

Age of Information under Energy Replenishment Constraints

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Abstract—We consider managing the freshness of status updates sent from a source (such as a sensor) to a monitoring node. The time-varying availability of energy at the sender limits the rate of update packet transmissions. At any time, the age of information is defined as the amount of time since the most recent update was successfully received. An offline solution that minimizes not only the time average age, but also the peak age for an arbitrary energy replenishment profile is derived. The related decision problem under stochastic energy arrivals at the sender is studied through a discrete time dynamic programming formulation, and the structure of the optimal policy that minimizes the expected age is shown. It is found that tracking the expected value of the current age (which is a linear operation), together with the knowledge of the current energy level at the sender side is sufficient for generating an optimal threshold policy. An effective online heuristic, Balance Updating (BU), that achieves performance close to an omniscient (offline) policy is proposed. Simulations of the policies indicate that they can significantly improve the age over greedy approaches. An extension of the formulation to stochastically formed updates is considered.

I. INTRODUCTION

For many monitoring applications (e.g. [1], [2], [3], [4]), where a source collects measured samples to be sent to a remote destination, the freshness of the last sample received is an important quality metric [5]. The *Age of Information*, i.e. the amount of time that has elapsed since the most recently received sample was formed, is useful as a freshness metric in such a scenario [6], [3], [7] and [8].

Age of Information was used as a metric of freshness of status updates in recent literature [3], [6]. In [3], modeling the generation intervals of updates as well as packet transmission durations as random, a queuing-theoretic system analysis was conducted. Using a first-come-first-served (FCFS) queuing discipline, the existence of an optimal rate at which a source must generate its information to keep its status as timely as possible at all its monitors was shown. It is notable that this rate differs from those that maximize throughput or minimize status packet delivery delay, and motivates further study of age of information as a metric of relevance to monitoring applications. Further work reported in [6] employed a time average age metric for the performance evaluation of status update systems. In [7], age of information in a cloud

system with randomly generated status updates is analysed. Following the framework of [7], average age as well as the peak age in M/M/1/1, M/M/2, M/M/1/2 and M/M/1/2* (the last corresponding to a queueing discipline where a new arrival to the system replaces the one in the queue) queueing models was analyzed in [8], [9]. It was observed that age is improved when the source node has the capability of packet management by discarding some samples.

In this paper, we turn the question around by asking when the sender should generate status updates, if the number of updates per time it is allowed to send is constrained by an arbitrary time-varying upperbound. While our analysis applies to an arbitrary bound on the total number of updates $N_U(t)$ that may be sent by time t , we motivate the problem formulation by modeling a sender with energy arrivals. Self-sustaining operation of energy harvesting sensor systems have been the focus of a plethora of studies in recent years (e.g.[10]). Accordingly, the number of updates that may be sent by time t depends on the total energy harvested by time t . In the continuous time setup of the problem, the objective is to determine the sequence of update instants that minimize the time average age of the most recent update, while respecting the energy constraints. The discrete time version is cast as a decision problem of whether or not to send an update in any given time slot. To the best of our knowledge, there has been no previous study of the age of information with respect to such constraints.

The contributions of the paper may be summarized as follows: The continuous time problem of optimizing status update instants in order to minimize average age of information for a given bounding function $N_U(t)$ (corresponding to an "energy harvest" profile) on the number of updates by time t is solved. The *online problem*, where the energy profile is a stochastic process with known statistics, investigated within a dynamic programming formulation. The study of the optimal threshold policy reveals what we believe is an interesting result: at any time (provided that energy is not fully depleted) the decision to send an update depends on the *expectation of the current age*. We conclude the paper by exhibiting a simple online heuristic which is motivated by the offline solution and appears to perform quite well as indicated by numerical studies.

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II. PROBLEM FORMULATION

Consider the operating scenario energy harvesting wireless device (such as a sensor node) S which continuously monitors the real time status of a system and repeatedly sends status update packets to a recipient device. Assume that the device can generate a status update packet at any arbitrary time, and transmit it instantly (an abstraction for the case when generation and transmission of a status update packet take negligibly small amount of time with respect to typical times between updates.) According to this assumption, status updates can be treated as time instants on the problem timeline. The recipient gets status updates at those particular time instants where S decides to send a status update packet. The most recent status update packet received by the recipient device gets outdated upon the reception of a new status update. The instantaneous "age" is the time that has elapsed since the most recent update. The goal of S is to time the status updates so that the average age is as small as possible. The problem is only meaningful under some constraint on the number of updates that can be sent. For illustration, we model S as an energy harvesting device which limits its energy consumption profile. In Section II-A and II-C, this situation is characterized in offline and online optimisation problems.

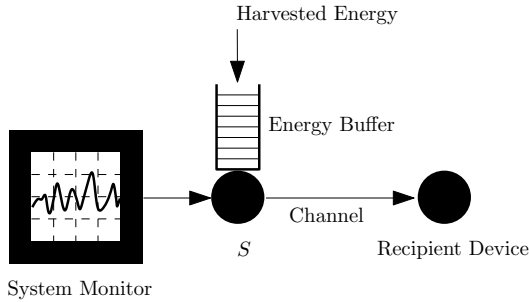


Fig. 1. The System Model.

A. Offline Problem

In the offline version of the problem, let us denote by $N_U(t)$ the counting process of updates sent by time t , defined for $t \in [0, T]$.

$$N_U(t) = \sum_k u(t - l_k) \quad (1)$$

where $u(t)$ is the unit step function and l_k is the time instant for the k th status update.

If all the status packets sent by S are received, the time difference $t - l_{N_U(t)}$ corresponds to the age of the freshest status packet at the recipient device by the time t . Let \bar{d} be the average age of status information available at the recipient device side. Then, the average age of status packets \bar{d} can be computed as in below:

$$\bar{d} = \frac{1}{T} \int_0^T t - l_{N_U(t)} dt. \quad (2)$$

It can be seen from Fig.2 that the age of status packets exhibits a sawtooth pattern and its average \bar{d} can be expressed as the sum of areas of triangles between status updates:

$$\bar{d} = \frac{1}{2T} \sum_k (\max\{l_{k+1}, T\} - l_k)^2 \quad (3)$$

From the above formulation of \bar{d} , one can deduce that status updates should be sent as frequently as possible to reduce \bar{d} which is desired to monitor the system accurately. However, in an energy harvesting device like S , the frequency of status updates is limited by the energy availability of the device which depends on both energy consumption and energy replenishment rate. Let us assume that each update requires one unit of energy, hence $N_U(t)$ is also the amount of energy used up to time t solely for updates. We will impose the condition that the device is "on" for the time window of interest, $[0, T]$, and, modeling a constant current drain while the device is on, there is an energy loss which is linear in time. Therefore:

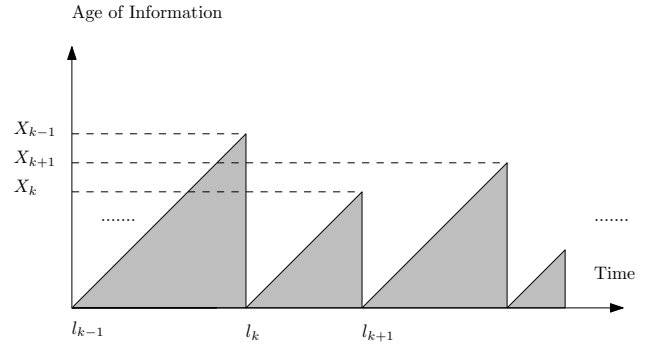


Fig. 2. Age of status update packets.

$$N_U(t) \leq M_U(t) = \max(0, \lfloor B_0 + E_h(t) - P_{ON}t \rfloor) \quad (4)$$

Note that due to the negative linear term, the RHS of (4) is not necessarily non-decreasing. As our formulation imposes the device to be "on" and consume a constant minimum power for the whole duration of interest, the RHS attaining the value zero for any t would force $N_U(t) = 0$ for all $t \in [0, T]$. However, we are interested in examining the (typical) case where $P_{ON}t$ is small with respect to the positive terms such that the RHS constitutes a nontrivial upperbound.

As $N_U(t)$ is a counting process, we can write a tighter bound $\hat{N}_U(t)$ on it by finding the largest counting process that fits within the RHS of (4).

$$N_U(t) \leq \hat{N}_U(t) = \min_{s \geq t} M_U(s) \quad (5)$$

In an operating regime where $P_{ON}t$ is appreciable with respect to the other terms (i.e. battery size is very small, and/or the energy replenishment rate is not sufficient to keep the device on continuously), it would be more appropriate to consider a formulation where we have the option of turning the device "off" or putting it in "sleep mode".

$\hat{N}_U(t)$ is the highest possible nondecreasing integer valued function below $M_U(t)$, hence it still constitutes an upper bound for $N_U(t)$. Let us decompose $\hat{N}_U(t)$ in terms of unit step functions as we did for $N_U(t)$.

$$\hat{N}_U(t) = \sum_l u(t - \hat{l}_k) \quad (6)$$

where \hat{l}_k is the earliest time instant to send the k th status update.

In the offline problem, the counting process $N_U(t)$ represents a solution which is constrained by another counting process $\hat{N}_U(t)$. Therefore, the offline problem to minimize the average age of status packets \bar{d} can be defined as follows:

$$\begin{aligned} \min. & \frac{1}{T} \int_0^T t - l_{N_U(t)} dt. \\ \text{s.t.} & N_U(t) \leq \hat{N}_U(t) \end{aligned}$$

Also note that even if a deterministic and constant delay were assumed to be seen by individual status update packets, the problem definition for that case would be equivalent to the above definition.

B. Offline Solution

To characterize the offline optimal solution for the offline problem in the previous section, we first state and prove an essential property of offline solutions. This property dictates that in optimal offline solutions, the frequency of status updates is nondecreasing in time.

Lemma 1. *Let X_k s represent the interval length between k th and $(k+1)$ st status updates. In any feasible $N_U(t)$ which minimizes \bar{d} , interval lengths X_k s are nonincreasing in k .*

Proof. Suppose that for some feasible $N_U(t)$, a status update interval (l_{k-1}, l_k) is shorter than the next status update interval (l_k, l_{k+1}) , i.e. $X_{k-1} < X_k$. In this case, without changing the interval length $l_{k+1} - l_{k-1}$, the time instant for the k th status update l_k can be shifted towards l_{k+1} which is the time instant of the next status update. This rearrangement does not violate energy constraints for any new value l'_k of l_k between l_k and l_{k+1} . The change in the value of \bar{d} can be calculated as follows:

$$\Delta \bar{d} = \frac{1}{2T} [(l'_k - l_{k-1})^2 + (l_{k+1} - l'_k)^2 - ((l_k - l_{k-1})^2 + (l_{k+1} - l_k)^2)]$$

Therefore, \bar{d} can be decreased for any $N_U(t)$ in which, status update intervals can increase as status update packets are sent.

Note that this result is also valid when $l_{k+1} = T$.

By using the above lemma, a policy using X_k^* as the status update interval after sending k th status update packet can be derived:

$$X_k^* = \max_{v \in (l_k, T)} \left(\frac{v - l_k}{\hat{N}_U(v) + 1 - k} \right) \quad (7)$$

Theorem 1. *Using X_k^* s as in Eq. 7 gives the optimal offline status update process.*

Proof. From the Lemma 1, it is known that X_k s do not increase by time in an optimal offline solution. Clearly, a status update process $N_U(t)$ satisfying this property, should be the one that keeps the frequency of status updates fixed until the end of problem horizon, i.e. $X_m = X_k$ for $m > k$. Therefore, such a status update process $N_U(t)$ can be written for time instants $t \geq l_k$ as in below:

$$N_U(t) = k + \lfloor \frac{t - l_k}{X_k} \rfloor \quad (8)$$

From the energy causality conditions $N_U(t)$ should be always smaller than $\hat{N}_U(t)$:

$$k + \lfloor \frac{t - l_k}{X_k} \rfloor \leq \hat{N}_U(t)$$

In the above inequality, both sides take integer values and the equality holds for a range of $\frac{t - l_k}{X_k}$ smaller than 1. Hence incrementing the RHS by 1 allows us to eliminate the floor operation for finding critical values of X_k . Accordingly, the inequality below can be written:

$$X_k \geq \frac{t - l_k}{\hat{N}_U(t) + 1 - k}$$

The above inequality should be satisfied for any time in (l_k, T) , hence X_k^* in Eq. 7 is an achievable lower bound for the optimal offline status update interval.

Now suppose that X_k is selected to be longer than X_k^* such as $X_k^* + \delta X_k$. When X_k is lengthened by δX_k , the rest of the status update intervals, i.e. X_m s for $m > k$, should be shortened by δX_k in total. However, the reduction in the age of information for updates $m > k$, cannot compensate the increase due to the lengthening of X_k by δX_k . The reason of this is that X_m s should be shorter than X_k by the Lemma 1 and hence a smaller area in the age of information pattern vanishes with their shortening. Therefore, in the optimal offline solution, it is not useful to have X_k s longer than X_k^* s and this shows that X_k^* s are optimal offline status update intervals. \square

Theorem 2. *Using X_k^* s as in Eq. 7 minimizes the longest status interval within $(0, T)$.*

Proof. The results in the Lemma 1 and Theorem 1 are also valid for an altered version of the offline problem where the objective function is replaced with the sum of n th powers of status update intervals, i.e. $\sum_k X_k^n$. Again, minimizing this objective function is also equivalent to minimizing the n th root of itself, i.e. $\sqrt[n]{\sum_k X_k^n}$ which is an L^n norm for the age of status updates. Therefore, as n goes to infinity, the objective function $\sqrt[n]{\sum_k X_k^n}$ goes to the longest of status update interval, i.e. a supremum norm for the age of status

updates, which shows that status update intervals as in Eq. 7 also minimize $\max_k X_k$.

$$\lim_{n \rightarrow \infty} \sqrt[n]{\sum_k X_k^n} = \max_k X_k$$

□

C. Online Problem

Suppose S has only real time knowledge of $\hat{N}_U(t)$ (the energy arrival profile), and its statistics. Also assume that S decides whether to send a status update packet at every t_s units of time and this status update packet can be successfully received with a probability of p . Then, a dynamic programming formulation can be employed to find the action minimizing the expected cumulative age of status updates $J(x, t)$ after some time t .

$$J(x(t), t) = \min_{\mu(t) \in \{0,1\}} E[c(x(t), \mu(t)) + J(\phi(x(t), \mu(t)), t + t_s) | \theta(t)] \quad (9)$$

where $x(t)$ is the present state, $\mu(t)$ represents the action to be taken, $c(x(t), \mu(t))$ is the cost of the action $\mu(t)$, $\phi(x(t), \mu(t))$ is the next state and $\theta(t)$ is the history before time t .

The cost function $J(x(t), t)$ where $t > T - t_s$ is defined as follows:

$$J(x(t), t > T - t_s) = \min_{\mu(t) \in \{0,1\}} E[c(x(t), \mu(t))]$$

Let us assume S takes actions depending on only its present state x and time t . In this case, the present history $\theta(t)$ can be eliminated from the stochastic dynamic programming equation.

As a part of the present state $x(t)$, the present energy level $e(t)$ should be taken into account to track the energy availability of S . The present energy level $e(t)$ changes until the next decision instant $t + t_s$ as follows:

$$e(t + t_s) = e(t) - \mu(t) - P_{ON}t_s - E_h(t) + E_h(t + t_s) \quad (10)$$

Whenever S has energy above 1 at a decision instant, it can decide on sending a status update packet. On the other hand, the expected cost of the decision $\mu(t)$ depends on the age of the most fresh status update packet at the recipient device. Let $d(t)$ represent the present age of status updates. Then, the expected cost of the decision $\mu(t)$, i.e. immediate cost function, can be expressed as follows:

$$E[c(x(t), \mu(t))] = E[d(t)](1 - \mu(t)p)t_s + \frac{1}{2}t_s^2 \quad (11)$$

Note that the expectation of $d(t)$ is sufficient to determine the expected cost of the decision $\mu(t)$ and its time evolution can be computed as in below:

$$E[d(t + t_s)] = E[d(t)](1 - \mu(t)p) + t_s \quad (12)$$

It can be concluded that the expectation $E[d(t)]$ is a part of the present state together with the present energy level $e(t)$ and assuming the energy harvesting is a stationary process, it is sufficient to have the pair as the present state information: $x(t) = (e(t), E[d(t)])$. Note that the main goal of the online problem is to minimize the time average of $E[d(t)]$ while being restricted by $e(t)$.

1) *Extension to Randomly Formed Updates:* This model can be extended to the case when the source only has access to status updates generated at random instants. In this case, the freshest update available at time t has a positive age $s(t)$ to which the current age $d(t)$ reduces once the update is successfully received. The discrete time analysis above (where a decision is made once every t_s time units) can be applied with minor modifications: if an update is generated, the sender may choose to send it the next slot, or keep it. At any decision point, it is sufficient to consider the latest single update that has not yet been sent, as any earlier (older) ones will be made obsolete by the arrival of the freshest. Hence, multiple updates occurring within the previous slot may be treated as one update without any effect on the optimal decision rule.

Let $I(t, t')$ be the indicator function of the event that a new update (i.e. at least one new update) became available to the sender in the time interval (t, t') , for $t' > t$. Then, $s(t + t_s)$ can be expressed as:

$$s((n+1)t_s) = s(nt_s)[1 - I(nt_s, (n-1)t_s)] + t_s \quad (13)$$

Suppose that the age $s(t)$ is known to S at time t and hence the state representation for the problem in the previous section can be enlarged to $x(t) = (e(t), E[d(t)], s(t))$. The immediate cost for this case can be rewritten as follows:

$$E[c(x(t), \mu(t))] = (E[d(t)](1 - \mu(t)p) + s(t)\mu(t)p)t_s + \frac{1}{2}t_s^2 \quad (14)$$

The expectation of $d(t)$ can be tracked by the following equation:

$$E[d(t + t_s)] = E[d(t)](1 - \mu(t)p) + s(t)\mu(t)p + t_s \quad (15)$$

Note that the expectation of $d(t)$ is always larger than $s(t)$, thus for $\mu(t) = 1$ and sufficiently small t_s , the expected age gets smaller, i.e. $E[d(t + t_s)] < E[d(t)]$.

D. Solution of the Online Problem

The optimal solution to the dynamic programming formulation of the online problem in Eq.9 can be obtained by evaluating the cost function $J(x(t), t)$ with the help of time evolutions in Eq. 10,12 and the immediate cost function in Eq. 11. Yet, considering the complexity of this computation, finding the optimal action might be impractical.

On the other hand, we will show (Thm 3) that the optimal policy is a threshold-type policy for $p = 1$, as well as for $p < 1$ for sufficiently small t_s , where the optimal decision function

$\mu^*(t)$ is a time-varying indicator function in the following form:

$$\mu^*(t) = \mathbb{1}_{(e(t) \geq 1)} \cap \{E[d(t)] \geq d^*(t)\} \quad (16)$$

To prove that $\mu^*(t)$ can be expressed as in Eq. 16, we first show that the cost function $J((e(t), E[d(t)]), t)$ is nondecreasing with $E[d(t)]$.

Lemma 2. $J((e(t), E[d(t)]), t)$ is a nondecreasing function of $E[d(t)]$.

Proof. When $t \geq T - t_s$, the cost function $J((e(t), E[d(t)]), t)$ is the immediate cost function in Eq.11 which increases with $E[d(t)]$.

$$J(x(t), t \geq T - t_s) = E[d(t)](1 - \mathbb{1}_{(e(t) \geq 1)}p)t_s + \frac{1}{2}t_s^2$$

Nondecreasing monotonicity of $J(x(t), t)$ for $t \geq T - t_s$ with respect to $E[d(t)]$ also implies the monotonicity of $J(x(t), t)$ by $t > T - 2t_s$ Eq. 9. To show that consider the time interval $T - t_s > t > T - 2t_s$ where $J(x(t), t)$ is computed as the minimum of expected costs, i.e. age of information, with decisions $\mu(t) = 0$ or $\mu(t) = 1$. In either of these decisions, $J(x(t), t)$ cannot decrease with increasing $E[d(t)]$ since both immediate cost function and the expected age of information for $t > T - t_s$ are monotonically nondecreasing with $E[d(t)]$. Therefore, $J(x(t), t)$ is nondecreasing function of $E[d(t)]$ for $t > T - 2t_s$. Similarly, by the monotonicity of $J(x(t), t)$, if the claim is true for $t > T - (m - 1)t_s$, it is true for $t > T - mt_s$ for any integer $m \geq 1$. Therefore, by induction, $J((e(t), E[d(t)]), t)$ is nondecreasing with $E[d(t)]$. \square

Theorem 3. If (i) $p = 1$, or (ii) $p < 1$ and t_s is chosen sufficiently small, the optimal policy is a threshold type policy as in Eq.16.

Proof. For $x(t) = (e(t), E[d(t)])$ where $e(t) > 1$, the difference between the expected costs of decisions $\mu(t) = 0$ and $\mu(t) = 1$, say $J_{0,1}(x(t), t)$, can be written as follows:

$$J_{0,1}(x(t), t) = E[d(t)]pt_s + E[J((e'(t+t_s), E[d(t)]+t_s), t+t_s)] - E[J((e'(t+t_s) - 1, (1-p)E[d(t)] + t_s), t+t_s)]$$

where $e'(t+t_s) = e(t) - P_{ON}t_s - E_h(t) + E_h(t+t_s)$

To prove that the optimal policy is a threshold type policy for $E[d(t)]$, it is sufficient to show that the difference $J_{0,1}(x(t), t)$ is nondecreasing in $E[d(t)]$ which means that if $\mu(t) = 1$ is the optimal decision for some $x(t) = (e(t), E[d(t)])$, then the optimal decision, for any $x'(t) = (e(t), E[d'(t)])$ where $E[d'(t)] > E[d(t)]$, is still $\mu(t) = 1$.

When $p = 1$, the positive part of $J_{0,1}(x(t), t)$ increases with $E[d(t)]$ by Lemma 3 but its negative part is independent from $E[d(t)]$, thus it is guaranteed that $J_{0,1}(x(t), t)$ is nondecreasing with $E[d(t)]$.

In order to show the monotonicity of $J_{0,1}(x(t), t)$ for $p < 1$, $J_{0,1}(x(t), t)$ can be expressed in terms of $J_{0,1}(x'(t+t_s), t+t_s)$.

Let us define $J_0(\cdot)$ and $J_1(\cdot)$ as the expected costs of decisions $\mu(t) = 0$ and $\mu(t) = 1$, respectively. Then, $J(\cdot) = \min(J_0(\cdot), J_1(\cdot))$ can be written in one of the following forms:

$$J(\cdot) = J_1(\cdot) - (-J_{0,1}(\cdot))_+$$

$$J(\cdot) = J_0(\cdot) - (J_{0,1}(\cdot))_+$$

Substituting these in the previous expression of $J_{0,1}(x(t), t)$, the expression below can be derived:

$$\begin{aligned} J_{0,1}(x(t), t) &= E[d(t)]pt_s + R(t_s) \\ &\quad - E[(-J_{0,1}((e'(t+t_s), E[d(t)] + t_s), t+t_s))_+] \\ &\quad + E[(J_{0,1}((e'(t+t_s) - 1, (1-p)E[d(t)] + t_s), t+t_s))_+] \end{aligned}$$

where

$$\begin{aligned} R(t_s) &= -pt_s + E[J((e'', (1-p)(E[d(t)] + t_s), t+2t_s)] \\ &\quad - E[J((e'', (1-p)E[d(t)] + 2t_s), t+2t_s)] \end{aligned}$$

and

$$e'' = e(t) - 2P_{ON}t_s - E_h(t) + E_h(t+2t_s) - 1$$

Note that $R(t_s)$ goes to zero as t_s goes to zero and accordingly its contribution in the above expression of $J_{0,1}(x(t), t)$ can be neglected for sufficiently small t_s compared to $E[d(t)]$. If $R(t_s)$ is omitted from the above expression, it can be seen that the nondecreasing monotonicity of $J_{0,1}(x, t)$ with respect to $E[d(t)]$ could be inherited from the monotonicity of $J_{0,1}(x', t+t_s)$. This means that when $J_{0,1}(x', t+t_s)$ is nondecreasing with respect to its expected age state, $J_{0,1}(x(t), t)$ should be also nondecreasing with respect to its expected age state. In addition to this, we know that $J_{0,1}(x(t), t)$ is nondecreasing with $E[d(t)]$ for $t > T - t_s$. Therefore, by the induction method, it can be concluded that $J_{0,1}(x(t), t)$ is a nondecreasing function of $E[d(t)]$ at any time t . \square

E. Suboptimal Online Solution: Balanced Updating

As an online heuristic, we introduce *Balanced Updating (BU)* policy where S registers the present state of expected age of status updates $E[d(t)]$. Basically, in this policy, S sends a status update whenever $E[d(t)]$ is higher than a threshold value based on an approximation of average interval of status updates that can be achieved in the remaining part of the period. Let \bar{P}_h show the average harvested power and assume that is higher than P_{ON} . The decisions of BU policy $\mu(t)^{BU}$ can be represented by a time varying indicator function as in the below:

$$\mu(t)^{BU} = \mathbb{1}_{(e(t) \geq 1)} \cap \{E[d(t)] \geq \bar{X}(t)\} \quad (17)$$

where

$$\bar{X}(t) = \frac{T-t}{e(t) + (T-t)(\bar{P}_h - P_{ON})} \quad (18)$$

Note that as T goes to infinity, the time varying threshold $\bar{X}(t)$ converges to $\frac{1}{\bar{P}_h - P_{ON}}$.

Let us define the simplest policy with $\mu(t) = \mathbb{1}_{(e(t) \geq 1)}$, which only checks whether S has sufficient energy to send a status, as *Greedy*. This simple non-adaptive policy is used as baseline in our comparative study below.

III. NUMERICAL AND SIMULATION RESULTS

The performance of BU against Greedy is studied in a numerical experiment. In this numerical experiment, setting t_s as a unit time, T has been taken as 100 time units and results have been averaged over 10^4 different realizations of energy profiles and packet failure/success events where P_{ON} has been set to 0.01 power units (i.e. unit energy/unit time). Energy arrivals are generated according to a Bernoulli process where energy harvested during each time unit is either zero or a constant satisfying a preset average harvested power level.

In Fig. 3 and 4, performances of BU and Greedy policies with respect to average and maximum age of information are compared for $p = 0.9$. As seen from these figures, the performances of BU and Greedy policies are close to each other at extreme values of harvested power. This result can be explained by the following observation: When average power is very low, a long time is needed between status updates and $E[d(t)]$ becomes so high that the conditions $(e(t) \geq 1)$ and $(E[d(t)] \geq \bar{X}(t))$ are satisfied concurrently most of the time although the threshold $\bar{X}(t)$ is relatively high for this case. On the contrary, at a high average power, the threshold $\bar{X}(t)$ is very low which guarantees the condition $(E[d(t)] \geq \bar{X}(t))$ holds almost always. The performance difference is much more apparent outside of these extremes. For example, Greedy policy requires 30%–50% more average power than BU policy to achieve the same average and maximum age of information when average harvested power is around 0.6 power units.

For $p = 1$, the optimal offline solution in Eq. 7 can be also applied for any numerical realization of the energy arrival process. In Fig.5, the policies BU and Greedy are compared with the performance of the offline optimal solution (the solution computed with full knowledge of the energy constraints at $t = 0$.)

IV. CONCLUSION

We considered the problem of optimizing the process of sending updates from a source to a receiver to minimize the time average age of updates, under constraints on the number of updates that may be sent by a given time. The solution also turns out to minimize peak age over all update packets sent. The constraints on the rate of updates are general but can model a sender whose battery gets replenished at arbitrary time instants.

Based on offline throughput maximizing solution and the cost-to-go function generated by a dynamic programming approach, an online heuristic (BU) is presented, and observed to achieve close to offline optimal performance. According to the numerical results obtained through simulations, the average age of information as well as the peak status age is improved, especially in low energy cases, if energy management policies proposed in this paper are employed.

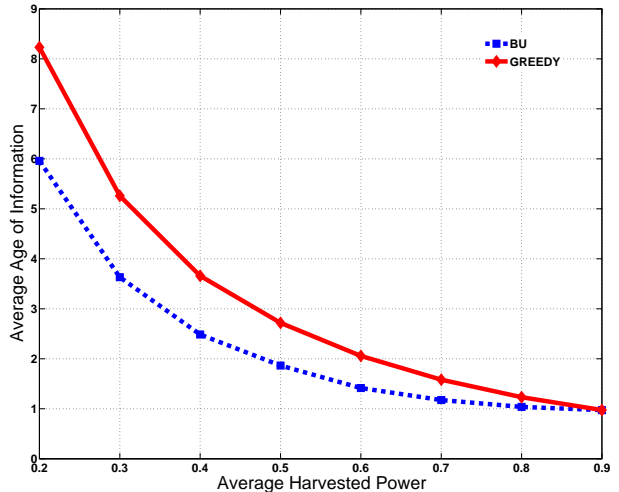


Fig. 3. Average age of information (in unit time) versus average harvested power (in unit power) comparison of BU and greedy policies for $p = 0.9$ taken over 10^4 different Bernoulli realizations of energy profiles and packet failure/success events where the probability of energy arrival is 0.1, $P_{ON} = 0.01$ power units and $T = 100$ time units.

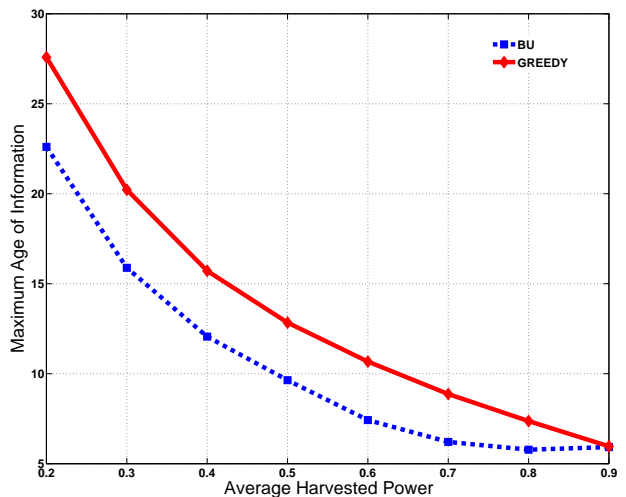


Fig. 4. Maximum age of information (in unit time) versus average harvested power (in unit power) comparison of BU and greedy policies for $p = 0.9$ taken over 10^4 different Bernoulli realizations of energy profiles and packet failure/success events where the probability of energy arrival is 0.1, $P_{ON} = 0.01$ power units and $T = 100$ time units.

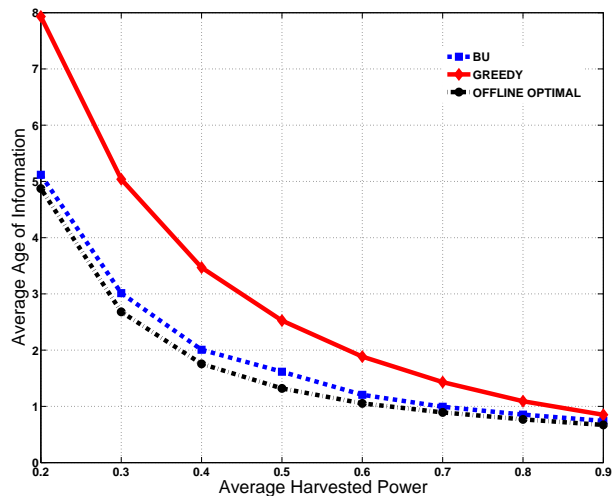


Fig. 5. Average age of information (in unit time) versus average harvested power (in unit power) comparison of offline optimal, BU and greedy policies for $p = 1$ taken over 10^4 different Bernoulli realizations of energy profiles and packet failure/success events where the probability of energy arrival is 0.1, $P_{ON} = 0.01$ power units and $T = 100$ time units.

Extension of the formulation to incorporate randomly formed updates, where the sender only has updates that are formed according to a stochastic process at the source side, was also briefly considered. Examination of the related dynamic programming formulation suggests a similar threshold that depends on the joint state of current energy, expected age at the receiver, and the current age at the source. In future work, it would be interesting to examine the behavior of this threshold with respect to the inter-update arrival distribution, specifically the three cases where update arrivals are memoryless, light-everywhere or heavy-everywhere. Finally, investigating the effect of delays during the transmission of status update messages which cause messages to be received out of order could be incorporated into the formulation constructed in this paper.

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