Rapid Granular Free-Surface Flow Past A Cylindrical Obstacle

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Introduction

When a supercritical granular avalanche flows down a chute and past a cylindrical obstacle (Fig. 1) we observe

- a detached bow shock wave, at which the velocity and depth are discontinuous, at the front of the cylinder.
- a particle-free ("vacuum") region on the lee-side, where the granular material separates from the obstacle.
- this particle-free region closes up rapidly in low Froude number flows and much more slowly at high Froude numbers.

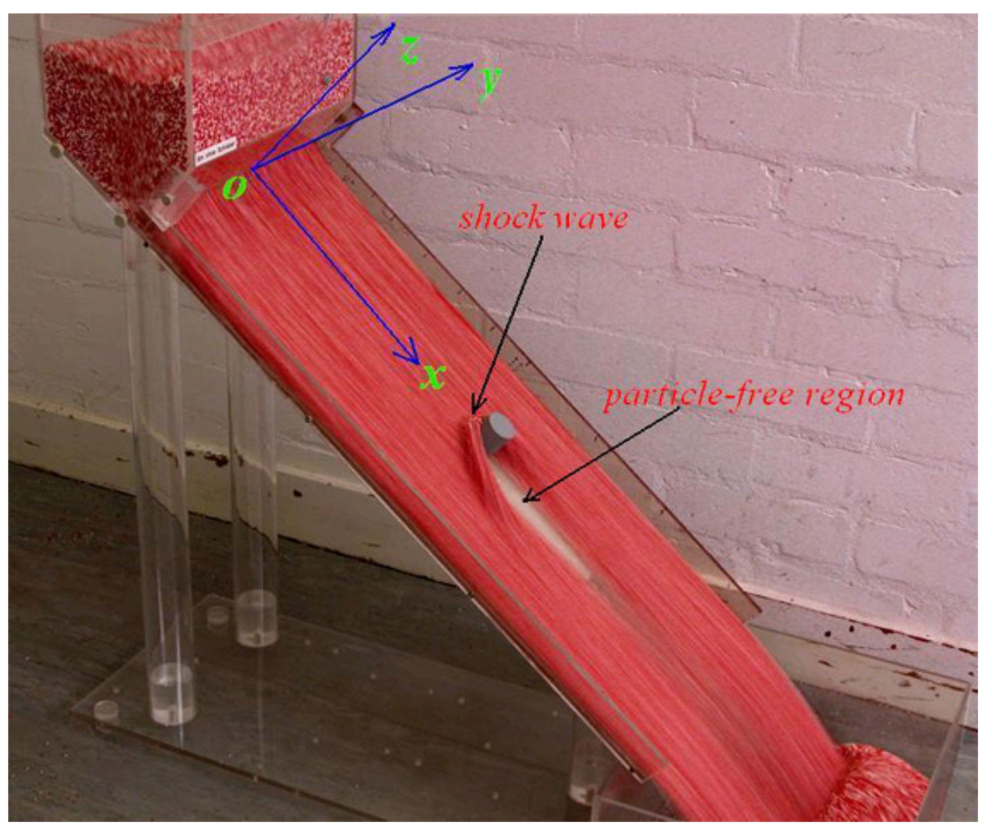

Figure 1. A photograph of the experimental set up, showing key flow features and the assumed coordinate system.

Governing equations & numerical methods

- To model such flows we adopt a fixed Cartesian coordinate system, $Oxyz$, shown in Fig. 1, with the $x-$axis is along the down slope direction at an angle ζ to the horizontal, the $y-$axis along the cross-slope direction, and the $z-$axis lies along the upward point normal.
- The avalanche has thickness h and velocity components $\boldsymbol{u} = (u, v, w)$ in the (x, y, z) directions, respectively.
- We use a hydraulic type model, first suggested by (Eglit, 1983; Savage & Hutter, 1989) and extended to two-dimensional flow past obstacles by Gray *et al.* (2003), to model the flow. The depth-integrated mass and momentum balances reduce to

$$\begin{aligned} \frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv) &= 0, \\ \frac{\partial}{\partial t}(hu) + \frac{\partial}{\partial x}(hu^2) + \frac{\partial}{\partial y}(huv) + \frac{\partial}{\partial x}\left(\frac{1}{2}\varepsilon h^2 \cos\zeta\right) &= hS^x, \\ \frac{\partial}{\partial t}(hv) + \frac{\partial}{\partial x}(huv) + \frac{\partial}{\partial y}(hv^2) + \frac{\partial}{\partial y}\left(\frac{1}{2}\varepsilon h^2 \cos\zeta\right) &= hS^y, \end{aligned} \tag{1}$$

where ε is the aspect ratio of the avalanche, the source terms are

$$\begin{aligned} S^x &= \sin\zeta - \mu(u/|\boldsymbol{u}|)\cos\zeta, \\ S^y &= \quad\;\; - \mu(v/|\boldsymbol{u}|)\cos\zeta, \end{aligned} \tag{2}$$

and μ is the coefficient of friction.

- A NOC (non-oscillatory central) finite-difference scheme (Jiang *et al.*, 1998) is used to solve this system.
- on an O-shape grid.
- and a wall boundary condition is applied at the body surface.

Laboratory experiments

- The plexiglass chute is 640mm long and 280mm wide, with an inclination angle $\zeta = 38°$.
- The center of the cylinder is located about 325mm down from the gate, the radius of the cylinder R is 15mm (0.5 in dimensionless).
- Two types of granular material are used: sand (0.1 - 0.2mm in size), and red and white 100's and 1000's (1mm in size) also known as "sprinkles".
- The formations of the shock wave and the particle-free region are captured by using a digital camera.

Low Froude number case

- We apply smaller size sand in the experiment.
- The inflow conditions are $h_0 = 0.4, u_0 = 0.847$, which implies that the inflow Froude number $\mathrm{Fr}_0 = 1.5$.
- A long-time exposure of the experiment is taken to show the steady flow around the cylinder (see Fig. 2).
- The computed thickness contours are shown in Fig. 3.
- The bow shock wave is generated about half a radius in front of the cylinder.
- The vacuum region separates from the lee-side of the obstacle and *re-attach* at the centreline about 3 radii downslope.

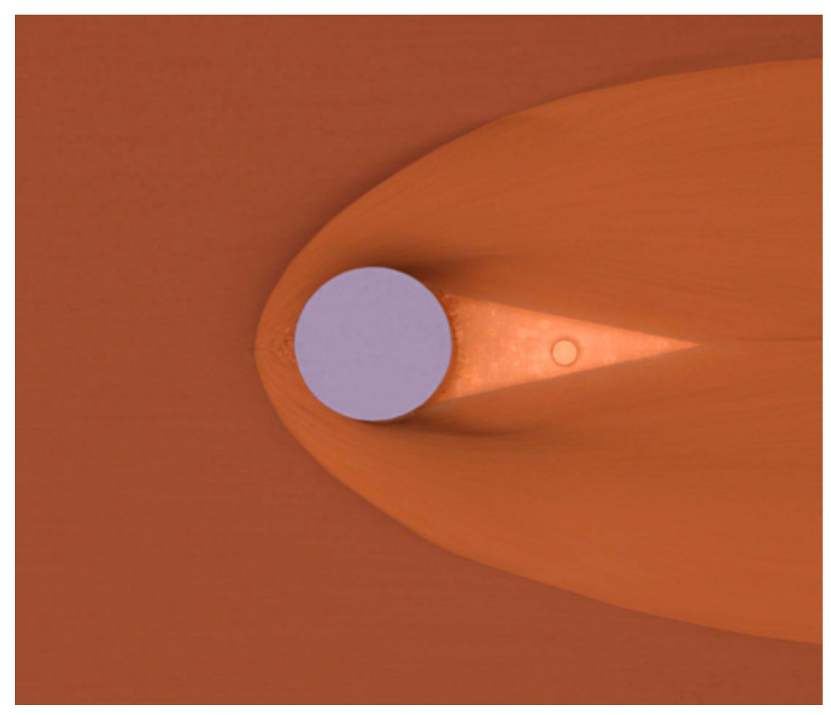

Figure 2. Long-time exposure image for the sand-case experiment (the shutter speed is 1/30 second).

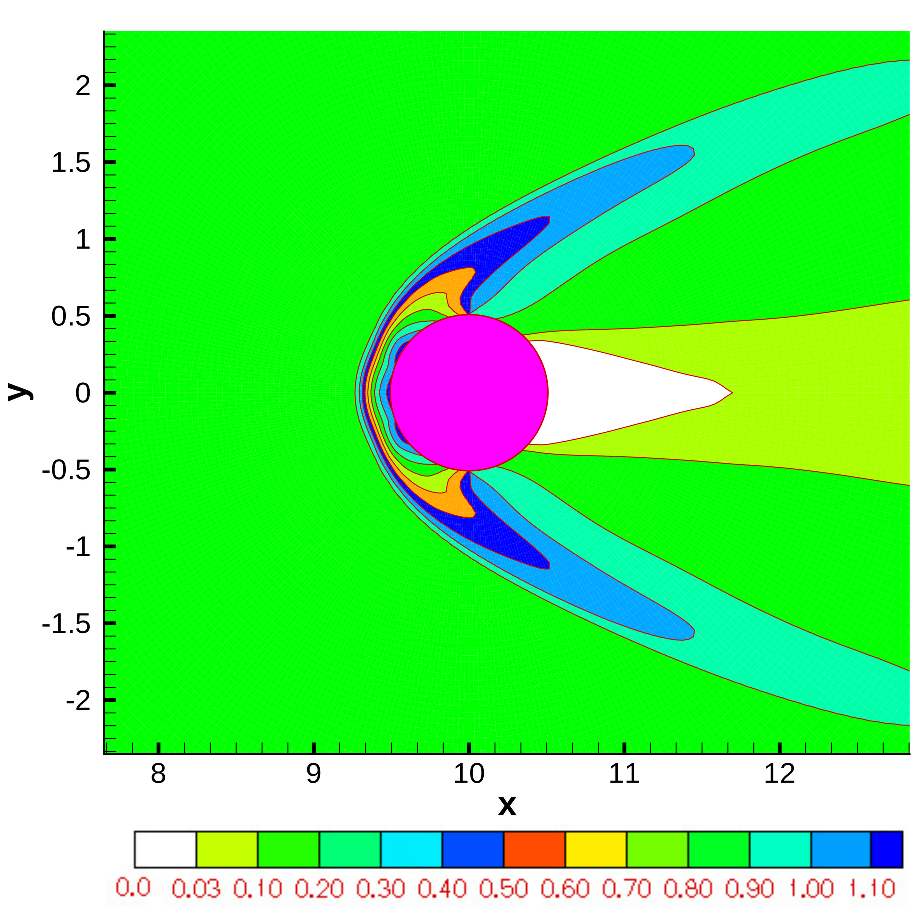

Figure 3. The h contours in the sand-case simulation ($h_0 = 0.4, u_0 = 0.847, \zeta = 38°$).

Separation from wall surface in simulations

- The velocity field is parallel to the thickness contours at the point of separation as shown in Fig. 4.

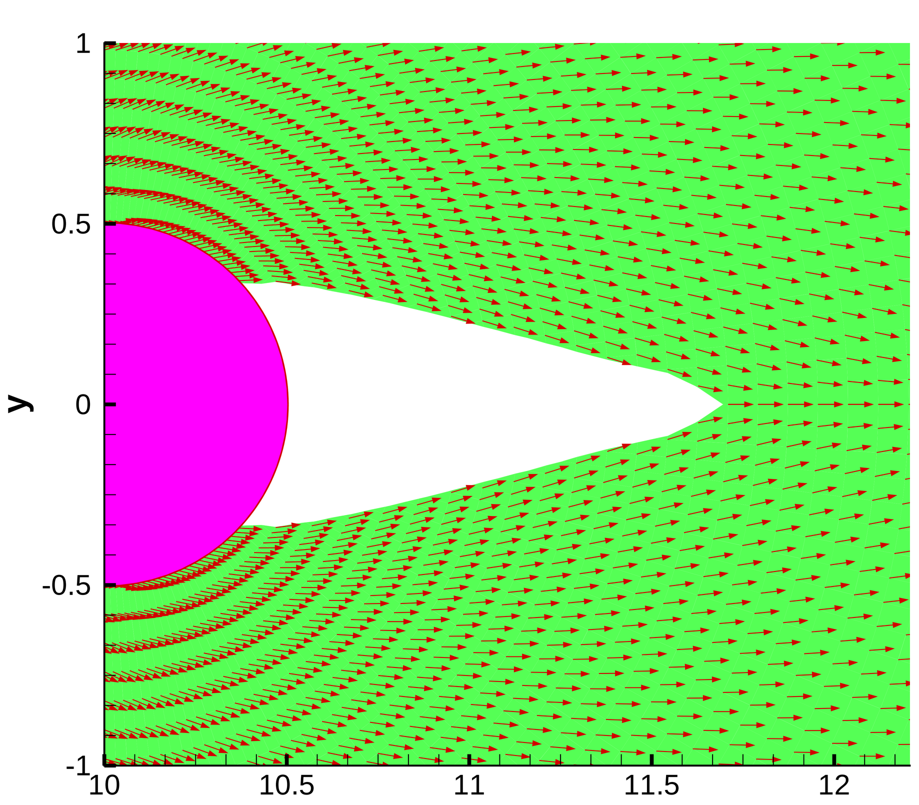

Figure 4. The velocity field near the wall surface in the sand-case simulation.

High Froude number case

- The red and white 100's and 1000's are used in the experiment.
- The inflow conditions are $h_0 = 0.3, u_0 = 1.1$, which implies an inflow Froude number $\mathrm{Fr}_0 = 2.3$.
- A long-time exposure for the case of 100's and 1000's is shown in Fig. 5.
- The numerical contours of the flow depth h are shown in Fig. 6.
- The vacuum region separates close to the middle of the cylinder and reattaches a long way downslope out of shot.

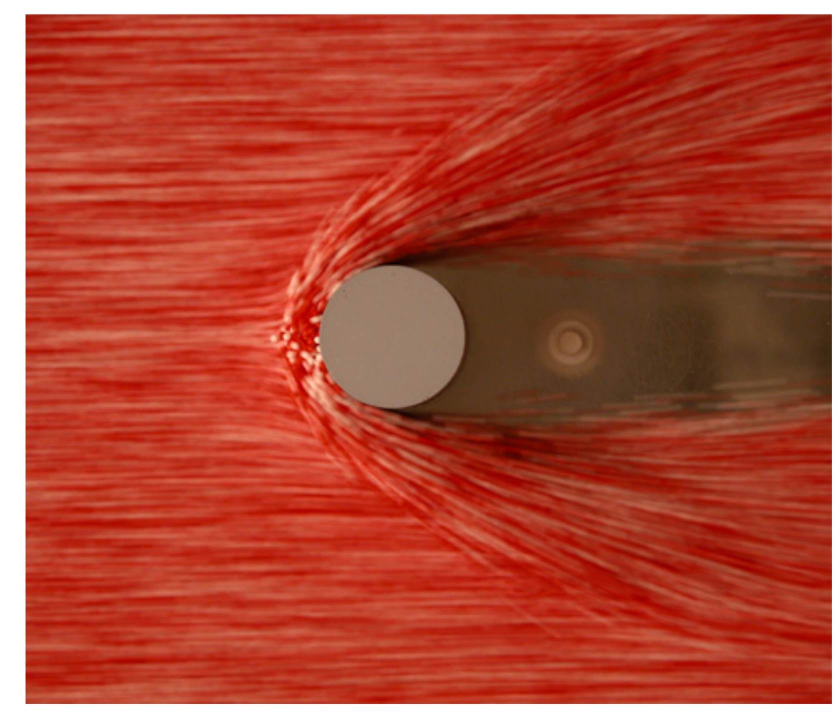

Figure 5. Long-time exposure image for the 100's and 1000's (the shutter speed is 1/60 second).

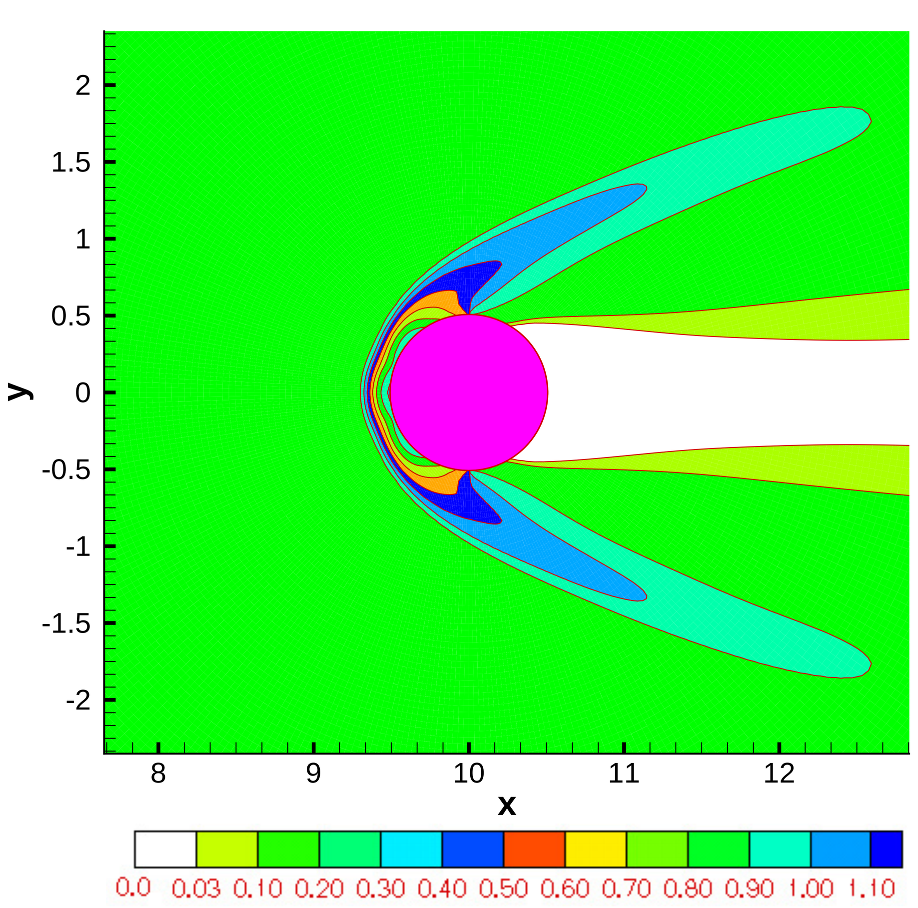

Figure 6. The h contours in the 100's and 1000's simulation ($h_0 = 0.3, u_0 = 1.1, \zeta = 38°$).

Conclusions

- The hydraulic model is able to qualitatively and quantitatively captures key features of the flow, such as the bow shock, separation and reattachment.
- The particle-free region becomes much bigger for the high Froude number case and the separation point is closer to the centre of the cylinder.

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