Reactor Thermo-fluids: Background

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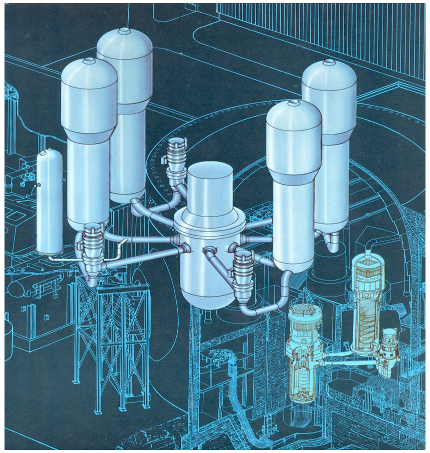
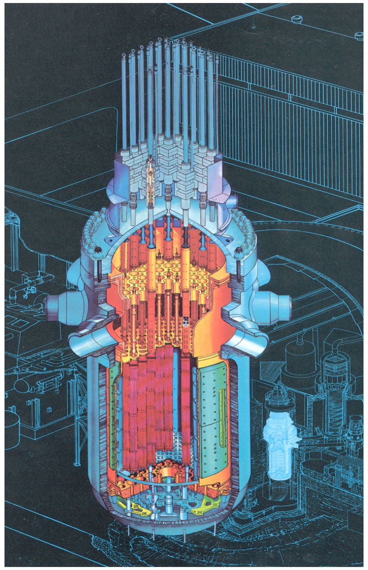
Objectives

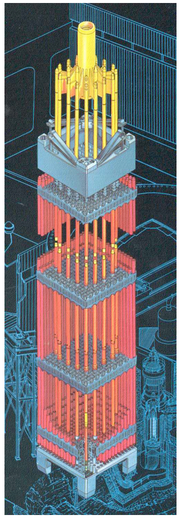
- To provide a summary of the main topics in heat transfer and fluid mechanics that are required for the later analysis of reactor thermo-fluids

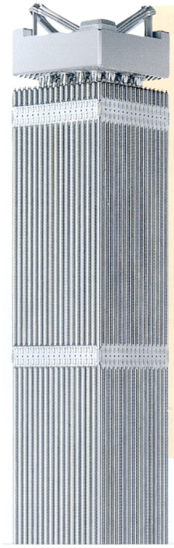
Topics

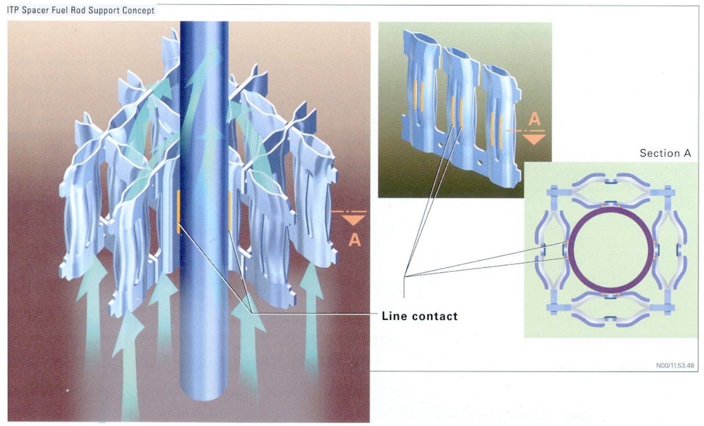
- Why is thermo-fluids so important?
- Basic heat conduction, thermal resistance
- Main types of fluid flow, physical properties of importance in fluid flow
- Dimensionless numbers
- Pressure drops in pipe flow
- Flow in circular and non-circular ducts, hydraulic diameter
- Convective heat transfer, Newton's 'Law of Cooling', heat transfer coefficients, correlations for heat transfer coefficients
- Calculations of heat transfer in mixed conductive - convective systems
- Boiling regimes











Heat Conduction: Governing equations

We will perform a 'heat balance' for an infinitesimal element of a solid (very much like the neutron balance we performed in our reactor physics analysis). At some point in the solid we will consider the net rate of flow of heat away from (towards) the point, the rate of generation of heat there, and the rate at which the amount of heat stored there (ie the temperature) is changing.
Symbolically we will have:

$\nabla \cdot \mathbf{q}$ the flow of heat away from the point (the divergence)

$\dot{q}'''$ the rate of heat generation (heat per unit time, per unit volume)

$\rho C_p \frac{dT}{dt}$ the rate of increase of heat energy per unit volume

Equating these we can write our balance as:

$$\rho C_p \frac{dT}{dt} = \dot{q}''' - \nabla \cdot \mathbf{q}$$

Heat Conduction: Governing equations II

Our equation involves both temperature and heat, and both are unknown. We need some relationship between them to allow one to be expressed in terms of the other.
Experimentally, it is observed that heat flows from hot places to colder places, at a rate proportional to the temperature gradient. We can thus write

$$\mathbf{q} = -k\nabla \mathbf{T}$$

This is known as Fourier's Law, and the constant of proportionality is termed the thermal conductivity. We see that the dimensions of k are:

$$k = \frac{\mathbf{q}}{\nabla \mathbf{T}} = \frac{Q}{L^2 t} \cdot \frac{L}{T} = \frac{Q}{t} \frac{1}{LT}$$

In SI units this would be Watts / metre / Celcius ($W\ m^{-1}\ C^{-1}$)

Heat Conduction: Governing equations III

We will insert Fourier's Law into our continuity equation:

$$\rho C_p \frac{dT}{dt} = \dot{q}''' - \nabla \cdot (-k\mathbf{\nabla T})$$

or, in terms of the Laplacian:

$$\rho C_p \frac{dT}{dt} = \dot{q}''' + k\nabla^2 T$$

This is our basic governing equation, the solution of which will give the temperature distribution in a solid.

Heat Conduction:Boundary Conditions

Our boundary conditions will in general be of three types:

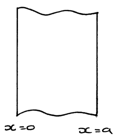
(i) The boundary temperature is given

(ii) The heat flux into the region of interest is given. This corresponds to a given normal component of temperature gradient:

$$q_n = \mathbf{n} \cdot \mathbf{q} = \mathbf{n} \cdot (-k\mathbf{\nabla T}) = -k\frac{\partial T}{\partial n}$$

(iii) Some relationship might be specified between the heat flux into the region, and the surface temperature. This is a common and very importance condition, to which we will return. For now we will consider only the first two types of boundary condition.

Example solutions of the heat conduction equation I



Consider a steady, source-free infinite slab of thickness a, with specified surface temperatures T_0 and T_a . Our governing equation simplifies from:

$$\rho C_p \frac{dT}{dt} = \dot{q}''' + k\nabla^2 T \quad \text{to} \quad \frac{d^2T}{dx^2} = 0$$

The general solution of this is obtained simply by integrating twice:

$$T = Ax + B$$

As usual, we apply our boundary conditions to determine our unknown constants.

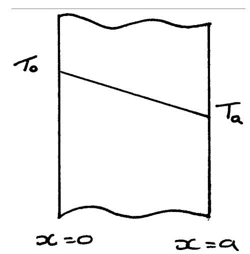
Applying the BC at $x = 0$: $T_0 = B$ giving us $T = Ax + T_0$

Applying the BC at $x = a$: $T_a = Aa + T_0$ from which $A = \frac{T_a - T_0}{a}$

Example solutions of the heat conduction equation II

Our full solution is thus

$$T = (T_a - T_0)\frac{x}{a} + T_0$$



The temperature gradient is

$$\frac{dT}{dx} = \frac{(T_a - T_0)}{a}$$

ie, the same at all x.

The heat flux through the slab is

$$\mathbf{q} = -k\frac{dT}{dx} = -k\frac{(T_a - T_0)}{a}$$

Example solutions of the heat conduction equation III

Consider a uniform cylindrical rod, subject to steady uniform internal heat generation:

With cylindrical symmetry, our governing equation

$$\rho C_p \frac{dT}{dt} = \dot{q}''' \quad + k\nabla^2 T$$

becomes

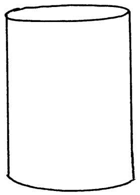
$$k\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = -\dot{q}'''$$

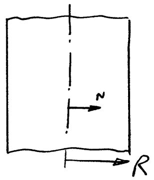
We will take boundary conditions as

$$\left(\frac{dT}{dr}\right)_{r=0} = 0 \quad \text{(symmetry)}$$

and

$$T(r=R) = T_s$$





Example solutions of the heat conduction equation IV

$$k\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = -\dot{q}'''$$

Or

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = -r\frac{\dot{q}'''}{k}$$

Integrating once:

$$r\frac{dT}{dr} = -\frac{r^2}{2}\frac{\dot{q}'''}{k} + A$$

Inserting $\frac{dT}{dr} = 0$ at $r = 0$ yields $A = 0$, and our equation is:

$$\frac{dT}{dr} = -\frac{r}{2}\frac{\dot{q}'''}{k}$$

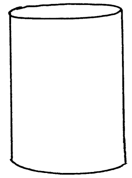
Integrating again:

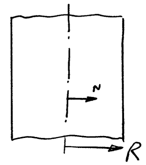
$$T = -\frac{r^2}{4}\frac{\dot{q}'''}{k} + B$$

Inserting $T = T_s$ at $r = R$, we have

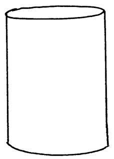
$B = T_s + \frac{R^2}{4}\frac{\dot{q}'''}{k}$, and our full solution is:

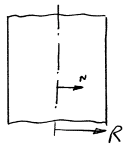
$$T = T_s + \frac{\dot{q}'''}{4k}\left(R^2 - r^2\right)$$





Example solutions of the heat conduction equation V





$$T = T_s + \frac{\dot{q}'''}{4k}\left(R^2 - r^2\right)$$

We could now, for example, evaluate the centre temperature:

$$T(r=0) = T_s + \frac{\dot{q}'''}{4k}\left(R^2 - 0^2\right) = T_s + \frac{\dot{q}'''R^2}{4k}$$

We could also evaluate the edge temperature gradient:

$$\left(\frac{dT}{dr}\right)_{r=R} = \left(\frac{\dot{q}'''}{4k}(-2r)\right)_{r=R} = -\frac{\dot{q}'''R}{2k}$$

The flow of heat out through the surface is then $+k\frac{\dot{q}'''R}{2k} = \frac{\dot{q}'''R}{2}$ per unit area

(ie, per unit perimeter, per unit length).

The flow of heat out through the surface per unit length is then $\frac{\dot{q}'''R}{2}2\pi R = \dot{q}'''\pi R^2$

As would be expected, this is seen to be equal to the total internal heat generation per unit length.

Thermal resistance

The concept of 'thermal resistance', the resistance to the flow of heat, is very convenient. Consider our solution to the slab conduction problem:

$$q'' = k\frac{(T_1 - T_2)}{L_{12}} = \frac{(T_1 - T_2)}{\frac{L_{12}}{k}}$$

We can identify the temperature difference as the driving force, and the quantity $\frac{L_{12}}{k}$

as a resistance to flow. There is obviously a close analogy with Ohm's law: $I = \frac{V}{R}$

The units of thermal resistance are W m^{-2} C^{-1}

Thermal resistance II

Just as for electrical conduction, we can add resistances. Consider two materials in contact:
Applying the idea of conservation of heat flow, for each individually we can write:

$\dot{q}'' = -\frac{(T_2 - T_1)}{\frac{L_{12}}{k_{12}}}$ for the first material, and $\dot{q}'' = -\frac{(T_3 - T_2)}{\frac{L_{23}}{k_{23}}}$ for the second

We can extract the interface temperature from both of these:

$T_2 = T_1 - \dot{q}''\frac{L_{12}}{k_{12}}$ from the first, and

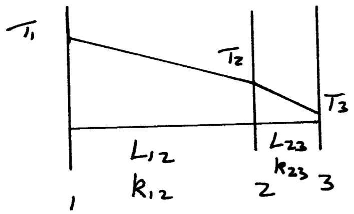
$T_2 = T_3 + \dot{q}''\frac{L_{23}}{k_{23}}$ from the second:

Equating these and tidying up:

$$T_3 - T_1 = -\dot{q}''\left(\frac{L_{12}}{k_{12\ 3}} + \frac{L_{23}}{k_{23}}\right)$$

or

$$\dot{q}'' = -\frac{(T_3 - T_1)}{\left(\frac{L_{12}}{k_{12\ 3}} + \frac{L_{23}}{k_{23}}\right)}$$



The Flow of Fluids

Important physical properties:

ρ	Density	$\frac{M}{L^3}$
u	Velocity	$\frac{L}{T}$
μ	Viscosity	$\frac{\mathrm{M}}{\mathrm{LT}}$

Dimensionless Numbers

The use of dimensionless numbers is widespread and helpful in fluid mechanics. The greatly aid, for example, the extraction of generally applicable conclusions from necessarily limited sets of experiments. The dimensionless numbers we will use are:

Re	Reynolds number	$\frac{\rho u d}{\mu} = \frac{\frac{\dot{m}}{A}d}{\mu}$	The relative importance of viscous forces and momentum
St	Stanton number	$\frac{h}{\rho u C_p}$	The relative importance of heat convection through a boundary layer and convection with the flow
Nu	Nusselt number	$\frac{hd}{k}$	The relative importance surface convection and conduction
f	Friction factor	$f = \frac{\tau}{\frac{\rho v^2}{2}}$	The ratio between wall shear stress and dynamic pressure
Pr	Prandtyl number	$\frac{\mu}{C_p k}$	Proportional to (momentum diffusivity) / (thermal diffusivity)

Laminar & Turbulent Flows

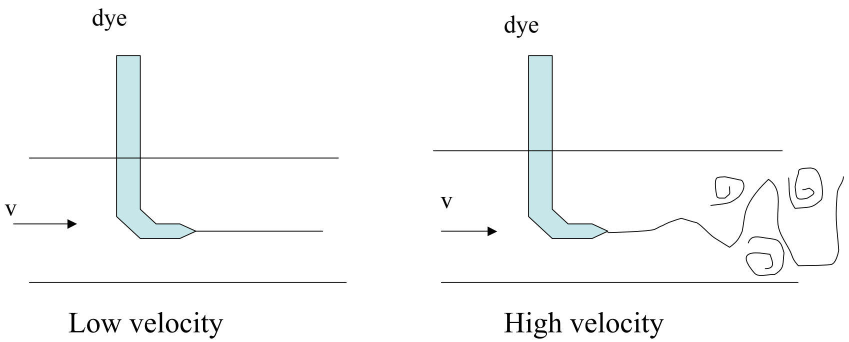
'Slow' flow in (eg) a pipe will be LAMINAR. A 'faster' flow will become TURBULENT.

How to characterise 'faster'? One of may applications of DIMENSIONLESS NUMBERS in fluids.

Laminar & Turbulent Flows II

The Reynolds Number is the best indicator of whether a flow will be turbulent or laminar.
For pipe flow the Reynolds Number is based on the pipe diameter.

The transition between laminar and turbulent flow occurs between Re = 10^3 and 10^4.
For flows of interest to us in reactors (at least under normal operating conditions) flows will be turbulent.



Pressure drops in pipe flow

For turbulent flow, this is obtained experimentally. The results of experimental measurements are plotted in terms of three dimensionless numbers:

-the friction factor ('the pressure drop')

-the reynolds number as the 'independent variable'; '(the flow speed')

-the pipe surface roughness as a parameter

The friction factor is a non-dimensionalised form of the wall shear stress (or equivalently, the pressure drop per unit length). It is defined as:

$$f \equiv \frac{\left(\frac{\Delta p}{L}\right) d}{\frac{1}{2}\rho v^2}$$

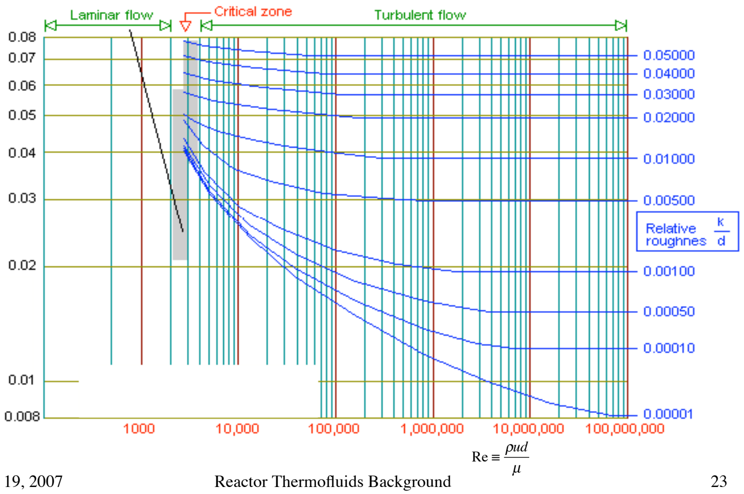
Expressing in terms of wall shear stress (for a circular pipe):

$\tau \pi d = \frac{\Delta p}{L}\pi d^2$, or $\frac{\Delta p}{L} = \frac{\tau}{d}$, so

$$f = \frac{\left(\frac{\tau}{d}\right) d}{\frac{1}{2}\rho v^2} = \frac{\tau}{\frac{1}{2}\rho v^2}$$

Friction Factor Chart

$$f \equiv \frac{2\left(\frac{\Delta p}{L}\right) d}{\rho u^2} = \frac{2\tau}{\rho v^2}$$



Convective Heat Transfer

It is a matter of everyday experience that a hot object left out in a cold wind gets cold. The hotter it is, the more rapidly its temperature reduces. This observation is known as Newton's Law of Cooling.
Algebraically, we would write

$$\dot{q}'' = h\left(T_{surface} - T_{fluid}\right)$$

The rate of heat loss per unit area is some constant times the difference between the solid's surface temperature and the **bulk** fluid temperature.

The constant of proportionality, h, is known as the **Heat Transfer Coefficient**

$$h = \frac{\dot{q}''}{\left(T_{surface} - T_{fluid}\right)} = \frac{Q}{L^2 tT}$$

and SI units Watts per square metre per degree $Wm^{-2}C^{-1}$

It is thus, dimensionally, 1/(thermal resistance).

Heat Conduction:Boundary Conditions re-visited

$$q_n = \mathbf{n}\cdot\mathbf{q} = \mathbf{n}\cdot(-k\nabla\mathbf{T}) = -k\frac{\partial T}{\partial n}$$

All boundary conditions are of the form: $\alpha T + \beta\frac{\partial T}{\partial n} + \gamma = 0$

$\beta = 0$ corresponds to $T = \frac{-\gamma}{\alpha}$; ie a prescribed temperature

α=0 corresponds to $\frac{\partial T}{\partial n} = \frac{-\gamma}{\beta}$; ie a prescribed surface hear flux. (We have used these already)

If none of $\alpha, \beta, \gamma = 0$, or

$\beta\frac{\partial T}{\partial n} = -\alpha T - \gamma$ it corresponds to convective heat transfer (and often termed the Robin condition):

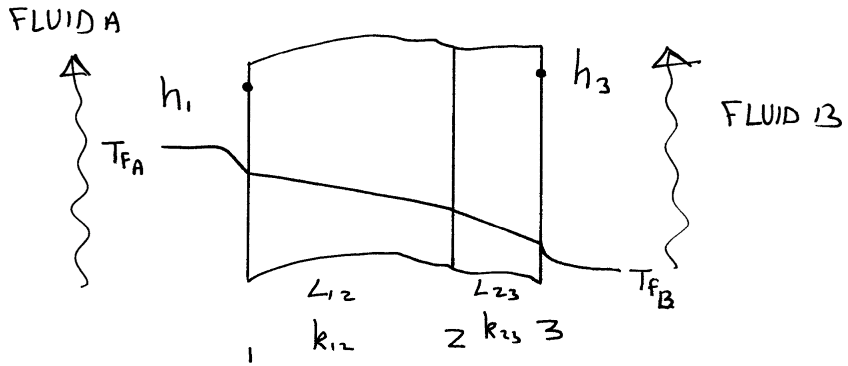
$k\frac{\partial T}{\partial n} = h(T - T_f)$ where we identify

$\beta = k$

$\alpha = -h$

$\gamma = hT_f$

Convective Heat Transfer II

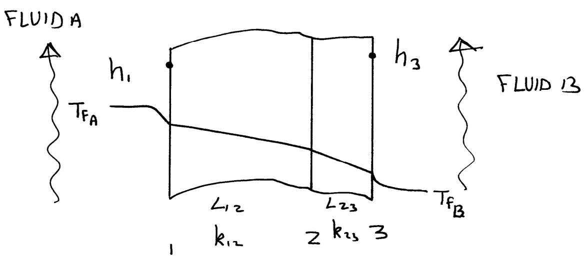


Convective Heat Transfer III

Using again the idea of thermal resistance, and just extending a little our earlier analysis of a composite solid, we can now analyse such a solid exposed to a fluid flow on both sides.

We can write for the flow of heat:

$$\dot{q}'' = \frac{(T_{fA} - T_{fB})}{\left(\frac{1}{h_1} + \frac{L_{12}}{k_{12}} + \frac{L_{23}}{k_{23}} + \frac{1}{h_3}\right)}$$



Once the heat flow is know, we can easily compute, say, the intermediate temperatures.

Heat transfer coefficients

Heat transfer coefficients plainly will depend on the fluid, and on the flow conditions. They are mostly determined experimentally.

Results from such experiments are invariably presented in dimensionless form. For example, for flow in circular pipes the Dittus-Boelter correlation is widely used:

$$\mathrm{Nu} = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.4}$$

or equivalently, as $\mathrm{St} = \frac{\mathrm{Nu}}{\mathrm{Re}\,\mathrm{Pr}}$:

$$\mathrm{St} = 0.023\,\mathrm{Re}^{-0.2}\,\mathrm{Pr}^{-0.6}$$

Hydraulic Diameter

Behaviour in non-circular passages may be approximated from measurements in circular ones by means of the hydraulic, or 'equivalent circular', diameter.

$$d_h \equiv \frac{4 \,.\, (\text{flow area})}{(\text{wetted perimeter})}$$

For example, flow in a rectangular duct of dimensions *a* and *b* would have a similar heat transfer coefficient to that in a circular pipe of diameter given by:

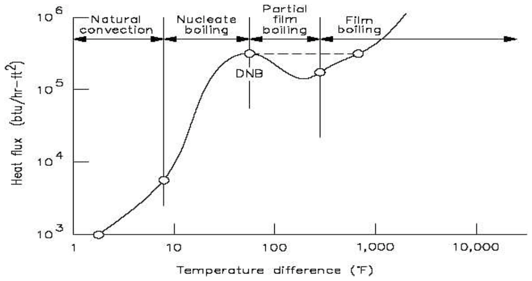
$$d_h \equiv \frac{4\,\text{ab}}{2\text{a}+2\text{b}} = \frac{2\,\text{ab}}{\text{a}+\text{b}}$$

For a square duct of side *a* we would have:

$$d_h \equiv \frac{2\,\text{a}^2}{2\text{a}} = a$$

(Naturally the 'equivalent diameter' for a circular pipe should be just the pipe diameter: It is.)

Boiling Heat Transfer



Nucleate boiling: Very good HT, high heat fluxes with small temperature differences

Film boiling: Vapour blankets the surface, and acts as an insulator. Very high temperatures are generated in a 'constant heat flux' device.

This 'Critical Heat Flux', or 'Departure from Nucleate Boiling, DNB, MUST be avoided.
The CHF depends in a complex way on steam/water conditions, temperatures, flow regime, surface conditions, ……

Thermofluids

Within-pin Heat Transfer, Heat Transfer to Coolant, and Core Pumping Power

Contents

1 INTRODUCTORY REMARKS

As we have seen in the reactor physics parts of the course, heat is generated essentially at the location of the fission event. Fissions occur only in the fuel; that is where the fissile material is. They occur at a rate (more or less) proportional to the local flux (neutron number density). The heat generation rate thus varies over the core with the same shape as the flux distribution.

On a local scale, there is a little variation in fission rate radially across individual pins. Neutrons enter the fuel from the moderator, and the fuel centre sees a slightly lower flux than the outer regions, simply because some neutrons get absorbed in the outer regions and do not make it to the centre. We will ignore this.

For purposes of thermal analysis, heat generation by the fission process is identical to heat generation by (say) electric resistance heating. We will model it as a heat generation rate of (energy per unit volume per unit time); in SI units, Joules per cubic metre, per second, or W m^{-3}.

It is helpful to have an order of magnitude impression of the rates involved.

It varies with reactor type, but in a power reactor, a fuel rod of about 15mm diameter will produce of order 40kW per metre. This corresponds to a volumetric generation rate of about $2x10^8$ W m^{-3}.

In (say) a 1kW electric fire, heat is generated in essentially a cylinder of conductor about 300mm long, about 30mm in circumference, and about 1/2mm thick. This also corresponds to a volumetric generation rate of about $2x10^8$ W m^{-3} considering the metal conductor only, and about 1/10 of this if averaged over the cylinder volume.

In the sections which follow, we will first study temperature distributions within a pin; essentially, how hot does the centre of the pin have to become to generate large enough temperature gradients to drive the heat generated within it out through the cladding into the coolant?

We will then turn to the variation of temperature along the length of the pin. Coolant enters the bottom of the reactor at a relatively low temperature. It picks up heat from the pins as it goes along, at a rate that follows the variation of fission rate and hence neutron flux. It leaves the core hotter. In order to give its heat to the coolant, the pin surface (cladding) must be hotter than the coolant. The amount hotter it needs to be depends on how much heat it needs to get rid of, and as we have seen, this rate varies markedly, following the neutron flux distribution. We will study this, and determine the cladding temperature variation along the length of the pin. The clad of course is one of the main barriers to fission product release, and avoiding its oxidation or melting is vital. In a similar way, the pin centre needs to be hotter than the cladding, by an amount also varying along the length of the pin, also following the flux distribution. We will study this. Too high temperatures in the fuel material must be avoided, to avoid both melting, and fission product release, which can become significant at temperatures some way below melting.

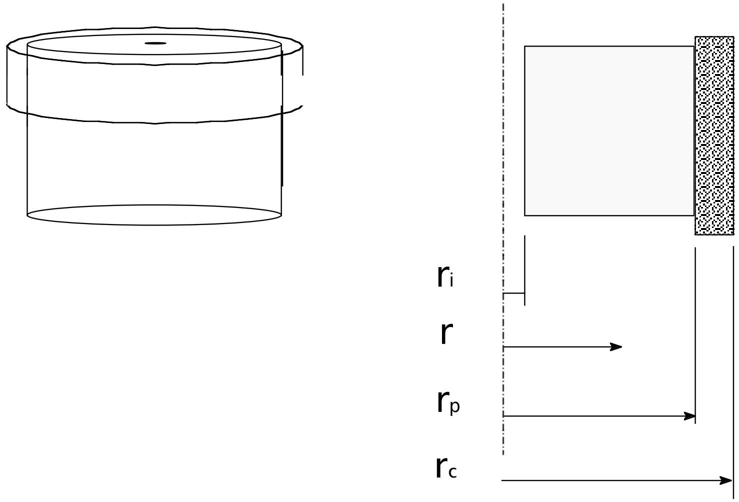
2 HEAT TRANSFER AND TEMPERATURE DISTRIBUTION WITHIN THE FUEL PIN

2.1 Introduction

The main constraints on pin power output are temperature-related; fuel melting, phase change, fission gas release, clad weakening...

Our objective here is to determine within-pin temperatures as a function of pin materials and geometrical parameters, and of the rate of power generation in the pin.

A typical segment of a pin (pellet) is:



The radius of the inner hole we will denote $r_{i,}$, the pellet outer radius r_p, and the clad outer radius r_c.

We will initially consider only the fuel material (pellet) itself, and then will include the pellet-clad gap, the clad itself, and the temperature drop through the boundary layer in the coolant.

2.2 Governing equation

$$\nabla^2 T = -\frac{\dot{q}'''}{k}$$

where we will take the rate of heat generation as being spatially uniform.

We will here neglect $\frac{\partial}{\partial z}$ terms relative to $\frac{\partial}{\partial r}$; most pins have lengths of order 1 m, and diameters of order 0.01m. Similarly, we will assume azimuthal symmetry, and neglect $\frac{\partial}{\partial \theta}$, although this second order term in particular can actually have very important effects.

The appropriate form of the Laplacian thus yields:

$$k\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = -\dot{q}'''$$

Here for now we will take *k* as a constant, not a function of *T*, but in practice, over the large range of temperatures existing in a nuclear fuel element, *k* actually does vary considerably.

For now we will neglect both the cladding and the fuel - clad gap.

2.3 Boundary conditions:

(i) We will take the pellet outer surface temperature as a 'given' for now.

$$\text{Pellet outer surface temperature} = T_P\left(r = r_P\right)$$

(ii) No heat transfer from the inner pellet surface:

$$\left(\frac{dT_P}{dr}\right)_{r=r_i} = 0$$

2.4 Solving for the temperatures

Our equation is:

$$k\frac{d}{dr}\left(r\frac{dT_P}{dr}\right) = -r\dot{q}'''$$

Integrating once:

$$kr\frac{dT_P}{dr} = -\frac{r^2}{2}\dot{q}''' + C_1$$

Applying the centre bc $\left(\frac{dT_P}{dr}\right)_{r=r_i} = 0$ gives:

$$C_1 = \frac{r_i^2}{2}\dot{q}'''$$

Using this, and tidying up, we obtain

$$k\frac{dT_P}{dr} = \frac{\dot{q}'''}{2}\left(\frac{r_i^2}{r} - r\right)$$

We are taking the surface temperature as 'known', so let us integrate with respect to *r* once more from the surface to some general radius *r:*

$$\int_r^{r_p} k\frac{dT_P}{dr}dr = \frac{\dot{q}'''}{2}\int_r^{r_p}\left(\frac{r_i^2}{r} - r\right)dr$$

so

$$\int_{T(r)}^{T(r_p)} kdT = \frac{\dot{q}'''}{2}\left[r_i^2\log r - \frac{r^2}{2}\right]_r^{r_p}$$

As noted, conductivity varies a lot over the large temperature ranges at issue. As will be seen from the above equation, it is the quantity $\int kdT$ over the relevant range that is most important, and this is commonly tabulated in datasheets and so on.

For now we will take *k* to be independent of temperature, allowing us to write

$$T_P(r) - T_{P_{r=r_p}} = \frac{\dot{q}'''}{2k}\left(r_i^2\log\frac{r}{r_p} + \frac{r_p^2 - r^2}{2}\right)$$

We are likely to be particularly interested in the peak temperature, at the edge of the inner hole. This is

$$T_P = T_{P_{r=r_p}} + \frac{\dot{q}'''}{2k}\left(r_i^2\log\frac{r_i}{r_p} + \frac{r_p^2 - r_i^2}{2}\right)$$

No central hole

If the pin has no central hole this reduces to:

$$T_P = T_{r=r_p} + \frac{\dot{q}'''}{4k}\left(r_p^2 - r^2\right)$$

The centre temperature is then given by:

$$T_P = T_{P_{r=r_p}} + \frac{\dot{q}'''r_p^2}{4k}$$

2.5 A typical example

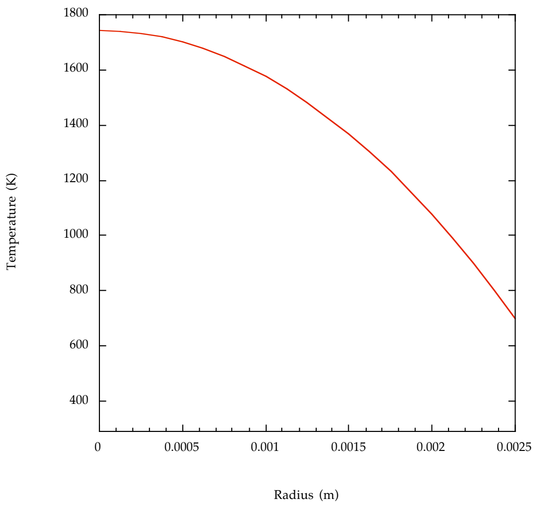
To give an idea of the kinds of magnitudes involved, consider a fast breeder reactor pin:

$\dot{q}'''$ (Wm^{-3})	2x10^9
r_p (m)	0.0025
k (Wm^{-1}K^{-1})	3 (UO_2)

For these conditions

$$T_{P\ r=0} - T_{P\ r=r_p} = 1042\text{ K}$$

Note how large a temperature difference exists over a distance of ~2 1/2 mm. Note also that our treatment of the conductivity as independent of temperature is questionable, when the temperature range is so large.



2.6 Linear rating

The linear rating, the heat release per unit length of pin, is of crucial importance.

$$\dot{q}' = \dot{q}'''\pi\left(r_p^{\ 2} - r_i^{\ 2}\right)$$

For no hole

$$\dot{q}' = \dot{q}'''\pi r_p^{\ 2}$$

and using our above result for the temperature distribution we can write $\dot{q}''' r_p^{\ 2}$ as $4k\left(T_{r=0} - T_{r=r_p}\right)$, giving

$$\dot{q}' = 4\pi k\left(T_{r=0} - T_{r=r_p}\right)$$

This is an important result. It shows:

•the linear rating depends on the temperature extremes; the inner fixed mostly by melting or associated materials constraints of the fuel material, and the outer essentially a function of how high a temperature the cladding can stand.

•the linear rating allowable is not a function of pin radius

•this might incline the design towards lots of small diameter pins, but note that for a fixed linear rating the surface heat flux rises inversely with pin radius. We need to be able to convect away this power. In particular, in a liquid cooled reactor we might go into ‘dryout’ conditions.

•the desirability of a high maximum permissible temperature is clear

•the desirability of a high k is clear

Typical values of peak linear rating would be ~40 kWm^{-1} in PWR, FBR, less in AGR, and much less in Magnox.

2.7 Gap heat transfer

This is very complicated.

Pellet and clad are generally, but not always, in mechanical contact. However, no surfaces are ‘smooth’, and contact is limited to the peaks of the two rough surfaces.

Heat transfer across the gap occurs by all of:

-conduction where these peaks touch

-convection in the gas filling the gap

-conduction through the gas filling the gap, and

-radiation from solid to solid.

The gap temperature drop is reduced by pressurising the pin with helium when it is made. Helium has a high thermal conductivity. However, fission gas release during operation dilutes this with less conductive gases.

Fission gas release, fuel swelling and creep, and clad creep and differential thermal expansion all occur during operation, and even more so during the course of accidents. The clad can be pressed down onto the fuel, or can be lifted away. All of this makes fuel thermal-mechanical modelling a major area of reactor analysis.

We will denote the clad inner radius to be r_p. We will simplify, and represent heat transfer across the gap via introducing a gap conductance (heat flux per unit area per unit temperature difference), and a temperature difference across the vanishingly small radial gap size:

$$\dot{q}'' = G\left(T_{P(r=r_p)} - T_{C(r=r_p)}\right)$$

where, when required, we will identify pellet, clad and later fluid (coolant) by subscripts P, C and F respectively.

The gap temperature drop is then

$$T_{P(r=r_p)} - T_{C(r=r_p)} = \frac{\dot{q}''}{G}$$

or in terms of linear rating:

$$T_{P(r=r_p)} - T_{C(r=r_p)} = \frac{1}{2\pi r_p}\frac{\dot{q}'}{G}$$

2.8 Clad heat transfer

The clad, of thickness t, extends from $r=r_p$ to $r=r_c$.

The governing equation is as before, but with no source term:

$$k_C \frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

Integrating once we obtain

$$r\frac{dT}{dr} = C$$

We know the heat flow at the inner boundary, so at $r=r_p$

$$\frac{\dot{q}'}{2\pi r_p} = -k_C\left(\frac{dT}{dr}\right)_{r=r_p}$$

and we can use his to evaluate the constant above:

$$C = -\frac{\dot{q}'}{k_C 2\pi}$$

giving us

$$r\frac{dT}{dr} = -\frac{\dot{q}'}{k_C 2\pi}$$

We integrate this from (say) the outer clad boundary, at $r=r_c$ to some general radius r:

$$\int_{T(r=r_c)}^{T(r)} dT = T(r) - T(r_c) = -\frac{\dot{q}'}{k_C 2\pi}\int_{r_c}^{r}\frac{1}{r}dr$$

or

$$T(r) - T(r_c) = -\frac{\dot{q}'}{k_C 2\pi}\ln\left(\frac{r}{r_c}\right)$$

The temperature difference through the clad is then

$$T(r_p) - T(r_c) = \frac{\dot{q}'}{k_C 2\pi}\ln\left(\frac{r_c}{r_p}\right)$$

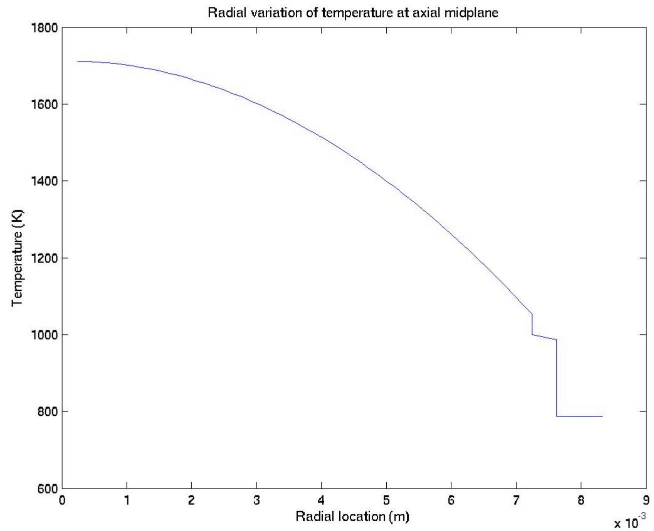
or in the more explicit notation:

$$T_{C_{r=r_p}} - T_{C_{r=r_C}} = \frac{\dot{q}'}{k_C 2\pi}\ln\left(\frac{r_c}{r_p}\right)$$

Again taking the FBR as an example:

r_p (m)	0.0035
t_{clad} (m)	~0.001 (cf PWR ~ 0.001, AGR ~0.0004), so
r_c (m)	0.0045
k_C ($Wm^{-1}K^{-1}$)	24 (stainless steel) (NB eg for Zr, conductivity is ~11)
$\dot{q}'$ (Wm^{-1})	40×10^3

This gives a clad temperature drop of ~67K. It is a big temperature difference to exist through a millimetre of metal; an indication of the high heat fluxes at issue. The drop would be less in say an AGR, where the clad is thinner, and the heat fluxes lower. It is still small compared to the temperature drop over the fuel radius itself.



2.9 Coolant heat transfer

We will represent the convective heat transfer to the coolant, as usual, via a heat transfer coefficient, h.

With this we have:

$$\dot{q}'' = h\left(T_{C(r=r_C)} - T_F\right)$$

or

$$T_{C(r=r_C)} - T_F = \frac{1}{h}\dot{q}''$$

2.10 Putting it all together

We can gather these various expressions for temperature drops.

We had:

For the within-pellet temperature:

$$T_P(r) = T_{P_{r=r_p}} + \frac{\dot{q}'''}{2k}\left(r_i^2 \log\frac{r}{r_p} + \frac{r_p^{\,2} - r^2}{2}\right)$$

For the drop through the gap:

$$T_{P(r=r_p)} = T_{C(r=r_p)} + \frac{1}{2\pi r_p}\frac{\dot{q}'}{G}$$

For the drop through the clad:

$$T_{C_{r=r_p}} = T_{C_{r=r_C}} + \frac{\dot{q}'}{k_C 2\pi}\ln\left(\frac{r_c}{r_p}\right)$$

For the drop through the boundary layer:

$$T_{C(r=r_C)} = T_F + \frac{1}{h}\dot{q}''$$

Putting these together, we can write an expression for the temperature within the pellet as:

$$T_P(r) = T_F + \frac{1}{h}\dot{q}'' + \frac{\dot{q}'}{k_C 2\pi}\ln\left(\frac{r_c}{r_p}\right) + \frac{1}{2\pi r_p}\frac{\dot{q}'}{G} + \frac{\dot{q}'''}{2k}\left(r_i^2 \log\frac{r}{r_p} + \frac{r_p^{\,2} - r^2}{2}\right)$$

where we can clearly see the contribution of the various temperature drops.

3 CHANNEL HEAT TRANSFER

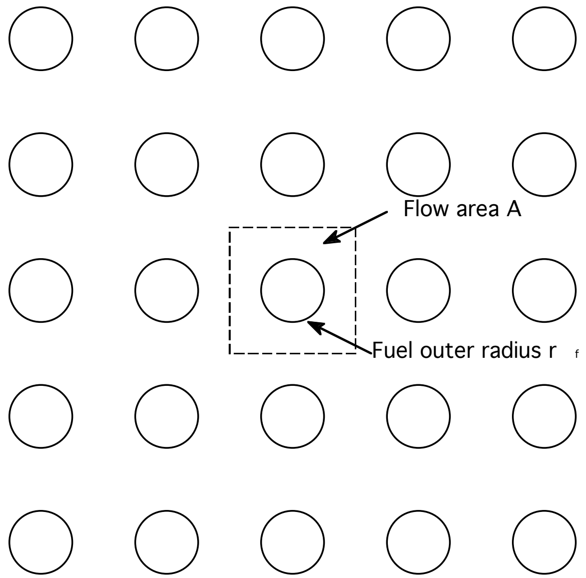
3.1 Introduction

Different reactor designs have different flow arrangements. In an AGR, flow is directed along a small number of channels, essentially axial holes through the graphite moderator, in which multiple pins are suspended. There is no crossflow between these channels, and the channel forms a logical unit of analysis. In a fast reactor the position is similar. A large number of pins (eg 256) are gathered into a subassembly, which is surrounded by a thin steel duct. Crossflow between pins in a single subassembly can occur, but not between subassemblies. In a PWR pins are similarly gathered into subassemblies, but there is no surrounding duct, and crossflow is easier.

In all cases we will perform the analysis by considering a single pin, and identifying some part of the cross sectional area for coolant flow which is that pin's fair share; we will term this a sub-channel. We will then, for example, be able to associate the power produced by that pin with the rate at which energy is gained by that fluid which flows in that pin's subchannel. Crossflow will mean that fluid from other subchannels enters the one at issue. Of course equally crossflow *from* our subchannel must occur, and these will, to a very good approximation, import and export equal energy flows.

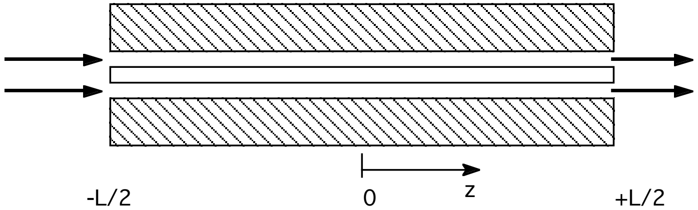
3.2 A characteristic 'flow sub-channel'

In cross section a typical set of pins is:



We identify and analyse a typical pin and associated flow area. We will term position along the channel the axial position, and use z as our axial coordinate. Because the flux is symmetrical about the axial mid-plane we will take the origin here, with the core thus extending from $-L/2$ to $+L/2$, where L is the total length of the core.

Viewed sideways, this is:



3.3 Variation with axial position of mean coolant temperature.

At a general location z we have:

$$\dot{m}C_p \frac{dT}{dz} = \dot{q}'(z)$$

(formally by application of the steady flow energy equation, SFEE, to an infinitesimal length of the channel).

So the temperature at any location is given by:

$$T(z) - T_{in} = \frac{1}{\dot{m}C_p} \int_{\frac{-L}{2}}^{z} \dot{q}'(z)\,dz$$

From our reactor physics we know that for a cylindrical reactor

$$\dot{q}'(z) = \hat{\dot{q}}' \cos\left(\frac{\pi z}{L}\right)$$

so using this

$$T(z) - T_{in} = \frac{\hat{\dot{q}}'}{\dot{m}C_p} \int_{\frac{-L}{2}}^{z} \cos\left(\frac{\pi z}{L}\right) dz$$

or

$$T(z) - T_{in} = \frac{\hat{\dot{q}}'}{\dot{m}C_p} \frac{L}{\pi} \left[\sin\left(\frac{\pi z}{L}\right) - \sin\left(\frac{-\pi \frac{L}{2}}{L}\right) \right] = \frac{\hat{\dot{q}}'}{\dot{m}C_p} \frac{L}{\pi} \left[\sin\left(\frac{\pi z}{L}\right) - \sin\left(\frac{-\pi}{2}\right) \right]$$

or

$$T(z) - T_{in} = \frac{\hat{\dot{q}}'}{\dot{m}C_p} \frac{L}{\pi} \left(\sin\left(\frac{\pi z}{L}\right) + 1 \right)$$

3.3.1 Coolant exit temperature

At the exit we have:

$$T_{out} - T_{in} = \frac{2\hat{\dot{q}}'}{\dot{m}C_p} \frac{L}{\pi}$$

3.3.2 Peak to Mean linear rating ratio

$$\bar{\dot{q}}' = \frac{\dot{m}C_p\left(T_{out} - T_{in}\right)}{L}$$

so the ratio is

$$\frac{\hat{\dot{q}}'}{\bar{\dot{q}}'} = \frac{\frac{\dot{m}C_p \pi}{2L}\left(T_{out} - T_{in}\right)}{\frac{\dot{m}C_p\left(T_{out} - T_{in}\right)}{L}} = \frac{\pi}{2} \approx 1.6$$

From an engineering / economics point of view this is unattractive. The ends of the fuel are little used; we are not making the most of the expensive fuel and heat transfer surface we have provided. This is one of the reasons that a lot is done to flatten this axial co-sinusoidal variation of flux, primarily by the use of neutron reflectors.

3.4 Variation of clad surface temperature, T_s

(Notation; we will use:

T	bulk mixed coolant temperature
T_c	clad surface temperature
T_p	for pellet edge temperature
T_0	pellet centre temperature)

At any location z the *difference* between clad surface temperature T_c and the bulk coolant temperature T is proportional to the local surface heat flux: That is

$$\left(T_c(z) - T(z)\right) 2\pi r_c h = \dot{q}'(z)$$

where h is the heat transfer coefficient and r_c the clad outer radius.

So

$$T_C(z) = T(z) + \frac{1}{2\pi r_C h}\dot{q}'(z)$$

We have just obtained an expression for $T(z)$, and using this, and our knowledge of $\dot{q}'(z)$, we can write

$$T_C(z) = T_{in} + \frac{\hat{\dot{q}}'}{\dot{m}C_p}\frac{L}{\pi}\left(\sin\left(\frac{\pi z}{L}\right) + 1\right) + \frac{1}{2\pi r_C h}\hat{\dot{q}}'\cos\left(\frac{\pi z}{L}\right)$$

or

$$T_C(z) = T_{in} + \frac{\hat{\dot{q}}'}{\pi}\left(\frac{L}{\dot{m}C_p}\left[\sin\left(\frac{\pi z}{L}\right) + 1\right] + \frac{1}{2r_C h}\cos\left(\frac{\pi z}{L}\right)\right)$$

3.4.1 Location of maximum cladding temperature

Differentiating the above we get:

$$\frac{\partial T_C(z)}{\partial z} = \frac{\hat{\dot{q}}'}{\pi}\left\{\frac{L}{\dot{m}C_p}\frac{\pi}{L}\cos\left(\frac{\pi z}{L}\right) - \frac{1}{2r_C h}\frac{\pi}{L}\sin\left(\frac{\pi z}{L}\right)\right\}$$

Setting to zero we have at the maximum-temperature location:

$$\frac{z}{L} = \frac{1}{\pi}\arctan\left(\frac{2r_C hL}{\dot{m}C_p}\right)$$

This (as the arctan is between 0 and $\pi/2$) lies somewhere between 0 and 0.5. The hottest cladding is somewhere above the axial mid-plane, as we would expect.

3.5 Variation of pellet temperature

We will repeat the above process, but now to determine the axial variation of pellet maximum temperature.

Pellet temperature is given by

(i) the clad temperature plus:

(ii) the through-clad temperature drop

(iii) the gap temperature drop

(v) the temperature difference between the pellet edge and centre (or edge of inner hole, if it has one).

We have expressions for all these quantities already, so putting it together (and inserting as required the particular axial flux variation, and expressing the one volumetric rate as a linear rate for tidiness) we have:

$$\begin{aligned} T_P(z) = T_{in} & \\ & + \frac{\hat{\dot{q}}'}{\pi}\left(\frac{L}{\dot{m}C_p}\left[\sin\left(\frac{\pi z}{L}\right) + 1\right] + \frac{1}{2r_C h}\cos\left(\frac{\pi z}{L}\right)\right) \\ & + \frac{\hat{\dot{q}}'\cos\left(\frac{\pi z}{L}\right)}{k_C 2\pi}\ln\left(\frac{r_c}{r_p}\right) \\ & + \frac{1}{2\pi r_p}\frac{\hat{\dot{q}}'\cos\left(\frac{\pi z}{L}\right)}{G} \\ & + \frac{\hat{\dot{q}}'\cos\left(\frac{\pi z}{L}\right)}{2k}\frac{1}{\pi\left(r_p^2 - r_i^2\right)}\left(r_i^2\log\frac{r_i}{r_p} + \frac{r_p^2 - r_i^2}{2}\right) \end{aligned}$$

We can, as for the clad, differentiate this to find the maximum pellet temperature location and magnitude:

$$\begin{aligned} 0 = & \frac{\hat{\dot{q}}'}{\pi}\left(\frac{L}{\dot{m}C_p}\left[\frac{\pi}{L}\cos\left(\frac{\pi z}{L}\right)\right] - \frac{1}{2r_C h}\frac{\pi}{L}\sin\left(\frac{\pi z}{L}\right)\right) \\ & - \frac{\pi}{L}\frac{\hat{\dot{q}}'\sin\left(\frac{\pi z}{L}\right)}{k_C 2\pi}\ln\left(\frac{r_c}{r_p}\right) \\ & - \frac{\pi}{L}\frac{1}{2\pi r_p}\frac{\hat{\dot{q}}'\sin\left(\frac{\pi z}{L}\right)}{G} \\ & - \frac{\pi}{L}\frac{\hat{\dot{q}}'\sin\left(\frac{\pi z}{L}\right)}{2k}\frac{1}{\pi\left(r_p^2 - r_i^2\right)}\left(r_i^2\log\frac{r_i}{r_p} + \frac{r_p^2 - r_i^2}{2}\right) \end{aligned}$$

Tidying this up we have

$$0 = -\sin\left(\frac{\pi z}{L}\right)\left\{\frac{1}{2r_C h} + \frac{1}{k_C 2}\ln\left(\frac{r_c}{r_p}\right) + \frac{1}{2r_p}\frac{1}{G} + \frac{1}{2k}\frac{1}{\left(r_p^{\,2} - r_i^2\right)}\left(r_i^2 \log\frac{r_i}{r_p} + \frac{r_p^{\,2} - r_i^2}{2}\right)\right\}$$

$$+\cos\left(\frac{\pi z}{L}\right)\left\{\frac{L}{\dot{m}C_p}\right\}$$

Or

$$\tan\left(\frac{\pi z}{L}\right) = \frac{\left\{\frac{L}{\dot{m}C_p}\right\}}{\left\{\frac{1}{2r_C h} + \frac{1}{k_C 2}\ln\left(\frac{r_c}{r_p}\right) + \frac{1}{2r_p}\frac{1}{G} + \frac{1}{2k}\frac{1}{\left(r_p^{\,2} - r_i^2\right)}\left(r_i^2 \log\frac{r_i}{r_p} + \frac{r_p^{\,2} - r_i^2}{2}\right)\right\}}$$

or

$$\left(\frac{z}{L}\right) = \frac{1}{\pi}\arctan\left(\frac{\left\{\frac{L}{\dot{m}C_p}\right\}}{\left\{\frac{1}{2r_C h} + \frac{1}{k_C 2}\ln\left(\frac{r_c}{r_p}\right) + \frac{1}{2r_p}\frac{1}{G} + \frac{1}{2k}\frac{1}{\left(r_p^{\,2} - r_i^2\right)}\left(r_i^2 \log\frac{r_i}{r_p} + \frac{r_p^{\,2} - r_i^2}{2}\right)\right\}}\right)$$

or

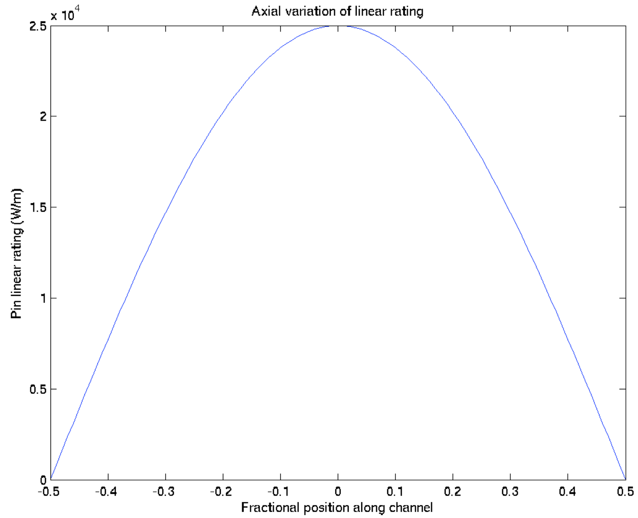
$$\left(\frac{z}{L}\right) = \frac{1}{\pi}\arctan\left(\frac{2L}{\dot{m}C_p\left\{\frac{1}{r_C h} + \frac{1}{k_C}\ln\left(\frac{r_c}{r_p}\right) + \frac{1}{r_p}\frac{1}{G} + \frac{1}{k}\frac{1}{\left(r_p^{\,2} - r_i^2\right)}\left(r_i^2 \log\frac{r_i}{r_p} + \frac{r_p^{\,2} - r_i^2}{2}\right)\right\}}\right)$$

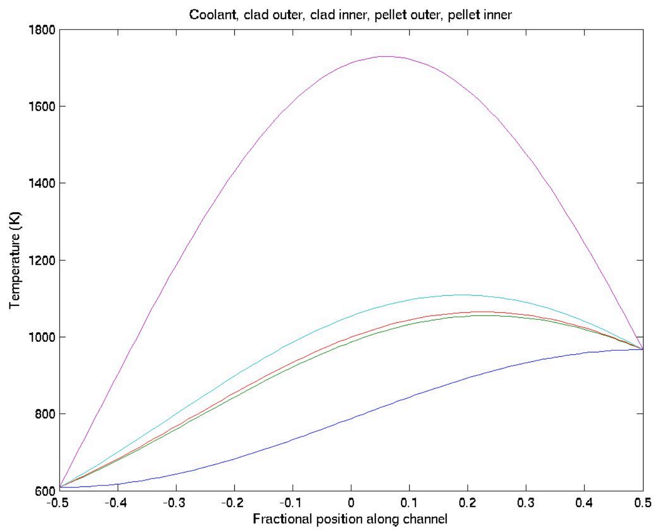
3.6 <u>Axial temperature distributions</u>

The following graph shows axial variation of each of linear power (neutron flux), coolant temperature, clad outer and inner surface temperature, and pellet outer and inner temperature.

These are important graphs, and you should make sure you are familiar with and understand their principal features.

(The graphs are for a typical gas cooled reactor; you are advised to consider how values are likely to differ for a water-cooled reactor.)





3.7 <u>Surface heat transfer coefficient and heat-transfer correlations</u>

3.7.1 Introduction

Actual reactor geometries and flow conditions are naturally very complex, and analytical results are not available.

The surface heat transfer coefficient depends on the flow conditions as well as on the fluid properties. For turbulent flows the relationship is naturally complex. Values of Stanton number, St (and hence of surface heat-transfer coefficient) have to be obtained from other sources, either experimental results or more advanced theoretical analysis. Simplified forms are often adequate for many engineering purposes.

3.7.2 Water and gas coolants

For steady, full-developed flow of a single-phase fluid in straight, smooth tubes, the 'Dittus-Boelter' correlation is often used:

$$St = 0.023\,\mathrm{Re}^{-0.2}\,\mathrm{Pr}^{-0.6}$$

Where Reynolds number:

$$\mathrm{Re} = \frac{\rho u d}{\mu}$$

Prandtl number:

$$\mathrm{Pr} = \frac{\mu C_p}{k}$$

and Stanton number:

$$\mathrm{St} = \frac{h}{\rho u C_p}$$

These correlations are, strictly, valid only for flows in which temperature and pressure differences are small enough for the fluid properties to be regarded as uniform.

Correlations derived from experiments or advanced analysis for flows in (round) tubes can be used also for flows in non-circular passages by employing the device of the 'equivalent diameter'. This is defined as

$$d_e = \frac{4\,x\,\text{flow area}}{\text{wetted perimeter}}$$

The value of St is obtained, for the non-circular passage, by using the value of Re calculated from

$$\mathrm{Re} = \frac{\rho u d_e}{\mu}$$

Other modifications of the above correlations may be needed to account for the artificial roughening of the hear-transfer surfaces of some reactor fuel pins. For AGR fuel pins, the following heat-transfer correlation is sometimes used:

$$St = 0.054\,\mathrm{Re}^{-0.2}\,\mathrm{Pr}^{-0.6}$$

Comparison with the normal form shows an increase of ~135% above the standard value for smooth surfaces. The usual height of the ribs is 0.3mm, so the increase in the heat-transfer surface area is very small (10%); the main factor causing the increase is the vortices created by the ribs. These considerably increase the transport rate of momentum and energy in the transverse direction, breaking up the 'laminar sublayer' that would occupy the thin region next to a smooth surface. The chief penalty to be paid is the increase in wall shear stress - roughly proportional to that in surface heat-transfer coefficient. To reflect this, the numerical factor in the friction factor equation (used in the pressure drop analysis) has to be raised (to about 0.108, from the 0.046 appropriate for smooth walls).

3.7.3 Liquid metal

The processes involved in liquid metal heat transfer are rather different (principally conduction takes a larger role) and different correlations are employed. There is a variety, reflecting design differences and so on. A typical one is:

$$Nu = 4.0 + 0.16(\theta)^{5.0} + 0.33(\theta)^{3.8}\left(\frac{Pe}{100}\right)^{0.86}$$

where θ is the pitch to diameter ratio of the fuel pins, and the Nusselt number:

$$\mathrm{Nu} = \frac{h d_e}{k}$$

and the Peclet number

$$\mathrm{Pe} = \mathrm{Re}\,\mathrm{Pr}$$

4 CORE PRESSURE DROP AND PUMPING POWER

4.1 Introduction

4.2 Pressure drop through the core

In normal operation, reactor coolants flow through the core at 'low' speed (Mach number <<1). Nevertheless, because of the large temperature rise the density changes significantly (at least for gaseous coolants), and this has to be allowed for.

Application of the force-momentum relation (equating the net force on the control volume to the increase in momentum flow rate) in the z-direction, ignoring the weight of the fluid, gives

$$-\frac{dp}{dz}A\delta z-\tau P\,\delta z=\dot{m}\left(u+\frac{du}{dz}\delta z-u\right)$$

Here, A is the flow area and P the wetted perimeter, over which a shear stress τ operates.

This simplifies to

$$\frac{dp}{dz}=-\frac{P}{A}\tau-\frac{\dot{m}}{A}\frac{du}{dz}$$

which we can integrate between axial locations 1, the core inlet, and 2, the core outlet.

$$\left(p_2-p_1\right)=-\frac{P}{A}\int_1^2\tau dz-\frac{\dot{m}}{A}\int_1^2\frac{du}{dz}dz$$

The first term in this expression is the pressure difference required to balance wall friction. The second is the pressure difference required to accelerate the fluid if its density changes. Here the change in density that causes it is principally due to the heating of the coolant as it passes through the core. Naturally, this density change is greatest for a gaseous coolant; it is less significant in water-cooled reactors.

The wall shear stress is found from the friction factor correlation such as

$$f=0.046\,\mathrm{Re}^{-0.2}$$

where

$$\tau=f\frac{\rho v^2}{2}=f\frac{1}{2}\left(\frac{\dot{m}}{A}\right)^2\frac{1}{\rho}$$

(Beware that various definitions of friction factor are used. The definition

$$f=\frac{2\left(\frac{\Delta p}{l}\right)d}{\rho u^2}$$

gives values numerically larger by a factor 4.)

Gas cooled reactors

For a gas, the wall shear stress will increase as we go up the channel, as the fluid gets hotter, becomes thus less dense and thus must travel faster. If the pressure change through the core is small compared to the mean absolute pressure we can express the density in terms of the temperature only:

$$\frac{1}{\rho}=\frac{RT}{\overline{p}}$$

We will take f to be independent of z. This is reasonable, as temperature and pressure changes will be such that $\mu^{0.2}$ varies little along the length of the core.

The pressure drop expression now becomes

$$\left(p_2-p_1\right)=-\frac{P}{A}f\frac{1}{2}\left(\frac{\dot{m}}{A}\right)^2\frac{R}{\overline{p}}\int_1^2 T dz-\frac{\dot{m}}{A}\int_1^2\frac{du}{dz}dz$$

For axially symmetric heat generation, the first integral collapses to the mean temperature, so for a channel of length L;

$$\left(p_2-p_1\right)=-\frac{P}{A}f\frac{1}{2}\left(\frac{\dot{m}}{A}\right)^2\frac{R}{\overline{p}}\frac{1}{2}\left(T_1+T_2\right)L-\frac{\dot{m}}{A}\int_1^2\frac{du}{dz}dz$$

Turning to the second integral:

2

Using this, and tidying up, gives for the pressure drop

$$\left(p_1-p_2\right)=\left(\frac{\dot{m}}{A}\right)^2\frac{R}{\overline{p}}\left\{\frac{PfL}{4A}\left(T_1+T_2\right)+\left(T_2-T_1\right)\right\}$$

We can obviously rearrange this in various ways. One helpful one is to use the mean density and temperature and temperature rise, giving

$$\left(p_1-p_2\right)=\left(\frac{\dot{m}}{A}\right)^2\frac{1}{\overline{\rho}}\left\{\frac{PfL}{2A}+\frac{\Delta T}{\overline{T}}\right\}$$

Water cooled reactors:

When the coolant is water, the change of density within the channel is small enough to allow the extraction of the average value from under the integral sign. The analysis is thus a little simpler. Eliminating the shear stress using the friction factor we have:

$$\left(p_2-p_1\right)=-\frac{P}{A}f\frac{1}{2}\left(\frac{\dot{m}}{A}\right)^2\int_1^2\frac{1}{\rho}dz-\frac{\dot{m}}{A}\int_1^2\frac{du}{dz}dz$$

and we will make the above approximation for the density:

$$\left(p_2-p_1\right)=-\frac{P}{A}f\frac{1}{2}\left(\frac{\dot{m}}{A}\right)^2\frac{2}{\left(\rho_1+\rho_2\right)}L-\frac{\dot{m}}{A}\int_1^2\frac{du}{dz}dz$$

The second integral simplifies initially as for the gaseous coolant, via:

$$
\begin{aligned}
(p_2 - p_1) &= -\frac{P}{A} f \frac{1}{2}\left(\frac{\dot{m}}{A}\right)^2 \frac{2}{(\rho_1+\rho_2)} L - \frac{\dot{m}}{A}(u_2 - u_1) \\
&= -\frac{P}{A} f \frac{1}{2}\left(\frac{\dot{m}}{A}\right)^2 \frac{1}{\bar{\rho}} L - \frac{\dot{m}}{A}\left(\frac{\dot{m}}{\rho_2 A} - \frac{\dot{m}}{\rho_1 A}\right) \\
&= -\frac{P}{A} f \frac{1}{2}\left(\frac{\dot{m}}{A}\right)^2 \frac{1}{\bar{\rho}} L - \left(\frac{\dot{m}}{A}\right)^2 \left(\frac{1}{\rho_2} - \frac{1}{\rho_1}\right) \\
&= -\frac{P}{A} f \frac{1}{2}\left(\frac{\dot{m}}{A}\right)^2 \frac{1}{\bar{\rho}} L - \left(\frac{\dot{m}}{A}\right)^2 \left(\frac{\rho_1 - \rho_2}{\rho_1 \rho_2}\right) \\
&\approx -\frac{P}{A} f \frac{1}{2}\left(\frac{\dot{m}}{A}\right)^2 \frac{1}{\bar{\rho}} L - \left(\frac{\dot{m}}{A}\right)^2 \left(\frac{\rho_1 - \rho_2}{\bar{\rho}^2}\right)
\end{aligned}
$$

or

$$(p_1 - p_2) = \left(\frac{\dot{m}}{A}\right)^2 \frac{1}{\bar{\rho}}\left[\frac{PfL}{2A} + \left(\frac{\Delta\rho}{\bar{\rho}}\right)\right]$$

where $\Delta\rho$ is the density decrease from channel entrance to exit.

4.3 Primary circuit pressure drop

The above has addressed just the pressure drop in the core. There is naturally a pressure drop in the rest of the primary circuit, as the coolant is pumped through the boilers. This will generally be of the same order as that through the core.

4.4 Calculation of circulator/pump power

Since the pressure drop in the whole primary coolant circuit is small compared to the absolute pressure, the power required to circulate the coolant can be calculated with reasonable accuracy by the simple expression

$$\dot{W} = \frac{1}{\eta}\dot{m}\frac{\Delta p}{\rho_p}$$

Here ρ_p is the (mean) value of the density in the pumps or circulators, which is not very different from that at entry to the core; η is the efficiency of the pumps/circulators.