Qualitative parameter inference: Automated Detection of Chaotic and Oscillatory Regimes

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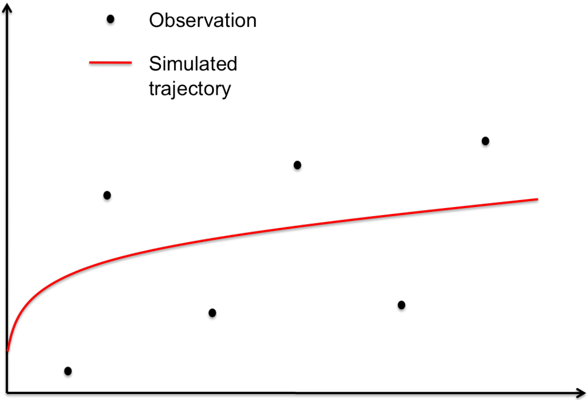
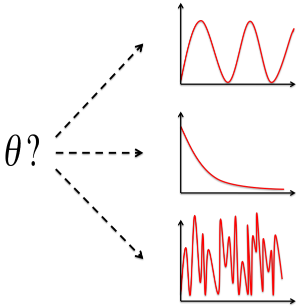
Motivation – Elusive behaviours

- Traditional quantitative driven parameter inference methods can fail for certain types of data.



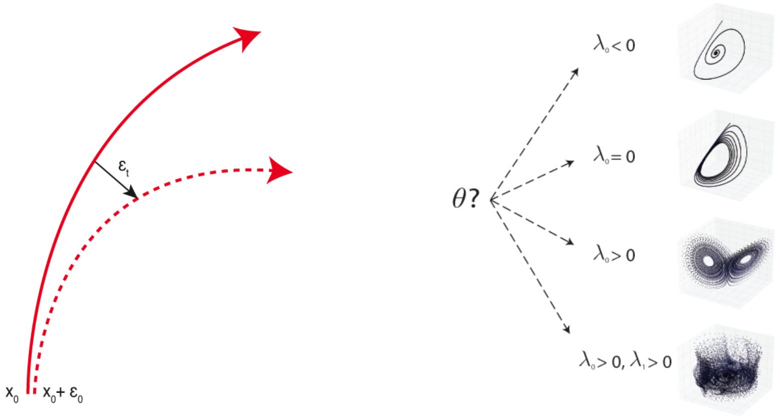
Motivation – System design

- System design – finding parameter combinations that give rise to desired types of behaviour.



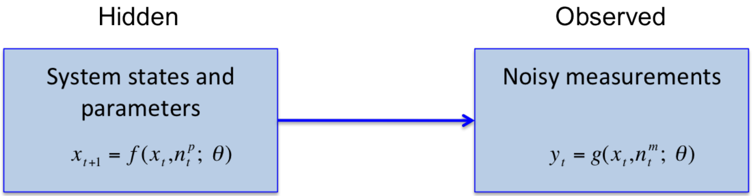
Outline

Encoding dynamical behaviour: Lyapunov exponents



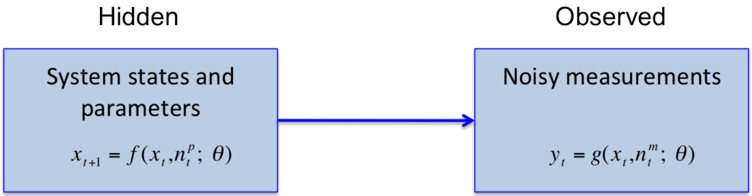
- Lyapunov spectra, $\{\lambda_i\}$, measure the long term average rate of contraction/expansion of nearby trajectories.
- Computationally expensive inference procedure.

Probabilistic inference



- Allows the estimation of hidden system parameters from a sequence of incomplete and noisy observations.
  - Posterior distribution $p(\theta_t|\mathbf{y}_{1:t})$

Probabilistic inference



- Allows the estimation of hidden system parameters from a sequence of incomplete and noisy observations.
  - Posterior distribution $p(\theta_t|\mathbf{y}_{1:t})$
- Also applicable to state and dual estimation problems.

The optimal solution

$$p(\theta_k|\mathbf{y}_{1:k}) = \frac{p(\theta_k|\mathbf{y}_k)p(\theta_k|\mathbf{y}_{1:k-1})}{p(\mathbf{y}_k|\mathbf{y}_{1:k-1})}$$

- Terms in the *Bayesian estimation update* correspond to multi-dimensional integrals.
- In general, closed form solutions are only available for linear systems.
    - Known as *Kalman filtering*.

Filtering for non-linear systems

- Models of biological systems are often non-linear.
- Our method requires the use of highly non-linear functions.

- The extended Kalman filter
- Particle filtering
- Sigma-point Kalman filters

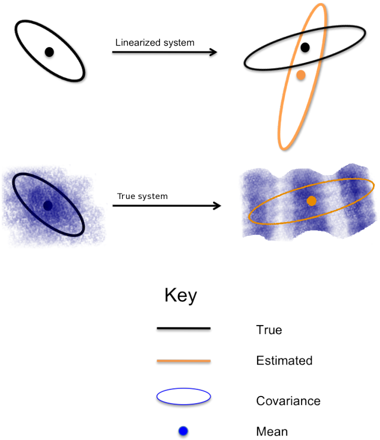
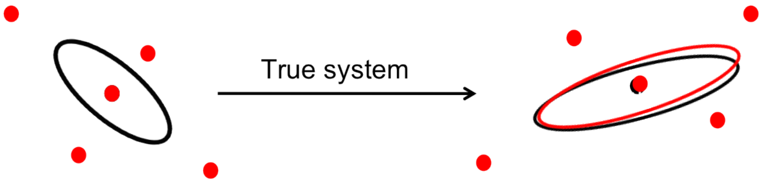


Figure adapted from *The Unscented Kalman Filter for Nonlinear Estimation* - E. A. Wan and R. van der Merwe

The Unscented Kalman Filter (Van der Merwe 2004)

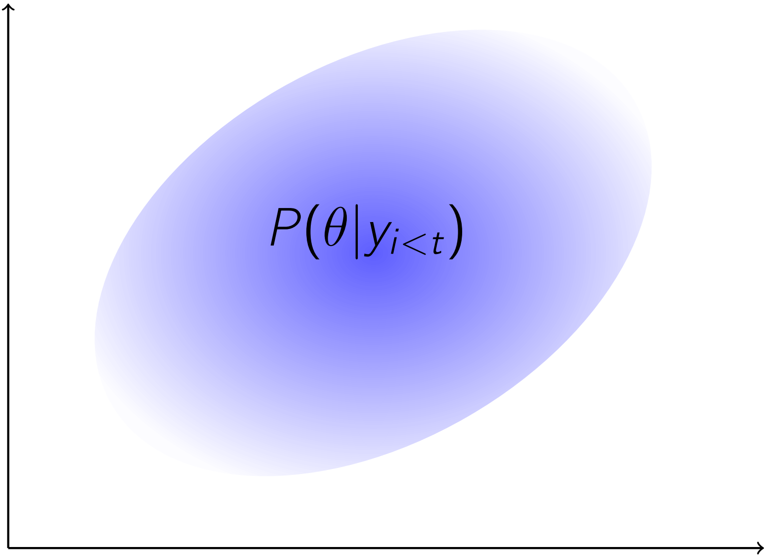
Intuition: *Probability density functions may be easier to approximate than highly non-linear systems.*



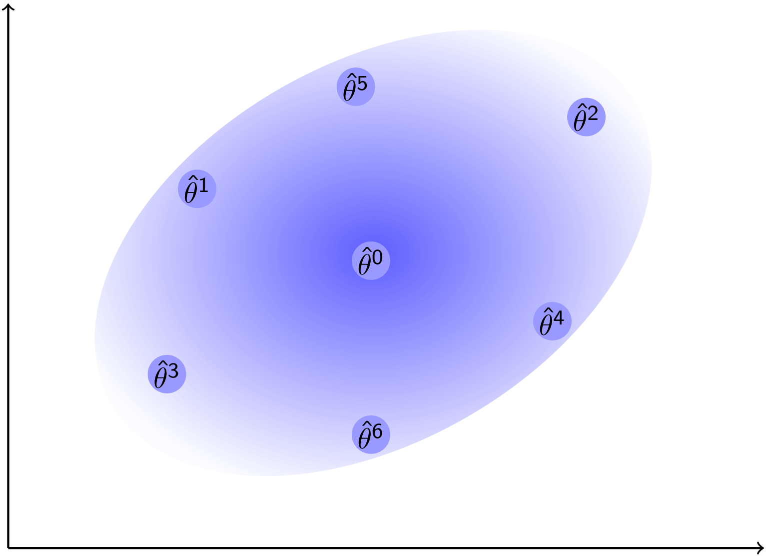
- Propagated means and covariances are accurate to third order in the Taylor expansion.
- Computationally very efficient.

Figure adapted from *The Unscented Kalman Filter for Nonlinear Estimation* - E. A. Wan and R. van der Merwe

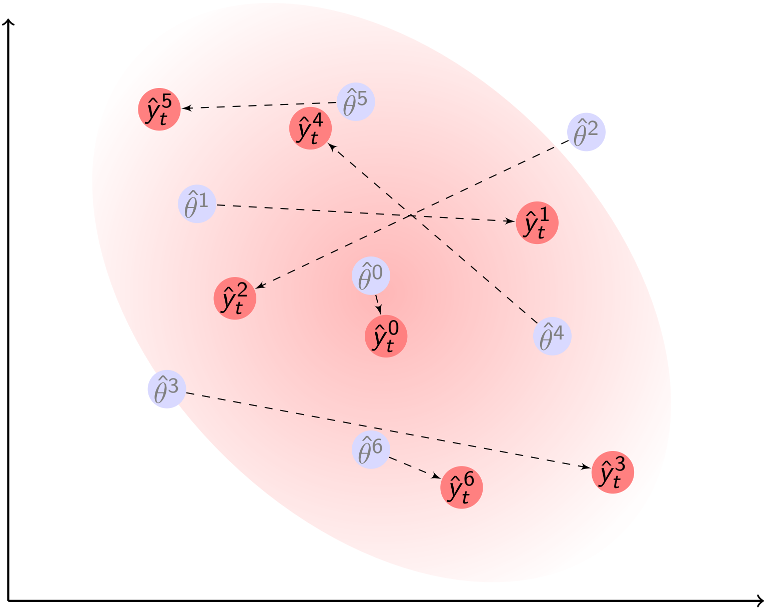
UKF t^{th} iteration



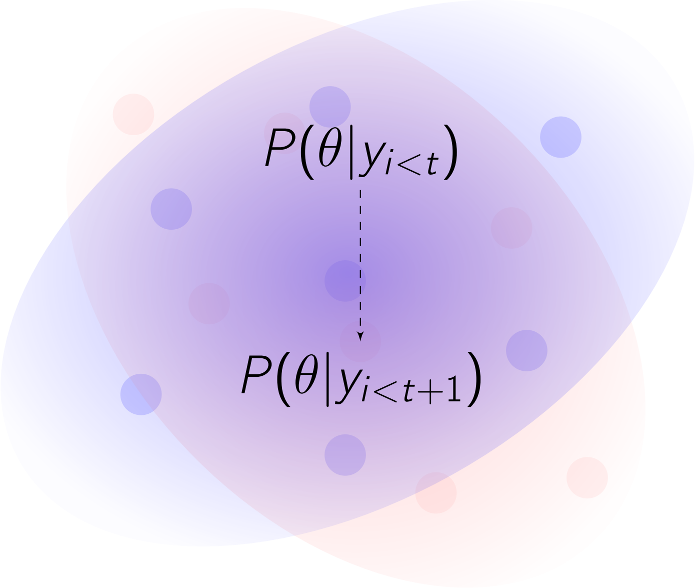
Summarize parameter distribution with sigma-points



Propagate sigma-points through the observation model g



Update parameter distribution using observation y_t



$$P(\theta | y_{i<t})$$

$$P(\theta | y_{i<t+1})$$

Adapting the UKF for qualitative inference

The idea

- Exploit the flexibility of the *observation function*, g, and *observations*, $\mathbf{y}_t$.
- Choose g to output the Lyapunov exponents of the model for parameter vector $\hat{\theta}^i$.
- Fix $\mathbf{y}_t = \Lambda_t = \Lambda$ as the constant desired Lyapunov spectrum.

$$\theta_{t+1} = \theta_t + v_t$$
$$y_t = g(x_t, \theta_t) + u_t \ ,$$

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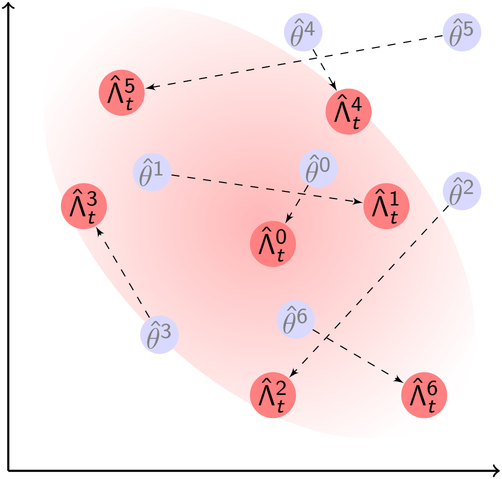

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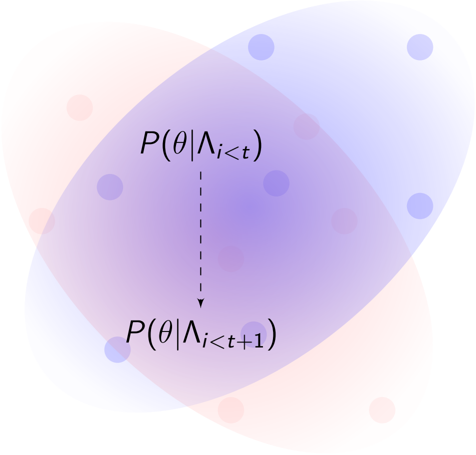
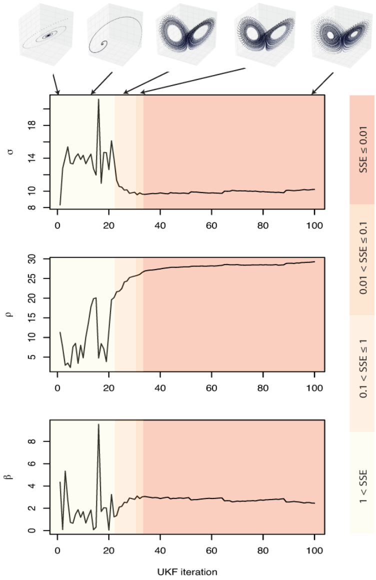
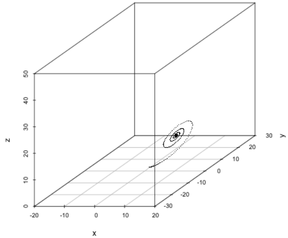


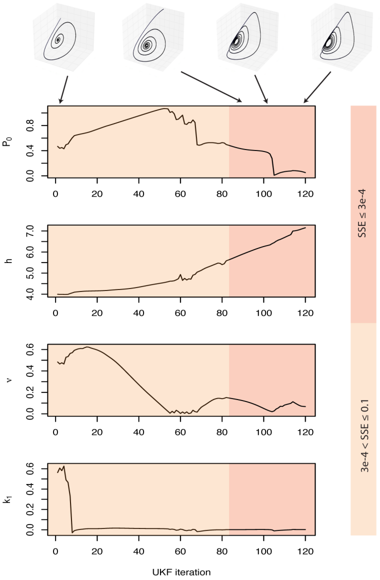
Results – Chaos in the Lorenz system



$$\dot{x} = \sigma(y - x)$$
$$\dot{y} = x(\rho - z) - y$$
$$\dot{z} = xy - \beta z,$$

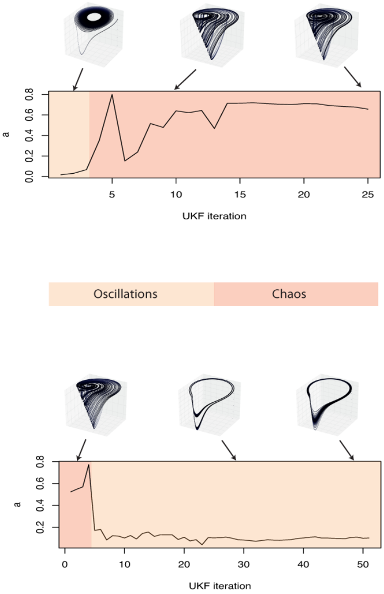


Results – Oscillations in a Hes1 regulatory model



$$\dot{M} = -k_{deg} M + 1/(1 + (P_2/P_0)^h)$$
$$\dot{P}_1 = -k_{deg} P_1 + \nu M - k_1 P_1$$
$$\dot{P}_2 = -k_{deg} P_2 + k_1 P_1$$

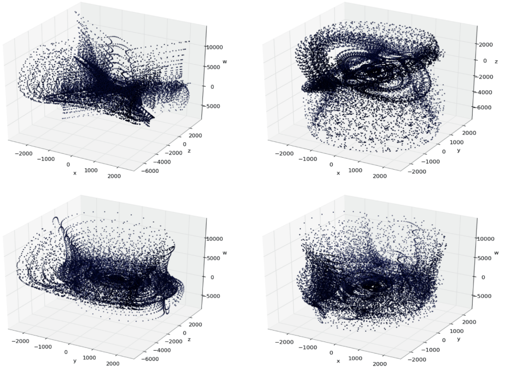
Results – Chaotification



$$
\begin{aligned}
\dot{x} &= y \\
\dot{y} &= ay - x - z \\
\epsilon \dot{z} &= b + y - c(e^{z} - 1),
\end{aligned}
$$

Tamaševičius et al. Eur. J. Phys. (2005)

Results – A very hyperchaotic attractor Qi et al. (2008)



- Very large Lyapunov exponents give trajectories similar properties to white noise.

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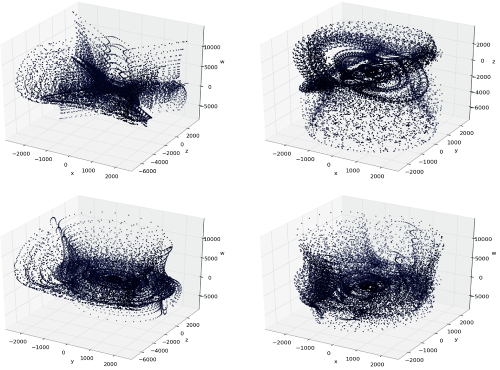

- Very large Lyapunov exponents give trajectories similar properties to white noise.
- Lyapunov spectrum$=(31.8, 16.8, -19.1, -71.4)$, over twice as large as previously found.

Summary

- The Unscented Kalman filter may be adapted for qualitative parameter inference.
- Parameters may be inferred such that a model exhibits a chosen Lyapunov spectrum.
- Example applications successfully identify chaotic, oscillatory and hyperchaotic regimes.

Ongoing work

- Can we infer model structure as well?

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- Can we infer model structure as well?
- Scan the space of 3 and 4 node networks for chaos/oscillations. Hope to identify common and necessary motifs for different types of behaviour.

Acknowledgements

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CRISP
combinatorial
responses
in stress pathways

BBSRC
bioscience for the future