Transmission of Correlated Sources Over Multiuser Channels with Receiver Side Information

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***Abstract*— We consider transmission of correlated sources over multiuser channels where the receiver(s) have access to correlated side information. Our goal is to characterize necessary and sufficient conditions for lossless transmission and uncover scenarios where separation of source and channel coding, either in the traditional 'informational' sense (where both source and channel encoders and decoders are designed independently) or in the 'operational' sense (where the encoders are independent, but the source and channel decoding is done jointly), is optimal. We first study a multiple access channel where the source signals are independent given the receiver side information. We prove an informational source channel separation theorem for this communication system. We next investigate source and channel coding for the compound multiple access and interference channels. We give general sufficient conditions for lossless transmission of each source for both channels, and then provide necessary conditions that hold under certain assumptions on the nature of the source and the receiver side information. For the interference channel, the necessary conditions hinge on a strong source-channel interference condition which depends not only on the channel but on the source and side information correlations as well. Our results suggest the optimality of informational or operational separation depending on the correlation structure of the side information and the amount of interference.**

I. Introduction

One of the most fundamental results of information theory is the optimality of source and channel separation for a point-to-point channel. This significant result promises modularity in communication system design without incurring any loss in the performance. However, optimality of source-channel separation breaks down for most multi-user scenarios with correlated sources. We only have a limited number of non-trivial scenarios where separation of source and channel coding is optimal, and a limited understanding of the fundamental relations.

One of the earliest papers showing the suboptimality of separation is [1] which considers transmitting correlated sources over a multiple access channel (MAC). The sufficient conditions for achievability of [1] are then shown not to be necessary by Dueck [2]. The 'correlation preserving mapping' technique of [1] used for achievability is later extended to source coding with side information via multiple access channels in [3], to broadcast channels with correlated sources in [4], and to interference channels in [5]. Optimality of separation for a network with independent, non-interfering channels is proven in [6]. A more intriguing case is the asymmetric MAC considered in [7] for which a source channel separation theorem holds for lossless reconstruction with or without casual perfect feedback at either or both of the transmitters. More recently, Tuncel considers broadcasting a common source to receivers with different correlated side information [8], and shows that, while 'informational separation', i.e., the classical source-channel separation, fails to achieve optimality, 'operational separation', in which source and channel encoders are separate, but decoding is done jointly, is optimal. Recently, [9] extends this technique to transmitting correlated sources over a broadcast network of non-interfering links.

In this paper, we first consider a multiple access channel (MAC) with correlated side information where the receiver has access to a correlated side information (see Fig. 1). We generalize the sufficient conditions for lossless transmission given in [1] to the receiver side information setup. Then, assuming that the sources are independent given the receiver side information, we show the optimality of informational source-channel separation. Next, we consider lossless transmission of correlated sources over a compound MAC (Fig. 2) and an interference channel (Fig. 3) with correlated side information at each receiver. For the compound MAC under certain assumptions on the side information, we show either 'informational separation' in the classical sense, or 'operational separation'. While the achievability results for compound MAC are also valid for the interference channel, we also prove converse results by introducing an extension of the strong interference condition to joint source-channel setting, and obtain informational and operational separation theorems for some special cases under this 'strong source-channel interference' condition.

In practical sensor network applications, the modularity provided by source-channel separation would be invaluable to reduce the complexity of the protocols used. However, in most cases separation results in a performance loss which can be critical for low power, bandwidth limited sensor networks. The operational separation, on the other hand, would reduce the complexity on the sensor side while allowing joint source-channel decoding on the receiver side. Since it is mostly the sensor side which has cost and complexity constraints, optimality of operational separation might lead to modular sensor network design without performance loss.

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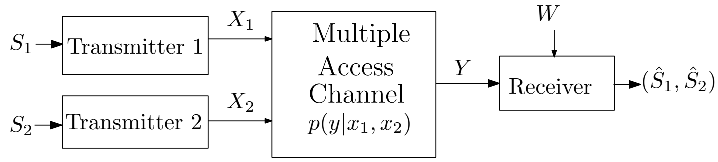

Fig. 1. Multiple access channel with correlated sources and correlated side information at the receiver.

II. MAC WITH CORRELATED SOURCES

We first consider a multiple access channel (MAC) with memoryless correlated sources (S_1, S_2) and a correlated side information W, jointly distributed according to $P_{S_1S_2W}$ over the alphabet $\mathcal{S}_1\times\mathcal{S}_2\times\mathcal{W}$ (see Fig. 1). Transmitter i ($i = 1, 2$) wishes to send a length-m source vector S_i^m losslessly in the Shannon sense to the receiver which also has a length-m vector of correlated side information W^m. The underlying discrete memoryless (DM) channel is characterized by $P_{Y|X_1X_2}$ with input alphabets $\mathcal{X}_1$ and $\mathcal{X}_2$ and the output alphabet $\mathcal{Y}$.

Definition 2.1: We say that *rate* (or bandwidth ratio) b is achievable if, for every $\epsilon > 0$, there exist m, n for which we have encoders

$$f_k : \mathcal{S}_k^m \to \mathcal{X}_k^n, \text{ for } k = 1, 2$$

and a decoder

$$g : \mathcal{Y}^n \times \mathcal{W}^m \to \mathcal{S}_1^m \times \mathcal{S}_2^m,$$

with decoder outputs $(\hat{S}_1^m, \hat{S}_2^m) = g(Y^n, W^m)$ such that the probability of error

$$P_e = Pr[(S_1^m, S_2^m) \neq (\hat{S}_1^m, \hat{S}_2^m)] < \epsilon$$

while $n/m = b$.

The following is a generalization of the achievability scheme given in [1] to the above case with receiver side information.

Theorem 2.1: For arbitrarily correlated sources (S_1, S_2) over DM MAC with receiver side information W and $b = 1$, i.e., when source and channel bandwidths match, lossless representation of S_1 and S_2 at the receiver is possible if,

$$\begin{aligned} H(S_1|S_2, W) &< I(X_1; Y|X_2, S_2, W), \\ H(S_2|S_1, W) &< I(X_2; Y|X_1, S_1, W), \\ H(S_1, S_2|W) &< I(X_1, X_2; Y|W), \end{aligned}$$

for some joint distribution $p(s_1, s_2, w, x_1, x_2, y) = p(s_1, s_2, w)\ p(x_1|s_1)p(x_2|s_2)p(y|x_1, x_2)$.

Note that, correlation among the sources and the side information, both condenses the left hand side and enlarges the right hand side compared to transmitting independent sources. While the reduction in entropies on the left side is due to Slepian-Wolf source coding, the increase on the right side is mainly due to the possibility of generating correlated channel codewords at the transmitters. Applying distributed source coding followed by MAC channel coding would result in the loss of this possible correlation among the channel codewords. However, when S_1-W-S_2 form a Markov chain, that is, the two sources are independent given the side information at the receiver, the receiver already has access to the correlated part of the sources and it is not clear whether additional channel correlation would help. The following theorem suggests that channel correlation is not necessary in this case and source-channel separation is optimal for any b.

Theorem 2.2: For arbitrarily correlated sources (S_1, S_2) over DM MAC with correlated side information W at the receiver, for which the Markov relation $S_1 - W - S_2$ holds, rate b is achievable if and only if,

$$\begin{aligned} H(S_1|W) &< b \cdot I(X_1; Y|X_2, Q), \\ H(S_2|W) &< b \cdot I(X_2; Y|X_1, Q), \\ H(S_1|W) + H(S_2|W) &< b \cdot I(X_1, X_2; Y|Q), \end{aligned}$$

for some joint distribution $p(q, x_1, x_2, y) = p(q)p(x_1|q)$ $p(x_2|q)p(y|x_1, x_2)$, with $|\mathcal{Q}| \leq 4$.

Proof: We can easily see that the achievability part follows from Slepian-Wolf compression at each encoder conditioned on the side information at the receiver while ignoring the other source, and transmitting these compressed source outcomes using an optimal MAC channel code with independent codewords. Proof of the converse can be found in [10]. ■

This is one of the few non-trivial separation theorems for MAC. Following [8], we refer to the classical separation achieved here as 'informational separation', where both the encoder and the decoder can apply separate source and channel coding.

III. COMPOUND MAC WITH CORRELATED SOURCES

We next consider a compound multiple access channel (MAC), where the two transmitters wish to transmit their correlated sources losslessly to two receivers simultaneously. We assume each receiver has its own side information W_i, $i = 1, 2$, as in Fig. 2, correlated with the sources, with the joint distribution $P_{S_1S_2W_1W_2}$ over the alphabet $\mathcal{S}_1\times\mathcal{S}_2\times\mathcal{W}_1\times\mathcal{W}_2$. The underlying discrete memoryless compound MAC is characterized by $P_{Y_1Y_2|X_1X_2}$ over the alphabet $\mathcal{X}_1\times\mathcal{X}_2\times\mathcal{Y}_1\times\mathcal{Y}_2$.

Definition 3.1: We say that rate b is achievable for the given DM compound MAC if, for every $\epsilon > 0$, there exist m, n for which we have encoders

$$f_k : \mathcal{S}_k^m \to \mathcal{X}_k^n, \text{ for } k = 1, 2$$

and decoders

$$g_k : \mathcal{Y}_k^n \times \mathcal{W}_k^m \to \mathcal{S}_1^m \times \mathcal{S}_2^m,$$

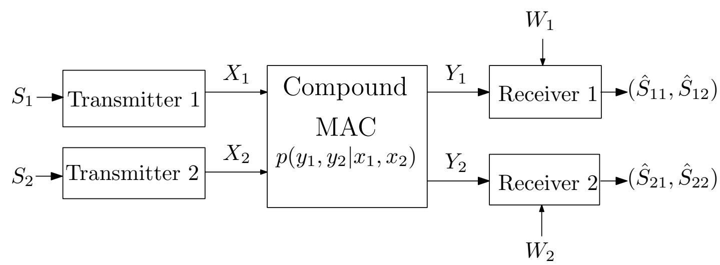

Fig. 2. Compound multiple access channel with correlated sources and correlated side information at the receivers.

with decoder outputs $(\hat{S}_{k1}^m, \hat{S}_{k2}^m) = g_k(Y_k^n, W_k^m)$ such that the probability of error at receiver k

$$P_k = Pr[(S_1^m, S_2^m) \neq (\hat{S}_{k1}^m, \hat{S}_{k2}^m)] < \epsilon$$

while $n/m = b$.

While the capacity region of compound MAC is given in [14] for independent sources and no receiver side information, source-channel matching conditions for correlated source transmission are not known. We first extend our achievability result for MAC in Theorem 2.1 to the compound MAC case.

Theorem 3.1: Lossless transmission of arbitrarily correlated sources (S_1, S_2) over a DM compound MAC with side information (W_1, W_2) is achievable if, for $k = 1, 2$,

$$\begin{aligned} H(S_1|S_2, W_k) &< I(X_1; Y_k|X_2, S_2, W_k), \\ H(S_2|S_1, W_k) &< I(X_2; Y_k|X_1, S_1, W_k), \\ H(S_1, S_2|W_k) &< I(X_1, X_2; Y_k|W_k), \end{aligned}$$

for some joint distribution of the form $p(s_1, s_2, w_1, w_2, x_1, x_2) = p(s_1, s_2, w_1, w_2)p(x_1|s_1)p(x_2|s_2)$.

Proof: The encoding technique in Theorem 2.1 which does not utilize the receiver side information also applies here. Receiver k decodes using its own side information W_k, leading to the above theorem. ■

Now, we consider two special cases of source and side information correlation. First suppose that (S_1, W_2) is independent of (S_2, W_1). This might model a scenario where receiver 1 (2) and transmitter 2 (1) are located close to each other, hence they have correlated observations, while two transmitters are far away.

Theorem 3.2: For lossless transmission of arbitrarily correlated sources (S_1, S_2) over a DM compound MAC with side information (W_1, W_2), where (S_1, W_2) is independent of (S_2, W_1), rate b is achievable if and only if, for $k = 1, 2$

$$\begin{aligned} H(S_1|W_k) &\le b \cdot I(X_1; Y_k|X_2, Q), \\ H(S_2|W_k) &\le b \cdot I(X_2; Y_k|X_1, Q), \\ H(S_1|W_k) + H(S_2|W_k) &\le b \cdot I(X_1, X_2; Y_k|Q), \end{aligned}$$

for some joint distribution $p(q, x_1, x_2, y) = p(q)p(x_1|q)$ $p(x_2|q)p(y|x_1, x_2)$, with $|\mathcal{Q}| \le 4$.

Proof: Proof of the theorem can be found in [15]. ■

Note that, even though the conditions in Theorem 3.2 resemble intersection of two multiple access regions of Theorem 2.2, unlike ordinary MAC, we do not have informational separation for the compound MAC in general. However, it is possible to prove 'operational separation' using the coding technique of [8]. Here, the encoders simply match typical source outcomes to typical channel codewords generated independently. The decoders on the other hand, apply joint source-channel decoding and find the index pairs that simultaneously result in jointly typical source and side information sequences, as well as jointly typical channel codewords and the received signal. The details of the achievability scheme as well as a converse can be found in [15]

We also consider the special case where $W_1 = W_2 = W$ and $S_1 - W - S_2$ form a Markov chain. This corresponds to the case where two receivers are close to each other, hence have the same side information.

Theorem 3.3: In lossless transmission of correlated sources S_1 and S_2 over a DM compound MAC with common receiver side information $W_1 = W_2 = W$ satisfying $S_1 - W - S_2$, rate b is achievable, if and only if, for k=1,2,

$$\begin{aligned} H(S_1|W) &< b \cdot I(X_1; Y_k|X_2, Q), \\ H(S_2|W) &< b \cdot I(X_2; Y_k|X_1, Q), \\ H(S_1|W) + H(S_2|W) &< b \cdot I(X_1, X_2; Y_k|Q), \end{aligned}$$

for some joint distribution $p(q, x_1, x_2, y) = p(q)p(x_1|q)$ $p(x_2|q)p(y|x_1, x_2)$, with $|\mathcal{Q}| \le 4$.

Proof: Similar to Theorem 2.2 achievability follows from Slepian-Wolf source compression at each encoder conditioned on the receiver side information, and transmitting the compressed source outcomes using an optimal compound MAC channel code. The converse on the other hand, follows similar to the proof given in [10]. ■

Note that, in the above case of equal side information, the proof suggests that we can achieve optimality with informational separation.

IV. Interference Channel with Correlated Sources

For the interference channel, even in the case of independent messages at the transmitters, and no side information at the receivers, the capacity region in general is not known. One of the best known achievable schemes is given in [11]. Exact capacity region can be characterized in the strong interference case [12], [13], where the capacity region coincides with the capacity region of the compound multiple access channel [14].

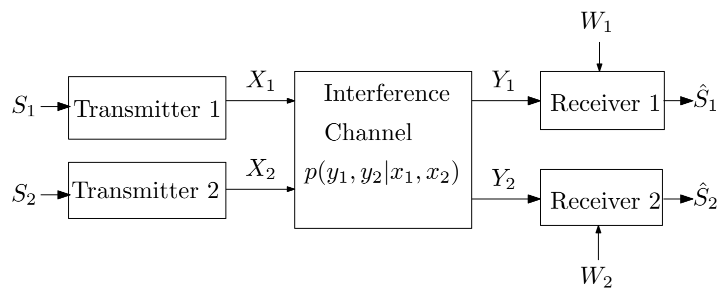

Fig. 3. Interference channel with correlated sources and correlated side information at the receivers.

In the interference channel scenario (see Fig. 3), we replace the decoder functions with

$$g_k^{(m,n)} : \mathcal{Y}_k^n \times \mathcal{W}_k^m \rightarrow \mathcal{S}_k^m, \tag{1}$$

where $\hat{S}_k^m = g_k^{(m,n)}(Y_k^n, W_k^m)$, and the probability of error expressions with

$$P_k^{(m,n)} = Pr\{S_k^m \neq \hat{S}_k^m)\}, \tag{2}$$

for $k = 1, 2$, while achievability definition for b remains the same.

In the case of correlated sources and receiver side information, sufficient conditions for lossless transmission over compound MAC given in Theorem 2.1 serve as sufficient conditions for the interference channel as well, since we can constraint both receivers to obtain reconstructions of both sources. However, to achieve necessary conditions for source-channel matching for the interference channel, we need to have further assumptions similar to the strong interference conditions in [12], [13].

We first consider the setup of Theorem 3.2 where the two source are independent while side information W_1 is correlated with source S_2, and side information W_2 is correlated with source S_1. We assume the following *strong source-channel interference conditions* hold:

$$b \cdot I(X_1; Y_1|X_2) \leq b \cdot I(X_1; Y_2|X_2) + I(S_1; W_2), \tag{3}$$
$$b \cdot I(X_2; Y_2|X_1) \leq b \cdot I(X_2; Y_1|X_1) + I(S_2; W_1), \tag{4}$$

for all input distributions of the form $p(x_1, x_2) = p(x_1)p(x_2)$.

The regular strong interference conditions given in [13] correspond to the case, where, for all input distributions at transmitter 1, the rate of information flow to receiver 2 is higher than the information flow to the intended receiver 1. A similar condition holds for transmitter 2 as well. This leads to the observation that, no performance is lost if both receivers decode the messages of both transmitters. Consequently, under strong interference condition, the capacity region of the interference channel is equivalent to the capacity region of compound MAC. However, in the joint source-channel coding scenario, the receivers have access to correlated side information. Thus while calculating the total rate of information flow to a particular receiver, we should not only consider the information flow through the channel, but also the mutual information that already exists between the source and the receiver side information. This idea is reflected in conditions (3)-(4). Using strong source-channel interference conditions, we obtain the following theorems.

Theorem 4.1: Consider lossless transmission of independent sources S_1 and S_2 over a DM interference channel with side information W_1 and W_2, where (S_1, W_2) is independent of (S_2, W_1). Under strong source-channel interference conditions in Eqn. (3)-(4), rate b is achievable if and only if, for $k = 1, 2$,

$$H(S_1|W_k) \leq bI(X_1; Y_k|X_2, Q),$$
$$H(S_2|W_k) \leq bI(X_2; Y_k|X_1, Q),$$
$$H(S_1|W_k) + H(S_2|W_k) \leq bI(X_1, X_2; Y_k|Q),$$

for some $|\mathcal{Q}| \leq 4$ and some input distribution of the form $p(q, x_1, x_2) = p(q)p(x_1|q)p(x_2|q)$.

Proof: While achievability follows from Theorem 3.2, converse proof also uses the strong source-channel interference conditions and is given in [15]. ■

The theorem proves the optimality of 'operational separation' for the interference channel when strong source-channel interference conditions hold. As in Section III it is possible to see that informational separation would achieve a strictly smaller rate. We further note that, for the considered setup, conditions in (3)-(4) are weaker than the usual strong interference conditions. That is, even if the channel interference is not strong, it may still be optimal for receivers to decode both messages. This enlarges the set of interference channels for which source-channel matching conditions can be characterized.

Next, we consider the second case in Section III, where the two receivers have access to the same side information W given which the sources are independent. In this case, while we still have correlation between the sources and the common receiver side information, the amount of information arising from this correlation is equivalent at both receivers since $W_1 = W_2$. This means that the usual strong interference channel conditions suffice to obtain the converse result. We have the following theorem for this case.

Theorem 4.2: Consider lossless transmission of correlated sources S_1 and S_2 over DM interference channel with common receiver side information W satisfying $S_1 - W - S_2$. Under strong interference conditions, i.e., for all product distributions on $\mathcal{X}_1 \times \mathcal{X}_2$, we have

$$I(X_1;Y_1|X_2) \leq I(X_1;Y_2|X_2), \quad (5)$$

$$I(X_2;Y_2|X_1) \leq I(X_2;Y_1|X_1), \quad (6)$$

rate b is achievable if and only if, for $k = 1, 2$,

$$\begin{aligned} H(S_1|W) &< bI(X_1;Y_k|X_2,Q), \\ H(S_2|W) &< b \cdot I(X_2;Y_k|X_1,Q), \\ H(S_1|W) + H(S_2|W) &< b \cdot I(X_1,X_2;Y_k|Q), \end{aligned}$$

for some input distribution of the form $p(q, x_1, x_2) = p(q)p(x_1|q)p(x_2|q)$.

Proof: Achievability follows from informational separation as in Theorem 3.3. The converse follows similar to Theorem 4.1. ■

V. Conclusion

We consider lossless transmission of correlated sources over discrete memoryless multiple access, compound multiple access and interference channels, where the receivers also have correlated side information. The problem of characterizing the necessary and sufficient conditions is open in the most general setting, however, we concentrate on certain scenarios where we can explicitly formulate the source-channel matching conditions for reliable transmission. Particularly, for the scenarios we investigate, we show that the optimal performance can be achieved by source-channel separation at both the encoder and the decoder, or at the encoder only. Beyond obtaining source-channel matching conditions for some interesting multi-user network scenarios, our results together with [8] and [9], are significant in pointing at a new paradigm in joint source-channel coding, where source-channel separation should be analyzed at the encoders and decoders separately. Optimality of separation at the encoders would lead us to the design of modular transmitters without performance loss while performing complex joint decoding at the receiver.

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