Problems with discrete events

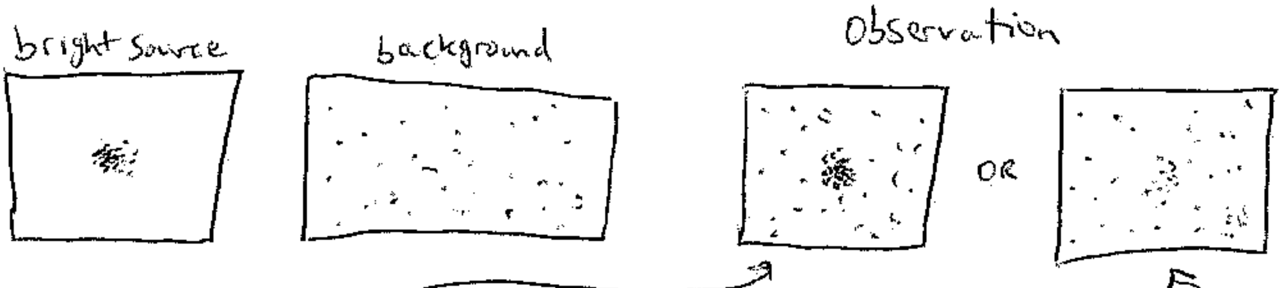
e.g. counting experiments: LHC, dark matter direct detection

Source detection: γ-ray / ν astronomy

discovery of ultrafaint galaxies in star catalogs.

Simple example:

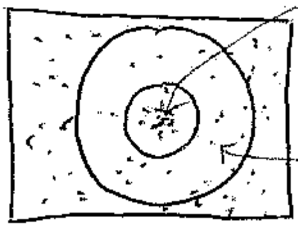
Source + background



Parameter estimation: what are the properties of the source?

Model selection: is there a source there?

Need: "data", "model", P("data" | "model")



$$\text{Data} = \begin{cases} n = \#\text{ counts in circle} \\ m = \#\text{ counts in ring} \end{cases}$$

Model: s = model prediction for # source counts in circle

b = " " " " background counts in ring

Poisson probability: has parameter μ

$$P(k|\mu) = \frac{e^{-\mu}\mu^k}{k!} = \text{probability of observing } k \text{ counts/events}$$

(k is an integer, $\mu \geq 0$ a real number)

Applies in the case where events occur independently of one another.

$$P(n,m|s,b) = P(n|s,b)\,P(m|s,b)$$ given model params, # counts in circle is independent of # counts in ring

$$P(m|s,b) = \frac{e^{-b}\, b^m}{m!}$$

$$P(n|s,b) = \frac{e^{-(s+b)}(s+b)^n}{n!} \; ???$$ Only if area of circle = area of ring

Let $\alpha = \dfrac{\text{area of circle}}{\text{area of ring}}$, assume background is uniform

so that (predicted # bg counts in circle) $= \alpha b$

then $$P(n|s,b) = \frac{e^{-(s+\alpha b)}(s+\alpha b)^n}{n!}$$

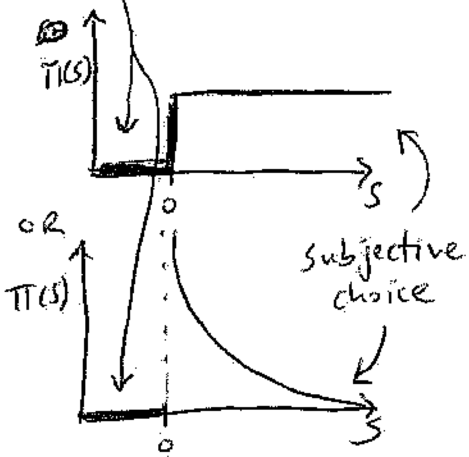
and $$P(\text{data}|\text{model}) = P(n,m|s,b) = \frac{e^{-b}\, b^m}{m!}\;\frac{e^{-(s+\alpha b)}(s+\alpha b)^n}{n!}$$

Likelihood, function of model params.

Posterior $$P(s,b|n,m) = \frac{P(n,m|s,b)\,\pi(s,b)}{\iint P(n,m|s,b)\,\pi(s,b)\,ds\,db}$$

prior on model params

physical constraint: $s \geq 0,\ b \geq 0$



Exercise: Find the "maximum likelihood estimates" of s and b.

i.e. what values $s=\hat{s}$ and $b=\hat{b}$ maximize likelihood function (for given data n, m)?

solve: $$\frac{\partial \ln P(n,m|\hat{s},\hat{b})}{\partial s} = 0 \quad \text{and} \quad \frac{\partial \ln P(n,m|\hat{s},\hat{b})}{\partial b} = 0$$

(be careful about $\hat{s} < 0$).