

# Correlated Gaussian Sources over Gaussian Weak Interference Channels

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**Abstract**—We consider the transmission of two correlated Gaussian sources over a Gaussian weak interference channel. Each transmitter has access to one component of a bivariate Gaussian source, and each of these components need to be reconstructed at the corresponding receiver, under the squared-error distortion measure. We are interested in characterizing the region of average distortion pairs achievable simultaneously at the two receivers. We derive an outer bound on the achievable region, and show that, under certain conditions, achievable distortion pair with uncoded transmission lies on this outer bound; therefore, it is optimal. In particular, optimality of uncoded transmission is shown to hold for all signal-to-noise ratio values below a certain threshold which depends on the correlation coefficient between the sources.

## I. INTRODUCTION

Emerging Internet of Everything (IoE) applications require simultaneous wireless transmission of sensor measurements for distributed monitoring and control. In most cases, these measurements come from an underlying continuous-amplitude physical signal, such as vibration, temperature, various health indicators, etc., and due to the increasing density of IoE devices, measurements at nearby sensors exhibit significant correlations. In order to model such a scenario, we consider lossy transmission of correlated Gaussian signals over a Gaussian interference channel.

This is a multi-user joint source-channel coding (JSCC) problem, and like most such problems, the optimal transmission strategy remains open. As opposed to the point-to-point setting, in multi-terminal JSCC problems, the optimality of separate source and channel coding breaks down. In recent years, there have been significant efforts towards understanding multi-user JSCC problems. In [1] the optimality of uncoded transmission is shown for transmission of correlated Gaussian sources over a Gaussian multiple access channel (MAC) below a signal-to-noise ratio (SNR) threshold. Uncoded transmission is the simplest JSCC scheme, in which the source samples are simply scaled and used as the channel input. In the case of distributed transmitters, this allows generating correlated channel inputs; hence, exploiting beamforming gains without any coordination among the transmitters (apart from synchronization). The optimality of uncoded transmission is also significant, as it achieves the optimal performance in a zero-delay fashion. A similar result is also proven for transmission of correlated Gaussian sources over a Gaussian broadcast channel (BC) in [2]. The optimal strategy for this latter scenario is characterised for all SNR values in [3]. Various other results involving MAC and BC settings can also be found in [4]–[9].

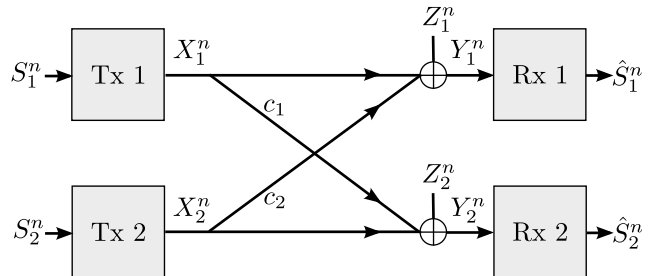


Fig. 1. Correlated Gaussian sources transmitted over a Gaussian weak interference channel, i.e.,  $c_i^2 \leq 1$ ,  $i = 1, 2$ .

In this paper, we show that the optimality of uncoded transmission also holds for Gaussian interference channels (GICs) (see Fig. 1) in the weak interference regime, in which  $c_1^2, c_2^2 \leq 1$ , up to a certain SNR threshold. This extends the results in [1], [2] to the GIC model, and, to the best of our knowledge, is the first such optimality result for GICs, apart from some trivial and asymptotic regimes studied in [10] and [11]. It is worth noting that, as opposed to the MAC and BC scenarios, the capacity region for the weak GIC is not known, apart from the sum-rate optimality of treating interference as noise in the noisy interference regime [12].

## II. SYSTEM MODEL

Consider a length- $n$  sequence of independent identically distributed (i.i.d.) zero mean bivariate Gaussian source  $\{S_{1k}, S_{2k}\}_{k=1}^n$  with a covariance matrix

$$\mathbf{K}_{S_1, S_2} = \begin{pmatrix} \sigma^2 & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 \end{pmatrix}. \quad (1)$$

Here, assuming equal variances for the sources and imposing  $\rho \in [0, 1]$  is without loss of optimality. Transmitter  $i$  observes the  $i$ -th source sequence, and encodes it with function  $f_i^n : \mathbb{R}^n \rightarrow \mathbb{R}^n$  such that  $X_i^n = f_i^n(S_i^n)$  for  $i = 1, 2$ . The corresponding channel input vectors  $X_i^n$  are subject to individual average power constraints

$$\mathbb{E}[|X_i^n|^2] = \frac{1}{n} \sum_{k=1}^n \mathbb{E}[|X_{ik}|^2] \leq P_i, \quad i = 1, 2. \quad (2)$$

The additive memoryless GIC is characterized by

$$Y_{1k} = X_{1k} + c_2 X_{2k} + Z_{1k}, \quad (3)$$

$$Y_{2k} = c_1 X_{1k} + X_{2k} + Z_{2k}, \quad (4)$$

where  $c_i > 0$ ,  $i = 1, 2$ , are the interference coefficients, and  $Z_{ik}$  is the i.i.d. zero-mean Gaussian noise term at the  $i$ -th terminal with variance  $N$ , i.e.,  $Z_{ik} \sim \mathcal{N}(0, N)$ . The decoding

function at the  $i$ -th receiver,  $\phi_i^n : \mathbb{R}^n \rightarrow \mathbb{R}^n$  for  $i = 1, 2$ , reconstructs an estimate of the sequence of interest  $S_i^n$ , i.e.,  $\hat{S}_i^n = \phi_i^n(Y_i^n)$ . The distortion is measured in terms of the mean square-error defined as

$$\delta_i^{(n)} \triangleq \frac{1}{n} \sum_{k=1}^n \mathbb{E}[(S_{ik} - \hat{S}_{ik})^2], \quad i = 1, 2. \quad (5)$$

Given  $\Omega \triangleq (\sigma^2, \rho, c_1, c_2, P_1, P_2, N)$ , we say that an average distortion pair  $(D_1, D_2)$  is *achievable* if there exists a sequence  $\{f_1^n, f_2^n, \phi_1^n, \phi_2^n\}$  satisfying the power constraints in (2), and achieve a mean square-error distortion of

$$\lim_{n \rightarrow \infty} \delta_i^{(n)} \leq D_i, \quad i = 1, 2. \quad (6)$$

We define the *achievable distortion region*,  $\mathcal{D}_{\text{IC}}(\Omega)$ , as the set of all achievable distortion pairs.

In this paper we are interested in the *weak interference regime*, that is, we assume  $c_i^2 \leq 1$ ,  $i = 1, 2$ . For convenience, we define  $i^c = \{1, 2\} \setminus i$ ,  $\log^+(x) \triangleq \max\{0, \log(x)\}$  and

$$\bar{P}_i \triangleq P_i + c_{i^c}^2 P_{i^c} + 2c_{i^c} \rho \sqrt{P_1 P_2}, \quad \text{for } i = 1, 2. \quad (7)$$

#### A. Main Result: Optimality of Uncoded Transmission

The main result of the paper is the optimality of uncoded transmission in the weak interference regime under certain conditions. In uncoded transmission, each transmitter sends a scaled version of its source directly over the channel, i.e.,  $X_i^n = \sqrt{\beta_i} P_i / \sigma^2 S_i^n$ , where  $\beta_i \in [0, 1]$  is the scaling factor. Then, a minimum mean square error (MMSE) estimator is used at each receiver to reconstruct the source of interest. In general, different distortion pairs can be achieved by employing only part of the available power at the transmitters to control the interference. If full power transmission is used, i.e.,  $\beta_i = 1$ , the achievable distortion pairs are given by

$$D_1^u = \sigma^2 \frac{c_2^2 P_2 (1 - \rho^2) + N}{\bar{P}_1 + N}, \quad D_2^u = \sigma^2 \frac{c_1^2 P_1 (1 - \rho^2) + N}{\bar{P}_2 + N} \quad (8)$$

The main result of the paper states that, under certain conditions on the available average transmit power values and the correlation coefficient  $\rho$ , the pair  $(D_1^u, D_2^u)$  lies on the boundary of the achievable distortion region  $\mathcal{D}_{\text{IC}}(\Omega)$ ; that is, uncoded transmission is optimal.

**Theorem 1.** *The pair  $(D_1^u, D_2^u)$  lies on the boundary of the achievable distortion region  $\mathcal{D}_{\text{IC}}(\Omega)$  if*

$$\min\{c_1, c_2\} \cdot \frac{\sqrt{P_1 P_2}}{N} \leq \frac{\rho}{1 - \rho^2}. \quad (9)$$

*Proof.* The proof is outlined in Section V.  $\square$

It follows from Theorem 1 that, in the case of a *symmetric GIC*, i.e.,  $P_1 = P_2 = P$  and  $c = c_1 = c_2$ , uncoded transmission achieves the optimal symmetric distortion of  $D_1 = D_2 = D^*$ .

**Corollary 1.** *In a symmetric GIC in the weak interference regime, i.e.,  $c = c_1 = c_2 \leq 1$ , uncoded transmission achieves the optimal symmetric distortion  $D^*$ , given by*

$$D^* = \sigma^2 \frac{c^2 P (1 - \rho^2) + N}{(1 + c^2 + 2c\rho)P + N}, \quad \text{if } c \cdot \frac{P}{N} \leq \frac{\rho}{1 - \rho^2}.$$

### III. BACKGROUND: BIVARIATE GAUSSIAN SOURCES OVER POINT TO POINT CHANNELS

In this section, we provide some results concerning the achievable distortion pairs when transmitting a bivariate Gaussian source sequence  $(S_1^n, S_2^n)$  over a point-to-point Gaussian channel. We assume that an average power constraint of  $P$  applies at the encoder, and the decoder reconstructs the sources with average distortion pair  $(D_1, D_2)$ . We denote the set of achievable distortion pairs by  $\mathcal{D}_{\text{PtP}}(\sigma^2, \rho, P, N)$ .

The set of achievable distortion pairs is characterized in [1], and shown to be given by the pairs  $(\hat{D}_1, \hat{D}_2)$  satisfying,

$$R_{S_1, S_2}(\hat{D}_1, \hat{D}_2) = \frac{1}{2} \log_2 \left( \frac{P + N}{N} \right), \quad (10)$$

where  $R_{S_1, S_2}(\hat{D}_1, \hat{D}_2)$  is the rate distortion function of a bivariate Gaussian source presented in [13].

Under certain conditions, stated in Proposition 1 below, some of the optimal pairs can be achieved by using a symbol-by-symbol linear transmission of the source samples as the channel input  $X_k$ , given by

$$X_k = \sqrt{\frac{P}{\sigma^2(\alpha^2 + 2\alpha\beta\rho + \beta^2)}} (\alpha S_{1,k} + \beta S_{2,k}), \quad (11)$$

for some  $\alpha, \beta \geq 0$ . The receiver employs MMSE estimation to achieve the optimal distortion pairs. The distortion pairs achieved by the linear encoding scheme are given as follows.

$$\begin{aligned} \hat{D}_1^u(\alpha, \beta, P) &= \sigma^2 \left[ \frac{P\beta^2(1 - \rho^2)}{(P + N)(\alpha^2 + 2\alpha\beta\rho + \beta^2)} + \frac{N}{P + N} \right], \\ \hat{D}_2^u(\alpha, \beta, P) &= \sigma^2 \left[ \frac{P\alpha^2(1 - \rho^2)}{(P + N)(\alpha^2 + 2\alpha\beta\rho + \beta^2)} + \frac{N}{P + N} \right]. \end{aligned}$$

The following proposition, proven in [1], states a sufficient condition for the optimality of linear transmission in the point-to-point setting.

**Proposition 1** ([1], Proposition III.1). *For any  $(\hat{D}_1, \hat{D}_2) \in \mathcal{D}_{\text{PtP}}(\sigma^2, \rho, P, N)$ , if*

$$\frac{P}{N} \leq \Gamma(\hat{D}_1, \sigma^2, \rho), \quad (12)$$

where the threshold  $\Gamma(\hat{D}, \sigma^2, \rho)$  is defined as

$$\begin{aligned} \Gamma(\hat{D}, \sigma^2, \rho) & \\ \triangleq & \begin{cases} \frac{\sigma^4(1 - \rho^2) - 2\hat{D}\sigma^2(1 - \rho^2) + \hat{D}^2}{\hat{D}(\sigma^2(1 - \rho^2) - \hat{D})}, & \text{if } 0 < \hat{D} < \sigma^2(1 - \rho^2), \\ +\infty, & \text{otherwise,} \end{cases} \end{aligned} \quad (13)$$

then, there exist  $\alpha^*, \beta^* \geq 0$  such that

$$\hat{D}_1^u(\alpha^*, \beta^*, P) \leq \hat{D}_1, \quad \text{and} \quad \hat{D}_2^u(\alpha^*, \beta^*, P) \leq \hat{D}_2.$$

### IV. TWO OUTER BOUNDS FOR THE GAUSSIAN IC

In this section we derive two outer bounds on the region of achievable distortion pairs  $(D_1, D_2)$  for the Gaussian IC. First, we note from the cut-set bound in [11, Lemma 1] that if  $(D_1, D_2) \in \mathcal{D}_{\text{IC}}(\Omega)$ , then for  $i = 1, 2$ ,

$$D_i \geq D_{i, \min} \triangleq \sigma^2 \frac{N}{\bar{P}_i + N}. \quad (14)$$

Here we present another outer bound on  $\mathcal{D}_{\text{IC}}(\Omega)$ . For this, we define the region  $\bar{\mathcal{D}}_{\text{IC}}(\Omega)$  as the set of all  $(D_1, D_2)$  pairs which satisfy

$$D_{i^c} \geq \Psi_i(D_i), \quad \text{for } i = 1, 2, \quad (15)$$

where

$$\Psi_i(\delta) \triangleq \sigma^2 \frac{N}{\bar{P}_{i^c} + N} \left( c_i^2 \frac{\sigma^2(1 - \rho^2)}{\delta} + 1 - c_i^2 \right).$$

**Lemma 1.** *If  $c_1^2, c_2^2 \leq 1$ , then  $\mathcal{D}_{\text{IC}}(\Omega) \subseteq \bar{\mathcal{D}}_{\text{IC}}(\Omega)$ .*

*Proof.* See Appendix A.  $\square$

A tighter outer bound can be constructed if certain conditions on the parameters  $\Omega$  are satisfied. We define the regions  $\bar{\mathcal{D}}_{\text{IC}}^i(\Omega)$ ,  $i = 1, 2$ , as the set of all  $(D_1, D_2)$  pairs which satisfy for some  $\alpha^*, \beta^* \geq 0$ ,

$$D_i = \hat{D}_i^u(\alpha^*, \beta^*, \bar{P}_i), \quad D_{i^c} \geq \Psi_i(\eta_i(D_i, a_{1i}, a_{2i})), \quad (16)$$

and

$$\frac{\bar{P}_i}{N} \leq \Gamma(D_i, \sigma^2, \rho), \quad (17)$$

where

$$\eta_i(\delta, a_1, a_2) \triangleq \sigma^2 - a_1(\sigma^2 - \delta)(2 - a_1) - a_2\sigma^2(2\rho - a_2) + 2a_1a_2\sqrt{(\sigma^2 - \delta)(\sigma^2 - \tilde{D}_{i^c}^*(\delta))},$$

and the coefficients  $a_{1i}$  and  $a_{2i}$  are given as follows

$$a_{1i} \triangleq \frac{(\sigma^2 - D_i)\sigma^2 - \rho\sigma^2\sqrt{(\sigma^2 - D_i)(\sigma^2 - \tilde{D}_{i^c}^*(D_i))}}{(\sigma^2 - D_i)\tilde{D}_{i^c}^*(D_i)},$$

$$a_{2i} \triangleq \frac{\rho\sigma^2 - \sqrt{(\sigma^2 - D_i)(\sigma^2 - \tilde{D}_{i^c}^*(D_i))}}{\tilde{D}_{i^c}^*(D_i)}, \quad (18)$$

where  $\tilde{D}_{i^c}^*(D_i) = \hat{D}_{i^c}^u(\alpha^*, \beta^*, \bar{P}_i)$ .

**Lemma 2.** *If  $c_1^2, c_2^2 \leq 1$ , then  $\mathcal{D}_{\text{IC}}(\Omega) \subseteq \bar{\mathcal{D}}_{\text{IC}}^1(\Omega) \cap \bar{\mathcal{D}}_{\text{IC}}^2(\Omega)$ .*

*Proof.* See Appendix B.  $\square$

## V. PROOF OF THEOREM 1

The optimality of uncoded transmission under the condition in (9) follows by showing the pair  $(D_1^u, D_2^u)$  lies on the boundary of the outer bound derived in Lemma 2. In particular, we note that if

$$\alpha^* = \sqrt{\frac{P_1}{\sigma^2}}, \quad \beta^* = \sqrt{c_2^2 \frac{P_2}{\sigma^2}} \quad (19)$$

then  $\hat{D}_1^u(\alpha^*, \beta^*, \bar{P}_1) = D_1^u$ . If condition (17) holds, then

$$(\hat{D}_1^u(\alpha^*, \beta^*, \bar{P}_1), \hat{D}_2^u(\alpha^*, \beta^*, \bar{P}_1)) = (D_1^u, \tilde{D}_2^*) = (D_1, \tilde{D}_2^*),$$

from the definition of  $\mathcal{D}_{\text{IC}}^1(\Omega)$ . Condition (17) can be equivalently written, after some manipulation, as

$$c_2 \frac{\sqrt{P_1 P_2}}{N} \leq \frac{\rho}{1 - \rho^2}. \quad (20)$$

Finally, substituting  $D_1^u$  and  $\tilde{D}_2(D_1)$  into (16) we have

$$D_2 \geq \Psi_1(\eta_1(D_1^u, a_{11}, a_{21})) = D_2^u, \quad (21)$$

which implies that uncoded transmission lies on the boundary of  $\mathcal{D}_{\text{IC}}^1(\Omega)$ , and therefore of  $\mathcal{D}_{\text{IC}}(\Omega)$ . Following the same reasoning when  $D_2 = D_2^u$ , we obtain  $D_1 \geq \Psi_2(\eta_2(D_2^u, a_{21}, a_{22})) = D_1^u$  if  $c_1\sqrt{P_1 P_2}/N \leq \rho/(1 - \rho^2)$ , that is,  $(D_1^u, D_2^u)$  lies on the boundary of  $\mathcal{D}_{\text{IC}}^2(\Omega)$ . Therefore uncoded transmission is optimal if condition (9) holds.

## VI. DISCUSSION AND CONCLUSION

It is worth noting that separate source and channel coding is optimal when the two sources are independent, i.e.,  $\rho = 0$  [14]. On the other hand, it is known that in most multi-user JSCC problems separate source and channel coding is suboptimal in general [15]. In the GIC model considered here, it is challenging even to characterise the achievable distortion pairs with separation, since we do not know all the achievable rate pairs in the weak interference regime.

The best achievable channel coding scheme for the interference channel is the well-known Han-Kobayashi scheme [16], which applies message splitting. In the case of correlated sources, the analysis of the Han-Kobayashi scheme is challenging as it requires the analysis of multi-terminal rate-distortion problems [17]. Instead, for the sake of comparison, we consider the scheme in which the receivers treat interference as noise (TIN). TIN is known to be sum-capacity achieving in the noisy interference regime [12], given by

$$c_2(c_1^2 P_1 + N) + c_1(c_2^2 P_2 + N) \leq N. \quad (22)$$

With a single message available at each destination, the optimal compression becomes simple point-to-point compression. Therefore, the achievable distortion pair by TIN is given by

$$D_i^{\text{TIN}} = \sigma^2 \left( 1 + \frac{P_i}{c_{i^c}^2 P_{i^c} + N} \right)^{-1}. \quad (23)$$

In Fig. 2, we plot the upper and lower bounds on the achievable symmetric distortion with respect to the correlation coefficient  $\rho$  in a symmetric weak GIC assuming  $P/N = 1.5$  and  $c = 0.4$ , which corresponds to the noisy interference regime, i.e., (22) is satisfied. We observe that, in general, the lower bound proposed in Lemma 1, which we denote by  $D_{epi}$ , is significantly tighter than the cut-set bound  $D_{cs}$ , given in (14). Observe that in the noisy interference regime, uncoded transmission and TIN achieve the same performance at  $\rho = 0$  while for  $\rho > 0$ , uncoded transmission outperforms TIN, and performs close to the  $D_{epi}$ , while TIN becomes highly suboptimal. For  $\rho$  larger than a certain threshold  $\rho_{th} = 0.4684$ , for which condition in (8) holds with equality, it follows from Lemma 9 that uncoded transmission achieves the minimum achievable distortion  $D^*$ . We can see that  $D_{epi}$  is not always tight in this regime, and the tighter bound derived in Lemma 2 is required to prove the optimality of uncoded transmission.

## APPENDIX A PROOF OF LEMMA 1

We define  $\Delta_i^{(n)}$  as the least distortion at which  $S_i^n$  can be reconstructed at receiver  $i$  when  $S_{i^c}^n$  is given as side

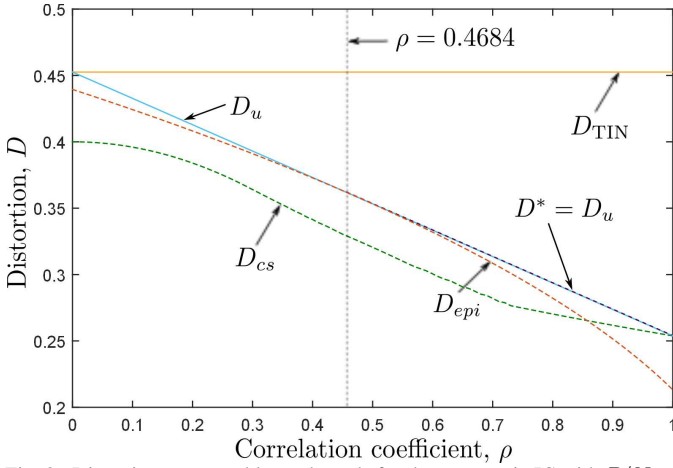


Fig. 2. Distortion upper and lower bounds for the symmetric IC with  $P/N = 1.5$  and  $c = 0.4$ .

information, and  $f_1^n(S_1^n)$  and  $f_2^n(S_2^n)$  are transmitted. Note that  $\Delta_i^{(n)} \leq D_i$ , and

$$I(S_i^n; Y_i^n | S_{i^c}^n) \geq nR_{S_i|S_{i^c}}(\Delta_i^{(n)}) \geq nR_{S_i|S_{i^c}}(D_i), \quad (24)$$

where  $R_{S_i|S_{i^c}}(\delta) \triangleq 1/2 \log^+(\sigma^2(1-\rho^2)/\delta)$  is the Wyner-Ziv rate-distortion function for  $S_i$  when  $S_{i^c}$  is available at the receiver. We prove the following bound,

$$D_{i^c} \geq \Psi_i(\Delta_i^{(n)}), \quad \text{for } i = 1, 2. \quad (25)$$

We prove (25) for  $i=1$ . The result for  $i=2$  follows similarly. We have

$$\begin{aligned} I(X_1^n; Y_2^n | X_2^n S_2^n) &= h(c_1 X_1^n + Z_2^n | X_2^n S_2^n) - h(Z_2^n) \\ &= h(X_1^n + 1/c_1 Z_2^n | X_2^n S_2^n) - h(1/c_1 Z_2^n) \\ &= h(X_1^n + \tilde{Z}_1^n + W^n | X_2^n S_2^n) - h(1/c_1 Z_2^n), \end{aligned} \quad (26)$$

where in (26) we define  $\tilde{Z}_1^n$  and  $W^n$  as i.i.d. sequences with Gaussian entries distributed as  $\tilde{Z}_{1k} \sim \mathcal{N}(0, N)$  and  $W_k \sim \mathcal{N}(0, N(1/c_1^2 - 1))$ , respectively, and independent of each other.

We have

$$\begin{aligned} &\exp\left(\frac{2}{n}h(X_1^n + \tilde{Z}_1^n + W^n | X_2^n S_2^n)\right) \\ &\geq \exp\left(\frac{2}{n}h(X_1^n + \tilde{Z}_1^n | X_2^n S_2^n)\right) + \exp\left(\frac{2}{n}h(W^n)\right) \\ &= \exp\left(\frac{2}{n}h(X_1^n + \tilde{Z}_1^n | X_2^n S_2^n)\right) + 2\pi e \left(\frac{1}{c_1^2} - 1\right) N \\ &\geq 2\pi e N \left[\exp\left(2R_{S_1|S_2}(\Delta_1^{(n)})\right) + 1/c_1^2 - 1\right], \end{aligned} \quad (27)$$

where (28) follows from the entropy power inequality, and (29) is due to the following inequality.

$$\begin{aligned} h(X_1^n + \tilde{Z}_1^n | X_2^n S_2^n) &= I(X_1^n; Y_1^n | X_2^n S_2^n) + h(Z_1^n) \\ &\geq I(S_1^n; Y_1^n | S_2^n) + h(Z_1^n) \\ &\geq nR_{S_1|S_2}(\Delta_1^{(n)}) + \frac{n}{2} \log(2\pi e N), \end{aligned} \quad (28)$$

where (30) follows from the data processing inequality, the Markov chain  $S_1^n - S_2^n - X_2^n$ , and since conditioning reduces

entropy. Then, by substituting (29) in (26) we obtain

$$I(X_1^n; Y_2^n | X_2^n S_2^n) \geq \frac{n}{2} \log\left(\frac{c_1^2 \sigma^2 (1 - \rho^2)}{\Delta_1^{(n)}} + 1 - c_1^2\right) \quad (31)$$

Now, we have the following:

$$\begin{aligned} I(X_1^n X_2^n; Y_2^n) &= I(S_2^n X_1^n X_2^n; Y_2^n) \\ &\geq I(S_2^n; Y_2^n) + I(X_1^n; Y_2^n | S_2^n X_2^n) \\ &\geq nR_{S_2}(D_2) + \frac{n}{2} \log\left(c_1^2 \sigma^2 (1 - \rho^2) / \Delta_1^{(n)} + 1 - c_1^2\right) \end{aligned} \quad (32)$$

where (32) follows due to the Markov chain  $(S_1^n S_2^n) - (X_1^n X_2^n) - Y_2^n$ , and (33) follows from (31) and standard rate-distortion arguments.

Finally, we upper bound the left hand side of (32) as follows,

$$I(X_1^n X_2^n; Y_2^n) \leq nI(X_1 X_2; Y_2) \leq \frac{n}{2} \log((\bar{P}_2 + N)/N)$$

where we have used the fact that correlated Gaussian inputs  $(X_1, X_2)$  maximize the mutual information, and that the correlation between the channel inputs is upper bounded by the correlation between the source sequences, similarly to [1, Lemma B.2].

Lemma 1 follows since we have  $D_{i^c} \geq \Psi_i(\Delta_i^{(n)})$ , and  $\Psi_i(\delta)$  is monotonically decreasing in  $\delta$ .

## APPENDIX B PROOF OF LEMMA 2

Let  $\delta_1^{(n)}, \delta_2^{(n)}$  satisfy (6) be an achievable distortion pair for some  $n$ , and an encoder decoder tuple  $\{f_1^n, f_2^n, \phi_1^n, \phi_2^n\}$ . We prove  $\mathcal{D}_{\text{IC}}(\Omega) \subseteq \bar{\mathcal{D}}_{\text{IC}}^i(\Omega)$ ,  $i = 1, 2$ , by deriving an upper bound on  $\Delta_i^{(n)}$  tighter than the upper bound  $\Delta_i^{(n)} \leq D_i$  used to derive Lemma 1 in Appendix A. We show the following upper bound

$$\Delta_i^{(n)} \leq \eta_i(\delta_i^{(n)}, a_{1i}, a_{2i}), \quad (34)$$

by relying on the average distortion with which  $S_{i^c}^n$  can be reconstructed at the  $i$ -th receiver from the received signal  $Y_i^n$ .

In this proof, we assume that  $(D_1, D_2)$  satisfy

$$D_{i,\min} \leq D_i \leq D_{i,\text{th}} \triangleq \frac{\sigma^2(1-\rho^2)\bar{P}_i + N}{\bar{P}_i + N}, \quad (35)$$

and consider the necessary conditions (14) if (35) does not hold. We will see later that this leads to a continuous outer bound. Here, we prove the lemma for  $i = 1$ , i.e.,  $\mathcal{D}_{\text{IC}}(\Omega) \subseteq \bar{\mathcal{D}}_{\text{IC}}^1(\Omega)$ , and  $\mathcal{D}_{\text{IC}}(\Omega) \subseteq \bar{\mathcal{D}}_{\text{IC}}^2(\Omega)$  follows similarly.

We need the following definition concerning the lowest distortion at which  $S_2^n$  can be reconstructed at receiver 1.

**Definition 1.** For every  $D_1 \geq D_{1,\min}$ , we define

$$\tilde{D}_2^*(D_1) \triangleq \inf \tilde{D}_2, \quad (36)$$

where the infimum is over all average-power limited encoders  $\{f_1^{(n)}, f_2^{(n)}\}$  and decoders  $\{\phi_1^{(n)}, \tilde{\phi}_2^{(n)}\}$  satisfying

$$\lim_{n \rightarrow \infty} \delta_1^{(n)} \leq D_1 \quad \text{and} \quad \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \mathbb{E}[(S_{2,k} - \tilde{S}_{2,k})^2] \leq \tilde{D}_2,$$

where  $\hat{S}_1^n = \phi_1^{(n)}(Y_1^n)$  and  $\tilde{S}_2^n = \tilde{\phi}_2(Y_1^n)$ .

It follows that  $\tilde{D}_2^*(D_1)$  is the unique solution to equation

$$R_{S_1, S_2}(D_1, \tilde{D}_2^*(D_1)) = \frac{1}{2} \log_2 \left( \frac{\bar{P}_1 + N}{N} \right). \quad (37)$$

The pairs  $(D_1, \tilde{D}_2^*(D_1))$  can be explicitly characterized using the results presented in Section III. Then, if

$$\frac{\bar{P}_1}{N} \leq \Gamma(D_1, \sigma^2, \rho), \quad (38)$$

the distortion pairs  $(D_1, \tilde{D}_2^*(D_1))$  are given by the pairs  $(\hat{D}_1, \hat{D}_2)$  in Proposition 1 we have, for some  $\alpha^*, \beta^* \geq 0$ ,

$$D_1 = \hat{D}_1^u(\alpha^*, \beta^*, \bar{P}_1) \quad \text{and} \quad \tilde{D}_2^*(D_1) = \hat{D}_2^u(\alpha^*, \beta^*, \bar{P}_1).$$

Next, we derive an upper bound on  $\Delta_1^{(n)}$  based on the pairs  $(D_1, \tilde{D}_2^*(D_1))$  by considering the linear estimator of  $S_1^{(n)}$  when receiver 1 has  $S_2^n$  as side-information, that is

$$\check{S}_{1,k} = a_{11}\hat{S}_{1,k} + a_{21}S_{2,k}, \quad k = 1, \dots, n, \quad (39)$$

where  $a_1, a_2 \geq 0$ .

We note that, without loss of generality, the set of decoders can be reduced to optimal MMSE estimators,  $\phi_i^{(n)}(Y_i^n) = \mathbb{E}[S_i^n | Y_i^n]$ ,  $i = 1, 2$ , since any achievable distortion is indeed achievable using the optimal decoding function. Then, we have the following direct version of [2, Lemma B.9].

**Lemma 3.** *If a scheme  $(f_1^{(n)}, f_2^{(n)}, \phi_1^{(n)}, \phi_2^{(n)})$ , satisfies the orthogonality condition, i.e.,  $\mathbb{E}[(S_{1,k} - \hat{S}_{1,k})\hat{S}_{1,k}] = 0$ ,  $k = 1, \dots, n$ , then,*

$$\frac{1}{n} \sum_{k=1}^n \mathbb{E}[\hat{S}_{1,k} S_{2,k}] \leq \sqrt{(\sigma^2 - \delta_1^{(n)}) (\sigma^2 - \tilde{D}_2^*(\delta_1^{(n)}))}.$$

Then, the upper bound on  $\Delta_1^{(n)}$  in (34) is derived as follows.

$$\begin{aligned} \Delta_1^{(n)} &\leq \frac{1}{n} \sum_{i=1}^n \mathbb{E}[(S_{1,k} - \check{S}_{1,k})^2] \\ &= \sigma^2 - 2a_{11}(\sigma^2 - \delta_1^{(n)}) - 2a_{21}\rho\sigma^2 + a_{11}^2(\sigma^2 - \delta_1^{(n)}) \\ &\quad + 2a_{11}a_{21} \frac{1}{n} \sum_{i=1}^n \mathbb{E}[\hat{S}_{1,k} S_{2,k}] + a_{21}^2\sigma^2 \end{aligned} \quad (40)$$

$$\begin{aligned} &\leq \sigma^2 - a_{11}(\sigma^2 - \delta_1^{(n)})(2 - a_{11}) - a_{21}\sigma^2(2\rho - a_{21}) \\ &\quad + 2a_{11}a_{21} \sqrt{(\sigma^2 - \delta_1^{(n)})(\sigma^2 - \tilde{D}_2^*(\delta_1^{(n)}))}, \end{aligned} \quad (41)$$

where (40) follows by substituting (39) and due to the orthogonality principle; and (41) follows due to Lemma 3 and because  $a_{11}, a_{21} \geq 0$ .

Similarly to the proof of Lemma (25) we have

$$\delta_2^{(n)} \geq \Psi_1(\Delta_1^{(n)}). \quad (42)$$

Therefore, since  $\Psi_1(\delta)$  is monotonically decreasing in  $\delta$ , from (41) we have

$$\delta_2^{(n)} \geq \Psi_1(\eta_1(\delta_1^{(n)}, a_{11}, a_{21})). \quad (43)$$

We choose  $a_{11}$  and  $a_{21}$  as in (18), such that satisfy

$\Psi_1(\eta_1(D_1, a_{11}, a_{21})) = D_2^u$  under the conditions in Theorem 1, and satisfy  $a_{11}, a_{21} \geq 0$  if (35) holds. Then, (16) follows similarly to [2, Proof of Lemma B.4] from the continuity of  $\Psi_1(\eta_i(\delta), a_1, a_2)$  in  $\delta$ , and since, if (35) holds, we can reduce to decoders satisfying

$$\lim_{n \rightarrow \infty} \delta_1^{(n)} = D_1 \quad \text{and} \quad \lim_{n \rightarrow \infty} \delta_2^{(n)} \leq D_2. \quad (44)$$

Finally, for  $\alpha = 0$ ,  $D_{2,\min} = \Psi_1(\eta_1(D_{1,\text{th}}, a_{11}, a_{21}))$ , and therefore, by considering  $D_2 \geq D_{2,\min}$  we have a continuous outer bound if (35) does not hold.

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