de Broglie-Bohm Theory ,Quantum to Classical Transition and Applications in Cosmology

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Abstract

Standard quantum mechanics has conceptual inconsistency regarding measurement and 'quantum to classical' transition. We reviewed different interpretations and the previous attempts to resolve this problem, with emphasis on de Broglie-Bohm pilot wave theory. Decoherence scheme and collapse process are discussed within the context of standard quantum theory and de Broglie-Bohm theory. We applied guidance equation to solve for late time solution of Guth-Pi upside-down harmonic oscillator model and found result concuring with that of classical approximation from quantum mechanics. We also reviewed the method for calculation of vacuam inflaton field fluctuation and result from de Broglie-Bohm theory. .

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Contents

1 Introduction

Quantum mechanics, although a very successful theory for all practical purposes, has conceptual issues regarding what is considered to be elements of reality [1]. The standard interpretation rejects the reality of objects, such as electrons, before a process known as *measurement*. According to Born rule of probability distribution, the measurement process has a special status in quantum mechanics as a process that causes a *collapse* of wave function into one of the possibilities. Quantum mechanics also states that a wave function contains all there is to describe a physical system. If we were to take such a postulates seriously, then the picture of the world would be disconnected. In particular, particles do not have localized positions but are rather described by a wave function that gives us probabilities of particles being in a particular positions.

The theory of de Broglie-Bohm is constructed to resolve such a disconnected picture by giving an individual particle a localized position $\mathbf{x}(t)$ and a velocity associated with it $\dot{\mathbf{x}}(t)$. The velocity of the particle (or as termed by Bell as particle beable) is a function of a wave function; in other words, it is "guided" by wave function. Within this picture, we reduced the role of a wave function from a complete description of physical system to that of a *field* set up by the configuration of the system. Thus, particles would simply follow the trajectories of such field the same way that a moving charged particle would follow a magnetic field line.

Given that we view the wave function as a guiding field; we can explore other possibilities such as quantum non-equilibrium. In this case, we would consider the situation where $\rho \neq |\psi|^2$ and how this condition could possibly be made into observable power spectrum from a process of inflaton field fluctuation that has been postulated to have occurred in the early universe during inflationary era. During such an era, the usual assumptions of the standard quantum theory, such as the requirement to have an observer to make a measurement, raised a serious question regarding the correctness of our picture of reality through such anthropocentric model. One needs to be clear of each statement of each action in previously ill defined in quantum theory.

The process of decoherence gives a partially fulfilled answer as to why we do not observe large physical objects in superpositions or interference between them. Such process eliminates the need to make a boundary between a classical apparatus and a quantum state of object are to be measured by treating all of them on equal footing i.e. states are to be treated as quantum states. The process of environmentally induced selection (einselection) which would cause decoherence to occur is, at the time, the best answer towards why a macroscopic object appears classical. However, the process only reduced the possible outcome of a measurement to a set of pointer states which are decohered, it still does not eliminate the special status of measurement to cause a collapse of the wave function. This difficulty is what makes the interpretation of de Broglie-Bohm theory more attractive, in which we inferred no collapse since such a process is not needed within this interpretation.

There is no point in choosing a particular theory if it explains only to the same level as its predecessor. However, de Broglie-Bohm theory has some features which are not explicitly explained in standard quantum theory, which is the non-local feature of multi-particle theory. While the correlation occurs in quantum theory it is not explicitly non-local. Other attempts with the theory have been to formulate a relativistic field theory, one would need to find the variable that is an ontology of the theory. For bosonic field theory, the obvious choice is field configuration which leads to considerable success in formulating the theory. For fermionic field theory, choosing the right ontology has been more difficult with limited success.

Since it is the assumption of the theory that we have continuity in trajectories of particle, it is a natural question to ask whether such trajectories would also be the same within appropriate classical limit, we will explore a one-dimensional model of Guth-Pi, of which shows that, indeed, the same trajectory is found between that prediction of pilot wave and the implied classical approximation from of standard quantum theory.

Lastly, I have attempted to include the point of view from various interpretations on the topic found in previous literature. However not all of them could have been included within. It is not because they are not important but because there have been fewer developments into applications of those interpretations into testable models. My hope is that the reader will gain a wider perspective into the field of interface between foundations of quantum theory and classical physics.

2 de Broglie-Bohm pilot wave theory

"We prize a theory more highly if, from the logical standpoint, it is not the result of an arbitrary choice among theories which, among themselves, are of equal value and analogously constructed." - Albert Einstein.

The pilot wave mechanics, was first proposed by de Broglie [2] and revived again by Bohm [3, 4]. As with quantum mechanics of Copenhagen interpretation, the pilot wave theory in the non relativistic viewpoint describes a wave function $\psi(\mathbf{x_1}, ..., \mathbf{x_k}, t)$ that obeys Schrödinger's equation

$$\imath\hbar\frac{\partial\psi(\mathbf{x_1}, ..., \mathbf{x_k}, t)}{\partial t} = (-\frac{\hbar^2}{2m}\nabla^2 + V(r))\psi(\mathbf{x_1}, ..., \mathbf{x_k}, t) \tag{1}$$

where we have 3k dimensions of configuration space.

In this case, pilot wave theory retains the same solution as the standard quantum mechanics when solving for $\psi(\mathbf{x_1}, ..., \mathbf{x_k}, t)$, therefore, it has the same prediction for non relativistic case. However, the main conceptual difference between the pilot wave theory and the standard quantum mechanics is the interpretation of $\psi(\mathbf{x_1}, ..., \mathbf{x_k}, t)$ within quantum mechanics. This is seen as a solution to Schrödinger's equation but with an ambiguous existence as Bohr's complementary interpretation as a wave function before a measurement and reduced to a localized wave package or a particle after a measurement. From the pilot wave theory's standpoint, $\psi(\mathbf{x_1}, ..., \mathbf{x_k}, t)$ is seen as an objective field. In order to obtain the velocity of particles which belongs to this field, consider continuity equation for fluid with density ρ and velocity $\mathbf{v}$

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0 \tag{2}$$

The quantity $\rho\mathbf{v}$ is equal to the current density $\mathbf{J}$. To obtain the guidance equation, one only has to consider the analogue in quantum mechanics seriously, where we instead consider $\mathbf{J}$ to be Noether's current

$$\mathbf{J} = i\frac{\partial\psi}{\partial x}\psi^* - \frac{\partial\psi^*}{\partial x}\psi \tag{3}$$

Now we simply equate $\rho = |\psi|^2$, then we can write the velocity as

$$\mathbf{v} = \frac{\mathbf{J}}{|\psi|^2} = \frac{i}{|\psi|^2}(\frac{\partial\psi}{\partial x}\psi^* - \frac{\partial\psi^*}{\partial x}\psi) \tag{4}$$

There are two possible interpretations of this result. The first is to follow Copenhagen interpretation, which means that such velocity does not exist and we simply ignore the result. The second interpretation is to take the result seriously, that the trajectories exist and particles are considered as real objects, independent of measurement or collapse of wave function. The latter is where de Broglie-Bohm's interpretation stands.

Quantum mechanics is itself a statistical theory. An example of this would be the Young's double slit experiment, it is expected to predict the pattern on the screen behind the slit as a probability equal to $|\psi|^2$. In view of quantum mechanics, only the statistics at large scale corresponding to $|\psi|^2$ is predictable, while on microscopic scale such as the trajectory for individual particle would be *principally impossible* to predict, as it is ingrained that *such trajectories do not exist before a measurement.* Since the trajectories do not exist, only the outcome at the points of the measurement have physical meaning. In the point of view of pilot wave theory, the wave function $\psi(x_1, ..., x_k, t)$ guides the particles, in which the statistical prediction resembles that of quantum mechanics $|\psi|^2$ but owing to classical statistical distribution in which the initial positions of particles inherit, thus following different trajectories as given by the guidance equation.

2.1 Quantum non-equilibrium and relaxation to equilibrium

The consequence of view point of pilot wave theory as outlined in previous section follows that we can have a situation where $\rho \neq |\psi|^2$, but this is not possible in the context of quantum mechanics [7, 6]. The inequality between ρ and $|\psi|^2$ is called *quantum non-equilibrium*. If a system starts off at equilibrium $\rho = |\psi|^2$, then it will continue to be in the state of equilibrium. Moreover, the evolution of the system from non-equlibrium into equlibrium is analogous to the evolution of a closed system which always increases in its total entropy [8, 7]. We can measure the difference between ρ and $|\psi|^2$ by *sub-quantum H-function*

$$H = \int dx\ \rho\ ln(\frac{\rho}{|\psi|^2}) \tag{5}$$

In the process of relaxation, H will tends to approach zero. We must emphasize the fact that it is not a priori requirement to have an equilibrium distribution in de Broglie-Bohm theory, for example, in the early universe, the distribution did not need to start off in equilibrium. However, it is expected that relaxation will take place at time as the system undergoes virulent process, we also expect most of the systems to be in an equilibrium state apart from those which have been "frozen" in non-equilibrium [6].

2.2 *Collapse* of the wave function?

One of the areas with the greatest difficulties within the realms of quantum mechanics is the process known as the"collapse" of the wave function. Suppose we have a wave function given by

$$|\Psi\rangle = \sum_{i=1}^{n} c_i|\psi_i\rangle \tag{6}$$

Suppose we have a hermitian operator $\hat{A}$ with eigenvalues λ_i

$$\hat{A}|\psi_i\rangle = \lambda_i|\psi_i\rangle \tag{7}$$

According to the Born rule, if we were to make a measurement for hermitian operator $\hat{A}$, then we would have the probability of obtaining eigenvalue λ_k as $|c_k|^2$, thus, effectively collapsing the wave function $\Psi \longrightarrow \psi_k$. Various conceptual difficulties made this scheme more difficult to accept as a fundamental theory. It is notable that the requirement to have a segregation between the measuring apparatus and the object to be measured is rather artificial. While the measuring apparatus is considered as an external classical system, the object is considered as a wave function.

The scheme of "collapse postulate" was proposed by Von Neumann [14] where we assign the basis vectors of "pointers" $|a_i\rangle$ in addition to the initial superposition, such that the pointer$|a_k\rangle$ corresponds to the outcome of measurement in the state $|\psi_k\rangle$. The wave function may be written as

$$|\Psi\rangle = \sum_{i=1}^{n} c_i|\psi_i\rangle|a_i\rangle \tag{8}$$

It can be seen that von Neumann's collapse model does not resolve the conceptual problem of measurement, but is merely a statement of a more consistent system which includes the measurement pointer. The model still asserts that the outcome of a measurement is intrinsically indeterministic and there is no clarification of what constitutes a *measurement* process.

An attempt to resolve the measurement problem within the context outlined above is given by Everett [15], we introduce the experience of observers to be part of the superposition of wave function. To illustrate this, we shall follow discussion in Bohm and Hiley [5]. Suppose we have a wave function of spin 'up' $|\uparrow\rangle$ and 'down' $|\downarrow\rangle$

$$|\Psi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle \tag{9}$$

Following von Neumann, we also have 'pointers' for the apparatus, which are the wave package corresponding to the 'up' and 'down' results of spin measurements.

$$|\Psi\rangle = \alpha|\uparrow\rangle|a_\alpha\rangle + \beta|\downarrow\rangle|\ a_\beta\rangle \tag{10}$$

Everett introduces an observer having two separate memories of measurement, $|O(\alpha)\rangle$ and $|O(\beta)\rangle$ such that we can express the whole wave function as

$$|\Psi\rangle = \alpha|O(\uparrow)\rangle|\uparrow\rangle|a_\alpha\rangle + \beta|O(\downarrow)\rangle|\downarrow\rangle|\ a_\beta\rangle \tag{11}$$

The interpretation is that, there exists an independent universal wave function $|\Psi\rangle$ as given above, which includes the two separate memories of observer $O(\uparrow)$ and $O(\downarrow)$. There exists two possible outcome of measurement $|\uparrow\rangle$ and $|\downarrow\rangle$ in which if the observer finds the outcome $|\uparrow\rangle$ he would 'branch out' into the the state $|O(\uparrow)\rangle|\uparrow\rangle|a_\alpha\rangle$, vice versa if he finds the outcome $|\downarrow\rangle$ he would be in the state $|O(\downarrow)\rangle|\downarrow\rangle|\ a_\beta\rangle$ with probability of being in the states $|\alpha|^2$ and $|\beta|^2$ respectively. Within the context of this interpretation, all possible outcomes occur, but the observer can only observe one of the possible outcomes at a time. There is no 'collapse' of wave function as it is assumed that the universal wave function is the complete description, independent of observer's experience. Each of the observer's experience would represent a different world complementary to the other one.

Within the context of de Broglie-Bohm theory, we must take the 'collapse' of wave function differently. We shall illustrate with the approach of [9, 11].

Within standard quantum theory, a simplified model of a measurement of a spin component of spin 1/2 particle can be given by the interaction

$$H = g(t)\sigma\frac{\partial}{\imath\partial x} \tag{12}$$

Where $g(t)$ is time dependent coupling and σ is the Puali matrix for chosen component. The initial state of wave function

$$\psi_m(0) = \phi(r)a_m \tag{13}$$

Where $\phi(x)$ is a narrow wave package centered at r=0 and spin index is given by m = 1,2. Solving Schroedinger's equation we find.

$$\psi_m(t) = \phi(x - (-1)^m h)a_m|h(t) = \int_c^t dt`g(t`) \tag{14}$$

As $t \to \infty$ the components of the wave package $\psi(x - (-1)^m h)$ will separate in space for two spin components m =1,2 .Measurement of spin will yield the value $+h$ or $-h$ with probability of obtaining outcome m as $|a_m|^2$.

In the point of view of de Broglie-Bohm theory, we can determine the trajectory from (4) , with $\rho(x,t) = \psi^*(x,t)\psi(x,t)$ and $j(x,t) = \psi^*(x,t)g\sigma\psi(x,t)$.

We find that

$$\frac{dx}{dt} = g\frac{\sum_m |a_m|^2|\phi(x-(-1)^m h)|^2(-1)^m}{\sum_m |a_m|^2|\phi(x-(-1)^m h)|^2} = \pm g \tag{15}$$

We easily retrain the same result $x = \pm h$. It is noteworthy to make a comment on the result obtained above using de Broglie-Bohm theory; firstly that the only process of measurement we have is a position measurement x, secondly and importantly, there is no "wave package reduction" or "collapse of wave function" but instead we obtained an *effective collapse* when there is no overlap between the two wave packages at time t_0 such that.

$$\psi_1(x,t_0)\psi_2(x,t_0) = 0 \tag{16}$$

When the condition (16) is satisfied, the trajectories can be considered independent of one another and we have an effective collapse.
Although the notion of a 'collapse' in quantum mechanics and 'effective collapse' could explain very well with simple experiments such as the double slit or spin measurement, in order to explain the macroscopic world proposals base on *decoherence* scheme have been employed, which we shall explain in section 3 of the paper

2.3 Non-locality

In the celebrated paper by Einstein, Podolsky and Rosen [1] the authors raised an objection to quantum mechanics in what is now known as *EPR Paradox*, that the wave function, as assumed by standard quantum mechanics to be 'complete' description of physical system, is in fact *incomplete* by their definition of completeness as
(1) "every element of physical reality must have counter part in physical theory"
(2) "If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity". Within the original EPR argument, the objection raised against the completeness of wave the function is as follows.

Suppose we have two systems **I,II** which are allowed to interact from time $t=0$ to $t=T$ such that the wave function at time $t > T$ is Ψ. Let A be a physical quantity corresponding to system **I**, such that $a_1, a_2, ...$ are the eigenvalues of A, the corresponding eigenfunction is $u(x_1)$and of system **II** as $\phi(x_2)$ we may express the wave function of system **I+II** as

$$\Psi = \sum_{n}^{\infty} u_n(x_1)\phi_n(x_2) \tag{17}$$

Suppose a measurement is made and eigenvalue a_k is obtained, then system of **I** and **II** has its wave packet into $u_k(x_1)\phi_k(x_2)$. Furthermore, if we were to measure instead a physical property B of system **I** with eigenvalues $b_1, b_2, ...$ and eigenfunction $v(x_1)$ the wave function of system **I+II** must now be written as

$$\Psi = \sum_{n}^{\infty} v_n(x_1)\psi_n(x_2) \tag{18}$$

If the result obtained from this measurement is b_r then the wave packet has been reduced to $v_r(x_1)\psi_r(x_2)$. The objection to having wave function as a complete description of reality is that one can, by arbitrary choice of measurement of property A or B on system **I** assign either ϕ_k or ψ_r on system **II** even though both systems are no longer locally interacting. Furthermore, the measurement on **I** results in immediate collapse of **II** into the corresponding reduced wave packet.

In the framework of de Broglie-Bohm theory, we fundamentally concur with the objection that the wave function Ψ is not the complete description of the system, as we also have trajectories of particles $x(t)$ assigned with the wave function which is continuous in time as in (4). Bohm [4] suggested that for n-body treatment within de Broglie-Bohm theory we have 3n dimension trajectories, such that each i particle has velocity defined by

$$\frac{d\mathbf{x_i}}{dt} = \nabla_i \frac{S(\mathbf{x_1}, \mathbf{x_2}, ..., \mathbf{x_n}, t)}{m} \tag{19}$$

and the wave function of the system is entangled $\psi(x_1, x_2, ..., x_n, t)$, any a measurement of particle x_i will bring instantaneous 'uncontrollable disturbance' to the wave function. Although the disturbance is expected to instantaneous, no useful information sent using this method in quantum equilibrium $\rho = |\psi|^2$ vice versa in non equilibrium real instantaneous communication may be established at statistical level [19, 20]

2.4 Pilot wave model for quantum field theory

The construction of pilot wave models, is base on defining the 'beables' of the theory. What we mean by a beable of the theory is an object, such as an electron with a localized position, that is *assumed* to exist independent of observation. In the case of non relativistic pilot wave mechanics, we defined the beable of the theory to be the trajectories of particles $\mathbf{x_1}, \mathbf{x_2}, ..., \mathbf{x_n}$. The usual choice of beable for the construction of pilot wave quantum field theory is the field configuration. There are some subtle reasons behind such choice, notably for bosonic field theory in which it is not possible to make the theory Lorentz invariant if we are to choose position of particles instead of field configuration to be the beable of the theory, although the choice of field configuration is also not Lorentz invariant, it is expected to behave indifferently to Lorentz invariant quantum field theory under the quantum equilibrium condition.The conventional choice of representation is of functional Schrödinger representation. We shall give only a summary of the important results for the construction of bosonic field theory here and a brief overview with problems of construction a fermionic field theory. Please see Struyve [11, 13] for more detailed overview.

Canonical quantization

We may represent a field that is dependent upon continuous coordinate variable

$\mathbf{x}$ by $\phi(\mathbf{x})$, where the field $\psi(\mathbf{x})$ is considered to be the *beable* of the system. The field velocity may be defined as

$$\frac{\partial\phi(t_0,\mathbf{x})}{\partial t} = \dot{\phi}(t,\mathbf{x}) \tag{20}$$

And the field momentum may be defined as

$$\Pi_\phi(\mathbf{x}) = \frac{\delta L}{\delta\dot{\phi}(\mathbf{x})} \tag{21}$$

Where L is the Lagrangian of the system. The Hamiltonian of the system can be written as,

$$H = \frac{1}{2}\sum_{r,s}\int d^3x d^3y \Pi_{\phi,r}(\mathbf{x}) h^\phi_{rs}(\mathbf{x},\mathbf{y})\Pi_{\phi s}(\mathbf{y}) + V(\phi) \tag{22}$$

where $h^\phi_{rs}(\mathbf{x},\mathbf{y})$ is the Hamiltonian density. We have the corresponding canonical relation,

$$[\hat{\psi}_r(\mathbf{x}),\hat{\Pi}(\mathbf{y})] = i\delta_{rs}\delta(\mathbf{x}-\mathbf{y}) \tag{23}$$

We define the field operators as $\hat{\psi}(\mathbf{x}) \rightarrow \psi_r\mathbf{x}$, $\hat{\Pi}_{\psi r}(\mathbf{x}) \rightarrow -i\frac{\delta}{\delta\psi_r(\mathbf{x})}$, where these operators act on wave functional $\Phi(\psi)$, which has inner product

$$\langle\Phi_1|\Phi_2\rangle = \int(\Pi_r D_{\phi_r)}\Phi_1^*(\psi)\Phi_2(\psi) \tag{24}$$

Time dependent wave functional is defined as

$$\langle\phi|\Phi(t)\rangle = \Psi(\phi,t) \tag{25}$$

which satisfies the functional Schrödinger's equation,

$$i\frac{\partial}{\partial t}\Psi(\phi,t) = H\Psi(\phi,t) \tag{26}$$

Using Hamiltonian defined in (22) we have

$$i\frac{\partial}{\partial t}\Psi(\phi,t) = (\frac{1}{2}\sum_{r,s}\int d^3x d^3y \Pi_{\phi,r}(\mathbf{x}) h^\phi_{rs}(\mathbf{x},\mathbf{y})\Pi_{\phi s}(\mathbf{y}) + V(\phi))\Psi(\phi,t) \tag{27}$$

The beable of this theory is chosen to be the field $\phi(\mathbf{x})$, thus just like in non-relativistic Bohmian mechanics that has particle positions $\mathbf{x}(t)$ as the beable with a velocity $\dot{\mathbf{x}}(t)$ as defined in (4), the field has a field velocity as

$$\dot{\phi}(\mathbf{x}) = \sum_r \int d^3y h^{\phi}_{rs}(\mathbf{x}, \mathbf{y}) \frac{\delta S}{\delta \phi_r(\mathbf{y})} \tag{28}$$

with $\Phi = \Phi e^{iS}$

Fermionic field theory

Further attempts have been made on constructing a fermionic field theory, notably by Holland and Valentini, differing on the choice of beables. Holland chose what he defined to be angular variables in momentum space $\alpha_{\mathbf{k}} = (\alpha_k, \beta_k, \gamma_k)$ with the wave functional as $\Psi(\alpha, t)$. Valentini's approach is to define *Grassman fields* as the beables of theory $u^\dagger, u$ guided by the wave functional $\Psi(u^\dagger, u, t)$. There are some difficulties relating to formulating a fermionic field theory within the theme of pilot wave, please see [13] for more detailed discussion of the problems.

3 Quantum to classical transition

Quantum theory is an atomistic theory, initially made to explain systems on atomic scale, such as electrons orbiting an atom or molecular structures. For all practical purposes of experiments within an atomic physics lab, the theory we have could explain the phenomenon which we construct an experiment to observe on those microscopic scale. Within those constructions, an idea to separate a *quantum system* which is the object of experiment such as the spin of electrons and *classical system* which includes the experimenter, the air in the lab and the whole macroscopic world.

Einstein was often quoted [16] for asking "Is there moon there when nobody looks?", the question may sound absurd but during the early days of quantum theory made perfect sense, displaying the absurdity in which postulates of quantum mechanics requires the world to be. If one were to take the Copenhagen interpretation seriously, the moon (at least the dark side of it) would be a wave function Ψ_M until an astronomer decides to look at it, not only that but distance quasars and unexplored galaxies without 'intelligible life' would also be some form of wave function.

Such question could have been avoided by a professional physicist, until one starts to consider a more absurd postulate; that the whole universe is a wave function [17, 18]. The question that can no longer be avoided is how such a system which behaves quantum mechanically could have spontaneous transition into classical system.

3.1 Decoherence approach

The analysis of quantum system in the framework of Copenhagen interpretation is usually as follows; there is an object to be measured which is described using quantum mechanics and the apparatus - this is a classical system use for measurement of the object. The 'classical apparatus' is a macroscopic object, and would not obey the law of quantum mechanics such as superposition of states or evolving according to Schrödinger's equation. But it is difficult to define line between a macroscopic 'classical system' and a macroscopic quantum system when system of 'macroscopic' number of electrons have been shown to behave quantum mechanically [29, 28]. Zurek [**?**, **?**, 26] proposed a scheme of *decoherence* , which explains how quantum system transforms into a classical system via interaction with the *environment*. We shall follow the approach as extensively reviewed in Zurek [24] and Schlosshauer [31].

Let us introduce a quantum which we are interested in making a measurement, we shall call this system **S** , which is in a superposition of states $|\phi_i\rangle; i = 1, ..., n$ such that $|\Psi\rangle = \sum_i^n |\phi_i\rangle$, we define a *pointer state* to be a particular state $|\phi_k\rangle$ which is singled out by the environment. An *einselection* is decoherence imposed selection of the preferred set of pointer states that remain stable in the presence of the environment. The novelty of decoherence is egalitarian treatment of the components of the system; measured system **S**, apparatus **A** and environment **E** ,are treated as quantum states. To demonstrate the scheme, let us introduce one bit system **S** with basis $|\uparrow\rangle, |\downarrow\rangle$ and apparatus with basis $|A_0\rangle, |A_1\rangle$ such that the basis of system **S** act on apparatus as

$$|\uparrow\rangle|A_0\rangle = |\uparrow\rangle|A_1\rangle, |\uparrow\rangle|A_1\rangle = |\uparrow\rangle|A_1\rangle \tag{29}$$

and

$$|\downarrow\rangle|A_1\rangle = |\downarrow\rangle|A_0\rangle, |\downarrow\rangle|A_0\rangle = |\downarrow\rangle|A_0\rangle \tag{30}$$

where $\langle A_0|A_1\rangle = 0$ If the system is initially a pure state

$$|\Psi\rangle = \alpha|\downarrow\rangle + \beta|\uparrow\rangle \tag{31}$$

such that $\alpha^2 + \beta^2 = 1$, a *premeasurement* is letting the system interact with the apparatus

$$|\Psi\rangle|A_0\rangle = (\alpha|\downarrow\rangle + \beta|\uparrow\rangle)|A_0\rangle = \alpha|\downarrow\rangle|A_0\rangle + \beta|\uparrow\rangle|A_1\rangle \tag{32}$$

Now if we introduce the environment **E** into the system with basis $|e_0\rangle, |e_1\rangle$ initially in state $|e_0\rangle$, the environment will 'perform a premeasurement' on the system

$$(\alpha|\downarrow\rangle|A_0\rangle + \beta|\uparrow\rangle|A_1\rangle)|e_0\rangle = \alpha|\downarrow\rangle|A_0\rangle|e_0\rangle + \beta|\uparrow\rangle|A_1\rangle|e_1\rangle \tag{33}$$

After the interaction of the measurement, the density matrix is now reduced to only the diagonal terms

$$\rho_E = \mathbf{Tr}_e(|\Psi\rangle\langle\Psi|) = |\alpha|^2|\downarrow\rangle\langle\downarrow|A_0\rangle\langle A_0| + |\beta|^2|\uparrow\rangle\langle\uparrow|A_1\rangle\langle A_1| \tag{34}$$

as long as we have the orthogonality condition on the environment basis

$$\langle e_0|e_1\rangle = 0 \tag{35}$$

When we have a condition that the density matrix is reduced to a diagonal term ("Schmidt state") only, such as in (34), the claim by Zurek implies that we have reduced a quantum system into a classical system; a result of measurement would not yield any superposition of states. Although the scheme of decoherence demonstrated within simplified system indicates how suppression of interference could occur as long as the condition (35) is satisfied. Further justification of how to segregate the systems of **S,A,E** would be useful.

De-correlation time

The central idea behind the decoherence approach is for the density matrix ρ to be reduced to only diagonal terms, the example we shown earlier for one bit system is rather oversimplified. Other cases have been considered , such as where the environment is a massless scalar field by Unruh and Zurek[35] or collection of harmonic oscillators model of Caldiera-Leggett [30]. Decoherence problem, viewed this way can be solved by finding the *master equation* $\frac{d\rho}{dt} = \hat{L}\rho$ where the density matrix ρ evolves in time.Thus, the process of decoherence is time-dependent and will give rise to a *de-correlation timescale*, in which the off-diagonal terms of density matrix are essentially *reduced* and the system of interest becomes 'classical' . For a one dimensional particle interacting with harmonic oscillators (or a field Φ) this is in the form of [23]

$$\tau_D = \tau_R \left(\frac{\hbar}{\delta x \sqrt{2 m k_B T}} \right)^2 \tag{36}$$

where δx is the separation of wave packet, k_B is Boltzmann's constant, T is temperature and τ_R is *relaxation time* which is proportional to $2m/\eta$ where $\eta = \epsilon^2/2$ came from interaction with the scalar field. In general, this will be dependent on the form of the Hamiltonian and not necessary in the same form as (36), but an order of magnitude consideration would give a large object with large mass, like planets or the moon a very short de-correlation time, while a small object such as an electron a much longer time, due to relative magnitude difference of masses $m_{moon}/m_{electron} \sim 10^{53}$.

3.2 decoherence in de Broglie-Bohm

The context in which decoherence has been applied to as a solution to 'spontaneous' emergence of classicality, has been under the framework of Copenhagen interpretation and moreover to substantiate the claim of Many Worlds Interpretation (MWI). One still fundamentally has not altered the special role of observer, but rather give a result that is consistent with experience of observers who never see macroscopic objects as a quantum superposition by suppressing the terms that allow interference. However, there still is a 'quantum to classical' boundary, such as that observers are treated as 'classical system' with special ability to record information when making a 'measurement' thus still collapsing (see chapter 2) the wave function.

Within de Broglie-Bohm's framework of ontological theory, there is a mixture of mutual agreement and disparity with the framework of decoherence. The fundamental hypothesis that de Broglie-Bohm holds is that particle beables do have continuous trajectory $\mathbf{x(t)}$, guided within a wave function or a wave packet like particles of dust sustaining in droplets of water. Since the hypothesis is that the only intrinsic unpredictability of the theory is only at the initial moment when the particle beable goes into one of the wavepacket by classically random process, if we were to view the process of decoherence, the view would be incompatible with MWI. However, for practical purposes, the same approach involving 'interaction' with the environment has been employed to resolve the issue.

Bohm and Hiley [5] proposed a scheme base on macroscopic object by collision with stream of particles and the 'shadow' position where the particles have been blocked. The wave function of the whole system has been modified, such that probability P of having *effective collapse* as in 16 is enhanced with increasing number n particles hitting it $P \sim e^n$. Appleby [27] has further applied the pilot wave model to the Caldeira-Leggett model of environmentally induced 'decoherence'. Appleby has shown that there are two requirements to consider the system as classical, to have approximately diagonal density matrix after an elapsed time and to need Bohmian velocity to lie within classical range. The criteria being used, is often employed in the analysis of quantum to classical transition, is the requirement for the Wigner function to be strictly non-negative.

It is worth commenting on the validity of Wigner function(see appendix A.) as a test for classicality here. While many authors (see Zurek) use the criteria that whenever the function is positive, one can consider the subjected system as classical, its use is unwarranted apart from the fact that when the function is positive, in some case it resembles the classical Gaussian probability distribution in phase space [21]. Further on, it has been shown that WKB Wigner function does not reproduce a classical limit and decoherence is necessary [32, 33]

4 Applications in Cosmology

The application of quantum theory in cosmology is a non-trivial matter, when one consider an object such as the entire universe, then issues that were plaguing the foundations of quantum mechanics could not be as easily brushed aside without having a serious conceptual inconsistency. While it is a debatable topic about the ontology of electron before a 'measurement', one find a much greater difficulty in requiring state of the universe to be 'measured' by an 'observer' or having decoherence by 'environment'. Apart from the pressing conceptual issues which are acknowledged in various works, quantum theory has been very well developed in terms of vacuum fluctuations of the inflaton field, which led to observable predictions on the CMB spectrum. We will highlight the issues here and review where pilot wave theory has been applied,our attempt to solve a toy model of one dimensional upside-down harmonic oscillator and the discussion of future applications.

4.1 Overview of Inflation

The first inflationary model was proposed by Starobinsky [38] with the inflation driven by quantum corrections to vacuum Einstein's equations. A simpler model was developed by Guth's "old inflation" [37], which solved the three classic cosmological puzzles of *flatness, horizons and monopoles* problems. We shall refer to Weinberg's "Cosmology" [39] for the detailed discussions of the problems. The model suffered from vacuum phase transition problem, which led to the introduction of the "new inflation" model by Linde [40], Albrecht and Steinhardt [41]. A "chaotic inflation" model was proposed by Linde [42] to resolve issues regarding the requirement for initial thermal equilibrium in early universe.

Regardless of the details of aforementioned inflationary models, all of them, by construction, satisfies 'slow-roll' condition. We assume that an *inflaton* scalar field ϕ exists during an early time - on an approximately flat plateau of potential $V(\phi)$. The scenario here is ϕ starts to slowly roll down this potential $V(\phi)$ towards the minimum value. The viability of this depends on $V(\phi)$ satisfying the flatness conditions [43]

$$\epsilon(\phi) << 1,\ |\eta(\phi)| << 1 \tag{37}$$

where the parameters are defined by

$$\epsilon(\phi) = \frac{M_{PL}^2}{2}(\frac{V'}{V})^2,\ \eta(\phi) = M_{PL}^2\frac{V''}{V} \tag{38}$$

and M_{PL} is defined to be Planck mass, $V' = \frac{dV}{d\phi}$. Inflation can only be sustained if the conditions above are satisfied, it is assumed that when $\epsilon(\phi) \sim 1$, then the inflation ends.

4.2 Guth-Pi "upside-down" harmonic oscillator

The key feature of inflationary model, as we identified in previous section, is to have a "slow-rollover" phase transition. Motivated by this problem, Guth and Pi has constucted a 'toy model' that emulates the slow-rollover process [36].

The model is characterized by having the potential

$$V(x) = \frac{1}{2}kx^2,\ k > 0 \tag{39}$$

With the initial condition that at t=0 the wave function is centered at x=0, which the Guth and Pi claimed that for "simplicity" the wave function is chosen to be "Gaussian". The equation of motion in one dimension is then governed by Schrödinger's equation

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} - \frac{1}{2}kx^2\psi(x,t) \tag{40}$$

The solution is

$$\psi(x,t) = A(t)e^{-B(t)x^2} \tag{41}$$

with parameters

$$a^2 = \frac{\hbar}{\sqrt{mk}},\ w^2 = \frac{k}{m},\ b = \frac{a}{\sin(2\phi)^{1/2}} \tag{42}$$

where ϕ is an additional parameter that gives distribution of x. Guth and Pi asserted that a describes a length scale comparing to Bohr's radius. Further on with parameterization of $A(t)$ and $B(t)$ in terms of above variables, they found that for late time $t \to \infty$, the wave function becomes

$$\psi(x,t) \sim (2/\pi)^{1/4} b^{-1/2} exp[-\frac{1}{2}(wt + i\phi)] exp[-e^{-2wt}\frac{x^2}{b^2} + \frac{ix^2}{2a^2}] \tag{43}$$

Guth and Pi then computed the position-momentum commutator $[x,p]$ to be negligible in region where $x^2 >> a^2$. Further on, they asserted that the wave function 'must not be described by a classical trajectory, but instead by a classical probability distribution' which is

$$f(x,p,t) = |\psi(x,t)|^2 \delta(p - \sqrt{mk}x) \tag{44}$$

whereas two additional criteria for this distribution function $f(x,p,t)$ are imposed upon

a) f(x,p,t) must satisfy classical equation of motion

$$\frac{\partial f}{\partial t} + \dot{x}\frac{\partial f}{\partial x} + \dot{p}\frac{\partial f}{\partial p} = 0 \tag{45}$$

b) Any expectation value from ψ is equal to $f(x,p,t)$ in the limit given.

The result shows that 'classical trajectories' which described by $f(x,p,t)$ can be parametrized by

$$x(t) = Ce^{wt} \tag{46}$$

After solving for C, which thehy assumed inherited Gaussian probability distribution of x, the trajectory can be written as

$$x(t) = \pm b e^{w(t-\tau)} \tag{47}$$

where τ is the time delay in the classical solution which can be written as $C = \pm b e^{-w\tau}$.

Pilot wave solution

In pilot wave theory, one can readily use the late time solution (43) to calculate the velocity with

$$\frac{dx}{dt} = \frac{i}{2|\psi|^2}(\psi\frac{\partial\psi^*}{\partial x} - \psi^*\frac{\partial\psi}{\partial x}) \tag{48}$$

with no additional assumption, we obtain

$$\frac{dx}{dt} = \frac{x}{a^2} \tag{49}$$

which can be integrated to obtain

$$x(t) = Cexp[\frac{t}{a^2}] \tag{50}$$

where C is constant of integration and using relations in (42) we can rewrite this as

$$x(t) = Cexp[\frac{twm}{\hbar}] \tag{51}$$

Within the framework of pilot wave theory, trajectory of the particle always exists and is a characteristic of the theory. The theory also asserts that initial distribution is classical, hence a Gaussian distribution as Guth and Pi has assumed. It could also give the same characteristic result to (47),(46), as m, h are taken as constants and can be absorbed into a rescaling of t. The trajectories then automatically inherit the classicality condition imposed a) and b). The interpretation we must have for pilot wave theory solution is ,however, different from that of standard quantum theory which Guth and Pi assumed. We take that the particle beable roll down from resting with continuous trajectory at all time and coincide with classical trajectory as described by Guth and Pi in the late time limit.

It is an interesting question to ask whether a 'trajectory' given in the classical limit as defined by quantum theory will always coincide with trajectories predicted by de Broglie-Bohm theory. It is more economical in theoretical inventory to imply one continuous trajectory than to assume that such trajectory does not exist *then* show that it is statistically equivalent to having continuous existence. So far within the same approximations from Schrödinger's equations we have found no conflicting results. Future work should identify a class of solutions ψ and approximations which we can *always* take such trajectories to be classical. One definition of classical limit is the requirement to have commuting operators (equivalent to Dirac's original definition of limits of $\hbar \to 0$). One is keen to simply test ψ with Wigner function for positive definite condition. However we must be careful as Wigner function is only a 'heuristic' classical test. When one obtains a 'classical' ψ at time $t = t'$ in which we obtain an agreement between de Broglie-Bohm's trajectory and a classical approximation, future work should be put into analyzing the behavior of trajectory predicted by de Broglie-Bohm from t=0 to t=t' whether the transition is smooth or of virulent nature.

4.3 Inflaton field fluctuation

One of the most remarkable features of the quantum field theory is that vacuum expectation value of a field in flat space is non-zero. However the standard treatment is to simply ignore the subject as "it has absolutely no meaning until someone measures it" [43]. One of the key ingredients of inflation is the inflaton field ϕ which is treated as a quantum field that undergoes fluctuations $\phi+\delta\phi$. The model in which this process has been developed in detail by Muhkanov et al ([44] see also chapter 8 of [45]) in which the vacuum fluctuation of inflaton field occurs during inflation. For selective modes of the field ,microscopic quantum fluctuations are amplified to galactic scale. We shall follow [43, 46, 47] accounts in literature. Starting with a scalar field coupled to gravity, with Einstein-Hilbert action

$$S = \int d^4x\sqrt{-g}(-\frac{1}{16\pi G}R + \frac{1}{2}\partial_u\phi\partial^u\phi - V(\phi) \tag{52}$$

The background space-time is assumed to be de Sitter

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2) \tag{53}$$

During accelerated expansion, the equation of motion of ϕ is given by

$$\ddot{\phi} + 3H\dot{\phi} - \frac{\nabla^2}{a^2}\phi + \frac{dV}{d\phi} = 0 \tag{54}$$

assuming a free field $V(\phi) = 1/2m^2\phi^2$.

We are interested in an expansion of ϕ in different modes of momentum

$$\phi(\mathbf{k}, t) = \sum_p \phi_{\mathbf{k}}(t)e^{i\mathbf{k}\cdot\mathbf{r}} \tag{55}$$

with solution as superposition of plane waves

$$\phi = \frac{1}{L^{3/2}}\sum_k \sqrt{\frac{1}{2E_k}}(a_{\mathbf{k}}e^{i(\mathbf{k}\cdot\mathbf{r}-E_pt)} + a^\dagger_{\mathbf{k}}e^{-i(\mathbf{k}\cdot\mathbf{r}-E_kt)}) \tag{56}$$

where L is the size of the box that we expand the modes into to avoid infrared wavelength problem. We can re-write the Fourier component as

$$\phi(\mathbf{k}) = w_k(t)a_{\mathbf{k}} + w^*_k(t)a^\dagger_{-\mathbf{k}} \tag{57}$$

if we consider the fluctuations to for different **k** modes, it can also be expanded as

$$\delta\phi(\mathbf{k}) = w_k(t)a_{\mathbf{k}} + w_k^*(t)a_{-\mathbf{k}}^{\dagger} \tag{58}$$

Now we must solve (54) for (58), $\delta\phi_{\mathbf{k}}$ can be treated as fields with different **k** modes. Within Hubble horizon, we can neglect the mass term, thus equation of motion (54) becomes

$$\ddot{\delta\phi} + 3H\dot{\phi} + \frac{k^2}{a^2}\delta\phi = 0 \tag{59}$$

Components w_k from must also satisfy the equation of motion

$$\ddot{w_k} + 3H\dot{w_k} + \frac{k^2}{a^2}w_k = 0 \tag{60}$$

Which has the solution yielding

$$w_k = \frac{1}{(2k^3L^3)^{1/2}}(iH + k/a)exp[\frac{ik}{aH}] \tag{61}$$

The vacuum expectation value can be computed by using w_k solution. Bunch-Davies vacuum is defined by $a_{\mathbf{k}}|0\rangle = 0$. The commutation relations between creation operator $a^{\dagger}$ and annihilation operator a_k are

$$[a_{\mathbf{k}}, a_{\mathbf{p}}^{\dagger}] = \delta_{\mathbf{kp}},\ [a_{\mathbf{k}}, a_{\mathbf{p}}^{\dagger}] = [a_{\mathbf{k}}, a_{\mathbf{p}}^{\dagger}] = 0 \tag{62}$$

With the commutation relations obtained, it is straightforward to compute vacuum expectation value using (58)

$$\langle 0||\delta\phi_{\mathbf{k}}|^2|0\rangle = |w_{\mathbf{k}}|^2 \tag{63}$$

In which we can define *spectrum* as

$$P_{\delta\phi} = \frac{L^3k^3}{(\sqrt{2}\pi)^2}k^3\langle|\delta\phi|^2\rangle \tag{64}$$

using (63,61), we then have the spectrum as

$$P_{\delta\phi} = \frac{H^2}{(2\pi)^2} + \frac{k^2}{(2a\pi)^2} \tag{65}$$

We assume that modes start as $k >> a(t)H$ in the ground state, as a(t) grows exponentially the wavelength will reach Hubble radius $k = aH$ where it exits the horizon, thus we can neglect the second term and we have standard result

$$P_{\delta\phi} = (\frac{H}{2\pi})^2 \tag{66}$$

Note that in literature, conformal time is often used and a Fourier integral is preferred over summation of **k** modes. Nevertheless the result obtained in (66) is standard.

Possibility of Non-equilibrium Vacuum

The result for the power spectrum obtained earlier is obtained by standard quantum theory. Valentini [6] suggested that the vacuum wave function with a Gaussian amplitude

$$\psi_{\mathbf{k}r} = \frac{1}{\sqrt{2\pi\triangle^2}} e^{-q_{\mathbf{k}r}^2/2\triangle^2} \tag{67}$$

In standard quantum theory $q_{\mathbf{k}r}$ is treated as a random variable. But with de Broglie-Bohm theory, it can be treated as a 'beable' of the theory with de Broglie-Bohm velocity field. For equilibrium distribution $\rho = |\psi|^2$, the result will be of standard quantum theory. However for non equilibrium distribution $\rho \neq |\psi|^2$, we will obtain a different power spectrum, characterized by 'non-equilibrium factor' $\xi(k)$ such that

$$P_{\phi}^{dBB} = \frac{H^2}{4\pi^2}\xi(k) \tag{68}$$

the factor $\xi(k)$ is at this moment introduced to be an arbitrary function to be later constrained. It can lead to possibility of breaking Herrison-Zeldovich scale invariance.

There are more subtle issues regarding the analysis of this scheme concerning the quantum-to-classical transition. Although the process yields an expectation value of vacuum $\langle |\delta_{\mathbf{k}}|^2 \rangle$ of $\mathbf{k}$ modes, they are still considered to be superposition states. According to quantum theory, there is no particular value until a measurement is made. This is a severe position because such expectation value, as we have shown, have observable effect in large scale density perturbation. Requiring a measurement to be made by modern day cosmologists would have intriguing implications.

The attempt to resolve this issue has largely been from the decoherence scheme [48, 49, 50, 51, 52], with different choices of the 'environment' to cause decoherence to distinguish the fields into pointer basis. One choice of 'environment' is using entanglement between the sub-Hubble and super-Hubble modes of oscillation [49] between the short wave length modes and the long wave length modes. Burgess et al suggested that decoherence would be possible using as weak interaction as gravity. However the identification of actual source is still a pending issue. It is arguable that decoherence is fool proof because either the entanglement between sub-Hubble and super-Hubble modes or an unidentified 'environment' should be effective. To test for classicality, most of the work examined the Wigner function. The assumption is that, a postive-definite outcome of Wigner function implies a Gaussian distribution, hence classical. Other claim of classicality [48] is to ignore the non-commutativity between the defined operators because of the "growing" and "decaying" modes on super-Hubble scale. A less popular test for classicality is the test for the entanglement [52]. The work claims that entanglement that does not occur in classical mechanics, hence would be useful to another test. Nambu found by using lattice field model, when expanding a patch to super-Hubble scale, disentanglement occurs and non-commutativity of the operators becomes negligible.

However, unsatisfied with the validity of standard procedure using the decoherence scheme [46, 55], Sudarsky el al proposed an alternative "collapse scheme" [56] base on Penrose's idea of gravitationally induced collapse. The exact mechanism is assumed to depend on the nature of yet undiscovered quantum gravity theory. However they could give also give a different power spectrum to (66) , but the result depends on what they introduced as collapse parameters.

In the framework of de Broglie-Bohm theory, we rely on the same decoherence procedure to cause an effective collapse, with a possibility of having modified power spectrum, if non-equilibrium was a precondition as in (68). It is still a questionable assumption to straightforwardly use operators derived for quantum theory, and a different formalism could invalidate the other assumptions followed.

5 Conclusion and further remarks

We have reviewed the recent development of de Broglie-Bohm theory here. The fundamental theme of the applications of the theory that is different from quantum theory of Copenhagen interpretation. It is the association of trajectories to particles, which gives a more continuous picture of the world. However, the true success of the theory still lies within non-relativistic regime with solutions to Schrödinger's equation. We have shown that trajectories from de Broglie-Bohm theory has remarkably followed that of 'classical approximation' from standard quantum theory. In particular we have shown to be the case with one-dimensional harmonic oscillator toy model for slow-roll condition of Guth-Pi.

For applications into field theory, many studies has been done in bosonic theory. However difficulty has been faced with fermionic theory, where the choice of beables is not as simple. The problem of choosing a beable of the theory seems to plague into applications where it is more subtle. It requires a more definite criteria for what could be considered as fundamental beables of the theory. We have especially focused on the quantum-to-classical transition, which we have found that both quantum theory and de Broglie-Bohm theory require decoherence process to cause an apparent suppression of superposition. The positive-definite condition of Wigner function has often been used as a criteria for a system to be classical, which is also claimed to resemble Gaussian statistics. Other criteria we have found includes condition in which non-commutativity can be neglected. EPR type entanglement is also another type of criteria. We lastly reviewed the process of inflaton field fluctuations which we derived the power spectrum and shown how non-equilibrium condition can possibly modify the result. The difficulty of pilot wave approach, as we identified in last section. It is that one can not simply take any wave function from quantum theory and assign trajectories to the beables. We argued that quantum theory ignores the microscopic details while de Broglie-Bohm theory is strictly describing microscopic configurations. The attempt wiithin quantum cosmology to use the same procedure that was successful in non-relativistic Schrödinger's equation. scale factor $a(t)$ and field configuration $\psi(x)$to obtain trajectories of the beables of the system [53] . The result found is incompatible with the experimental bound of power spectrum, thus disregarded.

Consequently, we are led to question why it seems that only true success has been with non-relativistic Schrödinger's equation. It is a questionable practice whether choosing an arbitrary variable to be a beable of the system will ever lead to correct physical law. Any of such variable, if deemed to be real, must be a conserved physical quantity. One would notice that, that the correctness of the guidance equation in non-relativistic system relies on Noether's theorem. Noether's current is a fundamental , conserved quantity. It is of no surprise that we could obtain the guidance equation (4). We consider it as a reasonable question to ask whether it is a general principle that a conserved quantity of the wave function such as Noether's current can always be assigned a trajectory by assuming that the quantity obeys continuity equation. If this is the case, then we have another way of finding such quantity. We must find the conserved Noether's current which is obtained by the considering the symmetry of the system. We can subsequently find the guidance equation for such quantity. If such principle is found to exist, then there will be no need to look for new conserved quantity. We will only need the correct form of continuity equation, Noether's current and the associated wave function in order to obtain the guidance equation.

A Wigner Function

Wigner function $W(x,p)$is a function in phase space. For a system with wave function $\psi(x)$ we may obtain the Wigner function by doing transformation

$$W(x,p) = \frac{1}{2\pi\hbar}\int_{-\infty}^{+\infty} due^{ipu/\hbar}\psi^*(x+\frac{u}{2})\psi(x-\frac{u}{2}) \tag{69}$$

Which also has useful properties

$$\int dxW(x,p) = |\bar{\psi}(p)|^2 \tag{70}$$

$$\int dpW(x,p) = |\psi(x)|^2 \tag{71}$$

where $\psi\bar{(}p)$ is a Fourier transform of $\psi(x)$. Because (69) is not strictly positive, it is considered to be a *quasi-probability distribution*. A positive probability is required for the function to describe a classical probability distribution, thus in literature it is often the case that Wigner function with negative values are considered to be in quantum state, this is however only a heuristic argument.

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