

Compound Multiple Access Channels with Partial Cooperation

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Abstract

A two-user discrete memoryless compound multiple access channel with a common message and conferencing decoders is considered. The capacity region is characterized in the special cases of physically degraded channels and unidirectional cooperation, and achievable rate regions are provided for the general case. The results are then extended to the corresponding Gaussian model. In the Gaussian setup, the provided achievable rates are shown to lie within some constant number of bits from the boundary of the capacity region in several special cases. An alternative model, in which the encoders are connected by conferencing links rather than having a common message, is studied as well, and the capacity region for this model is also determined for the cases of physically degraded channels and unidirectional cooperation. Numerical results are also provided to obtain insights about the potential gains of conferencing at the decoders and encoders.

This work was supported by NSF under grants CNS-06-26611 and CNS-06-25637, the DARPA ITMANET program under grant 1105741-1-TFIND and the ARO under MURI award W911NF-05-1-0246. The work of S. Shamai has been supported by the Israel Science Foundation and the European Commission in the framework of the FP7 Network of Excellence in Wireless COMMUNICATIONS NEWCOM++.

I. INTRODUCTION

In today's complex communication networks there are often multiple "signal paths" to utilize in delivering data between a given transmitter and receiver. Such signal paths may take the form of (generalized) feedback from the channel to the transmitters or additional (orthogonal) communication links between either the transmitters or the receivers. The first case corresponds to scenarios in which the additional signal paths share the spectral resources with the direct transmitter-receiver links (in-band signalling), while the second case refers to scenarios in which orthogonal spectral resources are available at the transmit and/or the receive side (out-of-band signalling).

In this work, we focus on the latter case discussed above and model the out-of-band signal paths as finite-capacity directed links. This framework is typically referred to as "conferencing" (or "partial cooperation") in the literature to emphasize the possibly interactive nature of communication on such links. Conferencing encoders in a two-user multiple access channel (MAC) have been investigated in [1] [2]¹ and in [3] for a two-user interference channel. These works show that conferencing encoders can create dependence between the transmitted signals by coordinating the transmission via the out-of-band links, thus mimicking multiantenna transmitters. Conferencing decoders have been studied in [4] for a relay channel and in [5] - [9] for a broadcast channel. Such decoders can use the out-of-band links to exchange messages about the received signals so as to mimic a multiantenna receiver (see also [10]).

This work extends the state of the art described above by considering the compound MAC with conferencing decoders and a common message (see Fig. 1) and then with both conferencing encoders and decoders (see Fig. 5). These models generalize the setup of a single-message broadcast (multicast) channel with two conferencing decoders studied in [5]² - [9], in that there are two transmitters that want to broadcast their messages to the conferencing receivers. Moreover, the transmitters can have a common message (Fig. 1) or be connected by conferencing links (Fig. 5). The model also generalizes the compound MAC with common messages studied, among other models, in [3], by allowing conferencing among the decoders. The main contributions of this work

¹It is noted that a MAC with conferencing encoders can be seen as a special case of a MAC with generalized feedback.

²Reference [5] also considers a broadcast channel with private messages to the two users.

are summarized as follows:

- The capacity region is derived for the two-user discrete-memoryless compound MAC with a common message and conferencing decoders in the special cases of physically degraded channels and unidirectional cooperation (Sec. IV);
- Achievable rate regions are given for the general model of Fig. 1 (Sec. V);
- Extension to the corresponding Gaussian case is provided, establishing the capacity region with unidirectional cooperation and deriving general achievable rates. Such achievable rates are also shown to be within some constant number of bits of the capacity region in several special cases (Sec. VI);
- The capacity region is determined for the compound MAC with both conferencing encoders and decoders as in Fig. 5 in the special cases of physically degraded channels and unidirectional cooperation (Sec. VII).

Finally, numerical results are also provided to obtain further insight into the main conclusions.

II. SYSTEM MODEL AND MAIN DEFINITIONS

We start by considering the model in Fig. 1, which is a discrete-memoryless compound MAC with conferencing decoders and common information (here, for short, we will refer to this channel as the CM channel). The CM channel is characterized by $(\mathcal{X}_1, \mathcal{X}_2, p^*(y_1, y_2|x_1, x_2), \mathcal{Y}_1, \mathcal{Y}_2)$ with input alphabets $\mathcal{X}_1, \mathcal{X}_2$ and output alphabets $\mathcal{Y}_1, \mathcal{Y}_2$. The i th encoder, $i = 1, 2$, is interested in sending a private message $W_i \in \mathcal{W}_i = \{1, 2, \dots, 2^{nR_i}\}$ of rate R_i [bits/channel use] to both receivers and, in addition, there is a common message $W_0 \in \mathcal{W}_0 = \{1, 2, \dots, 2^{nR_0}\}$ of rate R_0 to be delivered by both encoders to both receivers. The channel is memoryless and time-invariant in that the conditional distribution of the output symbols at any time $j = 1, 2, \dots, n$ satisfies

$$p(y_{1,j}, y_{2,j}|x_1^n, x_2^n, y_1^{j-1}, y_2^{j-1}, \bar{w}) = p^*(y_{1,j}, y_{2,j}|x_{1,j}, x_{2,j}) \quad (1)$$

with $\bar{w} = (w_0, w_1, w_2) \in \mathcal{W}_0 \times \mathcal{W}_1 \times \mathcal{W}_2$ being a given triplet of messages. Notation-wise, we employ standard conventions (see, e.g., [11]), where the probability distributions are defined by the arguments, upper-case letters represent random variables and the corresponding lower-case letters represent realizations of the random variables. The superscripts identify the number of samples to be

included in a given vector, e.g., $y_1^{j-1} = [y_{1,1} \cdots y_{1,j-1}]$. It is finally noted that the channel defines the conditional marginals $p(y_1|x_1, x_2) = \sum_{y_2 \in \mathcal{Y}_2} p^*(y_1, y_2|x_1, x_2)$ and similarly for $p(y_2|x_1, x_2)$. Further definitions are in order.

Definition 1: A $((2^{nR_0}, 2^{nR_1}, 2^{nR_2}), n, K)$ code for the CM channel consists of two encoding functions ($i = 1, 2$)

$$f_i: \mathcal{W}_0 \times \mathcal{W}_i \rightarrow \mathcal{X}_i^n, \quad (2)$$

a set of $2K$ “conferencing” functions and corresponding output alphabets $\mathcal{V}_{i,k}$ ($k = 1, 2, \dots, K$):

$$g_{1,k}: \mathcal{Y}_1^n \times \mathcal{V}_{2,1} \times \cdots \times \mathcal{V}_{2,k-1} \rightarrow \mathcal{V}_{1,k} \quad (3a)$$

$$g_{2,k}: \mathcal{Y}_2^n \times \mathcal{V}_{1,1} \times \cdots \times \mathcal{V}_{1,k-1} \rightarrow \mathcal{V}_{2,k}, \quad (3b)$$

and decoding functions:

$$h_1: \mathcal{Y}_1^n \times \mathcal{V}_{2,1} \times \cdots \times \mathcal{V}_{2,K} \rightarrow \mathcal{W}_0 \times \mathcal{W}_1 \quad (4a)$$

$$h_2: \mathcal{Y}_2^n \times \mathcal{V}_{1,1} \times \cdots \times \mathcal{V}_{1,K} \rightarrow \mathcal{W}_0 \times \mathcal{W}_2. \quad (4b)$$

Notice that the conferencing functions (3) prescribe K conferencing rounds between the decoders that start as soon as the two decoders receive the entire block of n output symbols y_1^n and y_2^n . Each conference round, say the k th, corresponds to a simultaneous and bidirectional exchange of messages between the two decoders taken from the alphabets $\mathcal{V}_{1,k}$ and $\mathcal{V}_{2,k}$, similarly to [1], [14]. It is noted that other works have used slightly different definitions of conferencing rounds [7], [16]. After the K conferencing rounds, the receivers perform decoding with functions (4) by capitalizing on the exchanged conferencing messages. Due to the orthogonality between the main channel and the conferencing links, the transmission from the users on one channel and conferencing/ decoding on the other can take place simultaneously.

Definition 2: A rate triplet (R_0, R_1, R_2) is said to be *achievable* for the CM channel with decoders connected by conferencing links with capacities (C_{12}, C_{21}) (see Fig. 1) if for any $\varepsilon > 0$ there exists, for all n sufficiently large, a $((2^{nR_0}, 2^{nR_1}, 2^{nR_2}), n, K)$ code with any $K \geq 0$ such that the probability of error satisfies

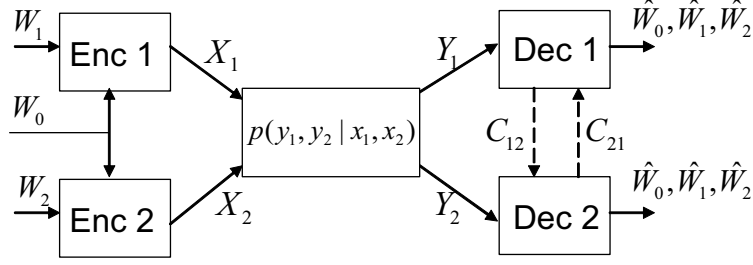


Fig. 1. A discrete-memoryless compound MAC channel with conferencing decoders and common information (for short, CM).

$$P_e \triangleq \frac{1}{2^{n(R_0+R_1+R_2)}} \sum_{\bar{w} \in \mathcal{W}_0 \times \mathcal{W}_1 \times \mathcal{W}_2} \Pr \left[\begin{array}{l} \{h_1(Y_1^n, V_2^k) \neq \bar{w}\} \cup \\ \{h_2(Y_2^n, V_1^k) \neq \bar{w}\} | \bar{w} \text{ sent} \end{array} \right] \leq \varepsilon \quad (5)$$

and the conferencing alphabets are such that

$$\sum_{k=1}^K |\mathcal{V}_{1,k}| \leq nC_{12} \text{ and } \sum_{k=1}^K |\mathcal{V}_{2,k}| \leq nC_{21}. \quad (6)$$

The *capacity region* $\mathcal{C}_{CM}(C_{12}, C_{21})$ is the closure of the set of all achievable rates (R_0, R_1, R_2) .

III. PRELIMINARIES AND AN OUTER BOUND

Similarly to [3], it is useful to define the rate region $\mathcal{R}_{MAC,i}(p(u), p(x_1|u), p(x_2|u))$ for the MAC seen at the i th receiver ($i = 1, 2$) as the set of rates

$$\mathcal{R}_{MAC,i}(p(u), p(x_1|u), p(x_2|u)) = \left\{ (R_0, R_1, R_2) : R_0 \geq 0, R_1 \geq 0, R_2 \geq 0, \right. \\ \left. R_1 \leq I(X_1; Y_i | X_2 U), \right. \quad (7a)$$

$$\left. R_2 \leq I(X_2; Y_i | X_1 U), \right. \quad (7b)$$

$$\left. R_1 + R_2 \leq I(X_1 X_2; Y_i | U), \right. \quad (7c)$$

$$\left. R_0 + R_1 + R_2 \leq I(X_1 X_2; Y_i) \right\}, \quad (7d)$$

where the joint distributions of the involved variables is given by

$$p(u)p(x_1|u)p(x_2|u)p(y_i|x_1, x_2).$$

If $C_{12} = C_{21} = 0$, the capacity region $\mathcal{C}_{CM}(0, 0)$ is given by [3]:

$$\mathcal{C}_{CM}(0, 0) = \bigcup \left\{ \bigcap_{i=1,2} \mathcal{R}_{MAC,i}(p(u), p(x_1|u), p(x_2|u)) \right\} \quad (8a)$$

$$= \bigcup \left\{ (R_0, R_1, R_2): R_0 \geq 0, R_1 \geq 0, R_2 \geq 0, \right. \quad (8b)$$

$$R_1 \leq \min\{I(X_1; Y_1|X_2U), I(X_1; Y_2|X_2U)\}, \quad (8c)$$

$$R_2 \leq \min\{I(X_2; Y_1|X_1U), I(X_2; Y_2|X_1U)\}, \quad (8d)$$

$$R_1 + R_2 \leq \min\{I(X_1X_2; Y_1|U), I(X_1X_2; Y_2|U)\}, \quad (8e)$$

$$R_0 + R_1 + R_2 \leq \min\{I(X_1X_2; Y_1), I(X_1X_2; Y_2)\} \left. \right\}, \quad (8f)$$

where the union is taken over all joint distributions of the form

$$p(u)p(x_1|u)p(x_2|u)p^*(y_1, y_2|x_1, x_2).$$

It is remarked that no convex hull operation is necessary in (8) as the region $\mathcal{C}_{CM}(0, 0)$ is convex [3] (see also [1], Appendix I).

We now derive an outer bound on the capacity region $\mathcal{C}_{CM}(C_{12}, C_{21})$. To this end, it is useful to define the capacity region achievable when the two receivers are allowed to fully cooperate (FC), thus forming a two-antenna receiver. In this case, we have

$$\mathcal{R}_{MAC,FC}(p(u), p(x_1|u), p(x_2|u)) = \left\{ (R_0, R_1, R_2): R_0 \geq 0, R_1 \geq 0, R_2 \geq 0, \right. \quad (9a)$$

$$R_1 \leq I(X_1; Y_1Y_2|X_2U), \quad (9b)$$

$$R_2 \leq I(X_2; Y_1Y_2|X_1U), \quad (9c)$$

$$R_1 + R_2 \leq I(X_1X_2; Y_1Y_2|U), \quad (9d)$$

$$R_0 + R_1 + R_2 \leq I(X_1X_2; Y_1Y_2) \left. \right\}, \quad (9e)$$

where the joint distributions of the involved variables is given by

$$p(u)p(x_1|u)p(x_2|u)p^*(y_1, y_2|x_1, x_2). \quad (10)$$

Proposition 3.1: We have $\mathcal{C}_{CM}(C_{12}, C_{21}) \subseteq \mathcal{C}_{CM-out}(C_{12}, C_{21})$ where (dropping the dependence on $p(u), p(x_1|u), p(x_2|u)$ to simplify the notation)

$$\mathcal{C}_{CM-out}(C_{12}, C_{21}) = \bigcup \{(\mathcal{R}_{MAC,1} + C_{12}) \cap (\mathcal{R}_{MAC,2} + C_{21}) \cap (\mathcal{R}_{MAC,FC})\} \quad (11a)$$

$$= \bigcup \left\{ (R_0, R_1, R_2): R_0 \geq 0, R_1 \geq 0, R_2 \geq 0, \right. \quad (11b)$$

$$R_1 \leq \min\{I(X_1; Y_1|X_2U) + C_{21}, I(X_1; Y_2|X_2U) + C_{12}, \\ \left. I(X_1; Y_1Y_2|X_2U)\}, \quad (11c)$$

$$R_2 \leq \min\{I(X_2; Y_1|X_1U) + C_{21}, I(X_2; Y_2|X_1U) + C_{12}, \\ \left. I(X_2; Y_1Y_2|X_1U)\}, \quad (11d)$$

$$R_1 + R_2 \leq \min\{I(X_1X_2; Y_1|U) + C_{21}, I(X_1X_2; Y_2|U) + C_{12}, \quad (11e)$$

$$\left. I(X_1X_2; Y_1Y_2|U)\}, \quad (11f)$$

$$R_0 + R_1 + R_2 \leq \min\{I(X_1X_2; Y_1) + C_{21}, I(X_1X_2; Y_2) + C_{12}, \\ \left. I(X_1X_2; Y_1Y_2)\}, \quad (11g)$$

in which the union is taken over all the joint distributions that factorize as (10).

Similarly to (8), region (11) can be proven to be convex following [1].

Proof: See Appendix I. ■

IV. CAPACITY REGION WITH PHYSICALLY DEGRADED CHANNELS AND UNIDIRECTIONAL COOPERATION

The next proposition establishes the capacity region $\mathcal{C}_{CM-DEG}(C_{12}, C_{21})$ in the case of physically degraded outputs.

Proposition 4.1: If the CM channel is physically degraded in the sense that $(X_1X_2) - Y_1 - Y_2$

forms a Markov chain, then the capacity region is obtained as

$$\mathcal{C}_{CM-DEG}(C_{12}, C_{21}) = \mathcal{C}_{CM-out}(C_{12}, 0) = \quad (12a)$$

$$= \bigcup \{(R_0, R_1, R_2): R_0 \geq 0, R_1 \geq 0, R_2 \geq 0, \quad (12b)$$

$$R_1 \leq \min\{I(X_1; Y_1|X_2U), I(X_1; Y_2|X_2U) + C_{12}\}, \quad (12c)$$

$$R_2 \leq \min\{I(X_2; Y_1|X_1U), I(X_2; Y_2|X_1U) + C_{12}\}, \quad (12d)$$

$$R_1 + R_2 \leq \min\{I(X_1X_2; Y_1|U), I(X_1X_2; Y_2|U) + C_{12}\}, \quad (12e)$$

$$R_0 + R_1 + R_2 \leq \min\{I(X_1X_2; Y_1), I(X_1X_2; Y_2) + C_{12}\}. \quad (12f)$$

Notice that here $p^*(y_1y_2|x_1, x_2) = p(y_1|x_1, x_2)p(y_2|y_1)$ due to degradedness.

Proof: See Appendix II. ■

Remark 4.1: A symmetric result clearly holds for the physically degraded channel $(X_1X_2) - Y_2 - Y_1$.

Establishment of the capacity region is also possible in the special case where only unidirectional cooperation is allowed, that is $C_{12} = 0$ or $C_{21} = 0$. This result is akin to [9] where a broadcast channel with two receiver under unidirectional cooperation was considered.

Proposition 4.2: In the case of unidirectional cooperation ($C_{12} = 0$ or $C_{21} = 0$), the capacity region of the CM channel is given by

$$\mathcal{C}_{CM}(0, C_{21}) = \mathcal{C}_{CM-out}(0, C_{21}) \quad (13)$$

or

$$\mathcal{C}_{CM}(C_{12}, 0) = \mathcal{C}_{CM-out}(C_{12}, 0). \quad (14)$$

Proof: Achievability follows by using the same scheme as in the proof of Proposition 4.1.

The converse is immediate. ■

V. GENERAL ACHIEVABLE RATES

Achievable rates can be derived for the general CM channel, extending the analysis of [5] from the broadcast setting with one transmitter to the CM channel. Notice that [5] uses a different

definition for the operation over the conferencing channels but this turns out to be immaterial for the achievable rates discussed below.

Proposition 5.1: The following region is achievable with one-round conferencing, i.e., $K = 1$:

$$\mathcal{R}_{OR}(C_{12}, C_{21}) = \bigcup \{(R_0, R_1, R_2): R_0 \geq 0, R_1 \geq 0, R_2 \geq 0, \quad (15a)$$

$$R_1 \leq \min\{I(X_1; Y_1 \hat{Y}_2 | X_2 U), I(X_1; Y_2 \hat{Y}_1 | X_2 U)\}, \quad (15b)$$

$$R_2 \leq \min\{I(X_2; Y_1 \hat{Y}_2 | X_1 U), I(X_2; Y_2 \hat{Y}_1 | X_1 U)\}, \quad (15c)$$

$$R_1 + R_2 \leq \min\{I(X_1 X_2; Y_1 \hat{Y}_2 | U), I(X_1 X_2; Y_2 \hat{Y}_1 | U)\} \quad (15d)$$

$$R_0 + R_1 + R_2 \leq \min\{I(X_1 X_2; Y_1 \hat{Y}_2), I(X_1 X_2; Y_2 \hat{Y}_1)\} \quad (15e)$$

subject to

$$C_{12} \geq I(Y_1; \hat{Y}_1 | Y_2) \quad (16a)$$

$$C_{21} \geq I(Y_2; \hat{Y}_2 | Y_1) \quad (16b)$$

with $|\hat{\mathcal{Y}}_i| \leq |\mathcal{Y}_i| + 1$, and the union is taken over all the joint distributions that factorize as

$$p(u)p(x_1|u)p(x_2|u)p^*(y_1, y_2|x_1, x_2)p(\hat{y}_1|y_1)p(\hat{y}_2|y_2).$$

Proof: (Sketch): The proof is similar to that of Theorem 3 in [5] and is thus only sketched here. A one-step conference ($K = 1$) is used. Encoding and transmission are performed as for a MAC with common information (see proof of Proposition 4.1). Each receiver compresses its received signal using Wyner-Ziv compression exploiting the fact that the other receiver has a correlated observation as well. The compression indices are exchanged during the one conferencing round via symbols $\mathcal{V}_{1,1}$ and $\mathcal{V}_{2,1}$. Decoding is then carried out at each receiver using joint typicality: For instance, receiver 1 looks for jointly typical sequences $(u^n(w_0), x_1^n(w_0, w_1), x_2^n(w_0, w_2), y_1^n, \hat{y}_2^n)$ with $w_i \in \mathcal{W}_i$, where \hat{y}_2^n is the compressed sequence received by the second decoder. ■

The achievable strategy of Proposition 5.1 is based on $K = 1$ round of conferencing. It is easy to construct examples where such a strategy fails to achieve the outer bound (11) as discussed in the example below.

Example 1. Consider a symmetric scenario with $R_0 = 0$ and equal private rates $R_1 = R_2 = R$ (i.e., $p^*(y_1, y_2|x_1, x_2) = p^*(y_2, y_1|x_1, x_2) = p^*(y_1, y_2|x_2, x_1) = p^*(y_2, y_1|x_2, x_1)$). Fix U to a constant

without loss of generality (given the absence of a common message) and the input distribution to $p(x_1)p(x_2)$. We are interested in finding the maximum achievable equal rate $R_1 = R_2 = R$. Assume that the conferencing capacities satisfy $C_{12} = H(Y_1|Y_2) = H(Y_2|Y_1)$ and $1/2 \cdot I(X_1X_2; Y_2|Y_1) \leq C_{21} < H(Y_1|Y_2)$. In this case, it can be seen that the maximum equal rate is upper bounded as $R \leq R_{out} = 1/2 \cdot I(X_1X_2; Y_1Y_2)$ by the outer bound (11), which corresponds to the maximum equal rate of a system with full cooperation at the receiver side. This bound can be achieved if both receivers have access to both outputs Y_1 and Y_2 . With the one-round strategy, since $C_{12} = H(Y_1|Y_2)$ receiver 1 can provide Y_1 to receiver 2 via Slepian-Wolf compression, but receiver 2 cannot do the same with receiver 1 since $C_{21} < H(Y_1|Y_2)$. Therefore, rate R_{out} cannot be achieved by this strategy, which in fact attains equal rate $R_{OR} = 1/2 \cdot I(X_1X_2; Y_1\hat{Y}_2) < R_{out}$ (recall (16)).

We now consider a second strategy that generalizes the previous one and is based on two rounds of conferencing ($K = 2$). As will be shown below, this strategy is able to improve upon the one-round scheme, while still failing to achieve the outer-bound (11) in the general case.

Proposition 5.2: The following rate region is achievable with two rounds of conferencing, i.e., $K = 2$:

$$\mathcal{R}_{TR}(C_{12}, C_{21}) = \text{co} \bigcup \{ \mathcal{R}_{TR,12} \cup \mathcal{R}_{TR,21} \} \quad (17)$$

where “co” indicates the convex hull operation, and we have

$$\mathcal{R}_{TR,12} = \{ (R_0, R_1, R_2) : R_0 \geq 0, R_1 \geq 0, R_2 \geq 0, \quad (18a)$$

$$R_1 \leq \min\{ I(X_1; Y_1|X_2U) + C_{21}, I(X_1; Y_2\hat{Y}_1|X_2U) \}, \quad (18b)$$

$$R_2 \leq \min\{ I(X_2; Y_1|X_1U) + C_{21}, I(X_2; Y_2\hat{Y}_1|X_1U) \}, \quad (18c)$$

$$R_1 + R_2 \leq \min\{ I(X_1X_2; Y_1|U) + C_{21}, I(X_1X_2; Y_2\hat{Y}_1|U) \}, \quad (18d)$$

$$R_0 + R_1 + R_2 \leq \min\{ I(X_1X_2; Y_1) + C_{21}, I(X_1X_2; Y_2\hat{Y}_1) \}, \quad (18e)$$

$\mathcal{R}_{TR,21}$ is similarly defined:

$$\mathcal{R}_{TR,21} = \{(R_0, R_1, R_2): R_0 \geq 0, R_1 \geq 0, R_2 \geq 0, \quad (19a)$$

$$R_1 \leq \min\{I(X_1; Y_1 \hat{Y}_2 | X_2 U), I(X_1; Y_2 | X_2 U) + C_{12}\}, \quad (19b)$$

$$R_2 \leq \min\{I(X_2; Y_1 \hat{Y}_2 | X_1 U), I(X_1; Y_2 | X_2 U) + C_{12}\}, \quad (19c)$$

$$R_1 + R_2 \leq \min\{I(X_1 X_2; Y_1 \hat{Y}_2 | U), I(X_1; Y_2 | X_2 U) + C_{12}\}, \quad (19d)$$

$$R_0 + R_1 + R_2 \leq \min\{I(X_1 X_2; Y_1 \hat{Y}_2), I(X_1; Y_2 | X_2) + C_{12}\}, \quad (19e)$$

subject to

$$C_{12} \geq I(Y_1; \hat{Y}_1 | Y_2)$$

$$C_{21} \geq I(Y_2; \hat{Y}_2 | Y_1)$$

with $|\hat{\mathcal{Y}}_i| \leq |\mathcal{Y}_i| + 1$, and the union is taken over all the joint distributions that factorize as

$$p(u)p(x_1|u)p(x_2|u)p^*(y_1 y_2 | x_1, x_2)p(\hat{y}_1 | y_1)p(\hat{y}_2 | y_2).$$

Proof: (Sketch): The proof is quite similar to Theorem 4 in [5], and here we only sketch the main points. Conferencing takes place via $K = 2$ conferencing rounds. Moreover, two possible strategies are considered, giving rise to the convex hull operation in (17) by time-sharing. The two corresponding rate regions $\mathcal{R}_{TR,12}$ in (18) and $\mathcal{R}_{TR,21}$ in (19) are obtained as follows. Consider $\mathcal{R}_{TR,12}$. Receiver 2 randomly partitions the message sets \mathcal{W}_0 , \mathcal{W}_1 and \mathcal{W}_2 into $2^{n\alpha_0 C_{12}}$, $2^{n\alpha_1 C_{12}}$ and $2^{n\alpha_2 C_{12}}$ subsets, respectively, for a given $0 \leq \alpha_i \leq 1$ and $\sum_{i=0}^2 \alpha_i = 1$, as in the proof of Proposition 4.1. Encoding and transmission are performed as for the MAC with common information. Receiver 1 compresses its received signal using Wyner-Ziv quantization as for the scheme discussed in the proof of Proposition 5.1. This index is sent in the first conferencing round (notice that $|\mathcal{V}_{1,1}| = nC_{12}$ and $|\mathcal{V}_{2,1}| = 0$). Upon reception of the compression index $\mathcal{V}_{1,1}$, receiver 2 proceeds to decoding via joint typicality and then sends the subset indices (see proof of Proposition 4.1) to receiver 1 via $\mathcal{V}_{2,2}$ (now, $|\mathcal{V}_{1,2}| = 0$ and $|\mathcal{V}_{2,2}| = nC_{21}$). The latter decoder performs joint-typicality decoding on the subsets of messages left undecided by the conferencing message $\mathcal{V}_{1,1}$ received by 1. The rate region $\mathcal{R}_{TR,21}$ is obtained similarly by simply swapping the roles of decoder 1 and decoder 2. ■

Example 1 (cont'd): To see the impact of the two-round scheme, here we reconsider Example 1 discussed above. It was shown that, for the scenario discussed therein, the one-round scheme is not able to achieve the outer bound R_{out} . However, it can be seen that the two-round scheme does indeed achieve the outer bound. In fact, receiver 1 can provide Y_1 to receiver 2 via Slepian-Wolf compression as for the one-round case, while receiver 2 does not send anything in the first conferencing round (\hat{Y}_2 is a constant). Now, receiver 2 decodes and sends the bin index of the decoded messages to receiver 1 in the second conferencing round according to the two-round strategy discussed above (receiver 1 is silent in the second round). Since $C_{21} \geq 1/2 \cdot I(X_1 X_2; Y_2 | Y_1)$ by assumption, it can be seen from Proposition 5.2 that the maximum equal rate achieved by the two round scheme is $R_{TR} = R_{out}$.

We finally remark that it is possible in principle to extend the achievable rate regions derived above to more than two conferencing rounds, following [6] [7]. This is generally advantageous in terms of achievable rates. While conceptually not difficult, a description of the achievable rate region would require cumbersome notation and is thus omitted here.

VI. GAUSSIAN COMPOUND MAC

Here we consider the Gaussian version of the CM channel:

$$Y_1 = \gamma_{11}X_1 + \gamma_{21}X_2 + Z_1 \tag{20a}$$

$$Y_2 = \gamma_{22}X_2 + \gamma_{12}X_1 + Z_2, \tag{20b}$$

with channel gains $\gamma_{ij} \geq 0$, independent white zero-mean unit-power Gaussian noise $\{Z_i\}_{i=1}^n$ and per-symbol power constraints $E[X_i^2] \leq P_i$. Notice that the channel described by (20) is not physically degraded.

The outer bound of Proposition 3.1 can be extended to (20) by using standard arguments. In particular, the capacity region of the Gaussian CM, $\mathcal{C}_{CM}^{\mathcal{G}}(C_{12}, C_{21})$ satisfies the following.

Proposition 6.1: We have $\mathcal{C}_{CM}^{\mathcal{G}}(C_{12}, C_{21}) \subseteq \mathcal{C}_{CM-out}^{\mathcal{G}}(C_{12}, C_{21})$ where:

$$\mathcal{C}_{CM-out}^{\mathcal{G}}(C_{12}, C_{21}) = \bigcup_{\substack{0 \leq P'_i \leq P_i \\ i=1,2}} \left\{ (R_0, R_1, R_2) : R_0 \geq 0, R_1 \geq 0, R_2 \geq 0, \right. \quad (21a)$$

$$R_1 \leq \min \{ \mathcal{C}(\gamma_{11}^2 P'_1) + C_{21}, \mathcal{C}(\gamma_{12}^2 P'_1) + C_{12}, \mathcal{C}(P'_1 (\gamma_{11}^2 + \gamma_{12}^2)) \}, \quad (21b)$$

$$R_2 \leq \min \{ \mathcal{C}(\gamma_{21}^2 P'_2) + C_{21}, \mathcal{C}(\gamma_{22}^2 P'_2) + C_{12}, \mathcal{C}(P'_2 (\gamma_{21}^2 + \gamma_{22}^2)) \}, \quad (21c)$$

$$R_1 + R_2 \leq \min \left\{ \begin{array}{l} \mathcal{C}(\gamma_{11}^2 P'_1 + \gamma_{21}^2 P'_2) + C_{21}, \mathcal{C}(\gamma_{22}^2 P'_2 + \gamma_{12}^2 P'_1) + C_{12}, \\ \mathcal{C}(P'_1 (\gamma_{11}^2 + \gamma_{12}^2) + P'_2 (\gamma_{21}^2 + \gamma_{22}^2) + \mathcal{K}) \end{array} \right\} \quad (21d)$$

$$R_0 + R_1 + R_2 \leq \min \left\{ \begin{array}{l} \mathcal{C}(\gamma_{11}^2 P'_1 + \gamma_{21}^2 P'_2 + \rho_1) + C_{21}, \mathcal{C}(\gamma_{22}^2 P'_2 + \gamma_{12}^2 P'_1 + \rho_2) + C_{12}, \\ \mathcal{C} \left(\begin{array}{l} P'_1 (\gamma_{11}^2 + \gamma_{12}^2) + P'_2 (\gamma_{21}^2 + \gamma_{22}^2) + \mathcal{K} \\ + \rho_1 (1 + P'_1 \gamma_{12}^2 + P'_2 \gamma_{22}^2) + \rho_2 (1 + P'_1 \gamma_{11}^2 + P'_2 \gamma_{21}^2) \\ - 2\sqrt{\rho_1 \rho_2} (P'_1 \gamma_{11} \gamma_{12} + P'_2 \gamma_{21} \gamma_{22}) \end{array} \right) \end{array} \right\}.$$

with

$$\mathcal{K} = P'_1 P'_2 (\gamma_{12} \gamma_{21} - \gamma_{11} \gamma_{22})^2 \quad (22a)$$

$$\rho_1 = (\gamma_{11} \sqrt{P_1 - P'_1} + \gamma_{21} \sqrt{P_2 - P'_2})^2 \quad (22b)$$

$$\rho_2 = (\gamma_{22} \sqrt{P_2 - P'_2} + \gamma_{12} \sqrt{P_1 - P'_1})^2 \quad (22c)$$

and $\mathcal{C}(x) \triangleq \frac{1}{2} \log(1 + x)$.

Proof: Similarly to Proposition 3.1, one can prove that the rate region (11) is an outer bound on the achievable rates. It then remains to be proved that a Gaussian joint distribution $p(u)p(x_1|u)p(x_2|u)$ with $X_i = \sqrt{P - P'_i}U + \sqrt{P'_i}V_i$, where U , V_1 and V_2 are independent Gaussian zero-mean unit-power random variables, is optimal. This can be done following the steps of [2], where the proof is given for a single MAC channel with common information (see also [15]). The proof is concluded with some algebra. \blacksquare

The achievable rates in Proposition 5.1 (for $K = 1$) and Proposition 5.2 (for $K = 2$) can also be extended to the Gaussian CM channel. In so doing, we focus on jointly Gaussian auxiliary random variables for Wyner-Ziv compression. While no general claim of optimality is put forth here, some conclusion on the optimality of such schemes can be drawn as discussed later in Sec. VI-A.

Proposition 6.2: The following rate region is achievable with one-round conferencing, $K = 1$:

$$\mathcal{R}_{OR}^G(C_{12}, C_{21}) = \bigcup_{\substack{0 \leq P'_i \leq P_i \\ i=1,2}} \{(R_0, R_1, R_2): R_0 \geq 0, R_1 \geq 0, R_2 \geq 0, \quad (23a)$$

$$R_1 \leq \min \left\{ \mathcal{C} \left(P'_1 \left(\gamma_{11}^2 + \frac{\gamma_{12}^2}{1 + \sigma_2^2} \right) \right), \mathcal{C} \left(P'_1 \left(\gamma_{12}^2 + \frac{\gamma_{11}^2}{1 + \sigma_1^2} \right) \right) \right\}, \quad (23b)$$

$$R_2 \leq \min \left\{ \mathcal{C} \left(P'_2 \left(\gamma_{21}^2 + \frac{\gamma_{22}^2}{1 + \sigma_2^2} \right) \right), \mathcal{C} \left(P'_2 \left(\gamma_{22}^2 + \frac{\gamma_{21}^2}{1 + \sigma_1^2} \right) \right) \right\} \quad (23c)$$

$$R_1 + R_2 \leq \min \left\{ \begin{array}{l} \mathcal{C} \left(\begin{array}{l} P'_1 \left(\gamma_{11}^2 + \frac{\gamma_{12}^2}{1 + \sigma_2^2} \right) + P'_2 \left(\gamma_{21}^2 + \frac{\gamma_{22}^2}{1 + \sigma_2^2} \right) \\ + \frac{\kappa}{1 + \sigma_2^2} \end{array} \right), \\ \mathcal{C} \left(\begin{array}{l} P'_2 \left(\gamma_{22}^2 + \frac{\gamma_{21}^2}{1 + \sigma_1^2} \right) + P'_1 \left(\gamma_{21}^2 + \frac{\gamma_{11}^2}{1 + \sigma_1^2} \right) \\ + \frac{\kappa}{1 + \sigma_1^2} \end{array} \right) \end{array} \right\} \quad (23d)$$

$$R_0 + R_1 + R_2 \leq \min \left\{ \begin{array}{l} \mathcal{C} \left(\begin{array}{l} P'_1 \left(\gamma_{11}^2 + \frac{\gamma_{12}^2}{1 + \sigma_2^2} \right) + P'_2 \left(\gamma_{21}^2 + \frac{\gamma_{22}^2}{1 + \sigma_2^2} \right) + \frac{\kappa}{1 + \sigma_2^2} \\ + \rho_1 \left(1 + \frac{P'_1 \gamma_{12}^2 + P'_2 \gamma_{22}^2}{1 + \sigma_2^2} \right) + \frac{\rho_2}{1 + \sigma_2^2} (1 + P'_1 \gamma_{11}^2 + P'_2 \gamma_{21}^2) \\ - \frac{2\sqrt{\rho_1 \rho_2} (P_1 \gamma_{11} \gamma_{12} + P_2 \gamma_{21} \gamma_{22})}{1 + \sigma_2^2} \end{array} \right), \\ \mathcal{C} \left(\begin{array}{l} P'_1 \left(\gamma_{12}^2 + \frac{\gamma_{11}^2}{1 + \sigma_1^2} \right) + P'_2 \left(\gamma_{22}^2 + \frac{\gamma_{21}^2}{1 + \sigma_1^2} \right) + \frac{\kappa}{1 + \sigma_1^2} \\ + \rho_2 \left(1 + \frac{P'_1 \gamma_{11}^2 + P'_2 \gamma_{21}^2}{1 + \sigma_1^2} \right) + \frac{\rho_1}{1 + \sigma_1^2} (1 + P'_1 \gamma_{12}^2 + P'_2 \gamma_{22}^2) \\ - \frac{2\sqrt{\rho_1 \rho_2} (P_1 \gamma_{11} \gamma_{12} + P_2 \gamma_{21} \gamma_{22})}{1 + \sigma_1^2} \end{array} \right) \end{array} \right\},$$

with (22) and quantization noise variances satisfying

$$\sigma_1^2 \geq \frac{1 + (\gamma_{11}^2 + \gamma_{12}^2)P_1 + (\gamma_{21}^2 + \gamma_{22}^2)P_2 + (\gamma_{12}\gamma_{21} - \gamma_{11}\gamma_{22})^2 P_1 P_2}{(2^{2C_{12}} - 1)(1 + \gamma_{12}^2 P_1 + \gamma_{22}^2 P_2)} \quad (24a)$$

$$\sigma_2^2 \geq \frac{1 + (\gamma_{11}^2 + \gamma_{12}^2)P_1 + (\gamma_{21}^2 + \gamma_{22}^2)P_2 + (\gamma_{12}\gamma_{21} - \gamma_{11}\gamma_{22})^2 P_1 P_2}{(2^{2C_{21}} - 1)(1 + \gamma_{11}^2 P_1 + \gamma_{21}^2 P_2)}. \quad (24b)$$

Proof: As stated above, we consider Gaussian auxiliary random variables and evaluate the region (15). In particular, the test channels for Wyner-Ziv compression are selected as $\hat{Y}_i = Y_i + Z_{q,i}$ where the compression noise $Z_{q,i}$ is zero-mean Gaussian with variance σ_i^2 and independent of Y_i . The proposition follows from some algebraic manipulation. ■

The one-round strategy can be generalized by enabling two rounds of conferencing ($K = 2$), obtaining the following achievable rate region:

Proposition 6.3: The following rate region is achievable with two rounds of conferencing, $K =$

2:

$$\mathcal{R}_{TR}^G(C_{12}, C_{21}) = \text{co} \bigcup_{\substack{0 \leq P'_i \leq P_i \\ i=1,2}} \{ \mathcal{R}_{TR,12}^G \cup \mathcal{R}_{TR,21}^G \} \quad (25)$$

with

$$\mathcal{R}_{TR,21}^G = \left\{ (R_0, R_1, R_2): R_0 \geq 0, R_1 \geq 0, R_2 \geq 0, \right. \quad (26a)$$

$$\left. R_1 \leq \min \left\{ \mathcal{C}(\gamma_{11}^2 P'_1) + C_{21}, \mathcal{C} \left(P'_1 \left(\gamma_{12}^2 + \frac{\gamma_{11}^2}{1 + \sigma_1^2} \right) \right) \right\} \right. \quad (26b)$$

$$\left. R_2 \leq \min \left\{ \mathcal{C}(\gamma_{21}^2 P'_2) + C_{21}, \mathcal{C} \left(P'_2 \left(\gamma_{22}^2 + \frac{\gamma_{21}^2}{1 + \sigma_1^2} \right) \right) \right\} \right. \quad (26c)$$

$$\left. R_1 + R_2 \leq \min \left\{ \begin{array}{l} \mathcal{C}(\gamma_{11}^2 P'_1 + \gamma_{21}^2 P'_2) + C_{21}, \\ \mathcal{C} \left(P'_1 \left(\gamma_{12}^2 + \frac{\gamma_{11}^2}{1 + \sigma_1^2} \right) + P'_2 \left(\gamma_{22}^2 + \frac{\gamma_{21}^2}{1 + \sigma_1^2} \right) + \frac{\kappa}{1 + \sigma_1^2} \right) \end{array} \right\} \right. \quad (26d)$$

$$\left. R_0 + R_1 + R_2 \leq \min \left\{ \begin{array}{l} \mathcal{C}(\gamma_{11}^2 P'_1 + \gamma_{21}^2 P'_2 + \rho_1) + C_{21}, \\ \mathcal{C} \left(\begin{array}{l} P'_1 \left(\gamma_{12}^2 + \frac{\gamma_{11}^2}{1 + \sigma_1^2} \right) + P'_2 \left(\gamma_{22}^2 + \frac{\gamma_{21}^2}{1 + \sigma_1^2} \right) + \frac{\kappa}{1 + \sigma_1^2} \\ + \rho_2 \left(1 + \frac{P'_1 \gamma_{11}^2 + P'_2 \gamma_{21}^2}{1 + \sigma_1^2} \right) + \frac{\rho_1}{1 + \sigma_1^2} (1 + P'_1 \gamma_{12}^2 + P'_2 \gamma_{22}^2) \\ - \frac{2\sqrt{\rho_1 \rho_2} (P'_1 \gamma_{11} \gamma_{12} + P'_2 \gamma_{21} \gamma_{22})}{1 + \sigma_1^2} \end{array} \right) \end{array} \right\} \right\}$$

$\mathcal{R}_{TR,12}$ is similarly defined:

$$\mathcal{R}_{TR,12}^G = \left\{ (R_0, R_1, R_2): R_0 \geq 0, R_1 \geq 0, R_2 \geq 0, \right. \quad (27a)$$

$$\left. R_1 \leq \min \left\{ \mathcal{C} \left(P'_1 \left(\gamma_{11}^2 + \frac{\gamma_{12}^2}{1 + \sigma_2^2} \right) \right), \mathcal{C}(\gamma_{12}^2 P'_1) + C_{12} \right\} \right. \quad (27b)$$

$$\left. R_2 \leq \min \left\{ \mathcal{C} \left(P'_2 \left(\gamma_{21}^2 + \frac{\gamma_{22}^2}{1 + \sigma_2^2} \right) \right), \mathcal{C}(\gamma_{22}^2 P'_2) + C_{12} \right\} \right. \quad (27c)$$

$$\left. R_1 + R_2 \leq \min \left\{ \begin{array}{l} \mathcal{C} \left(\begin{array}{l} P'_1 \left(\gamma_{11}^2 + \frac{\gamma_{12}^2}{1 + \sigma_2^2} \right) + P'_2 \left(\gamma_{21}^2 + \frac{\gamma_{22}^2}{1 + \sigma_2^2} \right) \\ + \frac{P'_1 P'_2 (\gamma_{12} \gamma_{21} - \gamma_{11} \gamma_{22})^2}{1 + \sigma_2^2} \end{array} \right), \\ \mathcal{C}(\gamma_{22}^2 P'_2 + \gamma_{12}^2 P'_1) + C_{12} \end{array} \right\} \right. \quad (27d)$$

$$R_0 + R_1 + R_2 \leq \min \left\{ \mathcal{C} \left(\begin{array}{l} P'_1 \left(\gamma_{11}^2 + \frac{\gamma_{12}^2}{1+\sigma_2^2} \right) + P'_2 \left(\gamma_{21}^2 + \frac{\gamma_{22}^2}{1+\sigma_2^2} \right) + \frac{\kappa}{1+\sigma_2^2} \\ + \rho_1 \left(1 + \frac{P'_1 \gamma_{12}^2 + P'_2 \gamma_{22}^2}{1+\sigma_2^2} \right) + \frac{\rho_2}{1+\sigma_2^2} (1 + P'_1 \gamma_{11}^2 + P'_2 \gamma_{21}^2) \\ - \frac{2\sqrt{\rho_1 \rho_2} (P_1 \gamma_{11} \gamma_{12} + P_2 \gamma_{21} \gamma_{22})}{1+\sigma_2^2} \end{array} \right), \left. \begin{array}{l} \\ \\ \mathcal{C}(\gamma_{22}^2 P'_2 + \gamma_{12}^2 P'_1 + \rho_2) + C_{12} \end{array} \right\},$$

with (22) and (24).

A. Discussion

Here we draw some conclusions on the optimality of the one and two-round schemes discussed above for the Gaussian CM channel. We start with the one-round scheme of Proposition 6.2 and notice that, by comparison with the outer bound (21), it can be easily seen that the scheme at hand is optimal in the asymptotic regime of large conferencing capacities $C_{12} \rightarrow \infty$ and $C_{21} \rightarrow \infty$. Further conclusions on the gap between the upper bound (21) and the performance achievable with one round of conferencing at the decoders can be drawn in two special cases. Consider first the case of a broadcast channel with conferencing encoders [5] [7], which is obtained as $R_0 = 0$ and $R_2 = 0$ and thus $P_2 = 0$ without loss of generality (a symmetric statement can be straightforwardly obtained for $R_0 = 0$ and $R_1 = 0$). In this case, we show below that the one-round scheme achieves the upper bound (21) to within half a bit, irrespective of the channel gains of the broadcast channel and the capacities of the conferencing links. To elaborate, notice that the outer bound (21) for the case at hand is given by

$$R_1 \leq R_{1,out} = \min\{\mathcal{C}(\gamma_{11}^2 P_1) + C_{21}, \mathcal{C}(\gamma_{12}^2 P_1) + C_{12}, \mathcal{C}((\gamma_{11}^2 + \gamma_{12}^2)P_1)\}, \quad (28)$$

whereas the rate achievable with one-round conferencing is given by

$$R_{1,OR} = \min \left\{ \mathcal{C} \left(\gamma_{11}^2 P_1 + \frac{\gamma_{12}^2 P_1}{1 + \sigma_2^2} \right), \mathcal{C} \left(\gamma_{12}^2 P_1 + \frac{\gamma_{11}^2 P_1}{1 + \sigma_1^2} \right) \right\}, \quad (29)$$

where

$$\sigma_1^2 = \frac{1 + (\gamma_{11}^2 + \gamma_{12}^2)P_1}{(2^{2C_{12}} - 1)(1 + \gamma_{12}^2 P_1)},$$

and

$$\sigma_2^2 = \frac{1 + (\gamma_{11}^2 + \gamma_{12}^2)P_1}{(2^{2C_{21}} - 1)(1 + \gamma_{11}^2 P_1)}.$$

Using these two expressions, we can prove the following proposition (see Appendix III for a full proof).

Proposition 6.4: We have $R_{1,OR} \geq R_{1,out} - \frac{1}{2}$. Moreover, for the symmetric channel case, i.e., $\gamma_{11}^2 = \gamma_{12}^2$, we have $R_{1,OR} \geq R_{1,out} - \frac{\log 3-1}{2}$.

Next, we consider the symmetric Gaussian CM channel, that is, we let $R_0 = 0$, $\gamma_{11}^2 = \gamma_{22}^2 = a$, $\gamma_{12}^2 = \gamma_{21}^2 = b$, and $P_1 = P_2 \triangleq P$. We also assume symmetric conferencing link capacities $C_{12} = C_{21} \triangleq C$. In such a case, the outer bound and the achievable rates with one-round conferencing are:

$$\mathcal{C}_{CM-out}^G(C) = \{(R_1, R_2) : R_1 \geq 0, R_2 \geq 0, \\ R_1 \leq \min\{\mathcal{C}(aP) + C, \mathcal{C}(bP) + C, \mathcal{C}((a+b)P)\}, \quad (30a)$$

$$R_2 \leq \min\{\mathcal{C}(bP) + C, \mathcal{C}(aP) + C, \mathcal{C}((a+b)P)\}, \quad (30b)$$

$$R_1 + R_2 \leq \min\{\mathcal{C}((a+b)P) + C, \mathcal{C}(2(a+b)P + (b-a)^2P^2)\}, \quad (30c)$$

and

$$\mathcal{R}_{OR}^G(C) = \{(R_1, R_2) : R_1 \geq 0, R_2 \geq 0, \\ R_1 \leq \min\left\{\mathcal{C}\left(\left(a + \frac{b}{1+\sigma^2}\right)P\right), \mathcal{C}\left(\left(b + \frac{a}{1+\sigma^2}\right)P\right)\right\}, \quad (31a)$$

$$R_2 \leq \min\left\{\mathcal{C}\left(\left(b + \frac{a}{1+\sigma^2}\right)P\right), \mathcal{C}\left(\left(a + \frac{b}{1+\sigma^2}\right)P\right)\right\}, \quad (31b)$$

$$R_1 + R_2 \leq \mathcal{C}\left(\left(a + b + \frac{a+b}{1+\sigma^2} + \frac{(b-a)^2P}{1+\sigma^2}\right)P\right), \quad (31c)$$

with $\sigma^2 \triangleq \frac{1+2(a+b)P+(b-a)^2P^2}{(1+(a+b)P)(2^{2C}-1)}$, respectively. The following result can be proved (see Appendix IV).

Proposition 6.5: $\mathcal{R}_{OR}^G \supseteq \{(R_1, R_2) : R_1 \geq 0, R_2 \geq 0, (R_1+\delta, R_2+(\Delta-\delta)) \in \mathcal{C}_{CM-out}^G(C) \text{ for all } \delta \in [0, \Delta]\}$ with $\Delta = \frac{\log(1+\beta)}{2}$ where $\beta \triangleq \frac{\max(a,b)}{\min(a,b)}$. Moreover, in the special case $a = b$, the gap Δ can be further reduced to $\Delta = \left(\frac{\log 3-1}{2}\right) \approx 0.293$ bits.

The proposition above is equivalent to saying that the total rate loss of using one round of conferencing relative to the sum capacity is less than $\frac{\log(1+\beta)}{2}$, which is a constant that depends only on the relative qualities of the direct channels and the cross channels.

Let us now consider the two-round scheme of Proposition 6.3. Since $\mathcal{R}_{TR}^{\mathcal{G}}(C_{12}, C_{21}) \supseteq \mathcal{R}_{OR}^{\mathcal{G}}(C_{12}, C_{21})$, all the conclusions above on the one-round scheme apply also to the two-round strategy. Alternatively, we can interpret these results as a finite bit limit on the potential gain of going from one round of conferencing to two rounds. Moreover, it should be noted that the two-round approach was defined as single-session in [16] and shown therein to be optimal among several classes of multi-session protocols for a broadcast channel with cooperating decoders. Finally, we can prove the following.

Proposition 6.6: The two-round scheme is optimal in the case of *unidirectional cooperation*: $\mathcal{R}_{TR}^{\mathcal{G}}(0, C_{21}) = \mathcal{C}_{CM-out}^{\mathcal{G}}(0, C_{21})$ and $\mathcal{R}_{TR}^{\mathcal{G}}(C_{12}, 0) = \mathcal{C}_{CM-out}^{\mathcal{G}}(C_{12}, 0)$, thus establishing the capacity of the Gaussian CM channel for this special case.

Proof: This result follows by comparing the achievable region with the outer bound (21). ■

Next, we comment on the *sum-rate multiplexing gain* of the Gaussian CM channel. Consider a symmetric system with $P_1 = P_2 \triangleq P$, $\gamma_{11} = \gamma_{22}$, $\gamma_{12} = \gamma_{21}$, and $C_{12} = C_{21} \triangleq C$. We are interested in studying the conditions on the conferencing capacity C such that the maximum multiplexing gain on the sum-rate, $\lim_{P \rightarrow \infty} \sup_{(R_1, R_2) \in \mathcal{C}_{CM}^{\mathcal{G}}(C, C)} (R_1 + R_2) / (\frac{1}{2} \log P) = 2$, corresponding to full cooperation, can be achieved. From the outer bound in (21), it is clear that C should scale at least as $\frac{1}{2} \log P$ as the sum rate is limited by $\mathcal{C}(P(\gamma_{11}^2 + \gamma_{21}^2)) + C$. By considering the achievable regions with one (23) or two (25) conferencing rounds, it can be also concluded that if C scales as $(1 + \epsilon) \log P$ with any $\epsilon > 0$, then the optimal multiplexing gain is indeed achievable. This is because with $C = \frac{1}{2}(1 + \epsilon) \log P$ the quantization noise variances in (24) are proportional to $P^{-\epsilon}$ and thus tend to zero for large P . It is noted that this result would hold even if the decoders used regular compression that neglects the side information at the other decoder, as in this case we would have $\sigma_i^2 = \frac{\gamma_{i1}^2 P + \gamma_{21}^2 P + 1}{2^{2C-1}}$, which is still proportional to $P^{-\epsilon}$ for $C = \frac{1}{2}(1 + \epsilon) \log P$.

As a final remark, extending the achievable rates defined above for the Gaussian channel (and assuming Gaussian channel and compression codebooks as done above) to more than two conferencing rounds would not lead to any further gain, since with Gaussian variables, “conditional” compression and compression with side information have the same efficiency (see [7] for a discussion).

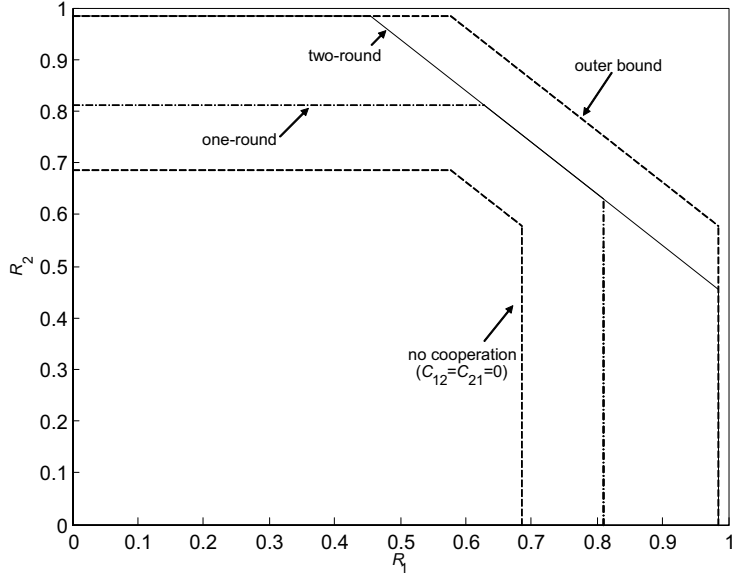


Fig. 2. Outer bound (21), rate region achievable with one-round (23) and two-round (25) strategies and with no cooperation ($C_{12} = C_{21} = 0$) for $R_0 = 0$, and a symmetric scenario with $P_1 = P_2 = 5\text{dB}$, $\gamma_{12}^2 = \gamma_{21}^2 = -3\text{dB}$, $\gamma_{11}^2 = \gamma_{22}^2 = 0\text{dB}$, $C_{21} = C_{12} = 0.5$.

B. Numerical results

Since the rate region expressions provided for the outer bound and the one-round and two-round achievable schemes give little insight, in this section we present numerical results to see how much gain is obtained via decoder cooperation. In Fig. 2, we consider a symmetric scenario with $P_1 = P_2 = 5\text{ dB}$, $\gamma_{12}^2 = \gamma_{21}^2 = -3\text{ dB}$, $\gamma_{11}^2 = \gamma_{22}^2 = 0\text{ dB}$, $C_{21} = C_{12} = 0.3$, and we plot the outer bound (21), the rate region achievable with one-round (23) and two-round (25) conferencing as well as with no cooperation ($C_{12} = C_{21} = 0$) (obtained from either (23) and (25)) for $R_0 = 0$ (so that selecting $P'_i = P_i$ is sufficient in all the capacity regions). It can be seen that cooperation via conferencing decoders enables the achievable rate region to be increased both in terms of sum-rate and individual rates. Moreover, the two-step strategy provides relevant gains with respect to the one-step approach, while still not achieving the outer bound (21).

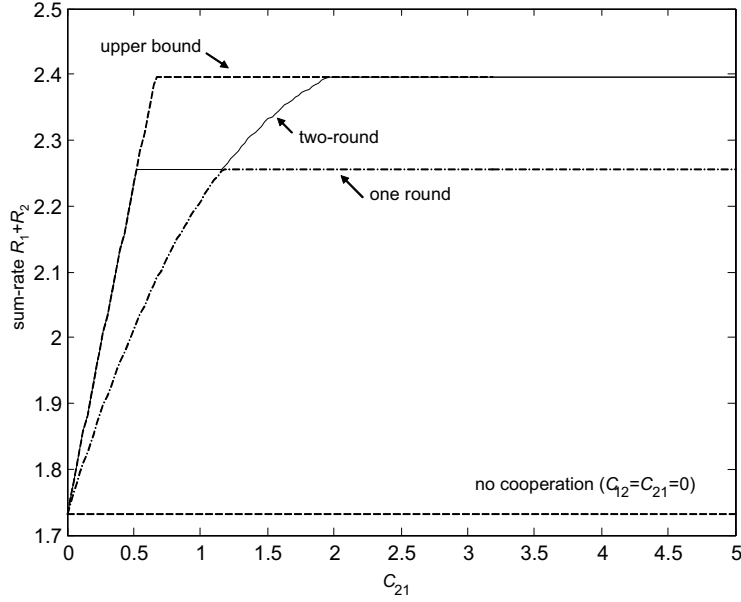


Fig. 3. Sum of the private rates $R_1 + R_2$ (with $R_0 = 0$) versus the conferencing link capacity C_{21} for the outer bound (21), the one-round (23) and two-round (25) strategies and with no cooperation ($P_1 = P_2 = 10dB$, $\gamma_{12}^2 = 0dB$, $\gamma_{22}^2 = 0dB$, $\gamma_{21}^2 = -3dB$, $\gamma_{11}^2 = -3dB$, $C_{12} = 0.2$).

Fig. 3 and Fig. 4 show the sum of the private rates $R_1 + R_2$ (with $R_0 = 0$) versus the conferencing link capacities C_{21} and C_{12} , respectively, for the outer bound (21), the achievable schemes with one-round (23) and two-round (25) conferencing and with no cooperation. In both figures, we consider cases in which receiver 1 has a worse signal quality than receiver 2 (stochastically degraded): $P_1 = P_2 = 10dB$, $\gamma_{12}^2 = 0dB$, $\gamma_{22}^2 = 0dB$, $\gamma_{21}^2 = -3dB$, $\gamma_{11}^2 = -3dB$. Fig. 3 shows the achievable sum-rates versus C_{21} for $C_{12} = 0.2$. It is seen that if $C_{21} = 0$ the upper bound coincides with the rate achievable with no cooperation, showing that if the link from the "good" receiver to the degraded receiver is disabled, the performance is dominated by the worse receiver and there is no gain in having $C_{12} > 0$. Increasing C_{21} enables the rate of the worse receiver to be increased via cooperation, thus harnessing significant gains with respect to no cooperation. In particular, it is seen that for C_{21} sufficiently small (here $C_{21} \lesssim 0.5$) the two-step strategy is

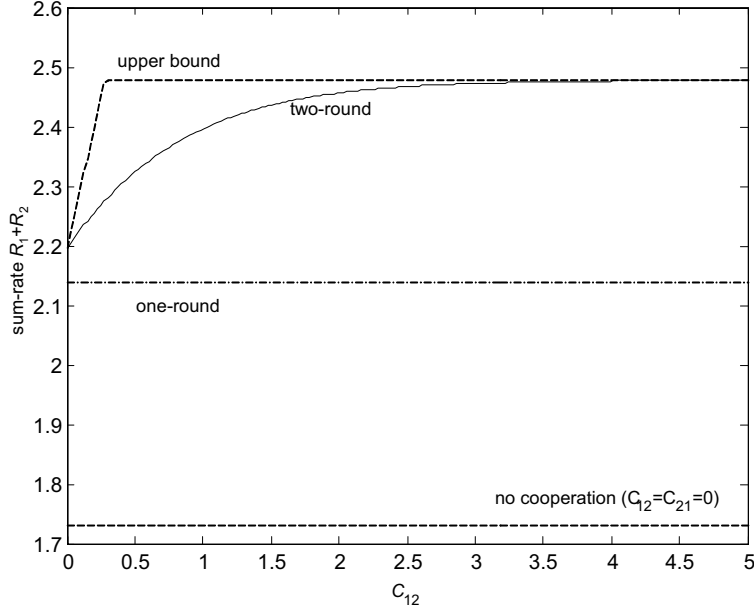


Fig. 4. Sum of the private rates $R_1 + R_2$ (with $R_0 = 0$) versus the conferencing link capacity C_{12} for the outer bound (21), the one-round (23) and two-round (25) strategies and with no cooperation ($P_1 = P_2 = 10dB$, $\gamma_{12}^2 = 0dB$, $\gamma_{22}^2 = 0dB$, $\gamma_{21}^2 = -3dB$, $\gamma_{11}^2 = -3dB$, $C_{21} = 0.8$).

optimal, since in this region the performance is dominated by the worse receiver whose achievable rate increases linearly with C_{21} due to cooperation via binning of the message set performed at the good receiver. The one-step protocol instead lags behind and its performance saturates at $\mathcal{C} \left(\gamma_{22}^2 P_2 + \gamma_{21}^2 P_1 + \frac{\gamma_{11}^2 P_1 + \gamma_{21}^2 P_2}{1 + \sigma_1^2} \right) \simeq 2.26$. Finally, for sufficiently large C_{21} , the achievable sum-rate at the worse receiver becomes larger than 2.26 and the performance tends to the sum-rate of the best receiver, $\mathcal{C}(\gamma_{22}^2 P_2 + \gamma_{12}^2 P_1) + C_{12} \simeq 2.4$, unless C_{12} is too large.

Further insight is shown in Fig. 4 where the rates are plotted versus C_{12} for $C_{21} = 0.8$. We notice that for $C_{12} = 0$ only the two-step protocol is able to achieve the upper bound, since in this regime it is optimal for the good receiver to decode and bin its decision. Moreover, similarly, increasing C_{12} enhances the gain of the two-round strategy over the one-round strategy up to the point where the performance is limited by the sum-rate at the worse receiver, i.e.,

by $\mathcal{C}(P_1(\gamma_{11}^2 + \gamma_{12}^2) + P_2(\gamma_{21}^2 + \gamma_{22}^2) + P_1P_2(\gamma_{12}\gamma_{21} - \gamma_{11}\gamma_{22})^2) \simeq 2.48$, which coincides with the upper bound.

VII. CONFERENCING ENCODERS AND DECODERS

In this section, we extend the capacity results of Sec. IV to the scenario in Fig. 5 in which instead of having a common message (as in the previous sections), the encoders are connected via conferencing links of capacity \bar{C}_{12} and \bar{C}_{21} . Here, each encoder has only one message W_i of rate R_i ($i = 1, 2$) to deliver to both decoders. We refer to this channel as a compound MAC with conferencing decoders and encoders (for short, the CME channel). Definitions of encoders and conferencing at the transmission side follows the standard reference [1] (see also [3]). A $((2^{nR_1}, 2^{nR_2}), n, \bar{K}, K)$ code for the CME channel consists of $2\bar{K}$ ‘‘conferencing’’ functions at the encoders, where \bar{K} is the number of conferencing rounds between the transmitters ($k = 1, 2, \dots, \bar{K}$):

$$\bar{h}_{1,k}: \mathcal{W}_1 \times \bar{\mathcal{V}}_{2,1} \times \dots \times \bar{\mathcal{V}}_{2,k-1} \rightarrow \bar{\mathcal{V}}_{1,k} \quad (32a)$$

$$\bar{h}_{2,k}: \mathcal{W}_2 \times \bar{\mathcal{V}}_{1,1} \times \dots \times \bar{\mathcal{V}}_{1,k-1} \rightarrow \bar{\mathcal{V}}_{2,k}. \quad (32b)$$

with alphabets $\bar{\mathcal{V}}_{i,k}$ ($k = 1, 2, \dots, \bar{K}$) satisfying the capacity budget on the conferencing links:

$$\sum_{k=1}^{\bar{K}} |\bar{\mathcal{V}}_{1,k}| \leq n\bar{C}_{12} \quad \text{and} \quad \sum_{k=1}^{\bar{K}} |\bar{\mathcal{V}}_{2,k}| \leq n\bar{C}_{21}, \quad (33)$$

and encoding functions:

$$f_1: \mathcal{W}_1 \times \bar{\mathcal{V}}_2^{\bar{K}} \rightarrow \mathcal{X}_1^n \quad (34a)$$

$$f_2: \mathcal{W}_2 \times \bar{\mathcal{V}}_1^{\bar{K}} \rightarrow \mathcal{X}_2^n. \quad (34b)$$

It is noted that encoding takes place after the \bar{K} conferencing rounds at the transmit side, similar to the operation at the receivers where decoding occurs after the K decoder-side conferencing rounds. Decoding and conferencing at the receiver side are defined as in Sec. II (by setting the common message W_0 to a constant). Achievability of a rate pair (R_1, R_2) is defined by requiring the existence of a code with such rates and with a vanishing probability of error on the two messages W_1 and W_2 . The capacity region of the CME channel is denoted as $\mathcal{C}_{CME}(\bar{C}_{12}, \bar{C}_{21}, C_{12}, C_{21})$.

An outer bound can be established similarly to Proposition 3.1.

Proposition 7.1: We have $\mathcal{C}_{CME}(\bar{C}_{12}, \bar{C}_{21}, C_{12}, C_{21}) \subseteq \mathcal{C}_{CME-out}(\bar{C}_{12}, \bar{C}_{21}, C_{12}, C_{21})$ with

$$\begin{aligned} \mathcal{C}_{CME-out}(\bar{C}_{12}, \bar{C}_{21}, C_{12}, C_{21}) &= \{(R_1, R_2): ((R_{12} + R_{21}), R_1 - R_{12}, R_2 - R_{21}) \\ &\in \mathcal{C}_{CM-out}(C_{12}, C_{21}) \text{ where } R_{12} = \min\{R_1, \bar{C}_{12}\} \\ &\text{and } R_{21} = \min\{R_2, \bar{C}_{21}\}\}, \end{aligned} \quad (35)$$

where $\mathcal{C}_{CM-out}(C_{12}, C_{21})$ is defined in (11). It is shown in [3] that with only conferencing encoders we have $\mathcal{C}_{CME}(\bar{C}_{12}, \bar{C}_{21}, 0, 0) = \mathcal{C}_{CME-out}(\bar{C}_{12}, \bar{C}_{21}, 0, 0)$.

Proof: See Appendix V. ■

The following capacity results can be established similarly to Proposition 4.1 and 4.2, respectively.

Proposition 7.2: If the CME channel is physically degraded such that $(X_1X_2) - Y_1 - Y_2$ forms a Markov chain, then the capacity region is obtained as

$$\mathcal{C}_{CME-DEG}(\bar{C}_{12}, \bar{C}_{21}, C_{12}, C_{21}) = \mathcal{C}_{CME-out}(\bar{C}_{12}, \bar{C}_{21}, C_{12}, 0). \quad (36)$$

Notice that here $p^*(y_1y_2|x_1, x_2) = p(y_1|x_1, x_2)p(y_2|y_1)$ due to degradedness. A symmetric result holds for the physically degraded channel $(X_1X_2) - Y_2 - Y_1$.

Proof: The converse follows from the same reasoning used in Proposition 4.1 and Proposition 6.3. Achievability is obtained by using a scheme similar to Proposition 4.1 with the only difference being that here transmission is performed according to the optimal strategy for a MAC with conferencing encoders [1] (see also Theorem 2 in [3]). It is noted that this strategy requires only one conferencing round at the encoders, $\bar{K} = 1$. ■

Proposition 7.3: In the case of unidirectional cooperation at the receiver side ($C_{12} = 0$ or $C_{21} = 0$), the capacity region is given by, respectively,

$$\mathcal{C}_{CME}(\bar{C}_{12}, \bar{C}_{21}, 0, C_{21}) = \mathcal{C}_{CME-out}(\bar{C}_{12}, \bar{C}_{21}, 0, C_{21}) \quad (37a)$$

or

$$\mathcal{C}_{CME}(\bar{C}_{12}, \bar{C}_{21}, C_{12}, 0) = \mathcal{C}_{CME-out}(\bar{C}_{12}, \bar{C}_{21}, C_{12}, 0). \quad (37b)$$

Proof: The proof is similar to those of Proposition 4.2 and Proposition 7.2.

It is finally noted that the outer bound and achievable rates derived in Sec. V and Section VI can also be extended to the CME channel and the Gaussian CME channel (20) following the same approach used to derive Propositions 7.2 and Proposition 7.3, that is, by considering the optimal coding strategy for the MAC with conferencing encoders [1] (which requires $\bar{K} = 1$). In terms of the rate regions, this simply amounts to using the same transformation from (R_0, R_1, R_2) to (R_1, R_2) discussed above (see also [3]). For instance, an outer bound on the Gaussian capacity region $\mathcal{C}_{CME}^{\mathcal{G}}(\bar{C}_{12}, \bar{C}_{21}, C_{12}, C_{21})$ can be obtained as

$$\begin{aligned} \mathcal{C}_{CME-out}^{\mathcal{G}}(\bar{C}_{12}, \bar{C}_{21}, C_{12}, C_{21}) = \{ & (R_1, R_2): ((R_{12} + R_{21}), R_1 - R_{12}, R_2 - R_{21}) \\ & \in \mathcal{C}_{CM-out}^{\mathcal{G}}(C_{12}, C_{21}) \text{ where } R_{12} = \min\{R_1, \bar{C}_{12}\} \\ & \text{and } R_{21} = \min\{R_2, \bar{C}_{21}\}\}, \end{aligned} \quad (38)$$

and similarly for the rate regions achievable with the one-round and two-round receiver-side conferencing strategies ((23) and (25)) coupled with the optimal transmit cooperation [1].

Remark 7.1: (Conferencing encoders vs. conferencing decoders) While no general capacity results have been derived that enable a conclusive comparison between the performance of conferencing encoders or decoders in the compound multiple access channel, some basic conclusions can be drawn based on the analysis above. To start with, conferencing decoders tend to behave like a multi-antenna receiver for large conferencing capacities and thus, as discussed in Sec. VI, have the potential for increasing the multiplexing gain of the sum-rate up to the maximum value of two. In contrast, it can be seen from the outer bound (38) that conferencing at the encoders alone does not have such a potential advantage, as the coherent power combining afforded by cooperating encoders is not enough to increase the multiplexing gain of the system³. However, this does not necessarily mean that decoder conferencing is always to be preferred to encoder conferencing. Consider for instance the case of unidirectional links, where say $\bar{C}_{21} = C_{21} = 0$, so that conferencing links

³It is noted that this conclusion would be significantly different for an interference channel, since in this case conferencing at the encoders has the capability of creating an equivalent two-antenna broadcast channel with single-antenna receivers, whose multiplexing gain is known to be two.

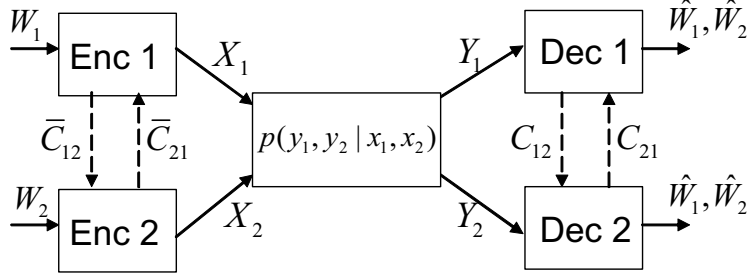


Fig. 5. A discrete-memoryless compound MAC channel with conferencing decoders and encoders (for short, CME).

exist only from encoder 1 to encoder 2 on the transmit side and from decoder 1 to decoder 2 on the receive side. In this case, the capacity region is given in Proposition 7.3, and one can see that, e.g., for a symmetric system ($\gamma_{11}^2 = \gamma_{22}^2$, $\gamma_{21}^2 = \gamma_{12}^2$ and $P_1 = P_2$), the conferencing link at the decoders alone never helps increase the achievable rates, while the conferencing link at the transmit side can always enlarge the achievable rate region. Further performance comparison is carried out numerically below.

A. Numerical results

In this section, we present a numerical example related to the scenario in Fig. 5 for the Gaussian CME channel (20). Fig. 6 shows the outer bound (38) evaluated for encoder-side ($\bar{C}_{12} = \bar{C}_{21} = 0$), decoder-side ($C_{12} = C_{21} = 0$) or both-side conferencing, along with the rate regions achievable with one-round and two-round strategies and with no cooperation for $P_1 = P_2 = 5dB$, $\gamma_{12}^2 = \gamma_{21}^2 = -3dB$, $\gamma_{11}^2 = \gamma_{22}^2 = 0dB$, and conferencing capacities (when non-zero) $\bar{C}_{21} = \bar{C}_{12} = C_{21} = C_{12} = 0.3$. Considering first the outer bounds, it can be seen that both conferencing at the encoders and decoders have the same potential in terms of increasing the rates R_1 and R_2 , whereas for this example the outer bound corresponding to decoder-side cooperation leads to a larger sum-rate $R_1 + R_2$. Comparison of achievable rates via one or two rounds of conferencing at the receiver side (recall that one round of encoder conferencing is enough to achieve all the rate points discussed

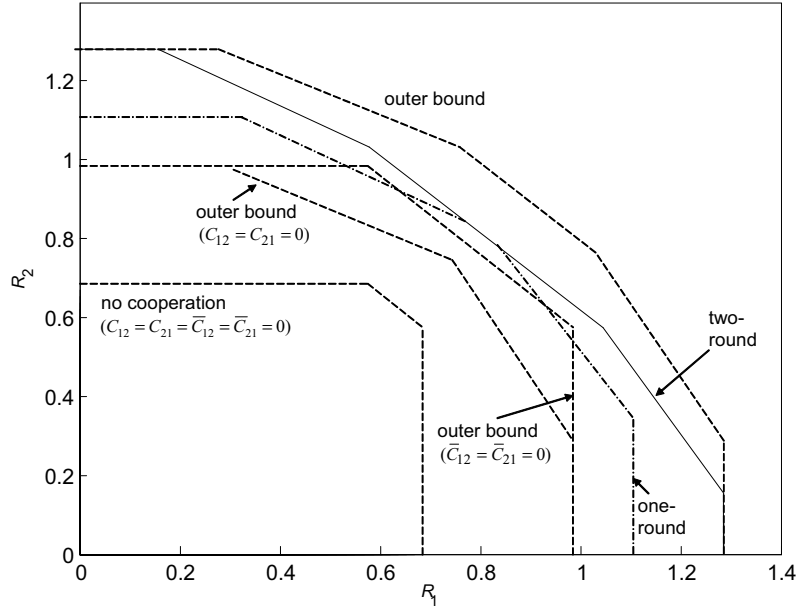


Fig. 6. Outer bound (38) evaluated for encoder-side ($\bar{C}_{12} = \bar{C}_{21} = 0$), decoder-side ($C_{12} = C_{21} = 0$) or both-side conferencing, along with the rate regions achievable with one-round and two-round strategies and with no cooperation for $P_1 = P_2 = 5dB$, $\gamma_{12}^2 = \gamma_{21}^2 = -3dB$, $\gamma_{11}^2 = \gamma_{22}^2 = 0dB$, and conferencing capacities (when non-zero) $\bar{C}_{21} = \bar{C}_{12} = C_{21} = C_{12} = 0.3$.

here) is similar to that seen in Fig. 2.

VIII. CONCLUSIONS

The model of conferencing encoders and/or decoders is a convenient framework that allows evaluation of the potential gains arising from cooperation at the transmitter or receiver side in a wireless network. From a practical standpoint, it accounts for scenarios where out-of-band signal paths exist at the two ends of a communication link, as is the case in wireless communication systems where nodes are endowed with multiple radio interfaces. In this work, we have contributed to the state of knowledge in this area by investigating a compound MAC with conferencing decoders and, possibly, encoders. The compound MAC can be seen as a combination of two single-message broadcast (multicast) channels from the standpoint of the transmitters, or two multiple access channels as seen by the receivers. The scenario at hand generalizes a number of previously studied

setups, such as MAC or compound MAC with common message or conferencing encoders and single-message broadcast channel with two conferencing decoders. A number of capacity results have been derived that have shed light on the impact of decoder and encoder conferencing on the capacity of the compound MAC. Among the conclusions, we have shown that in a compound Gaussian MAC, one round of conferencing at the decoders achieves the entire capacity region within a constant number of bits/s/Hz in several special cases. One round of conferencing at the transmitters is also optimal in all the cases where the capacity region is known. Moreover, comparing the performance of conferencing at the encoders and decoders, it has been pointed out that examples can be constructed where either one outperforms the other. However, in the Gaussian case, while conferencing at the decoders has the potential of increasing the sum-rate multiplexing gain to the optimal value of two by mimicking a multiantenna receiver, the same is not true of conferencing encoders, since coherent power combining afforded by cooperating encoders is not enough to increase the multiplexing gain beyond one (recall that the two decoders must estimate both messages).

As a possible extension of this work we mention the study of an interference channel, rather than the compound MAC, with conferencing decoders. As already pointed out in the paper, some of the conclusions here would be significantly different in this case, and the analysis could benefit from the techniques used in [17] [18] to study interference channels with no cooperation.

APPENDIX I

PROOF OF PROPOSITION 3.1

In order for rates (R_0, R_1, R_2) to be achievable, the probability of error P_e needs to satisfy (5) which, by the union bound, is implied by $P_{e,i} \leq \varepsilon/2$ for $i = 1, 2$ with

$$P_{e,1} = \frac{1}{2^{n(R_0+R_1+R_2)}} \sum_{\bar{w} \in \mathcal{W}_0 \times \mathcal{W}_1 \times \mathcal{W}_2} \Pr[h_1(Y_1^n, V_2^k) \neq \bar{w} | \bar{w} \text{ sent}]$$

and similarly for $P_{e,2}$. Consider the first receiver. By Fano's inequality, we have

$$H(W_0, W_1, W_2 | Y_1^n, V_2^K) \leq H(P_{e,1}) + n(R_0 + R_1 + R_2)P_{e,1} \triangleq n\delta_n \quad (39)$$

with $\delta_n \rightarrow 0$ as $n \rightarrow \infty$. It also follows that

$$H(W_1 W_2 | Y_1^n, V_2^K, W_0) \leq n\delta_n, \quad (40a)$$

$$H(W_1 | Y_1^n, V_2^K, W_0, W_2) \leq n\delta_n \text{ and} \quad (40b)$$

$$H(W_2 | Y_1^n, V_2^K, W_0, W_1) \leq n\delta_n. \quad (40c)$$

Now, from (39), we have

$$\begin{aligned} n(R_0 + R_1 + R_2) &\leq I(W_0, W_1, W_2; Y_1^n, V_2^K) + n\delta_n \\ &\leq I(W_0, W_1, W_2; Y_1^n) + I(W_0, W_1, W_2; V_2^K | Y_1^n) + n\delta_n \\ &\stackrel{(a)}{\leq} I(W_0, W_1, W_2; Y_1^n) + nC_{21} + n\delta_n \\ &\stackrel{(b)}{\leq} \sum_{i=1}^n I(X_{1,i}, X_{2,i}; Y_{1,i}) + nC_{21} + n\delta_n, \end{aligned}$$

where (a) follows from the fact that $I(W_0, W_1, W_2; V_2^K | Y_1^n) \leq H(V_2^K) \leq nC_{21}$ and (b) is obtained similarly to [1], Sec. 3.4. From (40), using similar arguments as in the above chain of inequalities, one can also obtain

$$\begin{aligned} n(R_1 + R_2) &\leq \sum_{i=1}^n I(X_{1,i}, X_{2,i}; Y_{1,i} | W_0) + nC_{21} + n\delta_n, \\ nR_1 &\leq \sum_{i=1}^n I(X_{1,i}; Y_{1,i} | X_{2,i}, W_0) + nC_{21} + n\delta_n \text{ and} \\ nR_2 &\leq \sum_{i=1}^n I(X_{2,i}; Y_{1,i} | X_{1,i}, W_0) + nC_{21} + n\delta_n. \end{aligned}$$

Now defining $U_i = W_0$, the proof is completed as in [3]. We can repeat the same arguments for receiver 2. Also the condition that $(R_0, R_1, R_2) \in \mathcal{R}_{MAC,FC}$ follows similarly considering full cooperation between the receivers.

APPENDIX II

PROOF OF PROPOSITION 4.1

Converse: The converse follows immediately from Proposition 3.1 and the data processing theorem. In fact, it is easy to see that, because of physical degradedness, receiver 1 cannot benefit

from V_2^K , which is a function of Y_2^n and Y_1^n via V_1^k . For instance, condition (39) now becomes

$$H(W_0, W_1, W_2|Y_1^n) = H(W_0, W_1, W_2|Y_1^n, V_2^K) \leq H(P_{e,1}) + n(R_1 + R_2)P_{e,1} \triangleq n\delta_n,$$

due to the Markov chain $(W_0, W_1, W_2) - Y_1^n - V_2^K$. Repeating the same arguments for the other conditions (40), the converse is then completed as in Proposition 3.1.

Achievability: Codeword generation at the transmitters is performed as for the MAC with common information [1] [13]:

Generate 2^{nR_0} sequences $u^n(w_0)$ of length n , with the elements of each being chosen independent identically distributed (i.i.d.) according to the distribution $p(u)$, $w_0 \in \mathcal{W}_0$. For any sequence $u^n(w_0)$, generate 2^{nR_i} independent sequences $x_i^n(w_0, w_i)$, $w_i \in \mathcal{W}_i$, again i.i.d. according to $p(x_i|u_i(w_0))$, for $i = 1, 2$.

At receiver 1, the message sets \mathcal{W}_0 , \mathcal{W}_1 and \mathcal{W}_2 are partitioned into $2^{n\alpha_0 C_{12}}$, $2^{n\alpha_1 C_{12}}$ and $2^{n\alpha_2 C_{12}}$ subsets, respectively, for given $0 \leq \alpha_i \leq 1$ and $\sum_{i=0}^2 \alpha_i = 1$. This is done by assigning each codeword in the message sets \mathcal{W}_0 , \mathcal{W}_1 and \mathcal{W}_2 independently and randomly to the index sets $\{1, 2, \dots, 2^{n\alpha_0 C_{12}}\}$, $\{1, 2, \dots, 2^{n\alpha_1 C_{12}}\}$ and $\{1, 2, \dots, 2^{n\alpha_2 C_{12}}\}$, respectively.

Encoding at transmitter i is performed by sending codeword $x_i^n(w_0, w_i)$ corresponding to the common message $w_0 \in \mathcal{W}_0$ and local message $w_i \in \mathcal{W}_i$ ($i = 1, 2$). Encoding at decoder 1 takes place after detection of the two messages W_0 , W_1 and W_2 (see description of decoding below). In particular, decoder 1 sends over the conferencing link 1-2 the indices of the subsets in which the estimated messages W_0 , W_1 and W_2 lie. Notice that this requires nC_{12} bits and $K = 1$ conferencing rounds (i.e., $|\mathcal{V}_{1,1}| = nC_{12}$). Also we emphasize again that the conferencing link 2-1 is not used ($|\mathcal{V}_{2,k}| = 0$).

Decoding at the first decoder is carried out by finding jointly typical sequences $(u^n(w_0), x_1^n(w_0, w_1), x_2^n(w_0, w_2), y_1^n)$ with $w_i \in \mathcal{W}_i$ [11]. As discussed above, once the first decoder has obtained the messages W_0 , W_1 and W_2 , it sends the corresponding subset indices to receiver 2 over the conferencing channels. Decoding at receiver 2 then takes place again based on a standard MAC joint-typicality encoder with the caveat that the messages W_0 , W_1 and W_2 are now known to belong to the reduced set given by the subsets mentioned above.

The *analysis of the probability of error* follows immediately from [1] [13]. In particular, as far

as receiver 1 is concerned, it can be seen from [1] [13] that a sufficient condition for the probability of error to approach zero as $n \rightarrow \infty$ is given by $(R_0, R_1, R_2) \in \mathcal{R}_{MAC,1}(p(u), p(x_1|u), p(x_2|u))$. Considering receiver 2, a sufficient condition is that the rates belong to the region

$$\{(R_0, R_1, R_2): R_0 \geq 0, R_1 \geq 0, R_2 \geq 0, \quad (41a)$$

$$R_1 \leq I(X_1; Y_2|X_2U) + \alpha_1 C_{12} \quad (41b)$$

$$R_2 \leq I(X_2; Y_2|X_1U) + \alpha_2 C_{12} \quad (41c)$$

$$R_1 + R_2 \leq I(X_1X_2; Y_2|U) + (\alpha_1 + \alpha_2)C_{12} \quad (41d)$$

$$R_0 + R_1 + R_2 \leq I(X_1X_2; Y_2) + C_{12}\}, \quad (41e)$$

for the given α_i . Taking the union over all allowed α_i in (41) concludes the proof.

APPENDIX III

PROOF OF PROPOSITION 6.4

We first prove that $R_{1,OR} \geq R_{1,out} - \frac{1}{2}$. We consider three separate cases and show that the statement of the theorem holds for each case separately. We define $P_a \triangleq \gamma_{11}^2 P_1$, $P_b \triangleq \gamma_{12}^2 P_1$, $\check{C}_{12} \triangleq 2^{2C_{12}} - 1$ and $\check{C}_{21} \triangleq 2^{2C_{21}} - 1$ for simplicity of notation. We remark that using this notation the compression noises (24) can be written for the case at hand as $\sigma_1^2 = \frac{1+P_a+P_b}{(1+P_b)\check{C}_{12}}$ and $\sigma_2^2 = \frac{1+P_a+P_b}{(1+P_a)\check{C}_{21}}$.

Case 1: Let

$$\check{C}_{21} \geq \frac{P_b}{1 + P_a} \quad (42)$$

and

$$\check{C}_{12} \geq \frac{P_a}{1 + P_b}. \quad (43)$$

In this case, the upper bound (28) is $R_{1,out} = \frac{1}{2} \log(1 + P_a + P_b)$ and for the achievable rate with one-round conferencing (29) we have

$$\begin{aligned} \mathcal{C} \left(P_a + \frac{P_b}{1 + \sigma_2^2} \right) &= \frac{1}{2} \log \left(1 + P_a + \frac{P_b}{1 + \frac{1+P_a+P_b}{P_{21}(1+P_a)}} \right) \\ &\geq \frac{1}{2} \log \left(1 + P_a + \frac{P_b^2}{1 + P_a + 2P_b} \right) \end{aligned} \quad (44)$$

$$\begin{aligned}
&= \frac{1}{2} \log \left(\frac{(1 + P_a + P_b)^2}{1 + P_a + 2P_b} \right) \\
&= \frac{1}{2} \log(1 + P_a + P_b) + \frac{1}{2} \log \left(\frac{1 + P_a + P_b}{1 + P_a + 2P_b} \right) \\
&\geq R_{out} - \frac{1}{2}.
\end{aligned}$$

where (44) follows from (42). Similarly, using (43), we can also show that $\mathcal{C} \left(P_b + \frac{P_a}{1 + \sigma_1^2} \right) \geq R_{out} - \frac{1}{2}$. It then follows that $R_{1,out} \geq R_{out} - \frac{1}{2}$.

Case 2: Now, let

$$\check{C}_{21} \leq \frac{P_b}{1 + P_a} \quad (45)$$

and

$$(1 + P_a)(1 + \check{C}_{21}) \leq (1 + P_b)(1 + \check{C}_{12}). \quad (46)$$

In this case, we have $R_{1,out} = \frac{1}{2} \log(1 + P_a)(1 + \check{C}_{21})$ and

$$\begin{aligned}
\mathcal{C} \left(P_a + \frac{P_b}{1 + \sigma_2^2} \right) &= \frac{1}{2} \log \left(1 + P_a + \frac{P_b}{1 + \frac{1 + P_a + P_b}{P_{21}(1 + P_a)}} \right) \\
&= \frac{1}{2} \log(1 + P_a) + \frac{1}{2} \log \left(1 + \frac{P_b \check{C}_{21}}{(1 + P_a) \check{C}_{21} + (1 + P_a + P_b)} \right) \\
&= \frac{1}{2} \log(1 + P_a) + \frac{1}{2} \log \left(\frac{(1 + P_a + P_b)(1 + \check{C}_{21})}{(1 + P_a) \check{C}_{21} + (1 + P_a + P_b)} \right) \\
&= \frac{1}{2} \log(1 + P_a) (1 + \check{C}_{21}) + \frac{1}{2} \log \left(\frac{1 + P_a + P_b}{(1 + P_a) \check{C}_{21} + (1 + P_a + P_b)} \right) \\
&\geq R_{out} + \frac{1}{2} \log \left(\frac{1 + P_a + P_b}{1 + P_a + 2P_b} \right) \quad (47) \\
&\geq R_{out} - \frac{1}{2}, \quad (48)
\end{aligned}$$

where (47) follows from (45). On the other hand, we also have

$$\begin{aligned}
\mathcal{C} \left(P_b + \frac{P_a}{1 + \sigma_1^2} \right) &= \frac{1}{2} \log \left(1 + P_b + \frac{P_a}{1 + \frac{1 + P_a + P_b}{P_{12}(1 + P_b)}} \right) \\
&= \frac{1}{2} \log \left[(1 + P_b) \left(1 + \frac{P_a}{1 + P_b + \frac{1 + P_a + P_b}{\check{C}_{21}}} \right) \right]
\end{aligned}$$

$$\geq \frac{1}{2} \log \left[(1 + P_b) \left(1 + \frac{P_a[(1 + P_a)(1 + \check{C}_{21}) - (1 + P_b)]}{(1 + P_b)[(1 + P_a)(1 + \check{C}_{21}) + P_a]} \right) \right] \quad (49)$$

$$\begin{aligned} &= \frac{1}{2} \log \left[\frac{(1 + P_a)(1 + \check{C}_{21})(1 + P_a + P_b)}{(1 + P_a)(1 + \check{C}_{21}) + P_a} \right] \\ &= R_{out} + \frac{1}{2} \log \left[\frac{1 + P_a + P_b}{(1 + P_a)(1 + \check{P}_{21}) + P_a} \right] \\ &\geq R_{out} + \frac{1}{2} \log \left[\frac{1 + P_a + P_b}{1 + 2P_a + P_b} \right] \end{aligned} \quad (50)$$

$$\geq R_{out} - \frac{1}{2}, \quad (51)$$

where (49) follows from (46); and (50) follows from (45). From (48) and (51), we see that the theorem holds for Case 2 as well.

Case 3: Let

$$\check{C}_{12} \leq \frac{P_a}{1 + P_b} \quad (52)$$

and

$$(1 + P_b)(1 + \check{C}_{12}) \leq (1 + P_a)(1 + \check{C}_{21}). \quad (53)$$

In this case, $R_{out} = \frac{1}{2} \log(1 + P_b)(1 + \check{C}_{12})$. Case 3 follows similarly to Case 2.

Now, for the symmetric channel case, i.e., $\gamma_{11}^2 = \gamma_{12}^2$, that is, if $P_a = P_b \triangleq P$, we have to prove that $R_{OR} \geq R_{out} - 0.29$. This follows similarly to the treatment above as

$$\begin{aligned} R_{OR} &\geq R_{out} - \frac{1}{2} \log \left[\frac{1 + P_a + P_b}{1 + 2P_a + P_b} \right] \\ &= R_{out} - \frac{1}{2} \log \left[\frac{1 + 2P}{1 + 3P} \right] \\ &\geq R_{out} - \frac{1}{2} (\log 3 - 1). \end{aligned} \quad (54)$$

APPENDIX IV

PROOF OF PROPOSITION 6.5

To prove the theorem, we show that the bounds for R_1 , R_2 and $R_1 + R_2$ in $\mathcal{C}_{OR}^{\mathcal{G}}(C)$ are all within $\frac{\log(1+\beta)}{2}$ bits of the corresponding bounds in $\mathcal{C}_{CM-out}^{\mathcal{G}}(C)$. We define $\check{C} \triangleq 2^{2C} - 1$. Without loss of

generality, we assume $b \geq a$, and define $x \triangleq aP$. Then from the definition of β , we get $bP = \beta x$.

The outer bound and the achievable rates with one-round conferencing can now be written as

$$\begin{aligned} \mathcal{C}_{CM-out}^{\mathcal{G}}(C) &= \{(R_1, R_2) : R_1 \geq 0, R_2 \geq 0, \\ &R_1 \leq \min\{\mathcal{C}(x) + C, \mathcal{C}((1 + \beta)x)\}, \end{aligned} \quad (55a)$$

$$R_2 \leq \min\{\mathcal{C}(x) + C, \mathcal{C}((1 + \beta)x)\}, \quad (55b)$$

$$R_1 + R_2 \leq \min\{\mathcal{C}((1 + \beta)x) + C, \mathcal{C}(2(1 + \beta)x + (\beta - 1)^2 x^2)\}, \quad (55c)$$

and

$$\begin{aligned} \mathcal{R}_{OR}^{\mathcal{G}}(C) &= \{(R_1, R_2) : R_1 \geq 0, R_2 \geq 0, \\ &R_1 \leq \mathcal{C}\left(\left(1 + \frac{\beta}{1 + \sigma^2}\right)x\right), \end{aligned} \quad (56a)$$

$$R_2 \leq \mathcal{C}\left(\left(1 + \frac{\beta}{1 + \sigma^2}\right)x\right), \quad (56b)$$

$$R_1 + R_2 \leq \mathcal{C}\left(\left(1 + \beta + \frac{1 + \beta}{1 + \sigma^2} + \frac{(\beta - 1)^2 x}{1 + \sigma^2}\right)x\right) \quad (56c)$$

with $\sigma^2 \triangleq \frac{1+2(1+\beta)x+(\beta-1)^2x^2}{(1+(1+\beta)x)C}$, respectively.

We first define functions A and B as

$$A(x) \triangleq 1 + (1 + \beta)x$$

and

$$B(x) \triangleq 1 + 2(1 + \beta)x + (\beta - 1)^2 x^2. \quad (57)$$

Consider the bound on R_1 . We analyze two cases separately. If $\check{C} \geq \frac{\beta x}{1+x}$, then the outer bound

is equivalent to $R_1 \leq \frac{1}{2} \log A$. On the other hand, the bound on the achievable R_1 is found as

$$\begin{aligned}
\frac{1}{2} \log \left(1 + x + \frac{\beta x}{1 + \frac{1+2(1+\beta)x+(\beta-1)^2x^2}{(1+(1+\beta)x)\check{C}}} \right) &= \frac{1}{2} \log \left(1 + x + \frac{\beta x}{1 + \frac{B}{A\check{C}}} \right) \\
&\geq \frac{1}{2} \log \left(1 + x + \frac{\beta^2 Ax^2}{\beta Ax + (1+x)B} \right) \\
&\geq \frac{1}{2} \log \left[A \frac{(\beta^2 + \beta)x^2 + \beta x + (1+x)^2 \left(1 + \frac{(\beta-1)^2}{\beta+1} x \right)}{\beta Ax + (1+x)B} \right] \\
&\geq \frac{1}{2} \log \frac{A}{1+\beta} \\
&= \frac{1}{2} \log A - \frac{1}{2} \log(1+\beta). \tag{58}
\end{aligned}$$

If $\check{C} \leq \frac{\beta x}{1+x}$, then the outer bound is equivalent to $R_1 \leq \frac{1}{2} \log((1+\check{C})(1+x))$. The achievable rate bound can be written as

$$\begin{aligned}
\frac{1}{2} \log \left(1 + x + \frac{\beta x}{1 + \frac{B}{A\check{C}}} \right) &= \frac{1}{2} \log \left(\frac{(1+x)(A\check{C} + B) + \beta A\check{C}x}{A\check{C} + B} \right) \\
&\geq \frac{1}{2} \log(1+x) \left(\frac{(1+x)B + A^2\check{C}}{\beta xA + (1+x)B} \right) \\
&\geq \frac{1}{2} \log(1+x) \frac{1+\check{C}}{1+\beta} \\
&= \frac{1}{2} \log(1+x)(1+\check{C}) - \frac{1}{2} \log(1+\beta). \tag{59}
\end{aligned}$$

Combining (58) and (59), we conclude that the difference between the achievable rate bound and the outer bound on R_1 is not more than $\frac{1}{2} \log(1+\beta)$ bits. The same result applies for the bounds on R_2 in the same way.

Next, we consider the bounds on the sum-rate. If $\check{C} \geq \frac{B-A}{A}$, then the outer bound on the sum-rate

is equivalent to $R_1 + R_2 \leq \frac{1}{2} \log B$. On the other hand, the bound on achievable sum-rate is

$$\begin{aligned}
& \frac{1}{2} \log \left[1 + (1 + \beta)x \left(1 + \frac{1}{1 + \sigma^2} \right) + \frac{(\beta - 1)^2 x^2}{1 + \sigma^2} \right] \\
& \geq \frac{1}{2} \log \left[1 + (1 + \beta)x \left(1 + \frac{1}{2 + \frac{A}{B-A}} \right) + \frac{(\beta - 1)^2 x^2}{2 + \frac{A}{B-A}} \right] \\
& = \frac{1}{2} \log \left(A + \frac{B - A}{2 + \frac{A}{B-A}} \right) \\
& = \frac{1}{2} \log \left(\frac{B^2}{2B - A} \right) \\
& \geq \frac{1}{2} \log B - \frac{1}{2}.
\end{aligned} \tag{60}$$

If $\check{C} \leq \frac{B-A}{A}$, then the sum-rate outer bound is equivalent to $R_1 + R_2 \leq \frac{1}{2} \log(1 + \check{C})A$. The achievable sum-rate bound is

$$\begin{aligned}
& \frac{1}{2} \log \left[1 + (1 + \beta)x \left(1 + \frac{1}{1 + \sigma^2} \right) + \frac{(\beta - 1)^2 x^2}{1 + \sigma^2} \right] \\
& \geq \frac{1}{2} \log \left[1 + (1 + \beta)x \left(\frac{2\check{C}A + B}{\check{C}A + B} \right) + \frac{(\beta - 1)^2 x^2 \check{C}A}{\check{C}A + B} \right] \\
& = \frac{1}{2} \log \left[A + \frac{A\check{C}(B - A)}{A\check{C} + B} \right] \\
& = \frac{1}{2} \log \left[\frac{AB(1 + \check{C})}{A\check{C} + B} \right] \\
& \geq \frac{1}{2} \log \left[A(1 + \check{C}) \frac{B}{2B - A} \right] \\
& \geq \frac{1}{2} \log A(1 + \check{C}) - \frac{1}{2}.
\end{aligned} \tag{61}$$

From (60) and (61), we see that the difference between the achievable sum-rate bound and the sum-rate outer bound is always within half a bit. The claim for the case $a = b$ can be similarly proved. This concludes the proof.

APPENDIX V

PROOF OF PROPOSITION 7.1

In order for rates (R_1, R_2) to be achievable, by Fano's inequality, we have for the first receiver (see also proof of Proposition 3.1):

$$H(W_1, W_2 | Y_1^n, V_2^K) \leq n\delta_n \quad (62)$$

with $\delta_n \rightarrow 0$ as $n \rightarrow \infty$. From the previous inequality, it also follows that ($i = 1, 2$)

$$H(W_1, W_2 | Y_1^n, \bar{V}_1^{\bar{K}}, \bar{V}_2^{\bar{K}}, V_2^K) \leq n\delta_n, \quad (63a)$$

$$H(W_1 | Y_1^n, \bar{V}_1^{\bar{K}}, \bar{V}_2^{\bar{K}}, V_2^K, W_2) \leq n\delta_n \text{ and} \quad (63b)$$

$$H(W_2 | Y_1^n, \bar{V}_1^{\bar{K}}, \bar{V}_2^{\bar{K}}, V_2^K, W_1) \leq n\delta_n, \quad (63c)$$

where $\bar{V}_1^{\bar{K}}, \bar{V}_2^{\bar{K}}$ represent the signals exchanged during the \bar{K} encoder-side conferencing rounds. Now, we can treat (62)-(63) similarly to the proof of Proposition 3.1 and using the approach in [1]. For instance, from (63a), we have

$$\begin{aligned} n(R_1 + R_2) &\leq I(W_1, W_2; Y_1^n, \bar{V}_1^{\bar{K}}, \bar{V}_2^{\bar{K}}, V_2^K) + n\delta_n \\ &\leq I(W_1, W_2; Y_1^n | \bar{V}_1^{\bar{K}}, \bar{V}_2^{\bar{K}}) + I(W_1, W_2; \bar{V}_1^{\bar{K}}, \bar{V}_2^{\bar{K}}) \\ &\quad + I(W_1, W_2; V_2^K | Y_1^n, \bar{V}_1^{\bar{K}}, \bar{V}_2^{\bar{K}}) + n\delta_n \\ &\stackrel{(a)}{\leq} I(W_1, W_2; Y_1^n | \bar{V}_1^{\bar{K}}, \bar{V}_2^{\bar{K}}) + n(\bar{C}_{12} + \bar{C}_{21}) + nC_{21} + n\delta_n \\ &\stackrel{(b)}{\leq} \sum_{i=1}^n I(X_{1,i}, X_{2,i}; Y_{1,i} | \bar{V}_1^{\bar{K}}, \bar{V}_2^{\bar{K}}) + n(\bar{C}_{12} + \bar{C}_{21}) + nC_{21} + n\delta_n, \end{aligned}$$

where (a) follows from the fact that $I(W_1, W_2; \bar{V}_1^{\bar{K}}, \bar{V}_2^{\bar{K}}) \leq H(\bar{V}_1^{\bar{K}}, \bar{V}_2^{\bar{K}}) \leq n(\bar{C}_{12} + \bar{C}_{21})$ and $I(W_1, W_2; V_2^K | Y_1^n, \bar{V}_1^{\bar{K}}, \bar{V}_2^{\bar{K}}) \leq H(V_2^K) \leq nC_{21}$, and (b) is obtained similarly to [1], Sec. 3.4. From (62) and the remaining inequalities in (63), using similar arguments as in the above chain of

inequalities, one obtains, respectively,

$$\begin{aligned}
n(R_1 + R_2) &\leq \sum_{i=1}^n I(X_{1,i}, X_{2,i}; Y_{1,i}) + nC_{21} + n\delta_n, \\
nR_1 &\leq \sum_{i=1}^n I(X_{1,i}; Y_{1,i} | X_{2,i}, \bar{V}_1^{\bar{K}}, \bar{V}_2^{\bar{K}}) + n\bar{C}_{12} + nC_{21} + n\delta_n \text{ and} \\
nR_2 &\leq \sum_{i=1}^n I(X_{2,i}; Y_{1,i} | X_{1,i}, \bar{V}_1^{\bar{K}}, \bar{V}_2^{\bar{K}}) + n\bar{C}_{21} + nC_{21} + n\delta_n.
\end{aligned}$$

Now defining $U_i = (\bar{V}_1^{\bar{K}}, \bar{V}_2^{\bar{K}})$, the proof is completed similarly to Proposition 3.1, and by repeating the same arguments for receiver 2.

REFERENCES

- [1] F. M. J. Willems, *Informationtheoretical Results for the Discrete Memoryless Multiple Access Channel*, Ph.D. thesis, Katholieke Universiteit Leuven, Leuven, Belgium, 1982.
- [2] S. I. Bross, A. Lapidoth and M. A. Wigger, "The Gaussian MAC with conferencing encoders," in *Proc. IEEE International Symposium Inform. Theory (ISIT 2008)*, Toronto, ON, Canada, July 6-11, 2008.
- [3] I. Maric, R. Yates, and G. Kramer, "Capacity of interference channels with partial transmitter cooperation," *IEEE Trans. Inform. Theory*, vol. 53, no. 10, pp. 3536-3548, Oct. 2007.
- [4] C. T. K. Ng, I. Maric, A. J. Goldsmith, S. Shamai, and R. D. Yates, "Iterative and one-shot conferencing in relay channels," in *Proc. IEEE Information Theory Workshop (ITW 2006)*, Punta del Este, Uruguay, Mar. 13-17, 2006.
- [5] R. Dabora and S. Servetto, "Broadcast channels with cooperating decoders," *IEEE Trans. Inform. Theory*, vol. 52, no.12, pp. 5438-5454, Dec. 2006.
- [6] R. Dabora and S. Servetto, "A multi-step conference for cooperative broadcast," in *Proc. IEEE International Symposium Information Theory (ISIT 2006)*, pp. 2190-2194, Seattle, WA, July 9-14, 2006.
- [7] S. C. Draper, B. J. Frey and F. R. Kschischang, "Interactive decoding of a broadcast message," *Proc. Forty-First Annual Allerton Conference on Communication, Control, and Computing*, Monticello, IL, 2003.
- [8] S. C. Draper, B. J. Frey and F. R. Kschischang, "On interacting encoders and decoders in multiuser settings," in *Proc. IEEE International Symposium Information Theory (ISIT 2004)*, Chicago, IL, June 27 - July 2, 2004.
- [9] S. Lasaulce and A. G. Klein, "Gaussian broadcast channels with cooperating receivers: the single common message case," in *Proc. IEEE International Conference Acoustics, Speech and Signal Processing (ICASSP 2006)*, Toulouse, France, May 2006.
- [10] V. Venkatesan, "Cooperation required between destinations in a two-source two-destination network to achieve full multiplexing gain," Semester thesis, ETH Zurich, Switzerland, Winter 2006.
- [11] T. Cover and J. Thomas, *Elements of Information Theory*, John Wiley & Sons, Inc., New York, 2006.
- [12] A. B. Carleial, "Interference channels," *IEEE Trans. Inform. Theory*, vol. 24, no. 1, pp. 60-70, Jan. 1978.

- [13] D. Slepian and J.K. Wolf, "A coding theorem for multiple access channels with correlated sources," *Bell Systems Tech. J.*, vol. 52, pp. 1037-1076, Sept. 1973.
- [14] Y. Liang and G. Kramer, "Rate regions for relay broadcast channels," *IEEE Trans. Inform. Theory*, vol. 53, no. 10, pp. 3517-3535, Oct. 2007.
- [15] O. Simeone, O. Somekh, G. Kramer, H. V. Poor and S. Shamai (Shitz), "Three-user Gaussian multiple access channel with partially cooperating encoders," to appear in *Proc. Asilomar Conference on Signals, Systems and Computers*, Monterey, CA, 2008.
- [16] A. Steiner, A. Sanderovich and S. Shamai (Shitz), "The multi-session multi-layer broadcast approach for two cooperating receivers," in *Proc. IEEE International Symposium Inform. Theory (ISIT 2008)*, Toronto, ON, Canada, July 6-11, 2008.
- [17] R. Etkin, D. Tse and Hua Wang, "Gaussian interference channel capacity to within one bit," submitted [arXiv:cs/0702045v1].
- [18] G. Bresler and D. Tse, "The two-user Gaussian interference channel: a deterministic view," *European Trans. Telecommunications*, vol. 19, no. 4, pp. 333-354, 2008.