Capacity Results for a Class of Deterministic Z-interference Channels with Unidirectional Receiver Conferencing

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Abstract— **We study the Z-interference channel in which there is an additional orthogonal link from the interference-free receiver to the interfered receiver. We call this channel model the Z-interference channel with unidirectional receiver conferencing. We find the capacity region when the Z-interference channel belongs to the class of deterministic Z-interference channels studied by El Gamal & Costa in 1982. Our results show that in the presence of unidirectional receiver conferencing, it is still optimal for the interfering transmitter to use superposition encoding to control the amount of interference it causes. For the interference-free receiver, it is optimal to forward part of the decoded message over the orthogonal cooperation link. We further note that the same scheme is also optimal for another class of Z-interference channels studied by Liu & Goldsmith in 2009.**

I. Introduction

The interference channel (IC), introduced in [1], is a simple network consisting of two pairs of transmitters and receivers. Each pair wishes to communicate at a certain rate with negligible probability of error, while the two communications interfere with each other. The problem of finding the capacity region of the IC is difficult and remains open except in some special cases [2]. The Z-interference channel (ZIC) is an IC in which one of the two transmitter-receiver pairs is interference-free. Although this is a simpler channel model than the IC, its capacity region is known only in some special cases [3, Section IV], [4, Section VI].

The classic IC model is often too simplistic to describe practical wireless networks. For example, in practical communication scenarios, the two receivers that are geographically close to each other, may decide to cooperate to enhance their useful signals and lessen the effect of interference. Thus, the Gaussian IC with receiver cooperation has been studied in [5]–[11]. Achievable rate regions and outer bounds have been characterized, which are then used to obtain capacity region approximations as well as degree-of-freedom results. As for the non-Gaussian case, the sum capacity has been found for a linear deterministic channel model in [12]. Due to the difficulty of finding the capacity region for the IC and the relay channel [13], finding the capacity region of the IC with receiver cooperation is an extremely difficult problem.

A more tractable channel model is to assume that the receivers cooperate over links of finite capacity that is orthogonal to the IC [14]–[19]. An especially simple channel model of IC with receiver cooperation is the ZIC with an orthogonal unidirectional link from the interference-free receiver to the interfered receiver, studied in [14]. We call this channel model the ZIC with unidirectional receiver conferencing. This channel model is simple because from the perspective of the interfered pair, the interference-free receiver acts as a relay that knows the interference completely but knows nothing about the message. Hence, it is expected that forwarding part of the interference to enable better interference cancelation at the interfered receiver is the optimal scheme for the interference-free receiver. The Gaussian model for the ZIC with unidirectional receiver conferencing is studied in [14] and the capacity region is found for the strong interference regime. In the weak to moderate interference regime, since the capacity region of the Gaussian ZIC is not known, the capacity region of the Gaussian ZIC with unidirectional receiver conferencing is also open.

In this work, rather than focusing on the Gaussian case as in [14], we focus on a class of *discrete* ZICs whose capacity region is fully known. More specifically, we study the class of deterministic ZICs in [3, Section IV], where it has been shown that superposition encoding achieves the capacity region. We generalize this channel model to incorporate receiver cooperation by adding an orthogonal link from the interference-free receiver to the interfered receiver.

We first establish the capacity region by proving the achievability and the converse. Our results show that for the interference-free transmitter, superposition encoding is still the optimal way to manage interference in the presence of unidirectional receiver conferencing. For the interference-free

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receiver, it is optimal to forward part of the message it has decoded over the orthogonal link. The benefit of the unidirectional receiver conferencing is that it increases the ability of the interfered receiver to decode the inner codeword of the interference. Then, we compare the capacity region with and without the unidirectional conferencing link to illustrate the benefit of receiver cooperation in reducing the interference, and hence, in enlarging the capacity region. Finally, we show that the same scheme is also optimal for another class of ZICs, whose capacity region was characterized in [4, Section VI].

II. System Model and Main Result

We consider the deterministic ZIC model, illustrated in Figure 1, which was introduced by El Gamal and Costa in [3, Section IV]. The ZIC is described by the conditional distribution $p(y_1|x_1)$ and the deterministic functions

$$Y_2 = f_2(X_2, V_1), \quad (1)$$
$$V_1 = g_1(X_1), \quad (2)$$
$$V_1 = h_2(X_2, Y_2). \quad (3)$$

The input and output alphabets are $\mathcal{X}_1$, $\mathcal{X}_2$, $\mathcal{Y}_1$ and $\mathcal{Y}_2$. Let W_1 and W_2 be two independent messages. Transmitter 1 has message W_1, which is intended for Receiver 1; and Transmitter 2 has message W_2, which is intended for Receiver 2. The difference between the channel model studied in this paper and that of [3] is that, there is an orthogonal link from Receiver 1 to Receiver 2 characterized by $p(\bar{y}|\bar{x})$, where the input and output alphabets are $\bar{\mathcal{X}}$ and $\bar{\mathcal{Y}}$, respectively. Let $\bar{C}$ denote the capacity of the orthogonal link, i.e.,

$$\bar{C} \triangleq \max_{p(\bar{x})} I(\bar{X};\bar{Y}).$$

.

An $(M_1, M_2, n, \epsilon_n)$ code for this channel model consists of two encoding functions at Transmitter 1 and Transmitter 2:

$$\tilde{f}_k : \{1, 2, \cdots, M_k\} \to \mathcal{X}_k^n, \quad k = 1, 2,$$

one encoding function at Receiver 1, whose output at time i depends on what it has received up to time $i-1$:

$$\bar{X}_i = \tilde{h}(Y_{11}, Y_{12}, \cdots, Y_{1(i-1)}),$$

and two decoding functions at Receiver 1 and Receiver 2:

$$\tilde{g}_1 : \mathcal{Y}_1^n \to \{1, 2, \cdots, M_1\},$$
$$\tilde{g}_2 : \mathcal{Y}_2^n \times \bar{\mathcal{Y}}^n \to \{1, 2, \cdots, M_2\}.$$

The probability of error is defined as

$$\epsilon_n = \frac{1}{M_1 M_2} \sum_{w_1, w_2} \Pr\left[\tilde{g}_1(Y_1^n) \neq w_1, \tilde{g}_2(Y_2^n, \bar{Y}^n) \neq w_2 | W_1 = w_1, W_2 = w_2\right].$$

A rate pair (R_1, R_2) is said to be achievable if there exists a sequence of $\left(2^{nR_1}, 2^{nR_2}, n, \epsilon_n\right)$ codes such that $\epsilon_n \to 0$ as $n \to \infty$. The capacity region of the deterministic ZIC with unidirectional receiver conferencing is the closure of the set of all achievable rate pairs.

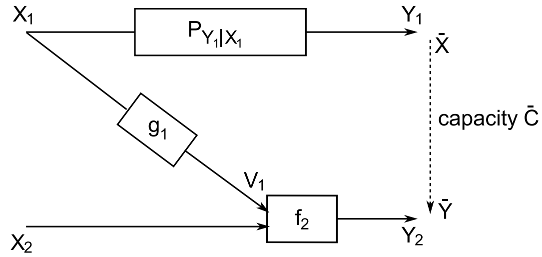

Fig. 1. The Deterministic ZIC with Unidirectional Receiver Conferencing.

The main result of the paper is the following theorem.

Theorem 1: The capacity region of the ZIC with an orthogonal link of capacity $\bar{C}$ from the interference-free receiver to the interfered receiver is

$$\bigcup_{p(x_1)p(x_2)} \{(R_1, R_2) : R_1 \leq I(X_1; Y_1), R_2 \leq H(Y_2|V_1)$$
$$R_1 + R_2 \leq H(Y_2) + I(X_1; Y_1|V_1) + \bar{C}\}$$

if the ZIC satisfies the conditions in (1)-(3).

III. Converse

In this section, we prove the converse part of Theorem 1. The proof is generalized from the converse proof in [3, Section IV].

For any sequence of codes $\left(2^{nR_1}, 2^{nR_2}, n, \epsilon_n\right)$, we have

$$\begin{aligned} nR_1 &= H(W_1) \\ &= I(W_1; Y_1^n) + H(W_1|Y_1^n) \\ &\leq I(X_1^n; Y_1^n) + n\epsilon_n && (4) \\ &\leq \sum_{i=1}^{n} I(X_{1i}; Y_{1i}) + n\epsilon_n && (5) \end{aligned}$$

where (4) follows from the data processing inequality using the Markov chain $W_1 \to X_1^n \to Y_1^n$ and Fano's inequality; and (5) follows from the memoryless nature of the channel and the fact that conditioning reduces entropy.

We further have

$$\begin{aligned} nR_2 &= H(W_2) \\ &= H(W_2|\bar{Y}^n, V_1^n) && (6) \\ &= I(W_2; Y_2^n|\bar{Y}^n, V_1^n) + H(W_2|Y_2^n, \bar{Y}^n, V_1^n) \\ &\leq I(X_2^n; Y_2^n|\bar{Y}^n, V_1^n) + n\epsilon_n && (7) \\ &= H(Y_2^n|\bar{Y}^n, V_1^n) - H(Y_2^n|X_2^n, \bar{Y}^n, V_1^n) + n\epsilon_n \\ &= H(Y_2^n|\bar{Y}^n, V_1^n) + n\epsilon_n && (8) \\ &= H(Y_2^n|V_1^n) + n\epsilon_n && (9) \\ &\leq \sum_{i=1}^{n} H(Y_{2i}|V_{1i}) + n\epsilon_n && (10) \end{aligned}$$

where (6) follows because both $\bar{Y}^n$ and V_1^n are functions, possibly random, of message W_1 and therefore are independent of W_2; (7) follows from the data processing inequality using the Markov chain $W_2 \to (X_2^n, \bar{Y}^n, V_1^n) \to Y_2^n$ and Fano's inequality; (8) follows from the deterministic condition in (1);

and finally (9) follows from the Markov chain $\bar{Y}^n \to V_1^n \to Y_2^n$.

We also have

$$
\begin{aligned}
&nR_1 - n\bar{C} + nR_2 \\
&\le H(W_1) - I(\bar{X}^n;\bar{Y}^n) + H(W_2) \\
&= H(W_1) - I(\bar{X}^n;\bar{Y}^n) + H(W_2|\bar{Y}^n) \quad (11)\\
&= H(W_1|\bar{Y}^n) + H(W_2|\bar{Y}^n) + I(W_1;\bar{Y}^n) - I(\bar{X}^n;\bar{Y}^n) \\
&\le H(W_1|\bar{Y}^n) + H(W_2|\bar{Y}^n) \quad (12)\\
&= I(W_1;Y_1^n|\bar{Y}^n) + H(W_1|Y_1^n,\bar{Y}^n) \\
&\qquad + I(W_2;Y_2^n|\bar{Y}^n) + H(W_2|Y_2^n,\bar{Y}^n) \\
&\le I(X_1^n;Y_1^n|\bar{Y}^n) + n\epsilon_n + I(X_2^n;Y_2^n|\bar{Y}^n) + n\epsilon_n \quad (13)\\
&= I(X_1^n;Y_1^n|\bar{Y}^n) + H(Y_2^n|\bar{Y}^n) - H(Y_2^n|X_2^n,\bar{Y}^n) + 2n\epsilon_n \\
&= I(X_1^n;Y_1^n|\bar{Y}^n) + H(Y_2^n|\bar{Y}^n) - H(V_1^n|X_2^n,\bar{Y}^n) + 2n\epsilon_n \quad (14)\\
&= I(X_1^n;Y_1^n|\bar{Y}^n) + H(Y_2^n|\bar{Y}^n) - H(V_1^n|\bar{Y}^n) + 2n\epsilon_n \quad (15)\\
&\le I(X_1^n;V_1^n,Y_1^n|\bar{Y}^n) + H(Y_2^n|\bar{Y}^n) - H(V_1^n|\bar{Y}^n) + 2n\epsilon_n \\
&= I(X_1^n;V_1^n|\bar{Y}^n) + I(X_1^n;Y_1^n|V_1^n,\bar{Y}^n) + H(Y_2^n|\bar{Y}^n) \\
&\qquad\qquad - H(V_1^n|\bar{Y}^n) + 2n\epsilon_n \\
&= H(V_1^n|\bar{Y}^n) - H(V_1^n|X_1^n,\bar{Y}^n) + I(X_1^n;Y_1^n|V_1^n,\bar{Y}^n) \\
&\qquad\qquad + H(Y_2^n|\bar{Y}^n) - H(V_1^n|\bar{Y}^n) + 2n\epsilon_n \\
&= -H(V_1^n|X_1^n,\bar{Y}^n) + I(X_1^n;Y_1^n|V_1^n,\bar{Y}^n) \\
&\qquad\qquad + H(Y_2^n|\bar{Y}^n) + 2n\epsilon_n \\
&= I(X_1^n;Y_1^n|V_1^n,\bar{Y}^n) + H(Y_2^n|\bar{Y}^n) + 2n\epsilon_n \quad (16)\\
&\le I(X_1^n;Y_1^n|V_1^n) + H(Y_2^n) + 2n\epsilon_n \quad (17)\\
&\le \sum_{i=1}^{n}(I(X_{1i};Y_{1i}|V_{1i}) + H(Y_{2i})) + 2n\epsilon_n \quad (18)
\end{aligned}
$$

where (11) follows from the same reasoning as in (6); (12) follows from the data processing inequality using the Markov chain $W_1 \to \bar{X}^n \to \bar{Y}^n$; (13) follows from the data processing inequality using the Markov chains $W_1 \to (X_1^n, \bar{Y}^n) \to Y_1^n$ and $W_2 \to (X_2^n, \bar{Y}^n) \to Y_2^n$ and Fano's inequality; (14) follows from the deterministic conditions in (1) and (3); (15) follows from the fact that X_2^n is independent of $(\bar{Y}^n, V_1^n)$; (16) follows from the deterministic condition in (2); (17) follows from the fact that conditioning reduces entropy and the Markov chain $\bar{Y}^n \to (X_1^n, V_1^n) \to Y_1^n$; and finally (18) follows from the same reasoning as in (5).

From (5), (10) and (18), for any sequence of codes $\left(2^{nR_1}, 2^{nR_2}, n, \epsilon_n\right)$ that satisfy $\epsilon_n \to 0$ as $n \to \infty$, we have proved the converse part of Theorem 1. Similar to [3], the time sharing random variable is omitted as the region in Theorem 1 is convex.

IV. Achievability

In this section, we prove the achievability part of Theorem 1. Let us define $\bar{R}_1 = \min(\bar{C}, R_1)$ and γ is a non-negative number that satisfies $\bar{R}_1 + \gamma \le R_1$. The main idea of the achievability scheme is for Transmitter 1 to split its message into three parts, W_{1s} of rate $\bar{R}_1$, W_{1p} of rate γ and $\tilde{W}_{1p}$ of rate $R_1 - \bar{R}_1 - \gamma$. Transmitter 1 encodes W_{1s} and W_{1p} into the inner codeword and $\tilde{W}_{1p}$ into the outer codeword. Receiver 1 upon decoding W_1, forwards W_{1s} to Receiver 2 using the orthogonal conferencing link. Receiver 2, upon receiving W_{1s} from Receiver 1, decodes W_{1p} and W_2 from its received signal Y_2^n while treating the outer codeword of Transmitter 1 as noise.

The details of the achievability scheme are as follows: we collect a block of $B-1$ messages, i.e., $(W_1(1), W_2(1))$, $(W_1(2), W_2(2)), \cdots (W_1(B-1), W_2(B-1))$ and transmit these over Bn channel uses. Let $W_1(0) = W_1(B) = W_2(B) = 1$. Further let δ be an arbitrarily small positive number.

Random Codebook Generation: Fix a product distribution $p(x_1)p(x_2)$. Transmitter 1 generates an inner codebook of $2^{n(\bar{R}_1+\gamma-\delta)}$ rows in an independent and identically distributed (i.i.d.) fashion with distribution $p(v_1)$. Conditioned on each inner codeword, it generates an outer codebook of $2^{n(R_1-\bar{R}_1-\gamma+\delta)}$ rows in a conditional i.i.d. fashion using $p(x_1)$. Transmitter 2 generates a codebook of 2^{nR_2} rows in an i.i.d. fashion using $p(x_2)$.

Encoding: For $j = 1, \cdots, B$, during channel uses $(j-1)n+1$ to $(j-1)n+n$, Transmitter 1 splits message $W_1(j)$ into three independent parts, $W_{1s}(j)$, $W_{1p}(j)$ and $\tilde{W}_{1p}(j)$ of rates $n\bar{R}_1 - \delta$, $n\gamma$ and $n(R_1 - \bar{R}_1 - \gamma + \delta)$, respectively. Suppose that $W_{1s}(j) = w_{1s}$, $W_{1p}(j) = w_{1p}$, $\tilde{W}_{1p}(j) = \tilde{w}_{1p}$ and $W_2(j) = w_2$. Transmitter 1 sends the $\tilde{w}_{1p}$-th codeword from the $(w_{1s}2^{n\gamma} + w_{1p})$-th outer codebook, and Transmitter 2 transmits the w_2-th codeword from its codebook. Receiver 1 transmits the $W_{1s}(j-1)$ that it has decoded from the previous block over the conferencing link, encoding against channel noise using a capacity-achieving code for the single user channel $p(\bar{y}|\bar{x})$.

Decoding: Receiver 1 uses its received signal, i.e., Y_1 at time slots $(j-1)n+1$ to $(j-1)n+n$ to decode $W_1(j)$ by decoding both the inner and outer codewords, this can be done as long as

$$
\begin{aligned}
R_1 &\le I(X_1;Y_1) \\
R_1 - \bar{R}_1 - \gamma + \delta &\le I(X_1;Y_1|V_1)
\end{aligned}
$$

Receiver 2 first uses its received signal $\bar{Y}$ at time slots $jn+1$ to $jn+n$ to decode $W_{1s}(j-1)$. This can be done with negligible probability of error since the rate of $W_{1s}(j-1)$ is less than the capacity of the conferencing link.

Now that Receiver 2 knows $W_{1s}(j-1)$, it decodes $W_{1p}(j-1)$ and $W_2(j-1)$ using its received signal Y_2 at time slots $(j-1)n+1$ to $(j-1)n+n$. This can be done as long as

$$
\begin{aligned}
R_2 &\le I(X_2;Y_2|V_1) \\
R_2 + \gamma &\le I(X_2,V_1;Y_2)
\end{aligned}
$$

After Fourier-Motzkin elimination, taking $B \to \infty$ and letting $\delta \to \infty$, we get that a rate pair (R_1, R_2) is achievable if it satisfies

$$R_1 \le I(X_1;Y_1), \tag{19}$$

$$R_2 \le I(X_2;Y_2|V_1), \tag{20}$$

$$R_1 + R_2 \le I(X_2,V_1;Y_2) + I(X_1;Y_1|V_1) + \bar{R}_1, \tag{21}$$

for some input distribution $p(x_1)p(x_2)$. Equivalently, a rate pair (R_1, R_2) is achievable if it satisfies (19), (20) and

$$R_1 + R_2 \le I(X_2,V_1;Y_2) + I(X_1;Y_1|V_1) + \bar{C}, \tag{22}$$

for some input distribution $p(x_1)p(x_2)$. This is because for any rate pair (R_1, R_2) that satisfies (19) and (20), it is not possible that it satisfies (22), but not (21).

Evaluating the achievable rate region for the deterministic ZIC in Figure 1, due to the deterministic condition in (1), we have $H(Y_2|X_2,V_1) = 0$. Hence, the achievability part of Theorem 1 is proved.

V. Benefits of Unidirectional Receiver Conferencing

Comparing the capacity results in Theorem 1 with that of [3, Theorem 2], we see that for the deterministic ZIC, the benefit of receiver conferencing from the interference-free receiver to the interfered receiver is that, for a fixed input distribution $p(x_1)p(x_2)$, the sum rate constraint is enlarged by the capacity of the conferencing link $\bar{C}$.

For a fixed input distribution $p(x_1)p(x_2)$, if $I(V_1;Y_2) \ge I(V_1;Y_1)$, the achievable rate region without the conferencing link is a rectangle. In this case, the conferencing link is useless. This is because for the deterministic ZIC that satisfies (1)-(3), the part of the interference that affects Receiver 2 is V_1^n. Since the orthogonal link from Receiver 1 to Receiver 2 can only forward interference, it should forward the information about V_1^n to enable a better decoding of V_1^n at Receiver 2. However, if Receiver 2 can already decode V_1^n better than Receiver 1, i.e., $I(V_1;Y_2) \ge I(V_1;Y_1)$, then the conferencing link does not provide any capacity gain.

If $I(V_1;Y_2) < I(V_1;Y_1) < H(Y_2)$, the achievable rate region without the conferencing link is a pentagon as shown by the solid line in Figure 2. With the existence of the conferencing link, the region is enlarged to the dashed line if $I(V_1;Y_2) + \bar{C} \le I(V_1;Y_1)$ and to the dotted line if $I(V_1;Y_2) + \bar{C} > I(V_1;Y_1)$.

Similarly, if $I(V_1;Y_1) \ge H(Y_2)$, the achievable rate region without the conferencing link is a trapezoid as shown by the solid line in Figure 3. With the existence of the conferencing link, the region is enlarged to the dashed line if $\bar{C} \le I(V_1;Y_1) - H(Y_2)$, to the dotted line if $I(V_1;Y_1) - H(Y_2) < \bar{C} \le I(V_1;Y_1) - I(V_1;Y_2)$, and to the dot-dashed line if $I(V_1;Y_2) + \bar{C} > I(V_1;Y_1)$.

In summary, the capacity region is enlarged by the existence of the conferencing link as the ability of Receiver 2 to decode V_1^n is improved from $I(V_1;Y_2)$ to $I(V_1;Y_2) + \bar{C}$.

As an example, we study the channel model in Figure 4, which reduces to [3, Figure 3] in the absence of the orthogonal unidirectional conferencing link, and compare the capacity

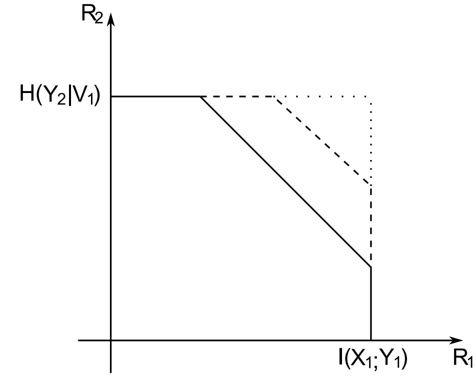

Fig. 2. $I(V_1;Y_2) < I(V_1;Y_1) < H(Y_2)$.

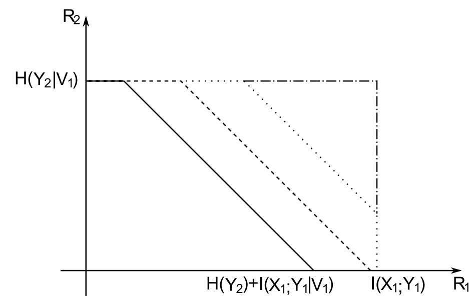

Fig. 3. $I(V_1;Y_2) \ge H(Y_2)$.

regions with and without the conferencing link from Receiver 1 to Receiver 2. The capacity region with the conferencing link of capacity $\bar{C}$ is

$$\bigcup_{p,q\ge 0, p+q\le 1} \Big\{R_1 \le h(p+\epsilon(1-p-q)) - (1-p-q)h(\epsilon),$$
$$R_2 \le 1,$$
$$R_1 + R_2 \le (1-q)h\left(\frac{p+\epsilon(1-p-q)}{1-q}\right) - (1-p-q)h(\epsilon) + 1 + \bar{C}\Big\}$$

where $h(x) \overset{\triangle}{=} -x\log x - (1-x)\log(1-x)$, $x \in [0,1]$. For $\epsilon = 0.4$, we plot the capacity region when the capacity of the conferencing link is equal to 0, 0.2, 0.5 and 0.8 in Figure 5. The improvement in the capacity region shows the usefulness of the conferencing link from Receiver 1 to Receiver 2.

VI. Discussions

Superposition encoding has been shown to be optimal in another class of ZICs in [4]. Furthermore, the optimality has been extended to the scenario of the ZIC with message side information where the the interfered receiver has side information about part of the interference it faces [20]. The ZIC with message side information and the ZIC with unidirectional receiver conferencing is closely related in the following sense: any achievability scheme of the ZIC with message side information serves as an achievability scheme for the ZIC with undirectional receiver conferencing where the interference-free receiver forwards $\bar{C}$ bits of the message it has decoded over

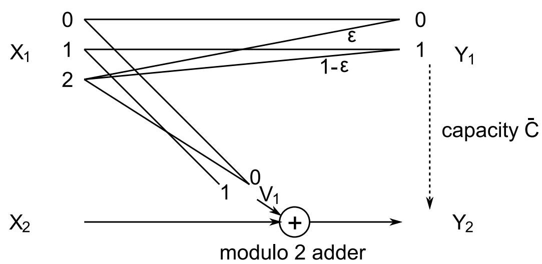

Fig. 4. An example of the class of ZICs with unidirectional receiver conferencing studied in this paper.

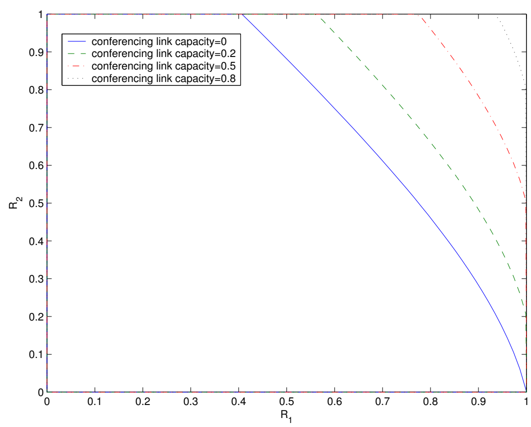

Fig. 5. Capacity region of the ZIC in Fig. 4 for various capacity values on the conferencing link.

the orthogonal cooperation link to the interfered receiver. As for the converse, replacing W_{1s} in the converse of the ZIC with message side information with $\bar{Y}^n$, we obtain a converse for the ZIC with unidirectional receiver conferencing. Hence, from the capacity region of a class of ZICs with message side information [20, Lemma 2], we obtain the capacity region of ZICs with unidirectional receiver conferencing if the ZICs satisfy Conditions 1 and 2 in [20]: Rate pair (R_1, R_2) is achievable if and only if,

$$R_1 \leq I(X_1; Y_1),$$
$$R_2 \leq I(U, X_2; Y_2),$$
$$R_1 + R_2 \leq I(X_1; Y_1|U) + I(U, X_2; Y_2) + \bar{C},$$

for some $p(u)p(x_1|u)$ where the mutual information terms are evaluated using the joint distribution

$$p(u, x_1, x_2, y_1, y_2) = p(x_1, u)p^*(x_2)p(y_1|x_1)p(y_2|x_1, x_2)$$

We see that in this scenario, similar to the deterministic ZIC, superposition encoding remains optimal and the benefit of the orthogonal link is that for a fixed input distribution $p(u, x_1)$, the sum rate constraint is increased by $\bar{C}$, the capacity of the orthogonal link.

VII. Conclusions

For two classes of Z-interference channels for which superposition encoding is optimal, we show that superposition encoding remains to be optimal in the presence of an orthogonal link from the interference-free receiver to the interfered receiver. Furthermore, it is shown that the optimal way to use the orthogonal unidirectional cooperation link is to forward part of the interference to the interfered receiver such that its ability to decode the inner codeword of the other transmitter is increased. In the capacity region expressions, for each fixed input distribution, the sum rate constraint is increased by the capacity of the orthogonal link.

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