Introduction to Rheology of complex fluids
Brief Lecture Notes

Introductory lecture

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Contents

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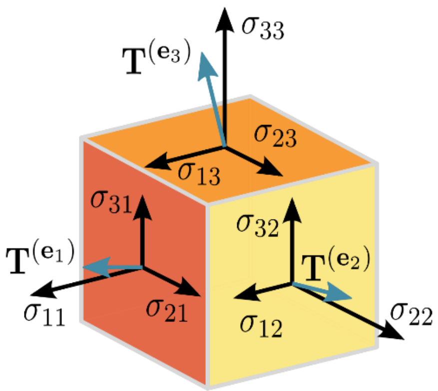
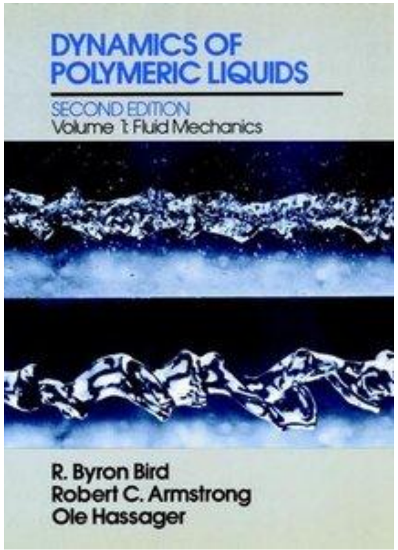
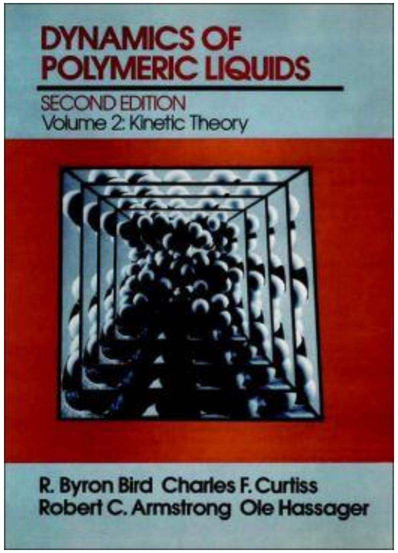
- **Simple Flows**
- **Material functions & Rheological Characterization**
- **Experimental Observations**
- **Generalized Newtonian Fluids**
- **Generalized Linear Viscoelastic Fluids**
- **Nonlinear Constitutive Models**



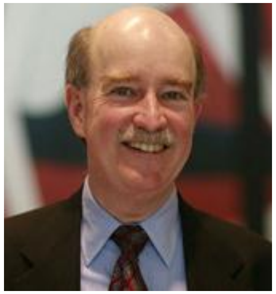
Basic Info: Suggested Books for Reading

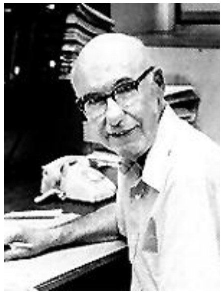




Title:
"Dynamics of polymeric Liquids. Vol.1 &2"

Authors:
R. Byron Bird, Robert Armstrong, Ole Hassager, Charles F. Curtiss



Publisher:
Wiley-Interscience

ISBN:
978-0471802457

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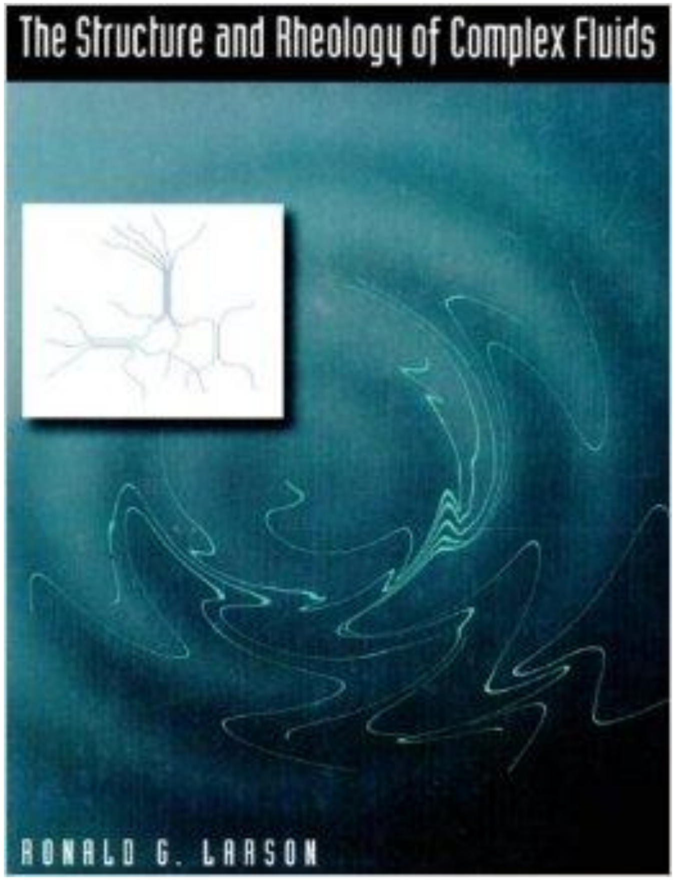

Title:
"The Structure and Rheology of Complex Fluids"

Author:
Ronald G. Larson

Publisher:
Wiley-VCH

ISBN:
978-0195121971

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What is rheology?

It is the science that studies:

- how complex fluids deform when subjected to forces (tensions).
- the rheological properties that describe the relationship between force and deformation.

Examples of properties: shear viscosity, extensional viscosity, shear modulus, coefficient of first and second normal stress difference, etc.

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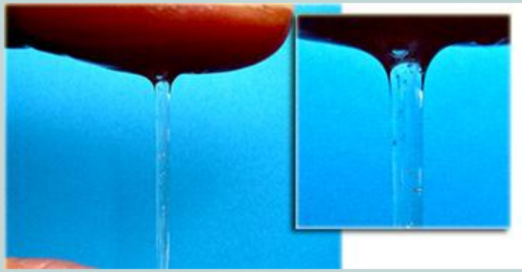
Some complex fluids

Food

- Emulsions (mayonnaise)
- Foams (ice cream, whipped cream)
- Suspensions (mustard, chocolate)
- Gels (cheese)

Biological fluids

- Suspensions (blood)
- Gel (mucin)
- Solutions (saliva)



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Some complex fluids

Personal Care Products

- Suspensions (nail polish)
- Solutions / Gels (shampoos, conditioners)
- Foams (shaving cream)

Electronic and optical materials

- Liquid Crystals (Display)
- Melts to protect electronic circuits

Pharmaceutical products

- Gels (creams)
- Aerosols (nasal sprays)

Plastic melts to produce

- Pipes, bottles
- Wrapping films
- Parts for home appliances, computers, clothes, etc.
- Parts for cars, airplanes and other transportation

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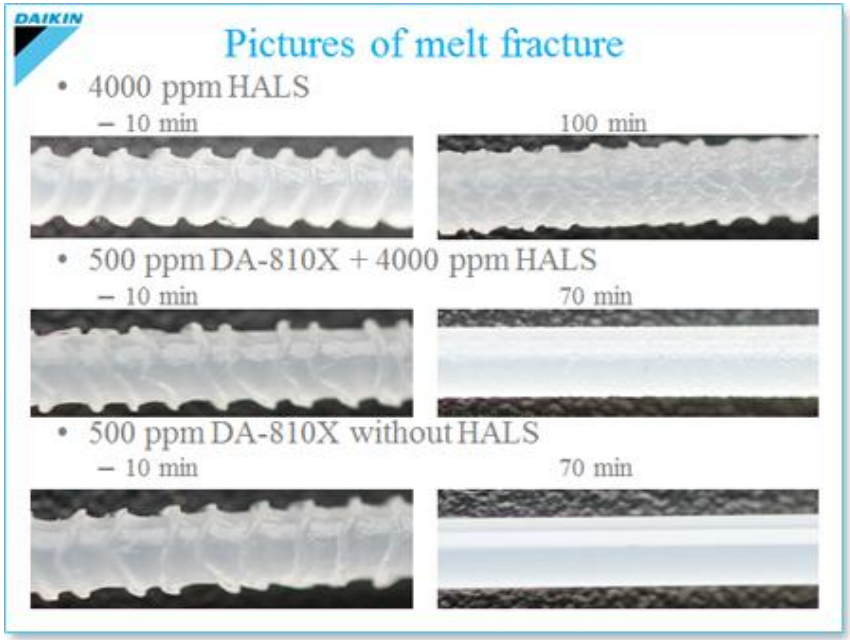
Where and Why is rheology useful?

Forming processes

Extrusion Process for production of pipes, preforms, etc.



Polymer Melts exhibit abnormal behavior (shark skin, spurt flow, melt fracture), when they flow at increasing velocities.

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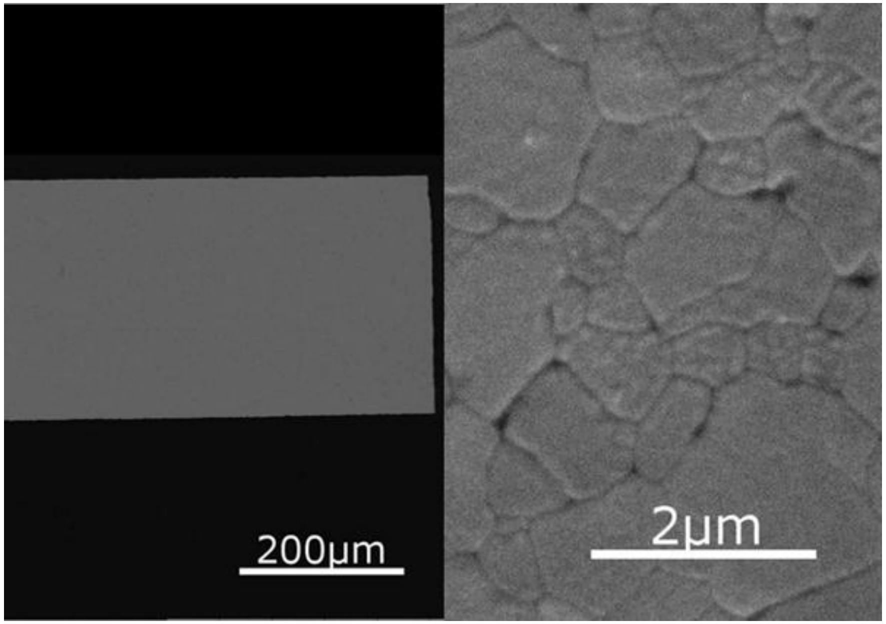
Where and Why is rheology useful?

Forming processes

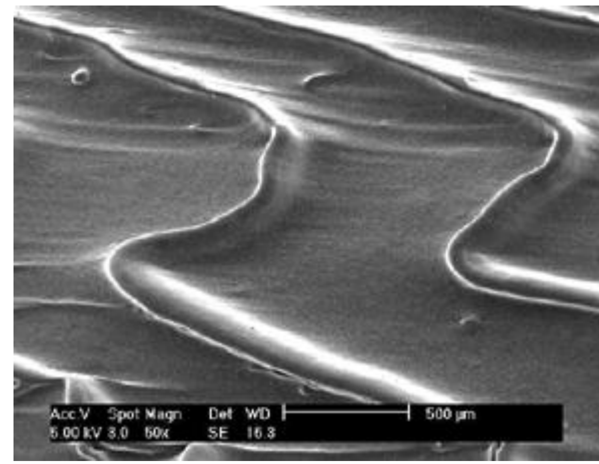


Calendering, used for producing high quality sheets of plastic

Where and Why is rheology useful?

Forming processes

http://www.reifenhauser-kiefel.com/



Blown Film Extrusion, for faster producing thinner and larger Films, e.g. wrapping films, greenhouse plastics, etc.

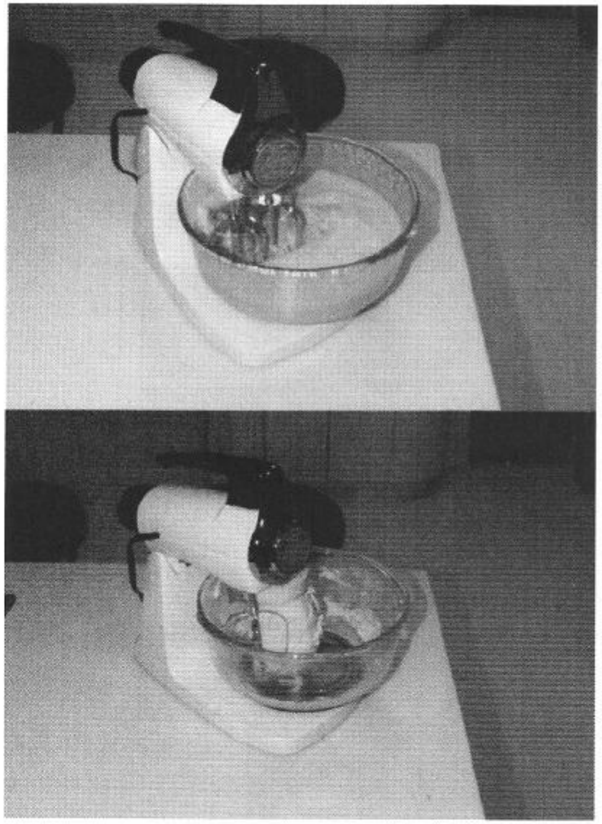
Some frequent observations

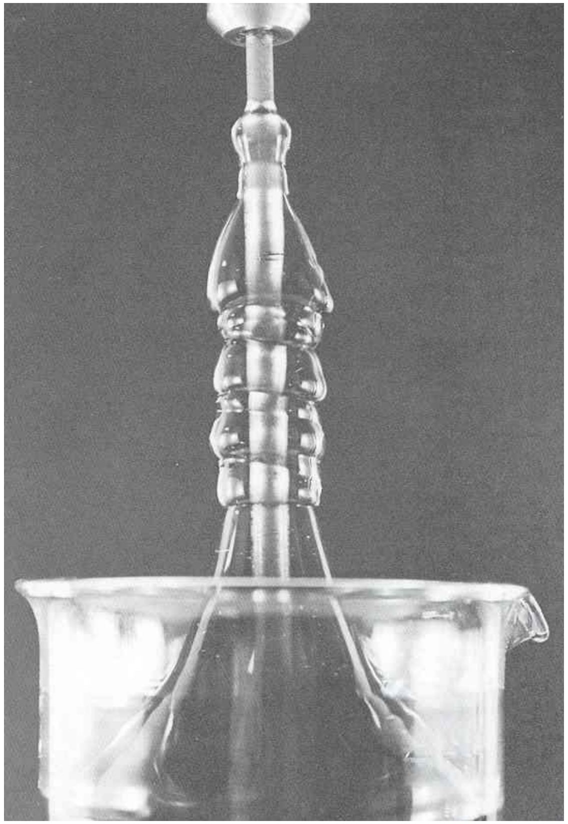
The Weissenberg effect

or Rod Climbing Effect

At low rotation rates or with a Newtonian fluid, the centrifugal force drives it away from the stirrer.

On the contrary, a complex fluid at higher rotation rates moves against the centrifugal force and climbs up the stirrer!

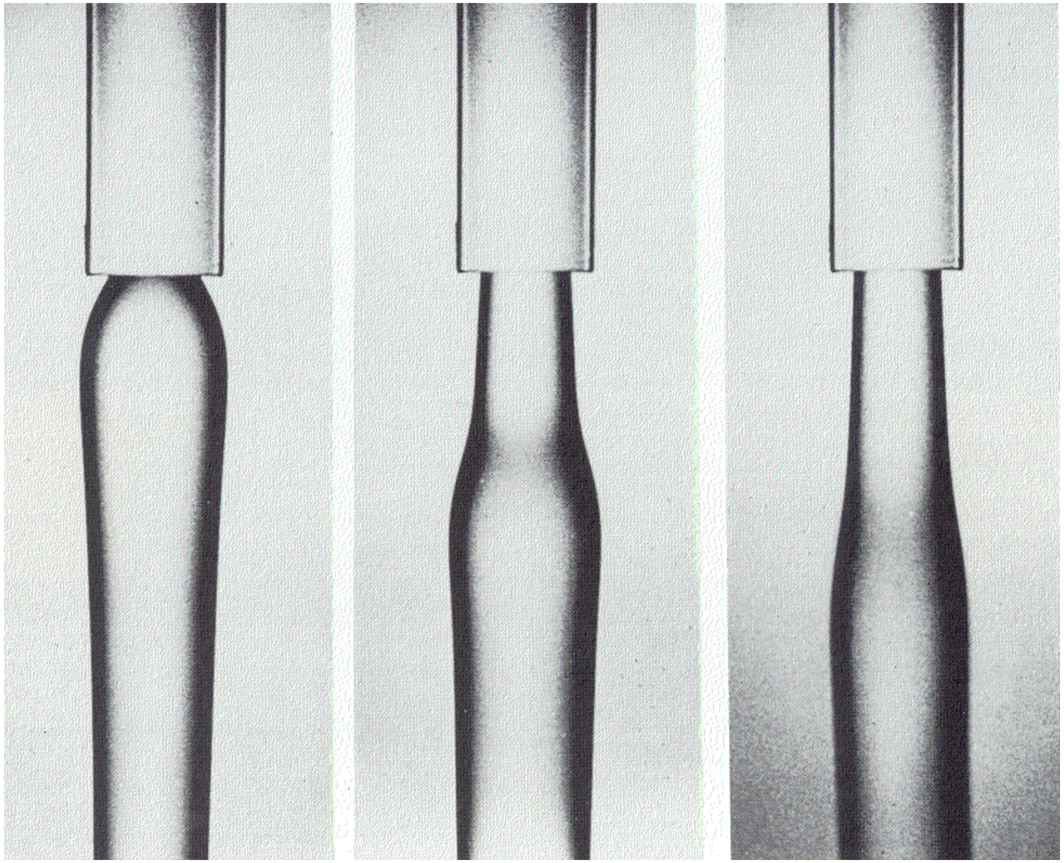




Extrudate Swell
(deformation opposite to that of water flow from a faucet)

When a polymeric fluid exits an extrusion die, the diameter of the fluid is increased up to an order of magnitude

It is caused by the relaxation of the extended macromolecular chains, as the polymer is extruded from the die



Intensity of vortices with increasing De (flow remains creeping)

Re = $5.7 \, 10^{-4}$ Re = $1.25 \, 10^{-2}$

(Boger, Hur, Binnington, JNNFM 1986)

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Tubeless Siphon Effect

When the conduit is raised above the fluid, the viscous fluid ceases to flow (N) while the polymer (P) continues its upward motion

[Reproduced from R. B. Bird, R. C. Armstrong and O. Hassager, *Dynamics of Polymeric Liquids. Vol I: Fluid Mechanics*, 2nd edition, Wiley-Interscience (1987), p. 74.]

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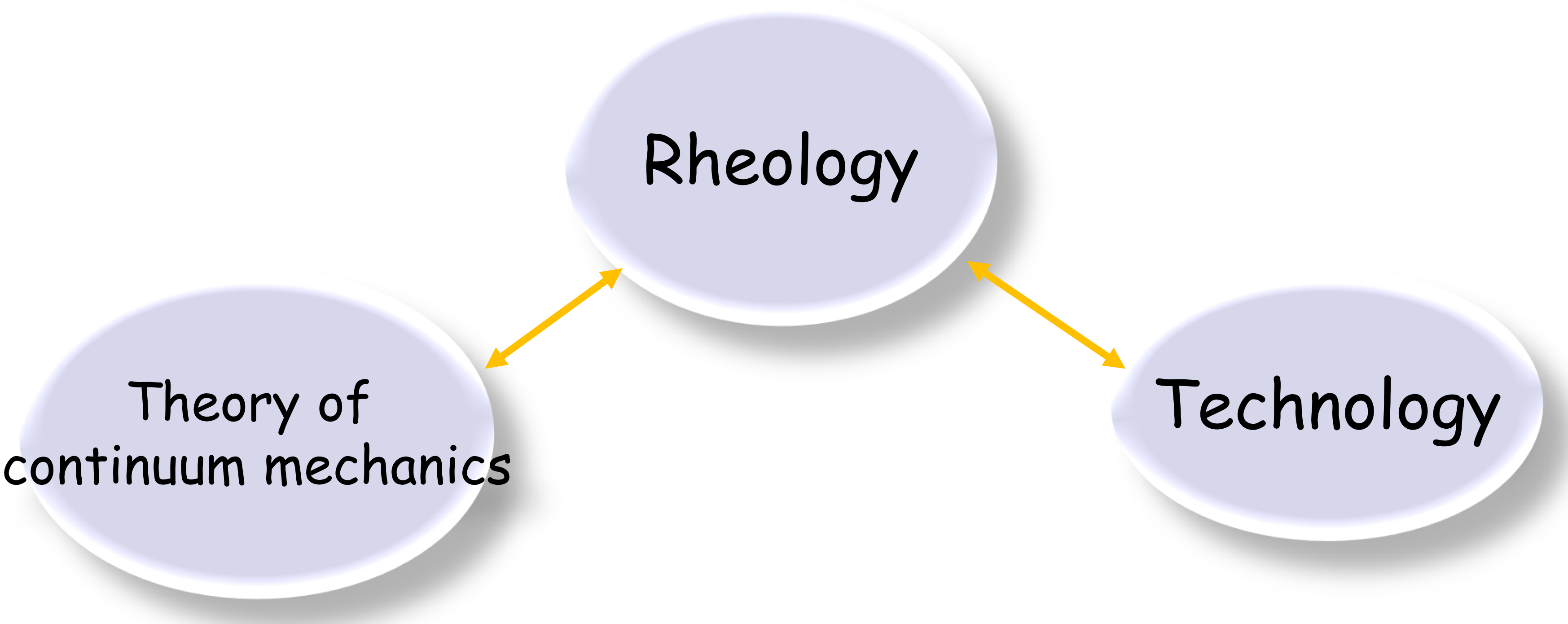
Application fields of rheology

- Polymer Rheology (solutions, melts)
- Rheology of liquid and solid suspensions
- Electro- and Magneto- Rheology
- Food Rheology
- Hemato-rheology & Biorheology
- Interfacial Rheology (interfacial properties)

Rheology as an Interdisciplinary Research Field



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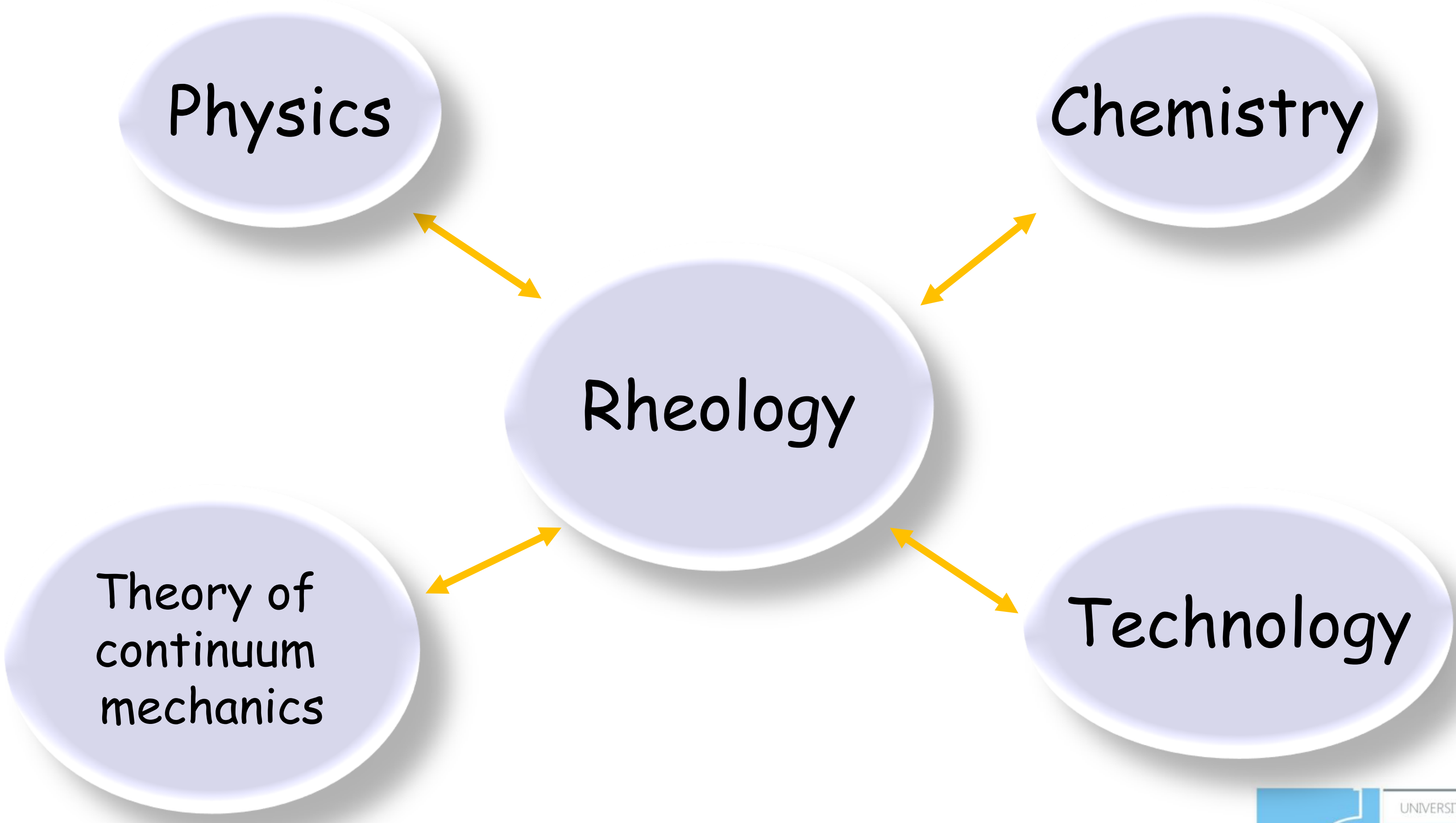
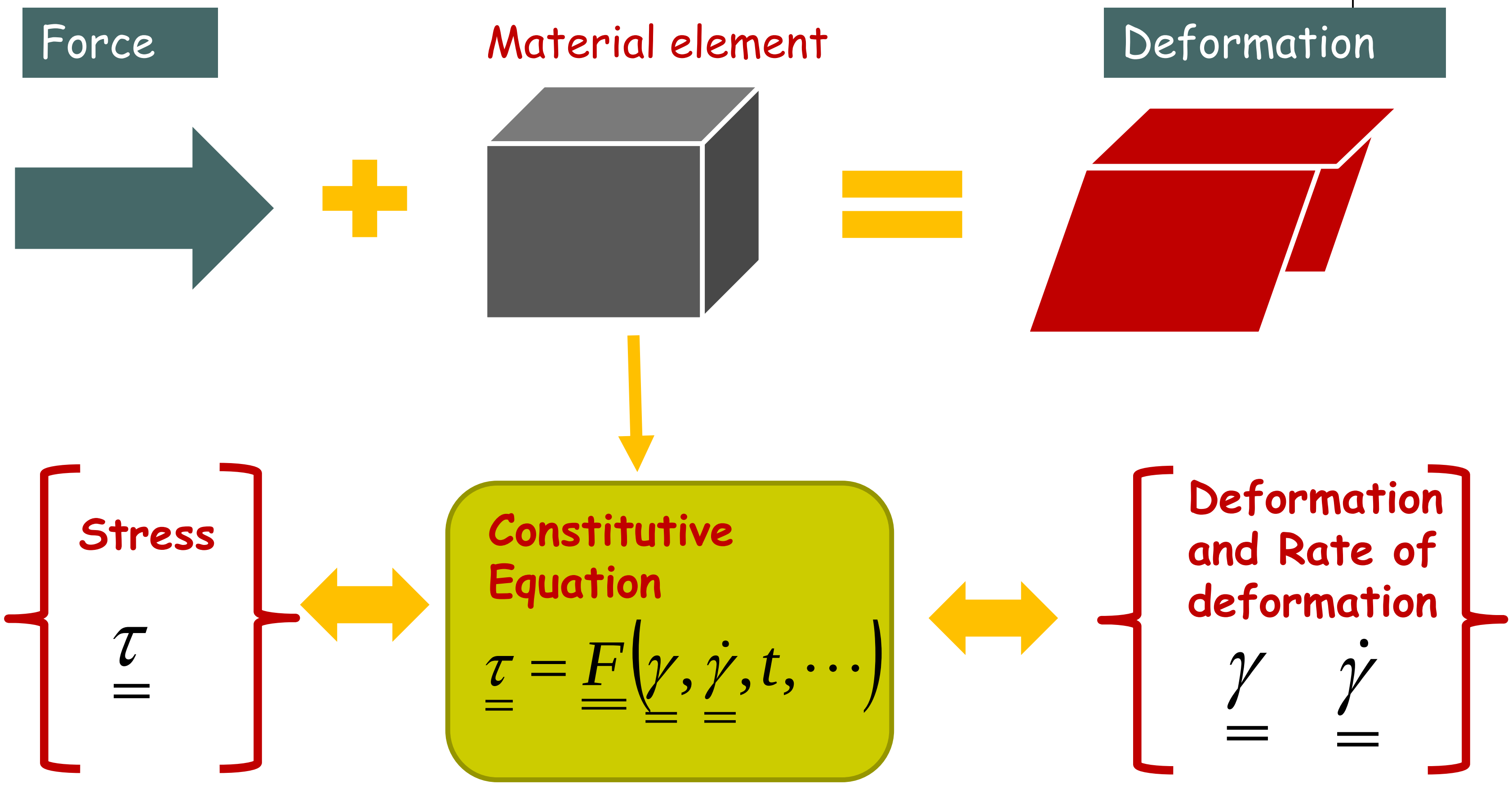
Rheology as an Interdisciplinary Research Field



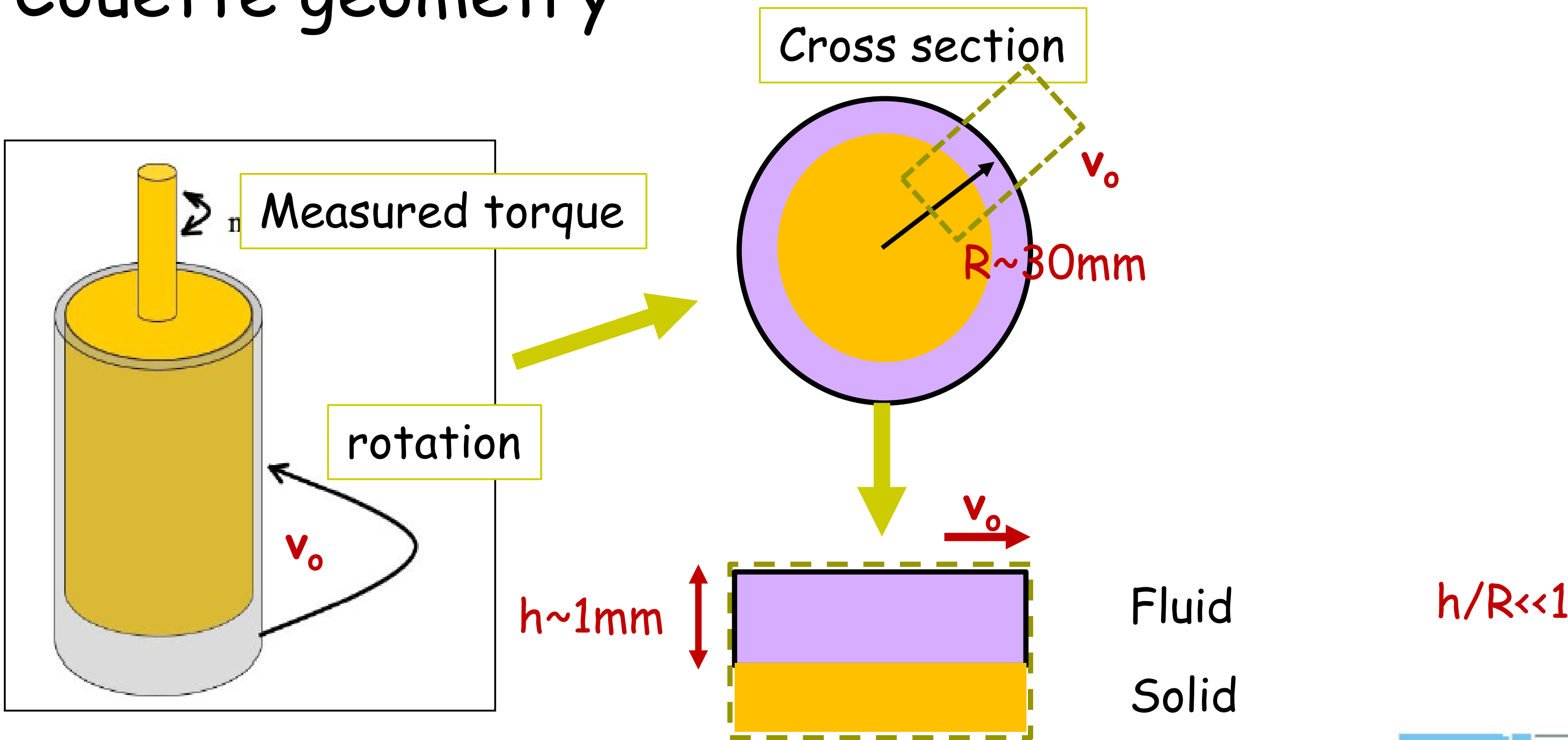
Basic Principles of Rheology



The simplest rheological experiment

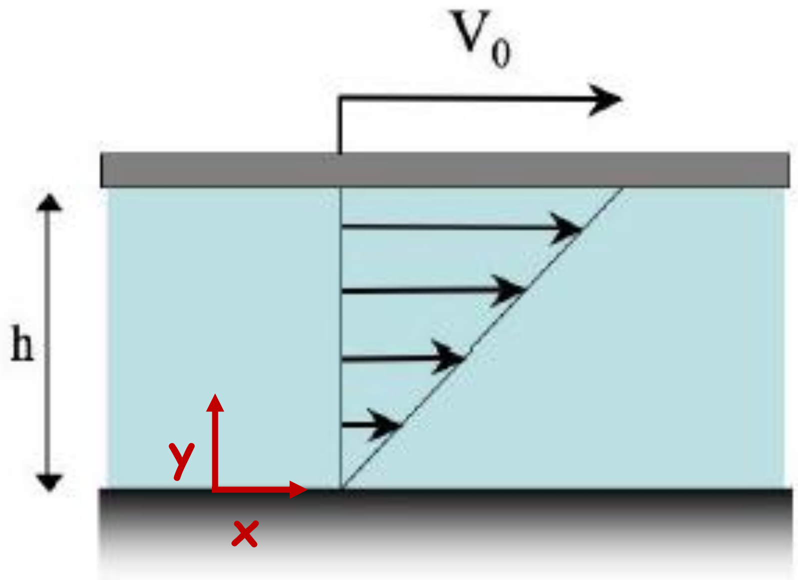
A standard flow

Couette geometry



The simplest rheological experiment

A standard flow



1D assumption

$$\underline{v} = (v_x, 0, 0) = v_x \underline{e}_x$$

Velocity

$$v_x(y) = \frac{v_o}{h} y$$

Rate deformation

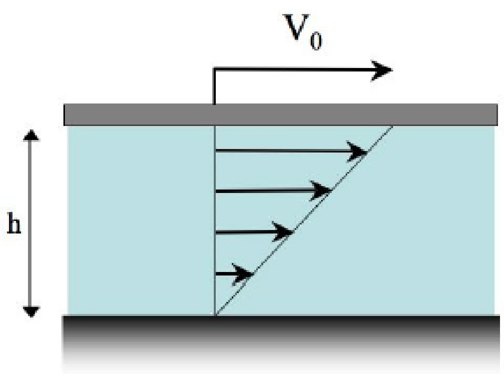
$$\dot{\gamma}_{xy} = \frac{d[v_x(y)]}{dy} = \frac{v_o}{h}$$

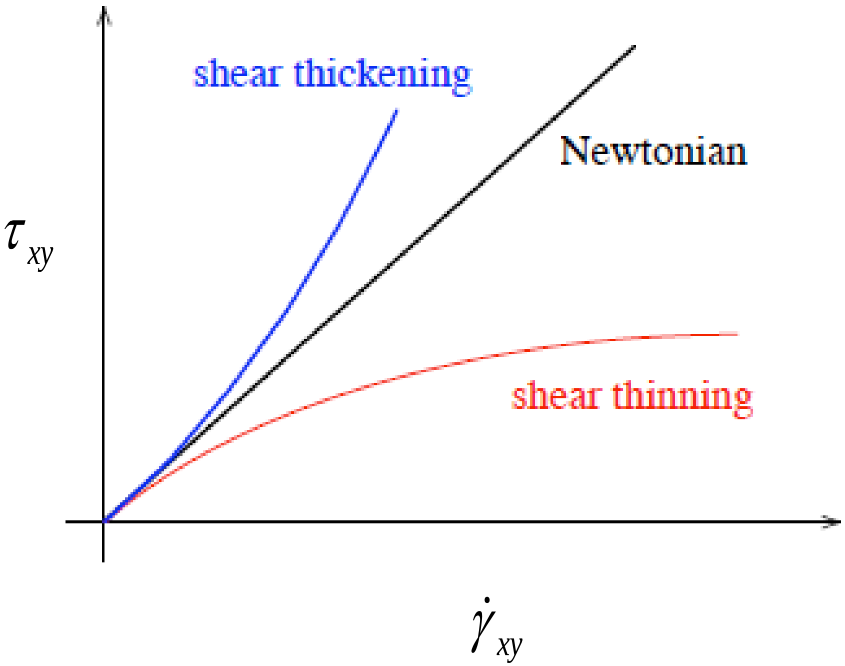
Shear stress

$$\tau_{xy} = \eta \dot{\gamma}_{xy} = \eta \frac{v_o}{h}$$

The simplest rheological experiment

A standard flow





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The origins go back to the 17th century…

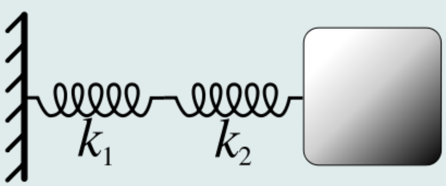
Robert Hooke (1660)

Hooke's law of elasticity

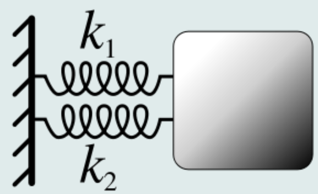
"As the extension, so the force"

*The most commonly encountered form of Hooke's law is probably the **elastic spring equation**, which relates the force exerted by a spring to the distance it is stretched by a **spring constant**, k, measured in force per length.*

$$F=-kx$$



Multiple springs in Series



Multiple springs in Parallel

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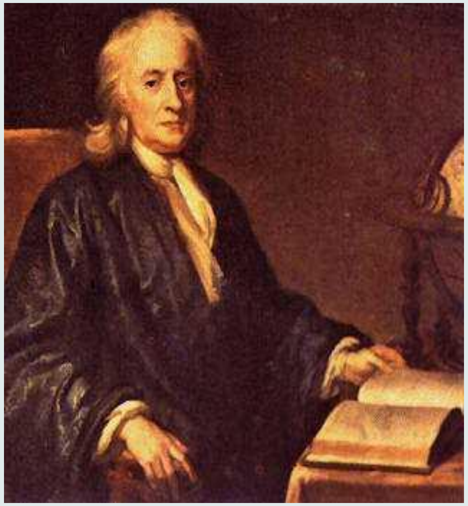
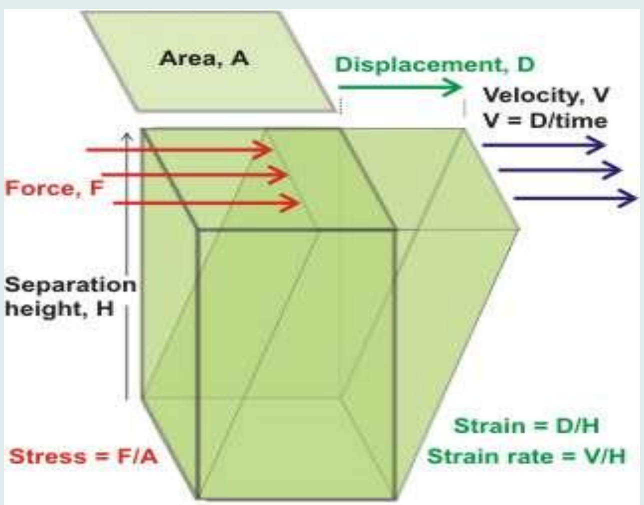
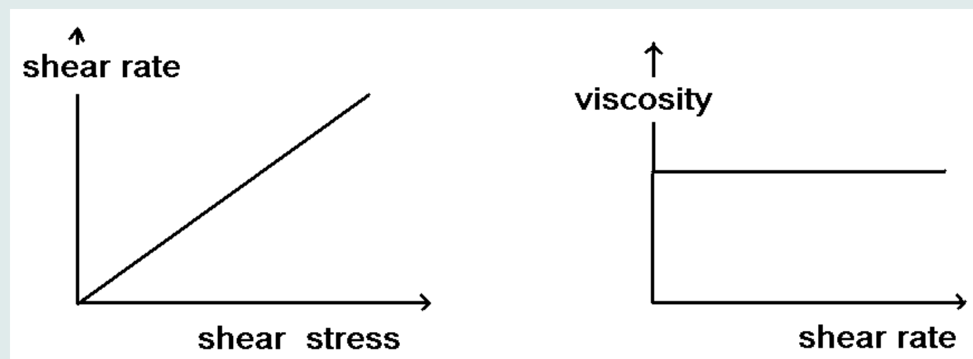
Isaac Newton (1685)



Newton's laws of motion

Newton's Second Law states that an applied force, F , on an object equals the rate of change of its momentum, P, with time.





Newtonian fluid

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Historic evolution of Rheology

1860	• **Linear Viscoelasticity** Maxwell Model	
1940	• **Kinetic Theory** Krammers (1944), Rouse (1953), Zimm (1956)	
1950	• **Oldroyd-B Model**	Continuum Mechanics
1960	• **Network Theories** K-BKZ (1963**)**	
1970	• **Reptation Theory & Tube Model** Edwards (1967), Doi-Edwards (1978)	Macroscopic Numerical Simulations
1980	• **Pom-Pom & Rolie-Poly Models**	Molecular Dynamics

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What are the differences between solids and fluids?

Phenomenological differences

Viscous Fluids

All viscous (Newtonian) fluids continuously **flow** under the influence of an applied stress.

Elastic Solids

Solids **deform** under the applied stress, but soon reach a new equilibrium position beyond which deformation stops. If the stress is removed, solids may regain their original form.

Viscoelastic Fluids

The viscoelastic fluids can exhibit **both viscous and elastic behavior**, depending on conditions.

Example:

- A simple fluid takes the shape of the container
- The solids retain their shape indefinitely
- Complex fluids maintain their shape for a long time and then take the shape of the container.

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What are the differences between solids and fluids?

Differences in the way we describe and model their behavior

Elastic solid

e.g. Steel

- Strong structure
- Stiff
- **Deforms**
  - Retains its shape (memory)
  - Conserves its internal structure
  - Lagrangian description, based on the position vector
  - Basic property: G′ (storage modulus) or G (shear modulus)

Viscoelastic fluids

Viscous fluids

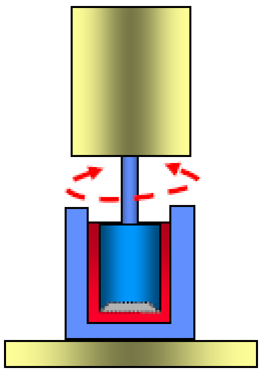
e.g. Water

- Weak structure
- Fluidity
- **Flows**
  - Losses its shape
  - Losses its internal structure
  - Eulerian description, based on velocity vector
  - Basic property: G′′ (loss modulus) or η (shear viscosity)

How do we measure deformation (rate) in solids and fluids?

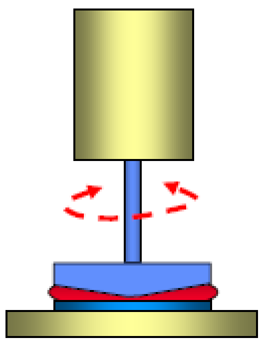
Rheometers

Concentric cylinders



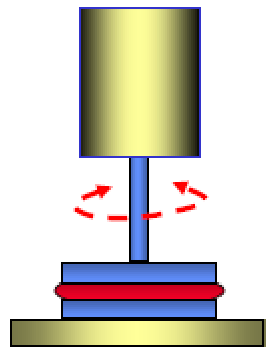
Fluids of low & medium viscosity

Cone & Plate



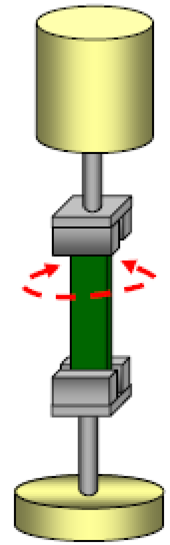
Fluids of high viscosity

Parallel Plates



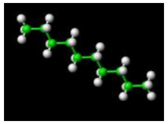
Low viscosity fluids to soft materials

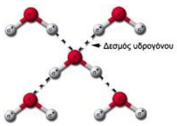
Rectangular Torsion

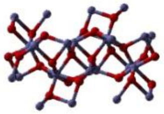


Soft & Stiff solids

Decane → **Water** → **Steel**







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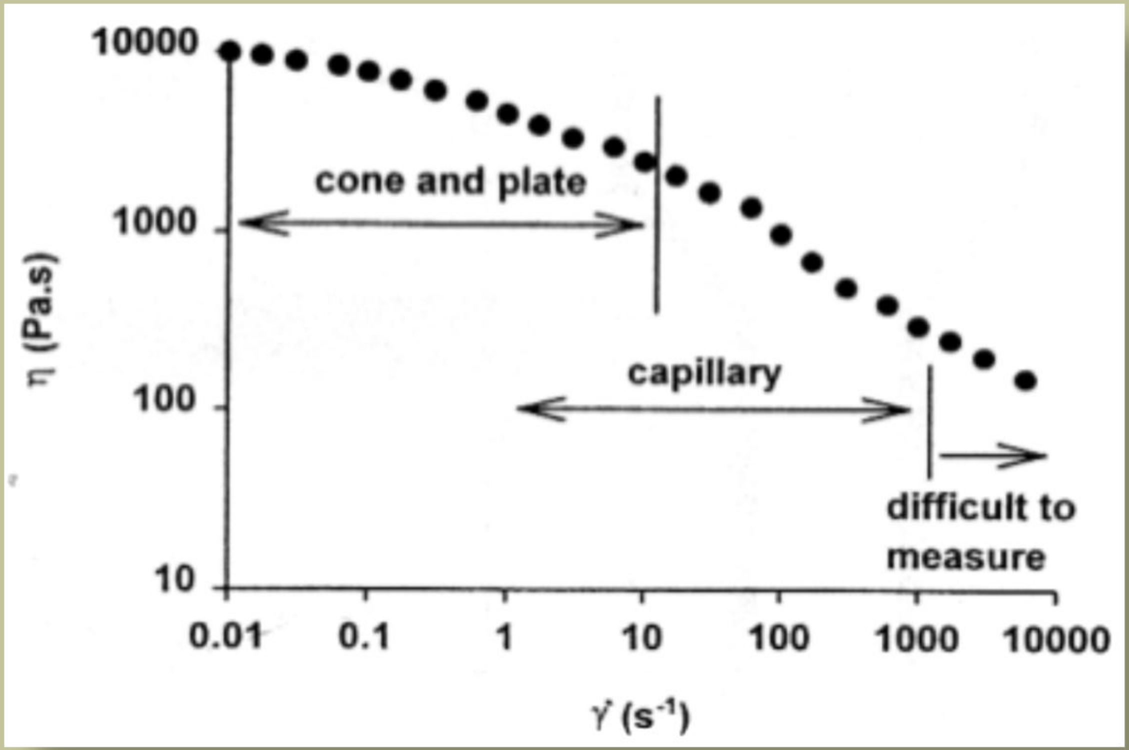
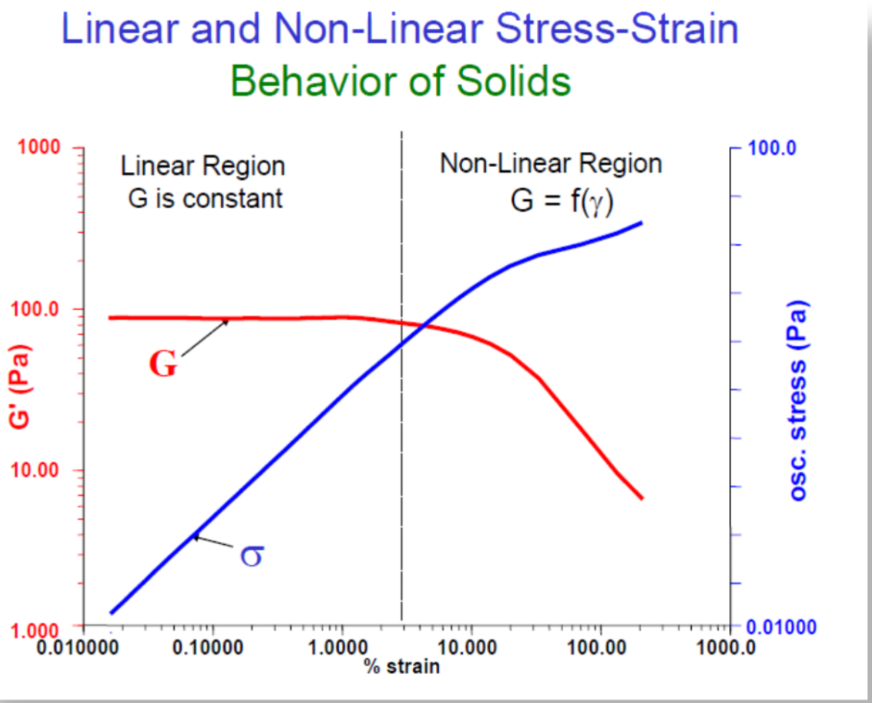
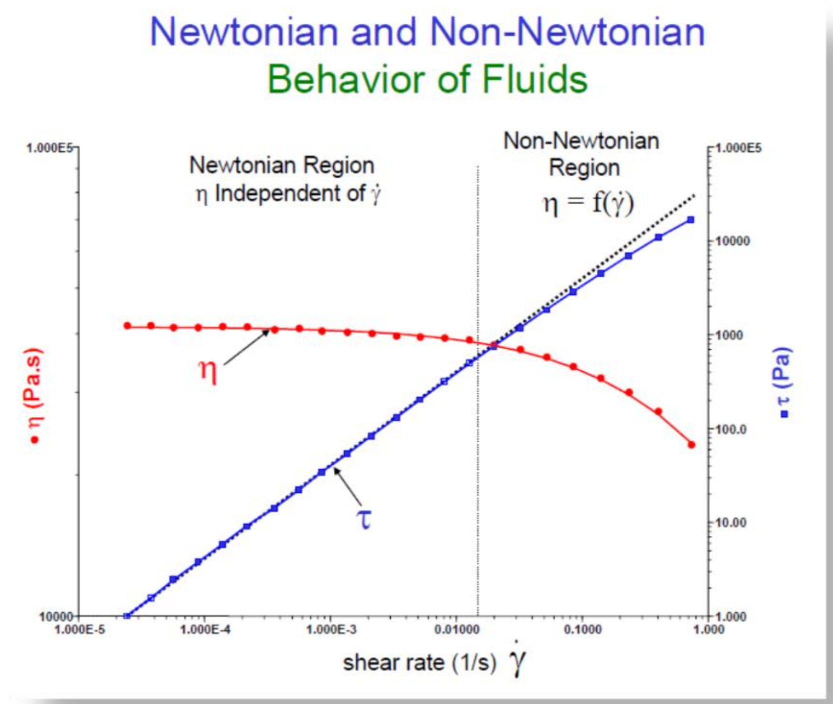
Viscometers

- Melt Flow Index
- Capillary Rheometer
- Coaxial Cylinder Viscometer (Couette)
- Cone and Plate Viscometer (Weissenberg rheogoniometer)
- Disk-Plate (or parallel plate) viscometer



What are the differences between solids and fluids?





Shear Modulus = Shear Stress / Shear Strain

$$\underline{\underline{\tau}} = G\underline{\underline{\gamma}}$$

Viscosity = Shear Stress / Shear Rate

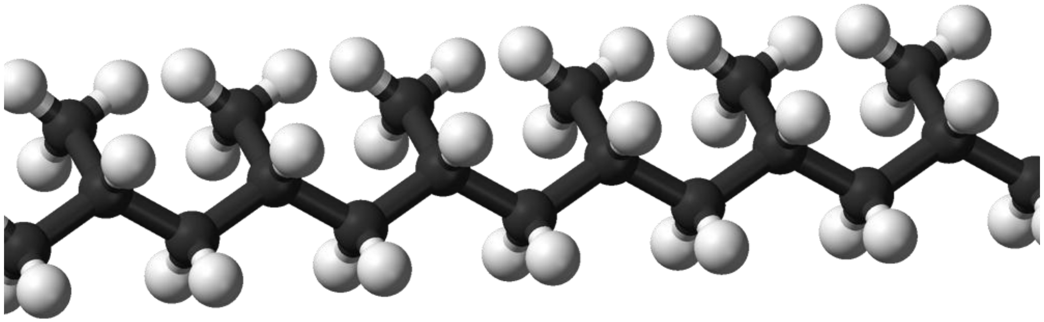
$$\underline{\underline{\tau}} = \eta\underline{\underline{\dot{\gamma}}}$$

What is viscoelasticity?

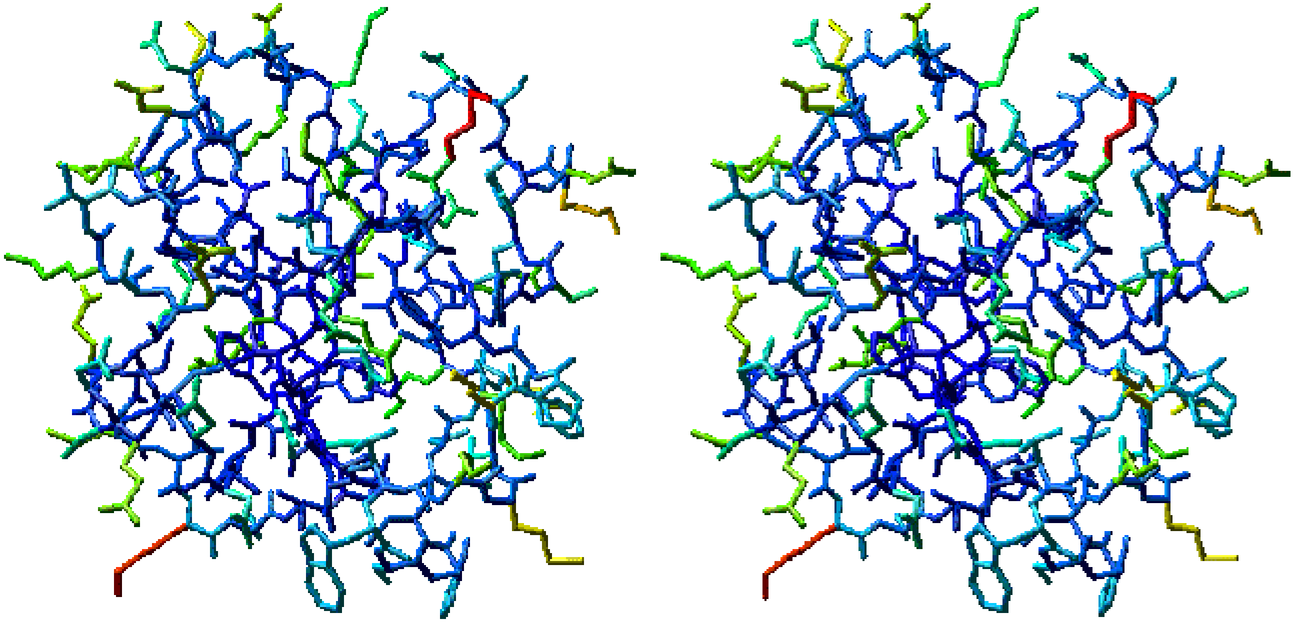
- **Viscoelasticity is the property of materials that exhibit both viscous and elastic characteristics when undergoing deformation.**
- **A viscous material, such as honey, resists to shear flow and deforms linearly in time when a constant shear stress is applied.**
- **An elastic material deforms when stretched and quickly tends to return to its original state after the removal of the stress.**
- **Viscoelastic materials have elements of both of these properties and, as such, a time-dependent deformation.**
- **Elasticity is usually the result of the extension of the molecular bonds along crystallographic planes arranged in a solid, whereas viscosity is the result of the motion of molecules in an amorphous material.**

viscoelasticity = fading memory

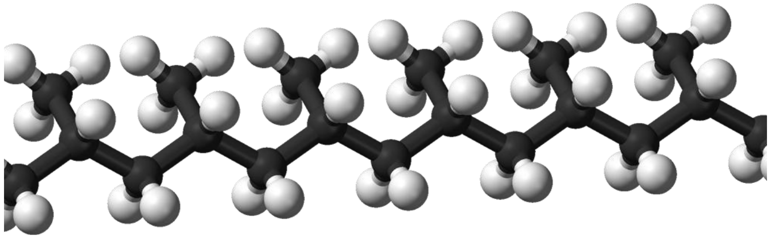
What are the origins of viscoelasticity?



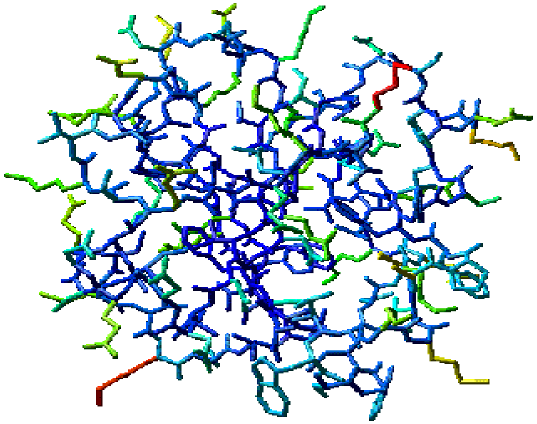
It originates in the structure of macromolecules



What are the origins of viscoelasticity?

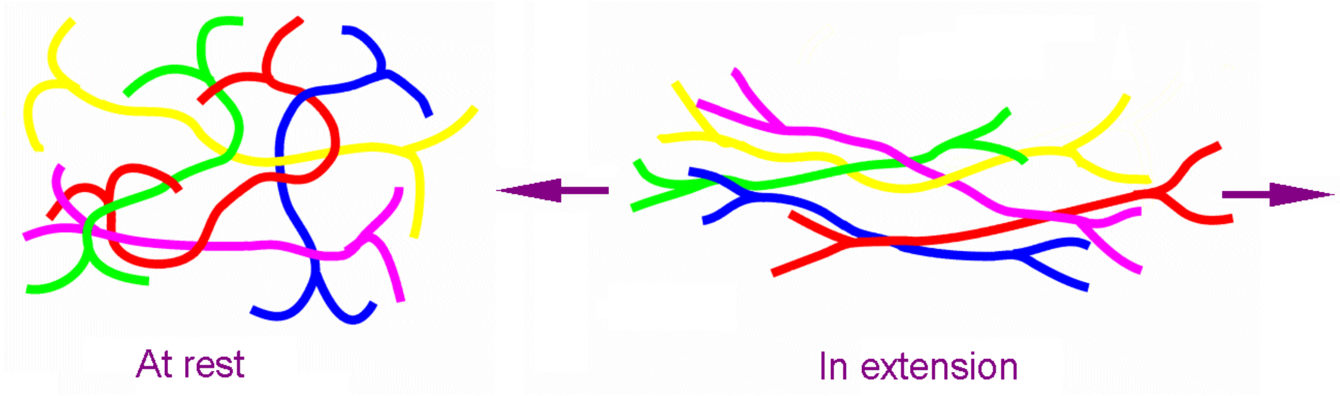


Polymer molecules form zig-zag structures



Entangled macromolecules (like spaghetti)

What are the origins of viscoelasticity?



- If a stress is exerted on a macromolecule, it would extend it, at least in part, in the direction of the applied force.
- Stretching a molecule increases the angles of the bonds between the atoms and causes rotation of the atoms within the chain.
- When the stress is removed the molecule tries to relax, i.e. to return to its original shape, but this is not always entirely possible.

The viscoelasticity level is determined by the De number

$$De = \frac{\lambda}{t_{flow}}$$

Deborah number

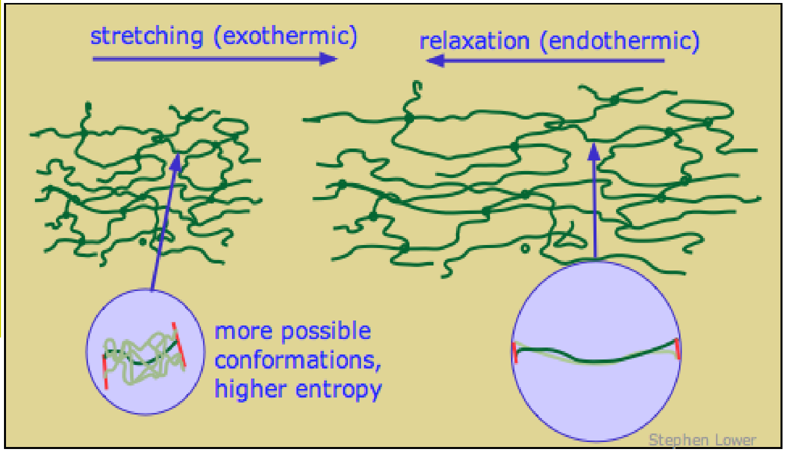
Material relaxation time

$$\lambda \quad \left(= \frac{viscosity}{elastic\ modulus} \right)$$

Characteristic flow time

$$t_{flow} = \dot{\gamma}^{-1}$$

The relaxation time is the time required for a polymeric system to return to equilibrium in response to a sudden disturbance.



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“The mountains flowed before the Lord”

[From Deborah’s Song, Judges, 5:5]

The Deborah number (De) was introduced by Marcus Reiner

M. Reiner is credited with naming the Deborah Number after the song of Deborah, Judges 5:5- "The mountains flowed before the Lord"(Fig. 3.10). It was first mentioned in his article "The Deborah Number" in the January 1964 issue of Physics Today.

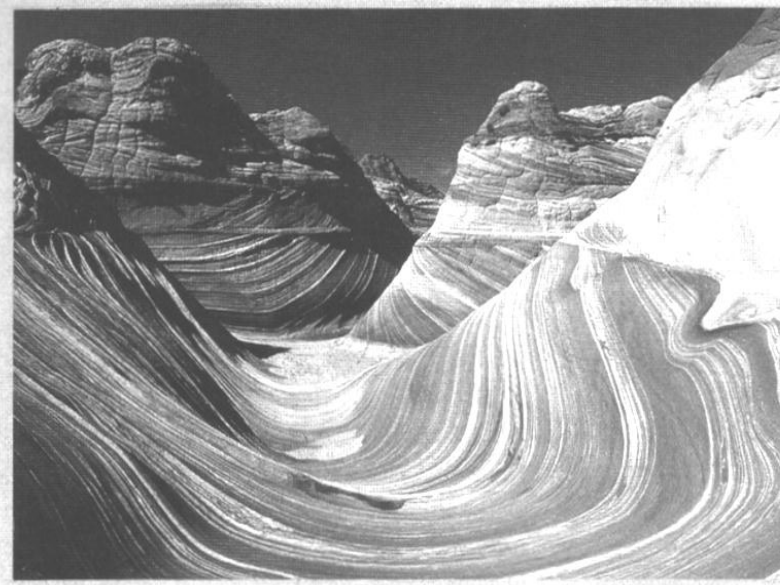

Figure. 3.10 Nestles Canyon, Arizona. Courtesy of Wolfgang Cohnen (©1997)

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What is a constitutive model?

It is either an algebraic, differential or integral relation, often implicit, that relates the stress tensor with the deformation $\underline{\underline{\gamma}}$ or rate of deformation $\underline{\underline{\dot{\gamma}}}$ tensors

$$\underline{\underline{\tau}} = \underline{\underline{F}}(\underline{\underline{\gamma}}, \underline{\underline{\dot{\gamma}}}, t, \cdots)$$

Viscous fluid

$$\underline{\underline{\tau}} = \underline{\underline{F}}(\underline{\underline{\dot{\gamma}}})$$

Elastic solid

$$\underline{\underline{\tau}} = \underline{\underline{F}}(\underline{\underline{\gamma}})$$

Viscoelastic fluid

$$\underline{\underline{\tau}} = \underline{\underline{F}}(\underline{\underline{\gamma}}, \underline{\underline{\dot{\gamma}}}, t, \cdots)$$

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Types of Materials

Rigid solid	$\underline{\underline{\gamma}} = \underline{\underline{0}}$
Linear Hookean solid	$\underline{\underline{\tau}} = G\underline{\underline{\gamma}}$
Non-linear Hookean solid	$\underline{\underline{\tau}} = G(\underline{\underline{\gamma}})\underline{\underline{\gamma}}$
Viscoelastic fluid	$\underline{\underline{\tau}} = \underline{\underline{F}}(\underline{\underline{\gamma}}, \underline{\underline{\dot{\gamma}}}, t, \cdots)$
Non-linear fluid	$\underline{\underline{\tau}} = \eta(\underline{\underline{\dot{\gamma}}})\underline{\underline{\dot{\gamma}}}$
Linear Newtonian fluid	$\underline{\underline{\tau}} = \eta\underline{\underline{\dot{\gamma}}}$
Inviscid fluid	$\underline{\underline{\tau}} = \underline{\underline{0}}$

Solids

Fluids

Constitutive models

$$\mathbf{T} = 0$$

(Euler 1748)

$$\mathbf{T} = 2\eta\mathbf{D}$$

(Newton / Navier-Stokes 1660 - 1845)

$$\lambda\frac{\partial \mathbf{T}}{\partial t} + \mathbf{T} = 2\eta\mathbf{D}$$

(Maxwell 1867)

$$\lambda\left(\frac{\partial \mathbf{T}}{\partial t} + (\vec{u}\cdot\nabla)\mathbf{T}\right) + \mathbf{T} = 2\eta\mathbf{D}$$

(convected + Maxwell)

$$\lambda\left(\frac{\partial \mathbf{T}}{\partial t} + (\vec{u}\cdot\nabla)\mathbf{T} - \nabla\mathbf{u}^T\cdot\mathbf{T} - \mathbf{T}\cdot\nabla\mathbf{u}\right) + \mathbf{T} = 2\eta\mathbf{D}$$

(Oldroyd 1950)

Notation

$\mathbf{T} \Leftrightarrow \underline{\underline{\tau}}$

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End of introductory lecture

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