Supergravity & Generalised Geometry

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24th September 2010

Submitted for partial fulfilment of the degree of Master of Science

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Declaration

I hereby declare that this thesis is entirely my own work, and has not been submitted, either in the same or different form, to this or any other university.

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Contents

Acknowledgements

I would firstly like to offer sincere thanks to Professor Daniel Waldram for his helpful supervision and guidance during this project. The whole Imperial College London theoretical physics department naturally deserves some recognition as well; for the stimulating and friendly environment in which I have enjoyed studying during this year.

A big thanks is also due to my flatmates for the last year; Brian, Toby and Dangerous Dave. This year has definitely been an enjoyable one, which is in no small way thanks to you guys. Finally I must of course express thanks to my family, and in particular my parents, without whom I wouldn't be here.

Abstract

We present generalised geometry as a natural mathematical language within which to expressing supergravity theories, by virtue of the ability it provides to composite the metric g and B field into a single object. We motivate this via the roots of supergravity in supersymmetry, before detailing the most tractable case of interest; the NS sector of type II supergravity. The important quantities and field equations are summarised, before we outline the relevant extended objects within the theory; the Dabholkar Harvey fundamental string and the NS 5 brane. The structure of generalised geometry is then investigated, with a view to the most useful facets in this context, before the novel tools provided by generalised geometry are outlined. It is hoped that this work can provide a clear and concise basis for further developments in this area.

Chapter 1

Introduction

Supergravity is, as the name would suggest, a field of research concerning theories of gravity incorporating some degree of supersymmetry; a conjectural symmetry between bosons and fermions. Supersymmetry aims to unify these two families of particles by postulating as of yet unobserved space-time symmetries, achieved in practice by extending the familiar Poincaré group of spacetime symmetries. With this in mind, the question which leads us to supergravity can be phrased in a very natural way; whilst we can arrive at general relativity as the result of gauging the Poincaré group, what results from gauging this new extended group of symmetries? The result is a theory of gravitation incorporating supersymmetry, and as we will see, some very useful properties.

1.1 Historical perspective

Investigations into supergravity first began in the mid 1970's through the work of Freedman, Van Nieuwenhuizen and Ferrara, at around the time that the implications of supersymmetry for quantum field theory were first beginning to be understood [1]. By promoting supersymmetry from a global to a local symmetry, supergravity appeared to offer a way to combine general relativity and the strong, weak and electromagnetic forces, generating considerable excitement. Although originally formulated in four dimensions, a number of subsequent developments established a unique supergravity in eleven dimensions as a potential 'theory of everything'.

- 11 dimensions were shown by Werner Nahm to be the largest number compatible with a single graviton and no particles with spin greater than two [2].
- Edward Witten demonstrated that a minimum of eleven dimensions are required to embed the gauge groups of the standard model in supergravity [3].
- Sherk, Julia and Cremmer succesfully found the 11 dimensional classical supergravity action [4].
- The $N = 8$ theory was found to predict rather than assume the correct charges for fundamental particles, and potentially offered to replicate much of the content of the standard model [5].

Indeed, Stephen Hawking argued in 1980 during his inaugural lecture as the Lucasian Professor of mathematics that a 'final' theory based on eleven dimensional $N = 8$ supergravity was near to completion, suggesting a 50% chance of success by the end of the century [6]. This optimism however proved to be short lived, and in particular it was not long before a number of gauge and gravitational anomalies were discovered; seemingly fatal flaws which would render the theory inconsistent [7].

Perhaps unsurprisingly, interest in supergravity subsequently waned. It was known that some of these difficulties could be circumvented by moving to a ten dimensional supergravity, however this would sacrifice the unique nature of the eleven dimensional theory. Progress in this once-exalted theory faltered for a number of years, until the first of a series of now famous developments began to rekindle interest through advances made in a parallel field; superstring theory.

1.1.1 The first superstring revolution

A series of rapid developments in the period from 1984-1985, the first superstring revolution provided convincing arguments that the five superstring theories, previously considered irrelevant, were promising candidates for unification of the known fundamental forces [8]. This both transformed the fortunes of superstring theory, and brought its low energy limit; supergravity, back to the fore, albeit in ten dimensions. In particular, Michael Greene, John Schwartz and David Gross demonstrated the existence of three (and only three) ten dimensional supergravities in which the previously fatal anomalies cancel; type I, type IIA and type IIB [9][7].

1.1.2 The second superstring revolution

It took until 1994 for the first in a new series of breakthroughs; now known as the second superstring revolution, for supergravity to experience a full resurgence. Spearheaded by the work of Edward Witten, it came to be understood that the five consistent superstring theories could be unified non-perturbatively into a single eleven-dimensional framework; M theory [10]. This is considered by many to be a contender for the much sought after 'final theory of everything', and has eleven dimensional supergravity as its low energy limit. Needless to say, this development has brought supergravity full circle, casting it squarely back into a field of active research.

1.2 Supermotivations

There are a number of compelling reasons to study supergravity, some of which we briefly outline.

1.2.1 Supergravity as an effective field theory

The modern perspective of supergravity is as a low energy effective theory to a more fundamental theory. The best candidate we have at present for this is string theory; seemingly the only viable candidate for a theory of quantum gravity. Supergravity allows us to investigate the low energy dynamics of string theory, where we can simplify by disregarding complications relating to renormalisability. This approach has yielded a number of insights; particularly into the various dualities in string theory, some of which were first understood through supergravity models and solutions. Some of the extended objects in string theory, such as D and p-branes, were also first discovered as solutions to supergravity models [11].

1.2.2 AdS/CFT

Also known as gauge/gravity correspondence, the AdS/CFT correspondence is a conjectured duality between certain string theories on curved spaces and conformal field theories. Pioneered by Maldacena in late 1997 [12], the AdS/CFT correspondence has subsequently engendered one of the biggest revolutions in our understanding of string theory. The correspondence also casts supergravity, as the low energy limit of these string models, into a powerful tool for computations in the strong coupling limit of the dual field theory [11].

1.2.3 Phenomenology

As we would expect from an effective field theory to string theory, supergravity has an important role to play in particle physics phenomenology. Supergravity theories in particular are known to have a number of useful properties in this context, particularly regarding problems induced by supersymmetric extensions of the standard model. These include the removal of the large cosmological constant induced by spontaneous supersymmetry breaking, and an explanation for the lack of experimental evidence for the goldstino particle predicted by supersymmetry;

when supersymmetry is broken in supergravity the goldstino is ‘eaten’ by the gravitino [13].

1.3 Overview

In this thesis we will primarily focus on type II supergravity in the Neveu-Schwarz sector; that is, the bosonic part of type II supergravity. Of course there are two type II supergravities; IIA and IIB, but as they differ only in fermion content and we are concerned solely with bosons, this is inconsequential. In particular we will focus on outlining the mathematics required to recast supergravity in the language provided by generalised geometry; a relatively new topic in differential geometry. As generalised geometry happens to provide a very natural formulation for expressing supergravity theories, it is hoped that this approach may yield new insights.

To this end, we will first look at supersymmetry, before supergravity etc. Subsequently we will introduce other useful stuff. (I’ll set up the rest of this small overview section once the rest of the chapter is finalised). In what follows some familiarity with general relativity is assumed, readers lacking this background may find Wald’s General Relativity a helpful resource in this regard. Throughout this thesis we will also make extensive use of the language of differential forms, for those who are unfamiliar in this regard we can thoroughly recommend M. Nakahara’s ‘Geometry, Topology and Physics’ as a resource.

Chapter 2

Supersymmetry

> *"Discovery of supersymmetry would be more profound than life on Mars"*
> - John H. Schwarz [11]

Supersymmetry, along with superstring theory, has arguably been the two defining features of the high energy physics of the last few decades. An enormous subject in its own right, it is also our starting point for understanding supergravity.

To this end we will first outline the roots of supersymmetry in the Coleman Mandula theorem, before motivating the subject with some modern day theoretical results. We subsequently look at the superalgebra and supermultiplets of N=1 and N=2 supersymmetry, and touch on the $N > 2$ case. Finally, we introduce global supersymmetry transformations, which lead us into the next chapter. For interested readers there are a number excellent review articles which further espouse the concepts developed within this chapter, we can particularly recommend [14] and [15].

2.1 A little background

The roots of the supersymmetry can be traced back to the 1967 paper 'All Possible Symmetries of the S Matrix' by Sidney Coleman and Jeffrey Mandula, which comprises their eponymous theorem [16]. Therein, they provided rigorous proof that under some reasonable assumptions, only certain symmetries of the S matrix, and thus fundamental physics, can exist. The symmetries in question are the familiar discrete C, P and T symmetries, internal global symmetries relating to conserved quantum numbers, and Poincaré invariance; essentially Lorentz invariance combined with spacetime translations.

Supersymmetry can be viewed as an attempt to circumvent this bound. This is achieved in practice by weakening one of these assumptions; namely that the Lie algebra of the S matrix symmetries must consist solely of commuting generators. To extend this algebra we allow also for the possibility of anticommuting generators, which transform in spinor representations of the Lorentz group. As each irreducible representation of this extended algebra must contain several irreducible representations of the Poincaré algebra; each corresponding to single particles, this extension implies an inherent grouping of particles. These groupings are known as supermultiplets and contain particles differing by spin $\frac{1}{2}$, which are related by the action of the anticommuting generators. If supersymmetry is a facet of nature, there must exist a raft of thus far unobserved particles to fully populate these supermultiplets, each a superpartner to a known fundamental particle. The existence of these superpartners has a profound effect on physics, as we will see.

2.2 Motivations

Aside from the aesthetic appeal of a single framework relating fermions and bosons, a number of quantitative results have spurred supersymmetry research [17].

- The possibility for a high energy unification of the strong and electroweak forces.

- A potential candidate for dark matter in superpartners.

- A solution to the hierarchy problem of the standard model.

Needless to say, evidence of supersymmetry is a hotly anticipated development in the future of physics.

It seems natural at this point to consider the possibility of other symmetries potentially arising from the introduction of these anticommuting generators. However the work of Haag, Lopuszański, and Sohnius has demonstrated that supersymmetry is the only additional symmetry of the S matrix permitted by this particular circumvention of the Coleman Mandula theorem [18]. The question remains as to whether further weakening of some the assumptions of this theorem may permit further, useful symmetries, however at present no meaningful examples have been found [19]. This leads us to the not unreasonable assertion that at present, supersymmetry should be considered the only viable extension of the known spacetime symmetries of physics.

2.3 The Poincaré group

Our starting point for the mathematics of supersymmetry is the familiar Poincaré algebra, the Lie algebra of the Poincaré group;

$$
\begin{aligned}
[P_\mu, P_\nu] &= 0 \\
[M_{\mu\nu}, P_\rho] &= i\omega_{\mu\rho}P_\nu - i\omega_{\nu\rho}P_\mu \\
[M_{\mu\nu}, M_{\rho\sigma}] &= i\omega_{\mu\rho}M_{\nu\sigma} - i\omega_{\mu\sigma}M_{\nu\rho} - i\omega_{\nu\rho}M_{\mu\sigma} + i\omega_{\nu\sigma}M_{\mu\rho}
\end{aligned}
$$

where P_μ is the generator of translations, $M_{\mu\nu}$ are the Lorentz generators, and $\omega_{\mu\rho}$ is the Minkowski metric. The Poincaré group is of course the full symmetry group of special relativity, including boosts, translations and rotations.

To include new symmetries we extend the Poincaré group via new anticommuting symmetry generators, and postulate their anticommutation relations. To ensure that these unobserved symmetries do not conflict with experimental results obtained thus far we must assume that supersymmetry is spontaneously broken, allowing the superpartners to be more massive than the energy scales probed thus far. A number of arguments suggest that supersymmetry breaking must be intrinsically related to the electroweak scale; roughly 0.1-1TeV [11]. If this is the case, the superpartners of known elementary particles should lie roughly within this range, and will hopefully be observable in the next few years at the CERN Large Hadron Collider (LHC) and possibly at Fermilab's Tevatron.

2.4 The SuperPoincaré group

The simplest extension of the Poincaré group is through the inclusion of the symmetry generators Q_α and $\bar{Q}_{\dot{\beta}}$. This defines the $N = 1$ supersymmetry algebra, uniquely determined by the relations;

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2\sigma^\mu_{\alpha\dot{\beta}} P_\mu, \tag{2.1}$$

$$[P^\mu, Q_\nu] = 0, \tag{2.2}$$

where $\bar{Q}_{\dot{\alpha}}$ is the hermitian adjoint of Q_α and $\sigma^\mu_{\alpha\dot{\beta}} = (\mathbb{1}, \sigma^i)$, for the Pauli matrices σ^i. It is important to note that as Q_α is related to the generator of spacetime translations P_μ, rather than an internal symmetry such as the $SU(3) \otimes SU(2) \otimes U(1)$ symmetry of the standard model, supersymmetry must also indeed also be a spacetime symmetry.

2.4.1 Supersymmetry representations

To properly investigate supersymmetry representations we consider the action of Q_α and $\bar{Q}_{\dot\beta}$ on a helicity eigenstate $|\lambda_0\rangle$. We find

$$Q|\lambda_0\rangle = |\lambda_0 - \frac{1}{2}\rangle, \text{ and } \bar{Q}|\lambda_0\rangle = |\lambda_0 + \frac{1}{2}\rangle. \tag{2.3}$$

From the anticommutation relations

$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0, \tag{2.4}$$

we can see that

$$Q_\alpha Q_\beta|\lambda_0\rangle = -Q_\beta Q_\alpha|\lambda_0\rangle = 0, \tag{2.5}$$

and the repeated action of Q_α and $\bar{Q}_{\dot\beta}$ can only create a finite number of states. In the $N = 1$ case this action creates multiplets containing only two states; $|\lambda_0\rangle$ and $|\lambda_0 + \frac{1}{2}\rangle$, denoted $(\lambda_0, \lambda_0 + \frac{1}{2})$. As this multiplet is not CPT invariant, we must add the CPT conjugate to ensure that the supermultiplets satisfy CPT symmetry.

2.4.2 $N = 1$ Supermultiplets

As we are only interested in physical states, we also restrict our attention to particles with a maximum spin of 2. The resulting $N = 1$ supermultiplets are

- Chiral; $(0, \frac{1}{2})$ and $(-\frac{1}{2}, 0)$, comprising a complex scalar and a Weyl fermion.
- Vector; $(\frac{1}{2}, 1)$ and $(-1, -\frac{1}{2})$, comprising a gauge boson and a Weyl fermion.
- Gravitino; $(1, \frac{3}{2})$ and $(-\frac{3}{2}, 1)$, comprising a gravitino and a gauge boson.
- Graviton; $(\frac{3}{2}, 2)$ and $(-2, -\frac{3}{2})$, comprising a graviton and a gravitino.

2.5 Extended supersymmetry

It is of course possible to continue this extension of the Poincaré group, postulating N supersymmetry generators Q^A_α $(A = 1...N)$, and the relation

$$\{Q^A_\alpha, \bar{Q}_{\dot{\beta}B}\} = 2(\sigma^\mu)_{\alpha\dot{\beta}}P_\mu\delta^A_B. \tag{2.6}$$

We also define

$$\{Q^A_\alpha, Q^B_\beta\} = \epsilon_{AB}Z^{AB} \tag{2.7}$$

where ϵ_{AB} is the Levi-Civita tensor, for some antisymmetric central charges Z^{AB} which commute with all the generators. The simplest extension from $N = 1$ supersymmetry, and the case of interest in this thesis, is $N = 2$. This holds the promise to unify gravity and electromagnetism, via a supermultiplet containing two gravitinos, the photon and the graviton. Excitement in this model was first triggered by a powerful breakthrough; an explicit photon-photon scattering calculation known to be divergent in the Maxwell-Einstein system yielded finite results due to cancellations from the new gravitino diagrams [5]. This set the scene for the original development of supergravity as a finite theory of quantum gravity.

2.5.1 $N = 2$ Supermultiplets

$N = 2$ supermultiplets take the form of $(\lambda_0, \lambda_0 + \frac{1}{2}, \lambda_0 + \frac{1}{2}, \lambda_0 + 1)$. They are

- Vector; $(0, \frac{1}{2}, \frac{1}{2}, 1)$ and $(-1, -\frac{1}{2}, -\frac{1}{2}, 0)$, comprising a gauge boson, two Weyl fermions and a complex scalar.

- Weyl; $(\frac{1}{2}, 1, 1\frac{3}{2})$ and $(-\frac{3}{2}, -1, -1, -\frac{1}{2})$, comprising a Weyl fermion, two gauge bosons and a gravitino.

- Graviton; $(1, \frac{3}{2}, \frac{3}{2}, 2)$ and $(-2, -\frac{3}{2}, -\frac{3}{2}, -1)$, comprising a gauge boson, two gravitinos and a graviton.

2.6 $N > 2$ Supersymmetries

As an aside, It is of course possible to continue this extension into higher degrees of supersymmetry. There are a number of important facets of these $N > 2$ supersymmetries, some of which we briefly state.

- Since renormalizable theories have $|\lambda| \leq 1$, $N = 4$ is the maximal supersymmetry for renormalizable field theories. This is known as Super Maxwell or Super Yang Mills theory, and is of special interest as it is UV finite, lacking divergences in the quantum theory [11].

- As we consider massless particles with $|\lambda| > 2$ to be unphysical, $N = 8$ is considered to be the maximum number of supersymmetries. For the interested reader there is a detailed discussion of the $|\lambda| > 2$ case in the 'soft photon' section in Volume 1 of Weinberg's The Quantum Theory of Fields, Chapter 13.

2.7 Supersymmetry transformations

It is easy to see from the action of Q_α and $\bar{Q}_{\dot{\beta}}$ in the previous section that supersymmetry transforms bosons into fermions and vice versa. In particular we can consider infinitesimal supersymmetry transformations

$$\delta_1 B \sim \epsilon_1 F \tag{2.8}$$

$$\delta_2 F \sim \bar{\epsilon}_2 \partial B \tag{2.9}$$

for some bosonic and fermionic fields B and F. The anticommuting parameter ϵ must have dimension $[\epsilon] = -\frac{1}{2}$ in mass units, since $[B] = 1$ and $[F] = \frac{3}{2}$. This then implies the presence of the derivative operator in the second transformation, to ensure dimensional consistency. It is the consideration of these variations that leads us from supersymmetry to supergravity, and into the next chapter.

Chapter 3

Supergravity I

> *"Gravity exists, so if there is any truth to supersymmetry then any realistic supersymmetry theory must eventually be enlarged to a supersymmetric theory of matter and gravitation, known as supergravity. Supersymmetry without supergravity is not an option, though it may be a good approximation at energies below the Planck Scale."*
> - Steven Weinberg, The Quantum Theory of Fields, Volume III

Supergravity is the logical conclusion of supersymmetry. As elucidated in previous chapters, it has also been and remains a very important topic in theoretical physics, having a key role to play in our understanding of string theory, M theory, the AdS/CFT correspondence and many other topics of active research [11][20][21]. It is hoped that formulating aspects of supergravity in the natural language provided by generalised geometry, the aim of this thesis, will provide further insight into these areas.

As such, we must first outline the basic features of type II supergravity as they stand today. In this chapter we establish the roots of supergravity via local supersymmetry transformations, before outlining the proceeding to the type II supergravity action. We subsequently elicit the behaviour of the two that couple to the NSNS sector of type II supergravity; the Dabholkar Harvey fundamental string and the NS 5 brane.

There is a large volume of supergravity literature in existence with a number of differing conventions, which can unfortunately render initial efforts into the subject a confusing experience. However there are a number of excellent resources that we can recommend for further study, particularly Bernard de Wit's lecture series and P. Van Nieuwenhuizen's 1980 review article 'Supergravity' [21] [5].

3.1 Local supersymmetry

In everything set out so far, we have implicitly assumed global supersymmetry. We now turn to consider the consequences of promoting supersymmetry from a global to a local symmetry, which as we will see, is the root of supergravity. From the relations

$$\delta_1 B \sim \epsilon_1 F, \tag{3.1}$$

$$\delta_2 F \sim \bar{\epsilon}_2 \partial B, \tag{3.2}$$

we can consider the effect of successive supersymmetry variations;

$$\{\delta_1, \delta_2\} B \sim a^\mu \partial_\mu B; \quad a^\mu = \bar{\epsilon}_2 \gamma^\mu \epsilon_1. \tag{3.3}$$

It is straightforward to see that these successive transformations result in spacetime translations. If we promote supersymmetry a local symmetry; i.e. $\epsilon^\alpha \longrightarrow \epsilon^\alpha(x)$, then we find translations $a^\mu \partial_\mu$ are generated which are necessarily spacetime dependent. These general coordinate transformations are of course the basis of general relativity, which leads us to the surprising statement that local supersymmetry necessarily implies gravity. This conclusion is of course the root of supergravity.

3.2 Vielbeins

As we have now established, the nature of local supersymmetry naturally predicates both gravity and fermions. Any description of supergravity must therefore be able to incorporate spinors in curved spacetime.

The most convenient way to achieve this is via the vielbein, or Cartan, formalism. In the standard formulation of general relativity, the metric $g_{\mu\nu}$ is the fundamental object. From this we can define the Christoffel symbols;

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2} g^{\alpha\beta}(g_{\beta\mu,\nu} + g_{\nu\beta,\mu} - g_{\mu\nu,\beta}), \tag{3.4}$$

and subsequently the Riemann tensor;

$$R^{\rho}{}_{\sigma\mu\nu} = \partial_{\mu}\Gamma^{\rho}_{\nu\sigma} - \partial_{\nu}\Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\sigma\nu} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\sigma\mu}. \tag{3.5}$$

However we can also consider that at each point of our spacetime manifold there exists a tangent space, isomorphic to d-dimensional Minkowski space $\mathbb{M}^{D}$. Due to the absence of curvature, Lorentz transforms are well defined in these tangent spaces. By formulating a gravitational theory in terms of these tangent spaces rather than the spacetime manifold itself, we can circumvent some of the issues created by this curvature. To leverage this we define a local orthonormal basis $e^{\mu}_{a}(x)$ $(a = 0, 1, .., D-1)$ of tangent vectors, such that

$$g_{\mu\nu} = \eta_{ab} e^{a}_{\mu} e^{b}_{\nu}, \tag{3.6}$$

where η_{ab} is a flat Minkowski metric, i.e. $\mathrm{diag}(+1, -1, \ldots, -1)$. These $e^{a}_{\mu}(x)$ are the vielbeins. They transform covariantly under general coordinate transforms and contravariantly under local coordinate transforms, as we would expect from their index structure. Bearing this in mind we can raise and lower via the general metric $g_{\mu\nu}$ and the Minkowski metric η^{ab};

$$e^a_\mu = \eta^{ab} g_{\mu\nu} e^\nu_b. \tag{3.7}$$

To ensure that the metric is invertible, we also define an inverse vielbein $\hat{e}^\mu_a$, such that

$$\hat{e}^\mu_a e^a_\nu = \delta^\mu_\nu. \tag{3.8}$$

We can think of vielbeins as elements parametrising the coset

$$\frac{GL(4)}{SO(3,1)}, \tag{3.9}$$

where $GL(4)$ matrices acting on the curved space indices parametrise general coordinate transformations, whilst the Lorentz group acts on the flat space indices. It is from this that we can see the importance of vielbeins in this context; as $GL(4)$ does not admit a spinorial representation, vielbeins are simply required to couple fermions into gravity.

3.3 Type II supergravity

We now turn out attention to type II supergravity; the 10 dimensional low energy limit to type II string theory. This theory is rooted in $N = 2$ supersymmetry, with 32 supersymmetries generated by two 16 dimensional Marjorana-Weyl fermions. There are two 'flavours' of type II supergravity, distinguished by the chirality of the supersymmetry generators $\epsilon^{1,2}$; in type IIA the generators have opposite chiralities, in type IIB they are the same.

For issues of simplicity, we are primarily concerned with the bosonic content common to both theories; known as the NSNS sector. This terminology originates from the Neveu-Schwarz-Neveu-Schwarz boundary conditions that which source these bosonic fields states in the string theory must obey. To concretely elucidate the properties of the theory, we require an action. Our starting point is therefore the bosonic part of the type II action;

$$S_{NS} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}e^{-2\Phi}(R + 4(\partial\Phi)^2 - \frac{1}{2}|H_3|^2). \tag{3.10}$$

Wherein;

- $g = \sqrt{\det g_{\mu\nu}}$
- the coupling constant κ is given by $2\kappa_{10}^2 = 16\pi g$,
- Φ is the scalar dilaton field,
- R is the Ricci scalar $R = g^{\mu\nu}R_{\mu\nu}$,
- H_3 is the Neveu-Schwarz 3 form field strength, defined as $H_3 = dB_2$, for some Kalb-Ramond field B_2.

It is important to note that this action is valid in the string frame, which differs by a scaling factor from the conventional Einstein frame used in general relativity. More explicitly; $g^E_{\mu\nu} = e^{-2\phi}g_{\mu\nu}$, for the string frame metric $g_{\mu\nu}$ and the Einstein frame metric $g^E_{\mu\nu}$.

It is also illuminating to consider that we can arrive at this action via compactification of the full 11 dimensional supergravity action on a torus, which seems to imply that in the strong coupling limit of type II theory, a new 11th dimension becomes accessible. This quirk of the theory was one of many things which were only fully understood in the light of M theory, and the events of the second superstring revolution.

3.3.1 Field equations

From the Euler-Lagrange equations it is a straightforward task to derive the field equations for this action;

$$R_{MN} - \frac{1}{4} H_{MRS} H_N{}^{RS} + \nabla_M \nabla_N \Phi = 0, \tag{3.11}$$

$$\nabla^2 (e^{2\Phi}) - \frac{1}{6} e^{-2\Phi} H_{MRN} H^{MNR} = 0, \tag{3.12}$$

$$\nabla_M (e^{2\Phi}) H^{MNR} = 0. \tag{3.13}$$

$$\tag{3.14}$$

Additionally, we have the Bianchi identity:

$$dH_3 = 0. \tag{3.15}$$

3.3.2 SUSY variations

As we would expect, the Type II supergravity supermultiplets contain a doublet of dilatinos $\lambda^{1,2}$, and a doublet of gravitinos $\delta\psi^{1,2}$. The supersymmetry variation of these doublets is then;

$$\delta\psi^1 = (D_M \epsilon)^1 = \left(\nabla_M + \frac{1}{4} H_M \right) \epsilon^1 \tag{3.16}$$

$$\delta\psi^2 = (D_M \epsilon)^2 = \left(\nabla_M - \frac{1}{4} H_M \right) \epsilon^2 \tag{3.17}$$

$$\delta\lambda^1 = \left(\partial\Phi + \frac{1}{2} H \right) \epsilon^1 \tag{3.18}$$

$$\delta\lambda^2 = \left(\partial\Phi - \frac{1}{2} H \right) \epsilon^2 \tag{3.19}$$

3.4 Fundamental strings

Having established the fundamentals of the NSNS sector of type II supergravity, our attention now turns to the two basic extended objects present in the theory. The first of these is the Dabholkar Harvey (DH) fundamental string, a singular supergravity solution which couples to the NSNS sector and preserves half of the spacetime supersymmetries [22][23].

The DH fundamental string corresponds to the most basic classical solution of string theory, and has been subsequently extended by Waldram and Sen to allow for the inclusion of momentum and charge flowing along the string [24][25]. Our starting point is the combined action;

$$
\begin{aligned}
S &= S_{NS} + S_{\sigma}, &(3.20)\\
&= S_{NS} - \frac{\mu}{2}\int d^2\sigma(\sqrt{\gamma}\gamma^{MN}\partial_M X^{\mu}\partial_N X^{\nu} g_{\mu\nu}e^{\alpha\Phi} + \epsilon^{MN}\partial_M X^{\mu}\partial_N X^{\nu} B_{\mu\nu}), &(3.21)
\end{aligned}
$$

for a sigma model action S_{σ}. This encodes the coupling of the string to the dilaton, metric and the 3 form field strength, with the parameter $\alpha = \frac{1}{\sqrt{2}}$. From this combined action we find a seemingly intractable system of nonlinear of equations of motion, however there exists a simple ansatz;

$$ds^2 = e^J[-dt^2 + (dx^1)^2] + e^K dx \cdot dx. \tag{3.22}$$

We can solve this for a single scalar function $E(r)$;

$$J = E(r) \quad , K = E(r) \quad \Phi = \alpha E(r) \quad B_{01} = -e^{E(r)}, \tag{3.23}$$

where x^1 is the direction along the string, and $r^2 = x \cdot x = \eta_{ij}x^i x^j (i, j = 2...9)$.

3.5 NS 5 branes

The other primary solution of interest is the NS 5 brane; an extended 5 dimensional object which is the electromagnetic dual of the fundamental string. The solutions arise from considerations related to Dirac's magnetic monopoles; in 3+1 dimensions we have the familiar relations

$$dF = 0, \quad d^*F = j_E, \quad Q_E = \int_{S^2_\infty} {}^*F. \tag{3.24}$$

Dirac conjectured that magnetic monopoles may exist, which would obey dual versions of these relations;

$$dF = j_M, \quad d^*F = 0, \quad Q_M = \int_{S^2_\infty} F. \tag{3.25}$$

We now consider an analogous situation in a D dimensional string theory. Firstly we require the 2 form potential B introduced previously, as we are now coupling to the world sheet of a string rather than the world line of a particle, through $\int_S B$. This generates the 3 form field strength $H = dB$, and the analogous equations become;

$$dH = 0, \quad d^*H = j_E, \quad Q_E = \int_{S^{D-3}_\infty} {}^*H. \tag{3.26}$$

The dual equations are;

$$dH = j_M, \quad d^*H = 0, \quad Q_M = \int_{S^3_\infty} H. \tag{3.27}$$

We are interested in objects with the fixed magnetic charge Q_M; so in $D = 10$ we can look for solutions which are spherically symmetric in the 4 directions bounded by S^3_∞, and independent of the other 5 spatial dimensions plus time. We have the field equation

$$d^*(e^{-2\Phi}H) = 0, \tag{3.28}$$

trivially as a result of spherical symmetry. From a generalisation of Birkhoff's theorem we can say that for a given mass M and charge Q_M there exists a unique solution. If

$$\left(\frac{Q_M}{M}\right) > \left(\frac{Q_M}{M}\right)_C, \tag{3.29}$$

where C indicates some critical value, the solution is a black p-brane with a hidden singularity. As we wish to avoid naked singularities we consider the so called extremal case, where;

$$\left(\frac{Q_M}{M}\right) = \left(\frac{Q_M}{M}\right)_C. \tag{3.30}$$

The solution in this case is supersymmetric and takes the form;

$$g_{MN} = e^{2\Phi}\delta_{MN}, \tag{3.31}$$

$$g_{\mu\nu} = \eta_{\mu\nu}, \tag{3.32}$$

$$H_{MNP} = -\epsilon^{Q}_{MNP}\partial_Q\Phi, \tag{3.33}$$

$$e^{2\Phi} = e^{2\Phi(\infty)} + \frac{Q_M}{2\pi^2 r^2}, \tag{3.34}$$

where the x^μ are tangent to the 5-brane, x^M are transverse, and $r^2 = x^M x^M$. This solution is the NS 5 brane. It carries the magnetic charge for the NS NS 2 form B, and is solitonic in nature; it is a localised classical solution to the supergravity field equations, rather than a D-branes or string.

Having now elucidated the key facets of the NS sector of type II supergravity, we turn our attention to generalised geometry; the succinct mathematical language which seems ideal for expressing these solutions.

Chapter 4

Generalised Geometry

In contrast to supergravity, generalised geometry is a relatively new area of research. It arises from generalised complex geometry, an area of mathematical research concerning geometric structures which generalise and unify the seemingly disparate notions of complex and symplectic geometry. The generalised structures in question were first introduced by Hitchin in the context of his work classifying low dimensional special geometries [26], and further developed by his students Gaultieri and Cavalcanti [27][28]. In generalised geometry we are primarily concerned with a few key aspects of generalised complex geometries; particularly the differential geometry of the generalised tangent spaces.

It is already known that this approach supercedes older concepts in bihermitian geometry [29], allowing a succinct description of the geometries investigated by Gates, Hull and Roček in the context of nonlinear sigma models [30]. By virtue of a natural $O(d,d)$ metric, generalised geometry also particularly offers a very natural framework within which to understand T-duality; a symmetry between quantum field theories central to the development of string theory [31]. The formalism of generalised geometry has also shed new light on other aspects of string theory, particularly regarding mirror symmetry [32], and through the development of generalised Calabi-Yau manifolds [26]. As if these successes were not enough however, we will see that in this chapter that the structure of generalised geometry additionally provides us with a remarkably natural language within which to describe various aspects of supergravity.

To this end, we will first detail the properties of generalised geometry via the linear algebra of so called ‘generalised tangent spaces’. Subsequently we will elicit the differential geometry of these spaces, through the Courant bracket. Finally we will outline the specific tools required to express supergravity in the language of generalised geometry; generalised metrics, generalised vielbeins and generalised Lie derivatives.

As generalised geometry is a relatively new area of research, there is unfortunately a dearth of easily accessible introductory material, especially for the physics orientated reader. That said however, we can recommend Zabzine’s lecture series as a useful starting point for many physicists [33], and Gaultieri’s thesis as a subsequent comprehensive mathematical overview [27].

4.0.1 Conventions

It is worthwhile stating at this point that we shall follow the usual conventions regarding the Lie derivative and interior product, so that for some vector fields X,Y;

$$\mathcal{L}_X = i_X d + d i_X, \quad \mathcal{L}_{[X,Y]} = [\mathcal{L}_X, \mathcal{L}_Y], \quad i_{[X,Y]} = [\mathcal{L}_X, i_Y]. \tag{4.1}$$

4.1 Generalised Tangent Spaces

As the equivalence class of atlases we can use to cover a manifold, differential structures play a fundamental role in differential geometry. Two of the most important differential structures in the context of theoretical physics are complex and symplectic, which, upon first inspection, appear unrelated. The central theme of generalised complex geometry however, is that both these structures should be considered special cases of a more general geometric structure, existing in the direct sum space $TM \oplus TM^*$ of the tangent and cotangent spaces. This we refer to as the generalised tangent space, elements of which are pairs $(X, \xi) = X + \xi$, of vector fields $X \in TM$ and 1-form fields $\xi \in TM^*$. As X and ξ have components X_i and ξ_i respectively, for $(i = 1...D)$ we can treat this formal sum as a generalised vector V with 2D components

$$V^I = \begin{pmatrix} X_i \\ \xi_i \end{pmatrix}. \tag{4.2}$$

4.1.1 Inner product & Flat metric

To aid in our understanding of these spaces, it is firstly important to investigate their linear algebra. For an n dimensional tangent space TM, we consider a $2D$ dimensional generalised tangent space $TM \oplus TM^*$. This is naturally endowed with a symmetric inner product:

$$\langle X + \xi, Y + \omega \rangle = \frac{1}{2}(\xi(Y) + \omega(X)), \tag{4.3}$$

which, in coordinates (dx^μ, ∂_μ), we can recast in matrix form:

$$\langle X + \xi, Y + \omega \rangle = \frac{1}{2} \begin{pmatrix} X & \xi \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} Y \\ \omega \end{pmatrix}. \tag{4.4}$$

We can express this more explicitly in a component wise fashion;

$$\langle X + \xi, Y + \omega \rangle = \frac{1}{2} \begin{pmatrix} X_i & \xi_i \end{pmatrix} \begin{pmatrix} 0 & \mathbb{1}_{D\mathrm{x}D} \\ \mathbb{1}_{D\mathrm{x}D} & 0 \end{pmatrix}_{IJ} \begin{pmatrix} Y_j \\ \omega_j \end{pmatrix}, \tag{4.5}$$

which defines a flat metric

$$\eta = \frac{1}{2}\begin{pmatrix} 0 & \mathbb{1}_{D\mathrm{x}D} \\ \mathbb{1}_{D\mathrm{x}D} & 0 \end{pmatrix}. \tag{4.6}$$

This has signature (D,D), and is invariant under the orthogonal group $O(TM \oplus TM^*) \cong O(D,D)$. This metric is congruent to the more conventional diag$(+1,-1)$ form, through Sylvester's law of inertia.

4.1.2 $SO(D,D)$ action

As we will show however, we can further reduce to the special orthogonal subgroup $SO(TM \oplus TM^*) \cong SO(D,D)$ by defining a canonical orientation on $TM \oplus TM^*$. Firstly note that we can decompose the highest exterior power as

$$\wedge^{2D}(TM \oplus TM^*) = \wedge^D TM \oplus \wedge^D TM^*, \tag{4.7}$$

and that there exists a natural pairing between $\wedge^k TM$ and $\wedge^k TM^*$;

$$(v^*, u) = \det(v_i^*(u_j)), \tag{4.8}$$

for $v^* = v_1^* \wedge ... \wedge v_k^* \in \wedge^k TM^*$ and $u = u_1 \wedge ... \wedge u_k \in \wedge^k TM$. This allows us to identify $\wedge^{2D} TM \oplus TM^* = \mathbb{R}$, so that we can specify a canonical orientation on $TM \oplus TM^*$ by some number in $\mathbb{R}$. To preserve both this orientation and the inner product we require the special orthogonal group $SO(TM \oplus TM^*) \cong SO(D,D)$.

4.2 The Courant Bracket

Our next step is to define a bracket operation on this generalised tangent space; the Courant bracket. This was first introduced in T. Courant and Weinstein's work on Dirac structures; essentially the real analogues of generalised complex structures [34][35]. Encoding the differential geometry of the generalised tangent space, the Courant bracket is central to our discussion of generalised geometry. It is defined as

$$[X+\xi, Y+\omega] = [X,Y] + \mathcal{L}_X\omega - \mathcal{L}_Y\xi - \frac{1}{2}d(i_X\omega - i_Y\xi), \tag{4.9}$$

where [X,Y] is the Lie bracket of two vector fields, and $X+\xi, Y+\omega \in TM \oplus TM^*$.

The Courant bracket essentially generalises the action of the Lie bracket, from sections of the tangent bundle to sections of the generalised tangent bundle.

4.2.1 Diffeomorphism Invariance

For a smooth manifold M the symmetries of the Lie bracket on the tangent bundle $\pi : TM \longrightarrow M$ can be described by the bundle automorphism (F, f); a pair of diffeomorphisms $F : TM \longrightarrow TM$ and $f : M \longrightarrow M$. Graphically we can represent this as

$$\begin{tikzcd} TM \arrow[r, "F"] \arrow[d, "\pi"'] & TM \arrow[d, "\pi"] \\ M \arrow[r, "f"'] & M \end{tikzcd}$$

where (F, f) preserves the commutativity of the diagram. If we dictate that F must preserve the Lie bracket, i.e.

$$F([X,Y]) = [F(X), F(Y)] \;\forall\; X, Y \in TM, \tag{4.10}$$

then $F = f_*$, the push forward of the tangent space, defined at a point p as;

$$f_* : TM_p \longrightarrow TM_{f(p)}. \tag{4.11}$$

As f is a diffeomorphism, the push forward f_* must also be a diffeomorphism. Otherwise stated, invariance under diffeomorphisms is the only symmetry preserving the Lie bracket on TM.

In an analogous fashion we can define a 'generalised' bundle automorphism (F, f) for $F : TM \oplus TM^* \longrightarrow TM \oplus TM^*$ and $f : M \longrightarrow M$, where the diagram

$$\begin{tikzcd} TM \oplus TM^* \arrow[r, "F"] \arrow[d] & TM \oplus TM^* \arrow[d] \\ M \arrow[r, "f"'] & M \end{tikzcd}$$

commutes. Under the requirement that F preserves the Courant Bracket;

$$F([X+\xi, Y+\omega]) = [F(X+\xi), F(Y+\omega)] \;\forall\; X+\xi, Y+\omega \in TM \oplus TM^*, \quad (4.12)$$

we see that $F = f_* \oplus f^*$, for the pushforward f_* of the tangent space and the analogous pull-back f^* of the cotangent space, is an automorphism of $TM \oplus TM^*$. This is as we would expect; the geometry we are describing here is invariant under diffeomorphisms.

4.2.2 B Field transformations

In contrast however to the action of the Lie bracket, the Courant bracket also possesses non-trivial automorphisms defined by forms. Through $X \to i_X B$, we can view a closed 2-form $B \in \wedge^2 T^*M$ as a map $TM \longrightarrow TM^*$. By exponentiating B we find

$$e^B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ B & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ B & 1 \end{pmatrix}, \quad (4.13)$$

which allows us to define an orthogonal bundle mapping;

$$e^B : X+\xi \to X+\xi+i_X B. \quad (4.14)$$

This can be thought of as a shear transformation, which fixes projections to TM and shears in the TM^* direction. It is then straightforward to see that

$$
\begin{aligned}
&[e^B(X+\xi), e^B(Y+\omega)] \\
&= [X+\xi+i_X B, Y+\omega+i_Y B] \\
&= [X+\xi, Y+\omega] + [X, i_Y B] + [i_X B, Y] \\
&= [X+\xi, Y+\omega] + L_X i_Y B - \frac{1}{2} d i_X i_Y B - L_Y i_X B + \frac{1}{2} d i_Y i_X B \\
&= [X+\xi, Y+\omega] + L_X i_Y - i_Y L_X B + i_Y i_X dB \\
&= [X+\xi, Y+\omega] + i_{[X,Y]} + i_Y i_X dB \\
&= e^B [X+\xi, Y+\omega] + i_Y i_X dB.
\end{aligned}
$$

Since B is closed, the Courant bracket is therefore invariant under the action of a closed 2-form. Furthermore, it can be proved that diffeomorphisms and B field transformations are the only automorphisms of the Courant bracket [27]. This implies that the automorphism group consists of the semi-direct product Diff$(M)\ltimes\Omega^2_{closed}(M)$; a generalisation of a direct product group which requires only one subgroup to be normal, required as the diffeomorphism group is not necessarily normal. It is worthy of mention at this point that the notational similarity between this 2-form and the 2-form B field detailed in the previous chapter is of course not accidental, they are the same object.

We can now see that through the consideration of a generalised tangent space, there exists a natural framework for describing geometries endowed with a metric, an action of $SO(D,D)$, 2-form gauge invariance and diffeomorphism invariance. As these geometries and symmetries arise in string theory and supergravity, generalised geometry seems an ideal construct within which to investigate both theories.

4.3 Generalised Metrics

The primary tool offered by generalised geometry in the context of type II supergravity is that of the generalised metric G. Originally introduced by Gualtieri, this is essentially the generalisation of a Riemannian metric on TM to $TM \oplus TM^*$, which composites the 2-form B field and the conventional metric g into a single object [27].

We arrive at this generalised metric by considering a reduction from the $O(D,D)$ structure of the generalised tangent bundle E to an $O(D) \times O(D)$ structure, achieved by splitting E into two d-dimensional orthogonal sub-bundles; $E = C_+ \oplus C_-$. With the requirement that η splits into a positive definite metric on C_+ and a negative definite metric on C_-, the subgroup then required to preserve each metric separately is $O(D) \times O(D)$.

The splitting $TM \oplus TM^* = C_+ \oplus C_-$ defines a positive definite generalised metric through

$$G = \eta|_{C_+} - \eta|_{C_-}. \tag{4.15}$$

We can view G as a symmetric automorphism of $TM \oplus TM^*$, since $G = G^*$ and $G : E \longrightarrow E$, with $\pm$ eigenspaces $C_\pm$.The simplest form of G is

$$G_0 = \begin{pmatrix} 0 & g^{-1} \\ g & 0 \end{pmatrix}, \tag{4.16}$$

where g is the usual Riemannian metric. However, we may write this in a more general form by accounting for B field transformations;

$$G = \begin{pmatrix} 1 & 0 \\ -B & 1 \end{pmatrix} \begin{pmatrix} 0 & g^{-1} \\ g & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ B & 1 \end{pmatrix} = \begin{pmatrix} g - Bg^{-1}B & Bg^{-1} \\ -g^{-1}B & g^{-1} \end{pmatrix}. \tag{4.17}$$

This metric provides a natural way to encode the g and B fields of type II supergravity, via the $O(D,D)/O(D) \times O(D)$ coset space.

4.4 Generalised Vielbeins

With this metric in mind we can now define generalised vielbeins, which transform in representations of the $O(D)\times O(D)$ product group rather than the Lorentz group of conventional vielbeins. There are several ways to construct these, the most straightforward way is to consider a basis of one forms $E_A \in E^*, (A = 1 \dots 2d)$. We require that G and η can be expressed in the form;

$$\eta = E^T \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix} E \qquad G = E^T \begin{pmatrix} \mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix} E. \tag{4.18}$$

Then for two sets of ordinary vielbeins $e^a_\pm$ with inverse $\hat{e}_{\pm a}$, satisfying

$$g_{mn} = e^a_{\pm m} e^b_{\pm n} \delta_{ab} \text{ and} \tag{4.19}$$

$$g^{mn} = \hat{e}^m_{\pm a} \hat{e}^n_{\pm b} \delta^{ab}, \tag{4.20}$$

or equivalently;

$$g = e^T_\pm e_\pm \text{ and} \tag{4.21}$$

$$g^{-1} = \hat{e}_\pm \hat{e}^T_\pm, \tag{4.22}$$

we find explicitly that

$$E = \frac{1}{\sqrt{2}} \begin{pmatrix} e_+ - \hat{e}^T_+ B & \hat{e}^T_+ \\ -(e_- + \hat{e}^T_- B) & \hat{e}^T_- \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \hat{e}^T_+ (g - B) & \hat{e}^T_+ \\ -\hat{e}^T_- (g + B) & \hat{e}^T_- \end{pmatrix}. \tag{4.23}$$

We have a basis for C_+ in the first D of these generalised vielbeins, and a basis for C_- in the second D generalised vielbeins.

4.5 Generalised Lie Derivatives

There are a few other pieces of differential geometry we must develop before we can ultimately proceed to formulate supergravity entirely in terms of generalised geometry. Given the importance of Lie derivatives in differential geometry, it seems natural to consider 'generalised' Lie derivatives. For two vector fields X and Y can derive the Lie derivative of Y with respect to X;

$$\mathcal{L}_X Y = d i_X Y + i_X dY, \tag{4.24}$$

by considering the infinitesimal transformations generated by X. Proceeding analogously, for two generalised vectors $V = X + \xi$ and $U = Y + \omega$ we can consider an infinitesimal transformation generated by X and a B field $d\xi$, and define a generalised Lie derivative;

$$\mathbb{L}_V U = \mathcal{L}_X Y + (\mathcal{L}_X \omega - i_Y d\xi). \tag{4.25}$$

Naturally, this reduces to the normal Lie derivative in the absence of any B fields. Whilst the normal Lie derivative on vector fields is equivalent to the Lie bracket, our generalised Lie derivative on generalised vectors fields is equivalent to the Dorfman bracket $[V, U]_D$ [36]. It is the antisymmetrisation of this Dorfman bracket that gives us the now familiar Courant bracket.

Chapter 5

Conclusions

The intention of this thesis was to investigate generalised geometry as a framework within which to express supergravity theories. This stems from the natural way in which generalised geometry incorporates the action of both a conventional metric and a 2 form B field. To this end we have firstly developed and motivated the NSNS region of type II supergravity; the sector in which the B field has the greatest role to play. This has been achieved firstly by assessing the roots of supergravity in supersymmetry, and subsequently via the investigation of type II supergravity and of the extended objects present in the theory. Finally we have elucidated the structure of generalised geometry, and outlined some useful 'generalised' tools. Given the natural way in which generalised geometry expresses the relevant quantities, we can conclude that the language it provides seems ideal for supergravity theories; almost tailor made.

5.1 Further work

The next step in this regard is to recast the NSNS sector of type II supergravity in the language of generalised geometry, utilising the tools and methods outlined herein. A natural continuation of this would of course be to investigate the entirety of type II supergravity in generalised terms. Further research is also ongoing into generalised geometry as the 'language' of T duality, further shedding light on aspects of string theory [31].

Extensions of generalised geometry are also possible, in the context of current theoretical physics the most important being exceptional generalised geometry. By replacing the natural O(D,D) action with the exceptional U-duality group $E_{D(D)}$, exceptional generalised geometry may provide an analogously natural language in which to express 11 dimensional supergravity theories, and thus potentially M theory [37] [38]. Ultimately, generalised geometry may prove to be a key development in the history of fundamental physics.

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