Competitive Analysis of Energy Harvesting Wireless Communication Systems

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Abstract— A competitive analysis for the online and offline optimization problems for a slotted energy harvesting (EH) wireless communication system is studied. The objective is to design online strategies that minimize the *competitive rate gap* that is defined as the maximum gap between the optimal rates that can be achieved by the offline and online policies over all possible energy arrival profiles. It is shown that the competitive rate gap is upper-bounded by the logarithm of the number of slots, and a myopic online transmission policy is proposed that achieves a lower rate gap.

I. INTRODUCTION

Energy harvesting (EH) technology is considered as a major component of future wireless networks and devices in order to reduce frequent battery replacements for exponentially increasing number of connected devices, to limit the growing carbon footprint of the wireless industry, and also to obliterate the dependence of wireless terminals on the power grid. Harvesting energy from the environment extends the lifetime of wireless devices, and provides them untethered mobility, as batteries can be charged without connecting to the power grid infrastructure. However, despite such advantages, designing EH communication systems bring its own challenges. Due to the stochastic nature of the energy arrivals, sources may eventually run out of energy, degrading the communication performances; or, being overly frugal for energy consumption might lead to battery overflows, and waste of harvested energy.

For many energy sources, such as solar, vibration or electromagnetic, the characteristics of the EH profile change over time. The time-varying nature of the available energy motivates the need for designing transmission polices that take into account the stochastic nature of the energy arrival process, while optimizing a desired performance criteria. The performance measure considered here is the average throughput. We model the EH process as a slotted packet arrival process, in which the energy arrives in packets at each time slot, and we study the problem of maximizing the achievable average throughput over a fixed number of time slots. We assume that the energy harvested during the course of the communication is used only in the power amplifier of the transmitter.

Previous work addressing the design of transmission policies for EH devices are typically classified based on the assumptions made on the transmitter's knowledge about the EH process [1]. In the *offline optimization* framework the transmitter is assumed to have access to all the future energy packet arrival instants and packet sizes. The optimal offline transmission policy maximizing the throughput for an EH point-to-point additive white Gaussian noise (AWGN) channel is first studied in [2], and later extended to battery capacity constraints and imperfections in [3] and [4]. Throughput maximizing offline strategies have also been extended to multi-terminal communication channels. The broadcast channel was studied in [5], the multiple-access channel in [6], the interference channel in [7], while the two-hop relay channel is considered in [8] and [9]. In addition to serving as theoretical upper bounds, offline designs have also been proven useful in inspiring online policies [10]. However, practical interest in offline polices is limited to scenarios for which the EH process is more or less deterministic, or is random, but can be accurately predicted. For example, solar based systems and shoe-mounted piezoelectric devices.

The *online optimization* framework, instead, assumes that the transmitter has only a statistical knowledge of the underlying EH process [1], [11]. In the online framework the optimization problem is modeled as a Markov decision process, and the optimal policy can be determined through dynamic programming. Most of the work in the literature on the online optimization show performance results that are very close to those achieved by optimal offline policies [12], [13]. However, it is not yet clear how much of these results can be attributed to the particular online policy chosen, or the stochastic model considered for the EH process.

In this work we aim at answering the following fundamental questions: Can the gap between the achievable offline and online throughputs be unbounded? If this is not the case, compared to the optimal offline throughput, can we quantify the loss of adopting an online policy, independent of the statistics of the EH process? Can we characterize a generic online policy that minimizes this gap? The answers to these important questions will determine the value of the knowledge about the EH process. If the gap between the optimal offline and online policies can be significantly large, more effort should be put into characterizing and learning the behaviour

of the underlying EH processes [14]. Moreover, identifying this gap independent of the EH statistics will also let us know the value of the offline results as a performance benchmark, a claim commonly used in the literature. To that end, we adopt a competitive analysis framework, for which the statistics of the EH process are not relevant.

The most related paper to our work is [15], in which the authors introduce a competitive analysis for an EH communication systems, and define the *competitive ratio* as the maximum ratio between the gain of the optimal offline algorithm and that of the online algorithm over all possible energy arrival profiles. They consider point-to-point communications over a slotted transmission interval and consider, both, arbitrary energy arrivals and time-varying channel coefficients which are known only causally at the transmitter. For this scenario, authors show that when all the energy arrives at the start of the transmission and only the fading coefficients are arbitrarily varying, the optimal competitive ratio over N slots is N. Then, they show that this same ratio is achieved for the general case of arbitrarily varying energy arrivals and fading coefficient. Here, we study the competitive rate gap rather than the competitive ratio for a static channel setting. We show that the optimal competitive rate gap for N time slots is upper-bounded by $\log_2(N)$. Other works addressing the design of online algorithms for fading channels under the competitive analysis framework are [16]–[20].

The remainder of the paper is organized as follows. The system model is described in Section II. The competitive analysis framework is developed in Section II-A. The case of two time slots is studied in Section III, and the extension to N time slots is done in Section IV. Numerical results are presented in V. Finally, concluding remarks are presented in Section VI.

II. SYSTEM MODEL

Consider wireless transmission from a source to a destination over an AWGN channel with zero mean and unit variance. The communications has a fixed time duration of T time units, which is divided into N slots of equal duration $\frac{T}{N}$. We consider the Shannon capacity function to relate the achieved instantaneous rate to the power; that is, if the transmission power at time t is $p(t)$ then the instantaneous rate is given by $r(p(t)) = \frac{1}{2}\log_2(1 + p(t))$, and the total number of bits transmitted over the period of time T is given by $\int_0^T r(p(t))d(t)$.

The source terminal harvests energy from the environment over time. The energy harvested during time slot $n-1$ is first stored into the battery, and is only available at the beginning of slot n, and is denoted by $E_n \in \{0, \mathbb{R}^+\}$, $n = 1, 2, \ldots, N$. Denote by U_n the energy allocated for transmission during time slot n.

It is well known that, due to the strict concavity of the capacity function, the rate in each slot is maximized by equally distributing the energy U_n over the whole slot duration $\frac{T}{N}$. Then, the total number of bits transmitted over slot n is found as follows.

$$D_n(U_n) = \frac{T}{2N}\log_2\left(1 + N\frac{U_n}{T}\right).$$

After N time slots, the rate achieved is $R = \frac{1}{T}\sum_{n=1}^{N} D_n(U_n)$. Due to the energy causality constraint, the total energy used by the end of slot n cannot be more than the energy harvested by the beginning of time slot n, $n = 1, 2, \ldots, N$, that is, U_m values have to satisfy:

$$\sum_{m=1}^{n} U_m \leq \sum_{m=1}^{n} E_m, \forall n \leq N.$$

A. Competitive Rate Gap

Power polices are typically classified based on the knowledge they assume at the transmitter about the underlying EH process into two categories: *offline* and *online policies*. Offline policies assume that the input EH sequence $\mathbf{E} = \langle E_1, ..., E_N \rangle$ is completely known in advance at the source. The optimal offline transmission policy for the point-to-point communication scenario considered here was first presented in [2]. We denote the offline rate as $R_O(\mathbf{E})$.

Online policies, instead, consider that the future energy arrivals are unknown. While the optimal offline policy is characterized as the solution of a convex optimization problem, the online problem falls into the category of Markov decision processes, and the optimal solution can be identified by dynamic programming. Alternative simple online policies can also be considered, such as the greedy policy, which uses all the available energy in the next slot, or the myopic policy which uses the offline optimization solution on the current available energy as if there will be no further energy arrivals, without claiming optimality.

In this study, we assume that there is no underlying known statistics for the EH process. Hence, we consider online policies $\mathcal{U}$ that make their decisions based only on the past energy arrivals and the transmission powers, namely

$$\mathcal{U}(\langle E_1, ..., E_N \rangle) = \langle U_1, ..., U_N \rangle,$$

where the energies spent at time slots $n = 1, ..., N$ are defined by the functions

$$U_n(\langle E_1, ..., E_n \rangle) : \{0, \mathbb{R}^+\}^n \to [0, B_n],$$

where B_n denotes the energy available for transmission at time slot n. Under this assumption, our goal is to study the rate gap between the rate achieved by the optimal offline policy $R_O(\mathbf{E})$ and the rate $R_{\mathcal{U}}(\mathbf{E})$ that is achieved by an online policy maximized over all possible energy profiles. We want to characterize the minimum value of this maximum rate gap, the competitive rate gap (g), defined as

$$g = \min_{\mathcal{U}} \max_{\mathbf{E} \in \{0, \mathbb{R}^+\}^N} R_O(\mathbf{E}) - R_{\mathcal{U}}(\mathbf{E}) \qquad (1)$$

This can be considered as a worst case guarantee: For example, if we can prove that this competitive rate gap has a finite value of G, then we can claim that there exists an

online policy that achieves rates within G bits per unit time of the offline policy independent of the energy arrival profile.

In general, solving (1) directly can be quite difficult, we instead derive upper- and lower-bounds on g. The lower-bounds can be obtained by consider only a subset of all the possible EH input sequences $\mathcal{S} \subset \{0, \mathbb{R}^+\}^N$, and minimizing over all the online policies $\mathcal{U}$, then $g \geq g_L$ with

$$g_L = \min_{\mathcal{U}} \max_{\mathbf{E} \in S} R_O\left(\mathbf{E}\right) - R_{\mathcal{U}}\left(\mathbf{E}\right)$$

Upper-bounds, can be obtained by fixing a particular the online policy $\mathcal{U}^*$ and solving

$$g_U = \max_{E^N \in \mathcal{E}} R_O - R_{\mathcal{U}^*}$$

If both upper- and lower-bound coincide then, we can conclude that the policy $\mathcal{U}^*$ is optimal. In that case, observe that an optimal policy satisfies

$$R_O\left(\mathbf{E}\right) - R_{\mathcal{U}^*}\left(\mathbf{E}\right) \leq g$$

for any input EH sequence, and thus the competitive rate gap informs us about the maximum number of bit/channel use that we may lose by adopting the online policy $\mathcal{U}^*$ instead of the optimal offline policy.

The competitive rate gap, here considered, is quite similar to the competitive ratio defined as

$$r = \min_{\mathcal{U}} \max_{\mathbf{E} \in \{0, \mathbb{R}^+\}^N} \frac{R_O\left(\mathbf{E}\right)}{R_{\mathcal{U}}\left(\mathbf{E}\right)}$$

The competitive ratio was addressed in [15] for a EH point to point slotted communication over a fading channels. There authors show, that if the power policy is online with respect to both; the EH input process and the channel fading process then the competitive ratio is equal to the number of slots $r = N$.

The competitive rate gap studied here complements the information provided by the competitive ratio. Observe that from the competitive ratio r one could wrongly conclude that, given that for an EH input sequence $\mathbf{E}$ it is satisfied that

$$\frac{R_O\left(\mathbf{E}\right)}{R_{\mathcal{U}^*}\left(\mathbf{E}\right)} \leq r$$

and, thus,

$$R_O\left(\mathbf{E}\right) - R_{\mathcal{U}^*}\left(\mathbf{E}\right) \leq (r-1) R_{\mathcal{U}^*}\left(\mathbf{E}\right)$$

then, the competitive rate gap is an increasing function of the online rate $R_{\mathcal{U}^*}\left(\mathbf{E}\right)$, and thus an increasing function of the total energy harvested. However, as we will show here the competitive rate gap is bounded and in particular it is satisfied that

$$R_O\left(\mathbf{E}\right) - R_{\mathcal{U}^*}\left(\mathbf{E}\right) \leq \log_2 N$$

for any EH input sequence $\mathbf{E}$.

III. Two Time Slots

We first consider the case of $N = 2$. In this case, we are able to determine the competitive rate gap and propose an online policy that achieves it. For convenience, let us introduce $\alpha_n\left(\langle E_1, ..., E_n\rangle\right) \in (0, 1]$, as the fraction of the available energy at time slot n, that is consumed in the same slot. Then, we can rewrite the energy consumed in slot n as

$$\begin{aligned} U_n &= \alpha_n B_n, \\ &= \alpha_n \left((1 - \alpha_{n-1}) \frac{U_{n-1}}{\alpha_{n-1}} + E_n \right), \quad n = 1, ...N. \quad (2) \end{aligned}$$

Theorem 1: The competitive rate gap for two time slots is given by

$$\begin{aligned} g &= \frac{1}{2} \log_2\left(\frac{4}{3}\right), \\ &= 0.2075 \text{ bits/s} \end{aligned}$$

and can be achieved by the online policy $\langle U_1, U_2 \rangle = \left\langle \frac{3}{4} E_1, \frac{1}{4} E_1 + E_2 \right\rangle$.

Proof: If $N = 2$, then the offline rate can be expressed as [2]

$$R_O = \begin{cases} \log_2\left(1 + \frac{E_1 + E_2}{T}\right), & \text{if } E_1 \geq E_2, \\ \frac{1}{2} \log_2\left(1 + 2\frac{E_1}{T}\right) + \frac{1}{2} \log_2\left(1 + 2\frac{E_2}{T}\right), & \text{if } E_1 \leq E_2. \end{cases}$$

whether, the online rate is given by

$$R_{\mathcal{U}} = \frac{1}{2} \log_2\left(1 + 2\frac{U_1}{T}\right) + \frac{1}{2} \log_2\left(1 + 2\frac{U_2}{T}\right)$$

where by using the notation previously introduced, we can write $U_1 = \alpha_1 E_1$ and $U_2 = \alpha_2 \left((1 - \alpha_1) \frac{U_1}{\alpha_1} + E_2 \right)$.

Following the procedure described in previous section, in order to determine the competitive rate gap, we need to find an upper- and a lower-bound on it. To find a **lower bound** g_L, consider the following two input EH sequences: $\mathbf{E}_1 = \lim_{E_1 \to \infty} \langle E_1, 0 \rangle$ and $\mathbf{E}_2 = \lim_{E_1 \to \infty} \langle E_1, E_1^2 \rangle$. Particularizing the online policy, we obtain

$$\begin{aligned} U_1\left(\mathbf{E}_1\right) &= U_1\left(\mathbf{E}_2\right) = \lim_{E_1 \to \infty} U_1\left(E_1\right) = \hat{\alpha}_1 E_1, \\ U_2\left(\mathbf{E}_1\right) &= \lim_{E_1 \to \infty} U_2\left(\langle E_1, 0 \rangle\right) = \hat{\alpha}_2^{(1)}\left(1 - \hat{\alpha}_1\right) E_1, \\ U_2\left(\mathbf{E}_2\right) &= \lim_{E_1 \to \infty} U_2\left(\langle E_1, E_1^2 \rangle\right) = \hat{\alpha}_2^{(2)} E_1^2 \end{aligned}$$

where, we have defined $\hat{\alpha}_1 = \lim_{E_1 \to \infty} \alpha_1\left(E_1\right)$, $\hat{\alpha}_2^{(1)} = \lim_{E_1 \to \infty} \alpha_2\left(\langle E_1, 0 \rangle\right)$ and $\hat{\alpha}_2^{(2)} = \lim_{E_1 \to \infty} \alpha_2\left(\langle E_1, E_1^2 \rangle\right)$. Then, the rate gaps $G_{\mathcal{U}}\left(\mathbf{E}\right) = R_O\left(\mathbf{E}\right) - R_{\mathcal{U}}\left(\mathbf{E}\right)$ are given by

$$G_{\mathcal{U}}\left(\mathbf{E}_1\right) = \lim_{E_1 \to \infty} G_{\mathcal{U}}\left(\langle E_1, 0 \rangle\right) = -\frac{1}{2} \log_2\left(4\hat{\alpha}_1 \hat{\alpha}_2^{(1)}\left(1 - \alpha_1\right)\right),$$

$$G_{\mathcal{U}}\left(\mathbf{E}_2\right) = \lim_{E_1 \to \infty} G_{\mathcal{U}}\left(\langle E_1, E_1^2 \rangle\right) = -\frac{1}{2} \log_2\left(\hat{\alpha}_1 \hat{\alpha}_2^{(2)}\right).$$

Now, we can compute the competitive rate gap lower-bound g_L as

$$g_L = \min_{0 \leq \hat{\alpha}_1, \hat{\alpha}_2^{(1)}, \hat{\alpha}_2^{(2)} \leq 1} \max\left(G_{\mathcal{U}}\left(\mathbf{E}_1\right), G_{\mathcal{U}}\left(\mathbf{E}_2\right)\right).$$

Observe that $G_{\mathcal{U}}(\mathbf{E}_1)$ and $G_{\mathcal{U}}(\mathbf{E}_2)$ decrease with $\hat{\alpha}_2^{(1)}$ and $\hat{\alpha}_2^{(2)}$. Then, g_L can be found by fixing $\hat{\alpha}_2^{(1)} = \hat{\alpha}_2^{(2)} = 1$. The optimal $\hat{\alpha}_1$ is found in the equality of $G_{\mathcal{U}}(\mathbf{E}_1)$, and $G_{\mathcal{U}}(\mathbf{E}_2)$ at $\alpha_1^* = \frac{3}{4}$, obtaining

$$ g_L = \frac{1}{2}\log_2\left(\frac{4}{3}\right). $$

To derive the competitive rate gap **upper-bound** g_U, let us fix the online policy by choosing $\langle U_1^*, U_2^* \rangle = \left\langle \frac{3}{4}E_1, \frac{1}{4}E_1 + E_2 \right\rangle$ or, equivalently, $\alpha_1^*(E_1) = \frac{3}{4}$ and $\alpha_2^*(\langle E_1, E_2 \rangle) = 1$, then

$$ g_U = \max_{E_1, E_2 \geq 0} G_{\mathcal{U}^*}(\langle E_1, E_2 \rangle). $$

Consider first that $E_2 < E_1$. Then, $G_{\mathcal{U}^*}(\mathbf{E})$ decreases if $0 \leq E_2 \leq \frac{E_1}{2}$ and increases if $E_2 \geq \frac{E_1}{2}$. Consequently, $G_{\mathcal{U}^*}(\mathbf{E})$ is maximized either at $E_2 = E_1$ or at $E_2 = 0$. In both cases, $G_{\mathcal{U}^*}(\mathbf{E})$ increases monotonically with E_1 and, we have

$$ G_{\mathcal{U}^*}(\mathbf{E}) \leq \lim_{E_1 \to \infty} G_{\mathcal{U}^*}(\langle E_1, 0 \rangle) = \lim_{E_1 \to \infty,} G_{\mathcal{U}^*}(\langle E_1, E_1 \rangle) $$
$$ = \frac{1}{2}\log_2\left(\frac{4}{3}\right). $$

Instead, if $E_1 < E_2$, $G_{\mathcal{U}^*}(\mathbf{E})$ increases monotonically with E_2 for any E_1. Moreover at $E_2 \to \infty$, $G_{\mathcal{U}^*}(\mathbf{E})$ increases monotonically with E_1. Let us define the ratio $\frac{E_2}{E_1} = \beta$, then $G_{\mathcal{U}^*}(\mathbf{E})$ satisfies

$$ G_{\mathcal{U}^*}(\mathbf{E}) = \max_\beta \lim_{E_1 \to \infty} G(\langle E_1, \beta E_1 \rangle), $$
$$ = \max_\beta \frac{1}{2}\log_2\left(\frac{4}{1+4\beta}\frac{4}{3}\beta\right), $$
$$ \leq \frac{1}{2}\log_2\left(\frac{4}{3}\right) $$

with equality as $\beta \to \infty$. Consequently, we conclude that

$$ g_U = \frac{1}{2}\log_2\left(\frac{4}{3}\right). $$

Given that, the upper-bound and the lower-bound coincide we have $g = g_U = g_L$. Moreover, the online policy $\alpha_1^*(E_1) = \frac{3}{4}$ and $\alpha_2^*(\langle E_1, E_2 \rangle) = 1$ achieves the competitive rate gap. ∎

IV. N TIME SLOTS

Next, we study the situation where the transmission is divided into N slots. In this case, we derive an upper-bound on the competitive rate gap and present an online policy which obtains a lower competitive rate gap.

Theorem 2: The competitive rate gap for a EH point-to-point communication over N time slots is upper-bounded by

$$ g \leq \log_2(N). $$

The online transmission policy $\mathcal{U} = \langle U_1, ..., U_N \rangle$, with

$$ U_n = \sum_{l=1}^{n} \frac{E_l}{N-l+1}, \quad n = 1, ..., N \quad (3) $$

achieves a lower competitive rate gap.

Before proving Theorem 2, we introduce a sufficient condition on the input EH sequences $\mathbf{E}$ that, if satisfied, ensures that the optimal offline policy consists on spending all the energy harvested in time slot $n-1$ in time slot n.

Lemma 1: If the energy harvested input sequence $\mathbf{E} = \langle E_1, ..., E_N \rangle$ satisfies

$$ E_n \geq \frac{1}{n-m}\sum_{i=m}^{n-1} E_i \text{ for all } m < n \text{ and } n = 1, .., N \quad (4) $$

then, the rate maximizing offline policy is $U_n = E_n$ for $n = 1, ..., N$ and the offline rate can be expressed as

$$ R_O(\mathbf{E}) = \frac{1}{N}\sum_{n=1}^{N} \log_2\left(1 + N\frac{E_n}{T}\right). $$

Proof: It was shown in [2] that if the overall transmission duration is T, then the optimal transmission policy is the one that yields the tightest piecewise linear energy consumption curve that lies under the energy harvesting curve $H(t) = \sum_{n < N\frac{t}{T}} E_n$ at all times t and touches the energy harvesting curve at $t = T$. Define $H_n = H(n\frac{T}{N})$ and the slope of the curves connecting $H(n\frac{T}{N})$ with $H(m\frac{T}{N})$ as

$$ s_{n,m} = \frac{N}{T}(H_n - H_m), $$

then, for the case of equal slot durations, the optimal energy consumption curve connects $H(T)$ with $H((N-1)\frac{T}{N})$ if the slope of the line connecting $H(T)$ and $H((N-1)\frac{T}{N})$ is larger or equal than slope of the lines connecting $H(T)$ and $H(m\frac{T}{N})$, for all $m < n$, namely $s_{N,N-1} \geq s_{N,m}$, for all $m < N-1$.

We, additionally, require the optimal energy consumption curves to touche the energy harvesting curve at $t = n\frac{T}{N}$ for all n, then, the EH input sequence must satisfy $s_{n,n-1} \geq s_{n,m}$, for all n and $m < n$ or, equivalently (4). ∎

Next, we prove Theorem 2.

Proof: Following the procedure descried in Section II-A, we can find an upper-bound g_U on the competitive rate gap by fixing a particular online policy. In this case, we use $\mathcal{U}$ in (3), which is equivalent to choosing $\alpha_n = \frac{1}{N-n+1}$ in (2). For the offline rate, we use an upper-bound, which we obtain by simply considering that at slot n, the energy harvested is $\hat{U}_n = \sum_{l=1}^{n} E_l$. The input EH sequence $\hat{\mathbf{E}} = \left\langle \hat{E}_1, ..., \hat{E}_N \right\rangle$ satisfies the condition in Lemma 1, as we can check simply by showing that

$$ \hat{U}_n \geq \frac{1}{n-m}\sum_{i=m}^{n-1} \hat{U}_i, \quad (5) $$

for all $m < n$ and $n = 1, .., N$. Observe that substituting (??) into (5), condition (5), can be rewritten as

$$ \sum_{l=1}^{n} E_l \geq \frac{1}{n-m}\sum_{i=m}^{n-1}\sum_{l=1}^{i} E_l, $$
$$ = \sum_{i=1}^{m} E_i + \frac{1}{n-m}\sum_{i=m+1}^{n-1}(i-m)E_l. $$

Now, observe also that for any EH input sequence $\mathbf{E}$, we have

$$\sum_{l=1}^{n} E_l \geq \sum_{i=1}^{n.-1} E_i,$$
$$\geq \sum_{i=1}^{m} E_i + \frac{1}{n-m} \sum_{i=m+1}^{n-1} (i-m) E_l,$$

and thus, condition (5) is always satisfied and the offline rate can be upper-bounded by

$$\hat{R}_O(\mathbf{E}) = \frac{1}{N} \sum_{n=1}^{N} \log_2 \left(1 + N \frac{\hat{U}_n}{T} \right).$$

We are now ready to formulate the competitive rate gap upper-bound problem, as

$$r_U = \max_{\mathbf{E} \in \{0, \mathbb{R}^+\}^N} \hat{R}_O(\mathbf{E}) - R_U(\mathbf{E}).$$

Define $\bar{G}_N(\mathbf{E}) = \hat{R}_O(\mathbf{E}) - R_U(\mathbf{E})$, for a N slots communication. In the following, we show that $\bar{G}_N(\mathbf{E})$ is maximum at $\mathbf{E} = \lim_{E_1 \to \infty} \langle E_1, 0, ..., 0 \rangle$. First, observe that $\bar{G}_N$ can be written as a function of $\bar{G}_{N-1}$ as

$$\bar{G}_N = \bar{G}_{N-1} + \frac{1}{N} \log_2 \left(\frac{1 + N \frac{\hat{U}_N}{T}}{1 + N \frac{U_N}{T}} \right).$$

Denote as $\mathbf{0}_l$ the length l zero vector, and the EH input sequence $\mathbf{E}_{i,j} = \langle E_j, ..., E_j \rangle$. By taking the first order derivative of $\bar{G}_N$ with respect to E_N, it can be shown, that $\bar{G}_N$ is a monotonically decreasing function of E_N. Consequently, $\bar{G}_N$ is maximized by choosing $E_N = 0$. This is,

$$\bar{G}_N(\mathbf{E}) \leq \bar{G}_N(\langle \mathbf{E}_{1,N-1}, 0 \rangle),$$

where

$$\bar{G}_N(\langle \mathbf{E}_{1,N-1}, 0 \rangle)$$
$$= \bar{G}_{N-1}(\mathbf{E}_{1,N-1}) + \frac{1}{N} \log_2 \left(\frac{1 + N \frac{\hat{U}_{N-1}}{T}}{1 + N \frac{U_{N-1}}{T}} \right). \quad (6)$$

Next, by taking the first order derivative of $\bar{G}_N(\langle E_{1,N-1}, 0 \rangle)$ with respect to E_{N-1}, we observe that $\bar{G}_N(\langle E_{1,N-1}, 0 \rangle)$ increases with E_{N-1}, if

$$\hat{U}_{N-2} < \frac{T}{N} + 2U_{N-2}, \quad (7)$$

and, otherwise, decreases. Consequently, $\bar{G}_N\left(\langle E_1^{N-1}, 0 \rangle\right)$ is maximized either as $E_{N-1} \to \infty$ or at $E_{N-1} = 0$, this is

$$\bar{G}_N(\mathbf{E}) \leq \begin{cases} \bar{G}_N(\langle \mathbf{E}_{1,N-2}, 0, 0 \rangle), & \text{if } \hat{U}_{N-2} \geq \frac{T}{N} + 2U_{N-2} \\ \bar{G}_N(\langle \mathbf{E}_{1,N-2}, \infty, 0 \rangle), & \text{otherwise.} \end{cases}$$

where

$$\bar{G}_N(\langle \mathbf{E}_{1,N-2}, \infty, 0 \rangle) = \bar{G}_{N-2}(\mathbf{E}_{1,N-2}) + \frac{2}{N} \log_2(2),$$
$$\bar{G}_N(\langle \mathbf{E}_{1,N-2}, 0, 0 \rangle) = \bar{G}_{N-2}(\mathbf{E}_{1,N-2})$$
$$+ \frac{1}{N} \log_2 \left(\frac{1 + N \frac{\hat{U}_{N-2}}{T}}{1 + N \frac{U_{N-2}}{T}} \right).$$

Next, we maximize $\bar{G}_N\left(\langle \mathbf{E}_1^{N-2}, \infty, 0 \rangle\right)$ and $\bar{G}_N\left(\langle \mathbf{E}_1^{N-2}, 0, 0 \rangle\right)$ with respect to E_{N-2}. First, observe that both increase with E_{N-2} if

$$\hat{U}_{N-3} < 2\frac{T}{N} + 3U_{N-3}, \quad (8)$$

and decrease otherwise. Then, observe that condition (7) can be rewritten in terms of E_{N-2}, as

$$E_{N-2} < 3 \left(\frac{T}{N} + 2U_{N-3} - \hat{U}_{N-3} \right). \quad (9)$$

Suppose $\frac{T}{N} + 2U_{N-3} > \hat{U}_{N-3}$, then condition (8) is always satisfied, and $\bar{G}_N(\langle E_{1,N-2}, \infty, 0 \rangle)$ is maximized in the equality of (9) whereas $\bar{G}_N(\langle E_{1,N-2}, 0, 0 \rangle)$ is maximized by letting $E_{N-2} \to \infty$. Evaluating both cases, we have

$$\bar{G}_N\left(\langle E_{1,N-3}, E_{N-2}^*, \infty, 0 \rangle\right) = \bar{G}_{N-3} + \frac{3}{N},$$
$$\bar{G}_N\left(\langle E_{1,N-3}, \infty, 0, 0 \rangle\right) = \bar{G}_{N-3} + \frac{3}{N} \log_2(3).$$

Suppose instead that $\hat{U}_{N-3} > \frac{T}{N} + 2U_{N-3}$, then condition (9) is never satisfied and, we only need to consider $\bar{G}_N(\langle E_{1,N-2}, 0, 0 \rangle)$, which is maximized either by letting $E_{N-2} \to \infty$ if $\hat{U}_{N-3} < 2\frac{T}{N} + 3U_{N-3}$ or otherwise at $E_{N-1} = 0$. Thus,

$$\bar{G}_N\left(\langle E_{1,N-2}, \infty, 0, 0 \rangle\right) = \bar{G}_{N-3} + \frac{3}{N} \log_2(3),$$
$$\bar{G}_N\left(\langle E_{1,N-2}, 0, 0, 0 \rangle\right) = \bar{G}_{N-3} + \frac{3}{N} \log_2 \left(\frac{1 + N \frac{\hat{U}_{N-3}}{T}}{1 + N \frac{U_{N-3}}{T}} \right).$$

Summarizing the results above, we have

$$\bar{G}_N(\mathbf{E}) \leq \begin{cases} \bar{G}_N(\langle \mathbf{E}_{1,N-3}, \infty, \mathbf{0}_2 \rangle) & \text{if } \hat{U}_{N-3} < 2\frac{T}{N} + 3U_{N-3}, \\ \bar{G}_N(\langle \mathbf{E}_{1,N-3}, \mathbf{0}_3 \rangle) & \text{if } \hat{U}_{N-3} > 2\frac{T}{N} + 3U_{N-3} \end{cases}$$

Following this procedure, for $j = 1$, we obtain

$$\bar{G}(\langle E_1, \infty, \mathbf{0}_{N-2} \rangle) = \frac{1}{N} \log_2 \left(\frac{1 + N \frac{E_1}{T}}{1 + \frac{E_1}{T}} \right) + \frac{N-1}{N} \log_2(N-1)$$

if $E_1 < (N-2)T$ and

$$\bar{G}(\langle E_1, \mathbf{0}_{N-1} \rangle) = \log_2 \left(\frac{1 + N \frac{E_1}{T}}{1 + \frac{E_1}{T}} \right)$$

if $E_1 > (N-2)T$. Consequently, the competitive rate gap is upper-bounded by

$$r_U = \max \bar{G}(\mathbf{E}),$$
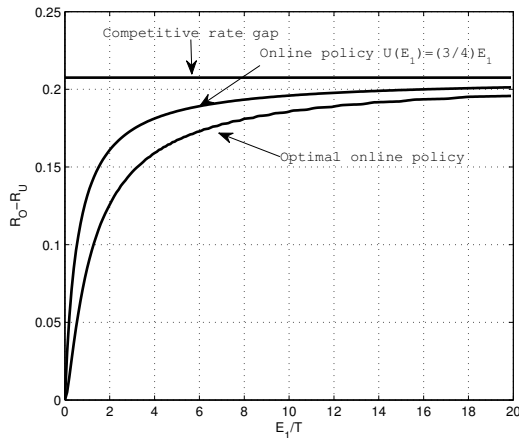$$\leq \bar{G}_N(\langle \infty, \mathbf{0}_{N-1} \rangle),$$
$$= \log_2(N).$$

∎

Fig. 1: Rate gap as a function of E_1, for $N = 2$.

V. NUMERICAL RESULTS

In this section, we illustrate the competitive rate gap for the case of two slots. Assume that the EH process is completely unknown to the source, then the optimal online policy consist on choosing $U_2(\langle E_1, E_2 \rangle) = E_1 + E_2 - U_1$ and $U_1(E_1)$ as the solution to

$$g(E_1) = \min_{U_1(E_1)} \max_{E_2} R_O - R_U. \qquad (10)$$

for all E_1. We solve (10) numerically. The resultant competitive rate gap as a function of the energy available in the first slot E_1 is depicted in Fig. 10, together with the bound given by the competitive rate gap $R_O - R_U < 0.2075$, and the rate gap obtained by using the competitive rate gap optimal online strategy here proposed $U_1(E_1) = \frac{3}{4}E_1$, and $U_2(\langle E_1, E_2 \rangle) = \frac{3}{4}E_1 + E_2$. Observe that, the competitive gap $g(E_1)$ is an increasing function with E_1 bounded by $0.207\,52$. Observe also that the proposed online strategy is indeed very close to the optimal online strategy, when the statistics of the EH process are ignored.

VI. CONCLUSIONS

In this paper, we study the competitive rate gap for EH communication systems. For a communication divided in two slots, we showed the competitive gap is 0.20 bits/channel use, namely, the maximum difference between the rate obtained with an offline power policy and an online power policy can be made as low as 0.20 bits/channel use, regardless of the amount of energy harvested in each slot. For a communication divided into N slots, we showed that the rate gap is upper-bounded by $\log_2 N$. We observed that the competitive rate gap upper-bound obtained here for the case of N time slots is lossy for $N = 2$, $0.207\,52 < \log_2(2) = 1$. In future immediate work we will address the problem of obtaining the competitive rate gap for the case of N slots, by improving the rate gap upper-bound and deriving a lower-bound on it. In addition, these results can be extended to EH terminals with battery capacity constraints, as well as, to fading channels. The extension to

multi-terminal communications, such as the multiple access, the broadcast, the relay and the interference channel are also of interest.

REFERENCES

[1] D. Gunduz, K. Stamatiou, N. Michelusi, and M. Zorzi, "Designing intelligent energy harvesting communication systems," *Communications Magazine, IEEE*, vol. 52, no. 1, pp. 210–216, January 2014.
[2] J. Yang and S. Ulukus, "Optimal packet scheduling in an energy harvesting communication system," *Communications, IEEE Transactions on*, vol. 60, no. 1, pp. 220–230, January 2012.
[3] K. Tutuncuoglu and A. Yener, "Optimum transmission policies for battery limited energy harvesting nodes," *Wireless Communications, IEEE Transactions on*, vol. 11, no. 3, pp. 1180–1189, March 2012.
[4] B. Devillers and D. Gunduz, "A general framework for the optimization of energy harvesting communication systems with battery imperfections," *Communications and Networks, Journal of*, vol. 14, no. 2, pp. 130–139, April 2012.
[5] O. Ozel, J. Yang, and S. Ulukus, "Optimal broadcast scheduling for an energy harvesting rechargeable transmitter with a finite capacity battery," *Wireless Communications, IEEE Transactions on*, vol. 11, no. 6, pp. 2193–2203, June 2012.
[6] J. Yang and S. Ulukus, "Optimal packet scheduling in a multiple access channel with energy harvesting transmitters," *Communications and Networks, Journal of*, vol. 14, no. 2, pp. 140–150, April 2012.
[7] K. Tutuncuoglu and A. Yener, "Sum-rate optimal power policies for energy harvesting transmitters in an interference channel," *Communications and Networks, Journal of*, vol. 14, no. 2, pp. 151–161, April 2012.
[8] D. Gunduz and B. Devillers, "Two-hop communication with energy harvesting," in *Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP), 2011 4th IEEE International Workshop on*, Dec 2011, pp. 201–204.
[9] O. Orhan and E. Erkip, "Throughput maximization for energy harvesting two-hop networks," in *Information Theory Proceedings (ISIT), 2013 IEEE International Symposium on*, July 2013, pp. 1596–1600.
[10] O. Ozel, K. Tutuncuoglu, J. Yang, S. Ulukus, and A. Yener, "Transmission with energy harvesting nodes in fading wireless channels: Optimal policies," *Selected Areas in Communications, IEEE Journal on*, vol. 29, no. 8, pp. 1732–1743, September 2011.
[11] C. K. Ho and R. Zhang, "Optimal energy allocation for wireless communications with energy harvesting constraints," *Signal Processing, IEEE Transactions on*, vol. 60, no. 9, pp. 4808–4818, Sept 2012.
[12] O. Ozel and S. Ulukus, "Achieving awgn capacity under stochastic energy harvesting," *Information Theory, IEEE Transactions on*, vol. 58, no. 10, pp. 6471–6483, Oct 2012.
[13] M. Kashef and A. Ephremides, "Optimal packet scheduling for energy harvesting sources on time varying wireless channels," *Communications and Networks, Journal of*, vol. 14, no. 2, pp. 121–129, April 2012.
[14] P. Blasco, D. Gunduz, and M. Dohler, "A learning theoretic approach to energy harvesting communication system optimization," *Wireless Communications, IEEE Transactions on*, vol. 12, no. 4, pp. 1872–1882, April 2013.
[15] R. Vaze, R. Garg, and N. Pathak, "Dynamic power allocation for maximizing throughput in energy-harvesting communication system," pp. 1–1, 2013.
[16] A. El Gamal, C. Nair, B. Prabhakar, E. Uysal-Biyikoglu, and S. Zahedi, "Energy-efficient scheduling of packet transmissions over wireless networks," in *INFOCOM 2002. Twenty-First Annual Joint Conference of the IEEE Computer and Communications Societies. Proceedings. IEEE*, vol. 3, 2002, pp. 1773–1782 vol.3.
[17] N. Chang and M. Liu, "Competitive analysis of opportunistic spectrum access strategies," in *INFOCOM 2008. The 27th Conference on Computer Communications. IEEE*, April 2008, pp. –.
[18] N. Buchbinder, L. Lewin-Eytan, I. Menache, J. Naor, and A. Orda, "Dynamic power allocation under arbitrary varying channels - an online approach," in *INFOCOM 2009, IEEE*, April 2009, pp. 145–153.
[19] ——, "Dynamic power allocation under arbitrary varying channels - the multi-user case," in *INFOCOM, 2010 Proceedings IEEE*, March 2010, pp. 1–9.
[20] R. Kannan, S. Wei, J. Zhang, and V. Athanasios, "Throughput optimization for cognitive radios under sensing uncertainty," in *Military Communications Conference, 2010*, Oct 2010, pp. 767–772.