can be achieved by adding a dilatation term to each boost generator. Let us emphazise that this is specific for the DSR formulation developed in [7].

First note that we are working in momentum and not configuration space. Momentum space $\mathcal{M}$ is the space of momentum four-vectors p_α. Greek letters take values $0, 1, 2, 3$ and Roman letters take values $1, 2, 3$. We use the metric signature $(+, -, -, -)$. The standard Lorentz generators are given by:

$$L_{\alpha\beta} = p_\alpha \frac{\partial}{\partial p^\beta} - p_\beta \frac{\partial}{\partial p^\alpha}.$$

As outlined above our strategy is to leave the rotation generators J^i unchanged, but to add an additional term to the generators of the boosts K^i. The additional term contains the dilatation generator $D = p_\alpha \frac{\partial}{\partial p^\alpha}$ and its action on momentum space is then obviously $D \circ p_\alpha = p_\alpha$. We define the modified boost generators as:

$$K^i := L^i_0 + \ell_p p^i D.$$

Note that L^i_0 is given by the above expression for the Lorentz generators and that the Planck length ℓ_p is the inverse of E_p. Since we leave the rotation generators unchanged, they simply follow their usual definition:

$$J^i := \epsilon^{ijk} L_{ij}.$$

It is important to note that despite the modification made, the above defined generators still satisfy the standard Lie algebra for the Lorentz group, namely:

$$\left[J^i, K^j\right] = i\epsilon^{ijk}K_k,\ \left[K^i, K^j\right] = -i\epsilon^{ijk}J_k,\ \left[J^i, J^j\right] = i\epsilon^{ijk}J_k.$$

Now that we have set up the algebra of the generators, our next task is to exponentiate the boost generators to find the corresponding action of the group, i.e the transformation laws of the boosts. Our hope might be that we end up with transformation laws which at least vaguely resemble the transformation laws of SR. To do this we first note that we can generate the modified, non-linear boost generators K^i, by using an energy dependent transformation $U(p_0) = exp(\ell_p p_0 D)$:

$$K^i = U^{-1}(p_0)L_0^i U(p_0).$$

Exponentiation then yields the sought-after representation of the Lorentz group:

$$R\left[\omega_{\alpha\beta}\right] = U^{-1}(p_0)e^{\omega^{\alpha\beta}L_{\alpha\beta}}U(p_0).$$

Working from this, we find that the boost transformations are now given by:

$$\begin{aligned}
p_0' &= \frac{\gamma(p_0 - vp_z)}{1 + \ell_p(\gamma-1)p_0 - \ell_p\gamma v p_z} \\
p_z' &= \frac{\gamma(p_z - vp_0)}{1 + \ell_p(\gamma-1)p_0 - \ell_p\gamma v p_z} \\
p_x' &= \frac{p_x}{1 + \ell_p(\gamma-1)p_0 - \ell_p\gamma v p_z} \\
p_y' &= \frac{p_y}{1 + \ell_p(\gamma-1)p_0 - \ell_p\gamma v p_z},
\end{aligned}$$

where γ is the usual Lorentz factor. Note that in the limit of small $|p_\alpha|$, these transformations reduce to the usual SR momentum space transformations (see [8], p. 29). This set of energy-dependent transformations looks similar to the standard SR transformations and forms in itself a neat and compact result, which should be encouraging.

In order to write down transformation laws which leave E_p invariant, we had to pay a heavy price. That is, that momentum space $\mathcal{M}$ now has a constant curvature and therefore has a deSitter or anti-deSitter geometry depending on which sign the curvature takes, see [4]. This has the profound consequence that translations on momentum space do not commute any longer.

To describe this situation, one could say that we have successfully avoided having to intoduce a preferred frame close to the Planck length, by instead had to introduce an observer-indepent energy scale. The price that we have to pay is that now our transformation laws have become non-linear.

2.4 Modification of the Poincaré-Algebra

Now that we have set up the Lorentz sector of our new algebra, the natural question to ask is: what do the commutation relations of the boost and rotation generators with the momentum generator look like? Or in other words, how does the rest of the Poincaré algebra change under the modifications made to the boost generators?

Without giving the full details here, we present the answer which was given in [7] and is summarized in [5]. Here P_α are the momentum generators and K_α are the boost generators as above:

$$\left[K^i, P^j\right] = i(\delta^{ij} P^0 - \frac{1}{\kappa} P^i P^j), \ \left[K^i, P^0\right] = i(1 - \frac{P^0}{\kappa}) P^i.$$

The remaining commutators which we have not written out explicitly are trivial. This is the algebra written in the Magueijo-Smolin basis and its Casimir is given by:

$$M^2 = \frac{P_0^2 - \vec{P}^{\,2}}{(1 - \frac{P_0}{\kappa})^2},$$

where M is the physical mass. As we have mentioned above another possible basis is the bi-cross-product basis. The relations between these two bases are simply given by:

$$p^i = P^i$$

$$p^0 = -\frac{\kappa}{2} \log(1 - \frac{2P^0}{\kappa} + \frac{\vec{P}^{\,2}}{\kappa^2}), \ P^0 = \frac{\kappa}{2}(1 - e^{-2p^0/\kappa} + \frac{\vec{p}^{\,2}}{\kappa^2}).$$

Here p_α are the momentum generators of the bi-cross-product basis. We then find the Casimir of the bi-cross-product basis to be:

$$m^2 = \left(2\kappa \sinh\left(\frac{p_0}{2\kappa}\right)\right)^2 - \vec{p}^2 e^{p_0/\kappa},$$

where m is a parameter which is not the physical mass.[4] We will discuss the significance of the Casimirs further down in section 3.1.

2.5 Configuration space

In our presentation of the structure of DSR theory we have so far only mentioned momentum space and completely ignored what is happening in configuration space. The reason for this is that it is hard to obtain a configuration space formulation of DSR and to the present day it is not known how to achieve, or, more importantly, how to interpret, such a formulation coherently. To make the transition to the configuration space, one needs to make use of another feature of quantum Hopf algebras which is totally unkown in the standard theory of Lie algebras. This is that every quantum Hopf algebra carries an additional structure known as the co-algebra. The co-algebra is dual to the algebra which we have derived above. The role which this co-algebra plays for the construction of DSR theory is crucial, because it allows us to construct the configuration space formulation of the theory. If the notion of a co-algebra was not included in the quantum Hopf algebra so naturally, we would not be able to derive any results for configuration space, which would clearly cast doubt on the usefulness of DSR as a physical theory. Without going through the whole calculation, see [5] for details, we will present the configuration space commutation relations which one finds, when using the Magueijo-Smolin basis:

$$\left[X^0, X^i\right] = -\frac{i}{\kappa} X^i,\ \left[X^i, X^j\right] = 0,$$

$$\left[P^0, X^i\right] = -\frac{i}{\kappa} P^i,\ \left[P^0, X^0\right] = -(1 - \frac{2P^0}{\kappa}),\ \left[P^i, X^j\right] = -i\delta^{ij},\ \left[P^i, X^0\right] = -\frac{i}{\kappa} P^i.$$

In a similar way, one can obtain the spacetime commutation relations for the bi-cross-product basis. See also [9], which provides a different way of writing a position space formulation of DSR.

One notices immediately from the first of the commutation relations, that we have introduced an non-commutative spacetime, which clearly is a very profound thing to happen. This also happens when we

[4]The physical mass is given by, $m_{phys}^2 = \frac{\kappa^2}{4}\left(1 - \left(-\frac{m}{2\kappa} + \sqrt{\frac{m^2}{4\kappa^2} + 1}\right)^4\right)^2$.

derive the configuration space commutation relations from the bi-cross-product basis. The interpretation of non-commutative geometry is still very much open for debate and we will not comment on it any further.

3 The physics of DSR

Now that we have outlined the general mathematical structure of DSR, we want to consider what kind of physics might possibly emerge from the theory. It should be made clear from the beginning that since there is no agreed formulation of DSR theory at the moment, we can also only speculate about the physics and have to restrict ourselves to rather general and qualitative remarks. We are going to look at two different aspects of the physics, which are modifications of the dispersion relation and the possibility of having an energy-dependent metric. The former is an immediate mathematical consequence of the particular form of our symmetry algebra and is therefore an integral part of the theory. In contrast to that the latter is less well founded and should be regarded more as a bold, but intriguing, guess rather than a mathematical necessity.

3.1 Modified dispersion relations

At the beginning of this article we have motivated and introduced the idea of an observer-independent energy or length scale. We have then looked at the fundamental principles and the transformation laws of SR and we have discussed how these need to be modified as a result of introducing an additional observer-independent scale. We will now consider modifications of the standard relativistic dispersion relation which is given by:

$$E^2 = m^2c^4 + p^2c^2.$$

In the light of our earlier discussion it is natural to expect that in the context of DSR this dispersion relation should somehow also include the invariant energy or length scale. The kind of modification which is suggested is of the following form:

$$E^2 - p^2c^2 - m^2c^4 = f\left(\frac{E}{E_p}\right),$$

where f is some function in powers of E/E_p, of order greater than 2. Modified dispersion relations of this type were first suggested in [1]. In fact, if we are given the κ-Poincaré algebra written in a particular basis, we may derive the precise form of the modified dispersion relation. Namely the dispersion relation is nothing but the Casimir of the algebra, and we have already listed the Casimirs for various bases in section 2.4. It should be emphasized again, that this means that any modified dispersion relation in DSR is observer-independent, i.e invariant under the non-linear transformations in momentum space. There are many different proposals for the precise form of f (just as many as there are proposals for possible bases of the κ-Poincaré algebra) and should the DSR idea turn out to be correct, f will have to be fixed by experiment. To make things a little more concrete, let us give the explicit form of the modified dispersion relation used in [10]:

$$E^2 - p^2c^2 - m^2c^4 \simeq \pm\frac{2}{n+1}E^2\left(\frac{E}{E_p}\right)^n. \tag{2}$$

On the right-hand side, we have introdced a modification, which is only given up to leading order, as indicated by the use of $\simeq$. The parameter n is the power of the leading correction. We would like to point out that possible modifications of dispersion relation for high energy photons, such as the ones produced in gamma ray bursts, can be tested in experiments which are already running or will become feasible in the near future. So DSR theory provides one of the first testable predictions made by any theory of the semiclassical regime, let alone the pure quantum gravity regime.

As a consequence of modifying the dispersion relation, one finds that now there is the possibility of having an energy/frequency dependent speed of light. Assuming that the relation $v = \partial E/\partial p$ for the speed of light still holds in the semiclassical regime, we find, see [10], for the explicit form of the dispersion relation given in equation (2) an energy dependence of the speed of light v which is given by:

$$v\left(E\right) = 1 \pm \left(\frac{E}{E_p}\right)^n.$$

It should be clear that in the limit of $E \to 0$, the speed v approaches the speed of light c. Notice that the modification of the dispersion relation and the frequency dependent speed of light are similar to the case of solids and the propagation of light in a medium. Having given this derivation of an energy-dependent speed of light, it should be said that recently evidence has been given that an energy-dependent speed of light might lead to conceptual problems of the theory, which cannot be easily overcome. We will discuss this in detail in section (5), but let us already make a brief comment here. Namely the main lesson to learn is that one has to be careful in making predictions about what is happening in configuration space, simply based on what we know from momentum space. More precisely, what we have done is that we have taken the modified dispersion relation, which is clearly written in terms of the momentum space variables, and by applying $v = \partial E/\partial p$, we have tried to derive v which is a position space quantity. This means that because an energy-dependent speed of light leads to conceptual problems, one might suspect that the transition between momentum and configuration space is not correctly done by simply using the formula $v = \partial E/\partial p$ and that it has to be replaced by some other procedure, which might or might not yield to a speed of light which is energy-dependent.

3.2 The metric and spacetime geometry in the DSR framework

In the previous sections we have tried to outline the DSR formalism and make some of the concepts commonly used in the literature more precise. It should be clear however, that there is not only one way of doing things, but that there are several different formulations of the theory. A key issue is the derivation and interpretation of the spacetime geometry, which derives from the DSR principles such as a non-commutative geometry, which we have discussed above. Since the issue about which spacetime geometry to adopt is far from settled, it is worth considering other possibilites. In particular we shall discuss a proposal which approaches the question from a different angle and provides a fresh view on the subject matter.

The standard question which is often asked in the DSR literature is: What is the geometry of spacetime in the semiclassical regime? This question clearly implies that there is only one geometry which is valid. However as we have seen in the above sections, the main result of introducing an observer-independent energy scale E_p was that the deformed transformation laws and the dispersion relation, picked up an energy dependence. This basic observation should make us wonder, if the geometry of spacetime, might not be energy-dependent itself. This question was asked in [11] and the details of how an energy dependence of the geometry of spacetime could appear are made more precise in that paper. The idea is actually quite simple; however the interpretation of the result is more tricky.

The starting point is to assume a energy-dependent metric $g(E)$, where the meaning of E has to be made precise. We will define E as the energy of a particle in spacetime as seen by someone observing the particle. Since E has kinetic energy contributions, this leads to the consequence that different observers may attribute different energies to one and the same particle. Put in a slightly different, but physically more intuitive, way: E is the energy with which a particle probes spacetime, according to an observer.

Now, going back to section (2.3), we remind ourselves, that the invariant energy scale E_p results in deformations of momentum space, which can be achieved via the action of a map U of momentum space

$\mathcal{M}$ into itself, $U : \mathcal{M} \to \mathcal{M}$. This means we have:

$$U \circ (E, p_i) = (U_0, U_i) = \left(f\left(\frac{E}{E_p}\right) E, h\left(\frac{E}{E_p}\right) p_i \right).$$

We can use the functions f and h to write down energy-dependent orthonormal frame fields:

$$e_0 = f^{-1}\left(E/E_p\right) \tilde{e}_0,\ e_i = h^{-1}\left(E/E_p\right) \tilde{e}_i,$$

and of course the metric written in terms of these is:

$$g\left(E\right) = \eta^{ab} e_a \otimes e_b.$$

$\tilde{e}_\mu$ are the frame fields, which describe the spacetime geometry at very low energies. It then goes without saying that in the limit of $E \to 0$, we recover the classical spacetime geometry which we deal with in GR and which we think as being energy-independent:

$$\lim_{E \to 0} g_{\mu\nu}\left(E\right) = g_{\mu\nu}^{classical}.$$

We have ended up with a one-parameter family of metrics. Due to its energy dependence this metric as been termed the rainbow metric. It should be noted that the introduction of an energy-dependent metric is a mathematical construct. This does not mean that with an increase in its kinetic energy the gravitational field around that particle gets stronger. The only thing that happens is that an observer sees the particle moving in a metric which depends on the energy of that particular particle.

The discussion of the rainbow metric has so far only dealt with a single particle. The situation becomes more complicated when one considers what happens when there are different particles at the same point in spacetime, but moving with different speeds and therefore possesing different energies according to an observer. The observer will associate different metrics with each of the particles, and the natural question arises of what overall metric the observer will see. The result should be a superposition of all the different metrics, but it is unclear how exactly the metrics superimpose.

4 DSR as a semiclassical limit of Quantum Gravity

In this section we will investigate in what regime DSR theory is the physically relevant theory and in particular we will try to determine more precisely how it could emerge as the semiclassical limit of a theory of quantum gravity. This will naturally lead us to a further study of quantum deformed Hopf algebras.

4.1 What does "taking the limit" mean?

As we have already mentioned a couple of times in this report, in the context of different physical regimes, DSR theory should be seen as the semiclassical limit, lying in the regime between SR and a theory of Quantum Gravity. We have also said that in taking an appropriate limit, it should be possible to recover DSR from the Quantum Gravity theory, whatever it may be, and we now want to define precisely how exactly such a limiting process has to take place.

Let us pause for a moment and think about what exactly we are trying to achieve. We are trying to go from a theory of QG which clearly takes into account all possible physics, i.e relativity, gravitational interactions and quantum mechanics, to SR which only takes into account relativistic physics. So in this transistion process we have to switch off gravitational interactions as well as quantum mechanical behavior. This can be done by letting the coupling constants G and $\hbar$ go to zero, i.e $G, \hbar \to 0$. However, we have to be careful about how we actually take such a limit, since as we have said earlier on, the quantity which determines whether quantum gravitational effects are important is the Planck energy, $E_p = \sqrt{\frac{G}{\hbar}}$.[5] We then have to take the limit in such a way, that it is a smooth limit and that E_p remains finite. So in short this is the limiting process which will take us from QG to DSR. Since we feel that this limiting process is an important issue, which is often ignored in the DSR literature, we want to elaborate on it further and in particular follow C. Rovelli's paper [12] where he gives a detailed analysis of the limiting process leading to the DSR regime, and which makes the above rather general argument more precise.

We start by noticing that any physical system can be characterised by three quantities, namely the length l, time t and the mass m. From these we can define a velocity $v = l/t$, an action $s = ml^2/t$ and $x = l^3/(mt^2)$ (which has the same units as Newton's constant G) that characterize the physical system. The magnitude of these three characteristic quantities with respect to c, $\hbar$ and G respectively determines

[5] We have set $c = 1$ here.

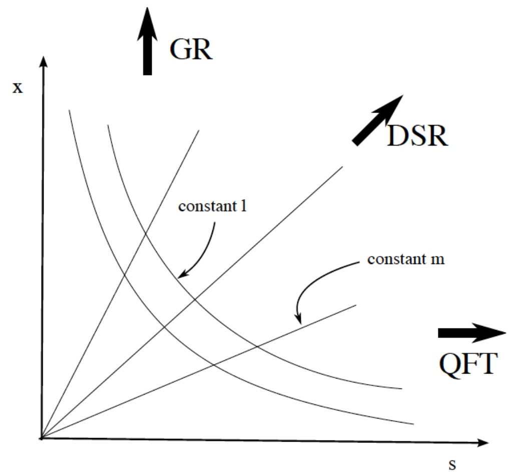


Figure 1: Illustration of the different physical regimes. The figure is taken from Rovelli's paper [12].

the physics which is relevant for describing the physical system under consideration. So for example we have: $v \ll c$ implies that relativistic effects can be neglected, $s \gg \hbar$ means that quantum effects are not relevant and $x \gg G$ results in gravitational effects becoming negligible. v, s and x characterize a 3 dimensional space of different physical regimes. Let us now constrain ourselves to a regime where relativistic effects have to be taken into account, i.e when v is comparable to c. This means that now we are effectively in a 2 dimensional space characterised by s and x only. We can also identify units of length and time, which leaves us with $s = ml$ and $x = l/m$. What we are interested in now is to investigate the different regimes which arise as we make l larger and larger. Therefore we consider two cases:

1. $s = const.$: for l getting larger this implies that m is getting smaller and this results in x getting larger. As a result gravity can be neglected and QFT (remember we are in the relativistc limit) is the dominating theory.

2. $x = const.$: for l getting larger this implies that m is also getting larger and therefore s also gets very large. Therefore we need to take gravity into account, but quantum effects can be neglected. So this is the GR regime.

An interesting question to ask now is: what happens when we take the large l-limit, with m being held constant? Clearly both s and x become large and so at some point both gravitational as well as quantum mechanical effects can be neglected. However, m is finite and arbitrary and could be of the order of the Planck mass M_p. We can thus get arbitrarily high densities $\rho = m/l^3$. The region of the described limit is precisely the semiclassical regime of DSR and the fact that it is a high density regime has led Rovelli to suggest that DSR should stand for DenSity Relativistic regime. For an illustration of the different physical regimes see figure (1).

So in summary, DSR is a regime where s and x are both very large, but m can be of the Planck mass order. This explains in detail the limiting process of taking $G, \hbar \to 0$ but at the same time leaving M_p constant.

4.2 Obtaining the κ-Poincaré algebra as a limit

4.2.1 The $2+1D$ case

We will now consider a Quantum Gravity theory in 2+1 dimensions with a positive cosmological constant. It is well known that the excitations of the ground state in such a theory transform under the representations of the algebra of the quantum deformed group $SO_q(3,1)$. The algebra of this quantum group has already been given explicitly in section (2.1) and we will not repeat it here. For mathematical convenience we will as before introduce a relation between z and q, namely $z = \ln q$. It should be noted that for $z = 0$ ($q = 1$), we recover the classical $SO(3,1)$ group. It has been shown that the canonical commutation relations of the Chern-Simons theory of $2+1D$ Quantum Gravity determine a relation between the z and the cosmological constant Λ and it is found that $z \propto \sqrt{\Lambda}$. We fix the constant of proportionality to be the Planck length ℓ_p, so we have $z = \ell_p\sqrt{\Lambda}$. We are now interested in a low-energy limit, which recovers flat spacetime. This limit will be $\Lambda\ell_p^2 \to 0$, i.e $\Lambda \to 0$. However, we know that z and Λ are related to each other and we therefore have to be careful when taking the limit. More precisely, we have to take the limit in such a way that κ, which is given by:

$$\kappa^{-1} \equiv \frac{z}{\sqrt{\Lambda}} = \ell_p,$$

stays fixed when taking the limit. The process of obtaining the κ-Poincaré algebra from the algebra of $SO_q(3,1)$ is called contraction and we will now show how it works in detail. We start with the algebra given in equation (1) and make the following identifications:

$$\begin{aligned} P_0 &= \sqrt{\Lambda}\hbar M_{0,3} \\ P_i &= \sqrt{\Lambda}\hbar M_{0,i} \\ J &= M_{1,2} \\ K_i &= M_{i,3}, \end{aligned}$$

where P_0 is the energy, P_i are the momentum generators, J is the generator of rotation and K_i are boost generators. Note that we are in $2+1$ dimensions and so $i = 1, 2$. Now we make these substitutions in the all the commutators of equation (1). For example for the first commutator,

$$[M_{2,3}, M_{1,3}] = \frac{1}{z} \sinh\left(zM_{1,2}\right) \cosh\left(zM_{0.3}\right)$$

we find

$$[K_2, K_1] = \frac{\kappa}{\hbar\sqrt{\Lambda}} \sinh\left(\frac{\hbar\sqrt{\Lambda}}{\kappa} J\right) \cosh\left(\frac{P_0}{\kappa}\right),$$

and we get similar expressions for all the other commutators. Now we apply the limiting procedure, $\Lambda \to 0$ while at the same time keeping κ constant, just as we have described above. This then leaves us with the following algebra:

$$\begin{aligned}
[K_i, K_j] &= -J\epsilon_{ij} \cosh\left(\frac{P_0}{\kappa}\right) \\
[J, K_i] &= \epsilon_{ij} K^j \\
[K_i, P_0] &= P_i \\
[K_i, P_j] &= \delta_{ij} \kappa \sinh\left(\frac{P_0}{\kappa}\right) \\
[J, P_i] &= \epsilon_{ij} P^j \\
[P_0, P_i] &= 0 \\
[P_2, P_1] &= 0.
\end{aligned}$$

This algebra is a κ-Poincaré algebra in $2+1$ dimensions written in the standard basis. The method of contration is described in [6, 13]. The Lorentz sector of this algebra written in the standard basis is deformed and at first sight this casts doubt on the usefulness of this algebra. However as discussed in section (2.1), we are at liberty to transform this algebra into a different one by an analytic transformation. This then allows us to retrieve an algebra with an undeformed Lorentz sector, such as the ones written in the bi-cross-product or Magueijo-Smolin basis in sections (2.3) and (2.4).

4.2.2 The $3+1D$ case

We will now consider the case of quantum gravity with a positive cosmological constant in 3+1 dimensions which of course is the most interesting case. For reference see [6]. It has been established that the symmetry group of $3+1D$ quantum gravity is the quantum group $SO_q(4,1)$ with $z = \ln q$ as before. Just like in the $2+1D$ case, it is found that for small Λ the relation $z = \Lambda \ell_p^2$ holds. Note that this is different to the relation for the $2+1D$ case and we now have a linear dependence between z and Λ. To find the κ-Poincaré algebra we contract the algebra of $SO_q(4,1)$, which means that we take the limit $\Lambda \to 0$ in such a way that the ratio:

$$\frac{z}{\Lambda} = \ell_p^2,$$

stays fixed. To carry out the contraction, one identifies the generators $M_{\mu\nu}$ of the algebra of $SO_q(4,1)$ with the generators of boost, rotations and energy and momentum. This is rather straight-forward and proceeds in complete analogy to what was done for the $2+1D$ case.[6] However there is a complication in the $3+1D$ case which is not obvious. Namely one has to renormalise the energy and the momentum as one takes the contraction limit. Without this renormalisation, the energy and momentum relation with the $M_{\mu\nu}$ generators would just look like:

$$P_\mu = \sqrt{\Lambda} M_{0,\mu}.$$

But now with the renormalisation, we need to change this relation to:

$$P_{\mu,ren} = \left(\frac{\sqrt{\Lambda}\ell_p}{\alpha} \right)^r \sqrt{\Lambda} M_{0,\mu},$$

where the label $_{ren}$ indicates that we are now dealing with a renormalised quantity, r is a parameter which governs the renormalisation and α is a constant. One can now make the subsitutions in all the commutators of the algebra of $SO_q(4,1)$ and take the $\Lambda \to 0$ limit. We find that the result of the contraction depends on the precise value of the parameter r. For $r > 1$ the contraction is singular and for $r < 1$, the contraction yields the usual undeformed Poincaré algebra which we consider to be trivial. Only for $r = 1$, does one obtain, upon defining $\kappa = (\alpha \ell_p)^{-1}$, the κ-Poincaré algebra which is what we were originally trying to achieve in the first place. This is a curious result and it is yet to be understood what kind of mechansim chooses the particular value of $r = 1$. As in the $2+1D$ case, the algebra obtained in this contraction has a modified Lorentz sector and one again needs to apply a change of basis to fix this.

[6] In the 3+1D case there will however we more than one rotation generator.

5 The non-locality problem

Our discussion so far has concentrated on the formal development of DSR theory and we have not spoken about any concrete, physically oberservable consequences that the theory of DSR might have. However if we are hoping to ever bring theory and experiment into contact then this is precisely what we need to do. In particular it has become obvious in a series of very recent papers that there is an issue with non-locality which arises as a possible prediction of DSR theory and which is already well within the experimental regime. This has cast heavy doubt on the validity of some aspects of DSR theory and we will now try to outline the argument and to provide some analysis of it.

5.1 The paradox

The issue of non-locality has been around in the DSR literature for quite some time and started out as a paradox known as the box-problem. In recent papers, this issue has become somewhat a central focus, not least due to a very clear description and analysis of the problem given by S. Hossenfelder in [14] and subsequently in [15]. We will now give a qualitative description of the problem following this paper. In (paper) an experiment is imagined which leads to effects of non-locality of macroscopic magnitude such that they should have been observed a long time ago already. Let us first begin with a precise definition of what we mean by non-locality in the context of DSR theory.

By locality/non-locality we mean whether different observers agree or disagree on whether two events happen at the same point in space time. In other words, if two or more events always happen at exactly the same point in spacetime, independent of the observer, then this is what we mean by a local theory. On the other hand a theory is then non-local, if the events happen at a specific point in spacetime only for one particular observer, but not for others. The experiment we are about to construct will lead to the conclusion that DSR theory is a non-local theory. More precisely, since there is no single formulation of DSR theory that everybody agrees on, one should specify that we are talking about DSR theories, which have an observer-dependent speed of light. To be more concrete, let us introduce a law which gives the phase velocity of a photon in terms of its energy E:

$$c\,(E) \approx \left(1 + \alpha \frac{E}{M_{pl}}\right) + \mathcal{O}\left(\frac{E^2}{M_{pl}^2}\right).$$

In this relation we consider only first order corrections in E/M_{pl} and α is a constant, which we will choose to be negative, so that the velocity decreases with increasing energy. The crucial point about this relation is that its functional form does not depend on the observer! This means that the relation is invariant under transformations, whereas the energy E is not. Let us now proceed with giving a description of the experiment.

We consider a gamma ray burst, which emits two different types of photons. One is a very high energy photon of energy $E_\gamma \approx 10GeV \approx 10^{-18}M_{pl}$ and the other is a very low energy photon which we will use as a reference. Both photons are detected using an earth-bound detector. The distance between the gamma ray burst and the earth is set to be $L \approx 4Gpc \approx 10^{26}m$. If we assume that the low-and high energy photons are emitted at the same time, then due to the energy dependence in the expression for the phase velocity, we find that there is a delay ΔT in the arrival of the two photons in the detector. The delay is given by:

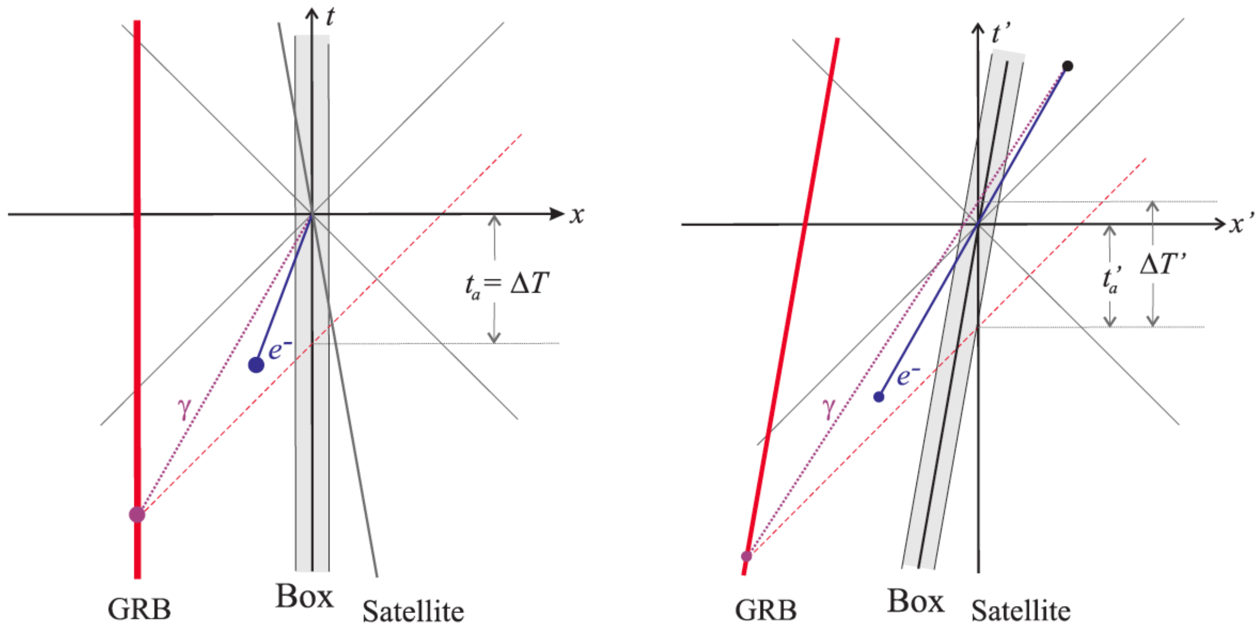
$$\Delta T = L\frac{E_\gamma}{M_{pl}} + \mathcal{O}\left(\frac{E_\gamma^2}{M_{pl}^2}\right).$$

One finds, using the values given above, that this delay is of the order of 1 second. We now add another detail to the experiment. Namely, we also consider an electron with energy $E_{e^-} \approx 10MeV$, which is emitted by a source in the direct vicinity of the detector on the earth. The electron is emitted in such a way that it arrives in the detector at exactly the same time as the high energy photon and therefore also a time difference $-t_{e^-} = \Delta T$ earlier than the low energy photon. We then also imagine that inside the detector the high energy photon and the electron scatter off each other and create some kind of macroscopic change, which is irreversible. The precise details of this event are not of importance here.

The second part of the experiment is to consider what a second observer will see when observing the above setup, in particular with regards to the arrival times of the different photons and the electron in the earth-bound detector. We can imagine such a second observer to be a team of physicists in a satellite and moving towards the gamma ray burst. If we assume that the satellite has a speed of $v_s = -10km$ with respect to the earth-bound detector, then this leaves us with a Lorentz factor of $\gamma_s \approx 1+10^{-9}$. From the rest frame of the satellite, the high energy photon and electron energies appear to be blue-shifted but the very low energy photon can still be seen as having very low energy. This is the reason why we call it a reference photon. The high energy photon's energy is:

$$E_\gamma^{'} = \sqrt{\frac{1-v_s}{1+v_s}}E_\gamma + \mathcal{O}\left(\frac{E_\gamma^2}{M_{pl}^2}\right).$$

Where the prime indicates that we are now in the satellite's rest frame as opposed to the earth-bound



(a) The worldlines of the particles as seen from the Earth's rest frame. The worldlines of the high-energy photon (doted line) and the electron meet inside the box and will trigger an event. The low-energy reference photon is indicated by the dashed line.

(b) The worldlines of the particles as seen from the satellite's rest frame. Note that in this case the worldlines of the high-energy photon and the electron do not meet inside the box.

Figure 2: The setup of the non-locality experiment. Note that in the figure $t_a = t_{e^-}$ and $t'_a = t'_{e^-}$. Both figures are taken from [14].

detector's rest frame for the unprimed quantities. Equally, upon transforming the difference in arrival time of the low energy photon and the electron t_{e^-} into the primed frame, we find:

$$t^{'}_{e^-} = \frac{L}{\gamma_s}\frac{1/c\,(E_\gamma) - 1}{1 - v_s}.$$

Since the high energy photon has a different energy in the primed frame, this means that we also have to calculate a new speed of light for this photon as seen in the satellite rest frame. Here the functional invariance of the expression $c\,(E)$ comes into play and we have:

$$c\left(E^{'}_\gamma\right) = 1 - \frac{E^{'}_\gamma}{M_{pl}} = 1 - \sqrt{\frac{1 - v_s}{1 + v_s}}E_\gamma + \mathcal{O}\left(\frac{E^2_\gamma}{M^2_{pl}}\right).$$

The distance travelled by the photons becomes $L^{'} = \gamma_s\,(v_s/c\,(E_\gamma) - 1)\,L$. Combining everything we find that the ΔT in the primed frame is:

$$\Delta T' = \frac{E'_\gamma}{m_{pl}} L' = \frac{1 - v_s}{1 + v_s} \Delta T + \mathcal{O}\left(\frac{E_\gamma^2}{M_{pl}^2}\right).$$

We see that whereas in the unprimed frame we had $-t_{e^-} = \Delta T$, we now find that in the primed frame we have:

$$\Delta T' - t'_{e^-} = \left(\frac{1 - v_s}{1 + v_s} - \frac{1}{\gamma_s (1 - v_s)}\right) \Delta T + \mathcal{O}\left(\frac{E_\gamma^2}{M_{pl}^2}\right).$$

Inserting our values for γ_s and v_s, this leaves us with $\Delta T' - t'_{e^-} \approx 10^{-5} \Delta T$ or in other words, the photon is lagging roughly one kilometer behind the electron. So we have found that in the satellite frame, the high energy photon and the electron do not arrive in the detector at the same time, as they did in the detector's rest frame. As a result they will not scatter off each other and will not cause a macroscopic event. This is how non-locality arises in DSR theory, if we assume an observer-indepent, energy-dependent phase velocity. The fact that we have not observed any such non-local effects allows us to make the conclusion that we can rule out an energy dependence of the form given above up to a very high precision. More precisely, since the centre of mass energy of the high energy photon and the electron is roughly $\approx 15MeV$, the scattering process probes spacetime distances of the order of $\approx 10fm$. If the difference in the distance between the arrival of the electron and the high energy photon in the detector is less than this distance probed by the scattering process, then non-locality is not an issue. So if $|\Delta T' - t'_{e^-}| < 10fm$ is fulfilled then the theory is still local. For boosts up to $\gamma_s = 30$, this allows us to find a bound on ΔT, namely $\Delta T < 10^{-23}s$ or alternatively we can also find $|\alpha| < 10^{-23}$. The last inequality rules out modifications in the phase velocity to first order in E/M_{pl}. This is the central conclusion drawn in paper [14].

5.2 Is there a way out of the dilemma?

Now that we have derived such strong restrictions on the existence of an energy-dependent speed of light, we want to see if there are ways out of this situation. First of all we want to consider if the quantum mechanical uncertainties Δt and Δx could be modified in such a way in DSR, that they hide any non-local behaviour. In other words, could it be that by some mechanism in DSR, the uncertainty Δt of the high energy photon gets so large that it is not possible to tell if the photon and the electron are at the same point in spacetime or not? In our discussion we will again follow [14].

So let us assume that there is a modification of the uncertainties, which is caused by the dispersion of the photon's wave-packet. If the wave-packet's width is initially σ_0, then we find that its spread with a modified dispersion relation is given by:

$$\sigma(t) = \sigma_0 \sqrt{1 + \left(\frac{2t}{M_{pl}\sigma_0^2}\right)^2}.$$

To fully hide any non-local effects, we need to satisfy the inequality, $|\Delta T' - t'_{e^-}| \ll |\Delta t'|$. This inequality needs to be satisfied in all rest frames, i.e for all possible observers. By working out the *algebraic details*, one finds that it is indeed not possible to satisfy this inequality.[7] This immediately leaves one with the conclusion, that non-local effects will not be completely hidden by quantum mechanical uncertainties and will therefore still be observable.

There is another possibility which could prevent non-locality from appearing in the above experiment. The point is that in our analysis we have used a standard (unmodified) Lorentz transformation to find the distance L' in the primed frame, i.e the rest frame of the satellite. As we have already seen in section (2.5) there is currently no agreement about the correct formulation of DSR theory in position space. Therefore there is the possibility that the Lorentz transformations in position space could also become energy-dependent. One could imagine that the transformation of L could depend on the energy just in such a way that ΔT now transforms *properly*.[8] Here *properly* means that ΔT transforms in such a way that the two particles, the high energy photon and the electron, also meet in the satellite frame and not only in the earth's rest frame. This would then make any non-local effect disappear and an energy-dependent speed of light would be possible again. When considering energy-dependent position space transformations, one should also think about the role that an energy-dependent metric might have in the resolution of this problem.

Before we close this section let us once again draw attention to the assumptions we have made in the derivation of non-local effects. The single most important one is that we assume that there is an energy-dependent but observer-independent speed of light in position space. One should bear in mind that this is indeed an assumption and has not been derived in any rigorous way. Indeed, since the formulation of DSR theory starts out in momentum space and since there is no reliable method for finding its position space formulation, assuming an energy-dependent speed of light is not really built on a solid foundation.

[7] By algebraic details we mean, considering the rest frames of all possible observers. For details see [14].

[8] Recall that it was the unusual transformation behaviour of ΔT which caused all the trouble.

The only reason why one might believe that the speed of light does, in fact depend on the energy, is because of the modified dispersion relation, which is an inherent feature of the theory of DSR.

6 Some final remarks

Now that we have outlined the general features of DSR and discussed some of its aspects in more detail, we want to take a step back and make a few final remarks which will help us understand the bigger picture.

What DSR does is almost analgous to what Einstein's theory of relativity does. What Einstein did was to take the Euclidean group of the dynamics in a Euclidean space and extend it to the Poincaré group, by adding boost generators, which allows for more general dynamics which take place on a Riemannian manifold. The Euclidean group is a subgroup of the Poincaré group. The idea behind DSR is to take the Poincaré group and yet again realise that it is part of a larger structure, namely a quantum group whose algebra is the κ-Poincaré algebra. In going from the Poincaré algebra to the κ-Poincaré algebra, we are not adding any additional generators and hence we do not extend the dynamics as is done in Einstein's relativity. The only thing we have done is to modify the generators and thereby obtain a more general algebra. It has been claimed, see [16], that the Riemannian geometry used in GR is then replaced by the more general Finslerian geometry. Then the action of the kinematical group preserves not a Minkowski line element, but a Finslerian line element which depends on both the variables of the tangent and the co-tangent space.

It is also interesting to note that in the regime of DSR, spacetime seems to reveal different physical properties, which are not significant when we are considering energies much lower than the Planck scale. Let us explain what we mean by *physical properties*. Firstly, the change in the dispersion relation, from the usual relativistic dispersion relation to a relation whose precise form depends on the energy. Secondly the possibility of having an energy-dependent speed of light and maybe even an energy-dependent metric. Even though a one-to-one analogy may not exist, such physical properties of spacetime might remind us of the properties that a crystal or any other solid body has. Just like the properties of a solid body undergo changes as we heat it up and as it undergoes phase transitions, the properties of spacetime change when we probe it at higher energies. If we take this analogy a bit more seriously, then we should consider spacetime to be more like some kind of medium with actual physical properties, rather than something that is physically hardly defined at all. The concept and structure of spacetime as it stands at the moment is not defined satisfactorily but the hope is to develop a theory in which spacetime is an emergent phenomenon. Emergent would mean that spacetime and its properties follow from a theory which does not make use of any kind of background (metric) on which it is formulated. Whether this is possible at all is not clear, but so far no theory has been formulated which could claim to be a complete theory.[9] If we try to describe spacetime as some kind of medium with physical properties, then this is in a sense similar to the idea of the 19th century Aether. It is of course well known that the problem of the classical Aether is that it introduces a preferred frame which was ruled out by the Michelson-Morely experiment to very high precision. If one assumes the existence of an Aether on the basis that spacetime should be a physical medium, i.e have physical properties, then there must be a mechansim, which prevents the introduction of a prefered frame. Possibly such a mechanism could be of quantum mechanical nature as was first suggested by P. Dirac in [17], with reference to his earlier paper [18]. We will not go into any more detail here, but we want to make the point that maybe some of the classical arguments, which led to the Aether being abandoned as an idea, need to be re-examined taking quantum mechanical arguments into account.

Deformed Special Relativity seems to provide a new and exciting way of thinking about the physics at energies close to the Planck scale. The elegance of DSR lies in the fact that it is based on a single and physically reasonable requirement, namely that all observers agree upon the exact value of the length scale below which Quantum Gravity becomes the dominating theory. This requirement does not lack in beauty much beside Einstein's Principle of Relativity and the invariance of the speed of light.

7 Acknowledgement

I would like to thank Professor Magueijo for the supervision of my dissertation project.

[9]Loop Quantum Gravity probably comes closest to a theory of emergent spacetime, although many conceptual and interpretational issues remain unsolved.

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