Wyner-Ziv Coding over Broadcast Channels

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***Abstract*—This paper deals with the design of coding schemes for lossy transmission of a source over a broadcast channel when there is correlated side information at the receivers. Using ideas from Slepian-Wolf coding over broadcast channels and dirty paper coding, new schemes are presented and their rate-distortion performance is derived. For the binary Hamming and quadratic Gaussian scenarios, when the source and the channel bandwidths are equal, it is shown that these schemes are sometimes optimal and that they can outperform both separate source and channel coding, and uncoded transmission.**

I. Introduction

Finding the capacity of a broadcast channel is a longstanding open problem in multiuser information theory. In a recent paper, the seemingly harder problem of characterizing the necessary and sufficient conditions for Slepian-Wolf (SW) coding over broadcast channels (BC) was completely solved [11]. SWBC involves lossless transmission of a source across a BC where each receiver has correlated side information.

In this paper, we consider a lossy extension of the SWBC problem, where the reconstruction of the source at the receivers need not be perfect. We shall refer to the problem setup as Wyner-Ziv (WZ) coding over BC. We present coding schemes for this problem and analyze the performance of our schemes in the binary Hamming and quadratic Gaussian cases. Unlike in [11], the schemes that derived here are not always optimal.

The new WZBC schemes that we present use ideas from SWBC [11] and dirty paper coding (DPC) [2], [4] as a starting point. The SWBC scheme is modified to a) allow quantization of the source, and b) additionally handle channel state information (CSI) at the encoder by using DPC. These modifications are then employed in layered transmission schemes, where there is common layer (CL) information destined for both receivers and refinement layer (RL) information meant for only one of the receivers. Since part of the information is decoded by both receivers, there are significant differences between optimal encoders and decoders in our DPC based layered scheme and those in DPC based schemes used in other contexts [1].

A simple alternative approach to the WZBC problem is to separate the source and channel coding. Both Gaussian and binary symmetric BC are degraded. Hence their capacity regions are known [3] and further, there is no loss of optimality in confining ourselves to two layer source coding schemes. Although the corresponding source-side information pairs are also degraded, only a partial characterization of the rate-distortion performance is available [9], [10]. By comparing the rate-distortion results with the capacity results, we obtain the distortion region achievable by separate source and channel coding.

For the two examples we consider, a second alternative if there is no bandwidth expansion or compression is uncoded transmission. This scheme is optimal in the absence of side information at the receivers in both the Gaussian and binary symmetric cases. However, in the presence of side information, the optimality breaks down.

The paper is organized as follows. In Section II, we formally define the problem and present relevant past work. Our main results are presented in Section III and Section IV: we develop extensions of the scheme in [11] and apply them to the binary Hamming and quadratic Gaussian cases. For these cases, we compare the derived schemes among themselves, with separate source and channel coding, and finally with uncoded transmission.

II. Problem Definition and Background

Let $(X, Y_1, Y_2) \in \mathcal{X} \times \mathcal{Y}_1 \times \mathcal{Y}_2$ be random variables (r.v.) denoting a source with independent and identically distributed (i.i.d.) realizations. The X sequence is to be transmitted over a memoryless broadcast channel defined by $p_{V_1V_2|U}(v_1, v_2|u), u \in \mathcal{U}, v_i \in \mathcal{V}_i, i = 1, 2$. Decoder i has access to side information Y_i in addition to the channel output V_i. A WZBC code (m, n, f, g_1, g_2) consists of

- an encoder – $f : \mathcal{X}^n \to \mathcal{U}^m$
- a decoder at each receiver i – $g_i : \mathcal{V}_i^m \times \mathcal{Y}_i^n \to \hat{\mathcal{X}}_i^n$

The rate of the code is $\kappa = \frac{m}{n}$ channel uses per source symbol. Single-letter distortion measures $d_i : \mathcal{X} \times \hat{\mathcal{X}}_i \to [0, \infty)$ are defined at each receiver. A distortion pair (D_1, D_2) is said to be achievable at rate κ (which we assume is rational) if for every $\epsilon > 0$, there exists n_0 such that for all integers $m > 0, n > n_0$ with $\frac{m}{n} = \kappa$, there exists a code (m, n, f, g_1, g_2) satisfying

$$\frac{1}{n}\mathsf{E}\left[\sum_{j=1}^{n} d_i(X_j, \hat{X}_{ij})\right] \leq D_i + \epsilon ,$$

where $\hat{X}_i^n = g_i(V_i^m, Y_i^n)$ and V_i^m denotes the channel output corresponding to $f(U_i^m)$.

In this paper, we present some general WZBC techniques and derive the corresponding achievable distortion regions. We study the performance of these techniques for the following cases.

- Binary Hamming: All source and channel alphabets are binary. The channels forming the BC are binary symmetric with transition probabilities p_1 and p_2. The side information sequences at the two receivers are noisy versions of the source corrupted by passage through virtual binary symmetric channels with transition probabilities β_1 and β_2. Reconstruction quality is measured by Hamming distance – $d_i(x, \hat{x}) = x \oplus \hat{x}$.
- Quadratic Gaussian: All r.v. are real-valued. The channels forming the BC are additive white Gaussian channels with noise variances $\sigma^2_{W_1}$ and $\sigma^2_{W_2}$. There is an input power constraint on the channel:

$$\frac{1}{m}\mathsf{E}\left[\sum_{j=1}^{m} U_j^2\right] \leq P \ ,$$

where $U^m = f(X^n)$. The source and side information are also jointly Gaussian. Without loss of generality, we assume that $\sigma^2_X = \sigma^2_{Y_1} = \sigma^2_{Y_2} = 1$ and $\mathsf{E}[XY_i] = \rho_i > 0$. $\sigma^2_{N_i} = 1 - \rho_i^2, i = 1, 2$ denotes the error in estimating X from Y_i. Reconstruction quality is measured by squared-error distance – $d_i(x, \hat{x}) = (x - \hat{x})^2$.

The problems considered in [6], [8], [11] can be seen as special cases of the WZBC problem. However the binary and Gaussian cases with non trivial side information have never, to our knowledge, been analyzed before. Nevertheless, separate source and channel coding and uncoded transmission are obvious strategies to compare our schemes with.

A. Separate Source and Channel Coding

In both the binary Hamming and the quadratic Gaussian cases, the channel and the side information are degraded: we can assume that one of the two Markov chains, $U - V_1 - V_2$ or $U - V_2 - V_1$, holds for the channel, and similarly either $X - Y_1 - Y_2$ or $X - Y_2 - Y_1$ holds for the source. The capacity region for degraded BC is known [3]. Further, since any information sent to the weaker channel can be decoded by the stronger channel, two layer source coding, which has been considered in [9], [10], is sufficiently general. For simplicity, we denote the r.v. associated with *g*ood channel by the subscript g and those associated with the *b*ad one by b, i.e., the channel variables satisfy the $U - V_g - V_b$. g is either 1 or 2 and b takes the other value. A distortion pair (D_b, D_g) is achievable by separate source and channel coding if there exist a channel input $U \in \mathcal{U}$ and an auxiliary r.v. $U_b \in \mathcal{U}_b$ satisfying $U_b - U - V_g - V_b$, source auxiliary r.v. $(Z_b, Z_g) \in \mathcal{Z}_b \times \mathcal{Z}_g$ satisfying either $(Y_b, Y_g) - X - Z_b - Z_g$ or $(Y_b, Y_g) - X - Z_g - Z_b$ and reconstruction functions $g_i : \mathcal{Z}_i \times \mathcal{Y}_i \to \hat{\mathcal{X}}, i = b, g$ such that $\mathsf{E}[d_i(X, g_i(Z_i, Y_i))] \leq D_i, i = b, g$ and

$$\kappa I(U_b; V_b) \geq I(X; Z_b|Y_b)$$

$$\kappa I(U; V_g|U_b) \geq \begin{cases} \left[I(X;Z_g|Y_g) - I(X;Z_b|Y_g)\right]^+, & X-Y_g-Y_b \\ I(X;Z_g|Y_g) - \min_{j=b,g} I(X;Z_j|Y_b), & X-Y_b-Y_g \end{cases}$$

In the original papers, [9], [10], the source coding bounds were on the total rates, whereas we use the marginal rates. When the channels are degraded, the two regions are identical. Also note that this characterization gives the complete region in the case of Gaussian source-channel pairs [10].

B. Uncoded Transmission

If $\kappa = 1$ and the source and channel alphabets are compatible, uncoded transmission is a possible strategy. The best distortion pairs achievable are

- Binary Hamming: $D_i = \min\{p_i, \beta_i\}, i = 1, 2$.
- Quadratic Gaussian: $D_i = \frac{\sigma^2_{N_i}\sigma^2_{W_i}}{\sigma^2_{W_i} + \sigma {N_i}^2 P}, i = 1, 2$.

C. Trivial Converse

At each terminal, no WZBC scheme can achieve a distortion less than the best distortion achievable by ignoring the presence of the other terminal. This gives the following converse.

- Binary Hamming: For $\alpha \in [0, 1]$ and $\beta \in [0, 1]$, let $g(\alpha, \beta) \triangleq h(\alpha * \beta) - h(\alpha)$ where $*$ denotes binary convolution: $a * b = \bar{a}b + a\bar{b}$. For $a \in [0, 1], \bar{a}$ denotes $1 - a$. A distortion pair is achievable only if

$$\min_{0 \leq q \leq 1, 0 \leq \alpha \leq \beta_i : q\alpha + \bar{q}\beta_i \leq D_i} qg(\alpha, \beta_i) \leq \kappa(1 - h(p_i)) \ .$$

- Quadratic Gaussian: A distortion pair (D_1, D_2) is achievable at power level P and rate κ only if

$$D_i \geq \frac{\sigma^2_{N_i}}{(1 + \frac{P}{\sigma^2_{W_i}})^\kappa}, i = 1, 2 \ .$$

III. Basic WZBC Schemes

In this section, we present the basic coding schemes that we shall then develop into the schemes that form the main contribution of this paper.

The first scheme, termed Scheme 0, is a simple extension of the scheme in [11] where the source is first quantized before transmission over the channel.

Theorem 1: (D_1, D_2) is achievable at rate κ if there exist r.v. $Z \in \mathcal{Z}$, $U \in \mathcal{U}$ and functions $g_i : \mathcal{Z} \times \mathcal{Y}_i \to \hat{\mathcal{X}}$ with $(Y_1, Y_2) - X - Z$ such that

$$I(X; Z|Y_i) \leq \kappa I(U; V_i), i = 1, 2 \quad (1)$$

$$\mathsf{E}[d_i(X; g_i(Z, V_i))] \leq D_i, i = 1, 2 \ . \quad (2)$$

Here and in what follows, we only present code constructions for discrete sources and channels. The constructions can be extended to the continuous case in the usual manner. Our coding arguments rely heavily on the notion of typicality. Given an r.v. $X \sim P_X(x), x \in \mathcal{X}$ the typical set at block length n is defined as [7]

$$T^n_\delta(X) \triangleq \{x^n \in \mathcal{X}^n : |\tfrac{1}{n}N(a|x^n) - P_X(a)| \leq \delta P_X(a), \forall a \in \mathcal{X}\} ,$$

where $N(a|x^n)$ denotes the number of times a appears in x^n.

Proof: The encoder constructs a source codebook $\mathcal{C}_Z \triangleq \{z^n(j), j = 1, \ldots, M\}$ by choosing sequences from $T^n_\delta(Z)$ uniformly at random. Similarly, it constructs a channel codebook $\mathcal{C}_U \triangleq \{u^m(j), j = 1, \ldots, M\}$ from $T^m_\delta(U)$. Given a source sequence X^n, the encoder finds $j^* \in \{1, \ldots, M\}$ such that $(X^n, z^n(j^*)) \in T^n_{\delta'}(X, Z)$ and transmits $u^m(j^*)$. The encoder declares an error if it cannot find j^*. The decoder at terminal i tries to find $j \in \{1, \ldots, M\}$ such that $(u^m(j), V^m) \in T^m_{\delta''}(U, V)$ and simultaneously $(Y^n, z^n(j)) \in T^n_{\delta''}(Y, Z)$. If no such j or more than one such j is found, an error is declared. If a unique j is found, coordinate-wise reconstruction is performed using g_i with Y_i^n and the decoded $z^n(j)$. For an appropriate choice of $(\delta, \delta', \delta'')$ we can show using standard typicality arguments that the distortion will be less than $D_i + \epsilon$ for sufficiently high n if (1) is satisfied. The requisite codebook size is $M \approx 2^{nI(X;Z)}$. ■

Next, we give a dirty-paper version of Theorem 1. Suppose that there is CSI available solely at the encoder, i.e., the broadcast channel is defined by the transition probability $p_{V_1V_2|US}(v_1, v_2|u, s)$ and the CSI $S^m \in T^m_\delta(S)$, where $S \in \mathcal{S}$ has some fixed distribution, is available non-causally at the encoder. Given a source and side information at the decoders (X, Y_1, Y_2), codes (m, n, f, g_1, g_2) and achievability of distortion pairs is defined as in the WZBC scenario except that the encoder now takes the form $f : \mathcal{X}^n \times \mathcal{S}^m \to \mathcal{U}^m$

Theorem 2: (D_1, D_2) is achievable at rate κ if there exist r.v. $Z \in \mathcal{Z}$, $T \in \mathcal{T}, U \in \mathcal{U}$ and functions $g_i : \mathcal{Z} \times \mathcal{Y}_i \to \hat{\mathcal{X}}$ with $(Y_1, Y_2) - X - Z$ and $T - (U, S) - (V_1, V_2)$ such that

$$I(X; Z|Y_i) \leq \kappa(I(T; V_i) - I(T; S)), i = 1, 2 \quad (3)$$

$$\mathsf{E}[d_i(X; g_i(Z, V_i))] \leq D_i, i = 1, 2\ . \quad (4)$$

Proof: This scheme will be referred to as Scheme 0 with DPC. The code construction is as follows. As before, a source codebook $\mathcal{C}_Z \triangleq \{z^n(j), j = 1, \ldots, M\}$ is chosen from $T^n_\delta(Z)$. A set of M bins $\mathcal{C}_T(j) = \{t^m(j, k), k = 1, \ldots, M'\}$, where each $t^m(j, k)$ is chosen randomly at uniform from $T^m_\delta(T)$, is also constructed. Given a source word X^n and CSI S^m, the encoder tries to find a pair (j, k) such that $(X^n, z^n(j)) \in T^n_{\delta'}(X, Z)$ and $(S^m, t^m(j, k)) \in T^m_{\delta'}(S, T)$. If it is unsuccessful, it declares an error. If it is successful, the channel input is drawn from the distribution $\prod_{l=1}^m p_{U|TS}(u_l|t_l(j, k), S_l)$. At terminal i, the decoder tries to find (j, k) such that $(Y^n, z^n(j)) \in T^n_{\delta''}(Y, Z)$ and $(V^m, t^m(j, k)) \in T^m_{\delta''}(V, T)$. If there is no such pair or more than one, the decoder declares an error. If decoding is successful, reconstruction proceeds as in Scheme 0. Again, using typicality arguments, it can be shown that the distortion constraints are met if (3) is satisfied. The requisite codebook sizes are $M \approx 2^{nI(X;Z)}$ and $M' \approx 2^{mI(S;T)}$. ■

Although both Scheme 0 and Scheme 0 with DPC are joint source-channel coding schemes, there is an apparent separation between source and channel coding in that the source and channel codebooks are independently chosen. Due to this quasi-independence we shall refer to source codes and channel codes separately when we discuss layered WZBC schemes.

IV. Layered WZBC Schemes

We improve the performance of Scheme 0 by layered coding, i.e., by not only transmitting a common layer (CL) to both receivers but also additionally transmitting a refinement layer (RL) to one of the two receivers. The channel codewords corresponding to the two layers are superposed by addition, denoted by '$+$'. The natural choices for binary and Gaussian channels are the XOR operation and real addition respectively. Superposition results in interference between the CL and RL channel codewords. To mitigate the interference, we devise two extensions of Scheme 0 one of which uses successive decoding (called Scheme CR to reflect that RL acts as interference while decoding CL) while the other uses DPC (called Scheme RC). For ease of exposition, we also rename the source and channel r.v. by replacing the subscripts 1 and 2 by c and r depending on whether the terminal referred to receives only the CL or whether it also receives the RL.

The CL is transmitted using either Scheme 0 or Scheme 0 with DPC. However, the expressions for channel capacity in (1) and (3) must be modified to account for the presence of the RL codeword. The RL is transmitted by separate source and channel coding. If Z_c and Z_r denote the source auxiliary r.v. for CL and RL, we shall require $(Y_c, Y_r) - X - Z_r - Z_c$. Due to the separability of the source and channel variables in the required inequalities, we can say that (D_c, D_r) is achievable if there exist (Z_c, Z_r) as above and reconstruction functions $g_i : \mathcal{Z}_i \times \mathcal{Y}_i \to \hat{\mathcal{X}}_i, i = c, r$ such that

$$R_{bb} \leq \kappa C_{bb} \quad (5)$$

$$R_{br} \leq \kappa C_{br} \quad (6)$$

$$R_{rr} \leq \kappa C_{rr} \quad (7)$$

$$\mathsf{E}[d_c(X, g_c(Z_c, Y_c))] \leq D_c \quad (8)$$

$$\mathsf{E}[d_r(X, g_r(Z_r, Y_r))] \leq D_r \quad (9)$$

where $R_{cc} = I(X; Z_c|Y_c), R_{cr} = I(X; Z_c|Y_r), R_{rr} = I(X; Z_r|Z_c, Y_r)$ (cf. [10]). C_{cc} and C_{cr} are the common input capacities of the effective c and r channels for transmitting the CL. C_{rr} is the capacity of the effective r channel for transmitting the RL. Characterizing these capacities for the two schemes is the task of the rest of the section.

A. Scheme CR

This is the simplest extension of Scheme 0, where RL is superposed over CL. We fix r.v. $U_c, U_r \in \mathcal{U}$ for transmitting CL and RL. The CL channel codebook is chosen from $T^m_\delta(U_c)$ while the RL channel codebook is chosen from $T^m_\delta(U_r)$. The channel input is the coordinatewise sum of the chosen pair of CL and RL codewords. In decoding the CL, the RL codeword acts as interference at both receivers. Once receiver r decodes CL, it can diminish the interference of the CL channel codeword while decoding RL. This gives

$$C_{cc} = I(U_c; V_c) \quad (10)$$

$$C_{cr} = I(U_c; V_r) \quad (11)$$

$$C_{rr} = I(U_r; V_r|U_c)\ . \quad (12)$$

B. Scheme RC

We begin by fixing U_r and constructing an RL codebook with elements from $T^m_\delta(U_r)$. The CL codeword is now encoded using Scheme 0 with DPC with the RL codeword acting as CSI. The bins $\mathcal{C}_T(j) = \{t^m(j, k)\}$ for Scheme 0 with DPC are chosen by fixing auxiliary r.v. (T, U_c) satisfying $T - (U_c, U_r) - (V_c, V_r)$. At both receivers, the CL codeword

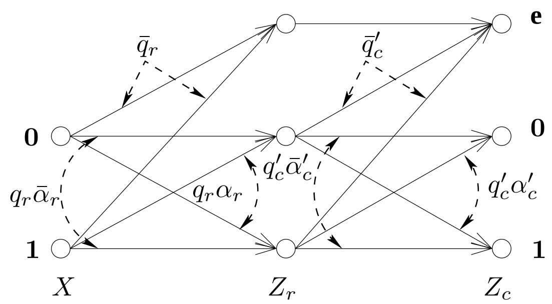


Fig. 1. Auxiliary r.v. for binary source coding. The edge labels denote transition probabilities.

is decoded first. As discussed in Section III, when the DPC decoding is successful, the receivers know the $t^m(j,k)$ chosen by the encoder. Since this codeword was chosen to be jointly typical with the RL codeword, the r receiver can use both $t^m(j,k)$ and V_r^m to decode the RL. The resulting capacity expressions are

$$C_{cc} = I(T;V_c) - I(T;U_r) \quad (13)$$

$$C_{cr} = I(T;V_r) - I(T;U_r) \quad (14)$$

$$C_{rr} = I(U_r;T,V_r)\ . \quad (15)$$

DPC is used here in a manner quite different from the way it has been used in other work on coding for BC [1]. Since most past work concentrates on sending private information only, the information that forms the CSI and the information that is dirty paper coded are meant for different receivers. Therefore, although the DPC auxiliary codewords are decoded at one of the receivers, unlike in our scheme, this is of no use to that receiver. This difference leads to an additional interplay in the choice of (T,U_c,U_r), as elaborated in Section IV-D for the quadratic Gaussian case.

We now specialize the results to the binary Hamming and quadratic Gaussian cases.

C. Binary Sources and Channels

To evaluate R_{cc}, R_{cr} and R_{rr}, we need to first fix the auxiliary r.v. Z_c, Z_r. We chose them both to be as shown in Figure 1, motivated by the optimality of the r.v. used by Wyner and Ziv [12]. While this choice might not be optimal, it has the advantage of being readily computable. Z_r can be viewed as the output of a generalized erasure channel, where both erasures and bit flips are allowed. The erasure probability is $\bar{q}_r$. Given there is no erasure, α_r is the flip probability. Similarly Z_c can also be viewed as the output of an erasure channel with erasure probability $\bar{q}_c$, where $q_c = q_r q_c'$ and flip probability $\alpha_c = \alpha_r * \alpha_c'$. The resulting source coding rates are shown in Table I.

For the channel coding part, we made the following choices. In Scheme CR, the auxiliary r.v. are $U_c \sim \mathrm{Ber}(\frac{1}{2})$ and $U_r \sim \mathrm{Ber}(\gamma)$, where $\mathrm{Ber}(\epsilon)$ denotes the Bernoulli distribution with $P[1] = \epsilon$. In Scheme RC, $U_r \sim \mathrm{Ber}(\frac{1}{2})$ while $U_c \sim \mathrm{Ber}(\gamma)$. T is defined as $U_c + U_r$, i.e., it is identical to the actual channel input. In both schemes, the parameter γ controls the tradeoff between C_{cc} and C_{br} on the one hand, and C_{rr} on the other. The resulting channel coding rates are presented in Table I.

TABLE I
BINARY SOURCES AND CHANNELS

	R_{cc}/C_{cc}	R_{cr}/C_{cr}	R_{rr}/C_{rr}
Source	$q_c g(\alpha_c,\beta_c)$	$q_c g(\alpha_c,\beta_r)$	$q_r g(\alpha_r,\beta_r) - q_c g(\alpha_c,\beta_r)$
CR	$1-h(\gamma * p_c)$	$1-h(\gamma * p_r)$	$h(\gamma * p_r) - h(p_r)$
RC	$h(\gamma) - h(p_c)$	$h(\gamma) - h(p_r)$	$1-h(\gamma)$

The performance of the various schemes for certain source-channel pairs for $\kappa = 1$ is presented in Figure 2. In both layered schemes, we need to choose which terminal receives RL, i.e., $(b = 1, r = 2)$ or vice versa. We tried both possibilities and for a given scheme the figures show the convex hull of the two. In computing the performance of separate source and channel coding, we use source auxiliary r.v. that are as presented in Figure 1. However, the alphabet size bounds in [9], [10] are much higher and therefore it might be possible to further improve the performance of separate coding. For the choice that we make, in all our examples, one of the new schemes always performs better than separate source and channel coding. In fact, in Figure 2 (b), we show a case where Scheme 0 (and consequently the derived schemes) is optimal, i.e., it attains the trivial converse. If the better channel also has the better side information, then separate coding performs as well as any of the new schemes.

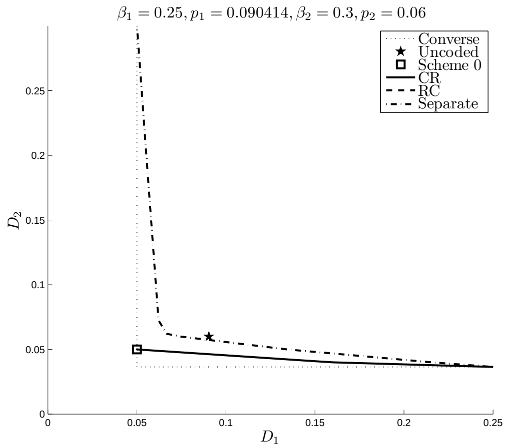
D. Gaussian Sources and Channels

In the Gaussian case, there is an additional power constraint. In all our schemes, the total power P is partitioned into P_c and P_r, the powers available for the common and refinement information. The auxiliary r.v. are $Z_r = X + S_r$ and $Z_c = X + S_c = Z_r + S_c'$ where S_r, S_c and S_c' are Gaussian r.v. satisfying $S_r \perp X$, $S_c \perp X$ and $S_c' \perp Z_r$. The variances of S_r and S_c are $\sigma^2_{S_r}$ and $\sigma^2_{S_c}$.

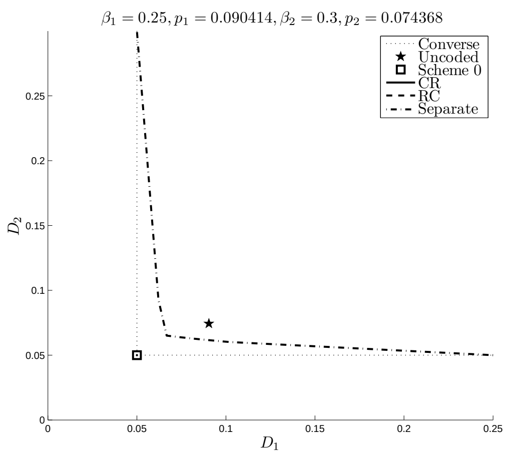
In the channel coding part, for both schemes U_c and U_r are chosen to be independent zero-mean Gaussian r.v. with variances P_c and P_r. In Scheme RC, $T = \gamma U_r + U_c$. Since each of the capacities has to be non-negative, γ has to lie in the interval $\left(\frac{1-\sqrt{1+\frac{P_r+\sigma^2_W}{P_c}}}{1+\frac{\sigma^2_W}{P_c}}, \frac{1+\sqrt{1+\frac{P_r+\sigma^2_W}{P_c}}}{1+\frac{\sigma^2_W}{P_c}}\right)$, $\sigma^2_W = \max[\sigma^2_{W_c}, \sigma^2_{W_r}]$. The flexibility in the choice of γ is a feature that is not present in other scenarios where DPC has been used for coding over BC [1]. In those scenarios, there is a unique value of γ, as specified in [2], that is optimal. In the layered WZBC schemes, γ can be used to tradeoff between (C_{cc}, C_{cr}) and C_{rr}. The source and channel coding rates are given in Table II.

A comparison of the various schemes for some source-channel pairs at rate 1 is given in Figure 3. For Gaussians, if $\sigma^2_{N_1}\sigma^2_{W_1} > \sigma^2_{N_2}\sigma^2_{W_2}$, it is enough to consider the case $b = 1, r = 2$. If the opposite inequality holds, $b = 2, r = 1$. If there is equality, then Scheme 0 achieves the converse.

In all the cases considered, one of the new schemes performs at least as well as separate coding. Here too, if the better channel has better side information, the new schemes offer no



(a)



(b)

Fig. 2. Performance comparison for binary sources and channels. In (a), Scheme RC has the same performance as Scheme 0 and Scheme CR is the best. In (b) schemes 0, CR and RC are optimal.

TABLE II
GAUSSIAN SOURCES AND CHANNELS

	$2^{2R_{cc}}/2^{2C_{cc}}$	$2^{2R_{cr}}/2^{2C_{cr}}$	$2^{2R_{rr}}/2^{2C_{rr}}$
Source	$1+\frac{\sigma^2_{N_c}}{\sigma^2_{S_c}}$	$1+\frac{\sigma^2_{N_r}}{\sigma^2_{S_c}}$	$(1+\frac{\sigma^2_{N_r}}{\sigma^2_{S_r}})(1+\frac{\sigma^2_{N_r}}{\sigma^2_{S_c}})^{-1}$
CR	$1+\frac{P_c}{P_r+\sigma^2_{W_c}}$	$1+\frac{P_c}{P_r+\sigma^2_{W_r}}$	$1+\frac{P_r}{\sigma^2_{W_r}}$
RC	$\frac{1+\frac{P}{\sigma^2_{W_c}}}{1+P_r(\frac{(1-\gamma)^2}{\sigma^2_{W_c}}+\frac{\gamma^2}{P_c})}$	$\frac{1+\frac{P}{\sigma^2_{W_r}}}{1+P_r(\frac{(1-\gamma)^2}{\sigma^2_{W_r}}+\frac{\gamma^2}{P_c})}$	$1+P_r(\frac{(1-\gamma)^2}{\sigma^2_{W_r}}+\frac{\gamma^2}{P_c})$

advantage over separate coding. Note that in case (a), uncoded performs better than any other scheme and therefore the union of the distortion regions of the layered schemes cannot be the complete region. In an upcoming paper, we combine the two digital schemes we just presented with analog or uncoded transmission to extract the benefits of both methods [5].

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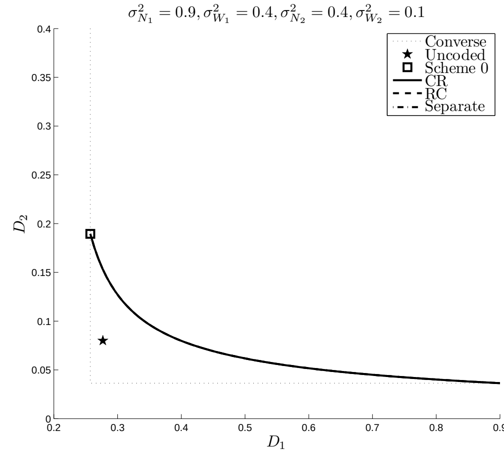
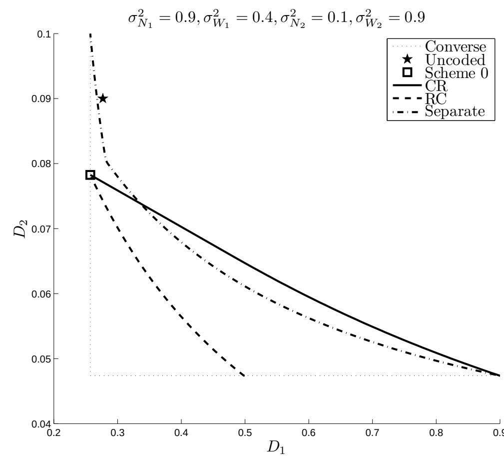


(a)



(b)

Fig. 3. Performance comparison for Gaussian sources and channels. In (a) schemes CR, RC and separate source and channel coding have the same performance, but uncoded transmission can achieve better distortion pairs. In (b) RC is the best scheme.

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