# Distortion Exponent of MIMO Fading Channels

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Abstract—In this paper, we consider transmission of a continuous amplitude source over a quasi-static MIMO Rayleigh fading channel. The performance metric is end-to-end distortion of the source caused both by the lossy compression and the channel errors. We are interested in the high SNR behavior expressed in the distortion exponent, which is the exponential decay rate of the average end-to-end distortion as a function of SNR. Our goal is to maximize this distortion exponent by considering joint source and channel coding techniques. We provide digital strategies that utilize layered source coding coupled with multi-rate channel coding either by progressive or by superposition transmission, as well as a hybrid digital-analog scheme. When either the transmitter or the receiver has one antenna, we show that we are able to achieve the optimal distortion exponent.

### I. INTRODUCTION

Many telecommunications applications require transmission of analog sources over wireless channels. Examples include digital TV, voice and multimedia transmission in cellular and wireless LAN environments or sensor networks where observations about some analog phenomena are transmitted to a fusion center over wireless links. Besides having analog sources, what is common in these applications is the random time-varying characteristics of the transmission media and the delay requirements. For these systems the appropriate performance is the end-to-end average distortion and achieving the optimal performance requires a cross-layer approach.

Multiple antenna systems can remarkably improve the performance of wireless communication systems by providing spatial multiplexing gain and/or spatial diversity gain. The tradeoff between these two gains is explicitly characterized in [1]. The best operating point on the diversity-multiplexing tradeoff curve depends on the application. In this paper we propose communication strategies that simultaneously operate at different points on the tradeoff curve in order to optimize the end-to-end distortion performance.

We consider a continuous amplitude, memoryless source that is to be transmitted over a MIMO quasi-static Rayleigh fading channel with minimum average distortion. We have stringent delay constraints, where each source block of K samples has to be transmitted over a block of N channel uses, during which the channel is constant. We define the corresponding bandwidth ratio as b=N/K, and analyze the system performance with respect to b.

In our scenario, Shannon's source-channel separation theorem does not hold and a joint optimization is required. We

know that digital transmission suffers from the 'threshold effect', i.e., error probability is bounded away from zero when the channel quality is worse than the attempted rate and digital transmission cannot utilize the increase in the channel quality beyond the threshold. For wireless systems where the channel quality varies randomly, it is desirable to design source and channel codes with graceful degradation in order to have reasonable performance over a wide range of channel states.

In this paper, we focus on the high SNR behavior of the average distortion. We apply layered successive-refinement source coding ideas to achieve the optimal distortion performance in the high SNR regime. As in [2], [3], we consider two different source coding strategies. In the first one, called layered source coding with progressive transmission (LS), each layer is successively transmitted in time. The second strategy, called broadcast strategy with layered source (BS), superimposes the codewords of each layer and transmits them simultaneously. We also discuss a hybrid layered digital-analog transmission strategy coupling LS strategy with analog transmission which we call hybrid LS (HLS).

We show that the rate allocation among layers in LS and hybrid LS can be optimized using the diversity-multiplexing tradeoff of the MIMO system. Furthermore, we argue that BS with infinite layers is able to achieve optimal distortion exponent for all bandwidth ratios, when either the transmitter or the receiver has one antenna (SIMO or MISO).

## II. SYSTEM MODEL

We assume a quasi-static MIMO fading channel with  $M_t$  transmit and  $M_r$  receive antennas. The channel model is

$$\mathbf{Y} = \sqrt{\frac{SNR}{M_t}} \mathbf{H} \mathbf{X} + \mathbf{Z},\tag{1}$$

where  $\mathbf{X} \in \mathcal{C}^{M_t \times N}$  is the transmitted codeword,  $\mathbf{Z} \in \mathcal{C}^{M_r \times N}$  is the complex Gaussian noise with i.i.d entries  $\mathcal{CN}(0,1)$ , and  $\mathbf{H} \in \mathcal{C}^{M_r \times M_t}$  is the channel matrix which has i.i.d. entries with  $\mathcal{CN}(0,1)$ . The channel is constant over a block of length N while independent from block to block.  $\mathbf{H}$  is assumed to be known by the receiver and unknown by the transmitter. The transmitted codeword is normalized in power so that it satisfies  $tr(E[\mathbf{X}^H\mathbf{X}]) \leq M_t N$ , i.e., average signal to noise ratio at each receive antenna is SNR. We define  $M_* = \min(M_t, M_r)$ .

Since we are interested in the high SNR regime, we will use the outage probability,  $P_{out}$ , instead of the channel error probability as it forms a tight lower bound for a finite block

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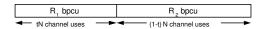


Fig. 1. Channel allocation for two-layered source coding strategy.

length of  $N > M_t + M_r - 1$ , and has the same exponential behavior [1]. For a family of codes with rate  $R = r \log SNR$ , r is defined as the multiplexing gain of the family, and

$$d(r) = \lim_{SNR \to \infty} \frac{\log P_{out}(SNR)}{\log SNR}$$
 (2)

as the diversity advantage. The diversity gain  $d^*(r)$  is defined as the supremum of the diversity advantage over all possible code families with multiplexing gain r. In [1], it is shown that there is a fundamental tradeoff between multiplexing and diversity gains and this tradeoff is explicitly characterized.

We consider an analog source denoted by s. For the analysis, we focus on a memoryless, complex Gaussian source with independent real and imaginary components each with variance 1/2. Generalization to other memoryless sources follows as discussed in [3]. The distortion-rate function for the complex Gaussian source is  $D(R) = 2^{-R}$ . Here we use compression strategies that meet the distortion-rate bound.

The decoder maps the received output of each block Y to an estimate  $\hat{\mathbf{s}} \in \mathcal{C}^K$  of the source. Average distortion ED(SNR)is defined as the average mean squared error between s and ŝ where the expectation is taken with respect to s, H and **Z**. Note that this average distortion is due to both the lossy compression of the source and the outages that occur over the channel. In this work we optimize the system performance in terms of the distortion exponent defined as [4]

$$\Delta = -\lim_{SNR \to \infty} \frac{\log ED}{\log SNR}.$$
 (3)

A distortion exponent of  $\Delta$  means that the expected distortion decays as  $SNR^{-\Delta}$  with increasing SNR when SNR is high.

A similar problem of minimizing end-to-end distortion for MIMO systems is explored in [7] as well, however, their analysis is limited to single layer source coding and only the integer multiplexing gains are considered for the optimization.

## III. LAYERED SOURCE WITH PROGRESSIVE TRANSMISSION

Although progressive transmission of images over lossy channels have been well studied, layered source coding with progressive transmission for improved distortion exponent over fading channels is first considered in [2]. The main idea is to do source coding in layers, where each layer is a refinement of the previous ones, and to transmit layers successively in time over the channel using codes with different rates. We will argue that, this corresponds to each layer operating at a different point of the diversity-multiplexing tradeoff curve of the MIMO system. This enables the receiver to get as many layers as it can depending on its current fading state.

Consider the two layer case, where the whole transmission block of N channel uses is divided into two as in Fig. 1. In

the first portion of tN channel uses  $(0 \le t \le 1)$ , base layer is transmitted at a channel rate of  $R_1$  bits per channel use (bpcu). In the second portion, we transmit the enhancement layer consisting of the successive refinement bits of the source at a rate of  $R_2$  bpcu. Although it might be suboptimal for finite number of layers, we consider equal channel allocation among the layers, that is t=1/2. It it possible to prove that in the limit of infinite layers, equal channel allocation achieves the same limiting performance as the optimal channel allocation.

For the transmission rates, we can impose the constraint  $R_1 \leq R_2$  since the enhancement layer is useless by itself. This constraint also guarantees that the base layer is not in outage whenever the enhancement layer is not. Upon successful reception of both portions, destination achieves a source description rate  $b(R_1 + R_2)/2$  bits per source sample. However, in case of an outage in the second portion only, it gets  $bR_1/2$  bits per source sample. These correspond to distortions of  $D(b(R_1 + R_2)/2)$  and  $D(bR_1/2)$ , respectively. In case of an outage at the base layer, the distortion is

Let  $P_{out}(R, SNR)$  be the outage probability at rate R and average received signal-to-noise ratio SNR, which we will denote as  $P_{out}^{R}$ . Then we can write the expected distortion expression for 2-level LS as:

$$\begin{split} ED(R_1,R_2,SNR) & \doteq (1-P_{out}^{R_2})D(b(R_1+R_2)/2) \\ & + (P_{out}^{R_2}-P_{out}^{R_1})D(bR_1/2)) + P_{out}^{R_1}, (4) \end{split}$$

where  $\doteq$  is used for exponential equality as defined in [1]. Apparent from the expected distortion expression, there is a tradeoff between the outage probability and the distortion of the corresponding layer. As shown in [3] for SISO, there exists an optimal rate pair  $(R_1, R_2)$  which results in the lowest average distortion for any specific SNR. We will see now how these results can be extended to MIMO.

In order to minimize expected distortion, we need to scale rates as  $R_1 = r_1 \log SNR$ , and  $R_2 = r_2 \log SNR$ . Then high SNR approximation for Eqn. (4) is found as

$$ED(r_1, r_2, SNR) \doteq SNR^{-\frac{b}{2}(r_1 + r_2)} + SNR^{-\frac{b}{2}r_1}SNR^{-d^*(r_2)} + SNR^{-d^*(r_1)}.$$
(5)

Optimal distortion exponent is achieved when all three exponents are equal. We have

$$\frac{b}{2}r_2 = d^*(r_2), \quad d^*(r_2) + \frac{b}{2}r_1 = d^*(r_1) = \Delta.$$

For n layers, we obtain the following set of equations

$$\frac{b}{n}r_n = d^*(r_n), (6)$$

$$\frac{b}{n}r_n = d^*(r_n),$$

$$d^*(r_n) + \frac{b}{n}r_{n-1} = d^*(r_{n-1}),$$
(6)

$$d^*(r_2) + \frac{b}{n}r_1 = d^*(r_1), \tag{8}$$

where  $\Delta = d^*(r_1)$ . These equations can be graphically illustrated on the diversity-multiplexing trade-off curve as

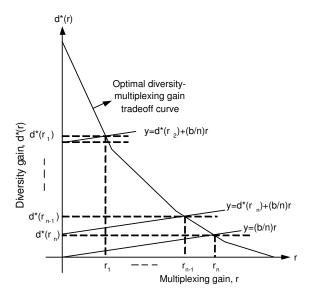


Fig. 2. Allocation of rates to the layers on diversity-multiplexing tradeoff curve.

shown in Fig. 2. This illustration suggests that the more layers we have, the higher we can climb on the trade-off curve and obtain a larger  $\Delta$ . Solving these equations for infinite layers in the case of  $2 \times 2$  MIMO, we find

$$\Delta = \begin{cases} 2(1 - e^{-b}) & \text{if } b \le \ln 2\\ 1 + 3(1 - e^{\frac{\ln 2 - b}{3}}) & \text{if } b \ge \ln 2. \end{cases}$$
 (9)

For a MISO system with  $M_r=1$ , we can express the optimal distortion exponent in a simpler closed form expression in terms of  $M_t$  and number of layers n as

$$\Delta = M_t \left[ 1 - \left( \frac{1}{1 + \frac{b}{nM_t}} \right)^n \right]. \tag{10}$$

In the limit, we get

$$\lim_{n \to \infty} \Delta = M_t (1 - e^{-b/M_t}). \tag{11}$$

Note that above equations hold for SIMO  $(M_t=1)$  if we replace  $M_t$  with the number of receive antennas  $M_r$  as they have the same diversity-multiplexing tradeoff curve. The distortion exponent achieved by LS with single and infinite layers with respect to bandwidth ratio can be seen in Fig. 3 and Fig. 4 for  $2\times 2$  and  $4\times 1$  MIMO, respectively.

## IV. HYBRID DIGITAL-ANALOG TRANSMISSION WITH LAYERED SOURCE

Hybrid digital-analog transmission protocols proposed in [5] provide "nearly robust" source-channel codes that perform well for a range of noise conditions. Here we combine this hybrid approach with progressive layered source coding to minimize the average end-to-end distortion in a MIMO fading channel. We call this strategy hybrid LS (HLS). Recently, [8] analyzed the distortion exponent for hybrid digital-analog space-time codes with one layer digital transmission, i.e., without layered source coding.

For  $b \geq 1/M_*$ , we reserve  $K/M_*$  channel uses for analog transmission as explained below. We divide the rest of the  $N-K/M_*$  channel uses to transmit digital source layers progressively as in the LS scheme. As before we assume that we can use optimal channel codes that operate on the MIMO diversity-multiplexing tradeoff curve.

Let  $\bar{\mathbf{s}} \in \mathcal{C}^{\bar{K}}$  be the reconstruction of the source  $\mathbf{s}$  upon successful reception of all the layers. We denote the reconstruction error as  $\mathbf{e} \in \mathcal{C}^K$  where  $\mathbf{e} = \mathbf{s} - \bar{\mathbf{s}}$ . We map this error to the transmit antennas where each component of the error vector is transmitted without coding in an analog fashion. Since  $rank(\mathbf{H}) < M_*$ , degrees of freedom of the channel is at most  $M_*$  at each channel use. Hence, at each channel use we utilize  $M_*$  of the  $M_t$  transmit antennas and in  $K/M_*$ channel uses we transmit all K components of the error vector e. Receiver first tries to decode all the digitally transmitted layers, and in case of successful reception of all the layers, it forms the estimate  $\hat{s} = \bar{s} + \tilde{e}$ , where  $\tilde{e}$  is the linear MMSE estimate of e based on the received signal during the  $K/M_*$ channel uses reserved for analog transmission. This analog portion is neglected unless all digitally transmitted layers can be decoded at the destination.

As an example we consider HLS with 2 source coding layers at rates  $R_1 \leq R_2$ . The expected distortion is similar to Eqn. (4) except that we have  $D((b-\frac{1}{M_*})R_1/2)$  instead of  $D(bR_1/2)$  term and  $D(b(R_1+R_2)/2)$  is replaced by

$$D\left(\frac{1}{2}(b-\frac{1}{M_*})(R_1+R_2)\right)\frac{1}{M_*}\sum_{i=1}^{M_*}\frac{1}{1+\frac{SNR}{M_*}\lambda_i}.$$

 $\lambda_i$  is the *i*-th eigenvalue of  $\mathbf{H}\mathbf{H}^{\dagger}$ , where  $\mathbf{H}$  is the channel matrix of the constrained  $M_* \times M_r$  system. The second part of the above expression is due to the analog transmission. The high SNR approximation for this expression in case of equal channel allocation among layers is

$$SNR^{-1-\frac{1}{2}(b-\frac{1}{M_*})(r_1+r_2)}$$

Note that in general for n layers, the effect of the analog portion to the distortion exponent analysis done in Eqn. (6-8) is to change the slopes of the curves from  $\frac{b}{n}$  to  $(b-\frac{1}{M_*})\frac{1}{n}$ , and replace the first equation with

$$1 + \frac{1}{n}(b - \frac{1}{M_n})r_n = d^*(r_n). \tag{12}$$

If we apply this analysis to the  $2\times 2$  MIMO system, we achieve the following distortion exponent for  $b\geq 1/2$ 

$$\Delta = 1 + 3 \cdot \left[ 1 - e^{-\frac{1}{3}(b - \frac{1}{2})} \right],\tag{13}$$

in the limit of infinite layers. For  $b < 1/M_*$  case, we apply the hybrid scheme proposed in [8] which superimposes a single source coding layer on uncoded transmission of  $M_*N$  source samples. Although [8] claims that this scheme is optimal for all MIMO systems in the specified bandwidth ratio range, we find that it achieves the upper bound only when the system

is limited to one degree of freedom,  $M_* = 1$ . In general, we find the corresponding distortion exponent as

$$\Delta = b/[1 - (M_* - 1)b].$$

Distortion exponent vs. bandwidth ratio relation of HLS is also included in Fig. 3 and Fig. 4 for  $2 \times 2$  and  $4 \times 1$  MIMO, respectively. Note that for  $b > 1/M_*$  the gain due to the analog portion, i.e., gain of HLS compared to LS, is more significant for small number of layers and decreases as the number of layers increases. Furthermore, for fixed n this gain decays to zero with increasing bandwidth ratio as well. When the degrees of freedom of the MIMO system is more than one, LS scheme performs better than the hybrid scheme for very small bandwidth ratios. We conclude that in MIMO systems with high bandwidth ratio, the main improvement in the distortion exponent performance is due to layered source coding.

In SISO systems, pure analog transmission achieves the optimal distortion exponent of  $\Delta=1$  for  $b\geq 1$  [3], however, in MIMO systems with any number of antennas, it is possible to show that analog transmission is still limited to  $\Delta=1$ . Hence analog transmission cannot utilize the increase in diversity either provided by multiple antennas, or by cooperation [2].

### V. Broadcast Strategy with Layered Source

Broadcast strategy for slowly fading channels is proposed and analyzed in [6] from the perspective of average throughput of the system. It is based on the idea that the transmitter views the fading channel as a degraded broadcast channel with a continuum of receivers each experiencing a different received signal-to-noise ratio corresponding to each fading level. In [3], [2] we combined the broadcast strategy with source compression by utilizing layered source coding and called it broadcast strategy with layered source (BS). Similar to LS, information is sent in layers, where each layer consists of the successive refinement information for the previous layers. However, in this case the different channel codes to which each layer of the source is mapped are superimposed, assigned different powers while still satisfying the total power constraint and sent throughout the whole transmission block. Power and rate allocation among the layers is optimized to minimize the average distortion. However, as it is mentioned in [6], in the general MIMO setting, channel ranking is not straightforward and only suboptimal strategies can be found. In this work we will only consider MISO and SIMO systems. Even for these cases, the problem of optimal rate and power allocation for minimum average distortion for a specific SNR level cannot be solved using the tools of [6] due to the nonlinear nature of the distortion function. However we will be able to obtain asymptotic results for the high SNR regime.

We start with MISO results, SIMO results follow similarly. We let  $\mathbf{h}$  denote the channel gain vector where  $\mathbf{h} \in \mathcal{C}^{1 \times M_t}$  for the MISO model and  $\mathbf{h} \in \mathcal{C}^{M_r \times 1}$  for the SIMO model.

Consider 2-level superposition coding. We superimpose enhancement layer signal  $X_2$  on the base layer signal  $X_1$ ,

where each layer uses a Gaussian codebook. Let the rates of the base and enhancement layers  $R_1$  and  $R_2$  scale as  $R_1 = r_1 \log SNR$ , and  $R_2 = r_2 \log SNR$ , respectively, and the corresponding average SNR at each receive antenna be  $SNR_1$  and  $SNR_2$ , respectively. Then we can write the received signal as

$$\mathbf{Y} = \sqrt{\frac{SNR_1}{M_t}}\mathbf{h}\mathbf{X_1} + \sqrt{\frac{SNR_2}{M_t}}\mathbf{h}\mathbf{X_2} + \mathbf{Z},$$
 (14)

where we have  $SNR_1 + SNR_2 = SNR$ .

The destination first tries to decode the base layer considering the enhancement layer as noise. This results in distortion D(0) in case of outage. If it can decode the base layer, but not the enhancement layer after subtracting the decoded portion, the distortion is  $D(bR_1)$ . Successful decoding of both layers results in a distortion of  $D(bR_1+bR_2)$ . Here we consider the fact that decoding the second layer reduces distortion if and only if the first layer can be decoded as well. The expected distortion, ED for BS can be written as follows.

$$ED(R_1, R_2, SNR) \doteq (1 - \bar{P}_{out}^2)D(bR_1 + bR_2) + (\bar{P}_{out}^2 - P_{out}^1)D(bR_1) + P_{out}^1,$$

where  $P^1_{out}$  is the outage probability of the first layer,  $P^2_{out}$  is the probability of the outage event of decoding the second layer after decoding and subtracting the first layer, and  $\bar{P}^2_{out} = \max(P^1_{out}, P^2_{out})$ . Now let the power assignment be  $SNR_1 = SNR - SNR^{1-(r_1+\epsilon)}$ ,  $\epsilon>0$ . We have

$$P_{out}^{1} = Pr \left\{ \log \left( 1 + \frac{||\mathbf{h}||^{2}SNR(1 - SNR^{-(r_{1} + \epsilon)})}{1 + ||\mathbf{h}||^{2}SNR^{1 - r_{1} - \epsilon}} \right) \right.$$

$$< r_{1} \log SNR \right\}$$

$$= Pr \left\{ ||\mathbf{h}||^{2} \left( SNR(1 - SNR^{-(r_{1} + \epsilon)}) - SNR^{1 - r_{1} - \epsilon} (SNR^{r_{1}} - 1) \right) < SNR^{r_{1}} - 1 \right\} (15)$$

$$\doteq Pr \left\{ ||\mathbf{h}||^{2} < \frac{SNR^{r_{1}} - 1}{SNR - SNR^{(1 - \epsilon)}} \right\}$$

$$\stackrel{\cdot}{=} SNR^{-d^{*}(r_{1})}.$$
(16)

and

$$P_{out}^{2} = Pr\{\log(1+||\mathbf{h}||^{2}SNR^{1-r_{1}-\epsilon}) < r_{2}\log SNR\}$$

$$\stackrel{\cdot}{=} Pr\{||\mathbf{h}||^{2} < SNR^{r_{2}+r_{1}-1+\epsilon}\}$$

$$\stackrel{\cdot}{=} SNR^{-d^{*}(r_{1}+r_{2})},$$
(18)

where  $d^*(r)$  is the diversity gain of the MISO system at

multiplexing gain r. We let  $\epsilon \to 0$  to get (17) and (19). We have  $\bar{P}_{out}^2 \doteq P_{out}^2$  in the high SNR regime.

Using the MISO diversity-multiplexing tradeoff curve, the high SNR approximation for ED can be found as

$$ED(r_1, r_2) \stackrel{:}{=} SNR^{-b(r_1+r_2)} + SNR^{-br_1+M_t(r_2+r_1+\epsilon-1)} + SNR^{M_t(r_1-1)}.$$

The distortion exponent will be characterized by the dominating term in the high SNR regime. By equating the three terms, we obtain a distortion exponent of

$$\Delta = M_t \left( 1 - \frac{M_t^2}{b^2 + b M_t + M_t^2} \right). \label{eq:delta_total_delta_total}$$

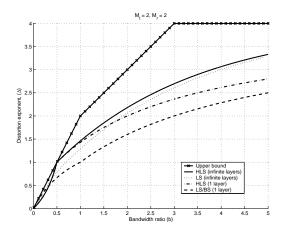


Fig. 3. Distortion exponent vs. bandwidth ratio for the 2x2 MIMO system. From top to bottom on the right hand side of the figure, the curves correspond to the Upper bound, HLS, LS with infinite layers, HLS with 1 layer and LS/BS with 1 layer, respectively.

Furthermore, generalization of the result to strategies with n layers of broadcast coding will give us the relation

$$\Delta = M_t \left( 1 - \frac{1 - b/M_t}{1 - (b/M_t)^{n+1}} \right). \tag{20}$$

Comparing Eqn. (10) and Eqn. (20) we see that as in the SISO case [3], the distortion exponent achieved by BS with the same number of layers is greater than that is achieved by LS. It is also seen that, in the limit of infinite layers we get

$$\Delta = \begin{cases} M_t & \text{if} \quad b \ge M_t, \\ b & \text{if} \quad b < M_t. \end{cases}$$
 (21)

This relation is included in Fig. 4 for  $4\times1$  MISO. Note that, since the diversity-multiplexing gain tradeoff curve is identical for SIMO and MISO systems with the same set of antennas, above result applies to the SIMO system as well. Comparison of this result with the upper bound which will be derived in the next section reveals that broadcast strategy achieves the MISO/SIMO upper bound in the limit of infinite layers.

### VI. UPPER BOUND

To find an upper bound for the distortion ratio of the MIMO system, we follow [3], where we assume that the transmitter has access to the perfect channel state information. Then during each channel block separation theorem holds. However, we restrict the transmitted signal vector at each channel use, i.e., each column of  $\mathbf{X}$  in Eqn. (1), to have a covariance matrix of  $M_t\mathbf{I}$ , which means that the channel state information is not utilized for power adaptation in time, across antennas or for beamforming. Then the instantaneous capacity,  $C(\mathbf{H})$  at channel state  $\mathbf{H}$  is  $\log \det(\mathbf{I} + SNR\mathbf{H}\mathbf{H}^{\dagger})$ . Corresponding minimum distortion can be found as

$$D(\mathbf{H}) = 2^{-bC(\mathbf{H})} = [\det(\mathbf{I} + SNR\mathbf{H}\mathbf{H}^{\dagger})]^{-b}.$$
 (22)

We take the expectation over all channel realizations and analyze the high SNR exponent of this expectation to find the following distortion exponent which we state without

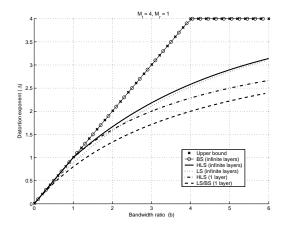


Fig. 4. Distortion exponent vs. bandwidth ratio for the 4x1 MIMO system. From top to bottom on the right hand side of the figure, the curves correspond to the Upper bound and BS (which coincide), HLS, LS with infinite layers, HLS with 1 layer and LS/BS with 1 layer, respectively.

proof due to space limitations. The proof is a straightforward extension of the outage probability analysis of [1].

$$\Delta = \sum_{k=1}^{M_*} \min\{b, 2i - 1 + |M_t - M_r|\}.$$
 (23)

### VII. DISCUSSION AND CONCLUSION

We analyzed the high SNR behavior of end-to-end distortion in a MIMO system where a continuous amplitude source is transmitted over a quasi-static Rayleigh fading channel. We characterized the distortion exponent for various strategies depending on the bandwidth ratio. We observed that layering in source coding brings a remarkable gain in the performance. Further improvement is possible by adding an analog transmitted portion, while this improvement is limited for increased number of layers. We show that the optimal distortion exponent is achievable for MISO/SIMO systems by using broadcast strategy with layered source. Although application of this strategy to the general MIMO system is not straightforward due to the lack of degradedness in the received signals, we are currently working on suboptimal strategies which would potentially increase the performance.

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