A Comparative Study of the de Broglie-Bohm Theory and Quantum Measure Theory Approaches to Quantum Mechanics

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1 Introduction

1.1 The Problem of the Foundations of Quantum Mechanics

1.1.1 The History of Quantum Mechanics

There is a long background to quantum mechanics and some would trace its development over a period of more than 100 years, from the early 1800s when Thomas Young first performed the two slit experiment to the 1920s when Heisenberg [1],[2],[3] and Schrödinger [4] published their formulations of the theory. It is difficult to name a date for the completion of quantum mechanics, since, although a successful and useable theory is well established, searches continue for a more satisfactory interpretation of this theory. The aim of this dissertation is to explore briefly two interpretations of quantum mechanics and to compare them against each other and against what has become known as the ‘standard’ interpretation. It will be useful to begin with a quick look at the development of quantum mechanics and why its interpretation is still an important issue for discussion.

Starting back in 1803, Thomas Young performed an experiment investigating the behaviour of particles as they passed through two slits. This has become a famous experiment with the well-known result that the particles display wave-like interference behaviour. Fast-forwarding 100 years, 1905 saw Einstein’s discovery of the photoelectric effect which showed the particle-like behaviour of light. This work led Einstein to the development of the quantum theory of light which, he soon realised, was equivalent to the earlier hypothesis of Planck that total energy was made up of indistinguishable energy elements. In 1923 came the discovery of the Compton Effect, that when electromagnetic waves are scattered by electrons, the wavelengths are changed in the exact way as if some other particle had collided with the electron at an angle. The following year de Broglie considered all these results and wondered whether the apparent dual nature of light could be extended to a general phenomenon, that all particles display wavelike behaviour. It turned out that this was the case and this phenomenon became known as wave-particle duality.

Since Einstein’s discovery of the photoelectric effect, it had become apparent that physics was entering a new era and many people wanted to contribute. Some notable names from the time include Bohr, Rutherford and Pauli. It was Heisenberg and Schrödinger who were, however, to make the next major leaps. It was well established that there were many micro-scale phenomena which were unexplainable by classical theories. Heisenberg and Schrödinger, working independently, wanted to produce a general theory that would be able to explain these phenomena. Heisenberg approached the problem using matrix techniques and, with help from Born and Jordan, proposed a theory of matrix mechanics. Schrödinger, meanwhile, took an approach based on wave mechanics. In his theory, the motion of particles was governed by a wave function which obeyed a wave equation, now known as the Schrödinger equation. Despite the different approaches, the two theories are mathematically equivalent and were united by Dirac in his transformation theory of quantum mechanics. Dirac’s formulation of quantum mechanics has been widely used since its publication [5]. Today Schrödinger’s mathematics is better known than Heisenberg’s and is often learnt alongside a mathematical basis using linear operators on a Hilbert space, the new basis was the work of von Neumann. The field of work concerned with providing an explanation for the micro-scale phenomena was soon after named ‘quantum mechanics’ by Born.

1.1.2 Problems with Quantum Mechanics

Development of the theory of quantum mechanics came from experimental results. Once developed, it continued to be rigorously tested by experiment. Despite the predictions successfully standing up to all tests, there have, from the beginning, been many who find the theory problematic. Most of the problems are associated with the description of reality provided by quantum mechanics and often come down to a matter of how the theory is to be interpreted. Before the problems of interpretation are discussed, the causes of these concerns should be noted.

Schrödinger's theory of wave mechanics introduced a new concept: that of the wave function. Despite the successes of Schrödinger's theory, there remained confusion over what the wave function actually was. Many ideas were proposed but the understanding that is still widely accepted today is that of Born. Born suggested that the square of the wave function denotes a probability density. Integrating this probability density over some region gives the probability of measuring the particle within that region. It is well known, even by those outside the physics community, that probability is at the heart of quantum mechanics and it is in the interpretation of the wave function that it is introduced.

Although it is now generally accepted as a fundamental part of the theory, the probabilistic nature of quantum mechanics has had critics. The most prominent such critic is Einstein who had concerns over the part chance plays in the theory. In a letter to Max Born [6] he aired his opinion that he did not think quantum mechanics was yet the "real thing" and he, "at any rate, am convinced that He does not throw dice", (or more famously, "God does not play dice"). There are still some today who believe that the theory is actually deterministic but that for the moment physicists remain unable to "see" what is determining it, hence quantum mechanics appears probabilistic. What was possibly of greater importance to Einstein, however, was the lack of an objective description of reality offered by quantum mechanics.

Another major problem posed by quantum mechanics is that known as the “phenomenon of wave function collapse”. As mentioned earlier, the wave function provides the probability of measuring the location of a particle; the probability density is given by the sum of the squares of the wave function and some interference term. With this description, there is no one way of thinking about the particle. It is not the case that the particle is at some position which cannot be determined, but there is disagreement over what the state of the particle is. Some physicists would argue that the particle does not exist when only a probability density is known while others would claim that it is in all possible states. “Wave function collapse” occurs when a measurement takes place and the probability density is given by just the sum of the squares of the wave function, there is no longer an interference term. At observation the wave collapses to an actual particle. In this case the state of the system can be known. Until observation, the system is said to be in a superposition of states. The idea of a superposition of states is behind the Schrödinger’s cat paradox.

1.1.3 Interpretation of Quantum Mechanics

Quantum mechanics seems to be unique in the issues it raises. On the one hand the mathematics cannot be easily faulted as it has continually stood up to tests, but on the other hand the problems involved in understanding the theory are hard to ignore. One solution is to say that the real problem is that of interpretation, and to date many differing interpretations have been proposed. It is difficult to pinpoint the reasons behind the choice of the interpretation of many physicists, but it seems likely that philosophical thought plays an influential role. The motivation behind some interpretations, for example de Broglie-Bohm theory, seems to be the desire to present quantum mechanics as a deterministic theory. Others appear to be concerned only with the results of the theory. For example instrumentalists consider that scientific theories should not be interpreted as stating truths or claiming objective correctness, rather they are instruments for the prediction of statements that can be tested by observation. The Copenhagen interpretation meanwhile claims that it is meaningless to ask questions of what is going on at the micro-level. Both therefore disallow questions external to what information the mathematics provides. Other interpretations produce elaborate new concepts in an attempt to explain away the problems of the theory. For instance, the Many Worlds interpretation claims that at the point of observation, the wave function continues to evolve according to the wave equation and all outcomes are obtained, each in a different world. Unfortunately there are too many differing interpretations to review them all here.

A common theme in the attempt to interpret quantum mechanics is that physicists consider the interpretation of the mathematical formalism of the theory by specifying the physical meaning of the mathematical entities in the theory. Smolin [7] suggests two ways the problems of quantum mechanics can be resolved other than through a new interpretation. These require providing either a completely new theory or else a new language for quantum theory: currently quantum phenomena are described using the language of classical physics. This dissertation will however not attempt to solve the problems of the foundations of quantum mechanics. Rather its purpose is merely to examine two of the many interpretations of quantum mechanics. In particular, it will consider the interpretations they provide of experiments, see how they face new advances in physics and attempt to compare them to determine whether one is more feasible than the other.

1.2 Outline of Dissertation

The two interpretations of quantum mechanics that will be compared here are the de Broglie-Bohm theory and Quantum Measure Theory. This is an original piece of work – while interpretations of quantum mechanics have been compared before, there is currently no comparison of these particular theories. This is probably due to the vast differences between the theories, both in what they say and in how developed they are. The de Broglie-Bohm theory can be traced back as far as quantum mechanics itself and there exists a vast amount of literature related to it. Quantum Measure Theory on the other hand is quite a new theory with much of its development occurring over only the past few years. Consequently there is relatively little written about Quantum Measure Theory.

My idea for this dissertation developed while attending two short lecture courses, one on each of the theories. There was something in both theories that I found interesting. In de Broglie-Bohm theory it was the idea that a particle does have a definite momentum and position, in Quantum Measure Theory it was the construction of a system from which quantum mechanics would follow. These lecture courses were my first introduction to the interpretation of quantum mechanics: before this point I had considered the problems of quantum mechanics interesting but had not given them any thought and was only aware of the understanding I had been taught when first learning quantum mechanics. I found the idea of interpretation interesting as much of my motivation in learning physics is concerned with how theories can be understood. Consequently I wanted to pursue these two, apparently very different, theories and consider how they compare to each other. Firstly a comparison might help me to determine which I considered more likely to be a “correct” interpretation of quantum mechanics but more interestingly, it would allow me to investigate how the theories work together. I wanted to know if they contradict each other, if understanding one helps with understanding the other and if they have a future in physics.

This dissertation will therefore explore both theories in some detail to understand how they differ. This will be done by examining the predictions each makes of experiment, attempting to understand how they are coping in new areas of physics and contemplating what considerations might one day help us decide which the better theory is. A wider aim is to encourage thinking about the value of a theory: that a theory may have a purpose beyond being true, and hence that both the de Broglie-Bohm theory and Quantum Measure Theory could have uses, even if they turn out not to give the best descriptions of quantum mechanics.

The structure of the remaining dissertation is fairly simple. To begin, an overview of the standard interpretation of quantum mechanics shall be given, both as a reminder of the ideas that are often ingrained from the start of studies in quantum theory and to highlight the differences of the other theories. A description of both the de Broglie-Bohm theory and Quantum Measure Theory will follow. There will be no time to go into a lot of detail with these descriptions, but all information necessary for this comparative study will be provided.

The theories will then be compared on two specific points: their predictions for the three-slit experiment and the theories of quantum gravity that result from each. In section 5 the three slit experiment will be explained and the predictions of particle behaviour made using the standard interpretation, de Broglie-Bohm theory and Quantum Measure Theory. In section 6, quantum gravity will be reviewed and the theories resulting from both de Broglie-Bohm theory and Quantum Measure Theory will be explored. Both these sections will end with a brief summary of what has been discussed and a comparison of the theories within that situation. The dissertation will close with some concluding remarks about what has been uncovered during the study.

2 The Copenhagen Interpretation

The first widely accepted interpretation of quantum mechanics was developed in the collaborative work of Heisenberg and Bohr in Copenhagen in the late 1920s and today is known as the Copenhagen Interpretation. Despite some continued opposition to the Copenhagen Interpretation, it remains the understanding most people are taught when beginning their studies in quantum mechanics. This is not a study of the Copenhagen Interpretation and so arguments for and against the interpretation will not be considered. Rather, the main points of the interpretation will be laid out so that the de Broglie-Bohm theory and Quantum Measure Theory can be compared to the 'standard' ideas of quantum mechanics as well as to each other and to try and make clear what is fundamental to quantum mechanics and what is an element of the Copenhagen Interpretation.

The essential features of the Copenhagen Interpretation are Heisenberg's Uncertainty Principle [8], Bohr's Principle of Complementarity and the phenomenon of wave function collapse. The uncertainty principle states that certain pairs of physical properties cannot both be known to arbitrary precision. The positions and momenta of a particle are such a pair and must obey $\Delta x \Delta p \geq \frac{\hbar}{2}$, Δx is the uncertainty in position and Δp the uncertainty in momentum. Knowing the position of a particle therefore leads to a large uncertainty in its momentum, and vice versa. The principle of complementarity says that there is no solution to the problem of whether quantum events should be described by waves or particles, the physics that is known is all that can be known in answer to any question. In the phenomenon of wave function collapse, when observed, the probability density of a system no longer involves an interference term and the wave collapses to a particle. The Copenhagen Interpretation therefore denies that a particle has a definite position or momentum and claims that models of quantum reality must include the classical concepts of both a particle and a wave.

The ideas behind the Copenhagen Interpretation fit very closely with the philosophical ideas of logical positivism which were also introduced in the 1920s. In brief, logical positivists believed that ‘statements of formal logic’ and ‘statements of science’ were significant, all other statements were nonsensical. Of course a definition of what is meant by a ‘statement of science’ is required to carry on discussion but is not required here. A comparison can be drawn with nonsensical statements and the idea of the Copenhagen interpretation that it is meaningless to have a discussion about a particle until the particle is observed and it can be measured. Some people have taken this even further, saying that the observer somehow creates reality, until observation the particle is unreal and only achieves actuality upon observation. Heisenberg went so far as to claim that it is senseless to speak of the path of the particle. This has led the Copenhagen Interpretation to be associated with the idea that the collapse of the wave function is a fundamental and irreducible concept and that we should not attempt to analyse the mechanism of the collapse.

It should be noted, before examining the de Broglie-Bohm theory and Quantum Measure Theory, that there is one big difference between these and the Copenhagen interpretation of quantum mechanics. The Copenhagen interpretation does not allow for any consideration of what is happening between measurements and therefore does not give an objective description of reality – this was one of Einstein’s concerns. De Broglie-Bohm theory and Quantum Measure Theory however attempt not only to correctly predict the results of experiments but also to provide a description of reality.

3 The de Broglie-Bohm Theory

3.1 History of the de Broglie-Bohm Theory

The de Broglie-Bohm theory as it is known and used today was developed over some 25 years predominantly by Louis de Broglie and David Bohm.

In the 1920s, de Broglie proposed a non-Newtonian dynamics to try and explain the quantum phenomena that were being observed. The idea grew from his work on the unification of the 'physics of particles' with the 'physics of waves' – the work for which he is probably most famous. Following this it was found that a particle diffracted by a screen does not touch the screen and yet does not continue to move in a straight line. This led de Broglie to the idea that, at the quantum level, Newton's laws of motion should be abandoned and replaced with a new form of dynamics.

As we shall see, de Broglie carried this work through to the discovery of the equation of motion required for his non-Newtonian mechanics. When he presented his work at the Solvay conference in 1927, however, other formulations of quantum theory were given preference and eventually de Broglie abandoned this particular area of his work. De Broglie's work was rediscovered in 1952 by Bohm who considered it to have great potential as an interpretation of quantum mechanics. It was Bohm's opinion that de Broglie had merely not followed his work through to its logical conclusion [9]. Bohm then took up the task and developed de Broglie's work into the full physical theory known today.

3.2 De Broglie's Dynamics

De Broglie's work included the unification of the principles of Maupertuis and Fermat. Maupertuis' principle is an integral equation that determines the path followed by a physical system without specifying the time parameterization of the path. Fermat's principle states that the path taken between two points by a light ray is the path that can be traversed in the least time. The unification of these principles led de Broglie to a guidance equation, what he considered to be the basis for his new dynamics. Meanwhile, Schrödinger was also working on his formulation of quantum mechanics and discovered that his wave equation was correct for de Broglie waves. De Broglie's dynamics [10] was then defined by 2 equations: the guidance equation (1) and the Schrödinger equation (2). The S-function in the guidance equation is the phase of the waves.

(1)
$$m_i \frac{dx_i}{dt} = \nabla_i S$$

(2)
$$i\frac{\partial \psi}{\partial t} = \sum_{i=1}^{N} -\frac{1}{2m_i}\nabla_i^{\,2}\psi + V\psi$$

De Broglie's next step was to suggest that for the non-relativistic Schrödinger case, the wave function is associated with an ensemble of identical particles. These particles are distributed in space according to the usual quantum distribution $|\psi|^2$. De Broglie recognised two purposes for the ψ-function: it determines the location of a particle and it influences the location of the particle by exerting a force on its orbit. He thus considered the ψ-function to be a pilot-wave that guides the particles to regions of high ψ intensity.

Valentini [11] advises that when analysing the de Broglie-Bohm theory, the motivation of de Broglie should be remembered. He was not trying to provide a completion of quantum theory or attempting to solve the measurement problem, for at this time quantum theory was in the early stages of development and the measurement problem was not known. Rather, his work was based only on trying to explain experimental evidence.

3.3 Bohm's Theory

When, in 1952, Bohm rediscovered de Broglie's work, he considered it incomplete and developed it himself into a full quantum theory [9]. He chose a slightly different approach to the dynamics from that of de Broglie. Whereas de Broglie's work resulted in a non-Newtonian first-order dynamics, Bohm preferred to look for a more classical dynamics for the theory. He used the first time derivative of the guidance equation and the time dependent Schrödinger equation to achieve a second-order theory analogous to Newton's second law. It was this second-order theory that Bohm regarded as the law of motion with de Broglie's guidance equation acting as a constraint on the initial momenta. Bohm's second-order law of motion (3) and a definition of the quantum potential term (4) are given below.

$$m_i \frac{d^2 x_i}{dt^2} = -\nabla_i \left(V + Q \right) \tag{3}$$

$$Q \equiv -\sum_{i=1}^{N} \frac{1}{2m_i} \frac{\nabla_i^2 |\psi|}{|\psi|} \tag{4}$$

The introduction of the quantum potential, Q, is an important aspect of Bohm's work. Bohm considered the novelty of quantum mechanics to lie not in its statistical or discrete aspects but rather in the state of the system. This state shows itself in the motion of the particles through this new quantum potential.

Bohm was able to show that, with de Broglie's dynamics and an assumption about the initial conditions, it is possible to derive the full phenomenology of quantum theory.

3.4 The de Broglie-Bohm Theory

A good introduction to the de Broglie-Bohm theory can be found in Bohm's two papers from 1952 [9], [12]. They will be summarized briefly to get an overview of the theory and to understand how it compares with the Copenhagen Interpretation of quantum mechanics.

As explained in section 2, the Copenhagen Interpretation of quantum mechanics (Bohm calls it the "usual interpretation") centres on Heisenberg's uncertainty principle and the assumption that the physical state of a system can be most completely specified by a wave function. Only probability densities can be calculated and the probability description is inherent in matter.

Bohm considers that we should attempt to investigate the truth of this assumption. To do this he claims an alternative interpretation of quantum theory, in terms of variables which can determine the precise behaviour of an individual system, is required. It would then be possible to conceive of an individual system in a precisely definable state whose changes with time are determined by definite laws. The variables that are required for such an interpretation are considered to be "hidden" by quantum mechanics. For example, in the de Broglie-Bohm theory, the position and momentum of a particle are considered "hidden" variables because they have definite values but it is not possible to know what these are. A hidden variables interpretation must lead to the same predictions for experiment as the

Copenhagen interpretation but Bohm hoped it would provide a broader conceptual framework for understanding quantum mechanics.

Bohr's mathematical formulation leads to an equation expressing the conservation of probability. An assumption of the statistical ensemble of particles with probability density $P(\underline{x}) = |\psi(\underline{x})|^2$ is then consistent if $\psi(\underline{x})$ satisfies both Schrödinger's equation and $\underline{v} = \frac{\nabla S(\underline{x})}{m}$. The probability density of Bohr's interpretation is then numerically equal to the probability density of the Copenhagen Interpretation. The probability description arises in the de Broglie-Bohm theory because in practice it is not possible to predict or control the precise location of a particle between measurements due to disturbances introduced by the measuring apparatus. The disturbances caused by the apparatus on the observed system are unpredictable and uncontrollable. Consequently Heisenberg's uncertainty principle holds in a hidden variables interpretation. It is not an essential feature of the interpretation as in the Copenhagen interpretation, but rather an effective practical limitation on the possible precision of measurements.

The de Broglie-Bohm interpretation differs from the Copenhagen interpretation in its claim that every particle has a definite "hidden" position and momentum that determines the result of each measurement. The precise details of these variables are, however, so complicated and uncontrollable that what can be known is restricted to a statistical description of the connection between the values of the variables and the directly observable results of measurements. The reality of quantum phenomena is explained in the de Broglie-Bohm theory by the 'pilot-wave' that guides the particles. This reality consists of particles in definite positions with definite momenta, as in classical theories.

4 Quantum Measure Theory

Quantum Measure Theory is a relatively new approach to quantum mechanics. The main motivation behind its introduction is to try and find an interpretation of quantum mechanics that can be easily reconciled with the theory of gravity. The space-time nature of reality is fundamental to gravity but the concept of "state" is redundant. Quantum Measure Theory is therefore based on the concept of "histories" rather than the concept of "state". This idea was first introduced by Dirac and Feynman who showed that the dynamics of quantum mechanics can be expressed in terms of a "sum over histories".

4.1 Sums over Histories

A good introduction to the work on sums over histories can be found in Feynman's paper [13]. The formulation of quantum mechanics in this paper follows closely from Dirac's work on the transformation theory. The transformation theory has two concerns: wave-particle duality and the idea that an object may undergo changes over a period of time. Feynman, basing his work on the Copenhagen interpretation, shows clearly how measurement affects whether a system is obeying classical or quantum laws. He claims that unless attempts are made to measure a particular quantity at a particular time, it is meaningless to ask what the value of that quantity is. For example, if a particle travels between two points, A and B, it is meaningless to ask the position of the particle between those points unless an actual measurement of position is taken. When the system is observed it obeys classical laws while when unobserved it obeys quantum laws. Feynman claims that Heisenberg, in his uncertainty principle, recognised that classical laws can be false and Schrödinger, Born and Jordan and Dirac used quantum laws in their work.

Referring back to the example of a particle travelling between two points, A and B, Feynman argues that the chances of finding the particle going from A to B through several possible routes may be represented as the square of a sum of complex quantities – one for each route, as long as no attempt is made to try to determine which route is taken. The sum of complex quantities is the amplitude of routes between A and B and can be thought of as a sum over histories. The histories are the possible routes the particle could have taken. If successive position measurements are taken at successive times, separated by small time ϵ, the successive values of position effectively define a path. The particle has taken just one of the possible routes and therefore behaves classically, as expected following observation.

4.2 The Structure of Quantum Mechanics

Quantum Measure Theory is considered to be more like a classical stochastic theory (behaviour described by the theory is non-deterministic) than any Hamiltonian theory. The general form of classical stochastic theories is the axiomatic theory of probability derived by Kolmogorov [14]. The axioms of Kolmogorov's theory are set out below.

> Let E be a collection of elements which we call elementary events and $\mathfrak{I}$ a set of subsets of E; the elements of the set $\mathfrak{I}$ will be called random events.
>
> I. $\mathfrak{I}$ is a field of sets
> II. $\mathfrak{I}$ contains the set E
> III. To each set A in $\mathfrak{I}$ is assigned a non-negative number P(A). This number P(A) is called the probability of the event A.
> IV. P(E) equals 1
> V. If A and B have no element in common, then $P(A+B) = P(A) + P(B)$

The difference between this and a measure theory is that in a measure theory, the non-negative number assigned to each set is not called the probability of the event but the measure of the event and is usually denoted by μ(A), for an event A. Axiom IV above does not hold in measure theory, the measure of the set of elementary events, E, can be greater than 1. In a classical theory the dynamics and initial state are encoded in the measure μ.

Quantum Measure Theory presents quantum mechanics with a structure similar to the form of stochastic theories and uses the idea of Dirac and Feynman that quantum mechanics can be formulated in terms of space-time histories. Quantum mechanics therefore consists of: a sample space Ω, of possible space-time histories, analogous to the elementary events, E; an event algebra ζ, whose elements are subsets of Ω, analogous to ℑ and a measure μ, on ζ, which encodes the dynamics and initial state of the theory, analogous to the probability of an event, P(A). It should be noted that the measure of Quantum Measure Theory is different from the measure of classical measure theories. The quantum measure μ(A) can be taken, roughly speaking, as the modulus squared of the sum of the quantum amplitudes of all the histories in event A.

4.3 Sum Rules

Interference is the clearest way to see the distinction between a quantum and a classical theory. The interference term between two disjoint events can be written as,

$$I\left(A,B\right) := \mu\left(\mathrm{A} \coprod B\right) - \mu\left(A\right) - \mu\left(B\right)$$

where $A \coprod B$ is the union of disjoint events A and B. In quantum theory the interference term does not equal zero due to quantum interference but in classical theory it is trivially zero.

The interference term can be extended to any number of disjoint events. The first three are given here.

$$I_1 \equiv \mu(A)$$

(5)

$$I_2 \equiv \mu(A \coprod B) - \mu(A) - \mu(B)$$

(6)

(7) $$I_3 \equiv \mu(A \coprod B \coprod C) - \mu(A \coprod B) - \mu(B \coprod C) - \mu(A \coprod C) + \mu(A) + \mu(B) + \mu(C)$$

Sorkin [15] associates each of these interference terms with a sum rule, obtained by setting the interference term to zero. Each sum rule can then be associated with a different ‘level’ of measure theory. If a theory satisfies I_1=0 then it is a level 0 theory, if a theory satisfies I_2=0 then it is a level 1 theory etc. The first of the sum rules (5) is uninteresting. The second (6) gives that the interference between two disjoint events is zero. This is the Kolmogorov Sum Rule and is satisfied by classical theories. The third (7) extends to the interference between three disjoint events being zero and is satisfied by quantum theories. Further sum rules define more general forms of measure theory which could be extensions to quantum theory. It is not likely, however, that such a theory will be postulated in the foreseeable future, if ever. As a result of these hierarchical sum rules it can be seen that classical theory is a special case of quantum theory.

Quantum Measure Theory demands a modification to the logic used in reasoning about reality, ‘classical logic’ [16]. An example shall be given to illustrate why a change in logic is required, but first a quick summary of classical logic shall be given. There are two principles of classical logic that will be considered: the law of non-contradiction and the law of excluded middle. The law of non-contradiction asserts that if one sentence is the negation of another, they cannot both be true. The law of excluded middle asserts that if one sentence is the negation of another, one of them has to be true. These principles can be derived from two inference rules of classical logic, the rule of 'not'-introduction and the rule of 'double negation elimination' [17].

The example considers two sets, A and B, which contain histories and together form the whole sample space. The reality of both classical logic and Quantum Measure Theory shall be discussed within this sample space and through consideration of the law of non-contradiction and the law of extended middle it shall be seen why Quantum Measure Theory demands a non-classical logic.

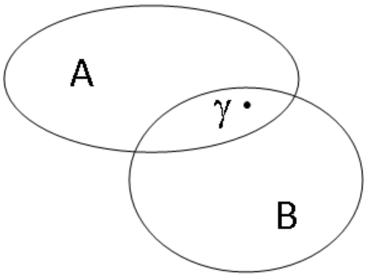

Figure 1 A sample space with a single history, γ.

Each of the sets A and B contain possible histories and together A and B form the whole sample space. In classical logic, reality is one history, γ, that will be somewhere in the sample space. The terminology that is used is such that '"A" is true' refers to the history, γ, being in set A, '"A" is false' refers to the history, γ, not being in set A and "not A" is the negation of "A". Using the law of non-contradiction and the law of excluded middle the following statements can be made.

If "A" is true and "B" is true then "A and B" is true.

If "B" is true then "not B" is false.

If "A" is false then "not A" is true.

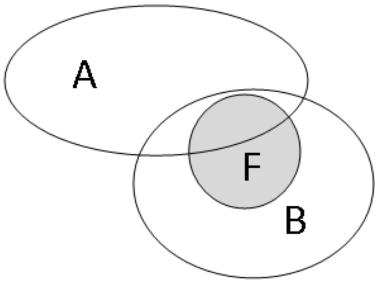

Figure 2 A sample space with a set of histories, F.

In Quantum Measure Theory, reality is a set of histories, F. Because the reality is a set of histories rather than a single history, the statements of classical logic made above do not all hold. Before looking at which statements do and do not hold, however, an explanation of the terminology in Quantum Measure Theory is needed. For reality as a set of histories, '"A" is true' means that the whole set of histories, F, is contained within the set A, '"A" is false' means that not all histories in F are contained within A and '"not A" is true' means that none of the histories in F are contained in A.

If "A" is true and "B" is true then "A and B" is true.	**Holds**
If "B" is true then "not B" is false.	**Holds**
If "A" is false then "not A" is true.	**Does not hold**

The last statement does not hold because the set F overlaps with the set A but is not completely contained within A. '"A" is false' holds because not all histories in F are contained in A. This does not however imply that '"not A" is true'. Neither all histories in F nor no histories in F are contained in A therefore neither '"A" is true' nor '"not A" is true' hold. Reality as a set of histories violates the law of excluded middle, both "A" and "not A" are true. It is for this reason that Sorkin demands a change in logic for Quantum Measure Theory.

Sorkin argues that by Cournot's principle[1], the set of histories that happens must not be contained in a set of measure zero. Using this, three rules for Quantum Measure Theory [16] can be written that must be obeyed by the "real" set of histories.

Given a "real" set of histories F,

1. if F is a subset of event X then X is true, otherwise X is false,
2. F must not be contained in an event of measure zero,
3. F must be minimal.

By minimal it is meant that the set of histories must be as small as possible, or contain as few elements as possible, subject to rule 2. The motivation for the third rule is obtain that "one history happens" in the classical limit. It will be found is that the "real" set of histories, F, contains just one element for classical theories. This will be seen in the example of the two-slit experiment in Quantum Measure Theory in section 5.1.2.

One last point that must be considered is the interpretation of trajectories. What can be said about particle trajectories in Quantum Measure Theory is dependent on the situation under discussion. It is the reality that determines what can be said. In a classical theory, only one history happens, or alternatively, the reality is a single history. Although the information available may not be sufficient to determine which particular history has occurred, it is known that only one happens. For example, in the two slit experiment, which is classical in Quantum Measure Theory (this shall be discussed later), it is not known which slit the particle travels through but it is known that it goes through only one. In a quantum theory however, the reality is a set of histories. An example of this is the three slit experiment which shall be explored in some detail in the next section. When the reality is a set of histories a definite trajectory cannot be calculated. Instead questions can be asked and the yes or no

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[1] Cournot's principle says that if the measure of an event is very small, $\mu(A)<<1$, then it can be assumed that this event does not happen, it is precluded. Early references to Cournot's principle can be found in Bernoulli's *Ars Conjectandi* (1713) [18].

responses to these questions give all the information that can be known. This is a complicated idea but hopefully will become clearer in the example of the three slit experiment.

5 The Three Slit Experiment

5.1 The Problem

The triple slit experiment shall be investigated using each of the Copenhagen, de Broglie-Bohm and Quantum Measure Theory interpretations of quantum mechanics. This will allow an easy comparison of the predictions that can be made by the three theories. It will also be of interest to see if the predictions differ and if so, how. This may then prove fruitful in a discussion of whether the theories can stand alongside each other or whether a strong leaning towards one suggests the others should be forgotten. Before these investigations, consideration should be given to why the three slit experiment is used rather than the two slit experiment. After all, in his Lectures on Physics [19] Feynman makes the claim that the double slit experiment is a phenomenon that cannot be explained in any classical way and "has in it the heart of quantum mechanics".

5.1.1 The Two Slit Experiment

As in Feynman's Lectures on Physics, the two slit experiment is often presented to introduce the oddities of quantum mechanics. It shall be assumed that the experiment is familiar and only a brief summary of the important results now follows.

In figure 3, the source of particles is located at A, the slits on a plane at B and the screen at which the particles are detected at C. It is taken that the slits are identical and the source of particles lies on the horizontal axis in the centre of the two slits.

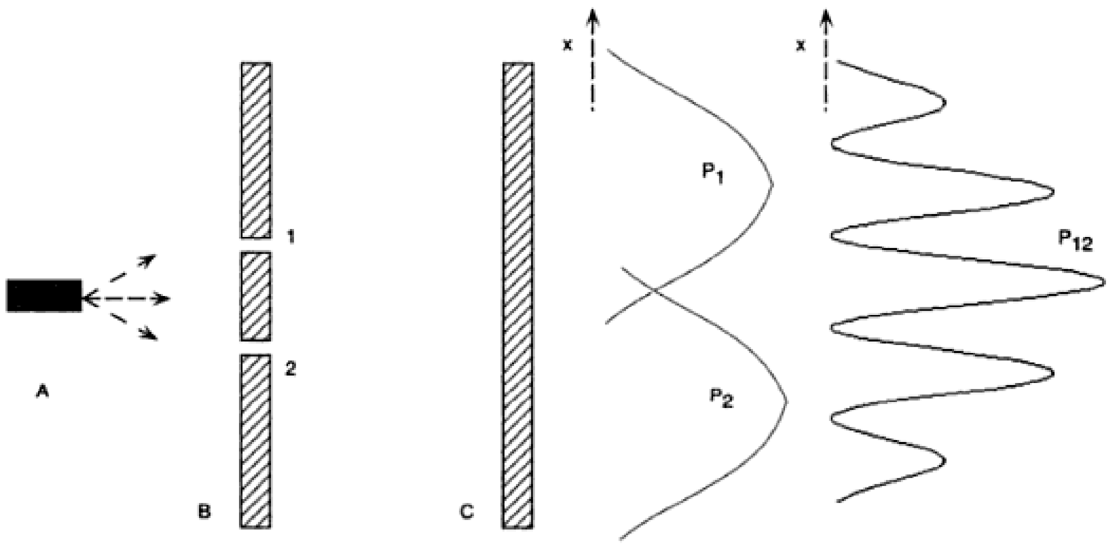

Figure 3 The two-slit experiment with distributions for when only slit 1 or only slit 2 is open (distributions P_1 and P_2) and when both slits are open (distribution P_{12}). (Taken from The Odd Quantum, page 83 [20])

The distributions shown in the diagram are the distributions of particles detected at the screen as a function of position x on the detector surface. P_1 and P_2 are the respective distributions for the cases where only slit 1 or only slit 2 is open. P_{12} is the distribution for when both slits are open.

Distributions P_1 and P_2 are exactly as would be predicted for particles passing through a single, small slit. Even in a classical framework it is expected that the distribution will broaden slightly as the particles traverse between the slit and screen. With both slits open it might be expected that the distribution would be given by the sum of P_1 and P_2 as the particles can only travel through one slit or the other but this is clearly not the case. P_{12} is not a sum of P_1 and P_2 but rather a distribution that would be observed due to the interference of waves travelling through both slits.

Further confusions are added when observations are made to try and determine which slit each particle travels through. A detector is placed at each slit as well as at the screen and the particles are sent through one at a time. It is found that the particles travelling through slit 1 have distribution P_1 and those travelling through slit 2 have distribution P_2. The overall distribution is the sum P_1+P_2 and no longer P_{12}. Making observations of the experiment has changed the outcome.

The conclusion then must be agreement with Feynman. Particles have a wavelike character, confirming the wave-particle duality of quantum mechanics and observation of a system changes the outcome. To consider why the three slit experiment must be used in the succeeding investigations, arguments from Quantum Measure Theory must be made.

5.1.2 The Classical Two Slit Experiment

A standard two slit set-up shall again be considered with two identical point slits A and B an equal distance from the central dotted line. Two points on the screen, D and E, will be analysed.

Figure 4 The Two Slit Experiment

Point D lies on the screen on the horizontal axis midway between the two slits. From experiments it is known that constructive interference occurs at point D such that when light is used as the source, a bright fringe is seen here. As a particle is free to travel through either of the slits, both the measure of a particle travelling through A and hitting the screen at D, $\mu(A\&D)$, and the measure of a particle travelling through B and hitting the screen at D, $\mu(B\&D)$, are non-zero. Because there is a known, positive detection of particles at D it must also be the case that the measure of a particle travelling through either A or B and hitting the screen at D, $\mu(A\coprod B\&D)$, is non-zero.

The rules of Quantum Measure Theory must now be applied. Event X is taken to be the particle travelling through the two slits and hitting the screen at D, X = {A, B & D}, it is known from experiment that this event is true. Using the rule 1 it therefore follows that the "real" set of histories, F, is a subset of X. The possible subsets of X are {A & D}, {B & D} and {A, B & D}. It has already been shown that the measures of the corresponding events are all non-zero. By rule 2 it is then the case that F could be contained in any of these sets. Rule 3 tells us that the set F must be minimal. {A, B & D} contains both {A & D} and {B & D} and so is not minimal. {A & D} and {B & D} are minimal, each containing just one element. The "real" set of histories, F, is either {A &D} or {B &D} and as each of these sets contains just one element, the reality is a single history; the particle will travel 'through A' or 'through B'.

Point E is taken as a point on the screen at which there is known destructive interference. Again the particles are free to travel through either of the slits so both $\mu(A\&E)$ and $\mu(B\&E)$ are non-zero measures. However due to the known absence of particles hitting the screen at E it must be the case that $\mu(A\coprod B\&E)$ is zero. As the particle cannot take the path 'through A or B' to reach E, rule 2 of Quantum Measure Theory tells us it cannot take either of the paths 'through A' or 'through B' to reach E.

The results of the two slit experiment in Quantum Measure Theory are that constructive and destructive interference occurs as is expected from experiment but that one history can happen. The logic is classical although the interference term ($\mu(A \coprod B) \neq \mu(A) + \mu(B)$) shows that the measure is not. In order to make comparisons between the different interpretations an experiment that is quantum in all three cases is required. The three slit experiment is the simplest such example.

5.2 The Copenhagen Solution

One of the main arguments of the Copenhagen Interpretation of quantum mechanics is that it is meaningless to ask about the position of a particle when it is not being detected. This means that if the trajectory of the particle is not being continually tracked, no information other than the start and end points of the trajectory can be known. However, continual tracking of the particle involves observing the particle at all points along its path which would alter the behaviour of the particle. What then can be asked about the Copenhagen interpretation of the three slit experiment?

Although nothing can be said of the trajectories, a few comments can be made about the expected distribution of particles on the screen. If only one of the slits was open the distribution of particles on the screen would be like P_1 and P_2 in the two slit example. Similarly, if detectors were placed at the slits then the particles through each of the slits would have this classical distribution. However, if two slits were open and the particles remained undetected until hitting the screen, the distribution would be like that of P_{12} in the two slit example with a wavelike interference pattern. The different combinations of two slits would not change the shape of the distribution but its position depends on which slits are open. In the case with three slits open and no detection, the expected interference pattern is slightly different from that resulting when two slits are open. From experiment it is known that the distribution should be something like the following.

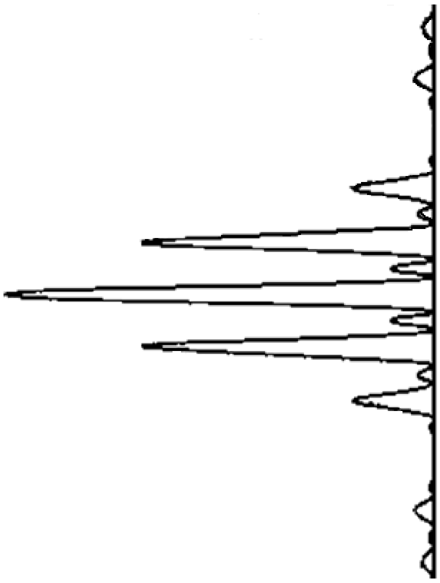

Figure 5 Three Slit Interference Pattern

For the purposes of comparison with other interpretations, it is meaningless to ask about the trajectories of the particles in the three slit experiment. All that can be known is that a wavelike interference pattern will be seen and can be described by a superposition of wave functions through all three slits. The pattern seen in the Copenhagen Interpretation should be the same as those predicted in the de Broglie-Bohm and Quantum Measure Theory interpretations because these try to give further explanation to the known, ‘standard’ results. Only by agreeing with these results can they become successful theories.

5.3 The de Broglie-Bohm Solution

Before investigating the triple slit experiment under the de Broglie-Bohm theory it will be useful to consider the work that has already been done with regards the double slit experiment.

5.3.1 The de Broglie-Bohm Two Slit Experiment

The following is based on the work of Philippidis *at al.* (1979) [21] who produced the first images of the possible trajectories of particles in the two slit experiment under the de Broglie-Bohm theory. The wave functions were calculated using the path integral method.

The origin lies on the dashed line, in the centre of the slits, denoted by x (figure 6). The electron source S_1 has coordinates (-X, 0) and the centres of slits have coordinates (0, Y), (0, -Y). The case for which a single particle passes through point *a* inside slit A at a distance δY from its centre shall be considered for calculation. The final point on the screen, D, has coordinates (x, Y+η) where η is measured from the centre of the slit.

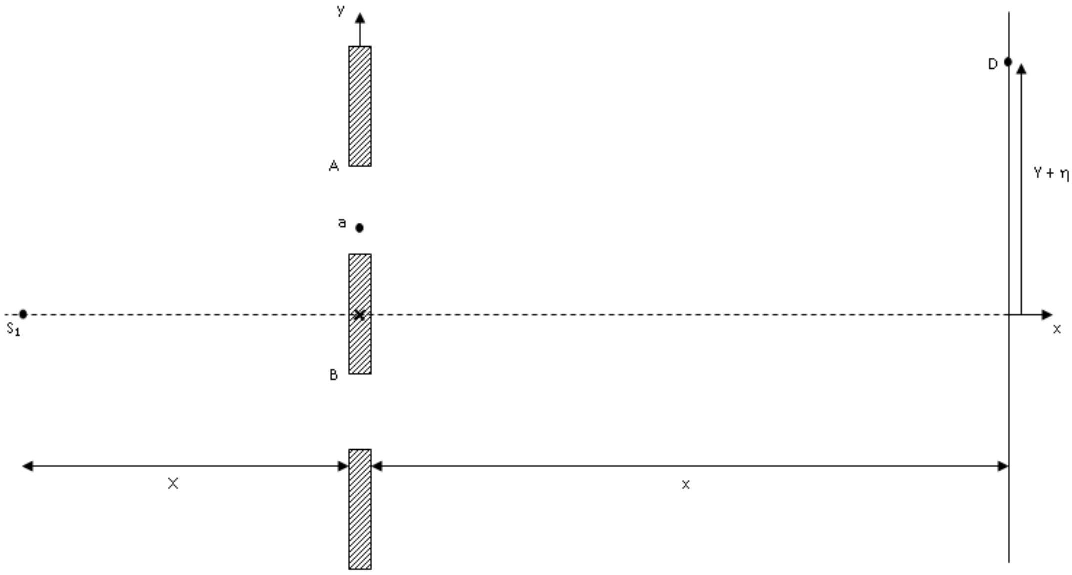

Figure 6 The Two Slit Experiment of Philippidis *et al.* (1979)

The kernel for the top slit, A, is simply written down,

$$K^{A}_{\delta Y}(-X,0,0;x,Y+\eta,t_D)=F(T,\tau)\exp\left[\frac{im}{2\hbar}\left[\frac{X^2+(Y+\delta Y)^2}{T}\right]\right]\exp\left[\frac{im}{2\hbar}\left[\frac{x^2+(Y+\eta-Y-\delta Y)^2}{\tau}\right]\right]$$

Where F(T, τ) is a normalizing factor, V_x is the velocity of the particle along the x-axis, $T=\frac{X}{V_X}$ is the time taken for the particle to move from the source to the slit and $\tau=\frac{x}{V_X}$ is the time taken for the particle to move from the slit to the screen.

The probability amplitude, ψ_A, is obtained by integrating over all positions of *a* within the slit. It is assumed that the slit is Gaussian so the probability amplitude is given by,

$$\psi_A=F(T,\tau)\int_{-\infty}^{\infty}K^{A}_{\delta Y}\exp\left[-\frac{\delta Y^2}{2\beta^2}\right]d(\delta Y)$$

Where β is the half-width of the slit. The final result is,

$$\psi_A=F(T,\tau)\exp\left[\frac{im}{2\hbar}\left[\frac{X^2}{T}+\frac{x^2}{\tau}\right]\right]\exp\left[\frac{im}{2\hbar}\left[\frac{Y^2}{T}+\frac{\eta^2}{\tau}\right]\right]\exp\left[\frac{(m^2/2\hbar^2\tau^2)(V_y\tau-\eta)^2}{im/\hbar T+im/\hbar\tau-1/\beta^2}\right]$$

Similarly for the bottom slit, B, we find,

$$\psi_B=F(T,\tau)\exp\left[\frac{im}{2\hbar}\left[\frac{X^2}{T}+\frac{x^2}{\tau}\right]\right]\exp\left[\frac{im}{2\hbar}\left[\frac{Y^2}{T}+\frac{(2Y+\eta)^2}{\tau}\right]\right]\exp\left[\frac{(m^2/2\hbar^2\tau^2)(V_y\tau+2Y+\eta)^2}{im/\hbar T+im/\hbar\tau-1/\beta^2}\right]$$

Where V_y is the velocity along the y-axis, $V_Y=\frac{Y}{T}$.

The trajectories of the particle are calculated by integrating the guidance equation which gives the relation between the S-function (phase) and particle velocity,

$$\nabla S=mv$$

The plot of particle trajectories through two Gaussian slits given by Philippidis *et al.* is shown below. It should be noted that the centre of this diagram shows the trajectories from the two slits do not cross or even touch. The reason for this shall be mentioned later but the phenomenon should be remembered as it will be important for predicting the trajectories of particles in the three slit experiment.

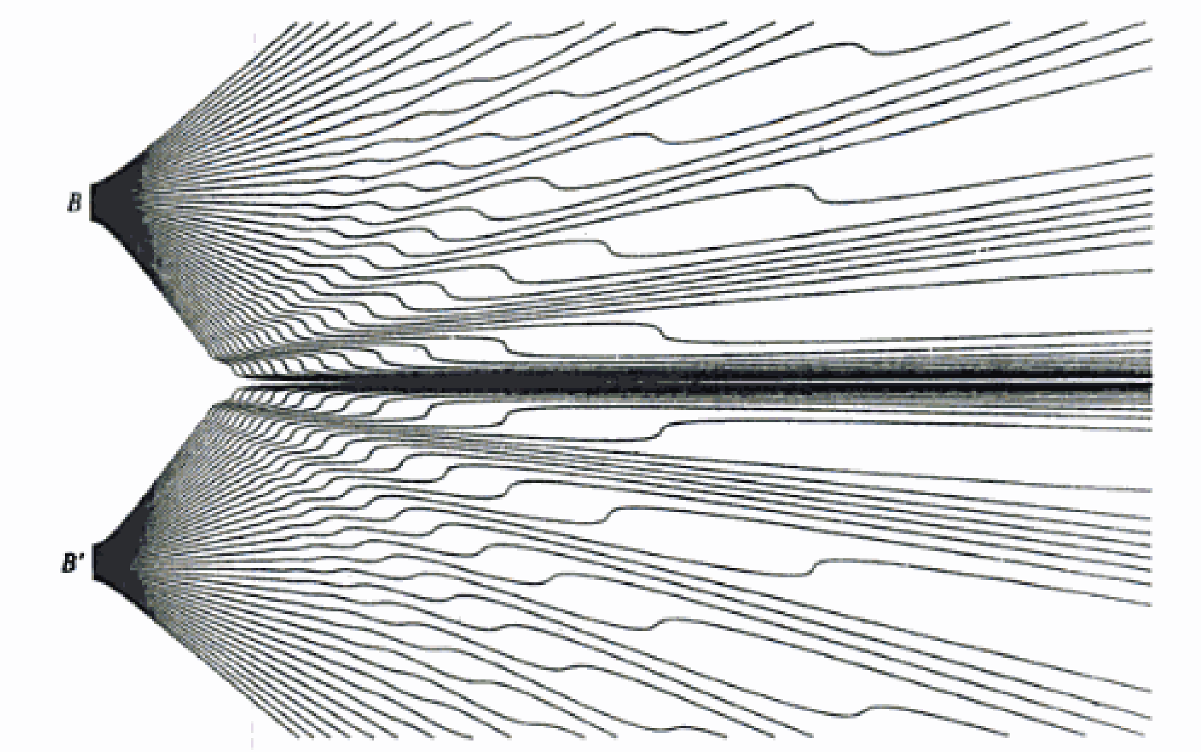

Figure 7 Trajectories for two Gaussian slits (from Philippidis *et al.* (1979) [21])

5.3.2 The Three Slit Experiment in de Broglie-Bohm Theory

In order to make predictions for the three slit experiment in de Broglie-Bohm theory, the work of Philippidis *et al.* [21] can be easily extended.

The origin is again on the dotted line in the centre of the slits, denoted by x (figure 8). The electron source S_1 has coordinates (-X, 0) and the centres of slits are at coordinates (0, Y), (0, -Y), (0,0). The case for which a single particle passes through point c inside slit C at a distance δY from is considered. The particle ends at a point D (on the screen) with coordinates (x, η) where η is measured from centre of slit.

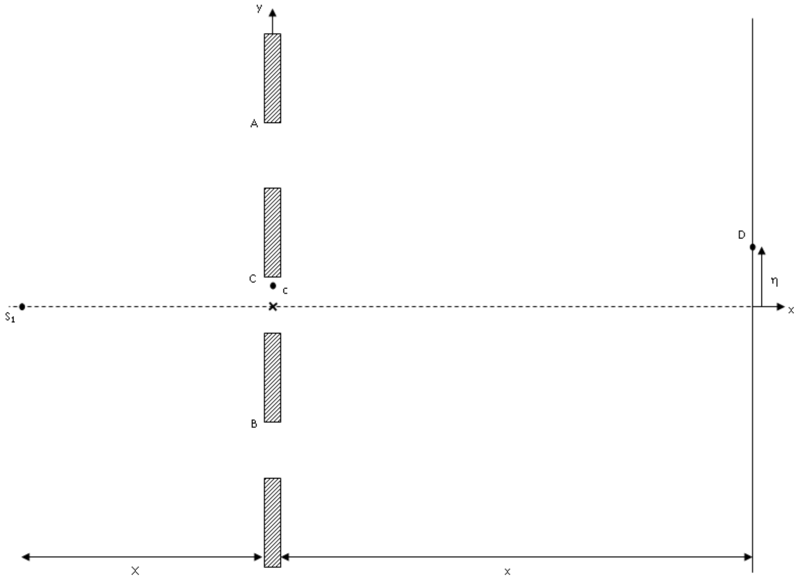

Figure 8 The three slit experiment

The kernel for a particle passing through the centre slit can easily be constructed using the form of the kernel for a particle passing through the top slit in the paper by Philippidis *et al.* [21],

$$K_{\delta Y}^{C}(-X,0,0;x,\eta,t_D)=F(T,\tau)\exp\left[\frac{im}{2\hbar}\left[\frac{X^2+\delta Y^2}{T}\right]\right]\exp\left[\frac{im}{2\hbar}\left[\frac{x^2+(\eta-\delta Y)^2}{\tau}\right]\right]$$

Again, F(T, τ) is a normalizing factor, V_x is the velocity of the particle along the x-axis, $T=\frac{X}{V_X}$ is the time taken for the particle to move from the source to the slit and $\tau=\frac{x}{V_X}$ is the time taken for the particle to move from the slit to the screen.

Assuming the slit to be Gaussian the probability amplitude is given by,

$$\psi_C=F(T,\tau)\int_{-\infty}^{\infty}K_{\delta Y}^{C}\exp\left[-\frac{\delta Y^2}{2\beta^2}\right]d(\delta Y)$$

Giving the final result,

$$\psi_C = F(T,\tau)\exp\left[\frac{im}{2\hbar}\left[\frac{X^2}{T}+\frac{x^2}{\tau}\right]\right]\exp\left[\frac{im}{2\hbar}\frac{\eta^2}{\tau}\right]\exp\left[\frac{(m^2/2\hbar^2\tau^2)\eta^2}{im/\hbar T+im/\hbar\tau-1/\beta^2}\right]$$

The trajectories would again be calculated by integrating $\nabla S = mv$. The phase, S, can be found from the wave functions by numerical computations.

5.3.3 De Broglie-Bohm Particle Trajectories

Unfortunately due to time limitations the particle trajectories cannot be calculated in this work. However, it is possible to make predictions as to the paths taken. In predicting the possible particle trajectories it is important to realise that in the de Broglie-Bohm theory, the trajectories cannot cross or even touch [22]. This is due to the uniqueness of the tangent vector associated with ∇S at each point in space and time.

The following initial assumptions can be made: the trajectories would be symmetric about the x-axis (perpendicular to the screen) and no trajectories would touch or cross. As a result of this, the only possible particle trajectory that reaches the point (x, 0) is a straight line from the source directly along the x-axis. A particle reaching this point must have come through the centre of the central slit.

Without carrying out the calculations this is the only prediction that can be known for sure. However, speculation could be made as to the origin of the other trajectories forming the central bright fringe. For example, it is probably a reasonable assumption that a majority, if not all such trajectories come through the central slit. Less can be predicted for the second order fringes as the position of the fringes is unknown. The top second order fringe could be formed by trajectories from just the top slit, the central slit, or both. Similarly the bottom second order fringe may be formed by trajectories from the bottom slit, the central slit, or both.

Limiting the problem to single point slits rather than finite slits and considering only the trajectories that hit the central point on the screen, a definite result for the de Broglie-Bohm theory is known. The only possible trajectory is a straight line along the x-axis from the source and through the central slit.

5.4 The Quantum Measure Theory Solution

The arrangement of the three slit experiment is essential to its evaluation in Quantum Measure Theory. The slits must be arranged and a final position on the screen chosen such that the amplitude for the particle to go through the central slit and land at the final position is -1 and the amplitude for the particle to go through either of the outside slits and land at the final position is +1.

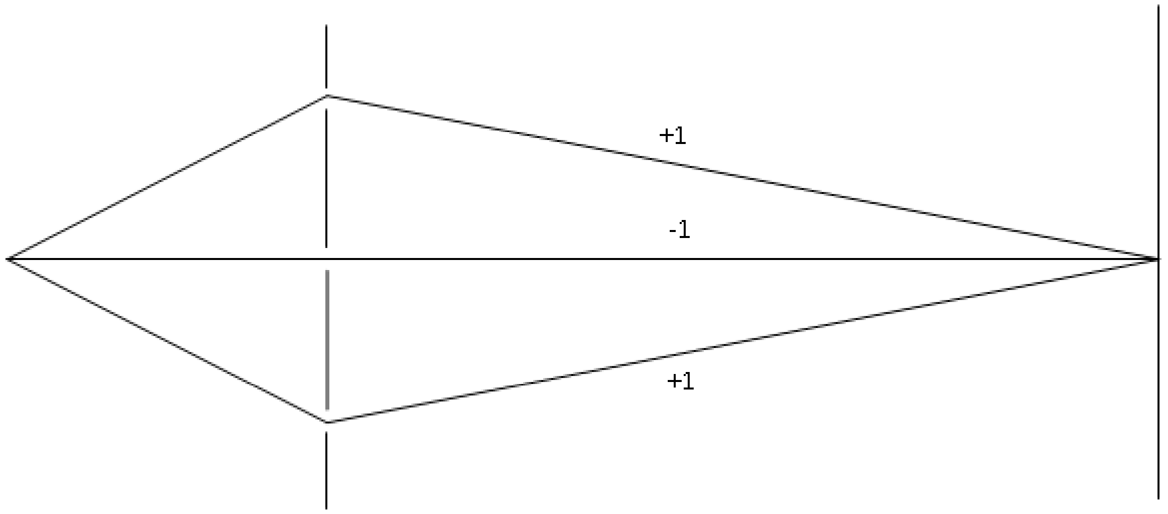

Figure 9 Trajectory amplitudes for the three slit experiment in Quantum Measure Theory

It follows directly from this that there are two events of measure zero: when the particle takes the path ‘through A or C’ or the path ‘through C or B’. The slits are labelled as they were for the de Broglie-Bohm approach, A and B are outside slits and C is the central slit. An abuse of notation shall be used such that μ(A) will be used to represent the measure of histories through slit A.

$$\mu(A \coprod C) = 0$$

$$\mu(C \coprod B) = 0$$

Before the predictions of Quantum Measure Theory for this experiment are made, the reasons why the set up of the apparatus is important should be explored. Consider if a change was made to the apparatus, for example if the slit spacing were increased. In this case it would no longer be that the amplitudes for the paths ending at the central point on the screen were ±1 as described above. There are then no events with obvious zero measures.

$$\mu(A \coprod C) \neq 0$$

$$\mu(C \coprod B) \neq 0$$

$$\mu(A \coprod B \coprod C) \neq 0$$

Again, following from the rules of Quantum Measure Theory, nothing is preventing the reality from being a single history; the path can be ‘through A’, ‘through B’ or ‘through C’. The three slit experiment would then be classical.

Assuming now that the experiment is set up as required, such that

$$\mu(A \coprod C) = 0$$

$$\mu(C \coprod B) = 0$$

The rules of Quantum Measure Theory can be applied to find the possible trajectories of the particle. These will be found under the conditions that the slits are point slits and only trajectories hitting the central point on the screen are considered.

As the real set of histories cannot be contained within a set of measure zero, the reality cannot be through a single slit A, B or C. This leaves just two possibilities, {A, B} and {A, B, C}. However the latter contains the former and hence cannot be minimal. The unique minimal preclusive reality is then {A, B} and the reality is all histories that pass through the outside slits. The trajectories will be definite single particle tracks because of the restriction to point slits. Due to the probabilistic nature of quantum mechanics it cannot be determined which of the two outside slits the particle travels through.

In section 4 the importance of the reality was mentioned in trying to determine the particle trajectories. It was said that in quantum theories the reality is a set of histories and nothing specific can be known about the particle trajectories. This will be examined quickly in the case of the three slit experiment as examples are the easiest way to try and understand exactly what Quantum Measure Theory says. It has been found that the unique minimal preclusive reality is the set of histories {A, B}, the particle trajectories pass through one of the outside slits. It has also been concluded that the particles do not take the single histories {A} or {B}. What then can be said of the trajectories other than that they are contained within the set of histories that pass through one of the outer slits? All the information that it is possible to obtain on the trajectories can be found from the yes/no answers to any question. For example,

Does the particle go through slit A? No
Does the particle go through slit B? No
Does the particle go through slit C? No
Does the particle go through slits A or C? No
Does the particle go through slits B or C? No
Does the particle go through slits A or B? Yes

The possible particle trajectories that pass through one of the outer slits and hit the centre of the screen cannot be calculated. Every possible history that goes through those particular slits is a trajectory for that history. However, it is possible to find the slit separation necessary to set up the experiment as required.

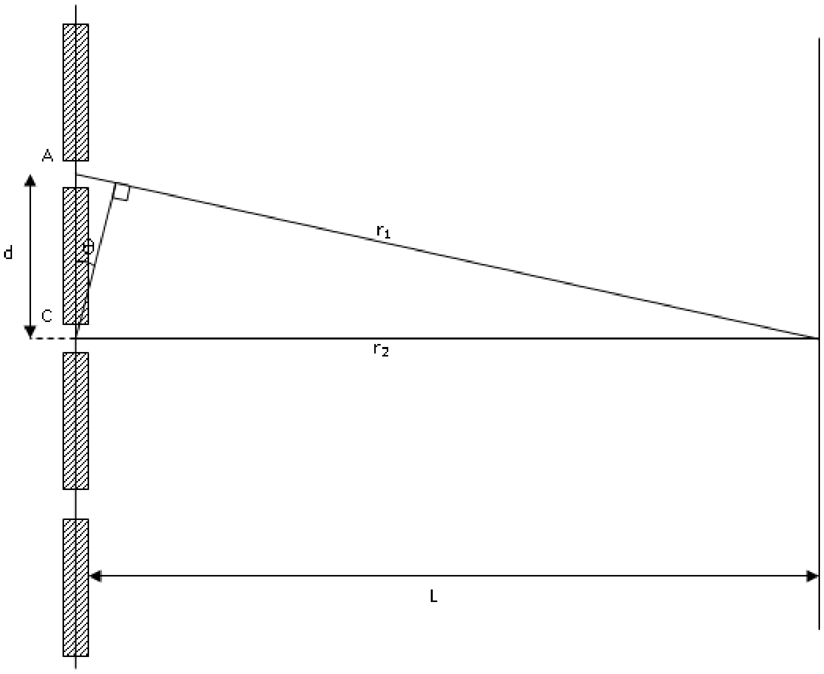

Figure 10 The set up of the triple slit experiment in Quantum Measure Theory

Due to the symmetry of the problem, it is only necessary to consider destructive interference of trajectories ‘through A or C’.

$r_2 = L$

$r_1 = \sqrt{d^2 + L^2}$

The path difference is $r_1 - r_2 = d\sin\theta$

For destructive interference the two trajectories must be out of phase at the screen,

$$d\sin\theta = \left(m + \frac{1}{2}\right)\lambda \qquad m = 0, \pm 1, \pm 2, \ldots$$

At the centre point of the screen $m = 0$ so $r_1 - r_2 = \frac{\lambda}{2}$.

Substituting in, $d = \sqrt{\frac{\lambda^2}{4} + \lambda L}$ but $\lambda \ll L$ and therefore $d = \sqrt{\lambda L}$.

As a comparison will be made between the predictions of the different interpretations, the wavelength shall be found from the wave functions calculated for the de Broglie-Bohm trajectories. Choosing an appropriate value for L will then allow the slit separation, d, to be found. The wave function at each of the three slits will be slightly different but considering the wave function at the central slit will give a good approximation to the peak wavelength of the wave packet emitted from the source. This should be an appropriate estimate for the purposes of this study.

At the slit $x = 0$, $\eta = 0$,

$$\psi_C = F(T,\tau)\exp\left[\frac{im}{2\hbar}\frac{X^2}{T}\right]$$

$$\frac{X}{T} = V_x \;,\; mV_x = p$$

$$\psi_C = F(T,\tau)\exp\left[\frac{ipX}{2\hbar}\right]$$

The apparatus in Quantum Measure Theory are set up such that the amplitude for the particle to travel through the central slit is -1. Ignoring the normalization term the probability amplitude for the central slit should then be set equal to -1.

$$\psi_C = -1$$

$$\frac{pX}{2\hbar} = \pi$$

Therefore, $\lambda = \dfrac{2\pi\hbar}{mV_x}$. This should be the expected result as it is the de Broglie wavelength.

The slit separation is therefore given by

$$d = \sqrt{\frac{4\pi\hbar L}{mV_x}}$$

In Quantum Measure Theory the exact conditions required for a quantum solution to the three slit experiment in can easily be found but the actual trajectories of the particles cannot be calculated. However, it must be the case that any particles hitting the central point on the screen travelled through one of the outer slits.

5.5 Summary

The Copenhagen Interpretation is unable to make many predictions about the three slit experiment. It is therefore hard to draw a comparison between this interpretation and others. However the Copenhagen Interpretation does indicate the particle distributions that are observed when the experiment is performed. Because these results are testable the distributions predicted by other interpretations must match those suggested by the Copenhagen Interpretation. For comparative purposes it should therefore be made clear what distribution is to be expected when the slits are point slits rather than finite slits and only the central point on the screen is considered. As is probably obvious, it is expected that a particle would be detected at this point on the screen. This limitation has been invoked by Quantum Measure Theory as it makes the calculations considerably simpler. It would be a good extension to this work, then, if slits of finite width could be considered in Quantum Measure Theory to determine that the distribution of particles at the screen does coincide with that of the Copenhagen Interpretation. Although the particle trajectories in Quantum Measure Theory are all possible histories through the outside slits, the trajectories contributing most to the amplitude will be those very close to the classical trajectory. An approximate calculation could then be made by considering just these trajectories.

As has been seen, it is possible to calculate trajectories in the de Broglie-Bohm theory although this has not been managed here. However, the task of calculating the exact trajectories taken is almost impossible. This is because precise information needs to be known concerning the exact point of emission of the particle and data of this detail is unobtainable without detection and detection would, of course, change the path of the particle.

The most obvious difference between the predictions of the de Broglie-Bohm theory and Quantum Measure Theory concern the slits through which the particles travel. For trajectories that hit the central point on the screen, the de Broglie-Bohm approach states that the particles must come through the central slit while the Quantum Measure Theory approach concludes that the reality is the set of histories through the outside slits! This contradiction of the theories is a very neat result. This does not help with determining the paths taken by the particles but it does open up many questions. An understandable first reaction might be to try and construct an experiment that will determine which of the theories is correct. However, it soon becomes clear that there is no such experiment as any measurements are taken would change the outcome. One conclusion that may be made is that both theories cannot be making the correct prediction. This suggests, then, that if it is found that one of the theories is the true theory of quantum mechanics, the other must be incorrect.

6 Quantum Gravity

6.1 Background to the Problem of Quantum Gravity

Through the history of Physics, unification of theories has proven very successful and gravity is now the only fundamental force not unified with quantum theory. It therefore seems the right step that physicists should be attempting for this unification. The field of physics concerned with this problem is quantum gravity. The aim of quantum gravity is not just to unite quantum mechanics and general relativity but must reduce to ordinary quantum mechanics in the limit of weak gravity and must reduce to “classical” general relativity in the limit of large actions. This is due to the independent success of the two theories. There are many theories that have been suggested in quantum gravity: some, theories of everything, try to unify gravity with the other fundamental forces while others simply try to quantize the gravitational field.

Initial attempts to combine general relativity with quantum mechanics involved modelling gravity as a field theory. Gravity particles attract each other and the sum of all the interactions results in many infinite values. However, unlike in quantum electrodynamics these infinities do not mathematically cancel - gravity is a non-renormalisable theory. The next intuitive step was to consider quantum gravity as an effective field theory with a high-energy cut-off. Gravity is found to be a good effective field theory but beyond the cut-off there is not necessarily a good description of nature and so in order to combine general relativity and quantum mechanics at these high energies, a totally new model might be required. It is often the assumption that a theory of quantum gravity for the high-energy states will be simple and elegant and as such, symmetries and findings from elsewhere may suggest ways to achieve a unified theory. However, there is no reason for this assumption other than desire.

Thus far it seems to be suggested that up to some high-energy limit, gravity and quantum theory can be combined. However there are some physicists who claim that reconciliation is very difficult because general relativity and quantum field theory are incompatible. Quantum field theory depends on particle fields embedded in the flat space-time of special relativity and general relativity models gravity as a curvature of space-time that changes as the gravitational mass moves. The conflict between the theories is concerned with the dynamics of space-time: quantum mechanics is formulated with a fixed, non-dynamical space-time background; general relativity is formulated in terms of a Riemannian geometry, assuming the metric to be a smooth and deterministic dynamical field. According to quantum mechanics, all dynamical entities are made up of quanta, so if general relativity states that space-time is dynamical then space-time must be quantized for a possible unification between the two theories. There are many approaches to the quantization of space-time but they will not be discussed here. For further reading see [23], [24], [25], [26], and [27]. It is enough for the purpose of this dissertation to recognise the problem.

As has been mentioned, there are many possible theories for quantum gravity. Two of the most popular approaches today are string theory and loop quantum gravity. String theory has emerged from attempts to build quantum gravity as a quantum field theory of the fluctuations of the metric over some background metric space, a covariant approach. The idea behind string theory is that all physical entities are understood as manifestations of a string and that gravity is one aspect of the dynamics of the string. Loop quantum gravity has emerged from a canonical attempt to construct a quantum theory in which the Hilbert space carries a representation of the operators corresponding to the metric but without the background metric being fixed. Loop quantum gravity tries to find a quantum version of general relativity by offering a precise mathematical description of quantum space-time. Examination of these two theories will not be taken further as discussion of quantum gravity in the de Broglie-Bohm theory and Quantum Measure Theory is of greater concern. However, according to Carlo Rovelli [24] there are only three main lines of research in quantum gravity and therefore it is then likely that either string theory or loop quantum gravity have developed from the same word as the theories that shall now be discussed.

6.2 Work Done on Quantum Gravity in de Broglie-Bohm Theory

The de Broglie-Bohm theory was being developed at the same time as the Copenhagen Interpretation. Its purpose was therefore to explain the quantum phenomena that had been observed at that time and provide an understanding of the mathematics. The original intention was not that the theory being formulated would be useful in the development of future theories although it would of course be hoped that a successful theory would be applicable throughout the development of physics. At the time of development of the de Broglie-Bohm theory the idea of uniting quantum mechanics and general relativity was very far off, however de Broglie-Bohm theories of quantum gravity have now been proposed.

On the topic of quantum gravity, supporters of the de Broglie-Bohm theory are split. Some think that using the de Broglie-Bohm interpretation of quantum mechanics solves the many problems posed when trying to combine general relativity with quantum mechanics while others believe that such claims are overblown and the issue should be readdressed in a different way. One of the more popular de Broglie-Bohm solutions to quantum gravity and its problems shall be discussed. This should allow an overview of how the de Broglie-Bohm interpretation of quantum mechanics fits with the new ideas emerging in physics.

When applying the de Broglie-Bohm interpretation to quantum gravity the canonical approach is used (this is the approach which led to the theory of loop quantum gravity). It is claimed that canonical quantum gravity has some conceptual problems that must be overcome. Two of these will be discussed: the problem of time indentified by Goldstein and Teufel [28] and Shtanov [29] and a problem due to a twofold description of measuring apparatus described by Shtanov [29]. It is agreed that such problems occur because of the 'standard' interpretation of quantum mechanics and vanish when the de Broglie-Bohm interpretation is applied. Before seeing how the de Broglie-Bohm interpretation solves these difficulties, the problems posed by Shtanov shall be considered in more detail.

The twofold description of measuring apparatus emerges when both the observed system and the system being used to observe (the measuring apparatus) are each described by wave functions. A wave function description is required for both systems as they each consist of particles of the same nature. This means that in the same way as the state of the observed system is dependent upon the apparatus observing it, the state of the apparatus is also dependent on some observer. This problem has famously led to the Schrödinger cat paradox. According to Shtanov, this must be overcome if a coherent physical description of reality is to be found and hence the problem must be solved before a successful theory of quantum gravity can be given.

In the de Broglie-Bohm formulation of quantum gravity, the universe is taken to be described by a time independent wave function. There is then a problem of how the wave function can correspond to the observed universe which evolves in time. This is the problem of time. Shtanov explains that attempts were made to resolve this issue by reducing the phenomenon of time to correlations between physical quantities. The choice of variable to be assigned the meaning of time, however, is arbitrary and time fails to be a universal concept.

According to Shtanov [29] these problems are solved in his theory as the de Broglie-Bohm interpretation offers a role for the wave function in all physical situations: to provide guidance laws for configuration variables. Goldstein and Teufel [28] claim simply that the de Broglie-Bohm interpretation offers the ontological clarity that is missing in the ‘standard’ interpretation and led to the problems in the first place, although exactly what they mean by this is not easy to comprehend. Considering no further detail, it shall be assumed that in the case of a de Broglie-Bohm theory of quantum gravity, the problem of time and the twofold description of the apparatus are resolved.

When the de Broglie-Bohm interpretation is applied to quantum gravity, the fundamental object of the theory is the geometry of three-dimensional space-like hypersurfaces. The evolution of these hypersurfaces is labelled by some time parameter and is controlled by the quantum potential, a quantity specific to de Broglie-Bohm theory. This splitting of four-dimensional space-time into an evolving three-dimensional space-like surface is the solution posed to the space-time problem mentioned earlier. This shall be discussed further later. What the time parameter is shall not be explored here but it is noted that it could raise questions in opposition to the de Broglie-Bohm theory of quantum gravity.

Detailed derivations of quantum gravity theories using the de Broglie-Bohm interpretation of quantum mechanics can be found in the papers of Shtanov [29] and Goldstein and Teufel

[28]. Both theories lead to some wave function of the universe, ψ, that obeys the Wheeler-DeWitt equation (8).

$$\left\{-\hbar^2\left[\kappa G_{ijkl}\frac{\delta}{\delta h_{ij}}\frac{\delta}{\delta h_{kl}}+\frac{1}{2\sqrt{h}}\frac{\delta^2}{\delta\phi^2}\right]+V\right\}\psi\left(h_{ij},\phi\right)=0 \tag{8}$$

V is the classical potential, h_{ij} is the three-metric of the closed three-dimensional space-like hypersurface, h is the determinant of h_{ij}, G_{ijkl} is the DeWitt metric, and ϕ is the scalar field.

Pinto-Neto [30] suggests two important points of de Broglie-Bohm quantum gravity: by analogy with the cases of non-relativistic particles and quantum field theory in flat space-time, it can be postulated that the three-metric of space-time hypersurfaces, the scalar field and associated canonical momenta will always exist, independent of observation. The guidance equation can be integrated to find the three-metric and scalar field for all values of the time parameter resulting in solutions that will depend on initial values at some initial hypersurface. It should be noted that the evolution of the fields differs from evolution classically due to the quantum potential term that appears in the formulation of de Broglie-Bohm theory.

Many physicists still consider this de Broglie-Bohm approach to be flawed. A particular concern is with the splitting of space-time. It has previously been argued that the continuity of space-time causes problems for quantum gravity. The splitting of space-time into an evolving three-space seems to solve this problem in the de Broglie-Bohm approach however it poses its own difficulties. For some people, attempting to divide space and time is an impossible task and it is meaningless to try and talk about such a division. Others argue that due to Heisenberg’s uncertainty principle, if the precise three-geometry of space is known at one instant then the time rate of change of that three-geometry cannot also be known at that same instant.

From this brief consideration of de Broglie-Bohm approaches to quantum gravity, the important points for the discussions of this dissertation are as follows. There exist plausible, de Broglie-Bohm theories of quantum gravity but there are also strong criticisms of these theories, including from proponents of the de Broglie-Bohm theory. None of these theories of quantum gravity have been able to persuade more than a handful of physicists.

6.3 Ideas Presented in Quantum Measure Theory

One of the motivations behind the development of Quantum Measure Theory is to provide an interpretation of quantum mechanics that can easily be used in the formulation of a theory of quantum gravity. Gravity is a theory of a fully four-dimensional space-time and therefore does not depend on a system being in a particular state at a particular moment in time. The idea of Quantum Measure Theory is then that quantum theory should depend on histories, not states, thus it follows that any consequent theory of quantum gravity will also be histories based. As yet no theory has been presented as complete but the process behind setting up such a theory can be explored.

A quantum theory of gravity based upon Quantum Measure Theory will have, at its centre, the basic axiomatic structure seen earlier: a sample space, event algebra and a measure. The difficult questions are what each of these is in quantum gravity. As gravity is concerned with space-time this seems a good place to start, and for continuity it is then sensible to consider the Lorentzian geometries of general relativity. If a Lorentz geometric space-time forms the sample space for quantum gravity, it must follow that the events in the event algebra are covariant. What then is the measure in such a theory? As has been discussed previously, the quantum measure is related to a “sum over histories” or path integral. However performing such a sum over Lorentzian geometries is incredibly difficult. But without a measure the theory does not work.

Retracing the steps taken to try to set up a quantum theory of gravity, some changes can be made to produce a more successful structure for the theory. Again the sample space must include space-times as these are fundamental to general relativity, however rather than considering Lorentzian geometries on the space-times this new attempt will still take the sample space over all space-time but will consider discrete manifolds rather than Lorentzian geometries. This is looking to be a more successful approach which will not lead to the problems with the measure that were experienced with the first attempt. A measure for this approach has, however, not yet been found and progress must still be made before a theory of quantum gravity can be given. The idea that space-time is fundamentally discrete is widely accepted by physicists although there is only circumstantial evidence for it.

Taking space-time as a discrete manifold has opened the path to quantum gravity to a great many people. The question they are all trying to answer is what are these discrete manifolds? Two suggestions that have been proposed shall now be considered briefly. The first of these is the causal set theory of Rafael Sorkin [31], [32], one of the founders of Quantum Measure Theory. A causal set is a locally finite, partially ordered set which satisfies the conditions of transitivity and acyclicity. A causal set should relate to the continuous metric space of general relativity via two correspondences. The first of these states that, any region of finite volume of continuous space-time consists of a finite number of elements of some underlying causal set. The number of elements corresponds to the volume of the region when measured in fundamental units. The second correspondence is that the macroscopic light-cone structure of space-time reflects the order relation of the underlying causal set. For further introductory reading on causal sets see references [31], [32], [33]. The second suggestion for the discrete manifold is described by Regge calculus. The idea of Regge calculus is to approximate the curvature in the geometry of space-time. A simplified understanding can be achieved by considering a geodesic dome. Each face of the dome is a flat triangle but from afar the dome appears to be spherical. The edge between two of the triangles is straight and can be flattened while maintaining the connection between the triangles. However as soon as three triangles are connected the whole cannot be flattened, there is curvature. Space-time thus works like the geodesic dome, it has curvature which appears smooth from a distance but the finer detail shows it to be divided into small, flat, discretised parts.

While no full theory of quantum gravity has been developed using Quantum Measure Theory, there is another suggestion coming forward that also uses the histories approach. This is an idea presented by James Hartle who claims [34] that the “notion of a set of space-time alternatives is a portion (coarse-graining) of the histories into an exhaustive set of exclusive classes”. The idea is that due to quantum mechanical interference between individual histories in a set, not all sets of histories can be assigned probabilities. Considering instead a set of alternative coarse-grained histories, there is negligible quantum mechanical interference and consequently probabilities can be assigned to individual members of the set. Restricting therefore to coarse-grained histories, or macro-events, there is no interference and the measure is effectively classical, obeying the Kolmogorov rule.

Although considering the macroscopic rather than microscopic level, Hartle is making a good case that the quantum mechanics required for quantum gravity should be based in histories. It thus seems that although no definite theories have yet been presented, Quantum Measure Theory may have the potential as the underlying requirement for a successful theory of quantum gravity.

6.4 Summary

As no successful theory of quantum gravity has yet been found it is not possible to conclude that one approach is correct and the others false. Instead, a comparison of the de Broglie-Bohm and Quantum Measure Theory approaches to the problem will be made and consideration given to whether one of the two seems more likely to provide a good theory of quantum gravity. This comparison will take account of the successes of each theory and the problems they produce. It shall be considered whether the two approaches are mutually exclusive and an opinion given on which of the two theories appears most plausible.

An approach to quantum gravity using the de Broglie-Bohm theory of quantum mechanics is claimed to solve all apparent conceptual problems of quantum gravity. There remains some opposition to this interpretation of quantum mechanics but it seems that a successful theory of quantum gravity should increase the acceptance of the de Broglie-Bohm theory. This is probably not the case. Although the interpretation provides a solution to some of the problems presented by quantum gravity, other problems are generated and no obvious solution is posed. The separation between the non-relativistic and relativistic formulations of the de Broglie-Bohm theory is the first of two dilemmas that need to be addressed. There are proponents of de Broglie-Bohm theory who are satisfied that the problem of a relativistic theory has been sorted but opponents are unconvinced and the question remains how general relativity can be combined with a non-relativistic quantum theory. Unfortunately it does not appear that much work is being carried out to try and resolve this disagreement and while it remains the de Broglie-Bohm theory of quantum gravity will not receive the acclaim its proponents would hope for. The second dilemma is concerned with space-time. As has been seen, the de Broglie-Bohm approach to the problem of space-time in quantum gravity is to split it into a three-dimensional space like hypersurface that evolves in time. There is much opposition to claims that this is a reasonable solution to the problem and a strong argument has been given from the point of view of Heisenberg's uncertainty principle.

Quantum Measure Theory has not yet achieved so much opposition to its claims for quantum gravity but that is not to say it does not provide its own problems. First, however, the successes of the approach will be reviewed. The histories approach that Quantum Measure Theory takes to quantum mechanics is intended to reconcile the space-time of quantum mechanics with that of general relativity. This is not in itself a success of the theory but rather a motivation behind its formulation. The work of Hartle [34] does however suggest that a histories approach to the problem of quantum gravity should be well supported and, independent of the reasons for taking such an approach, it will prove one of the greatest successes of a potential successful theory. The approach of Quantum Measure Theory is forced to take a sample space that is a discrete manifold over all space-time in order to be solvable. Although it is now generally acknowledged that space-time must be accepted as a discrete manifold, the problem remains of finding what this discrete manifold is. Currently there are different possibilities postulated but until one can be found to be true a successful theory of quantum gravity cannot be given by Quantum Measure Theory.

If a theory of quantum gravity were found using one of these interpretations of quantum mechanics will it be necessary to reject the other interpretation completely? As physics currently stands there is no way to give a correct answer to this question but it also seems that there is no reason why the success of one of these approaches to quantum gravity will overthrow the other. The thinking behind this is based in the vast differences of the two approaches. The completion and acceptance of a theory in Quantum Measure Theory, for example, does not have to rule out the usefulness of maintaining the de Broglie-Bohm approach. Although an unsuccessful theory cannot be used by leading physicists to make predictions and advance the field, it can sometimes prove useful in the understanding of, particularly conceptual, problems. This use of a ‘wrong’ theory can often be adopted in the early stages of learning to encourage thinking in that area of physics and the ideas can be rectified and the true theory learnt later. In this particular case, assuming again that Quantum Measure Theory produces a theory of quantum gravity the de Broglie-Bohm approach to the problem may be more useful to explain some of the problems of quantum gravity.

In the above discussion the example has been used that Quantum Measure Theory provides a successful theory of quantum gravity. But is this a more likely scenario than de Broglie-Bohm theory providing a successful theory? Based on what has been reviewed concerning both the approaches it seems likely that Quantum Measure Theory will be the more successful in the area of quantum gravity. The reasoning is that the histories approach to the space-time problem seems more acceptable than the splitting of space-time into an evolving space like hypersurface. It is also probably an influential factor that de Broglie-Bohm theory was originally formulated at the same time as the Copenhagen interpretation. Its purpose was to explain the phenomena of quantum mechanics successfully. In the area of quantum gravity the approach involving the de Broglie-Bohm theory is therefore trying to manipulate an old theory that has had continuous opposition to fit new and evolving fields of physics. There is of course nothing to suggest that this cannot work but against the new area of Quantum Measure Theory which has been developed with a theory of quantum gravity in mind, the de Broglie-Bohm approach seems out-of-date and doomed to fail.

7 Concluding Remarks

Both the de Broglie-Bohm theory and Quantum Measure Theory agree with the standard tried and tested results of quantum mechanics, as any quantum mechanical theory should. They do however differ from each other, and from the ‘standard’ Copenhagen interpretation, in the explanations they offer for the situations considered. The Copenhagen interpretation does not attempt to give any explanation for why the mathematics is as it is not, nor does it try to explain what is physically happening in the cases of quantum phenomena. Instead, it takes the line that it is meaningless to ask what is going on. Quantum Measure Theory is, like the Copenhagen interpretation, a non-deterministic theory agreeing with Heisenberg’s uncertainty principle that, for example, if the position of a particle is known then its momentum cannot be determined. Quantum Measure Theory does not try to rewrite the mathematics of quantum mechanics but instead to provide a more fundamental understanding of it. Through Quantum Measure Theory it should therefore be possible to construct the necessary tools that are currently used in evaluating quantum events, for example a Hilbert space. An extra step that Quantum Measure Theory takes is to claim that a new form of logic is required. This has been accepted in this dissertation but it is likely that the need to modify classical logic will meet some opposition. The de Broglie-Bohm theory is in contrast a deterministic theory. Particles have both a determined position and determined momentum and the wave function is guided by a pilot wave satisfying a particular, new dynamics.

Section 5 established that in the case of the three-slit experiment both de Broglie-Bohm theory and Quantum Measure Theory are able to make predictions for the particle trajectories. The ‘don’t ask’ aspect of the Copenhagen Interpretation prevents it from making such predictions. In neither de Broglie-Bohm theory nor Quantum Measure Theory have calculations been made due to time limitations. However, the actual trajectories taken by a particle could not be found anyway. In de Broglie-Bohm theory this is because the initial conditions cannot be known in enough detail, whilst in Quantum Measure Theory it is because it is only possible to know that the particle travels through one of two slits. In the particular case where the three slits have been limited to point slits and only particles reaching the central point on the slit are considered, the de Broglie-Bohm theory and Quantum Measure Theory are found to have contradictory results. De Broglie-Bohm theory tells us that the particles must travel through the central slit while Quantum Measure Theory claims they must come through one of the outside slits. This disagreement suggests that if one of the theories is found to be “correct” then the other must be wrong.

The two theories can be applied to the problem of quantum gravity but in very different ways. Both overcome the problems of attempting to unite quantum theory with general relativity but leave new problems in their wake. An approach using de Broglie-Bohm theory easily overcomes the initial conceptual problems and appears to have a solution to the problem of space-time. This solution is however widely criticised and it seems implausible. Quantum Measure Theory, motivated by the desire to unite quantum theory with gravity, uses a histories approach to quantum mechanics to resolve the space-time problem. However, for this approach to end successfully the form of discrete manifolds over space-time must be found. The two approaches to quantum gravity do not directly contradict each other and it can be considered that the difference in their approaches contributes a great deal to the field of quantum gravity even if neither has yet produced a complete theory.

How about the two problems of the foundations of quantum mechanics mentioned in the introduction? The de Broglie-Bohm theory overcomes the problem of probability through its deterministic nature. Every particle has a definite position and momentum at all times; it is just that these are usually “hidden”. Consequently there is then no need for the phenomenon of wave function collapse, as the location of the particle is definite whether or not the particle is being observed. However, in order to agree with the mathematics that is known to work, the de Broglie-Bohm theory claims that in practice the measuring apparatus cause disturbances that prevent prediction of the precise particle location – they keep it “hidden” until the point of observation, of wave function collapse. Quantum Measure Theory does not deal directly with these issues. Rather it is more concerned with providing a foundation for the mathematics that already exists in quantum mechanics. As a non-deterministic theory, Quantum Measure Theory agrees with the Copenhagen interpretation that quantum mechanics is probabilistic and that until the point of observation the states of the system are in a superposition. However, eventually a new explanation may be possible, exploiting Quantum Measure Theory’s approach employing histories rather than states.

Neither of these interpretations therefore really solves all the problems of the foundations of quantum mechanics, at least not for the moment. Quantum Measure Theory does not try to tackle these problems directly, but rather accepts them as part of quantum mechanics as it currently is. The de Broglie-Bohm theory does try to take on these problems, but cannot provide a theory independent of probability. Rather the probabilistic aspect of quantum mechanics is shifted from being a fundamental part of the theory to being a consequence of the measuring apparatus used to observe the systems. It appears that the de Broglie-Bohm theory has to concede that probability is essential to quantum mechanics in order to agree with the standard mathematics.

Making a direct comparison between the de Broglie-Bohm theory and Quantum Measure Theory allows their differences to become very apparent. However, the two areas considered in this work, the triple slit and quantum gravity, do not show any particular reason why one should be preferred over the other. Both make sensible predictions for the trajectories of particles through the three slits, and each proposes the beginnings of a theory of quantum gravity that has gained support and resolves a lot of the initial problems of unifying quantum theory with general relativity. What has come across is that the approaches taken by Quantum Measure Theory appear to be simpler than those of de Broglie-Bohm theory. Each of these theories involves the acceptance of new ideas, either a new dynamics or a set of new axioms, but the mathematics that follows once these ideas have been accepted seems less laboured in Quantum Measure Theory. Quantum Measure Theory also does not ask for the acceptance of "hidden" variables, something else that complicates an understanding of de Broglie-Bohm theory. What must not be forgotten however is the level to which this dissertation has gone. Calculations have not been made in either theory and the details of each have been left for the reader to discover independently. It would therefore be too quick to claim by an Ockham's razor argument that Quantum Measure Theory is the preferred interpretation.

Further research into both the triple slit experiment and quantum gravity may help if a decision on a preferred interpretation had to be made. In the triple slit experiment, doing the calculations for the possible trajectories in both theories would allow a comparison of the mathematics that each entails. It would also allow a more detailed comparison of the predictions of the theories, as it would not be necessary to make such tight restrictions. Unfortunately the experiment can never be constructed in such a way as to determine which theory makes the correct predictions. In quantum gravity a detailed analysis of the research in each theory would show up more of their strengths and weaknesses. This could then lead to a more direct comparison of the de Broglie-Bohm theory and Quantum Measure Theory and consequently stronger opinions as to which is the better approach.

Neither de Broglie-Bohm theory nor Quantum Measure Theory has had problems with attempting to construct theories of quantum gravity, so it seems unlikely that there will emerge new fields of physics that stop either of these interpretations in their tracks. Such future theories will however probably be decisive in determining whether physicists not directly concerned with the foundations of quantum mechanics pay attention to the de Broglie-Bohm theory or Quantum Measure Theory. At the moment, there is no reason for a physicist working outside the foundations of quantum mechanics to try to understand these two theories, as the mathematics that is currently available in quantum mechanics is highly successful, well-known and easy to use. It is only when a situation arises for which this standard mathematics is not sufficient that other physicists will feel the need to consider other interpretations of quantum mechanics. This then raises the question why one should bother with the foundations of quantum mechanics, given that the work done is not thought necessary by other physicists. Aside from the intrinsic interest in trying to understand some of the most fundamental physics used today, there is also the hope that these theories could prove vital in the future. One should thus be aware that their development today could aid the next leaps forward in physics.

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