Lossless Transmission of Correlated Sources over a Multiple Access Channel with Side Information

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Abstract

In this paper, we consider lossless transmission of arbitrarily correlated sources over a multiple access channel. Characterization of the achievable rates in the most general setting is one of the longstanding open problems of information theory. We consider a special case of this problem where the receiver has access to correlated side information given which the sources are independent. We prove a source channel separation theorem for this system, that is, we show that there is no loss in performance in first applying distributed source coding where each encoder compresses its source conditioned on the side information at the receiver, and then applying an optimal multiple access channel code with independent codebooks. We also give necessary and sufficient conditions for source and channel separability in the above problem if there is perfect two-sided feedback from the receiver to the transmitters. These two communication scenarios constitute examples of few non-trivial multi-user scenarios for which separation holds.

1. Introduction

We consider a wireless sensor network, where correlated sensor observations are transmitted to an access point through a multiple access channel (MAC). We assume that sensors simultaneously observe some correlated phenomena, e.g. temperature, pressure, etc., and wish to transmit their observations in a lossless fashion to an access point by sharing the same communication channel. This model represents many dense sensor network applications where physical proximity results in correlated observations. We assume that while sensor observations at each instant are correlated among each other, they are memoryless, i.e., observations at different time instants are independent and identically distributed. We allow mismatch among the source and channel code lengths, i.e., we assume that sensors transmit a block of m observations in n uses of the channel, resulting in a bandwidth ratio of $b = n/m$.

The problem in this general setting is known to be hard problem and remains open despite ongoing research efforts. Shannon's source channel separation theorem [1] which states that, there is no loss in applying optimal source compression followed by optimal channel coding in the point-to-point communication scenario, fails to hold in the multiple access case. This means that the optimal strategy requires a joint source-channel coding approach.

The first attempt to find sufficient conditions for transmission of correlated sources over MAC's was due to Cover et al. [2], where a counter example proving the invalidity of the source-channel separation theorem for MAC was given. Authors also proposed a coding technique (achievability result) using the dependency structure of the correlated sources. This result showed that instead of removing the correlation among the sources, we can utilize

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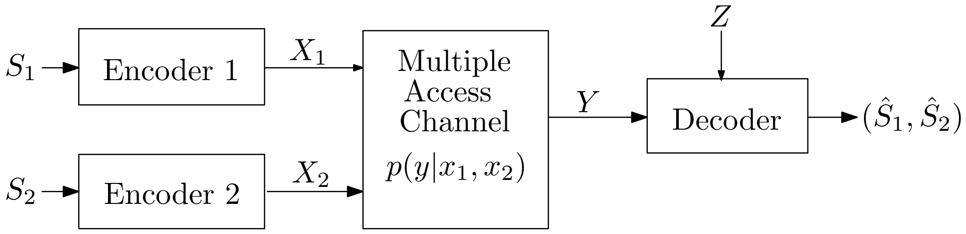

Figure 1. Discrete memoryless MAC for lossless transmission of arbitrarily correlated sources (S_1, S_2) to a receiver which has correlated side information Z**.**

the dependency to design correlated channel codes and in certain cases transmit the sources reliably even though this would not be possible with distributed compression followed by independent channel coding. However, Dueck [3] later showed that the achievable rate region of [2] does not necessarily give the full capacity region. Recently, [4] gave a finite letter outer bound on the capacity of MAC with correlated sources.

One simplification to the original problem might be that the sensor observations are rare events that do not occur simultaneously with high probability. This makes it possible to successfully use random access schemes which would result in low collision probability, hence perform very close to the optimal. Another simplifying assumption would be to have orthogonal channels from each sensor to the access point. This can be achieved by standard time/frequency/code division algorithms (TDMA/ FDMA/ CDMA). This model is considered in [5] and [6], where separation theorems are proven for lossless and lossy cases, respectively. Although both simplifications (rare event and orthogonal channel assumptions) make it possible to obtain a full characterization of the achievable schemes, the insights they provide about the general problem are limited as they orthogonalize the channel, which in many cases would be suboptimal. Surprisingly, to our knowledge, there is only one source-channel separation for non-trivial MAC in the literature, which is given in [7] for lossless transmission of correlated sources over asymmetric MAC.

In this paper, we consider two scenarios under which separate source and channel coding is optimal. We assume the receiver has access to a correlated side information with the source observations, such that, given this side information the sources are independent. Since our assumption does not necessarily lead to the independence of the sources, but only to conditional independence given the remote side information, it is not immediately clear whether the source-channel separation applies here. We argue that, unlike [2], using source correlation for channel code design can not improve the performance, since, loosely speaking, the correlated part of the sources is already available at the receiver in the form of side information. Indeed, we prove a separation theorem which states that optimal Slepian-Wolf source coding where each transmitter compresses its source conditioned on the remote side information at the receiver, followed by a capacity achieving channel code for the MAC is optimal. We also extend our separation theorem to the case where perfect feedback in the form of channel output is available to the encoders. Although the single letter characterization of the capacity region of discrete memoryless (DM) MAC with feedback is not known explicitly, we prove the optimality of separate source and channel codes using directed information [11].

The rest of the paper is organized as follows. In Section 2, we introduce the system model, define the problem, and state our two separation theorems. Proof of the separation theorem for the two user case without feedback is given in Section 3 and a proof of the separation theorem for the perfect two-sided feedback case is provided in Section 4.

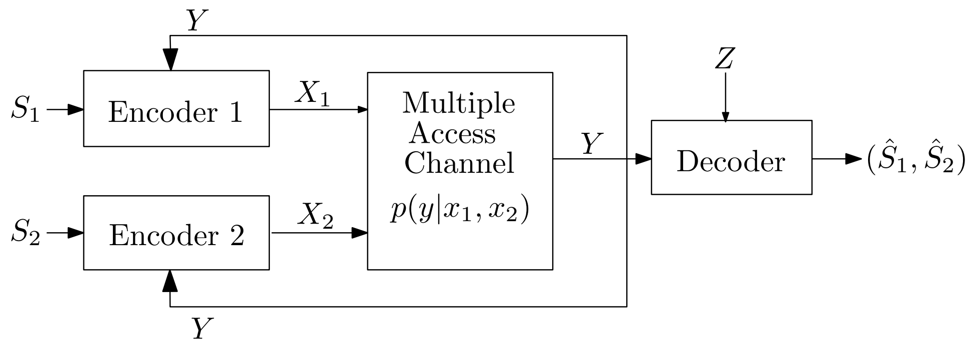

Figure 2. Discrete memoryless MAC with perfect two-sided feedback. We want to transmit arbitrarily correlated sources (S_1, S_2) losslessly to the receiver which observes the correlated side information Z.

2. System Model and The Main Results

2.1. Case I: No Feedback

We first concentrate on the two user case with no feedback, however, generalization to more then two users follows easily based on the arguments here. Two correlated sources and a correlated side information, $\{S_{1,k}, S_{2,k}, Z_k\}_{k=1}^{\infty}$, respectively, are generated i.i.d. according to a joint pmf $p(s_1, s_2, z)$ over a finite alphabet $\mathcal{S}_1 \times \mathcal{S}_2 \times \mathcal{Z}$. The correlated source vectors $S_1^m = (S_{1,1}, \ldots, S_{1,m})$ and $S_2^m = (S_{2,1}, \ldots, S_{2,m})$ are available at the two separate transmitters, and the side information, $Z^m = (Z_1, \ldots, Z_m)$, is available at the receiver. The transmitters know the joint pmf, but do not have access either to each other's samples or to the receiver side information. The transmitters encode the corresponding source sequences into two channel codewords $X_1^n = (X_{11}, \ldots, X_{1n})$ and $X_2^n = (X_{21}, \ldots, X_{2n})$, respectively, and transmit these codewords over a discrete memoryless (DM) multiple access channel to a receiver who observes the output vector $Y^n = (Y_1, \ldots, Y_n)$. The input and output alphabets $\mathcal{X}_1$, $\mathcal{X}_2$ and $\mathcal{Y}$ are all finite. The DM channel is characterized by the conditional distribution $P_{Y|X_1,X_2}(y|x_1, x_2)$.

The receiver forms estimates of the source vectors S_1^m and S_2^m, denoted as $\hat{S}_1^m$, $\hat{S}_2^m$, based on its received signal Y^n and the side information Z^m (see Fig. 1). We assume that S_1 and S_2 are independent given side information Z, i.e., the joint distribution has the form

$$p(s_1, s_2, z) = p(s_1|z)p(s_2|z)p(z).$$

The transmitters are required to convey the corresponding information vectors S_1^m and S_2^m to the decoder losslessly in the Shannon sense. Due to lossless transmission requirement, the reconstruction alphabets are same as the source alphabets.

The capacity region of the DM multiple access channel with independent inputs is denoted by $\mathcal{C}_{MAC}$. We define the interior of the capacity region, $\text{int}(\mathcal{C}_{MAC})$, as the largest open set contained in $\mathcal{C}_{MAC}$.

Theorem 2.1. *(Theorem 14.3.3, [8]) The capacity region $\mathcal{C}_{MAC}$, of a discrete memoryless multiple access channel characterized by the probability transition matrix $p(y|x_1, x_2)$ is given by the closure of the set of all (R_1, R_2) pairs satisfying*

$$\begin{aligned} R_1 &< I(X_1; Y|X_2, Q), \\ R_2 &< I(X_2; Y|X_1, Q), \\ R_1 + R_2 &< I(X_1, X_2; Y, Q), \end{aligned}$$

for some joint distribution $p(q)p(x_1|q)p(x_2|q)p(y|x_1, x_2)$ with $|\mathcal{Q}| \le 4$.

Following [10], we will call the bandwidth ratio, $b = n/m$, of the system as the *rate* of the joint source-channel coding scheme. The rate of the code characterizes the channel uses needed, on the average, to transmit each source sample.

Definition 2.1. *We say that the rate b is* achievable *with no feedback if, there exist sequences of encoders*

$$f_i^{(m,n)} : \mathcal{S}_i^m \to \mathcal{X}_i^n, \text{ for } i = 1, 2$$

and decoders

$$g^{(m,n)} : \mathcal{Y}^n \times \mathcal{Z}^m \to \mathcal{S}_1^m \times \mathcal{S}_2^m,$$

with decoder outputs $(\hat{S}_1^m, \hat{S}_2^m) = g^{(m,n)}(Y^n, Z^m)$ such that the probability of error

$$P_e^{(m,n)} = Pr[(S_1^m, S_2^m) \neq (\hat{S}_1^m, \hat{S}_2^m)] \to 0$$

as $n, m \to \infty$ while $n/m = b$.

We first provide necessary and sufficient conditions for achieving rate b when S_1 and S_2 are independent given side information Z, i.e., $S_1 - Z - S_2$ form a Markov chain. This condition on the sources leads to the Markov relations: $Z - S_1 - X_1$ and $Z - S_2 - X_2$. We also have a larger Markov chain

$$Z - (S_1, S_2) - (X_1, X_2) - Y.$$

The first main result of this paper is summarized in the following theorem.

Theorem 2.2. *For lossless transmission of correlated sources S_1 and S_2 over a DM MAC with no feedback and with receiver side information Z for which $S_1 - Z - S_2$, rate b is achievable if*

$$\left(\frac{1}{b}H(S_1|Z), \frac{1}{b}H(S_2|Z)\right) \in int(\mathcal{C}_{MAC}). \tag{1}$$

Conversely, if rate b is achievable, then

$$\left(\frac{1}{b}H(S_1|Z), \frac{1}{b}H(S_2|Z)\right) \in \mathcal{C}_{MAC}. \tag{2}$$

The proof will be provided in Section 3. The generalization of Thm. 2.2 to more than two users simply follows along the same lines of the proof in Section 3, and is stated below.

Theorem 2.3. *Consider lossless transmission of arbitrarily correlated M sources S_i ($i = 1, \ldots, M$) over a DM MAC with receiver side information Z such that the joint distribution $p(s_1, \ldots, s_M, z)$ is of the form*

$$p(s_1, \ldots, s_M, z) = p(z) \prod_{i=1}^{M} p(s_i|z).$$

Without feedback, rate b is achievable if

$$\left(\frac{1}{b}H(S_1|Z), \ldots, \frac{1}{b}H(S_M|Z)\right) \in int(\mathcal{C}_{MAC}).$$

Conversely, if rate b is achievable, then

$$\left(\frac{1}{b}H(S_1|Z), \ldots, \frac{1}{b}H(S_M|Z)\right) \in \mathcal{C}_{MAC},$$

where $\mathcal{C}_{MAC}$ denotes the capacity region of the M-user MAC.

2.2. Case II: Perfect two-sided feedback

Next we consider the problem stated in 2.1 with two-sided perfect feedback. Now, the channel output is available to both transmitters casually, as illustrated in Fig. 2. At each time instant i, each transmitter can access the previous channel output symbols $Y^{i-1} \triangleq (Y_1, \ldots, Y_{i-1})$. Therefore, each encoder function $f_i^{(m,n)}$ is composed n encoders ,$f_{ij}^{(m,n)}$, for $i = 1, 2$ and $j = 1, \ldots, n$, such that,

$$\begin{aligned} f_{i1}^{(m,n)}(S_i^m) &= X_{i1}, & (3)\\ f_{ij}^{(m,n)}(S_i^m, Y^{j-1}) &= X_{ij}, \text{ for } j = 2, \ldots, n. & (4) \end{aligned}$$

The definitions for error probability and achievability of rate b are similar to the case with no feedback. Before we state our result, we briefly summarize the achievable rate region $\mathcal{R}_L$ and then the capacity region $\mathcal{C}_{fMAC}$ for DM MAC with perfect two-sided feedback given in [12]. These regions do not assume single letter characterization, instead they can be written in terms of the *directed information* between the transmitter sequences and the output sequence. The directed information flowing from a sequence X^n to a sequence Y^n was introduced by Massey [11] as

$$I(X^n \to Y^n) \triangleq \sum_{i=1}^{n} I(X^i; Y_i | Y^{i-1}). \tag{5}$$

The directed information flowing from X^n to Y^n when casually conditioned on Z^n is defined as

$$I(X^n \to Y^n \| Z^n) \triangleq \sum_{i=1}^{n} I(X^i; Y_i | Y^{i-1}, Z^i). \tag{6}$$

If we have casual conditioning followed by usual conditioning, we write

$$I(X^n \to Y^n \| Z^n | U^n) \triangleq \sum_{i=1}^{n} I(X^i; Y_i | Y^{i-1}, Z^i, U^n). \tag{7}$$

Following is an achievable rate region for DM MAC with casual feedback given in [12].

Theorem 2.4. *(Lemma 5.4, [12]) The closure $\mathcal{R}_L$ of the rate pairs (R_1, R_2) satisfying*

$$\begin{aligned} R_1 &\le \frac{1}{L} I(X_1^L \to Y^L \| X_2^L | Q), & (8)\\ R_2 &\le \frac{1}{L} I(X_2^L \to Y^L \| X_1^L | Q), & (9)\\ R_1 + R_2 &\le \frac{1}{L} I(X_1^L, X_2^L \to Y^L), & (10) \end{aligned}$$

is contained within the capacity region $\mathcal{C}_{fMAC}$, where L is a positive integer and the joint probability distribution $p(x_{1i}, x_{2i}, y_i | x_1^{i-1}, x_2^{i-1}, y^{i-1}, q)$ factors as

$$p(x_{1i} | x_1^{i-1}, y^{i-1}, q) \cdot p(x_{2i} | x_2^{i-1}, y^{i-1}, q) \cdot p(y_i | x_{1i}, x_{2i}).$$

for $i = 1, \ldots, L$.

Theorem 2.5. *([12]) $\mathcal{C}_{fMAC} = \lim_{L \to \infty} \mathcal{R}_L$.*

We can now state the second result of this paper, which is the source-channel separation theorem in the case of casual, perfect two-sided feedback.

Theorem 2.6. *Consider lossless transmission of correlated sources S_1 and S_2 over a DM MAC with perfect two-sided feedback, where receiver has side information Z for which $S_1 - Z - S_2$ form a Markov chain. Rate b is achievable if*

$$\left(\frac{1}{b}H(S_1|Z), \frac{1}{b}H(S_2|Z)\right) \in int(\mathcal{C}_{fMAC}). \tag{11}$$

Conversely, if rate b is achievable, then

$$\left(\frac{1}{b}H(S_1|Z), \frac{1}{b}H(S_2|Z)\right) \in \mathcal{C}_{fMAC}. \tag{12}$$

3. Proof of Theorem 2.2

Proof. The direct part is straightforward. Consider rate pair (R_1, R_2) satisfying

$$H(S_1|Z) < R_1, \tag{13}$$

$$H(S_2|Z) < R_2, \tag{14}$$

and

$$\left(\frac{R_1}{b}, \frac{R_2}{b}\right) \in \mathcal{C}_{MAC}. \tag{15}$$

While (13-14) form sufficient conditions for lossless compression of the source pair (S_1, S_2) with respect to the receiver side information Z using Slepian-Wolf source coding theorem [9], when (15) is satisfied, the compressed rates can be reliably transmitted over the multiple access channel, guaranteeing the achievability of rate b.

We next prove the converse. We assume $P_e^{(m,n)} \to 0$ for a sequence of encoders $f_i^{(m,n)}$ $(i = 1, 2)$ and decoders $g^{(m,n)}$ as $n, m \to \infty$ with a fixed rate $b = n/m$. We will use Fano's inequality, which states

$$\begin{aligned} H(S_1^m, S_2^m|\hat{S}_1^m, \hat{S}_2^m) &\leq 1 + mP_e^{(m,n)} \log|\mathcal{S}_1 \times \mathcal{S}_2|, \\ &\triangleq m\delta(P_e^{(m,n)}), \end{aligned} \tag{16}$$

where $\delta(P_e^{(m,n)})$ is a non-negative function that goes to zero as $P_e^{(m,n)} \to 0$. We also obtain

$$H(S_1^m, S_2^m|\hat{S}_1^m, \hat{S}_2^m) \geq H(S_i^m|\hat{S}_1^m, \hat{S}_2^m), \tag{17}$$

$$\geq H(S_i^m|Y^n, Z^m), \tag{18}$$

for $i = 1, 2$, where the first inequality follows from the chain rule and the nonnegativity of entropy for discrete sources, and the second inequality follows from the data processing inequality. Then, for $i = 1, 2$,

$$H(S_i^m|Y^n, Z^m) \leq m\delta(P_e^{(m,n)}). \tag{19}$$

We have

$$\begin{aligned}
\frac{1}{n}I(X_1^n;Y^n|X_2^n,Z^m) &\geq \frac{1}{n}I(S_1^m;Y^n|Z^m,X_2^n), && (20)\\
&= \frac{1}{n}[H(S_1^m|Z^m,X_2^n)-H(S_1^m|Y^n,Z^m,X_2^n)], && (21)\\
&= \frac{1}{n}[H(S_1^m|Z^m)-H(S_1^m|Y^n,Z^m,X_2^n)], && (22)\\
&\geq \frac{1}{n}[H(S_1^m|Z^m)-H(S_1^m|Y^n,Z^m)], && (23)\\
&\geq \frac{1}{b}H(S_1|Z)-\frac{1}{n}m\delta(P_e^{(m,n)}), && (24)
\end{aligned}$$

where (20) follows from the Markov relation $S_1^m - X_1^n - Y^n$ given (X_2^n, Z^m); (22) from the Markov relation $X_2^n - Z^m - S_1^m$; (23) from the fact that conditioning reduces entropy; (24) from the memoryless source assumption and from (16) which uses Fano's inequality.

Then we obtain

$$\begin{aligned}
\frac{1}{n}I(X_1^n;Y^n|X_2^n) &\geq \frac{1}{b}\left[H(S_1|Z)-\delta(P_e^{(m,n)})\right],\\
&\geq \frac{1}{b}\left[H(S_1|Z)-\epsilon\right], && (25)
\end{aligned}$$

for any $\epsilon > 0$ and large enough m, n. On the other hand, we also have

$$\begin{aligned}
I(X_1^n;Y^n|X_2^n,Z^m) &= H(Y^n|X_2^n,Z^m)-H(Y^n|X_1^n,X_2^n,Z^m), && (26)\\
&= H(Y^n|X_2^n,Z^m)-\sum_{i=1}^{n}H(Y_i|Y^{i-1},X_1^n,X_2^n,Z^m), && (27)\\
&= H(Y^n|X_2^n,Z^m)-\sum_{i=1}^{n}H(Y_i|X_{1i},X_{2i},Z^m), && (28)\\
&\leq \sum_{i=1}^{n}H(Y_i|X_{2i},Z^m)-\sum_{i=1}^{n}H(Y_i|X_{1i},X_{2i},Z^m), && (29)\\
&= \sum_{i=1}^{n}I(X_{1i};Y_i|X_{2i},Z^m), && (30)
\end{aligned}$$

where (27) follows from the chain rule; (28) from the memoryless channel assumption; and (29) from the chain rule and the fact that conditioning reduces entropy.

Now, we follow the similar steps as in the converse proof of DM MAC in [8]. We introduce a new time-sharing random variable $\bar{Q}$, where $\bar{Q} = i$, $i \in \{1, 2, \ldots, n\}$ with probability $1/n$. Then we can write

$$\begin{aligned}
\frac{1}{n}I(X_1^n;Y^n|X_2^n,Z^m) &\leq \frac{1}{n}\sum_{i=1}^{n}I(X_{1i};Y_i|X_{2i},Z^m), && (31)\\
&= \frac{1}{n}\sum_{i=1}^{n}I(X_{1\bar{q}};Y_{\bar{q}}|X_{2\bar{q}},Z^m,\bar{Q}=i), && (32)\\
&= I(X_{1\bar{Q}};Y_{\bar{Q}}|X_{2\bar{Q}},Z^m,\bar{Q}), && (33)\\
&= I(X_1;Y|X_2,Q), && (34)
\end{aligned}$$

where $X_1 \triangleq X_{1\bar{Q}}$, $X_2 \triangleq X_{2\bar{Q}}$, $Y \triangleq Y_{\bar{Q}}$, and $Q \triangleq (Z^m, \bar{Q})$. Since S_1^m and S_2^m, and therefore X_{1i} and X_{2i}, are independent given Z^m, for $q = (z^m, i)$ we have

$$\begin{aligned} Pr\{X_1 = x_1, X_2 = x_2 | Q = q\} &= Pr\{X_{1i} = x_1, X_{2i} = x_2 | Z^m = z^m, \bar{Q} = i\} \\ &= Pr\{X_{1i} = x_1 | Z^m = z^m, \bar{Q} = i\} \\ &\quad \cdot Pr\{X_{2i} = x_2 | Z^m = z^m, \bar{Q} = i\} \\ &= Pr\{X_1 | Q = q\} \cdot Pr\{X_2 | Q = q\}. \end{aligned}$$

Hence, the probability distribution is of the form in Theorem 2.1.

Combining these two chains of inequalities we can obtain

$$\frac{1}{b}[H(S_1|Z) - \epsilon] \le I(X_1; Y | X_2, Q), \tag{35}$$

Similarly, we can also get

$$\frac{1}{b}[H(S_2|Z) - \epsilon] \le I(X_2; Y | X_1, Q). \tag{36}$$

For the joint mutual information we can write the following set of inequalities.

$$\begin{aligned} \frac{1}{n} I(X_1^n, X_2^n; Y^n | Z^m) &\ge \frac{1}{n} I(S_1^m, S_2^m; Y^n | Z^m), &(37)\\ &= \frac{1}{n}[H(S_1^m, S_2^m | Z^m) - H(S_1^m, S_2^m | Y^n, Z^m)], &(38)\\ &= \frac{1}{n}[H(S_1^m | Z^m) + H(S_2^m | Z^m) - H(S_1^m, S_2^m | Y^n, Z^m)], &(39)\\ &\ge \frac{1}{n}[H(S_1^m | Z^m) + H(S_2^m | Z^m) - H(S_1^m, S_2^m | \hat{S}_1^m, \hat{S}_2^m), &(40)\\ &\ge \frac{1}{b}\left[H(S_1|Z) + H(S_1|Z) - \delta(P_e^{(m,n)})\right], &(41) \end{aligned}$$

where (37) follows from the Markov relation $(S_1^m, S_2^m) - (X_1^n, X_2^n) - Y^n$ given Z^m; (39) from the Markov relation $S_2^m - Z^m - S_1^m$; (40) from the fact that $(S_1^m, S_2^m) - (Y^n, Z^m) - (\hat{S}_1^m$ and $\hat{S}_2^m)$ form a Markov chain; (41) from the memoryless source assumption and from (16) which uses Fano's inequality.

Then we can get

$$\begin{aligned} \frac{1}{n} I(X_1^n, X_2^n; Y^n | Z^m) &\ge \frac{1}{b}\big[H(S_1|Z) + H(S_1|Z) - \delta(P_e^{(m,n)})\big], \\ &\ge \frac{1}{b}\left[H(S_1|Z) + H(S_2|Z) - \epsilon\right], &(42) \end{aligned}$$

for any $\epsilon > 0$ and large enough m, n. Similar to the previous case, we can also show that

$$I(X_1^n, X_2^n; Y^n | Z^m) \le \sum_{i=1}^{n} I(X_{1i}, X_{2i}; Y_i | Z^m), \tag{43}$$

which finally leads to

$$\frac{1}{b}\left[H(S_1|Z) + H(S_2|Z) - \epsilon\right] \le I(X_1, X_2; Y | Q). \tag{44}$$

Altogether, taking the limit as $m, n \to \infty$ and letting $\epsilon \to 0$ we can obtain the following converse result:

$$\frac{1}{b} H(S_1|Z) < I(X_1; Y | X_2, Q), \tag{45}$$

$$\frac{1}{b}H(S_2|Z) < I(X_2;Y|X_1,Q), \quad (46)$$

$$\frac{1}{b}\left[H(S_1|Z)+H(S_2|Z)\right] < I(X_1,X_2;Y|Q), \quad (47)$$

for some joint probability density $p(q,x_1,x_2,y) = p(q)p(x_1|q)p(x_2|q)p(y|x_1,x_2)$. Although the cardinality of random variable Q is larger than the cardinality of Q in Theorem 2.1, we know that this would not enlarge the region due to Carathéodory theorem [8].

□

4. Proof of Theorem 2.6

Achievability is straightforward and follows from the lines of the proof of Theorem 2.2. To prove the converse, we have

$$\begin{aligned}
I(X_1^n \to Y^n \| X_2^n | Z^m) &= \sum_{i=1}^{n} I(X_{1i};Y_i|Y^{i-1},X_2^i,Z^m), && (48)\\
&\geq \sum_{i=1}^{n} I(S_1^m;Y_i|Y^{i-1},X_2^i,Z^m), && (49)\\
&= \sum_{i=1}^{n} I(S_1^m;Y_i,X_2^i|Z^m,Y^{i-1}) - I(S_1^m;X_2^i|Z^m,Y^{i-1}), && (50)\\
&= \sum_{i=1}^{n} I(S_1^m;Y_i,X_2^i|Z^m,Y^{i-1}), && (51)\\
&\geq \sum_{i=1}^{n} I(S_1^m;Y_i|Z^m,Y^{i-1}), && (52)\\
&= I(S_1^m;Y^n|Z^m), && (53)\\
&= H(S_1^m|Z^m) - H(S_1^m|Y^n,Z^m), && (54)
\end{aligned}$$

where (48) follows from the definition given in (7); (49) from the fact that $S_1^m - X_{1i} - Y_i$ form a Markov chain given (Y^{i-1}, X_2^i, Z^m); (50) from the chain rule; (51) follows from the Markov relation $S_1^m - (Z^m, Y^{i-1}) - X_2^i$; (52) and (53) follow from the chain rule and the non-negativity of the mutual information.

Finally, using the same steps following (23) of the no-feedback case, we can write

$$\frac{1}{n}I(X_1^n \to Y^n \| X_2^n | Z^m) \;\geq\; \frac{1}{b}\left[H(S_1|Z)-\epsilon\right]$$

for any $\epsilon > 0$ and large enough m, n. Similar steps follow for the other transmitter and the joint directed information and we can obtain the following set of inequalities.

$$\begin{aligned}
\frac{1}{b}H(S_1|Z) &\leq \frac{1}{n}I(X_1^n \to Y^n \| X_2^n | Z^m),\\
\frac{1}{b}H(S_2|Z) &\leq \frac{1}{n}I(X_2^n \to Y^n \| X_1^n | Z^m),\\
\frac{1}{b}\left[H(S_1|Z)+H(S_2|Z)\right] &\leq \frac{1}{n}I(X_1^n, X_2^n \to Y^n),
\end{aligned}$$

where the joint probability distribution factorizes as

$$p(x_{1i},x_{2i},y_i|x_1^{i-1},x_2^{i-1},y^{i-1},z^m) = p(x_{1i}|x_1^{i-1},y^{i-1},z^m)\cdot p(x_{2i}|x_2^{i-1},y^{i-1},z^m)\cdot p(y_i|x_{1i},x_{2i}).$$

Now, note that, from Theorem 2.4, the right hand side is achievable for any n and Z^m, hence the conditions of the theorem are necessary as well.

5. Conclusion

In this work, we consider lossless transmission of arbitrarily correlated sources over a discrete memoryless multiple access channel (MAC) with and without perfect two sided feedback. Assuming that the receiver has access to a correlated side information given which the sources are independent, we prove a source-channel separation theorem for both cases, that is, we show that, there is no loss in first applying Slepian-Wolf source coding where each user compresses its source conditioned on the remote side information at the receiver, and then transmitting the compressed source signals over the underlying MAC using channel codes operating on the boundary of the MAC capacity region. This result may be particularly valuable for sensor network applications where the modularity brought by source-channel separation would make simpler design architectures possible without rendering any performance loss.

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