Real-time Broadcasting over Block-Fading Channels

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I. Abstract

Broadcast transmission from a base station (BS) to a group of users is studied. It is assumed that the BS receives data at a constant rate and transmits these messages to the whole set of users within a certain deadline. The channels are assumed to be block fading and independent over blocks and users. Our performance measure is the total rate of received information at the users within the transmission deadline. Three different encoding schemes are proposed, and they are compared with an informed transmitter upper bound in terms of the average total reception rate for a set of users with varying channel qualities. It is shown that no single transmission strategy dominates for all channel setups, and the best broadcasting technique depends on the distribution of the average channel conditions over the users.

II. Introduction

Consider a satellite or a base station (BS) broadcasting to a set of users distributed over a geographical area. We assume a block fading channel model in which the channel state information (CSI) is available only at the receiver. At the beginning of each channel block the transmitter is provided with an independent message whose rate is controlled by an external source. We assume for simplicity that all the messages have the same fixed rate. For example, these messages might correspond to the video packets of a live event whose rate is fixed by the recording unit, and cannot be changed.

The goal of the BS is to broadcast these data packets to all the users in the system. Each user wants to receive as many packets as possible. We further assume a delay constraint on the transmission, that is, M messages that arrive gradually over M channel blocks need to be transmitted by the end of the last channel block. Hence, the last message sees only a single channel realization, while the first packet can be transmitted over the whole span of M channel blocks.

Performance measure is the total decoded rate at the users. Note that, for a finite number of M packets and M channel blocks, it is not possible to average out the effect of fading due to the delay constraint, and there is always a non-zero outage probability for any message [1]. Hence, we cannot talk about a capacity region in the Shannon sense. We will study the cumulative mass function (c.m.f) of the total decoded rate as well as the behavior of the average total decoded rate over a set of users with varying average channel quality.

It is important to identify a transmission scheme that performs well over the whole set of users. In a narrow-beam satellite system, for instance, the average signal-to-noise ratio (SNR) experienced by users in different parts of the beam footprint changes little (in clear sky conditions), while in a cell-based broadcasting system the SNR experienced by users in different parts of the cell may vary significantly with the distance from the BS. Hence, it is important for the BS to adapt the encoding technique to the specific channel characteristics.

The simplest transmission scheme is to transmit each message only over the following channel block. In this scheme, for any given user each packet will be received with equal probability. However, for the users with low SNR, this scheme might lead to a very low average rate. Instead, on the other extreme, BS can transmit only the first message over all channel blocks, increasing the probability of its correct decoding at the users that are located at the cell boundary. In general, the resources for each channel block can be distributed among all the available messages. This can be achieved in various ways. In particular we will consider time-division, superposition and joint encoding schemes, and compare numerically the c.m.f. of the number of successfully decoded packets for each of these schemes. We also introduce an upper bound considering the availability of the CSI at the transmitter.

III. System Model

We consider broadcasting over a block fading channel that is constant for a block of n channel uses. We assume that the BS receives one new message at the beginning of each channel block. We consider broadcasting of M messages over M channel blocks. Assume that message W_t is available at the beginning of channel block t, $t = 1, \dots, M$. Each message is chosen randomly with uniform distribution from the set $W_t \in \{1, \dots, 2^{nR}\}$. Equivalently, each message W_t has rate R. All the messages are addressed to a population of N users.

The channel from the BS to user j in block t is given by

$$\mathbf{y}_j[t] = h_j[t]\mathbf{x}[t] + \mathbf{z}_j[t],$$

where $h_j[t]$ is the channel state, $\mathbf{x}[t]$ is the length-n channel input vector of BS, $\mathbf{z}_j[t]$ is the vector of independent and identically distributed (i.i.d.) unit-variance Gaussian noise, and

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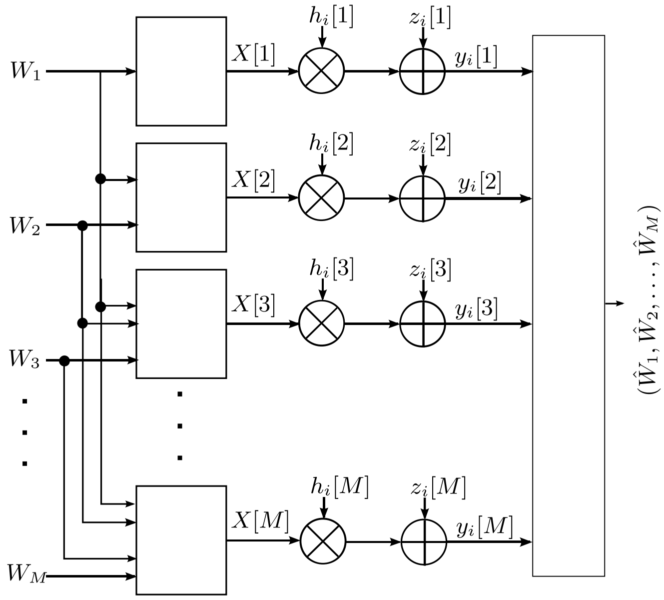

Fig. 1. Equivalent channel model for the sequential transmission of M messages over M blocks of the fading channel to a single receiver.

$\mathbf{y}_j[t]$ is the length-n channel output vector of user i. We assume that the channel coefficients $h_j[t]$ are i.i.d. with zero-mean unit variance complex Gaussian. These instantaneous channel gains are known at the receiving end of each link, while the BS only has a statistical information. We have a short-term average power constraint of P, i.e., $E[\mathbf{x}[t]\mathbf{x}[t]^\dagger] \leq nP$ for $t = 1, \ldots, M$.

The channel from the source to each receiver can be seen as a multiple access channel (MAC) with a special message hierarchy [2], in which the encoder at each channel block acts as a separate transmitter and each user tries to decode as many of the messages as possible. See Fig. 1 for an illustration of this channel model. We denote the instantaneous channel capacity to user j over channel block t by C_t^j:

$$C_t^j \triangleq \log_2(1 + \phi_j[t]P). \tag{1}$$

Note that C_t^j is a random variable, and due to the random nature of the channel, it is not possible to guarantee any non-zero rate to any user at any channel block. We consider the c.m.f. of the total average decoded rate at each user as our performance measure.

IV. Single User Scenario

In this section we focus on a single user. For simplicity of notation, we drop the subscripts indicating the user index to simplify the notation. We introduce three different transmission schemes.

A. Time Sharing Transmission

One of the resources that the encoder can allocate among different messages is the total number of channel uses within each channel block. While the whole of the first time slot has to be dedicated for message W_1, as it is the only available message, the second time slot can be divided among the messages W_1 and W_2, and so on so forth. Assume that the encoder divides the channel block t into t portions $\alpha_{1t}, \ldots, \alpha_{tt}$ such that $\alpha_{it} \geq 0$ and $\sum_{i=1}^t \alpha_{it} = 1$. In channel block t, $\alpha_{it}n$ channel uses is allocated for the transmission of message W_i. A constant power P is used throughout the block. Then the total amount of received mutual information relative to message W_i is:

$$I_i^{tot} \triangleq \sum_{t=i}^{M} \alpha_{it} \log_2 (1 + \phi[t]P). \tag{2}$$

Different time allocations among the messages lead to different c.m.f. for the total decoded rate. For simplicity, we assume equal time allocation among all available messages, that is, for $i = 1, \ldots, M$, we have $\alpha_{it} = \frac{1}{t}$ for $t = i, i+1, \ldots, M$, and $\alpha_{it} = 0$ for $t = 1, \ldots, i$. Hence,

$$I_i^{tot} = \sum_{t=i}^{M} \frac{1}{t} \log_2 (1 + \phi[t]P). \tag{3}$$

In this scheme the messages that arrive earlier are allocated more resources, and hence, are more likely to be decoded. We have $I_i^{tot} > I_j^{tot}$ for $1 < i < j < M$. Hence, the probability of decoding exactly m messages is:

$$\eta(m) \triangleq Pr\{I_{m+1}^{tot} < R < I_m^{tot}\}. \tag{4}$$

B. Superposition Transmission

Next we consider *superposition encoding (SE)*. In SE the source generates a Gaussian codebook of size 2^{nR} for each message to be transmitted in each block. In channel block t, it transmits the superposition of the t codewords, chosen from t different codebooks generated independently, corresponding to messages $\{W_1, \ldots, W_t\}$. The codewords are scaled such that the average total transmit power is P in each block. In the first block, only information about message W_1 is transmitted with average power $P_{11} = P$; in the second block we divide the total power P among the two messages, allocating P_{12} and P_{22} for the codewords corresponding to W_1 and W_2, respectively. In general, over channel block t we allocate average power P_{it} for the codeword corresponding to message W_i, while $\sum_{i=1}^t P_{it} = P$. We let $\mathbf{P}$ denote the $M \times M$ upper triangular power allocation matrix such that $\mathbf{P}_{i,t} = P_{it}$.

Let $\mathcal{S}$ be any subset of the set of messages $\mathcal{M} = \{1, \ldots, M\}$. We define $C(\mathcal{S})$ as follows:

$$C(\mathcal{S}) \triangleq \sum_{t=1}^{M} \log_2 \left(1 + \frac{\phi[t] \sum_{s \in \mathcal{S}} P_{st}}{1 + \phi[t] \sum_{s \in \mathcal{M} \setminus \mathcal{S}} P_{st}}\right). \tag{5}$$

This provides an upper bound on the total rate of messages in set $\mathcal{S}$ that can be decoded jointly at the user considering the codewords corresponding to the remaining messages as noise.

The receiver first checks if any of the messages can be decoded alone by considering the other transmissions as noise. If a message can be decoded, the corresponding signal is subtracted and the algorithm is run over the remaining signal. If no message can be decoded alone, then the receiver considers joint decoding of message pairs, followed by triplets, and so on so forth. This optimal decoding algorithm for superposition coding to find the total decoded rate at the receiver is outlined in Algorithm 1 below. The user calls the algorithm with $Rate = 0$ and $\mathcal{M} = \{1, \ldots, M\}$ initially. While Algorithm 1

Algorithm 1 Total_Decoded_Rate ($Rate$, $\mathcal{M}$, $\mathbf{P}$)

$Old\_Rate = 0$
for $i = 1$ to $|\mathcal{M}|$ **do**
  if $iR \le \max_{\mathcal{S}:\mathcal{S}\subseteq\mathcal{M},|\mathcal{S}|=i} C(\mathcal{S})$ **then**
    $Rate = Rate + iR$
    $\mathcal{M} = \mathcal{M} \backslash \mathcal{S}$
  end if
end for
if $(\mathcal{M} \neq \emptyset)\&(Old\_Rate < Rate)$ **then**
  `Total_Decoded_Rate` ($Rate$, $\mathcal{M}$,$\mathbf{P}$)
else
  Output= $Rate$
end if

gives us the maximum decoded total rate, it is hard in general to find a closed form expression for the average decoded total rate, and optimize it over power allocation matrices. Hence, we focus here on two special cases. In *memoryless transmission (MT)* scheme, we consider a diagonal power allocation matrix $\mathbf{P}$, that is, each message is transmitted over a single channel block. In *equal power allocation (EPA)* scheme, we divide the total average power P among all the available messages at each channel block. The power allocation matrix $\mathbf{P}$ takes the following form:

$$\mathbf{P}^{EPA} = \begin{pmatrix} P & \frac{P}{2} & \frac{P}{3} & \cdots & \frac{P}{M} \\ 0 & \frac{P}{2} & \frac{P}{3} & \cdots & \frac{P}{M} \\ \vdots & 0 & \frac{P}{3} & \cdots & \frac{P}{M} \\ \vdots & \vdots & 0 & \cdots & \frac{P}{M} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \ldots & \ldots & 0 & \frac{P}{M} \end{pmatrix} \quad (6)$$

where $\mathbf{P}^{EPA}_{j,t}$ is the power allocated to message j in block t.

In MT, messages can be decoded independently, and joint decoding is not needed. W_t can be decoded if and only if

$$\log_2(1 + \phi[t]P) \ge R. \quad (7)$$

Due to the i.i.d. nature of the channel over blocks, successful decoding probability is constant over messages. We define

$$p \triangleq Pr\left\{\phi[t] > \frac{2^R - 1}{P}\right\} = \int_{\frac{2^R-1}{P}}^{\infty} f_\Phi(\phi)d\phi = e^{-\frac{2^R-1}{P}}, \quad (8)$$

where $f_\Phi(\phi)$ is the p.d.f. of $\phi[t]$. The probability that exactly m messages are decoded is given by

$$\eta(m) = \binom{M}{m} p^m (1-p)^{M-m}. \quad (9)$$

Note that, we have a closed-form expression for $\eta(m)$ in MT for any regime of the channel SNR. If we let the number of messages M go to infinity, then (9) can be approximated with a Gaussian distribution, i.e.,

$$\eta(m) \simeq \frac{1}{\sqrt{2\pi Mp(1-p)}} e^{-\frac{(m-Mp)^2}{2Mp(1-p)}}. \quad (10)$$

Then the average achievable rate is

$$\overline{R} = RE[m] = R\sum_{m=0}^{M} m\eta(m) \simeq RMp, \quad (11)$$

where the approximation is tighter for higher values of M.

C. Joint Encoding Transmission

In the superposition scheme, we generate independent codebooks for each message available at the BS at each channel block and transmit the superposition of the corresponding codewords. Another possibility is to generate a single multiple-index codebook for each channel block. We call this the *joint encoding (JE)* scheme.

In the JE scheme, the transmitter generates a t dimensional codebook to be used in channel block t for $t = 1, \ldots, M$. That is, for channel block t, we generate a codebook of size $s_1 \times \cdots \times s_t$ such that $s_i = 2^{nR}$, $\forall i \in \{1, \ldots, t\}$, with Gaussian distribution, and index them as $x_t^n(m_1, \ldots, m_t)$ where $m_i \in [1, 2^{nR}]$ for $i = 1, \ldots t$. The receiver uses joint typicality decoder and tries to estimate as many messages as possible at the end of block M. With high probability, it will be able to decode the first m messages correctly if,

$$(m - j + 1)R \le \sum_{t=j}^{m} C_t \quad (12)$$

for all $j = 1, 2, \ldots, m$.

As a comparison, we illustrate the achievable rate regions for the MT and JE schemes in the case of $M = 2$ in Fig. 2. In the case of memoryless transmission, a total rate of $2R$ is achieved if both capacities C_1 and C_2 are above R. We achieve a total rate of R if only one of the capacities is above R. On the other hand, in the case of joint encoding, we tradeoff a part of the region of rate R for rate $2R$, that is, we achieve a rate of $2R$ instead of rate R, while rate 0 is achieved rather than rate R in the remaining part.

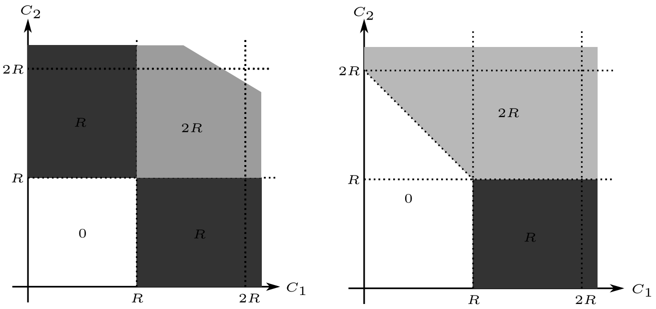

Fig. 2. Total decoded rate regions in the (C_1, C_2) domain in the case of $M = 2$ messages for independent encoding (on the left) and joint encoding (on the right) schemes.

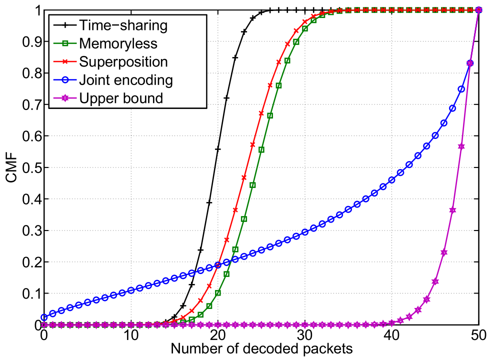

Fig. 3. CMF of the number of decoded packets for the different techniques considered. In the simulations we set $R = 1$ bit/sec/Hz, $M = 50$ and $P = 1.44$ dB.

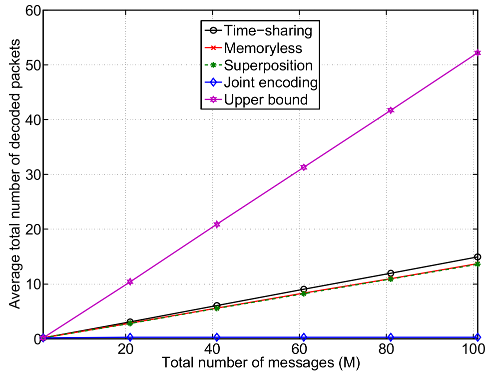

Fig. 4. Average total rate achieved plotted against the total number of packets M for a transmission rate $R = 1$ bit/s/Hz, $P = -3$ dB.

We define functions $f^m(R)$, for $m = 0, 1, \ldots, M$, as follows:

$$f^m(R) = \begin{cases} 1, & \text{if } (m-j+1)R \leq \sum_{t=j}^{m} C_t, j = 1, \ldots, m \\ 0, & \text{otherwise.} \end{cases}$$

Exactly m messages, $m = 0, 1, \ldots, M$, can be decoded if,

$$C_m \geq R \tag{13}$$
$$C_{m-1} + C_m \geq 2R \tag{14}$$
$$\cdots$$
$$C_1 + \cdots + C_m \geq mR, \tag{15}$$

and

$$C_{m+1} < R \tag{16}$$
$$C_{m+1} + C_{m+2} < 2R \tag{17}$$
$$\cdots$$
$$C_{m+1} + \cdots + C_M < (M-m)R. \tag{18}$$

Then the probability of decoding exactly m packets can be written as,

$$\eta(m) = Pr\left\{f^m(R) = 1 \text{ and } f^{m+1}(R) = 0\right\}. \tag{19}$$

Then (19) can be calculated as in Eqn. (13) at the bottom of the page, where we have defined $x^+ = \max\{0, x\}$, and $f_{C_1 \cdots C_m}(c_1 \cdots c_m)$ is the joint p.d.f. of $C_1, \ldots, C_m$, equal to the product of the marginal p.d.f.'s due to independence.

D. Informed Transmitter Upper Bound

We study an upper bound on the average decoded rate obtained by assuming the availability of the CSI at the BS. There will still be a non-zero probability of outage for each message due to the short-time power constraint which prevents power allocation among time slots; but the CSI allows the BS to transmit over the channel blocks in a manner to maximize the total decoded rate at each channel realization.

Assume that a total of $m \leq M$ packets can be decoded at some channel realization. We can always have the first m messages to be the successfully decoded ones by reordering. At any channel realization, the first m messages can be decoded successfully if and only if [2],

$$R \leq C_m + C_{m+1} + \cdots + C_M,$$
$$2R \leq C_{M-1} + C_m + \cdots + C_M,$$
$$\cdots$$
$$mR \leq C_1 + C_2 + \cdots + C_M.$$

We can equivalently write these conditions as

$$R \leq \min_{i \in \{1, \ldots, m\}} \left[\frac{1}{m-i+1} \sum_{j=i}^{M} C_j\right]. \tag{20}$$

Then, for each channel realization, the upper bound on the total decoded rate is given by $m^* R$, where m^* is the greatest m value that satisfies (20). We obtain the upper bound on the average total decoded rate by averaging $m^* R$ over the channel realizations.

E. Numerical Results for Single User Scenario

In this subsection we provide several numerical results comparing the proposed transmission schemes and the upper bound. In Fig. 3 the c.m.f. of the number of decoded packets is shown for the different techniques for $M = 50$ and $P = 1.44$

$$\eta(m) = \int_R^\infty \int_{(2R-x_m)^+}^\infty \cdots \int_{(mR-x_m-\cdots-x_2)^+}^\infty f_{C_1\cdots C_m}(x_1, \ldots, x_m) dx_1 \cdots dx_m$$
$$\times \int_0^R \int_0^{2R-x_{m+1}} \cdots \int_0^{(M-m)R-x_{m+1}-\cdots-x_{M-1}} f_{C_{m+1}\cdots C_{M-m}}(x_{m+1}, \ldots, x_{M-m}) dx_{m+1} \cdots dx_M \tag{13}$$

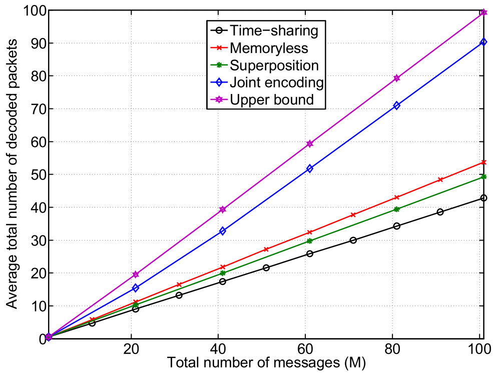

Fig. 5. Average total rate achieved plotted against the total number of packets M for a transmission rate $R = 1$ bit/s/Hz, $P = 2$ dB.

dB. From the figure it is evident that MT outperforms SE and TS, as its c.m.f. lays below the other two. On the other hand, the improvement of the JE scheme with respect to the other methods depends on the performance metric we choose. For instance, JE has the lowest probability to decode more than m packets, for $m \leq 15$, while the same scheme has the highest probability to decode more than m packets for $m \geq 22$.

In Fig. 4 and 5 the total average rate is plotted against the total number of messages M for channel SNR values equal to -3 and 2 dB, respectively, and a message rate of $R = 1$. While JE outperforms other schemes at $SNR = 2$ dB, it has the poorest performance at $SNR = -3$ dB. The opposite applies to the TS scheme.

V. Broadcast Scenario

In Section IV we have focused on the average rate decoded by a single user. We now consider the broadcasting scenario in which the BS wants to broadcast M messages to a group of users which are located at different distances from the BS. In this case we model the average channel power at node i as $d_i^{-\alpha}$, where d_i is the distance from BS to node i and α is the path loss exponent. Note that each proposed transmission scheme has a different behavior in terms of the c.m.f. of the received messages at different channel SNR values. A technique that may perform well at a given channel SNR, may perform poorly, compared to other schemes, at another SNR value. In the broadcast scenario, what becomes important is the range of average channel SNR values of the receivers, and to use a transmission scheme that performs well over this range. For instance, in a system in which all users have the same average SNR, which is the case for a narrow-beam satellite system where the SNR within the beam footprint has variations of at most a few dB on average, the transmission scheme should perform well around the average SNR of the beam. A similar situation may occur in a microcell, where the relatively small radius of the cell implies a limited variation in the average SNR range experienced by the users at different distances from the BS instead. Instead, in the case of a macrocell, in which the SNR may vary significantly from

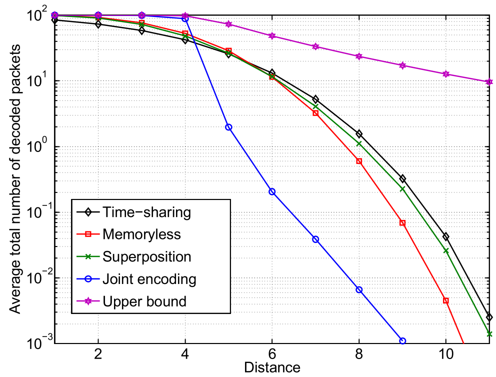

Fig. 6. The average total number of decoded messages against distance for the proposed schemes and the upper bound. In the simulations we set $R = 1$ bit/sec/Hz, $M = 100$ and $P = 20$ dB.

the proximity of the BS to the edge of the cell, the BS should adopt a scheme which performs well over a larger range of SNR values. For a given scenario the transmitter can choose the transmission scheme based on this average behavior.

A. Numerical Results for Broadcast Scenario

We present numerical results assuming that the users are placed at increasing distances from the BS. The average number of decoded messages is plotted against the distance from the base station in Fig. 6. We see that there is no scheme that outperforms the others in the whole range of distances considered. In the range up to $d = 4$ the JE scheme achieves the highest total number of decoded packets while for $d \geq 6$ the TS scheme outperforms the others. We see how the upper bound is tighter at smaller values of d. This is because the channel knowledge at the BS becomes more important as the SNR decreases.

VI. Conclusions

We have considered a BS broadcasting to a set of users, with the BS being provided with an independent message at a fixed rate at the beginning of each channel block. We have used the average total decoded rate as our performance metric. We have considered time-division, superposition and joint encoding schemes, and compared numerically their performances. An upper bound has also been introduced considering the availability of the CSI at the transmitter. We have showed that no single transmission strategy dominates for all channel setups, and the best broadcasting technique depends on the distribution of the average channel conditions over the users.

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