SOURCE AND CHANNEL CODING FOR COOPERATIVE RELAYING

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ABSTRACT

We consider the end-to-end distortion achieved by a half-duplex relay system where a continuous amplitude source is transmitted over a quasi-static Rayleigh fading channel. We investigate layered source coding (LS) where different source layers are sent at different times. The relay only forwards the important layers, leading to higher multiplexing gains. Alternatively, different source layers can be superimposed using a broadcast code (BS). The third strategy we consider is uncoded transmission (UT) followed by amplify-and-forward relaying. In all cases we perform a high SNR analysis and find the optimal distortion exponent which is the exponential decay rate of expected distortion with increasing SNR. We show that layered compression and broadcast coding is optimal in distortion exponent sense for large bandwidth expansions.

1. INTRODUCTION

The wireless communication technology has entered a new era where it evolved from a system offering mainly voice service to one that provides services with rich multimedia content. The increased demand for different services at the application layer has resulted in higher transmission rate and diversity requirements in the physical layer. Considerable improvements in the capacity and error rates have been realized by multiple-antenna techniques. User cooperation diversity is another spatial diversity technique that is especially valuable when the number of antennas on a mobile are limited.

In [1], Sendonaris et.al. show that cooperation provides higher rates and increased robustness against channel variations. Since then many different cooperative relaying protocols have been offered with varying complexities and performances. It is later shown in [2] that even with considerably simple protocols such as amplify-and-forward (AF) and decode-and-forward (DF), it is possible to achieve higher diversity levels.

This work is partially supported by NSF Grant No. 0430885.

Most of the previous work on cooperation focuses on optimizing the channel coding performance, i.e., different cooperation protocols and/or more practical codes are designed to achieve higher diversity, thus lower probability of error. However, due to diversity-multiplexing trade-offs present in cooperative systems, it is critical to analyze the performance of these protocols in terms of their effects on the QoS of higher layer applications which require high data rates as well as robustness against channel failures. The natural candidate for such a QoS metric for multimedia communications is the end-to-end distortion. The losses in the multiplexing gain that are traded-off for higher diversity level become important when the total average distortion is considered. Thus, an optimal channel code that maximizes diversity might result in a poor performance in the overall distortion sense.

In this paper we find the minimum expected distortion (ED) achieved by different cooperation protocols, and compare their high SNR behavior. We focus on the distortion exponent ($\triangle$) defined as [3]

$$\triangle = -\lim_{SNR\to\infty} \frac{\log ED}{\log SNR}. \quad (1)$$

Distortion exponent captures the high SNR behavior of expected distortion in a similar manner diversity does for probability of outage. A distortion exponent of $\triangle$ means that the optimal expected distortion achieved by the system decays as $SNR^{-\triangle}$ when SNR is high.

Determining optimal $\triangle$ requires joint source-channel optimization which is analytically intractable. We consider two different cooperation protocols based on source-channel separation where layered source coders are utilized. The first one, layered source coding (LS) is based on transmitting different layers of compressed source successively in time, with the relay terminal utilized only for part of the transmitted source layers. The second one, broadcast strategy (BS) uses broadcast codes to superimpose each layer. We consider AF type cooperation, however our results can easily be applied to decode-and-forward or compress-and-forward type protocols. We further consider uncoded transmission (UT) where the source is only scaled and transmitted. We find the corresponding distortion exponents for varying bandwidth expansions and compare with an upper bound we derive. We show that BS is optimal in the high SNR regime for bandwidth expansion ratios larger than 4. We also numerically compute the expected distortion for arbitrary SNRs. The results obtained here for Gaussian sources can be generalized to other continuous amplitude sources with squared error distortion [5].

2. SYSTEM MODEL

We assume that a memoryless, complex Gaussian source with unit variance, available at the source terminal, is transmitted over a relay channel (Fig. 1). The aim of the destination is to construct an estimate with minimum mean-squared distortion. The fading coefficients h_1, h_2, and h_3 (Fig. 1) capture the effect of flat fading, and are independent, circularly symmetric complex Gaussian random variables with variance $1/2$ in each dimension. Additive noise components at the receivers are modelled as zero-mean, mutually independent, circularly symmetric, complex Gaussian random sequences each with variance N_o. The fading coefficients are known at the corresponding receivers, but not at the transmitters.

The relay is half-duplex and there exist average power constraints on both the source and the relay. For simplicity we assume both constraints are equal to P. Since the transmitters do not have channel state information they transmit with constant power P and thus the average received signal-to-noise ratio is $SNR = P/N_o$.

The quasi-static Rayleigh fading channels are assumed to be constant over a block of N channel uses, during which K source samples are to be transmitted to the destination. This corresponds to a bandwidth expansion ratio of $l = N/K$. We assume that K is large enough to consider the source as ergodic, but slow channel variations result in non-ergodic channels.

The analysis in [4] assumes real source and a bandwidth expansion ratio of 1. Here we characterize the system performance corresponding to different protocols as a function of the bandwidth expansion. The relation between $\triangle$ and l illustrates the effect of l on the expected distortion. We further demonstrate the benefits of layered compression on the distortion exponent.

3. DISTORTION EXPONENT FOR DIRECT TRANSMISSION

Before we move on to relaying strategies incorporating layered compression, we consider direct transmission. In DT, relay is not utilized and data is transmitted at a predetermined rate of R bits per channel use. Due to bandwidth expansion, this corresponds to a source coding rate of lR

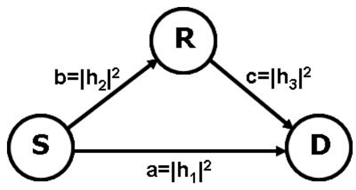

Fig. 1. Illustration of the cooperative system model.

bits per source sample. Let P_{out}^{DT} be the probability of outage at rate R and signal-to-noise ratio SNR. Then the expected distortion corresponding to DT at the specified rate and signal-to-noise ratio can be written as

$$\begin{aligned} ED &= (1 - P_{out})D(lR) + P_{out} \\ &\approx 2^{-lR} + \frac{2^R - 1}{SNR}, \end{aligned} \qquad (2)$$

where we used the high SNR approximation of outage probability at channel rate R, and the distortion-rate function of a complex Gaussian source at source coding rate lR. Constant R results in a non-diminishing distortion with increasing SNR, however if we scale our rate as $R = r \log SNR$, the expected distortion at the destination will decay to zero as SNR goes to infinity. Then we can rewrite Eqn. 2 as $ED \approx SNR^{-lr} + SNR^{r-1}$, which is dominated by the slowest decay. Thus, the optimal multiplexing gain is achieved when $-lr = r - 1$, and this corresponds to the distortion exponent, $\triangle = \frac{l}{l+1}$. In [5], we show that by layered compression we can improve the distortion exponent of DT substantially. By sending layers successively in time (LS) we can achieve $\triangle = 1 - e^{-l}$, while by using broadcast codes (BS) we can achieve $\triangle = 1$ for $l \geq 1$ and $\triangle = l$ for $l < 1$, both in the limiting case of infinite layers.

4. LAYERED CODING STRATEGY FOR COOPERATION

In this section we find the distortion exponent of AF relaying with layered coding strategy (LS). In AF protocol of [2], the source and the relay cooperate for each information bit irrespective of their importance. However, we know that distortion-rate function, in general, has an exponential decay, so correct transmission of the first bits of the compressed data would decrease the distortion more than the same amount of additional bits. Based on this idea, 'partial cooperation' protocol [4] divides the information bits into two layers: first layer is the base layer for which the terminals cooperate, and the second layer, composed of the successive refinement bits, is sent directly without cooperation.

Here we will first analyze two-level LS strategy where terminals cooperate for base layer and the enhancement layer is sent directly. Let the base and enhancement layers be transmitted at rates R_1 and R_2, respectively, and let P_{out}^1 and P_{out}^2 be the corresponding outage probabilities which depend on the rates and SNR. Let the total time allocated for AF be $2\alpha N$ channel uses (which is equally divided among the source and the relay) and the time for DT be βN channel uses, where $2\alpha + \beta = 1$. Then, we can write the expected distortion and its high SNR approximation as

$$\begin{aligned} ED &= (1 - P_{out}^2) D(\alpha l R_1 + \beta l R_2) \\ &\quad + (P_{out}^2 - P_{out}^1) D(\alpha l R_1) + P_{out}^1, \end{aligned}$$

$$\approx 2^{-l(\alpha R_1 + \beta R_2)} + \left(\frac{2^{R_2} - 1}{SNR}\right) 2^{-l\alpha R_1} + \left(\frac{2^{R_1} - 1}{SNR}\right)^2. \tag{3}$$

Following the arguments made for DT, we let both rates increase with SNR as $R_i = r_i \log SNR$, $i = 1, 2$. Then we equate the rates of decays for SNRs in the three terms, and minimize r_1 over nonnegative α and β that satisfy $2\alpha + \beta = 1$. We find the optimal distortion exponent from $\triangle = 2 - 2r_1$.

Motivated by the gains obtained for a direct link with higher number of layers, here we will analyze layered coding strategy (LS) with more than two layers. We will divide the whole transmission block of N channel uses into $2k+m$ portions (Fig. 2). The first $2k$ portions are grouped into pairs, where the first k layers are sent by AF and the last m portions are used for direct transmission of the last m layers. Note that each layer is composed of the successive refinement bits for the previous ones. The length of each cooperation portion is $\alpha_i N$ channel uses, during which data is transmitted at a rate of R_i ($i = 1, 2, \ldots, k$) bits per channel use. Similarly, for the direct transmission portions of length $\beta_i N$ channel uses, the rates are R_i' ($i = 1, \ldots, m$). Note that we have

$$2\sum_{i=1}^{k} \alpha_i + \sum_{j=1}^{m} \beta_j = 1. \tag{4}$$

To obtain a decaying expected distortion, we need $R_i = r_i \log SNR$, $i = 1, 2, \ldots, k$, and $R_i' = r_i' \log SNR$, $i = 1, 2, \ldots, m$.

We write the ED expression similar to the two-layer AF case and find its high SNR approximation. Similarly, r_1 is found to be

$$\begin{aligned} r_1 &= 2^{k-1} \prod_{i=1}^{k} \frac{1}{2 + l\alpha_i} \left(1 + \prod_{j=1}^{m} \frac{1}{1 + l\beta_j}\right) \\ &\geq \frac{1}{2}\left(1 + \frac{l p_{AF}}{2k}\right)^{-k} \left(1 + \left(1 + \frac{l p_{DF}}{m}\right)^{-m}\right), \end{aligned} \tag{5}$$

where $\sum_{i=1}^{k} \alpha_i = p_{AF}$ and $\sum_{i=1}^{m} \beta_i = p_{DT}$, and we used the arithmetic mean-geometric mean inequality. We minimize r_1 to obtain optimal $\triangle = 2 - 2r_1$. For the equality to hold,

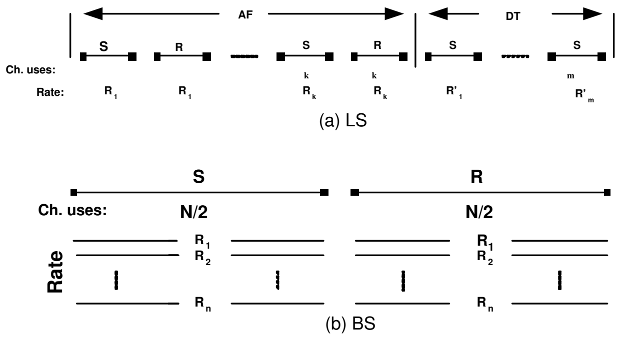

Fig. 2. Illustration of the LS and BS schemes.

we need $\alpha_1 = \cdots = \alpha_k = \alpha$ and $\beta_1 = \cdots = \beta_m = \beta$ with α and β satisfying $2k\alpha + m\beta = 1$. For any finite (k, m) pair, the optimum r_1 can be found by solving this constrained minimization problem.

Obviously $k = 0, m = 1$ and $k = 1, m = 0$ cases correspond to ordinary DT and AF, respectively. In the limiting case, i.e., when $k \to \infty$ and $m \to \infty$, while $p_{AF} \to p_{AF}^*$ and $p_{DF} \to p_{DT}^*$ such that $2p_{AF}^* + p_{DT}^* = 1$, after some manipulation, we can write r_1 as

$$r_1 = \frac{1}{2}[e^{-\frac{l}{2}p_{AF}^*} + e^{-\frac{l}{2}(2 - p_{AF}^*)}]. \tag{6}$$

Optimization over p_{AF}^* results in the following optimal distortion exponent in the limit, i.e, with infinitely many layers for both direct transmission and cooperation,

$$\lim_{\substack{m\to\infty \\ k\to\infty}} \triangle = \begin{cases} 1 - e^{-l} & \text{if} \quad l \leq \ln 3, \\ 2 - 4 \cdot 3^{-3/4} e^{-l/4} & \text{if} \quad l > \ln 3. \end{cases} \tag{7}$$

5. BROADCAST STRATEGY

The main idea of broadcast strategy (BS) is that the transmitter views the fading channel as a degraded Gaussian broadcast channel with a continuum of receivers each experiencing a different signal-to-noise ratio depending on the fading realization [6]. In broadcast relaying, similar to LS information is sent in layers, where each layer consists of the successive refinement source bits for the previous layers. However, in this case the codes corresponding to different layers are superimposed, assigned different power levels and sent throughout the whole transmission block. Compared to LS, power distribution and interference among different layers are traded off for increased multiplexing gain.

We first study 2-level superposition coding, where we superimpose a code at rate R_1 for the base layer on a code at rate R_2 for the successive refinement information. The superimposed code will be transmitted by the source and amplified and forwarded by the relay. Note that the terminals

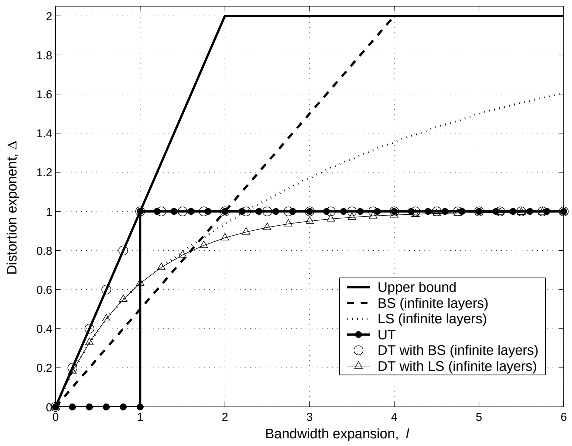

Fig. 3. The relation between the distortion exponent and the bandwidth expansion.

cooperate for all source bits. Similar to LS, we scale these rates with increasing SNR as $R_i = r_i \log SNR$, $i = 1, 2$. Powers assigned to these layers, P_1 and P_2, respectively, sum up to P, the common power constraint. The destination first tries to decode the base layer and considers the second layer as noise. If it can decode the base layer, but cannot decode the second layer after subtracting the decoded portion, the achieved distortion is $D(lR_1/2)$. Successful decoding of both layers results in a distortion of $D((lR_1 + lR_2)/2)$. The factor $1/2$ is due to the half-duplex assumption.

Let $P_1 = (1 - \beta)SNR$ and $P_2 = \beta SNR$, where β is a function of SNR. This additional β parameter controls the power allocation among the layers as a function of SNR. Let P^1_{out} and P^2_{out} be the outage probabilities of the first and the second layers, respectively, at rates R_1 and R_2 and with power allocation according to β. It can be shown that, for an outage probability less than 1, we need to have $\beta < 2^{-R_1} = SNR^{-r_1}$. Letting $\beta = SNR^{-x}$ where $x > r_1$, we get the high SNR approximation for ED as

$$ED = SNR^{-l(r_1+r_2)/2} + SNR^{2(r_2+x-1)}SNR^{-lr_1/2} + SNR^{2(r_1-1)}. \quad (8)$$

Similar analysis of this exponential form as in the LS case, results in an optimal value of $\triangle = \frac{2(l^2+4l)}{l^2+4l+16}$. Furthermore, generalization of this result to n layers of broadcast coding will give

$$\triangle = 2l\frac{(4^n - l^n)}{(4^{n+1} - l^{n+1})}. \quad (9)$$

In the limit of infinitely many layers, we get

$$\lim_{n\to\infty} \triangle = \begin{cases} 2 & \text{if} \quad l \geq 4, \\ l/2 & \text{if} \quad l < 4. \end{cases} \quad (10)$$

6. UNCODED TRANSMISSION

In [5], it is shown that uncoded transmission is optimal for direct transmission over a quasi-static fading channel in the distortion exponent sense if $l \geq 1$. Here we will find the distortion exponent of an uncoded cooperative system.

For $l \geq 2$, the source transmits each uncoded source sample scaled by the power constraint in one use of the channel. The source is kept silent during $N/2 - K$ channel uses, while it scales its transmission power to $lP/2$ to keep the power per channel use constant. The relay scales the signals it receives during the first K channel uses of the source to $lP/2$ and retransmits them. The relay is also silent in the last $N/2 - K$ channel uses of its half. For $1 \leq l < 2$ the source transmits K source samples directly scaling its power to lP, while for $l < 1$ it only transmits first N source samples.

When $l \geq 2$ it is possible to cooperate for each source sample. The optimum estimator can be shown to achieve

$$D = \frac{1}{1 + a\frac{lSNR}{2} + \frac{bc(lSNR/2)^2}{1+(b+c)lSNR/2}}.$$

Let $x = lSNR/2$. Then ED can be found by

$$\begin{aligned} ED &\geq E\left[\frac{1}{1 + ax + bx}\right], \quad (11) \\ &= \frac{e^{1/x}}{x}\int_0^\infty E_1\left(\frac{1+bx}{x}\right)db, \end{aligned}$$

where $E_1(z) = \int_z^\infty \frac{e^{-t}}{t}dt$ is the exponential integral and satisfies $\frac{dE_2(z)}{dz} = -E_1(z)$, with $E_2(z) = \int_z^\infty \frac{e^{-t}}{t^2}dt$. Then the average distortion can be rewritten as

$$ED = \frac{e^{1/x}}{x}E_2(\frac{1}{x}). \quad (12)$$

Substituting the value of x back into the equation and using the series expansion for the exponential integral, we find $\triangle \leq 1$. Using [5] we have $\triangle \geq 1$. Then we can conclude that the distortion exponent, $\triangle = 1$ for uncoded cooperation when $l \geq 2$. For $1 \leq l < 2$, DT can achieve $\triangle = 1$, which serves as an upper bound for the cooperation scenario as well. For $l < 1$, since it is impossible to transmit all source samples we have $\triangle = 0$.

Note that, although UT reaches the optimal distortion exponent for DT with $l \geq 1$, it cannot improve this performance when cooperation through relaying is possible. Uncoded transmission lacks the adaptivity required to utilize the additional degree of freedom brought by the relay.

7. UPPER BOUND

For an upper bound to $\triangle$, assume that the source samples are available at the relay a priori and also the relay and the source have the side information that tells them which one of the two has a better channel to the destination. Then for each time slot, the terminal with the best channel state transmits at the maximum rate that the channel can sustain for the given state. The expected distortion for this idealized case is a full-duplex lower bound for the cooperative scheme we consider. We can write the expected distortion as

$$\begin{aligned} ED &= E_{a,c}[\frac{1}{(1+max(a,c)SNR)^l}], \\ &= \int_0^\infty \frac{e^{-a}}{(1+aSNR)^l}da - \int_0^\infty \frac{e^{-2a}}{(1+aSNR)^l}da, \end{aligned}$$

We will use the following series expansion of the exponential integrals for noninteger l [7]:

$$E_l(z) = \Gamma(1-l)z^{l-1} - e^{-z}\sum_{i=0}^{\infty}\frac{z^i}{(1-l)(2-l)\dots(1+i-l)},$$

Expanding both integrals and considering the high SNR case, we simplify the expected distortion expression to

$$ED \approx \Gamma(1-l)(1+2^{l-1})SNR^{-l} + \frac{SNR^{-2}}{(1-l)(2-l)}.$$

(The integer values of l can be dealt with using the Euler expansion [7].) Now it is easy to see that this upper bound gives

$$\triangle = \begin{cases} 2 & \text{if} \quad l \geq 2, \\ l & \text{if} \quad l < 2. \end{cases} \tag{13}$$

8. DISCUSSION OF THE RESULTS

In Fig. 3 we show how $\triangle$ changes as a function of l for various relaying and direct transmission protocols discussed. We observe that BS with infinite layers achieves the full-duplex upper bound for $l \geq 4$. Although LS performs better than BS for small l values, further improvement in BS is possible by partitioning time to have a separate slot for direct transmission of less important source bits (again using superposition codes), similar to LS. However, we should note that the superiority of BS comes with a more complex encoder-decoder pair, as BS requires SNR-dependent power allocation among layers, superimposition of codewords and sequential decoding. In Fig. 4 we numerically computed the minimum average distortion of various schemes as a function of SNR for $l = 3$. The corresponding theoretical values of $\triangle$ from our analysis for DT with BS, AF with LS and AF with BS, all with 2 layers, are 0.923, 1, 1.135, respectively, while uncoded cooperation has $\triangle = 1$. We observe that the numerical results of Fig. 4 are compatible with these theoretical values which implies that high SNR calculations hold even for moderate SNR values. Even though uncoded cooperation performs better for the SNR range studied, AF with BS eventually has lower distortion since its $\triangle$ is larger.

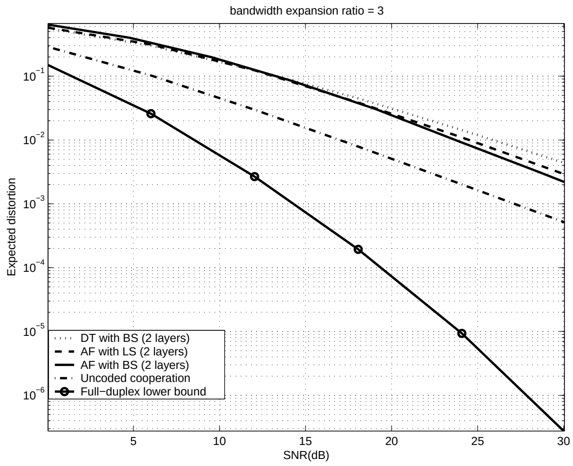

Fig. 4. Expected distortion vs. SNR with bandwidth expansion $l = 3$.

9. CONCLUSION

We consider the expected distortion(ED) of a cooperative system that transmits a continuous source over a quasi-static fading channel. We analyze the minimum achievable ED of various source and channel cooperation strategies which use layered source coding and an amplify-and-forward type relaying. We focus on the distortion exponent ($\triangle$), which is the exponential decay rate of ED, and show that the optimal performance can be approached by separate source and channel coding for certain bandwidth expansion ratios.

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