Interference Channel and Compound MAC with Correlated Sources and Receiver Side Information

Deniz Gündüz, Elza Erkip
Dept. of Electrical and Computer Engineering,
Polytechnic University, Brooklyn, NY
dgundu01@utopia.poly.edu, elza@poly.edu

***Abstract*— We consider discrete memoryless compound multiple access and interference channels with correlated sources and correlated side information at the receivers, and investigate necessary and sufficient conditions for lossless transmission. We first give sufficient conditions for the most general setting, and then show that these conditions are also necessary for both channels under certain assumptions on the side information and the interference. In particular, we generalize the notion of strong interference to take into account the correlation among the sources and side information. We prove the optimality of 'informational' or 'operational' source-channel separation for certain special cases. While informational separation results in independent source and channel encoding and decoding; operational separation corresponds to separation at the encoder, while decoding is done jointly. To our knowledge, these results constitute the first source-channel separation results for interference and compound multiple access channels with correlated sources and side information.**

I. Introduction

Consider two wireless sensors observing correlated data, where each wants to transmit its measurement either to its private collection node, or to both of the collection nodes. Furthermore, each collection node may have its own observation correlated with either or both of the sensor data. These scenarios can be modeled as interference channel (IC) or compound multiple access channels (MAC) with correlated sources and correlated side information depending on whether data is only intended for the private collection node or both.

It is known that, Shannon's source-channel separation theorem breaks down in most multi-user networks. Characterization of the necessary and sufficient conditions for lossless transmission is an open problem for many multiuser scenarios, in particular for compound MAC and interference channels. On the other hand, optimality of source-channel separation is desirable as it provides modularity in system design without any loss in the performance. Motivated by the potential benefits of separate source and channel coding, in this paper, we investigate scenarios under which separation either only at the encoder or at both the encoder and the decoder is optimal for IC and compound MAC.

It is shown in [1] that separating source and channel coding is not optimal when transmitting correlated sources over MAC. Sufficient conditions for achievability are provided in [1] using a 'correlation preserving mapping' technique. Optimality of separation for a network with independent, non-interfering channels is proved in [2]. In [3], we consider transmission of correlated sources over MAC, where the receiver has a side information given which the sources are independent, and show the optimality of source-channel separation under this assumption. Tuncel considers Slepian-Wolf coding over broadcast channels in [4], and shows that 'operational separation', in which source and channel encoding is done separately, while decoding is joint, is optimal, while 'informational separation', the classical source-channel separation, fails to achieve optimality. Coleman et al. extended this joint decoding idea to transmitting correlated sources over a broadcast network without multiple access interference [5].

In this paper, we first provide general sufficient conditions for lossless transmission of correlated sources over a compound MAC with receiver side information, which are also sufficient for the corresponding IC. We then provide converse results under certain assumptions on the source and side information structure. Our results show that, for certain scenarios, it is possible to achieve optimality either by informational separation, or by operational separation. Based on the results of this paper, we argue that, in multi-user networks, even though informational separation fails through operational separation, we might still achieve a certain degree of modularity without performance loss.

II. System Model

Let $\{S_{1t}, S_{2t}, W_{1t}, W_{2t}\}_{t=0}^{\infty}$ be a discrete memoryless sequence generated according to the joint probability distribution p_{S_1,S_2,W_1,W_2} over the alphabet $\mathcal{S}_1 \times \mathcal{S}_2 \times \mathcal{W}_1 \times \mathcal{W}_2$. Here, S_1 and S_2 are the two correlated source sequences available at transmitter 1 and 2, respectively, where the source S_i is intended for either only receiver i, or both receivers ($i = 1, 2$). Receiver i has access to correlated side information W_i. Let $\{S_1^m, S_2^m, W_1^m, W_2^m\}$ denote length-m vectors of source and side information sequences. Each transmitter wants to transmit its source sequence S_i^m losslessly to its respective receiver(s). The underlying channel is characterized by the probability density function $p_{Y_1,Y_2|X_1,X_2}(y_1, y_2|x_1, x_2)$ with input alphabets $\mathcal{X}_i$ and output alphabets $\mathcal{Y}_i$, $i = 1, 2$ (see Fig. 1). Depending on whether receiver i is interested in only source S_i or both sources, the channel corresponds to a discrete memoryless (DM) interference channel (IC), or compound multiple access channel (MAC), respectively.

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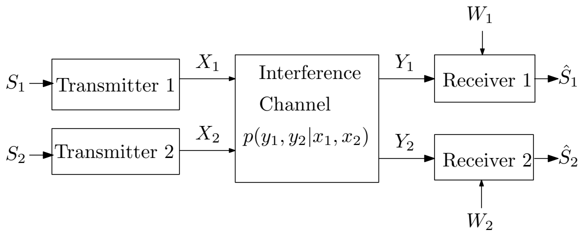

Fig. 1. IC with correlated sources and correlated side information at the receivers. When both decoders are interested in obtaining estimates for both sources, this model corresponds to compound MAC.

For our setup the notion of compound MAC is generalized to take into account correlated sources and receiver side information. Note that, in regular compound MAC we are only concerned with channel coding and both receivers are interested in the same message set. However, in the source-channel coding scenario with side information, the messages received can be different as long as the lossless reconstruction of the sources is possible, as the side information are different in general.

Definition 2.1: A *rate* b is said to be *achievable* for the DM IC, if there exist sequences of encoders

$$f_i^{(m,n)} : \mathcal{S}_i^m \to \mathcal{X}_i^n, \tag{1}$$

with $X_i^n = f_i^{(m,n)}(S_i^m)$, and sequences of decoders

$$g_i^{(m,n)} : \mathcal{Y}_i^n \times \mathcal{W}_i^m \to \mathcal{S}_i^m, \tag{2}$$

with $\hat{S}_i^m = g_i^{(m,n)}(Y_i^n, W_i^m)$, such that for the probability of error at each receiver, defined as

$$P_i^{(m,n)} : Pr\{S_i^m \neq \hat{S}_i^m\}, \tag{3}$$

we have $\lim_{m,n\to\infty} P_i^{(m,n)} = 0$ for $i = 1, 2$, while $n/m = b$.

In the compound MAC case, we replace the decoder functions with

$$g_i^{(m,n)} : \mathcal{Y}_i^n \times \mathcal{W}_i^m \to (\mathcal{S}_1^m, \mathcal{S}_2^m), \tag{4}$$

where $(\hat{S}_{i1}^m, \hat{S}_{i2}^m) = g_i^{(m,n)}(Y_i^n, W_i^m)$, and the probability of error expression with

$$P_i^{(m,n)} = Pr\{(S_1^m, S_2^m) \neq (\hat{S}_{i1}^m, \hat{S}_{i2}^m)\}, \tag{5}$$

while the achievability definition remains same.

For IC, even in the case of independent messages at the transmitters, and no side information at the receivers, the capacity region in general is not known. The most general case for which the exact capacity region can be characterized is the so-called strong interference channel [6], where the region coincides with the capacity region of a compound MAC [8]. In this case, it is optimal for both receivers to decode both messages. In Section III we present necessary and sufficient conditions for achievability of rate b for compound MAC and then in Section IV we extend these conditions to IC under 'strong source-channel interference' condition.

III. Compound Multiple Access Channel

We first give an achievability result for transmitting correlated sources over compound MAC with correlated side information.

Theorem 3.1: For lossless transmission of arbitrarily correlated sources (S_1, S_2) over a compound MAC with receiver side information (W_1, W_2), rate $b = 1$ is achievable if, for $k = 1, 2$,

$$\begin{aligned} H(S_1|S_2, W_k) &< I(X_1; Y_k|X_2, S_2, W_k), \\ H(S_2|S_1, W_k) &< I(X_2; Y_k|X_1, S_1, W_k), \\ H(S_1, S_2|W_k) &< I(X_1, X_2; Y_k|W_k), \end{aligned}$$

for some input distribution of the form $p(s_1, s_2, x_1, x_2) = p(s_1, s_2)\ p(x_1|s_1)p(x_2|s_2)$.

Proof: The proof follows from a generalization of the 'correlation preserving mapping' technique of [1] and is skipped for brevity. ■

When $S_1 - W_1 - S_2$ and $S_1 - W_2 - S_2$ constitute two Markov chains, that is, the sources are independent given either of the side information, we have the following result.

Theorem 3.2: For lossless transmission of arbitrarily correlated sources (S_1, S_2) over DM compound MAC with side information W_1 and W_2 at the receivers satisfying $S_1 - W_1 - S_2$ and $S_1 - W_2 - S_2$, rate b is achievable if, for $k = 1, 2$

$$\begin{aligned} H(S_1|W_k) &\leq bI(X_1; Y_k|X_2, Q), \\ H(S_2|W_k) &\leq bI(X_2; Y_k|X_1, Q), \\ H(S_1|W_k) + H(S_2|W_k) &\leq bI(X_1, X_2; Y_k|Q), \end{aligned}$$

for some $|\mathcal{Q}| \leq 4$ and input distribution of the form $p(q, x_1, x_2) = p(q)\ p(x_1|q)p(x_2|q)$.

Proof: Proof is given in Appendix I. ■

The achievability scheme used in Theorem 3.2 uses channel codebooks independent of source realizations and follows the 'operational separation' idea of [4], where we have separate source and channel coding at the encoders, while decoding is done jointly. We remark here that, this is not a straightforward extension of the compound MAC capacity region [8], as due to the existence of side information at the receivers, transmitters do not necessarily send the same message to both receivers.

We next prove that the conditions in Theorem 3.2 are also necessary for the compound MAC when (S_1, W_2) is independent of (S_2, W_1). This might model a scenario where receiver 1 (2) and transmitter 2 (1) are located close to each other, hence they have correlated observations, while two transmitters are far away.

Theorem 3.3: For lossless transmission of arbitrarily correlated sources (S_1, S_2) over a DM compound MAC with side information W_1 and W_2, where (S_1, W_2) is independent of (S_2, W_1), if rate b is achievable, then conditions of Theorem 3.2 are satisfied.

Proof: Proof is given in Appendix II. ■

Theorems 3.2 and 3.3 together form the necessary and sufficient conditions for lossless transmission of independent sources over a compound MAC when each receiver has side information correlated with one of the sources. Note that, this also proves the optimality of operational separation in this setting. Since we need to transmit each source to two receivers one with correlated side information and the other without any side information following [4], in general, informational separation does not hold.

Finally, we consider the special case where $W_1 = W_2 = W$ and $S_1 - W - S_2$ form a Markov chain. This corresponds to the case where two receivers are close to each other, hence have the same side information.

Theorem 3.4: For lossless transmission of correlated sources S_1 and S_2 over a DM compound MAC with common receiver side information $W_1 = W_2 = W$ satisfying $S_1 - W - S_2$, if rate b is achievable, then conditions of Theorem 3.2 are satisfied.

Proof: Proof follows similar to [3], and is skipped for brevity. ■

In this case of common side information at the receivers, informational separation turns out to be optimal, where, each transmitter compresses its source conditioned on the receiver side information W, and transmits its compressed data using an optimal compound MAC channel code.

IV. Interference Channel

In this section, we extend the results in Section III to interference channels (IC). We first note that the conditions of Theorem 3.1 and Theorem 3.2 provide sufficient conditions for achievability of IC as well, under the source and side information structures stated in the theorems. Our main goal is to extend the necessary conditions of Theorem 3.3 and Theorem 3.4 to IC. This will require defining a 'strong source-channel interference' condition.

The regular strong interference conditions given in [9] corresponds to the case that, for all input distributions at transmitter 1, the rate of information flow to receiver 2 is higher than the information flow to the intended receiver 1. A similar condition holds for transmitter 2 as well. This leads to the observation that, no performance is lost if both receivers decode the messages of both transmitters. Consequently, under strong interference condition, the capacity region of the IC is equivalent to the capacity region of compound MAC. However, in the joint source-channel coding scenario, the receivers have access to correlated side information. Thus while calculating the total rate of information flow to a particular receiver, we should not only consider the information flow through the channel, but also the mutual information that already exists between the source and the receiver side information. This idea is reflected in the following *strong source-channel interference conditions* stated for the setup of Theorem 3.3 where the two sources are independent while the side information W_1 is correlated with source S_2, and the side information W_2 is correlated with source S_1.

$$b \cdot I(X_1; Y_1|X_2) \leq b \cdot I(X_1; Y_2|X_2) + I(S_1; W_2), \quad (6)$$

$$b \cdot I(X_2; Y_2|X_1) \leq b \cdot I(X_2; Y_1|X_1) + I(S_2; W_1), \quad (7)$$

for all distributions of the form $p(w_1, w_2, s_1, s_2, x_1, x_2) = p(w_1, w_2, s_1, s_2)p(x_1|s_1)p(x_2|s_2)$. Then we can obtain the following theorem.

Theorem 4.1: Consider lossless transmission of S_1 and S_2 over a DM IC with side information W_1 and W_2, where (S_1, W_2) is independent of (S_2, W_1). Under strong source-channel interference conditions of (6)-(7), rate b is achievable if and only if conditions in Theorem 3.2 hold.

Proof: Proof is given in Appendix III. ■

Next, we consider the case in Theorem 3.4, where the two receivers have access to the same side information W given which the sources are independent. In this case, while we still have correlation between the sources and the common receiver side information, the amount of mutual information arising from this correlation is equivalent at both receivers since $W_1 = W_2$. This means that the usual strong interference channel conditions suffice to obtain the converse result. We have the following theorem for this case.

Theorem 4.2: For lossless transmission of correlated sources S_1 and S_2 over the strong IC with common receiver side information $W_1 = W_2 = W$ satisfying $S_1 - W - S_2$, rate b is achievable if and only if, conditions in Theorem 3.4 hold.

Proof: Proof follows similar to the proof of Theorem 3.4 and [3] where we incorporate the strong source-channel interference conditions. ■

V. Conclusion

We consider transmission of correlated sources over interference and compound multiple access channels where the receivers also have correlated side information. For compound MAC, while informational separation (separation of source and channel coding in the usual sense) is suboptimal in the most general setting, we show that, under certain assumptions on sources and side information, it is possible to prove the optimality of either the informational separation or operational separation, a less constraining separation theorem, where source and channel encoders act as independent components, while decoding is done jointly. Under the strong source-channel interference conditions we define, we can extend this optimality result to interference channels as well. Our results show that, in many multi-user network scenarios, while informational separation does not apply, it might be possible to achieve optimality by only separating source and channel encoding. This separation of encoding can bring modularity to the transmitter design which might be invaluable especially in low complexity sensor network applications.

Appendix I
Proof of Theorem 3.2

We will use the coding technique introduced in [4], where we use separate encoding at the transmitters and joint source-channel decoding at the receivers. We fix $\delta_k > 0$ and $\gamma_k > 0$ for $k = 1, 2$, and P_{X_1} and P_{X_2} such that

$$H(S_1|W_k) \leq bI(X_1; Y_k|X_2) - \epsilon,$$

$$H(S_2|W_k) \leq bI(X_2; Y_k|X_1) - \epsilon,$$

$$H(S_1|W_k) + H(S_2|W_k) \leq bI(X_1, X_2; Y_k) - \epsilon, .$$

are satisfied for $k = 1, 2$. For $b = n/m$ and $k = 1, 2$, at transmitter k, we generate $M_k = 2^{m[H(S_k)+\epsilon/2]}$ i.i.d. length-m source codewords and i.i.d. length-n channel codewords using probability distributions P_{S_k} and P_{X_k}, respectively. These codewords are revealed to the receivers as well, and denoted by $s_k^m(i)$ and $x_k^n(i)$ for $1 \leq i \leq M_k$.

Encoder: Each source outcome is directly mapped to a channel codeword as follows: Given a source outcome S_k^m at transmitter m, we find the smallest i_k such that $S_k^m = s_k^m(i_k)$, and transmit the codeword $x_k^n(i_k)$. An error occurs if no such i_k is found at either of the transmitters $k = 1, 2$.

Decoder: At receiver k, we find the unique pair (i_1^*, i_2^*) that simultaneously satisfies

$$(x_1^n(i_1^*), x_2^n(i_2^*), Y_k^n) \in \mathcal{A}^{(n)}_{[X_1,X_2,Y]_{\delta_k}},$$
$$(s_1^m(i_1^*), s_2^m(i_2^*), W_k^m) \in \mathcal{A}^{(m)}_{[S_1,S_2,W_k]_{\gamma_k}},$$

where $\mathcal{A}^{(n)}_{[X]_\delta}$ is the set of weakly δ-typical sequences. An error is declared if (i_1^*, i_2^*) pair is not uniquely determined.

Probability of error: We define the following error events:

$$E_1 = \bigcup_{k=1,2} \{S_k^m \neq s_k^m(i), \forall i\}$$
$$E_2(k) = \{(s_1^m(i_1), s_2^m(i_2), W_k^m) \notin \mathcal{A}^{(m)}_{[S_1,S_2,W_k]_{\gamma_k}}\}$$
$$E_3(k) = \{(X_1^n, X_2^n, Y_k^n) \notin \mathcal{A}^{(n)}_{[X_1,X_2,Y]_{\delta_k}}\}$$
$$E_4(k) = \{\exists (j_1, j_2) \neq (i_1, i_2) : (s_1^m(j_1), s_2^m(j_2), W_k^m) \in \mathcal{A}^{(m)}_{[S_1,S_2,W_k]_{\gamma_k}}\}$$
$$E_5(k) = \{\exists (j_1, j_2) \neq (i_1, i_2) : (x_1^n(j_1), x_2^n(j_2), Y_k^n) \in \mathcal{A}^{(n)}_{[X_1,X_2,Y]_{\delta_k}}\}$$

Here, E_1 denotes the error event where either of the encoders fails to find a unique source codeword in its codebook that corresponds to its current source outcome. When such a codeword can be found, $E_2(k)$ denotes the error event that the sources S_1^m, S_2^m and the side information W_k at receiver k are not jointly typical, whereas $E_4(k)$ denotes the error event that a source codeword pair different from the current realization is jointly typical with W_k. On the other hand, $E_3(k)$ denotes the error event that channel codewords that match the current source realizations are not jointly typical with the channel output at receiver k, while $E_5(k)$ is the event that some other channel codeword pair is jointly typical with Y_k^n.

Using the union bound, we have

$$\begin{aligned} P_k^{(m,n)} &\leq Pr(E_1) + Pr(E_2(k)) + Pr(E_3(k)) \\ &+ \sum_{\substack{j_1\neq i_1,\\ j_2=i_2}} Pr\left\{(s_1^m(j_1), s_2^m(j_2), W_k^m) \in \mathcal{A}^{(m)}_{[S_1,S_2,W_k]_{\gamma_k}}\right\} \\ &\cdot Pr\left\{(x_1^n(j_1), x_2^n(j_2), Y_k^n) \in \mathcal{A}^{(n)}_{[X_1,X_2,Y]_{\delta_k}}\right\} \\ &+ \sum_{\substack{j_1=i_1,\\ j_2\neq i_2}} Pr\{\cdot\} + \sum_{\substack{j_1\neq i_1,\\ j_2\neq i_2}} Pr\{\cdot\} \end{aligned} \quad (8)$$

It can be shown that $Pr(E_i) \to 0$ for $i = 1, 2, 3$ as $m, n \to \infty$. We can also obtain:

$$\begin{aligned} &\sum_{\substack{j_1\neq i_1,\\ j_2=i_2}} Pr\left\{(s_1^m(j_1), s_2^m(j_2), W_k^m) \in \mathcal{A}^{(m)}_{[S_1,S_2,W_k]_{\gamma_k}}\right\} \\ &\cdot Pr\left\{(x_1^n(j_1), x_2^n(j_2), Y_k^n) \in \mathcal{A}^{(n)}_{[X_1,X_2,Y]_{\delta_k}}\right\} \\ &\leq 2^{m[H(S_1)+\frac{\epsilon}{2}]-m[I(S_1;S_2,W_k)-\lambda]-n[I(X_1;Y_k|X_2)-\lambda]} \\ &= 2^{-m[H(S_1|W_k)-bI(X_1;Y_k|X_2)-(b+1)\lambda-\frac{\epsilon}{2}]} \\ &= 2^{-m[\frac{\epsilon}{2}-(b+1)\lambda]}. \end{aligned}$$

A similar bound can be found for the second sum in (8). On the other hand, we also have

$$\begin{aligned} &\sum_{\substack{j_1\neq i_1,\\ j_2\neq i_2}} Pr\left\{(s_1^m(j_1), s_2^m(j_2), W_k^m) \in \mathcal{A}^{(m)}_{[S_1,S_2,W_k]_{\gamma_k}}\right\} \\ &\cdot Pr\left\{(x_1^n(j_1), x_2^n(j_2), Y_k^n) \in \mathcal{A}^{(n)}_{[X_1,X_2,Y]_{\delta_k}}\right\} \\ &\leq 2^{m[H(S_1)+\epsilon/2]+m[H(S_2)+\epsilon/2]} \\ &\cdot 2^{-m[I(S_1;S_2,W_k)+I(S_2;S_1,W_k)-I(S_1;S_2|W_k)]-\lambda]} \\ &\cdot 2^{-n[I(X_1,X_2;Y_k)-\lambda]} \\ &\leq 2^{-m[H(S_1|W_k)+H(S_2|W_k)-bI(X_1,X_2;Y_k)-(b+1)\lambda-\epsilon]} \\ &= 2^{-m[\epsilon-(b+1)\lambda]}. \end{aligned}$$

Choosing $\lambda < \frac{\epsilon}{2(b+1)}$, we can make sure that all these three terms also vanish as $m, n \to \infty$. Any rate pair in the convex hull can be achieved by time sharing, hence the time-sharing random variable Q.

APPENDIX II
PROOF OF THEOREM 3.3

We have

$$\begin{aligned} \frac{1}{n} I(X_1^n; Y_1^n|X_2^n) &\geq \frac{1}{n} I(S_1^m; Y_1^n|X_2^n), && (9) \\ &= \frac{1}{n}[H(S_1^m|X_2^n) - H(S_1^m|Y_1^n, X_2^n)], && (10) \\ &\geq \frac{1}{n}[H(S_1^m) - H(S_1^m|Y_1^n)], && (11) \\ &\geq \frac{1}{b} H(S_1) - \epsilon, && (12) \end{aligned}$$

for any $\epsilon > 0$ and large enough m, n, where (9) follows from the conditional data processing inequality since $S_1^m - X_1^n - Y_1^n$ forms a Markov chain given X_2^n; (11) from the independence of S_1^m and X_2^n and the fact that conditioning reduces entropy; (12) from the memoryless source assumption, and from Fano's inequality.

For the joint mutual information, we can write the following set of inequalities.

$$\frac{1}{n}I(X_1^n, X_2^n; Y_1^n) \geq \frac{1}{n}I(S_1^m, S_2^m; Y_1^n), \quad (13)$$
$$= \frac{1}{n}I(S_1^m, S_2^m, W_1^m; Y_1^n), \quad (14)$$
$$\geq \frac{1}{n}I(S_1^m, S_2^m; Y_1^n | W_1^m), \quad (15)$$
$$= \frac{1}{n}[H(S_1^m, S_2^m | W_1^m) - H(S_1^m, S_2^m | Y_1^n, W_1^m)],$$
$$= \frac{1}{n}[H(S_1^m) + H(S_2^m | W_1^m) - H(S_1^m, S_2^m | Y_1^n, W_1^m)], \quad (16)$$
$$\geq \frac{1}{b}\left[H(S_1) + H(S_2|W_1)\right] - \epsilon, \quad (17)$$

for any $\epsilon > 0$ and large enough m, n, where (13) follows from the data processing inequality since $(S_1^m, S_2^m) - (X_1^n, X_2^n) - Y_1^n$ form a Markov chain; (14) from the Markov relation $W_1^m - (S_1^m, S_2^m) - Y_1^n$; (15) from the chain rule and the non-negativity of the mutual information; (16) from the independence of S_1^m and (S_2^m, W_1^m); (17) from the memoryless source assumption and Fano's inequality.

It is also possible to show that

$$\sum_{i=1}^{n} I(X_{1i}; Y_{1i} | X_{2i}) \geq I(X_1^n; Y_1^n | X_2^n), \quad (18)$$

and similarly for other mutual information terms. Then, using the above set of inequalities and letting $\epsilon \to 0$, we obtain

$$\frac{1}{b}H(S_1) \leq \frac{1}{n}\sum_{i=1}^{n} I(X_{1i}; Y_{1i}|X_{2i}),$$
$$\frac{1}{b}H(S_2|W_1) \leq \frac{1}{n}\sum_{i=1}^{n} I(X_{2i}; Y_{1i}|X_{1i}),$$
$$\frac{1}{b}(H(S_1) + H(S_2|W_1)) \leq \frac{1}{n}\sum_{i=1}^{n} I(X_{1i}, X_{2i}; Y_{1i}),$$

for any product distribution on $\mathcal{X}_1 \times \mathcal{X}_2$. We can write similar expressions for the second receiver as well. Then the necessity of the conditions of Theorem 3.2 can be argued simply by inserting the time-sharing random variable Q following the same steps as in [10].

APPENDIX III
PROOF OF THEOREM 4.1

We only write the bound involving $I(X_1^n, X_2^n; Y_1^n)$, the others follow similar to the proof of Theorem 3.3. First, we prove the following lemma which will be critical in our converse proof.

Lemma 3.1: If (S_1, W_2) is independent of (S_2, W_1) and strong source-channel interference conditions (6)-(7) hold, then we have

$$I(X_2^n; Y_2^n | X_1^n) \leq I(X_2^n; Y_1^n | X_1^n) + I(S_2^m; W_1^m),$$

for all m, n satisfying $n/m = b$.

Proof: Condition (7) implies

$$I(X_2; Y_2|X_1, U) - I(X_2; Y_1|X_1, U) \leq \frac{1}{b}I(S_2; W_1), \quad (19)$$

for all U satisfying $U - (X_1, X_2) - (Y_1, Y_2)$.

Using this and following the technique of the Lemma in [7], we can obtain

$$I(X_2^n; Y_2^n | X_1^n) - I(X_2^n; Y_1^n | X_1^n)$$
$$= I(X_{2n}; Y_{2n} | X_1^n, Y_2^{n-1}) - I(X_{2n}; Y_{1n} | X_1^n, Y_2^{n-1})$$
$$+ I(X_2^{n-1}; Y_2^{n-1} | X_1^n, Y_{1n}) - I(X_2^{n-1}; Y_1^{n-1} | X_1^n, Y_{1n})$$
$$\leq \frac{n}{b}I(S_2; W_1) = I(S_2^m; W_1^m).$$

■

Then we can obtain

$$\frac{1}{n}I(X_1^n, X_2^n; Y_1^n) = \frac{1}{n}[I(X_1^n; Y_1^n) + I(X_2^n; Y_1^n | X_1^n)],$$
$$\geq \frac{1}{n}[I(S_1^m; Y_1^n) + I(X_2^n; Y_2^n | X_1^n) - I(S_2^m; W_1^m)], \quad (20)$$
$$\geq \frac{1}{n}[H(S_1^m) - H(S_1^m | Y_1^n) + I(S_2^m; Y_2^n | X_1^n) + H(S_2^m | W_1^m) - H(S_2^m)], \quad (21)$$
$$\geq \frac{1}{n}[H(S_1^m) - \epsilon + H(S_2^m | X_1^n) - H(S_2^m | Y_2^n, X_1^n) + H(S_2^m | W_1^m) - H(S_2^m)], \quad (22)$$
$$\geq \frac{1}{b}[H(S_1) + H(S_2|W_1)] - 2\epsilon, \quad (23)$$

for any $\epsilon > 0$ and large enough m, n, where (20) follows from the data processing inequality and Lemma 3.1; (21) from data processing inequality since $S_2^m - X_2^n - Y_2^n$ form a Markov chain given X_1^n; (22) from data processing inequality and Fano's inequality; (23) from independence of S_2^m and X_1^n and again from Fano's inequality.

The rest of the proof closely resembles the proof of Theorem 3.3.

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