$$BF_{01} = \frac{\int p_1(x|\psi, \phi = \phi_0)\, p_1(\psi, \phi = \phi_0)\, d\psi}{\int p_1(x|\psi, \phi)\, p_1(\psi, \phi)\, d\psi d\phi} = \frac{p_1(x|\phi = \phi_0)}{p_1(x)}.$$

Bayes:

$$p_1(x|\phi = \phi_0) = \frac{p_1(\phi = \phi_0|x)\, p_1(x)}{p_1(\phi = \phi_0)}$$

So we find

$$BF_{01} = a\frac{p_1(\phi = \phi_0|x)}{p_1(\phi = \phi_0)}$$

$a = 1/\Delta\Omega_k$

This is the **Savage-Dickey Density Ratio**, SDDR

The numerator is a posterior, not a likelihood, so may be complicated in practice. However, if we have sampled it, we can estimate the SDDR.

If $f(\Delta\phi, \phi_0)$ is the fraction of samples within $\Delta\phi$ of ϕ_0

$$p_1(\phi = \phi_0|x) \simeq \frac{f(\Delta\phi, \phi_0)}{2\Delta\phi}$$