An Introduction to Dynamical Supersymmetry Breaking

Thomas Walshe

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Abstract

We present an introduction to dynamical supersymmetry breaking, highlighting its phenomenological significance in the context of $4D$ field theories. Observations concerning the conditions for supersymmetry breaking are studied. Holomorphy and duality are shown to be key ideas behind dynamical mechanisms for supersymmetry breaking, with attention paid to supersymmetric QCD. Towards the end we discuss models incorporating DSB which aid the search for a complete theory of nature.

Contents

1 Introduction

The success of the Standard Model (SM) is well documented yet it is not without its flaws. The SM is an accurate effective low-energy theory at energy scales up to 100GeV but at higher energies it is believed the SM will not persist. The SM cannot be the ultimate theory valid at arbitrary energy scales.At the Planck scale ($M_{Pl} \approx 10^{19}$GeV) the SM must be modified to incorperate the effects of gravity. But why is the weak scale m_W tiny compared to the Planck scale, $m_W/M_{Pl} \approx 10^{-17}$? This is known as the Hierarchy Problem. Put another way, the Higgs scalar mass suffers from quadratic divergences which put its 'natural' value at Λ (presumably M_{Pl}) due to radiative corrections.But the Higgs mass must be of order the electroweak scale, as the scale of the electroweak interactions derive from this elementary scalar field. Clearly it cannot be equal to its natural value of M_{Pl}. So it appears that if we could somehow cancel these troublesome divergences, we can ensure the electroweak scale is of the accepted value. The theory of Supersymmetry provides a suitable mechanism where the cancellation of quadratic divergences is not only possible but is a natural consequence.The systematic cancellation of quadratic divergences to the Higgs vev is a consequence of a symmetry relating fermions and bosons - Supersymmetry. But then we are left with the question of how is the Higgs vacuum expectation value (vev) so small,how can we explain the origin of the electroweak scale? It is through Dynamical Supersymmetry Breaking that we can generate the observed hierarchy.

The universe we occupy is not supersymmetric, supersymmetry must be broken by some mechanism. We could introduce supersymmetry breaking explicitly in the Lagrangian of our theory by including terms not respecting supersymmetry, but any attempt following this approach fails in addressing the hierarchy problem. In explicit supersymmetry breaking, the supersymmetry breaking scale Λ_S would be of the same order as the natural scale of the tree level theory and thus leave us with unanswered questions. Λ_S could be be of order M_{Pl},in which case the relevance of supersymmetry to physics at ordinary energies m_W would be indirect, only making predictions about particle properties and relations. Λ_S could be of order $\sim m_W$ in which case we could have a globally supersymmetric description of physics at energies much less than M_{Pl}. To generate the observed hierarchy we require supersymmetry to be respected at tree level and that its breaking be a consequence of extremely small corrections. Supersymmetric non-renormalization theorems [4,10], teach us that perturbative effects cannot be responsible for these small corrections and therefore we seek a deeper understanding of the non-perturbative gauge dynamics in our theories. These non-perturbative effects which are suppressed by roughly $e^{-8\pi^2/g^2}$,where g is the coupling, do lead to dynamical supersymmetry breaking, giving us the hierarchy. The idea of dynamical supersymmetry breaking was first contemplated by Witten [19], and developed by many through the years.

As we will see, the vacuum energy is an order parameter for supersymmetry breaking, a consequence of the supersymmetry algebra. Following the superfield formalism of [3], the vacuum energy is determined by the potential of the supersymmetric theory.It is through analysis of this potential that one can establish supersymmetry breaking. This potential is in turn determined by two quantities, the Kähler potential, (containing kinetic terms for matter fields), and the superpotential,a holomorphic[1] function of the matter fields containing their Yukawa interactions. Holomorphy is one of the great powers of supersymmetric theories [10]. Together with the symmetries of the theory we can establish the superpotential and physical degrees of freedom, exactly what we need for studying dynamical supersymmetry breaking. Holomorphy is also the reason behind the supersymmetric non-renormalization theorem (sec.3.3).

In the early days of model building it was thought that the intimate link between supersymmetry breaking and the presence of an R symmetry was necessary [5].But models of supersymmetry breaking are now known which do not possess any R symmetries [5]. The most

[1]Holomorphic means it is a function of Φ_i and not $\Phi_i^\dagger$, equivalent to analytic

basic criteria for SUSY breaking is if a theory has a spontaneously broken global symmetry and no flat directions, the theory breaks supersymmetry.'t Hooft anomaly matching conditions may be used to argue whether a global symmetry is broken or not, eg. $SU(5)$ [6].Indeed, since the first basic model of SUSY breaking proposed by O'Raifeartaigh [21], the tools and techniques used for the search of SUSY breaking have come a long way. A key milestone in this development was the realisation by Affleck,Dine, and Seiberg that the gauge dynamics of a supersymmetric theory can generate a non-perturbative superpotential W_{ADS} [26], which can in turn lead to SUSY breaking. They studied SUSY QCD of SU(N) with F flavors,and for $F < N$, W_{ADS} is generated by either instanton effects or gaugino condensation. For $F \geq N$ Seiberg showed that duality is a vital tool for establishing SUSY breaking [32]. Once again 't Hooft anomalies played a role here as Seiberg employed the anomaly matching conditions to ensure consistency between dual theories.

Once supersymmetry breaking is established in a theory, the next question is how can we use this to describe what we already see/know? The literature is full of articles detailing the Supersymmetric extension of the SM, also known as the Minimal Supersymmetric SM (MSSM) [1,2].Here the approach to account for supersymmetry breaking is to start with a supersymmetric Lagrangian and then to introduce supersymmetry breaking explicitly through supersymmetry violating soft terms. What is meant by soft is that these terms do not reintroduce the infamous quadratic divergences of past, but logarithmic divergences which are not as problematic. Yet in order to obtain a particle spectrum consistent with the SM these soft parameters are highly constrained obeying specific relations [1]. Furthermore, in order to successfully generate the hierarchy, we employ dynamical supersymmetry breaking with supersymmetry not being broken by explicit devices. Thus, how is supersymmetry breaking communicated down to the SM? Is it that the SM particles themselves take part in the dynamical breaking or do they rely on a 'hidden' sector for this information? The two frontrunner's for possible models of supersymmetry breaking communication are the *Planck-scale-mediated supersymmetry breaking* (PMSB) scenario and the *gauge-mediated supersymmetry breaking* (GMSB) scenario. PMSB postulates that the mediating interactions are gravitational in origin, so the SUSY-breaking sector connects with the MSSM only through gravitational strength interactions. On the other hand,in GMSB models, the ordinary gauge interactions are responsible for the appearance of soft SUSY breaking terms in the MSSM. The main concern regarding these two models is the generation of an acceptable particle spectrum consistent with the SM. The most striking aspect of GMSB is that it is the gravitino which is the lightest superpartner (LSP) which may play an interesting role at the LHC if GMSB is the correct model [1].

In the next section we introduce general concepts involved in the study of dynamical supersymmetry breaking. The first two subsections deal with basic supersymmetry, with the remainder of the section developing ideas related to its breaking. Section 3 studies the most straight forward scenarios for supersymmetry breaking to occur, O'Raifeartaigh models and the Fayet-Iliopoulos mechanism. At the end of this section we elaborate upon the holomorphic nature of supersymmetry with its important consequence of non-renormalization in perturbation theory explained. Section 4 explains the mechanisms behind the dynamical breaking of supersymmetry, with our case study being supersymmetric QCD. The Affleck-Dine-Seiberg superpotential is presented in all its glory along with non-perturbative phenomenon in various regiemes. Theories with a greater number of flavor than color are then put under scrutiny as to how they evolve to dynamically break supersymmetry. To illustrate the various mechanisms presented in section 4, we present some simple models exhibiting dynamical supersymmetry breaking in section 5. A generalisation of these models leads to many more theories with desired properties. [In section 6 we discuss our current view on nature which encompasses the ideas presented so far. Finally, our conclusions are drawn about what we have found during our investigation of dynamical supersymmetry breaking.]

2 General Arguments

2.1 Supersymmetry Algebra

In this report we consider global $N = 1$ supersymmetry. The reason for restricting ourselves to global supersymmetry is because it allows the supersymmetry breaking scale to be low, leading to possible hierarchies and thus allowing us to address phenomological problems such as flavor changing neutral currents [16]. As Λ_S may be low, there is hope that the new physics involved may be seen in the forseeable future at the LHC. Promoting supersymmetry to a local symmetry involves allowing the anticommuting parameters $\xi^\alpha, \overline{\xi}_{\dot{\alpha}}$ of supersymmetry to become spacetime dependent, leading us to the rich and complex theory of 'Supergravity'. Such a theory deserves separate treatment in its own review and we will briefly refer to it in certain cases of interest. Our restriction to $N = 1$ theories is based on the fact that only these theories have chiral fields, where fields of helicity $\pm\frac{1}{2}$ transform differently, as is necessary for a faithful description of nature. A related topic to this is the interesting question of whether it is possible for supersymmetries with $N > 1$ to be spontaneously broken down to theories with $N = 1$. This is not possible for global supersymmetry [19], but it is possible for supergravity.

A supersymmetry transformation, generated by Q, transforms a bosonic state into a fermionic state and vice versa.

$$Q\,|boson\rangle = \sqrt{E}\,|fermion\rangle\,, Q\,|fermion\rangle = \sqrt{E}\,|boson\rangle \tag{1}$$

This generator Q must be an anticommuting spinor as it relates states differing by spin 1/2. Its hermitian conjugate, $\overline{Q}$, is also a supersymmetry generator, and together they satisfy an algebra of anticommutation and commutation relations of the following form:

$$Supersymmetryalgebra = \begin{cases} \{Q_\alpha, \overline{Q}_{\dot{\alpha}}\} = 2\sigma_{\alpha\dot{\alpha}}P^\mu, & \{Q_\alpha, Q_\beta\} = \{\overline{Q}_{\dot{\alpha}}, \overline{Q}_{\dot{\beta}}\} = 0, \\ [P^\mu, Q_\alpha] = [P^\mu, \overline{Q}_{\dot{\alpha}}] = 0, & [P^\mu, P^\nu] = 0 \end{cases} \tag{2}$$

$$\sigma^\mu_{\alpha\dot{\alpha}} = (1, \sigma^i), \overline{\sigma}^{\mu\dot{\alpha}\alpha} = (1, -\sigma^i) \tag{3}$$

Here the σ^i are the usual Pauli matrices:

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{4}$$

So for every fermion/boson of the SM there is a corresponding boson/fermion superpartner, and if nature respects supersymmetry then these superpartners would be degenerate in mass and gauge charge.But nature clearly does not respect supersymmetry as no superpartner of any SM particle has been found.Supersymmetry must be broken!

In order to preserve the appealing features of supersymmetry, its breaking must be spontaneous, rather than explicit. This means that the Lagrangian is supersymmetric , but the vacuum state is not invariant under a supersymmetric transformation. The vacuum state $|0\rangle$ is not invariant under supersymmetry if $Q_\alpha |0\rangle \neq 0$ and $\overline{Q}_{\dot{\alpha}} |0\rangle \neq 0$. Note that due to the first of the relations in the supersymmetry algebra (2) above, it follows the the energy (Hamiltonian operator) is given by the sum of squares of supersymmetry generators

$$H = P^0 = \frac{1}{4}(Q_1\overline{Q}_1 + \overline{Q}_1 Q_1 + Q_2\overline{Q}_2 + \overline{Q}_2 Q_2) \tag{5}$$

Thus, if supersymmetry remains unbroken in the vacuum state, it follows that $H|0\rangle = 0$ and the vacuum has zero energy. Conversely, if supersymmetry is spontaneously broken in the vacuum state, then the vacuum must have positive energy, since

$$\langle 0| H |0\rangle = \frac{1}{4}(\left\|\overline{Q}_1 |0\rangle\right\|^2 + \|Q_1 |0\rangle\|^2 + \left\|\overline{Q}_2 |0\rangle\right\|^2 + \|Q_2 |0\rangle\|^2) > 0 \tag{6}$$

if the Hilbert space is to have positive norm. Therefore this leads us to state that the vacuum energy $\langle 0| H |0\rangle$ is an order parameter for supersymmetry breaking: supersymmetry is spontaneously broken if and only if the vacuum has non-zero energy. Furthermore as already touched upon, we wish this breaking to be dynamical, arising from an exponentially small effect which naturally leads to hierarchies.

2.2 Superfield formalism

Superfields provide an efficient and elegant description of $N = 1$ supersymmetric theories. Excellent introductions to the superfield formalism can be found in [3,6,34].The idea is that 'superfields', living in a 'superspace' that has anticommuting coordinates in addition to space-time coordinates, allow us to assemble all the fields in a given supermultiplet into one entity - the superfield. Matter fields combine onto chiral superfields

$$\Phi = \phi + \sqrt{2}\theta\psi + \theta^2 F. \tag{7}$$

Here ϕ is the scalar superpartner of the fermion psi. F is the auxiliary field of the chiral supermultiplet which ensures the supersymmetry algebra closes off shell, as on-shell the algebra is automatically satisfied due to the equations of motion. From this auxiliary field,in conjunction with the vector superfields auxiliary component, D, we can calculate the potential of a supersymmetric theory.It must be noted that auxiliary fields are not dynamical.The afformentioned vector superfield contains the gauge bosons and their superpartners

$$V = -\theta\sigma^\mu\overline{\theta}A_\mu + i\theta^2\overline{\theta\lambda} - i\overline{\theta}^2\theta\lambda + \frac{1}{2}\theta^2\overline{\theta}^2 D, \tag{8}$$

where we have used Wess-Zumino gauge [35]. A_μ is the gauge field and λ is its superpartner, the gaugino. Once again the auxiliary field D is introduced to ensure the closing of the supersymmetry algebra offshell. Note we have suppressed all spinor indices above.

The most general supersymmetric theory can be written as

$$\mathcal{L} = \int d^4\theta K(\Phi^\dagger, e^{V\cdot T}\Phi) + \frac{1}{g^2}\int d^2\theta \mathcal{W}^\alpha\mathcal{W}_\alpha + h.c. + \int d^2\theta W(\Phi) + h.c. \tag{9}$$

The Kähler potential contains kinetic terms for the matter fields and also contributes gauge interaction terms to the scaler potential.The second term in equation (9) is the kinetic term for the gauge fields; $\mathcal{W}_\alpha = -\frac{1}{4}\overline{\mathcal{D}\mathcal{D}}\mathcal{D}_\alpha V$ is the supersymmetric generalisation of the gauge field strength $\mathcal{F}^{\mu\nu}$ and $\mathcal{D}_\alpha$ is a superspace derivative. The final term in (9) is the superpotential, from which the chiral auxiliary field contribution to the scalar potential comes.

The superpotential is holomorphic in the chiral fields obeying non-renormalisation theorems. It is possible to determine this superpotential in some weakly coupled theories however strongly coupled theories do not often lend themselves to detailed analysis.The K$\ddot{a}$hler potential can be a general function of both Φ and $\Phi^\dagger$.One must be careful with the K$\ddot{a}$hler potential ensuring it doesn't have singularities, or if it does, we must understand these singularities.As an example,in theories with chiral superfields only[7], the scalar potential is given by

$$V = g^{\alpha\dot{\alpha}} \partial_\alpha W \partial_{\dot{\alpha}} W^\dagger \tag{10}$$

where

$$g_{\alpha\dot{\alpha}} = \partial_\alpha \partial_{\dot{\alpha}} K \tag{11}$$

is the metric induced by the K$\ddot{a}$hler potential.It is clear that if this potential has a singularity, the potential will blow up at that point.If this theory respects supersymmetry, V must vanish,leading us to conclude that the supersymmetric ground states are related to the critical points of W.If no point exits in field space where the condition $\partial_\alpha W(\Phi^\alpha) = 0$ is not satisfied then supersymmetry is broken.

Upon expansion of our initial Lagrangian (9),the equations of motion for the auxiliary fields F and D are given by

$$\begin{aligned} F_i &= \tfrac{\partial}{\partial \Phi_i} W, \\ D^a &= g_i \textstyle\sum_i \Phi^{\dagger i} T^a \Phi^i. \end{aligned} \tag{12}$$

Here T^a denotes the gauge generators in the representations under which the chiral superfields Φ_i transform.Hence the full scalar potential from (9) is

$$\begin{aligned} V &= F^{\dagger i} F_i + \tfrac{1}{2} D^a D^a \\ &= W_i^\dagger W^i + \tfrac{1}{2} g^2 (\phi^* T^a \phi)^2 \end{aligned} \tag{13}$$

Note in the D-term contribution I changed notation by representing the chiral field Φ by its scalar component ϕ as is often done.

2.3 Flat Directions

Typically a supersymmetric theory will have a field configuration where the scalar potential vanishes.If we set the superpotential to zero, we have a pure gauge theory whose possible supersymmetric vacuum states are the points where the D-terms vanish.These points are the so called flat directions.Whenever the potential has such flat directions,the theory is said to possess a *moduli* space. For an SU(N) theory with F flavors, $F < N$, the auxiliary D field is given by

$$D^a = -g(\phi^{*in}(T^a)^m_n\phi_{mi} - \overline{\phi}^{in}(T^a)^m_n\overline{\phi}^*_{mi}. \tag{14}$$

As we wish to find configurations where the scalar potential $V = \frac{1}{2}D^aD^a$ vanishes, we are lead to finding configurations where $D_a = 0$ are satisfied. We begin by tracing field bilinears over the flavor index i, which gives the two following $N \times N$ positive semi-definite Hermitian matrices of maximal rank F,

$$\begin{aligned} d^n_m &= \left\langle \phi^{*in}\phi_{mi} \right\rangle, \\ \overline{d}^n_m &= \left\langle \overline{\phi}^{in}\overline{\phi}^*_{mi} \right\rangle. \end{aligned} \tag{15}$$

The supersymmetric vacuum then gives

$$D^a = -gT^{am}_n(d^n_m - \overline{d}^n_m) = 0. \tag{16}$$

As T^a forms a complete basis for traceless matrices, it follows that

$$d^n_m - \overline{d}^n_m = \alpha I. \tag{17}$$

By diagonalising d^n_m through an $SU(N)$ gauge transformation, $U^\dagger dU$, we form a new diagonal basis where at least $N-F$ eigenvalues of d are zero, with the rest being positive semi-definite

$$d = \begin{pmatrix} \nu_1^2 & & & & & & \\ & \nu_2^2 & & & & & \\ & & \ddots & & & & \\ & & & \nu_F^2 & & & \\ & & & & 0 & & \\ & & & & & \ddots & \\ & & & & & & 0 \end{pmatrix}, \tag{18}$$

where $\nu_i^2 \geq 0$. $\overline{d}^n_m$ is also diagonal on this basis with $N-F$ zero eigenvalues, leading to the relation, for $F < N$,

$$\overline{d}^n_m = d^n_m. \tag{19}$$

Invariance under $SU(F) \times SU(F)$ flavor transformations for these matrices follows from

$$\begin{aligned} \phi_{mj} &\to \phi_{mi} V^i_j, \\ d^n_m &\to V^{*j}_k \left\langle \phi^{*kn} \right\rangle \langle \phi_{mi} \rangle V^i_j, \\ &\to \langle \phi^{*in} \phi_{mi} \rangle = d^n_m. \end{aligned} \tag{20}$$

Thus, up to a flavor transformation, we can parameterise the classical moduli space by

$$\left\langle \overline{\phi}^* \right\rangle = \langle \phi \rangle = \begin{pmatrix} \nu_1 & & \\ & \ddots & \\ & & \nu_F \\ 0 & \dots & 0 \\ \vdots & & \vdots \\ 0 & \dots & 0 \end{pmatrix} \tag{21}$$

For $F \geq N$, a similar procedure to the above provides

$$d = \begin{pmatrix} |\nu_1|^2 & & & \\ & |\nu_2|^2 & & \\ & & \ddots & \\ & & & |\nu_N|^2 \end{pmatrix}, \tag{22}$$

and likewise for $\overline{d}^n_m$, but with eigenvalues $|\overline{\nu}_i|^2$. Again, using $SU(F) \times SU(F)$ flavor transformations, we put $\langle \phi \rangle$ and $\left\langle \overline{\phi} \right\rangle$ into the form

$$\langle \phi \rangle = \begin{pmatrix} \nu_1 & & & 0 & \dots & 0 \\ & \ddots & & \vdots & & \vdots \\ & & \nu_N & 0 & \dots & 0 \end{pmatrix}, \left\langle \overline{\phi} \right\rangle = \begin{pmatrix} \overline{\nu}_1 & & \\ & \ddots & \\ & & \overline{\nu}_N \\ 0 & \dots & 0 \\ \vdots & & \vdots \\ 0 & \dots & 0 \end{pmatrix}, \tag{23}$$

with $|\nu_i|^2 - |\overline{\nu}_i|^2 =$ independent of i. The study of the moduli space is a very important aspect to dynamical supersymmetry breaking. Their classical behavior is modified in the quantum regime leading to dynamical supersymmetry breaking in some models (section 4.3). Of course, including chiral interactions as well as gauge will constrain the moduli space as now both F- and D-terms must vanish for supersymmetry to remain unbroken.We adopt a two step approach similar to [8]:turn off the superpotential finding the D flat directions, then turn on the superpotential including non-perturbetive effects due to the gauge dynamics and see how the moduli space is affected. This approach makes use of the ADS dynamically generated superpotential which we shall discuss in section 4.2.

2.4 Global Symmetries

Global symmetries play an important role in the analysis of non-perturbative effects in gauge theories. The form of any possible non-perturbative contribution to the dynamics of our theory is restricted by these symmetries. In supersymmetric theories a new kind of global symmetry is often introduced called an R-symmetry. Symmetries which do not commute with the supersymmetry generators are called R-symmetries. Under an R-symmetry, the fermionic superspace coordinates transform as

$$\theta \rightarrow e^{i\alpha}\theta. \tag{24}$$

A general superfield with R-charge q transforms as

$$\Phi(x,\theta,\overline{\theta}) \rightarrow e^{-iq\alpha}\Phi(x, e^{i\alpha}\theta, e^{-i\alpha}\overline{\theta}). \tag{25}$$

As this symmetry does not commute with supersymmetry, the various component fields of a superfield will transform differently under the R-symmetry. For chiral superfields, the component fields transform as

$$\phi_j \rightarrow e^{iq_j\alpha}\phi_j \quad \psi_j \rightarrow e^{i(q_j-1)\alpha}\psi_j \quad F_j \rightarrow e^{i(q_j-2)\alpha}F_j. \tag{26}$$

The vector superfield on the otherhand is neutral under R-symmetry implying the gaugino in a vector superfield transforms as $\lambda \rightarrow e^{-i\alpha}\lambda$. Using the convention of letting the R-charge of the superspace coordinates θ_β equal 1 leads us to conclude that the superpotential in a theory with R symmetry, has R charge 2. Superpotential contributions to the action of our theory explicitly break R-symmetry if R-charge $\neq 2$.

Consider a theory involving chiral superfields only, $\phi_i, i = 1, ..., n$, with non-singular canonical Kähler potential $K = \sum_i \phi_i \phi_i$. As before for supersymmetry to remain unbroken we require $F_i = \frac{\partial W}{\partial \phi_i} = 0$ to be satisfied for all fields. If there are no global symmetries these conditions provide n equations for n unknowns, which for generic W are soluble. Hence supersymmetry should remain intact. If the theory has a global non-R symmetry, the end result is the same, supersymmetry is safe by the following argument. If there are k global symmetry generators, W may be written as a function of $n-k$ variables[2]. As W is now independent of k variables, k of the conditions above are automatically satisfied. The remaining equations give $n-k$ constraints for $n-k$ unknowns, typically having solutions. If the theory has a spontaneously broken R-symmetry then things work out differently.Because W carries R-charge 2 and some ϕ_m with R-charge q_m receives a vev, W can be expressed as

$$W = \phi_m^{2/q_m} f(X_i) \quad where \quad X_i = \phi_i \phi_m^{-\frac{q_i}{q_m}}. \tag{27}$$

f is a generic function of the $n-1$ variables X_i. For supersymmetry to remain unbroken, the n equations

$$\frac{\partial f}{\partial X_i} = 0 \tag{28}$$

and

$$f(X_i) = 0 \tag{29}$$

must be satisfied. But here we have n equations in $n-1$ variables and thus generically cannot be solved leading to supersymmetry breaking.

A sufficient but not necessary condition for supersymmetry breaking to occur is that if a theory has a spontaneously broken global symmetry and no flat directions, then supersymmetry is broken [18]. Consider a theory with a spontaneously broken global symmetry and no flat directions. As the global symmetry is spontaneously broken, there is a massless Goldstone boson. If supersymmetry were unbroken, this Goldstone boson would be part of a chiral supermultiplet containing an additional massless scalar. This new massless scalar describes motions along a flat direction of zero potential, contradicting our initial assumption of the theory not having any flat directions. We must drop the assumption of unbroken supersymmetry to avoid this contradiction. It may seem from the above discussion that a broken R-symmetry leads

[2]corresponding to the appropriate combinations of the ϕ_i which are singlets of the global symmetry

to broken supersymmetry. However, this criteria is only applicable to models with generic superpotentials respecting the symmetry of the theory [5]. Non-renormalisation theorems protect the superpotential in perturbation theory but the non-perturbative corrections are often non-generic in form (section 4). Non-generic superpotentials can break supersymmetry in the absence of an R-symmetry. The necessity of theories with a generic superpotential to have a spontaneously broken R-symmetry is awkward. This implies the existence of the R-axion (Goldstone boson), which must obey strong phenomenological constraints. One approach to solving this troublesome axion is to build a model with supersymmetry broken at relatively low energy such as in [16]. Other possible approaches to solving this problem involve studying the effects of higher dimension operators on the R axion mass or introducing a new strong gauge group called R color which could be a possible source for R axion mass, [5].

2.5 The Goldstino

The Goldstone theorem as applied to the Standard Model states that the spontaneous breaking of a bosonic global symmetry is accompanied by the appearance of massless Goldstone bosons. These massless particles are created through the vacuum coupling to the symmetry current. In an analogous fashion, we expect the spontaneous breaking of global supersymmetry to be accompanied by a massless Golstone fermion, Goldstino for short. This Goldstino will be created from the vacuum by the supersymmetry current J^{μ}_{α},

$$\langle 0| J^{\mu}_{\alpha} |\psi_{\beta}\rangle = f(\gamma^{\mu})_{\alpha\beta}, \tag{30}$$

where f describes the coupling of the supercurrent the the Goldstino $|\psi_{\beta}\rangle$ [19].

Proof of the Goldstino's existence may be obtained by considering the vev of the anticommutator of the generator Q_{α}, and the supercurrent, J^{μ}_{α}. This vev is related to the energy-momentum tensor, $T_{\mu\nu}$ through

$$\langle 0| \{Q_{\alpha}, J^{\mu\dagger}_{\dot{\alpha}}(x)\} |0\rangle = \sqrt{2}\sigma^{\nu}_{\alpha\dot{\alpha}} \langle 0| T^{\mu}_{\nu}(x) |0\rangle = \sqrt{2}\sigma^{\nu}_{\alpha\dot{\alpha}} E\eta^{\mu}_{\nu}, \tag{31}$$

where E is the vacuum energy density. A detailed analysis of this can be found in [6], where it is shown that the left hand side of the above equation is proportional to the integral of a total divergence:

$$\langle 0| \{Q_\alpha, J^{\mu\dagger}_{\dot\alpha}\} |0\rangle = \int d^4x \partial_\rho(\langle 0| T(J^\rho_\alpha(x) J^{\mu\dagger}_{\dot\alpha}(0) |0\rangle). \tag{32}$$

This integral can only be non-zero if there is a contribution from the surface. The surface contribution comes from a massless particle which contributes to the two-point function, or in other words from the goldstino. An important result of the above analysis is a fundamental relation between the vacuum energy density E and the coupling f:

$$E = f^2 \tag{33}$$

This simple formula reinforces the criteria for supersymmetry breaking in that for broken supersymmetry we expect a goldstino, i.e non-zero f, implying a non-zero energy density. Equivalently for supersymmetry breaking we expect a non-zero vacuum energy density, implying a non-zero f which leads to the existence of the goldstino.

Equation (30) played a key role in Wittens discussion, [19], on the general conditions facilitating dynamical supersymmetry breaking. Theories which have supersymmetry unbroken at tree level implies $f = 0$ for all possible massless fermions present at tree level. He wondered could it be possible for loop processes in the theory to give a non-zero f to some massless fermion. The answer is no, f remains 0 to all orders in perturbation theory, but the reasons behind this do not necessarily apply non-perturbatively. Hence Witten saw the potential of non-perturbative effects leading to supersymmetry breaking, allowing us to obtain a possible hierarchy of scales.

Through consideration of the fermion mass matrix of a general supersymmetric gauge theory, one finds a eigenvector of the mass matrix with eigenvalue zero,

$$\begin{pmatrix} \langle D^a \rangle / \sqrt{2} \\ \langle F_i \rangle \end{pmatrix}, \tag{34}$$

where we identify the auxiliary components of vector and chiral superfields in this eigenvector. We see that this eigenvector is nontrivial only if at least one of $\langle D^a \rangle$ or $\langle F_i \rangle$ is nonzero. From this eigenvector we identify the corresponding massless fermion field, the goldstino, as being a linear combination of the chiral and gauge fermions whose auxiliary fields F and D develop vevs.

$$\Psi_{goldstino} \sim \langle F_i \rangle \psi_i + \langle D^a \rangle \lambda^a \tag{35}$$

Once again we see that some F or D fields must acquire vevs in order for supersymmetry to be broken.

2.6 The Witten Index

An important concept in the study of supersymmetry and its breaking is the Witten index, [20]. As we see from equation(1), the action of the supersymmetry generator pairs fermionic and bosonic states of nonzero energy. As we vary the parameters of our theory, states of zero energy may move to non-zero energy or vice versa. The key point is that the states always move in fermion-boson pairs, hence the difference between the number of bosonic and fermionic states of zero energy remains constant. This lead Witten to consider the following index:

$$Tr(-1)^F = n_B^{E=0} - n_F^{E=0}. \tag{36}$$

In the case where the index is nonzero, there is at least one state of zero energy, and supersymmetry is unbroken. If the index is zero then we cannot distinguish between the following two possibilities: either $n_B^{E=0} = n_F^{E=0} = 0$ with supersymmetry broken, or $n_B^{E=0}$ and $n_F^{E=0}$ are equal but nonzero with supersymmetry unbroken. For the latter case, it may be possible to vary the parameters of the theory continuously, thereby allowing states to leave zero energy. It may be that all zero energy states leave zero energy. Hence we may have supersymmetry breaking for some set of parameters and not others, but in order to determine exactly how the theory behaves, more dynamical information is needed for classification of the ground state.

The variation of parameters just discussed must be continuous in nature. If they were not, say some non-zero parameter were set to zero, the asymptotic behavior of the potential may change. Supersymmetry may be asymptotically restored at infinity. This leads to a discontinuous change in $Tr(-1)^F$. A related idea is that of 'runaway directions' of [7] where supersymmetry may be restored at infinity for some particular field. Thus, this leads us to recognise the power of the Witten index. It is a topological invariant of the theory, independent of the parameters. The index may be calculated for any convenient choice of parameters; whichever regime one chooses, weak/strong, the result is valid generally. Witten calculated the index for supersymmetric Yang-Mills (SYM) and found it to be non-zero, thus SYM theories do not break supersymmetry. SYM with massive matter also does not break supersymmetry as this theory flows towards pure SYM upon integrating out the massive matter. The Witten index tells us nothing about vectorlike theories with flat directions. This is because the zero modes associated with the flat directions lead to a continuous spectrum of states rendering the index ill-defined at the classical level. Hence the question of supersymmetry breaking in vectorlike theories with flat directions remains an open one after consideration of the Witten index. As we'll see, there do exist such supersymmetry breaking models.

3 Tree Level Supersymmetry Breaking

3.1 O'Raifeartaigh Models

The simplest example of spontaneous supersymmetry breaking is based on an O'Raifeartaigh model [21], consisting of a theory of chiral superfields with the absence of gauge fields. Tree level supersymmetry breaking occurs, with the Lagrangian of the model being supersymmetric while the classical potential gives the vacuum a non-zero energy. With the superpotential being a polynomial in the chiral fields, we must have a linear term present in the superpotential, otherwise supersymmetry would be respected at the origin of the moduli space. This is a general feature of all O'Raifeartaigh models. The supersymmetry breaking will occur as a consequence of one or more auxiliary F components acquiring a vev. With broken supersymmetry we expect the presence of a Goldstino (section 2.5), and in O'Raifeartaigh models this is the fermionic partner to whichever F component acquired a vev, or for multiple F components obtaining vevs, the Goldstino would be as in equation (35), a linear combination.

The simplest O'Raifeartaigh model to consider is given by the superpotential

$$W = -k^2\Phi_1 + m\Phi_2\Phi_3 + \frac{y}{2}\Phi_1\Phi_3^2, \tag{37}$$

where there is also an R-symmetry with Φ_1, Φ_2, Φ_3 having R-charge $2, 2, 0$ respectively.Using $\overline{F}_i = -W_i$ it is straight forward to find $\overline{F}_1 = k^2 - \frac{y}{2}\phi_3^2$, $\overline{F}_2 = -m\phi_3$ and $\overline{F}_3 = -m\phi_2 - y\phi_1\phi_3$, leading us to the corresponding scalar potential

$$\begin{aligned} V &= |F_1|^2 + |F_2|^2 + |F_3|^2 \\ &= |k^2 - \tfrac{y}{2}\phi_3^2|^2 + |m\phi_3|^2 + |m\phi_2 + 2y\phi_1\phi_3|^2 \end{aligned} \tag{38}$$

We see that it is impossible to solve $F_1 = 0$ and $F_2 = 0$ simultaneously and thus the vacuum energy is non-zero with supersymmetry broken. For large enough m, the global minimum of the potential lies at $\phi_1 = \phi_3 = 0$ with ϕ_1 undetermined. Analysis of the particle spectrum shows ψ_1 is the goldstino as it is the auxiliary field F_1 which acquires a vev. The goldstinos scalar partner is also massless at tree level however quantum corrections to the potential result in ϕ_1 gaining a positive mass. The scalar ϕ_1 is necessarily massless at the tree level due to the model having directions of flat (non-zero) potential. The particle spectrum depends on $\langle\phi_1\rangle$, as vacua corresponding to different values of ϕ_1 are physically different. Around $\phi_1 = 0$ the scalar mass spectrum is

$$0, 0, m^2, m^2, m^2 + yk^2, m^2 - yk^2, \tag{39}$$

while for fermions we have

$$0, m, m. \tag{40}$$

For theories with tree level supersymmetry breaking there is a useful sum rule [22], which governs the tree-level squared masses of the particles in the theory. The supertrace of the tree-level squared mass eigenvalues

$$STr(m^2) \equiv \sum_j (-1)^{2j}(2j+1)Tr(m_j^2), \tag{41}$$

where j is the spin of the particle in question, satisfies the rule

$$STr(m^2) = 0 \tag{42}$$

and our O'Raifeartaigh model above does obey this rule.

3.2 Fayet-Iliopoulos Mechanism

The Fayet-Iliopoulos mechanism [25], induces supersymmetry breaking by allowing a non-zero D-term vev. D-type breaking can only occur if the gauge symmetry includes an Abelian $U(1)$ factor, which permits us to introduce a term linear in the auxiliary field of the gauge supermultiplet into the Lagrangian of the theory

$$\mathcal{L}_{F-I} = -\kappa D. \tag{43}$$

This is based upon the observation that the $\theta^2\overline{\theta}^2$ component (the auxiliary D component) of the vector superfield is both supersymmetric and gauge invariant in an Abelian theory. Let us elaborate upon this generic mechanism for supersymmetry breaking.

Consider a supersymmetric $U(1)$ gauge theory with two chiral superfields, Q and $\overline{Q}$, having $U(1)$ charges 1 and -1 respectively. We wish to illustrate D-type breaking so we take the K*ä*hler potential to be

$$K = Q^\dagger e^V Q + \overline{Q}^\dagger e^{-V}\overline{Q} + 2\kappa V \tag{44}$$

with V being the $U(1)$ vector superfield and κ is the Fayet-Iliopoulos term of dimension mass squared. There is also a non-zero superpotential given by

$$W = mQ\overline{Q}. \tag{45}$$

The scalar potential [3] of the theory is

$$V_{scalar} = |mQ|^2 + |m\overline{Q}|^2 + \frac{1}{8}|Q^\dagger Q - \overline{Q}^\dagger \overline{Q} + 2\kappa|^2. \qquad (46)$$

The F-terms of Q and $\overline{Q}$, obtained from the superpotential, are the first two terms in V_{scalar} respectively. The last term comes from the D-term of the vector supermultiplet. If $\kappa = 0$, there is a supersymmetric vacuum for $Q = \overline{Q} = 0$. For $\kappa \neq 0$ the vacuum energy determined by V_{scalar} is necessarily positive with broken supersymmetry. There are two cases of interest: $m^2 > \kappa/2$ and $m^2 < \kappa/2$. For $m^2 > \kappa/2$, the supersymmetry breaking is purely of D-type and the $U(1)$ gauge symmetry remains intact. As in equation (35), the role of the Goldstino is played by the gaugino. For $m^2 < \kappa/2$, the scalar field $\overline{Q}$ gains a negative mass and acquires a vev, thus breaking the gauge symmetry. The breaking of supersymmetry is of mixed D- and F-type, with the gaugino and fermionic components of $Q, \overline{Q}$ mixing to give the Goldstino. A detailed analysis of the spectrum can be found in [3], and the model above aptly illustrates the simple fact that non-vanishing vacuum expectation values of auxiliary fields induce supersymmetry breaking, while non-zero vacuum expectation values of dynamical scalar fields lead to the breaking of gauge symmetry.

3.3 Holomorphicity and non-renormalistation theorems

Holomorphicity of the superpotential is crucial to our understanding of supersymmetric theories. This property restricts the form of the superpotential allowing us to study the theory much deeper than an ordinary nonholomorphic potential would allow us. Powerful non-renormalisation theorems can be proved using this property, the superpotential is not renormalised in perturbation theory but can receive non-perturbative corrections generated through dynamical effects.

The first proof of the non-renormalisation theorem was provided by [11], who studied Feynman diagrams in superspace using supergraph techniques. An intuitive proof was latter provided by [12], who employed the holomorphicity of the superpotential. The idea is to promote the coupling constants of the superpotential to background chiral fields. These newly promoted fields are assigned transformation laws with respect to the symmetry of the theory, thus enlarging the symmetry of the original theory. By requiring the effective Lagrangian to be invariant under this enhanced symmetry we are lead to 'selection rules'. Next, as our effective superpotential must be holomorphic with respect to all fields, the superpotential is holomorphic in the coupling constants (background fields). These facts allow us to witness the non-renormalisation of the superpotential in perturbation theory. Also, the effective action generated by the non-perturbative dynamics is not generic in relation to the symmetries due to the restrictions imposed by holomorphy [4].

At this point we must note that the effective action we shall be considering is the Wilsonian effective action, distinct from the 1PI effective action. The two actions are equal in the absence of interacting massless particles, however the latter may not satisfy the property of holomorphicity or may have IR ambiguities when such particles are present, not so for the Wilsonian effective action.

As a simple illustration of the standard perturbative non-renormalisation theorem, let us consider the basic Wess-Zumino model, with tree level superpotential

$$W_{tree} = m\phi^2 + \lambda\phi^3. \tag{47}$$

The symmetries of the theory are, including background field transformations,

	$U(1)$	$\times$	$U(1)_R$
ϕ	1		1
m	-2		0
λ	-3		-1

It is these symmetries that lead to selection rules and together with holomorphy, we are lead to the effective superpotential of the form

$$W_{eff} = m\phi^2 h(\frac{\lambda\phi}{m}) = \sum_n a_n \lambda^n m^{1-n} \phi^{n+2} \tag{48}$$

for some function h. Through consideration of the weak coupling limit $\lambda \rightarrow 0$ which restricts $n \geq 0$ and the massless limit $m \rightarrow 0$ which restricts $n \leq 1$, it turns out that $W_{eff} = W_{tree}$, confirming that the superpotential is not renormalised.

For supersymmetry to fulfill its promise of solving the hierarchy problem we require its breaking to be dynamical and furthermore that the theory at tree level be unbroken. But the perturbative non-renormalisation theorem implies the superpotential cannot receive any perturbative contributions, facilitating a possible breaking of supersymmetry. But non-perturbative contributions generated through dynamical effects can lead to the required breaking and highlight to us the fact that the non-renormalisation theorem is violated beyond perturbation theory. The form of these non-perturbative contributions were studied in [4] and it was found that the effective action generated by the non-perturbative dynamics is not generic in relation to the symmetries due to the restrictions imposed by holomorphy.

4 Non-perturbative Gauge Dynamics

The power of supersymmetry lies in the properties of holomorphicity and nonrenormalisation of perturbative theories. The latter property guides us to consider the nonperturbative dynamics of our theory, but what kind of nonperturbative phenomena can be responsible for supersymmetry breaking? We first study supersymmetric QCD and then consider the nonperturbative dynamics leading to dynamical supersymmetry breaking. A very good introduction can be found in [6,9,32].

4.1 Supersymmetric QCD

Supersymmetric QCD is a SU(N) gauge theory with F flavors (consider first $F < N$), implying we have 2NF chiral supermultiplets. There are F flavors of quarks, Q_i in the fundamental (N) representation and $\tilde{Q}_{\tilde{i}}$ in the antifundamental $(\overline{N})$ representation$(i, \tilde{i} = 1, ...F)$. There is a global $SU(F) \times SU(F) \times U(1) \times U(1)_R$ symmetry with quantum numbers

	$SU(N)$	$SU(F)$	$SU(F)$	$U(1)$	$U(1)_R$
Q	$\boldsymbol{N}$	$\boldsymbol{F}$	$\mathbf{1}$	1	$\frac{F-N}{F}$
$\overline{Q}$	$\overline{\boldsymbol{N}}$	$\mathbf{1}$	$\overline{\boldsymbol{F}}$	-1	$\frac{F-N}{F}$

where the bar over N/F indicates the antifundamental representation. The last two global symmetries are a baryon number like transformation and a R transformation, with charges chosen so that only the non-R symmetry $U(1)$ is anomalous in the quantum theory. We have a classical D-flat moduli space given by equation (21). At a generic point in the moduli space the $SU(N)$ gauge symmetry is broken to $SU(N-F)$, leaving us with F^2 light degrees of freedom. These remaining F^2 degrees of freedom can be described by a $F \times F$ matrix field

$$M_i^j = \overline{\phi}^{jn} \phi_{ni} \tag{49}$$

where i, j are flavor indices and n is a color index. One must compute the effective action for these light degrees of freedom in order to determine if supersymmetry breaking occurs, i.e. are the flat directions lifted.

4.2 ADS superpotential

Supersymmetry breaking requires that we generate a superpotential for these light degrees of freedom. The generated superpotential must necessarily respect the original flavor symmetries of the theory. The $SU(F) \times SU(F)$ symmetry restricts the action to be a function of $det(M)$ only. The $U(1)_R$ symmetry then determines the form of the superpotential uniquely. We are left with the Affleck-Dine-Seiberg (ADS) superpotential

$$W_{ADS} = (N-F)(\frac{\Lambda^b}{detM})^{\frac{1}{N-F}} \quad (50)$$

where b is the coefficient of the β function for the gauge coupling at one-loop, and for supersymmetric QCD, $b = 3N - F$. Λ (the holomorphic intrinsic scale of our gauge theory) only enters as powers of Λ^b in order to preserve the periodicity of θ_{YM}[3]. Various consistency checks on the moduli space can be performed by constructing effective theories with fewer colors or flavors, or by adding mass terms for some of the flavors. By considering mass perturbations to the superpotential we can verify the coefficient of $N-F$ above as being correct. Consistency checks involve matching the running holomorphic gauge coupling at the mass thresholds and ensuring the two theories agree. For example, adding a mass to one flavor in a $SU(N)$ gauge theory leaves $F-1$ flavors. We could obtain the effective superpotential from the $SU(N)$ gauge with $F-1$ flavors directly or we could consider the full $SU(N)$ gauge with F flavors, integrating out the particle with mass. In either case the effective superpotentials should agree. In this case, matching the gauge coupling of the effective theory to the underlying theory at scale m gives $\Lambda_{N,F-1}^{3N-(F-1)} = m\Lambda^{3N-F}$. A similar argument applies for adding a large vev, ν, to some flavor, except in this case, the gauge symmetry breaks to $SU(N-1)$ with $F-1$ flavors. Adding masses for all F flavors provides us with the following vacuum solution for M_i^j

$$M_i^j = (m^{-1})_i^j (detm\Lambda^{3N-F})^{\frac{1}{N}} \quad (51)$$

We shall meet this relation again when we consider the moduli space for $F \geq N$.

It is understood that instanton effects are suppressed by

$$e^{-S_{inst}} \propto \Lambda^b. \quad (52)$$

So as

$$W_{ADS} \propto \Lambda^{b/(N-F)} \quad (53)$$

[3] θ_{YM} is the coupling for the CP violating $F\tilde{F}$ term in the Yang-Mills Lagrangian

we see that for $F = N-1$ it is possible for instantons to generate W_{ADS}. Another hint of such a phenomena comes from the fact that for $F = N-1$ the gauge group is completely broken. Not so for $F < N-1$. In this case the gauge group breaks to $SU(N-F)$, with gaugino condensation generating the ADS superpotential. An important point about this dynamically generated superpotential is that it does indeed remove the classical vacuum degeneracy. However as $W_{ADS} \propto \frac{1}{detM^{1/N-F}}$, this leads to a potential sloping to zero for $detM \to \infty$. Hence for $F < N$, our quantum theory does not have a ground state.

4.3 Theories with $F \geq N$

The classical moduli space for $F \geq N$ can be represented by equation (23) in section (2.3). At a generic point on this moduli space, the gauge group is broken completely, leaving us with $2NF - (N^2 - 1)$ massless chiral supermultiplets (recall from section (2.1) we originally had $2NF$ chiral supermultiplets). These light degrees of freedom are described by the gauge invariant meson and baryon operators

$$M_i^j = \overline{\phi}^{jn} \phi_{ni}, \tag{54}$$

$$B_{i_1,\ldots,i_N} = \phi_{n_1 i_1} \ldots . \phi_{n_N i_N} \epsilon^{n_1,\ldots,n_N}, \tag{55}$$

$$\overline{B}^{i_1,\ldots i_N} = \overline{\phi}^{n_1 i_1} \ldots . \overline{\phi}^{n_N i_N} \epsilon_{n_1,\ldots,n_N}. \tag{56}$$

Up to flavor transformations the moduli may be written as

$$\langle M \rangle = \begin{pmatrix} \nu_1\overline{\nu}_1 & & & & & \\ & \ddots & & & & \\ & & \nu_N\overline{\nu}_N & & & \\ & & & 0 & & \\ & & & & \ddots & \\ & & & & & 0 \end{pmatrix}, \tag{57}$$

$$\langle B_{1,...,N} \rangle = \nu_1 ... \nu_N, \tag{58}$$

$$\left\langle \overline{B}_{1,...,N} \right\rangle = \overline{\nu}_1 ... \overline{\nu}_N, \tag{59}$$

with all other components set to zero. The rank of M is at most N; if it is less than N, either B or $\overline{B}$ (or both) vanish. If M has rank $k < N$, then $SU(N)$ is broken to $SU(N-k)$ with $F-k$ massless flavors. The baryon fields and their superpartners were absent in our discussion of $F < N$ theories. Classically these fields satisfy

$$detM = B\overline{B}. \tag{60}$$

Having established the degrees of freedom for our classical theory with given moduli space, we now ask what happens in the quantum theory.

For $F \geq N$ W_{ADS} cannot be generated. There is a quantum moduli space where flat directions persist, possibly distinct from the classical moduli space. The classical moduli space contains singularities at points where the gauge symmetry is enhanced. For example, if $\nu_i = \overline{\nu}_i = 0$ for all i, the gauge symmetry is completely unbroken. We have a singularity. These singular points can be interpreted as massless particles - gluon's, which must be included in the effective description for the Lagrangian to become smooth at that point. In the quantum theory it is natural to ask whether or not these singularities persist and if so, what of their interpretation?

For the case $F = N$ we have a confining theory and analysis shows that the classical constraint equation (60) is modified to

$$detM - B\overline{B} = \Lambda^{2N}, \tag{61}$$

which is of the correct form to be a instanton effect. This new constraint gives us a smooth moduli space with no singularities present, i.e. the origin is absent. Far from the origin, the difference between the classical and quantum moduli space becomes small. This theory is an example of a theory exhibiting 'complementarity', no phase transition is invoked upon going from a Higgs phase to a confining phase, which is to be expected as there are scalars in the fundamental representation of the gauge group. As the origin is not present on the quantum moduli space, chiral symmetry breaking occurs. It is possible to obtain different patterns of global symmetry breaking at different points on the quantum moduli space. For example, at $M_i^j = \Lambda^2 \delta_i^j$, $B = \overline{B} = 0$, we have the following breaking pattern

$$SU(F)_L \times SU(F)_R \times U(1)_B \times U(1)_R \to SU(F)_d \times U(1)_B \times U(1)_R, \tag{62}$$

where d indicates the chiral symmetry is broken to a diagonal subgroup. Another point is $M = 0$, $B = -\overline{B} = \Lambda^N$, where the chiral symmetry is broken to

$$SU(F)_L \times SU(F)_R \times U(1)_R. \tag{63}$$

We use a Lagrange multiplier, X, to implement the new modified constraint with a superpotential $W = X(detM - B\overline{B} - \Lambda^{2N})$.

For $F = N+1$, the theory is once again confining with the classical constraint on the moduli space satisfied. For $F = N + 1$ flavors the baryons are flavor antifundamentals (antibaryons are fundamentals) due to being antisymmetrised in $N = F - 1$ colors,

$$B^i = \epsilon^{i,i_1,..,i_N} B_{i_1,..i_N}, \tag{64}$$

$$\overline{B}_i = \epsilon_{i,i_1,..,i_N} \overline{B}^{i_1,..,i_N}, \tag{65}$$

which leads to a new form of the classical constraints due to the new notation,

$$\begin{aligned} detM \left(\tfrac{1}{M}\right)_i^j - B_i \overline{B}^j &= 0 \\ M_j^i B_i = M_j^i \overline{B}^j &= 0. \end{aligned} \tag{66}$$

The quantum moduli space is identical to the classical one but the singularities present are interpreted differently than in the classical theory. Before we associated singularities with massless gluon's, now we interpret them as massless mesons and baryons. At the origin of the moduli space the global chiral symmetry remains intact, and theories exhibiting behavior involving confinement without chiral symmetry breaking are know as 's-confining theories', and in this case is another example of complementarity. The correct superpotential for the confined description of supersymmetric QCD with $F = N + 1$ flavors is:

$$W = \frac{1}{\Lambda^{2N-1}}[B^i M_i^j \overline{B}_j - detM]. \tag{67}$$

This can be verified by adding a mass term for some flavor and obtaining the correct form for the superpotential of the effective theory with $F = N$ flavors.

Theories with $F > N + 1$ lose asymptotic freedom for $F > 3N$ and for this range of F, the theory is in a non-Abelian free electric phase. For the region $\frac{3}{2}N < F < 3N$, Seiberg, [15], argued that the theory flows to an infrared fixed point and that in the vicinity of this point the theory admits a dual 'magnetic' description. This dual magnetic theory would have the same global symmetries as the electric theory, but a different gauge group - $SU(F - N)$. The field content of the magnetic theory consists of F flavors of quarks q and $\overline{q}$, along with a gauge singlet dual mesino:

	$SU(F-N)$	$SU(F)$	$SU(F)$	$U(1)$	$U(1)_R$
q	$\overline{\boldsymbol{(F-N)}}$	$\overline{\boldsymbol{F}}$	$\mathbf{1}$	$\frac{N}{F-N}$	$\frac{N}{F}$
$\overline{q}$	$\overline{\boldsymbol{(F-N)}}$	$\mathbf{1}$	$\boldsymbol{F}$	$\frac{-N}{F-N}$	$\frac{N}{F}$
M(mesino)	$\mathbf{1}$	$\boldsymbol{F}$	$\overline{\boldsymbol{F}}$	0	$2\frac{F-N}{F}$

The magnetic theory also flows to a fixed point, and in the presence of an allowed tree level superpotential

$$W = Mq\overline{q}, \tag{68}$$

the magnetic theory flows to the same fixed point of the electric theory.

The exact β-function of supersymmetric QCD satisfies [23]

$$\beta(g) = -\frac{g^3}{16\pi^2}\frac{3N - F + F\gamma(g^2)}{1 - N\frac{g^2}{8\pi^2}}, \tag{69}$$

where $\gamma(g^2)$ is the anomalous dimension of the quark mass term, given by

$$\gamma(g^2) = -\frac{g^2}{8\pi^2}\frac{N^2-1}{N} + O(g^4). \tag{70}$$

As was pointed out in [36], there is a IR fixed point lying below the point at which asymptotic freedom is lost in the electric theory and such a fixed point does exist for $\frac{3}{2}N < F < 3N$ [15]. At this fixed point the theory is scale invariant in the absence of any masses, leading to the theory actually being conformally invariant. Upon extension of the supersymmetry algebra to a superconformal algebra, one can identify the R-charge of the table in section (4.1) as being the superconformal R-charge. The superconformal algebra allows one to deduce the dimension of various gauge invariant operators in the theory, and hence leads to the verification of the IR fixed point being a conformal theory for the region $\frac{3}{2}N < F < 3N$. The phase of the theory in this region is a non-Abelian Coulomb phase. In this region we have dual descriptions of the physics as already noted.

In order for the descriptions of the physics to be consistent, there must be a mapping between the two sets of operators that parameterise the two moduli spaces of the dual theories. The mesons of the electric theory map to the magnetic theory singlets as they become identical in the infrared. In the magnetic theory we form baryon fields in a similar way to the electric theory

$$b^{i_1,..,i_{F-N}} = q^{n_1 i_1} ... q^{n_{F-N} i_{F-N}} \epsilon_{n_1,..,n_{F-N}}, \tag{71}$$

$$\overline{b}_{i_1,..i_{F-N}} = \overline{q}_{n_1 i_1} ... \overline{q}_{n_{F-N} i_{F-N}} \epsilon^{n_1,..,n_{F-N}}. \tag{72}$$

We then have the following mapping between baryon fields

$$B_{i_1,..i_N} \leftrightarrow \epsilon_{i_1,..i_N,j_1,..,j_{F-N}} b^{j_1,..,j_{F-N}}, \tag{73}$$

$$\overline{B}^{i_1,..,i_N} \leftrightarrow \epsilon^{i_1,..,i_N,j_1,..,j_{F-N}} \overline{b}_{j_1,..,j_{F-N}}. \tag{74}$$

The magnetic theory loses asymptotic freedom when $F \leq 3N/2$ Thus, for $N+2 \leq F \leq 3N/2$, the magnetic variables are free in the infrared while the electric variables are very strongly coupled and the theory is thus in a free non-Abelian magnetic phase. For $F > 3N$, the electric theory is free in the IR with the magnetic theory strongly coupled, thus we are in a free non-Abelian electric theory. These two phases are dual to eachother, while the non-Abelian Coulomb phase in the region $\frac{3}{2}N < F < 3N$ is self-dual, admitting two equivalent descriptions of the physics.

5 Models of Dynamical Supersymmetry Breaking

Models exhibiting dynamical supersymmetry breaking can be put into one of two classes: calculable or non-calculable. In calculable models we can completely determine the low-energy theory including the ground state energy and particle spectrum. These have immediate application in trying to find what model best describes nature. Non-calculable models on the other hand are not as fruitful in said areas, although they do give insights into arguments for establishing dynamical supersymmetry breaking. The key element in distinguishing the two cases is whether we can establish the correct degrees of freedom for the theory, thereby allowing us to determine a possible superpotential (perturbative or non-perturbative) along with the Kähler potential.

The Kähler potential can be highly problematic in both classes of DSB models. First, the main reason why non-calculable models are just that, is because the Kähler potential itself cannot be calculated. The nature of the ground state of the theory and the potential remain out of reach. Secondly, even if it can be calculated, we must worry if it has any singularities. From equations (10) and (11) see that a singular Kähler potential can lead to systems with no ground state, even if $\partial_a W(\phi^a) = 0 \forall a$ is satisfied [7]. Finally its behavior with respect to perturbation theory must be questioned. The Kähler potential is not a holomorphic function of its arguments like the superpotential and so is not protected by any non-renormalisation theorems or the like. Perturbative or indeed non-perturbative effects can influence the potential complicating any analysis. If we just wish to restrict ourselves to the establishment of supersymmetry breaking then we may take a minimal form for the Kähler potential whereby it is not singular in the fields comprising the low-energy theory. Taking a Kähler potential of canonical form, $K_{can} = \overline{X}X$, can greatly simplify calculations.

In the study of supersymmetry breaking it is helpful if the scale of supersymmetry breaking,Λ_s, is lower than the scale of the gauge dynamics,Λ, [16]. The non-perturbative effects are characterised by this strong coupling scale Λ. At lower energies than Λ, the degrees of freedom of the gauge group can be integrated out allowing us to proceed with our analysis. Generally, it is necessary to have a tree level superpotential for supersymmetry breaking to occur, and by taking the couplings in this superpotential to be small, the supersymmetry breaking scale will be lowered. Thus allowing a understanding of the Kähler potential as the resulting theory is weakly coupled. This is the case in the 3-2 model below. The $SU(5)$ model on the other hand is an example of a non-calculable model where the Kähler potential is unknown. Here the scale of supersymmetry breaking is of the same order as Λ.

5.1 3-2 model

The simplest example of a theory exhibiting dynamical supersymmetry breaking is known as the 3-2 model of Affleck, Dine, and Seiberg [26]. It is based upon a $SU(3) \times SU(2)$ gauge group with two global $U(1)$ symmetries. Matter content consists of the following chiral supermultiplets

	$SU(3)$	$SU(2)$	$U(1)$	$U(1)_R$
Q	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{3}$	1
L	$\mathbf{1}$	$\mathbf{2}$	-1	-3
$\overline{U}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\frac{-4}{3}$	-8
$\overline{D}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\frac{2}{3}$	4

Classically, this model has a moduli space parameterised by three invariants: $X = QL\overline{D}$, $Y = QL\overline{U}$,and $Z = Q^2\overline{UD}$. The most general renormalisable superpotential consistent with the symmetries is

$$W_{tree} = \lambda QL\overline{D} = \lambda X, \tag{75}$$

admitting a non-anomalous R-symmetry and $U(1)$ symmetry. A key point is that the above superpotential lifts all classical flat directions. Now we ask what non-perturbative behavior is there?

Considering first the simpler case where the $SU(3)$ interactions are much stronger than the $SU(2)$ interactions ($\Lambda_3 \gg \Lambda_2$), we find there is an $SU(3)$ instanton generated superpotential

$$W_{dyn} = \frac{\Lambda_3^7}{Z} \tag{76}$$

which is just the usual ADS superpotential obtained when considering a $SU(3)$ gauge theory with 2 flavors as in this case. Adding the dynamically generated superpotential to the tree-level superpotential we obtain

$$W_{exact} = \lambda X + \frac{\Lambda_3^7}{Z}. \tag{77}$$

This theory dynamically breaks supersymmetry. A quick way to see this is to note that W_{dyn} pushes Z away from the origin, spontaneously breaking the R-symmetry. Thus, our earlier criteria for supersymmetry breaking to occur are satisfied, i.e no flat directions with a broken global symmetry.

In the limit $\Lambda_2 \gg \Lambda_3$, supersymmetry breaking is associated with the quantum deformation of a classical moduli constraint due to the $SU(2)$ dynamics, rather than a dynamically generated superpotential arising from the $SU(3)$ dynamics. As $F = N$ for the $SU(2)$ gauge group, we employ a Lagrange multiplier field to impose the modified constraint for confinement with chiral symmetry breaking. The full superpotential which breaks supersymmetry is

$$W_{exact} = \mu[Pf(QL)(QQ) - \Lambda_2^4] + \lambda X, \tag{78}$$

where μ is our Lagrange multiplier field.[4] The fields in parenthesis are the physical degrees of freedom confined by the $SU(2)$ dynamics. This modified constraint removes the origin from the quantum moduli space thus allowing supersymmetry to be broken.

We should really consider the full superpotential

$$W = \mu[Pf(QL)(QQ) - \Lambda_2^4] + \frac{\Lambda_3^7}{Z} + \lambda X, \tag{79}$$

with no approximation about which gauge group is stronger than the other. The description of the 3-2 model can be found for arbitrary ratio Λ_2/Λ_3, exhibiting broken supersymmetry[14].

5.2 SU(5) theory

The $SU(5)$ model is not accessible to the analysis performed above. We are restricted to simply establishing supersymmetry breaking using arguments independent of the non-perturbative gauge dynamics we are familiar with. We lack knowledge of the ground state due to the supersymmetry breaking scale being comparable to the strong coupling scale of the $SU(5)$ gauge group. The matter content of the theory is that of a single antisymmetric tensor, A, and an antifundamental, $\overline{F}$. These fields have the charges $A(1,1)$ and $\overline{F}(-3,-9)$ under a $U(1) \times U(1)_R$ global symmetry. A key point to note in our discussion is that it is not possible to construct any holomorphic gauge invariants for this theory, and hence this theory has no classical flat directions.

The argument for supersymmetry breaking in the $SU(5)$ model was provided by Affleck, Dine, and Seiberg [18]. They argue that the complexity of solutions to 't Hooft's anomaly matching conditions imply the global symmetry of the theory is in fact broken. If the anomaly of a theory cannot be saturated, then that theory can only make sense as a spontaneously broken theory. The authors of [18] found that even the simplest solutions of the anomaly

[4] The Pfaffian is the square root of the determinant of an antisymmetric $N \times N$ matrix: $\mathrm{Pf}(M) = \epsilon^{i_1 \ldots i_N} M_{i_1 1_2} \ldots M_{i_{N-1} i_N}$.

matching conditions were quiet complicated, leading them to the conclusion that some, if not all, of the global symmetry is broken. Thus following our earlier criteria for supersymmetry breaking, namely the absence of flat directions in conjunction with a broken global symmetry, we conclude that supersymmetry is broken. The Witten index can be calculated for $SU(5)$ and indeed vanishes, consistent with our discussion above.

5.3 SU(7) with confinement

In this section we briefly highlight a model considered in [12] which is a very good example of how a theory can break supersymmetry through confinement. The theory is of a $SU(7)$ gauge group with two antisymmetric tensors A^i and six antifundamentals $\overline{Q}_a$, where $i = 1, 2$ and $a = 1, .., 6$. So the behavior of this theory is that of supersymmetric QCD with $F = N + 1$ and a confining superpotential. Confinement in this case breaks supersymmetry by the following argument.

When a theory confines the physical degrees of freedom are gauge-invariant fields. When we consider the low-energy regieme of the theory, our description must be in terms of these confined fields. If we have a tree-level superpotential which can lift the flat directions of the theory, then after confinement, the superpotential must be recast in terms of the new physical confining fields. It is through this re-expression can the low energy theory take the form of a O'Raifeartaigh model, thus leading to supersymmetry breaking. For our $SU(7)$ model above there is a tree level superpotential lifting flat directions given by

$$W = A^1\overline{Q}_1\overline{Q}_2 + A^1\overline{Q}_3\overline{Q}_4 + A^1\overline{Q}_5\overline{Q}_6 + A^2\overline{Q}_2\overline{Q}_3 + A^2\overline{Q}_4\overline{Q}_5 + A^2\overline{Q}_6\overline{Q}_1. \tag{80}$$

There are two confined degrees of freedom for this theory in $H = A\overline{Q}^2$ and $N = A^4\overline{Q}$, leading to the following confining superpotential

$$W = \frac{1}{\Lambda^{13}} N^2 H^2. \tag{81}$$

Upon confinement the superpotential becomes

$$W = H^1_{12} + H^1_{34} + H^1_{56} + H^2_{23} + H^2_{45} + H^2_{61} + \frac{1}{\Lambda^{13}} N^2 H^2, \qquad (82)$$

an O'Raifeartaigh type model with broken supersymmetry.

5.4 Generalisation of these models

A straight forward generalisation of the 3-2 model is based on the $SU(N) \times SU(2)$ model [13]. Matter content is similar to before

	$SU(N)$	$SU(2)$
Q	$\boldsymbol{N}$	$\mathbf{2}$
L	$\mathbf{1}$	$\mathbf{2}$
$\overline{U}$	$\overline{\boldsymbol{N}}$	$\mathbf{1}$
$\overline{D}$	$\overline{\boldsymbol{N}}$	$\mathbf{1}$,

with N odd. The classical moduli space of vacua is parameterised by the invariants $Z = Q^2\overline{UD}$, $X_1 = QL\overline{D}$, and $X_2 = PL\overline{U}$ and the gauge group is generically broken to $SU(N-2) \subset SU(N)$. There is a tree level renormalisable superpotential given by

$$W_{tree} = \lambda X_1. \qquad (83)$$

Also, the non-anomalous $U(1)_R$ and $U(1)$ remain invariant under this superpotential as before. For $\Lambda_2 < \Lambda_N$, the $SU(2)$ plays the role of spectator, providing a classical gauge potential which lifts flat directions in field space. In this regime, the non-perturbative $SU(N)$ dynamics lead to the exact superpotential

$$W = (N-2)\left(\frac{\Lambda_N^{3N-2}}{Z}\right)^{1/(N-2)} + \lambda X_1, \qquad (84)$$

breaking supersymmetry. For the regime where $SU(2)$ is strongly coupled, [13] found a confined description of the dynamics with non-perturbative effects breaking supersymmetry. Another possible generalisation of the $3-2$-model is based upon the $SU(N) \times SP(\frac{1}{2}(N-1))$

model of [14,40], where the absence of flat directions along with a dynamically generated superpotential leads to supersymmetry breaking.

There exists an infinite class of models generalising the $SU(5)$ model of section 3.2, which have an $SU(2N+1)$ gauge group [26]. Matter transforms as an antisymmetric tensor A and $2N-3$ antifundamentals $\overline{F}_i$, $i = 1, ..., 2N-3$. There exists D-flat directions for $N > 2$, which are lifted due to an R-symmetry preserving superpotential

$$W = \lambda_{ij} A \overline{F}_i \overline{F}_j. \tag{85}$$

This model is not calculable as the scale of the gauge dynamics is comparable to the scale of supersymmetry breaking, $\Lambda_{2N+1} \sim \Lambda_S$. To make progress we analyse the model ignoring the tree level superpotential, leading to a model with classical flat directions along which the effective theory reduces to the $SU(5)$ theory given in section 3.2. We already know supersymmetry is broken in this effective $SU(5)$ theory, and through consideration of the equality of couplings, in the effective theory and full theory at the energy scale threshold, we deduce that supersymmetry must be broken in the full theory too.

It is possible to analyse theories with field content obtained by decomposing an $SU(N)$ theory with an antisymmetric tensor A and $N-4$ antifundamentals $\overline{F}_i$, where $N > 5$ and is odd. The $SU(N)$ theory is decomposed into a $SU(N-M) \times SU(M) \times U(1)$ subgroup, where the two gauge groups can have different number of flavors depending on N and M. An example of such a decomposition is the '4-3-1' model of [41]. An interesting feature of such models is that in various limits (e.g.$\Lambda_{N-M} < \Lambda_M$ etc.), each of the three mechanisms of dynamical supersymmetry breaking can be seen, i.e. dynamically generated superpotential, quantum modification of moduli space, and confinement. Also, the consequences of the interplay between the gauge dynamics of the various groups can have interesting results. Dynamics of one gauge group can effect the dynamics of the other gauge group. For example, one gauge group may confine, after which the number of flavors in the other may change. Product group theories dynamically breaking supersymmetry add further candidates which may lead us to a more compelling model.

5.5 Non-chiral model with classical flat directions

We now introduce a model first considered by Intriligator and Thomas [14,37] and Izawa and Yanagida [38,39] - the ITIY model. This model is non-chiral, which in the classical theory has flat directions. Yet it breaks supersymmetry due to non-perturbative quantum effects lifting the flat directions. This model is a $SU(2)$ gauge theory with 4 fundamentals Q^i and 6 singlets S_{ij}, with a tree level superpotential

$$W = \sum_{ij} \lambda S_{ij} M_{ij} \tag{86}$$

where $M_{ij} = Q_i Q_j$. There is a global $SU(4)$ flavor symmetry, under which the Q_i transform as fundamentals and the S_{ij} transform in the antisymmetric representation. The meson fields M_{ij} describe the $SU(2)$ D-flat directions. Classically these meson fields satisfy the constraint

$$\epsilon_{ijkl} M^{ij} M^{kl} = 0. \tag{87}$$

Due to the tree level superpotential above, the meson flat directions are lifted through the F-term equations for the S_{ij} fields setting all the mesons to zero. But the singlet flat directions remain, the fields S_{ij} are arbitrary.

The quantum dynamics are described by the mesons M_{ij} and the singlets S_{ij}. Non-perturbative effects lead to the constraint (87) being modified by the introduction of a term dependent on the $SU(2)$ gauge scale Λ. We implement this constraint as before using a Lagrange multiplier, X, giving us the effective superpotential

$$W_{eff} = \lambda S_{ij} M^{ij} + X(\epsilon_{ijkl} M^{ij} M^{kl} - \Lambda^4). \tag{88}$$

From this superpotential, the F-term equations for the singlet fields still set all M_{ij} to zero. But this is in conflict with the F-term condition of the Lagrange multiplier X. This contradiction tells us that supersymmetry is broken in this non-chiral model. As discussed in section 2.6, due to the flat directions associated with the S fields, the Witten index argument fails in this case.

5.6 Meta-Stable Vacua

Consider the following theory of a single chiral field X, with superpotential

$$W = fX + \frac{1}{2}\epsilon X^2, \tag{89}$$

with ϵ a small parameter. If we are considering behavior near the origin, we may take as our Kähler potential

$$K = X\overline{X} - \frac{c}{|\Lambda|^2}(X\overline{X})^2 + ..., \tag{90}$$

with c positive. We see from our given superpotential, there is a supersymmetric vacuum at

$$\langle X \rangle_{susy} = -f/\epsilon, \tag{91}$$

and as ϵ is a small parameter, $\langle X \rangle_{susy}$ is very far from the origin. But for X close to the origin, we find

$$V(X, \overline{X}) = (K_{X\overline{X}})^{-1}|f + \epsilon X|^2 = |f|^2 + \overline{f}\epsilon X + f\overline{\epsilon}\overline{X} + \frac{4c|f|^2}{|\Lambda|^2}|X|^2 + ... \qquad (X \approx 0, \epsilon \ll 1). \tag{92}$$

Taking the derivative of V with respect to $\overline{X}$, we find a local minimum with broken supersymmetry at

$$\langle X \rangle_{meta} = -\frac{\overline{\epsilon}|\Lambda|^2}{4c\overline{f}}. \tag{93}$$

For $|\epsilon| \ll \sqrt{c}|f/\Lambda|$, this supersymmetry breaking vacuum is very far away from the supersymmetric vacuum (91). This supersymmetry breaking vacuum is called a meta-stable vacuum as it can be very long lived.

For generic models of supersymmetry breaking, it is necessary to have a global $U(1)_R$ symmetry [5]. For meta-stable supersymmetry breaking this may be extended to the existence of an approximate R-symmetry. The idea of metastable states is meaningful only when they are parametrically long lived, and for them to be applicable to any description of nature, the lifetime of our metastable state should be longer than the age of the Universe. A dimensionless parameter, ϵ, is employed to ensure the metastable state is long lived. A theory with an approximate R-symmetry has this parameter ϵ, such that for $\epsilon = 0$ the theory has an R-symmetry with broken supersymmetry. For non-zero ϵ, this R-symmetry is broken. For sufficiently small ϵ the supersymmetry breaking ground states are not ruined, the expectation values in these states are slightly deformed. Yet because $\epsilon \neq 0$, there are supersymmetric ground states, but these ground states are at field values proportional to an inverse power of ϵ and thus are very far out in field space. The tunneling between the supersymmetric states and non-supersymmetric states is sufficiently suppressed. [42,43] argue that metastable supersymmetry breaking is inevitable , leading to simpler models of supersymmetry breaking free of the classical requirements for supersymmetry breaking. Models of metastable supersymmetry breaking do not suffer from the problematic gaugino masses and light R-axions as in generic theories with an R-symmetry.

6 Gauge Mediated Supersymmetry Breaking

The MSSM incorporates supersymmetry breaking through the inclusion of soft terms [1,2], but these soft terms cannot arise at tree level as they leave no explanation as to the origin of the supersymmetry breaking scale or hierarchy. Returning to our O'Raifeartaigh model of equation (30), the parameter k is put in by hand which determines Λ_S as $\sqrt{F_1}$. This appears artificial as k will have to be fine tuned in order to generate the MSSM soft terms of correct magnitude. Also, there is no candidate gauge singlet within the MSSM, whose F-term could develop a vev. Tree-level D-type supersymmetry breaking is also unsatisfactory as a possible explanation of supersymmetry breaking in the MSSM. One could image $U(1)_Y$ as a possible source for the Fayet-Iliopoulos mechanism. Unfortunately, this fails due to an inconsistent particle spectrum. Up to this point, we have been concentrating on how supersymmetry can be broken through non-perturbative dynamics. In this section we briefly discuss strategies at communicating supersymmetry breaking to the MSSM. A good review on this topic may be found in [33].

It is expected that supersymmetry breaking occurs in a 'hidden sector', whose particles share very small couplings with the 'visible sector' chiral multiplets of the MSSM. The supersymmetry breaking is communicated between the two sectors through interactions they share, and it is this mediation of the breaking that gives rise to the soft terms of the MSSM. The two proposals for the mediating interactions are that they might be gravitational or they might be due to the ordinary electroweak and QCD gauge interactions. The Planck-scale-mediated supersymmetry breaking scenario (PMSB) leads to the soft terms in the visible sector being

$$m_{soft} \sim \frac{\langle F \rangle}{M_{Pl}}, \tag{94}$$

where supersymmetry breaking is due to a vev $\langle F \rangle$ in the hidden sector. This equation arises on dimensional grounds, we expect supersymmetry to be restored ($m_{soft} \to 0$)in the limits $M_{Pl} \to \infty$, and also $\langle F \rangle \to 0$. For $m_{soft} \sim 100GeV$, we would expect the scale of supersymmetry breaking in the hidden sector to be roughly $\sqrt{\langle F \rangle} \sim 10^{10} - 10^{11} GeV$. This scale is rather high, being beyond direct experimental reach.

An alternative to gravity mediation is gauge-mediated supersymmetry breaking (GMSB) [30], where the MSSM soft terms arise in loop diagrams involving new messenger particles. These messengers are chiral in nature, coupling to the vev $\langle F \rangle$ responsible for supersymmetry breaking, thus acquiring superpartner mass splittings, and also having $SU(3)_C \times SU(2)_L \times U(1)_Y$ charge necessary for connection to the MSSM. One finds

$$m_{soft} \sim \frac{\alpha_a}{4\pi} \frac{\langle F \rangle}{M_{mess}}, \tag{95}$$

where $\alpha_a/4\pi$ is a loop factor from Feynman diagrams, and M_{mess} is the mass scale for the messenger fields. To give m_{soft} of the right order, Λ_S could be as low as $\sim 10^4 GeV$ if M_{mess} and $\sqrt{\langle F \rangle}$ are roughly of the same order. We see that there is a possibility that gauge mediation may involve physics at scales much lower that supergravity models. Models of GMSB also have the added bonus of suppressing the problematic flavor-changing-neutral-currents (FCNCs) and CP violating effects that contradict experiment [27,31], if the mediating gauge interactions are flavor blind. Indeed, the problem of FCNCs in supergravity models was one of the motivations for considering gauge mediation [28].

Including gravity, the spin-2 graviton has the gravitino as its superpartner, a spin-$\frac{3}{2}$ fermion. Upon spontaneous breaking of supersymmetry, the super-Higgs mechanism allows the gravitino to *eat* the goldstino and thus gain mass. In PMSB models this mass is comparable to the masses of the MSSM particles $\sim 100 GeV$. The gravitino in this scenario is not witnessed in collider physics as its interactions are of gravitational strength. GMSB models in contrast predict the gravitino to be the lightest-supersymmetric-partner (LSP), with all superpartners in the MSSM decaying to it. In this case the gravitino may play a role in collider physics as through the super-Higgs mechanism, it inherits the non-gravitational interactions of the goldstino. The possibility of distinct experimental signatures is an exciting prospect.

During the 1980's, supersymmetric gauge dynamics were not well understood, leading to supergravity models being the main focus of attention. Then through the work of Seiberg and many others, gauge mediation was recognised as a distinct possibility in the 1990's. Yet questions remained whether any theories with dynamical supersymmetry breaking could accommodate the standard model. Then in 1993, Dine and Nelson [16], provided a model which served as an important "existence proof". They employed a global $SU(7)$ symmetry which was explicitly broken revealing the standard model as a $SU(3) \times SU(2) \times U(1)$ gauged subgroup. Dine et al. [40], further developed this idea of identifying the standard model gauge group with the unbroken global symmetry group of the supersymmetry breaking sector, with the only drawback being the complicated nature of their model. Their study of low energy dynamically broken supersymmetry automatically suppressed dangerous flavor-changing neutral currents and lead to a highly predictive model.

7 Conclusion

Dynamical supersymmetry breaking leads to a possible solution of the gauge hierarchy problem in theoretical physics. We have shown that non-perturbative dynamics are responsible for the breaking of supersymmetry as a result of non-renormalisation theorems in the perturbative regieme. But these non-perturbative effects are often not of generic form as a result of the interplay between holomorphy and global symmetries. As a result, R-symmetries may not play as a important role as first thought, due to the lack of generic form. But metastable supersymmetry breaking adopts a different view on R-symmetry, understanding it as being an approximate symmetry. This forces us to reconsider the role of R-symmetry in nature. One aspect which the study of dynamical supersymmetry lacks is there is no organising principle or algorithm which can tell us what theories exhibit dynamical supersymmetry breaking. Generalising simple models or considering dual models can lead to more theories of dynamical supersymmetry breaking but these generalisations are often unrelated, with differing mechanisms leading to broken supersymmetry in different regiemes. The Witten index can be a useful tool in confirming supersymmetry breaking although it is limited to chiral theories due to its inability to say anything about massless non-chiral models. The ITIY vector-like model underlined the vulnerability in the classical requirements for dynamical breaking, as does metastable symmetry breaking. Tree level supersymmetry breaking fails to address the hierarchy, but O'Raifeartaigh models do play a vital role in candidate models of nature, as once supersymmetry is fed down to the visible sector, the effective theory may take the form of an O'Raifeartaigh model. The various phenomena responsible for dynamical supersymmetry breaking were presented in section 4. The ADS superpotential has no vacuum state and thus must conspire with a tree level superpotential in order to break supersymmetry, such as in the 3-2 model. Duality, as presented by Seiberg, can often clarify phases in supersymmetric field theories. It also provides a useful method for establishing whether supersymmetry is broken or not by questioning the dual theory. The generalisation of basic models leads to a wider variety in theories with supersymmetry broken dynamically and this can only be good in the search for the most accurate model of nature. Problems in gauge mediated supersymmetry breaking such as the μ-problem require further attention. While techniques have been developed to address such problems, [40], one suspects the final theory is less cumbersome and more elegant.

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9 References

[1] S. Martin, "A Supersymmetry Primer", hep-ph/9709356.

[2] H. Haber, "Introductory Low-Energy Supersymmetry", hep-ph/9306207.

[3] J. Wess & J. Bagger, "Supersymmetry and Supergravity", (Princeton Series in Physics, 1992).

[4] N. Seiberg, "Naturalness Versus Supersymmetric Non-renormalisation Theorems", hep-ph/9309335.

[5] A.E.Nelson & N.Seiberg, "R Symmetry Breaking Versus Supersymmetry Breaking", hep-ph/9309299.

[6] J.Terning, "Modern Supersymmetry - Dynamics & Duality", (Oxford,2006).

[7] K. Intriligator & N. Seiberg, "Lectures on Supersymmetry Breaking", hep-ph/0702069.

[8] Y.Shadmi & Y.Shirman, "Dynamical Supersymmetry Breaking", hep-th/9907225.

[9] M.Dine, "Supersymmetry and String Theory:Beyond the Standard Model", (Cambridge,2007).

[10] N.Seiberg, "The Power of Holomorphy - Exact Results in 4D SUSY Field Theories", hep-th/9408013.

[11] M.T.Grisaru, W.Siegel & M.Roček, "Improved Methods for Supergraphs", Nucl. Phys. B159 (1979) 429.

[12] W.Skiba, "Dynamical Supersymmetry Breaking", hep-th/9703159.

[13] K.Intriligator & S.Thomas, "Dual Descriptions of Supersymmetry Breaking", hep-th/9608046.

[14] K.Intriligator & S.Thomas, "Dynamical Supersymmetry Breaking on Quantum Moduli Spaces", hep-th/9603158

[15] N.Seiberg, "Electric-Magnetic Duality in Supersymmetric Non-Abelian Gauge Theories", hep-th/9411149.

[16] M.Dine & A.E.Nelson, "Dynamical Supersymmetry Breaking at Low Energies", hep-ph/9303230.

[17] E.Poppitz & S.P.Trivedi, "Dynamical Supersymmetry Bre aking", hep-th/9803107.

[18] I.Affleck, M.Dine & N.Seiberg, "Dynamical Supersymmetry Breaking in Chiral Theories", Phys. Lett. 137B (1984) 187.

[19] E.Witten, "Dynamical Breaking of Supersymmetry", Nucl. Phys. B185 (1981) 513.

[20] E.Witten, "Constraints on Supersymmetry Breaking", Nucl. Phys. B202 (1982) 253.

[21] L.O'Raifeartaigh, "Spontaneous Supersymmetry Breaking for Chiral Scalar Superfields", Nucl. Phys. B96 (1975) 331.

[22] S.Ferrara, L.Girardello & F.Palumbo, "General Mass Formula in Broken Supersymmetry", Phys. Rev. D20 (1979) 403.

[23] V.A.Novikov, M.A.Shifman, A.I.Vainshtein & V.I.Zakharov, "The Beta Function in Su-

persymmetric Gauge Theories. Instantons versus Traditional Approach", Phys. Lett. B166 (1986) 329.

[24] K.Intriligator, N.Seiberg & S.H.Shenker, "Proposal For a Simple Model of Dynamical Supersymmetry Breaking", hep-ph/9410203.

[25] P.Fayet & J.Iliopoulos, "Spontaneously Broken Supergauge Symmetries and Goldstone Spinors", Phys. Lett. B51 (1974) 461.

[26] I.Affleck, M.Dine & N.Seiberg, "Dynamical Supersymmetry Breaking in Four Dimensions and its Phenomenological Implications", Nucl. Phys. B256 (1985) 557.

[27] F.Gabbiani, E.Gabrielli, A.Masiero, & L.Silvestrini, "A Complete Analysis of FCNC and CP Constraints in General SUSY Extensions of the Standard Model", hep-ph/9604387.

[28] M.Dine, A.E.Nelson & Y.Shirman, "Low Energy Dynamical Supersymmetry Breaking Simplified", hep-ph/9408384.

[29] G.Dvali, G.F.Giudice & A.Pomarol, "The μ-Problem in Theories with Gauge-Mediated Supersymmetry Breaking", hep-ph/9603238.

[30] G.F.Giudice & R.Rattazzi, "Theories with Gauge-Mediated Supersymmetry Breaking", hep-ph/9801271.

[31] J.E.Kim & H.P.Nilles, "The μ-Problem and the Strong CP-Problem", Phys.Lett. B138 (1984) 150.

[32] K.Intriligator & N.Seiberg, "Lectures on Supersymmetric Gauge Theories and Electric-Magnetic Duality", hep-th/9509066.

[33] M.A.Luty, "2004 TASI Lectures on Supersymmetry Breaking", hep-th/0509029.

[34] A.Salam & J.Strathdee, "Super-Gauge Transformations", Nucl. Phys. B76 (1974) 477.

[35] J.Wess & B.Zumino, "Supergauge Transformations in Four Dimensions", Nucl. Phys. B70 (1974) 39.

[36] T.Banks & A.Zaks, "On the Phase Structure of Vector-Like Gauge Theories with Massless

Fermions", Nucl. Phys. B196 (1982) 189.

[37] K.Intriligator & S.Thomas, "Dynamical supersymmetry breaking on quantum moduli spaces", Nucl.Phys. B473 (1996) 121.

[38] K.I.Izawa & T.Yanagida, "Dynamical supersymmetry breaking in vector - like gauge theories", Progr. Theor. Phys. 95 (1996) 829.

[39] K.I.Izawa & T.Yanagida, "Dynamical Supersymmetry Breaking in Vector-like Gauge Theories", hep-th/9602180.

[40] M.Dine, A.E.Nelson, Y.Nir & Y.Shirman, "New Tools for Low Energy Dynamical Supersymmetry Breaking", hep-ph/9507378.

[41] L.Randall, "Models of Dynamical Supersymmetry Breaking", hep-ph/9706474.

[42] K.Intriligator, N.Seiberg & D.Shih, "Supersymmetry Breaking, R-Symmetry Breaking and Metastable Vacua", hep-th/0703281.

[43] K.Intriligator, N.Seiberg & D.Shih, "Dynamical SUSY Breaking in Meta-Stable Vacua", hep-th/0602239.