Energy Harvesting Communication System with a Finite Set of Transmission Rates

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Abstract—**A point-to-point communication system in which the transmitter is equipped with an energy harvesting (EH) device and a rechargeable battery of limited size is considered. The harvested energy profile is assumed to be known in advance, that is, the problem is studied in the offline optimization framework. A practical communication system is considered, in which the possible transmission rates, and equivalently, power levels, belong to a finite set of discrete values. This problem is formulated as a convex optimization problem, which can be solved numerically. In order to provide further insights into the nature of the optimal transmission policy, an alternative solution is provided based on the solution of the continuous version of this problem, which itself has a "shortest path" interpretation. We propose an optimal algorithm, which permits us to easily build the solution of the discrete case from the continuous case, using an equivalence notion between different transmission policies, and several equivalence- preserving transformations.**

I. INTRODUCTION

Energy harvesting (EH) has recently emerged as a promising technology to extend the lifetime of communication networks. Today's battery limited wireless devices have limited lifetimes, which can be extended using rechargeable batteries and EH technology. Thanks to the recent advances in the efficiency of EH devices, the environment offers an unlimited energy supply that can be harvested and used for communication. More than simply being a sustainable and green energy source, harvested ambient energy permits us to extend the lifetime of a device to its material constraint limits. Nevertheless, depending on the characteristics of the energy source and the environment, harvested energy quantity can vary significantly over space and time. It is thus essential that this limited energy be used in the most efficient way.

The design of EH communication systems has motivated a substantial amount of research activity in recent years [1]–[10]. Assuming that the profile of the harvested energy is known non-causally, i.e., offline optimization, the optimal transmission strategy has been studied for point-to-point systems with discrete [1] as well as continuous [2] energy arrivals. Follow-up work has extended this model and solutions to fading channels [3], various multi-user scenarios [11], multi-hop relay networks [6] as well as an EH receiver model [12]. In online optimization, only a statistical knowledge is assumed about the energy harvesting and the data arrival processes, and the optimization problem is modelled as a Markov Decision Process with the long-term average throughput as the performance measure [8], [9], [13], [14].

In online optimization problems, possible transmission rates in the system are generally assumed to be discrete in order to limit the dimension of the state space; and hence, the complexity of the corresponding dynamic programming solution [8], [13], [14]. However, to the best of our knowledge, no prior work has so far considered a discrete set of available transmission rate/ power levels in offline optimization. In practice, devices can transmit only from a predetermined discrete and finite set of transmission rates. Assuming that the transmitter intends to keep the error probability constant throughout the transmission, this corresponds to a finite and discrete set of transmission power values as well. As a consequence the existing literature that studies the continuous problem does not reflect the real performance of EH communication systems in practice.

Our goal in this paper is to characterize the optimal transmission policy for any given EH profile and rate-power function for a target error probability, taking into account the discrete values of power levels. This optimization problem is shown to be convex; and hence, lends itself to efficient numerical solutions; however, the main contribution of our paper is a low-complexity algorithm based on the geometric interpretation of the optimal transmission policy. This algorithm allows us to obtain the optimal solution of the discrete problem directly from the optimal solution of the continuous problem, which itself has an attractive "shortest-path" interpretation.

The rest of the paper is organized as follows. The system model and the problem description is presented in Section II. Section III is dedicated to various definitions that will be used in the rest of the paper to prove the optimality of the proposed algorithm. In Section IV we consider the solution of the DP when the optimal solution of the OP is a constant power transmission curve, and an algorithm that provides the transmission policy with discrete set of transmission powers is presented. In Section V we generalize this algorithm to any scenario, and prove its optimality. Section VI provides some numerical results illustrating the loss due to the discreteness of the transmission rates. Finally, we conclude our paper in Section VII.

II. SYSTEM MODEL

We consider an EH transmitter communicating data over a point-to-point channel. Similarly to [1] the EH process is characterized by a packet arrival model; that is, the transmitter

receives an energy packet of H_n units of energy at time instant s_n, for $n = 1, \ldots, N$. We assume that the energy arrival instants and the packet sizes are known in advance by the transmitter, i.e., offline optimization. We let $s_1 = 0$, and define $s_{n+1} = T$ as the deadline for the operation of the transmitter. The time period between two consecutive energy packet arrivals s_n and s_{n+1} is called *epoch* n. We then define the length of epoch n as $T_n \triangleq s_{n+1} - s_n$, for $n = 1, \ldots, N$.

Following [2] and [4], we assume that the transmitter is equipped with a finite-capacity battery; that is, the battery can store up to H_{max} units of energy. We assume that the harvested energy packet E_n is first stored into the battery, and can then be used for transmission after time s_n; hence, we can assume that $E_n \leq H_{max}, \forall n$.

We consider continuous-time transmission, where $p(t) \geq 0$ is the instantaneous transmission power at time t. To the best of our knowledge, in all the previous literature on EH communication systems, zero-error communication is assumed, and the transmitter is allowed to transmit at any desired rate at any point in time. The instantaneous rate is then assumed to be a monotonically increasing function of the instantaneous transmission power. This corresponds to a continuous optimization problem, in which the transmitter optimizes its transmission power over time period $[0, T]$, where $p(t)$ at time t can take any non-negative value. However, in practical communication systems, a transmitter is limited to a set of pre-defined modulation schemes and coding rates; and hence, the transmission rate can only be chosen from a discrete set of rates. While the transmission power can be chosen arbitrarily, once the modulation scheme and the coding rate is fixed, increasing the transmission power improves the reliability, but not the transmission rate.

Motivated by this practical constraint, here we assume that the transmitter can choose from a finite set of transmission rates. We also assume that the transmitter has a fixed target error probability throughout its transmission, which fixes the corresponding transmit power for each rate. Equivalently, we can consider a finite set of transmission power levels that the transmitter can choose from, denoted by $\mathcal{P} = \{p_k, k = 1, \ldots, K\}$. For a fixed target error probability, the rate corresponding to each transmit power level is given by the rate-power function $r(p)$, which is assumed to be a non-negative, increasing, strictly concave function.

We consider the problem of maximizing the total transmitted data by time T. Considering there are no retransmissions, this corresponds to maximizing the throughput, since the target error probability is fixed. Our model also assumes that there is an infinite amount of data to be transmitted in the data-backlogged system.

We use the cumulative curve approach to state the problem. We denote by $E(t)$ the *transmitted energy curve*, which represents the energy that has been used for transmission until time t. We define $H(t)$ as the *harvested energy curve*, which denotes the total energy harvested by time t. Assuming that the battery is empty for $t < 0$, we have $E(0) = 0$, and since one cannot use more energy than that has been harvested, i.e.,

the *energy causality constraint*, we have $E(t) \leq H(t)$ for $t \geq 0$. In order to harvest as much energy as possible from the environment; and thus, to maximize its use, we want to avoid battery overflows. Defining $M(t) \triangleq H(t) - H_{max}$ as the *minimum energy curve*, the *no-battery-overflow constraint* can be achieved imposing $E(t) \geq M(t)$ for $t \geq 0$.

We first consider the problem without any limitations on the possible transmission rate or power values; that is, $\mathcal{P} = \mathbb{R}^+$, where $\mathbb{R}^+$ denotes the set of non-negative real numbers. We call this the *original problem (OP)*. It is well-known that the optimal transmission strategy keeps the transmission power fixed within each epoch due to the strict concavity of the $r(\cdot)$ [1]. Denoting the transmission power in epoch i by p_i, the OP is formulated as follows:

$$\max_{p_n \geq 0} \sum_{n=1}^{N} T_n r(p_n) \tag{1}$$
$$\text{s.t} \sum_{n=1}^{L} T_n p_n \leq \sum_{n=1}^{L} H_n \qquad L = 1, \ldots, N,$$
$$\sum_{n=1}^{L+1} H_n - \sum_{n=1}^{L} T_n p_n \leq H_{max} \qquad L = 2, \ldots, N-1.$$

This is a convex optimization problem, and as such, it has a unique solution. More importantly, the optimal solution has a "shortest path" interpretation [2]. The optimal power allocation strategy $E(t)$, is given by the shortest path from the origin $(0, 0)$, to the point $(T, H(T))$ that lies between $H(t)$ and $M(t)$ for all $t \geq 0$. This interpretation also allows us to design a simple algorithm that identifies the optimal power allocation scheme [1], instead of being limited to numerical solutions.

Now we consider the case in which the transmission power can only be chosen from a finite set of values, $\mathcal{P} = \{p_1, \ldots, p_K\}$. We assume that $0 \in \mathcal{P}$. We call this the *discrete problem (DP)*. Denoting the time within epoch i that the transmitter transmits with power p_k by $t_{i,k}$, the optimisation problem can be written as follows:

$$\max_{t_{n,k} \geq 0} \sum_{n=1}^{N} \sum_{k=1}^{K} t_{n,k} r(p_k) \tag{2}$$
$$\text{s.t} \sum_{n=1}^{L} \sum_{k=1}^{K} t_{n,k} p_n \leq \sum_{n=1}^{L} H_i, \qquad L = 1, \ldots, N,$$
$$\sum_{n=1}^{L+1} H_n - \sum_{n=1}^{L} \sum_{k=1}^{K} t_{n,k} p_n \leq H_{max}, \quad L = 2, \ldots N-1,$$
$$\sum_{k=1}^{K} t_{n,k} = T_n, \qquad n = 1, \ldots, N.$$

Remark 1: As opposed to OP, in which p_n are the optimisation variables, in DP, the optimisation variables are $t_{n,k}$. Similarly to OP, the DP is also a convex optimisation problem.

While the above problem can be solved numerically, our goal in the rest of the paper is to come up with a low complexity algorithm, similar to the shortest path algorithm for OP, which will provide a better understanding of the

limitations and the consequences of limited set of transmission rates in an EH communication system.

III. NECESSARY DEFINITIONS

Let $E(t)$ be any piecewise-linear transmitted energy curve, consisting of M linear components. Then $E(t)$ can be described by a set $\mathcal{E} = \{(p_m, t_m), m = 1, \ldots, M\} \in (\mathbb{R}^+ \times \mathbb{R}^{+*})^M$, where $\sum_{m=1}^{M} t_m = T$, and $\mathbb{R}^{+*}$ denotes the set of positive real numbers. The set $\mathcal{E}$ is ordered, in the sense that the transmitter first transmits at power p_1 for t_1 time units, then at power p_2 for t_2, and so on so forth. Note that the same power level can appear multiple times in this set. In the rest of the paper we will use $E(t)$ and $\mathcal{E}$ interchangeably to denote the same transmission strategy. We have:

$$E(t) = \sum_{i=1}^{j(t)} p_i t_i + p_{j(t)+1} \left(t - \sum_{i=1}^{j(t)} t_i \right),$$

where

$$j(t) \triangleq \max \left\{ j : \sum_{i=1}^{j} t_i \le t \right\}.$$

For a given $r(p)$, the total amount of transmitted data by time t is then given by

$$D_E(t) = \sum_{i=1}^{j(t)} r(p_i) t_i + r(p_{j(t)+1}) \left(t - \sum_{i=1}^{j(t)} t_i \right).$$

Definition 1: We say that two energy curves $E_1(t)$ and $E_2(t)$, with their respective descriptions $\mathcal{E}_1$ and $\mathcal{E}_2$, are *equivalent* over a period T, denoted by $\mathcal{E}_1 \overset{T}{\Leftrightarrow} \mathcal{E}_2$, if they transmit the same amount of data using the same amount of energy:

$$\mathcal{E}_1 \overset{T}{\Leftrightarrow} \mathcal{E}_2 \text{ iff } E_1(T) = E_2(T) \text{ and } D_{E_1}(T) = D_{E_2}(T).$$

There are many ways to obtain equivalent policies, and we define two such operations: re-ordering (ReO) and redistributing (ReD), that permit to modify a transmitted energy curve description without loss of equivalence.

$$ReD : (\mathbb{R}^+ \times \mathbb{R}^{+*})^M \times \mathbb{N}^+ \longmapsto (\mathbb{R}^+ \times \mathbb{R}^{+*})^{aM},$$
$$ReD(\mathcal{E}, a) = \left\{ \left(\left(p_1, \frac{t_1}{a} \right), \ldots, \left(p_1, \frac{t_1}{a} \right), \ldots \right. \right.$$
$$\left. \left. \left(p_M, \frac{t_M}{a} \right), \ldots, \left(p_M, \frac{t_M}{a} \right) \right) \right\}. \quad (3)$$

$$ReO : (\mathbb{R}^+ \times \mathbb{R}^{+*})^M \times (\mathbb{N}^*)^2 \longmapsto (\mathbb{R}^+ \times \mathbb{R}^{+*})^M.$$

For $\mathcal{E} = \{(p_1, t_1), \ldots, (p_a, t_a), \ldots, (p_b, t_b), \ldots, (p_M, t_M)\}$, we have

$$ReO(\mathcal{E}, a, b) = \{(p_1, t_1), \ldots, (p_b, t_b), \ldots,$$
$$(p_a, t_a), \ldots, (p_M, t_M)\}. \quad (4)$$

Re-distributing an energy curve $\mathcal{E}$ divides each of its epoch into a smaller epochs with the same transmission power. In other words, re-distributing can be seen as increasing the

'resolution'. *Re-ordering* an energy curve is simply switching the order of two epochs.

Proposition 1: Given $\mathcal{E} = \{(p_m, t_m), m = 1, \ldots, M\}$, we have the following equivalences over the period T:

$$ReD(\mathcal{E}, a) \overset{T}{\Leftrightarrow} \mathcal{E}, \forall a \in \mathbb{N}^+, \quad (5)$$
$$ReO(\mathcal{E}, a, b) \overset{T}{\Leftrightarrow} \mathcal{E}, \forall a, b \in \{1, \ldots, M\}. \quad (6)$$

Proof: These two operations change neither the total energy used during T, nor the amount of transmitted data. ∎

Remark 2: Even if an equivalent transmitted energy curve is obtained with these operations, it is possible that energy causality, or no-battery-overflow constraints are violated.

For a given energy curve $\mathcal{E} = \{(p_m, t_m), m = 1, \ldots, M\}$, we can use operation $ReO(\mathcal{E})$ to re-order the transmission policy into:

$$\text{MinC}(\mathcal{E}) = \{(p_m, t_m), m = 1, \ldots, M\} \text{ with } p_{m+1} \ge p_m \ \forall m.$$

Note that $\text{MinC}(\mathcal{E})$ is the *minimum* transmitted energy curve equivalent to $\mathcal{E}$, such that $\text{MinC}(\mathcal{E})(t) \le E(t) \ \forall t$.

Similarly, using ReO we can obtain:

$$\text{MaxC}(\mathcal{E}) = \{(p_m, t_m), m = 1, \ldots, M\} \text{ with } p_{m+1} \le p_m \ \forall m.$$

Then, $\text{MaxC}(\mathcal{E})$ is the *maximum* transmitted energy curve equivalent to $\mathcal{E}$, such that $\text{MaxC}(\mathcal{E})(t) \ge E(t) \ \forall t$.

Next we define a 'two-slope curve with respect to another curve' in Definition 2. This will be used to state Proposition 2, which claims that we can transform any curve that starts and ends at the same points with a constant transmission rate curve, into a 'two-slope curve' with respect to the latter, using the equivalence-preserving transformations ReO and ReD.

Definition 2: For a given energy cure $E_1(t)$, we say that an energy curve $E_2(t)$ is a *two-slope curve with respect to $E_1(t)$* if it uses a maximum of two different transmission power levels between any consecutive time instants that it crosses $E_1(t)$. The time intervals defined by the time instants at which the two curves cross are called the *two-slope intervals*.

In the next section we will show that if the optimal solution of OP for a given $r(t)$ and EH profile is a constant transmission power policy with p^*, then the optimal solution of DP for the same system uses the two closest power levels to p^*, one higher and one lower, available in $\mathcal{P}$. To prove this result, we first need to explain how any transmitted energy curve which uses the same amount of energy as the constant power policy with p^* is equivalent to a two-slope curve, as illustrated in Figure 1. Then on each two-slope interval we will be able to show that it is better to use the two closest power levels to p^* available in $\mathcal{P}$. This will prove our statement.

Proposition 2: Consider two energy curves $E^*(t)$ and $E(t)$, with their corresponding descriptions $\mathcal{E}^*$ and $\mathcal{E}$, respectively. Let $E^*(t)$ be a constant transmission rate curve with $\mathcal{E}^* = \{(p^*, T)\}$, and $E(T) = p^* T$. It is always possible to transform $\mathcal{E}$ into an equivalent two-slope curve with respect to $\mathcal{E}^*$ using only ReO and ReD. Moreover, one can always do this such that the new curve never crosses above or below $\mathcal{E}$.
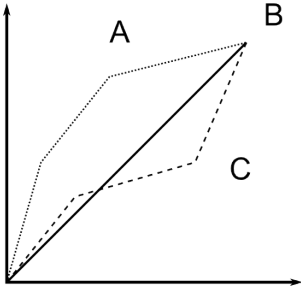
Fig. 1: Curve C is a two-slope curve with respect to B built from curve A using ReO.

The proof is omitted due to space limitations. See Fig. 1 for an example of such a transformation.

IV. CONSTANT RATE TRANSMISSION

We now consider the case in which the solution of OP is a constant power transmission scheme; i.e., $\mathcal{E}^* = (p^*, T)$. We want to find out the optimal transmission scheme if the transmitter is constrained to power levels from the set $\mathcal{P}$. Assume that we are constrained to use only two different power levels from $\mathcal{P}$. Let $\mathcal{D}(p_i, p_j)$ denote the maximum transmitted data if we use power levels $p_i, p_j \in \mathcal{P}$. We can prove the following proposition.

Proposition 3: Consider transmission power values $p_1, \ldots, p_4 \in \mathcal{P}$ such that: $p_1 < p_2 < p^* < p_3 < p_4$. Then, we have

$$\mathcal{D}(p_2, p_3) > \mathcal{D}(p_1, p_3), \tag{7}$$

$$\mathcal{D}(p_2, p_3) > \mathcal{D}(p_3, p_4). \tag{8}$$

The proof is omitted due to space limitations.

We can now give the rule for replacing a constant power energy curve with the optimal energy curve that uses power levels from a finite set.

Theorem 1: For an EH transmission system with $r(p)$, if the solution of OP is given by a constant power transmission curve with p^*, the optimal transmitted energy curve that uses only the values in $\mathcal{P}$ is obtained as follows:

1) If $p^* \in \mathcal{P}$ then $p(t) = p^* \; \forall t \in [0, T]$.
2) If $\max \mathcal{P} < p^*$ then $p(t) = \max \mathcal{P} \; \forall t \in [0, T]$.
3) If $\min \mathcal{P} > p^*$ then $p(t) = \min \mathcal{P} \; \forall t \in [0, \frac{p^* T}{\min \mathcal{P}}]$, and $p(t) = 0 \; \forall t \in (\frac{p^* T}{\min \mathcal{P}}, T]$ (or the inverse).
4) In all the other cases, we use only two power values in $\mathcal{P}$: the smallest value that is higher than p^*, denoted by p_h, and the highest value smaller than p^*, denoted by p_l. Their respective transmission durations are given by $t_l = \frac{T p_h - p^* T}{p_h - p_l}$ and $t_h = \frac{p^* T - T p_l}{p_h - p_l}$. We have $p(t) = p_l \; \forall t \in [0, t_l]$ and $p(t) = p \; \forall \in [t_l, t_l + t_h]$ (or the inverse).

Proof: The first three points are obvious. To prove the last one, we select any possible solution. We re-order and re-distribute it such that you get a two-slope curve with respect to p^*. This is always possible due to Proposition 2. Then, consider the first two-slope interval. Let the power values used

in this interval be p_1 and p_2 with $p_1 < p_2$. Since the two curves intersect at the end of the first two-slope interval, we have $p_1 < p^* < p_2$. From Proposition 3, we know that $\mathcal{D}(p_1, p_2) < \mathcal{D}(p_1, p_h) < \mathcal{D}(p_l, p_h)$. Hence, by replacing p_1 and p_2 with p_l and p_h, we can improve the amount of data transmitted over this two-slope interval.

We can apply the same argument to all the other two-slope intervals. We end up using only the power levels p_l and p_h, and their respective durations are found as given above.

Note that once we reduce the transmission strategy to the two power values p_l and p_h, it is always possible to come up with an equivalent curve that lies between the $H(t)$ and $M(t)$ curves. Note that, by switching between p_l and p_h, we can get arbitrarily close to the constant transmission rate curve. ∎

V. THE GENERAL SCENARIO

Now we address the general problem, that is, the solution of DP when the solution of OP can be an arbitrary piecewise linear function. We know that the optimal continuous solution for the OP uses constant power transmissions during each epoch, some of them being equal. We denote this scheme by $\mathcal{E}_{OP}^* = \{(p_{i,OP}^*, t_{i,OP}^*), i = 1, \ldots, N\}$, where N is the number of epochs. We assume that there exists in $\mathcal{P}$ at least one power value larger than the highest power value used in $\mathcal{E}_{OP}^*$, and at least one power value smaller than the smallest power used in $\mathcal{E}_{OP}^*$. The latter is always satisfied since $0 \in \mathcal{P}$.

Algorithm 1

$\mathcal{E}_{DP} = \{\}$
for $n = 1 : N$ **do**
  if $p_{n,OP}^* \in \mathcal{P}$ **then**
    $p_{n,DP} \leftarrow p_{n,OP}^*$
    $t_{n,DP} \leftarrow t_{n,OP}^*$
  else
    find $p_h = \min\{\mathcal{P}\}$ s.t. $p_h > p_{n,OP}^*$.
    find $p_l = \max\{\mathcal{P}\}$ s.t. $p_l < p_{n,OP}^*$.
    $t_{h,DP} \leftarrow \frac{p_{n,OP}^* t_{n,OP}^* - t_{n,OP}^* p_l}{p_h - p_l}$
    $t_{2,DP} \leftarrow \frac{t_{n,OP}^* p_h - p_{n,OP}^* t_{n,OP}^*}{p_h - p_l}$
  end if
  $\mathcal{E}_{DP} \leftarrow \mathcal{E}_{DP} \bigcup \{(p_h, t_{h,DP}), (p_l, t_{l,DP})\}$
end for

This algorithm builds epoch after epoch the best approximation to the solution of the OP. Note that the new transmitted energy curve obtained from this algorithm touches the $H(t)$ and $M(t)$ curves at exactly the same points as the solution of the OP. Since $H(t)$ and $M(t)$ are step functions, the optimal transmitted energy curve of DP remains bounded between the two, and satisfy both energy causality and no-battery-overflow constraints.

A. Proof of optimality

Due to the space limitation, the detailed proof of the optimality of Algorithm 1 is not included here. However, we describe its main challenges. The difficulty here is to

understand why we are allowed to generalize the result of the previous theorem over each epoch $[s_i, s_{i+1}]$ to get global optimality. In fact, the previous theorem gives a rule that ensures the optimality over the energy curves starting and ending at the same points (i.e., using the same amount of energy during a period previously referred as T). Any other transmitted energy curve using power levels in $\mathcal{P}$ that would not be equal to the optimal solution of problem OP at the time instants s_i does not follow from Theorem 1. In such a case, it is not possible to directly use the theorem over the epochs defining the problem.

Therefore, the main challenge in the proof of the optimality of Algorithm 1 is to show that any other feasible energy curve using power levels in $\mathcal{P}$ can be transformed into an equivalent energy curve equal to $\mathcal{E}_{OP}^*$ at instants s_i using ReD and ReO. That is, we will have $E(s_i) = E_{OP}^*(s_i)$ for all i. Once this is achieved, we know that the optimality of a transmission strategy that uses only two power levels within each epoch follows from Theorem 1.

As before, once the two optimal power levels for each epoch are identified, we can obtain an equivalent transmitted energy curve using ReD and ReO such that both $M(t)$ and $H(t)$ constraints are satisfied.

VI. NUMERICAL RESULTS

In this section we present some numerical results that illustrate the loss in terms of transmitted data by constraining the transmitter to a discrete set of transmission power values. We consider a scenario with seven epochs, in which the sizes of the arriving energy packets have the following values in terms of the energy unit used: 5, 15, 20, 5, 15, 15, 18. The durations of the respective epochs are: 5, 5, 10, 10, 5, 5, 2 time units. The battery capacity is taken to be $H_{max} = 20$.

We first find the solution of the OP with continuous transmission power values, described by the set $\mathcal{E}_{OP} = \{(1,5), (2.6,5), (1.35,10), (1.35,10), (3,5), (3,5), (9,2)\}$. Even if this set is equivalent to the re-distributed set $\mathcal{E}_{OP}^{'} = \{(1,5), (2.6,5), (1.35,20), (3,10), (9,2)\}$, the first description permits to visualize the initial epochs defining the energy arrivals. We then use our algorithm on different sets of available transmit power levels.

In our simulations we run our algorithm for a set of available powers from 0 to 10 with an increasing number of available power values, K. We start with $K = 2$, i.e., only two available power levels 0 and 10 can be used; and finish by allowing $K = 11$ power levels $[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10]$. For $K = 3$ to $K = 10$, the available power levels are chosen uniformly over the range $[0, 10]$. For example, for $K = 4$, we have $\mathcal{P} = [0, 3.33, 6.66, 10]$. Our simulations give the results presented in Table I. The first column indicates the ratio of the total transmitted data with K power levels uniformly distributed over $[0, 10]$ to the solution of the OP, which allows arbitrary transmission power values.

We observe that the general trend is that the total transmitted data decreases as less and less power levels are available to the transmitter. This is due to the fact that the transmitter is not

Data Ratio	K
0.441	2
0.658	3
0.812	4
0.882	5
0.917	6
0.951	7
0.980	8
0.987	9
0.986	10
0.989	11

TABLE I: The ratio of the total transmitted data, with K power levels uniformely distributed over [0,10], to the solution of OP.

able to exploit the available energy in the most efficient way. Table I illustrates another important fact. While the number of available power levels affects the overall performance, the exact values of these levels are also important. For example, if we compare the two cases $K = 9$ and $K = 10$, we see that the former transmits more data despite it has less power levels. This observation points to an important design criteria for EH transmission systems. For a simple EH sensor transmitter limited to a finite number of transmission rate and power values, the values of these can be designed optimally based on the characteristics of the underlying EH process, if it is known before the deployment of the sensor.

VII. CONCLUSIONS

We have considered a point-to-point communication system with an EH transmitter which can store the harvested energy in a finite-capacity battery. We have considered the practical constraint that the transmitter can only choose from a finite number of transmission rate values, or equivalently, a finite set of transmission power levels for a fixed target error probability. We have shown that the corresponding optimization problem is convex, and hence, can be solved numerically. Then, in order to obtain further insights into the nature of the optimal transmission policy, we have proposed an algorithm that allows us to extend the results of the continuous problem (i.e., transmission power and rate can take any non-negative value) to the discrete case. We have shown that, with a finite number of energy arrivals, once the optimal solution of the continuous problem is obtained through a simple shortest path algorithm, it is sufficient to replace each optimal power level for the continuous case with the two closest power levels from the available set. This new transmission scheme will automatically satisfy both the energy causality and the no battery overflow constraints.

By relaxing the idealized continuous transmission rate/ power assumptions for EH communication systems, we have made a new step forward to understand the potential limits of practical communication systems with harvested energy devices. We also note that our results directly apply to the optimal scheduling problems with data packet arrivals over time studied in [15], when the transmitter is limited to a discrete set of transmission power levels and rates.

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