Stochastic modelling and estimation of renewable energy production data with applications in operational decision making

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Motivation

We can help tackle **climate change** by a more effective use of renewable energy sources. The goal of my research is to improve existing methods of stochastic modelling and statistical inference to quantify risk and uncertainty of **renewable energy sources** in a more reliable way.

Figure: Wind farm.

Electricity features

Since electricity is generally traded for consumption, it is considered a commodity. However, in contrast to other types of commodities, it has some unique features ([1]).

- Non-storability (supply and demand must always match).
- Seasonality (higher demand in winter months due to the need of heating and longer use of lights).
- Periodic behaviour (higher demand in the peak time, i.e., Monday to Friday between 8 am and 8 pm).
- Mean reversion (over time the electricity prices will tend to their average).
- Large and heteroscedastic volatility.

Empirical data

I work with a set of German data consisting of daily prices of spot and futures contracts over about 10 years.

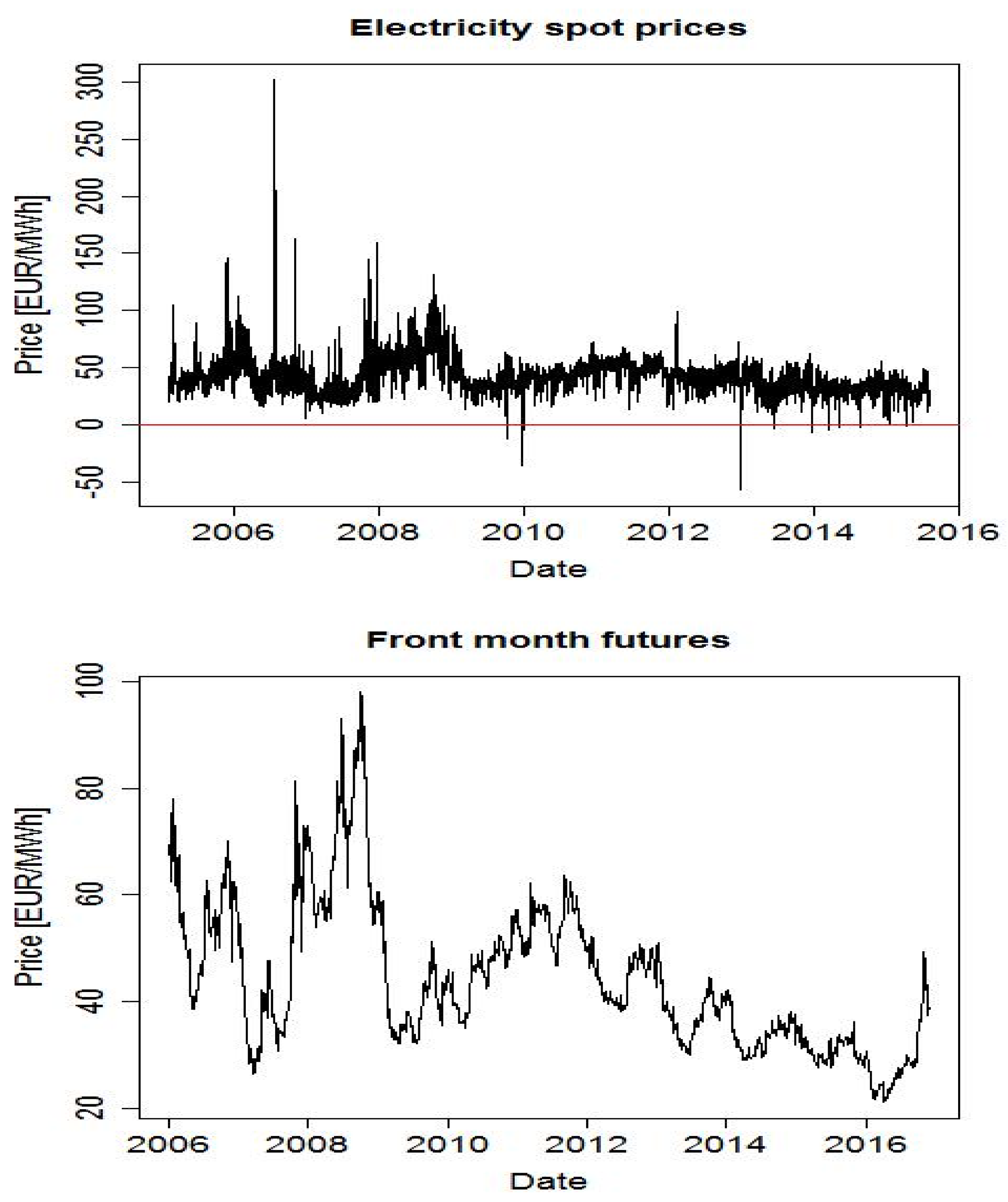

Figure: Data for German electricity spot and futures prices.

The arithmetic model for spot prices

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\in\mathbb{R}}, \mathbb{P})$ be a probability space, where the filtration $\{\mathcal{F}_t\}_{t\in\mathbb{R}}$ satisfies the 'usual conditions'. Let $S(t)$ be the spot price. Following [2], I proposed the arithmetic model

$$S(t) = \Lambda(t) + Z(t) + Y(t),$$

where $\Lambda(t) + Z(t)$ is the long-term factor, while $Y(t)$ describes the short-term behaviour, which includes the **impact of renewables**.

Model terms

- $\Lambda(t)$ – a deterministic seasonality/trend function.
- $Z(t)$ – a Lévy process with zero mean (under the physical measure).
- $Y(t) = \int_{-\infty}^{t} g(t-s)\sigma_{s-} dL_s$ with a kernel $g(t-s)$ such that $\lim_{t\to\infty} g(t-s) = 0$.

Here σ_t is a cádlág stochastic process describing the volatility of $Y(t)$. Similarly to [3], I defined it as

$$\sigma_t = \int_{-\infty}^{t} j(t-s) dV_s,$$

where j is a deterministic, positive function and $V(t)$ – a Lévy subordinator. I assumed that σ_t is independent from the driving Lévy process $L(t)$.

The arithmetic model for futures prices

By the no-arbitrage arguments, one can define the price of a futures contract with maturity T as

$$f(t, T) = \mathbb{E}_{\mathbb{Q}}[S(T)|\mathcal{F}_t],$$

where $0 \le t \le T < \infty$ and $\mathbb{Q}$ is a risk neutral probability measure (see eg. [2]).

In the arithmetic case the forward price at the time t equals

$$f_t(T) = \Lambda(T) + Z(t) + (T-t)\mathbb{E}_{\mathbb{Q}}[Z(1)] + \int_{-\infty}^{t} g(T-s)\sigma_{s-} dL_s + \mathbb{E}_{\mathbb{Q}}[L_1] \int_{t}^{T} g(T-s)\mathbb{E}_{\mathbb{Q}}[\sigma_s|\mathcal{F}_t]\, ds.$$

In the long run futures prices can be approximated by

$$f_t(T) \approx \Lambda(T) + Z(t) + (T-t)\mathbb{E}_{\mathbb{Q}}[Z(1)] + \mathbb{E}_{\mathbb{Q}}[L_1] \frac{\mathbb{E}_{\mathbb{Q}}[V_1]}{\delta} \int_{0}^{\infty} g(y) dy.$$

Current and future work

The most important step is to fit the proposed model to the empirical data in order to find the most appropriate form of $\Lambda(t)$, $Z(t)$ and $Y(t)$. I am especially interested in the **impact of renewables** on electricity prices. Futhermore, I am going to compare the results to an alternative approach, in which one models futures prices directly using ambit processes (see eg. [4]).

References

- R. ter Haar, "On modelling the electricity futures curve," 2010.
- F. E. Benth, C. Klüppelberg, G. Müller, and L. Vos, "Futures pricing in electricity markets based on stable CARMA spot models," *Energy Economics*, vol. 44, pp. 392–406, 2014.
- O. E. Barndorff-Nielsen, F. E. Benth, and A. E. D. Veraart, "Modelling energy spot prices by volatility modulated Lévy-driven Volterra processes," *Bernoulli*, vol. 19, no. 3, pp. 803–845, 2013.
- F. Barndorff-Nielsen, O., Benth and A. Veraart, "Modelling Electricity Futures by Ambit Fields," *Advances in Applied Probability*, vol. 745, no. 46(3), pp. 719–745, 2014.

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