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An Unfitted Finite Element Method for the Approximation of Void Electromigration

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Overview of the Problem

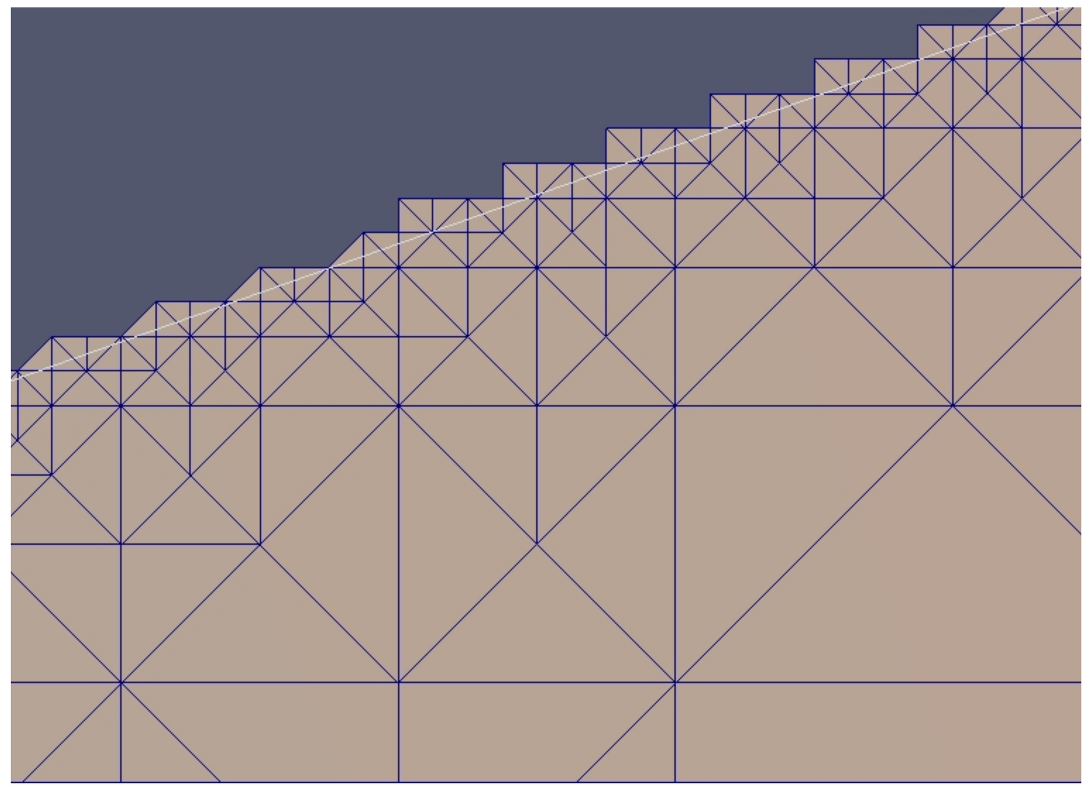
Microelectronic circuits usually contain small voids or cracks, and if those defects are large enough to sever the line, they cause an open circuit. A numerical method for investigating the migration of voids in the presence of both *surface diffusion* and electric loading is presented.

We simulate a **bulk-interface coupled system**, with a moving interface governed by a fourth-order geometric evolution equation and a bulk where the electric potential is computed. We present an *ad hoc* algorithm to identify 'cut', 'inside' and 'outside' bulk elements for each time step. The F. E. method, programmed in C++, shows **no need of re-meshing or deforming the bulk**, and **equidistribution of vertices for the interface** (see also [4]).

Bulk-Interface coupling: cut/inside/outside elements

Adapting [2], the algorithm proceeds as follows:

- Follow the interface curve and mark all triangles T s.t. $T \cap \Gamma \neq \emptyset$ as 'cut'.
- Mark all elements at $\partial\Omega$ as 'outside', and take them as advancing front.
- Continue by marking all the neighbours of the elements in the front as 'outside', updating the new front accordingly, till the 'cut' layer is reached.
- Finally, mark the 'cut' elements as 'outside'; all not-yet-marked elements are 'inside' elements, and are to be discarded for the subsequent step of the Laplace equation.



Unfitted FEM with local adaptivity.

Software in use

Dune

Distributed and Unified Numerics Environment

DUNE is C++-based, *open-source* software based on the following main principles:

- separation of data structures and algorithms by abstract interfaces;
- efficient implementation of these interfaces using generic programming techniques;
- reuse of existing finite element packages with a large body of functionality.

Governing equations

Let $\Omega = (-L_1, L_1) \times (-L_2, L_2)$ be the conductor, with boundary $\partial\Omega$.
At any time $t \in [0,T]$, let $\Gamma(t) \subset \Omega$ be the boundary of the void $\Omega^-(t)$ inside the conductor Ω. Then $\Gamma(t) = \partial\Omega^-(t)$ and $\Omega^+(t) := \Omega \setminus \Omega^-(t)$.
The evolution of the interface $\Gamma(t)$ (see also [1] and [3]) is then given by

$$\mathcal{V} = -\frac{\partial}{\partial s^2}(\alpha_1 \varkappa - \alpha_2 \phi), \tag{1}$$

where $\mathcal{V}$ is the normal velocity, s is the arc-length, $\varkappa$ is the curvature, α_1 and α_2 are parameters related to the *surface diffusion* and to the magnitude of the electric field respectively, and $\phi(t)$ the electric potential satisfies a Laplace equation in $\Omega^+(t)$. Precisely:

$$-\triangle\phi = 0 \quad \text{in } \Omega^+(t) \qquad \frac{\partial\phi}{\partial\nu} = 0 \quad \text{on } \Gamma(t), \tag{2a}$$

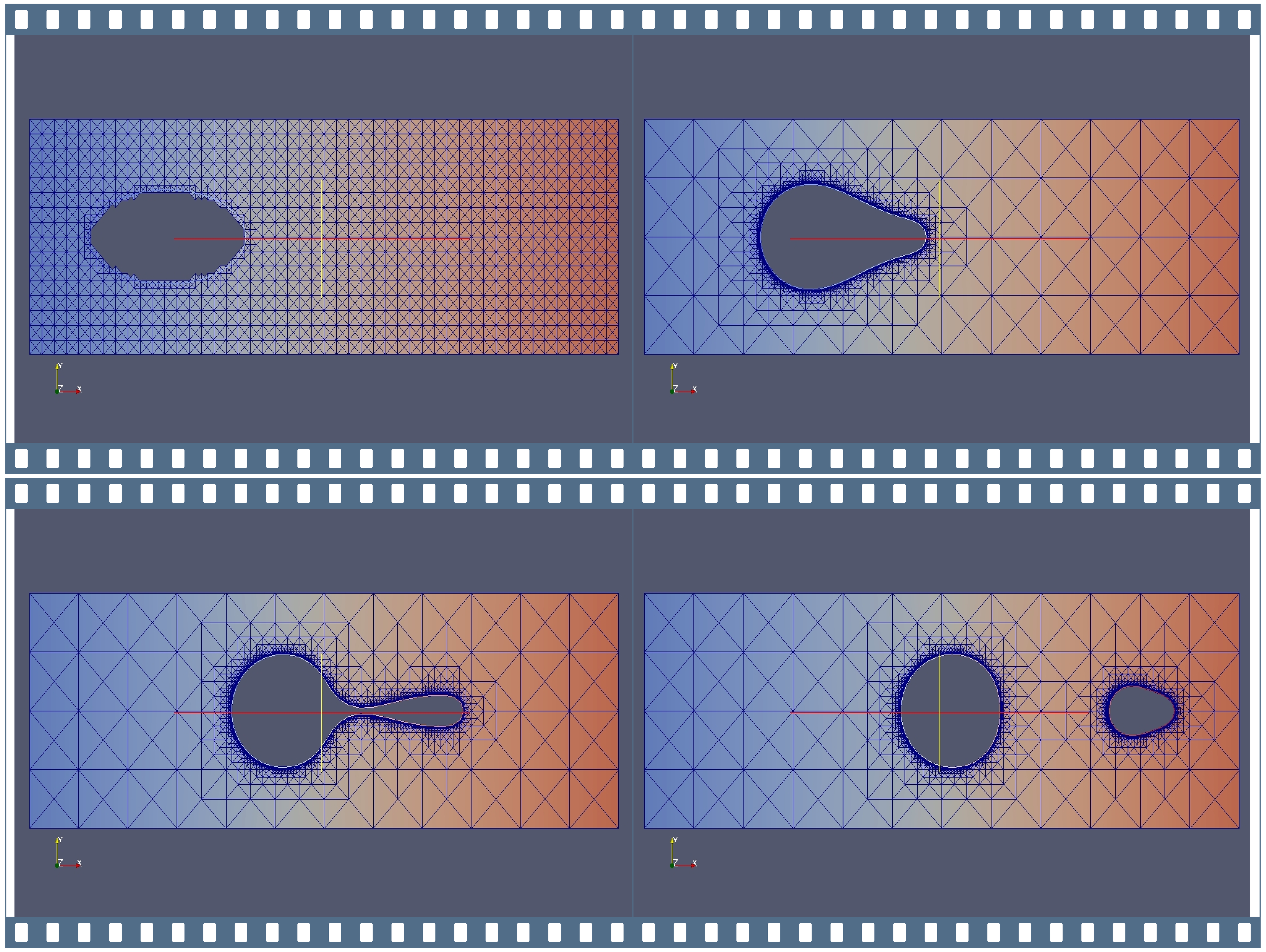
$$\frac{\partial\phi}{\partial\nu_{\partial\Omega}} = 0 \text{ on } \partial_1\Omega \qquad \phi = g^{\pm} \quad \text{on } \partial_2^{\pm}\Omega, \tag{2b}$$

ν being the unit normal to $\Gamma(t)$. In the above $\partial\Omega = \partial_1\Omega \cup \partial_2\Omega$, where $\partial_1\Omega \cap \partial_2\Omega = \emptyset$ and

$$\partial_2\Omega = \partial_2^-\Omega \cup \partial_2^+\Omega \quad \text{with} \quad \partial_2^{\pm}\Omega := \{\pm L_1\} \times [-L_2, L_2],$$

and ν is the outward unit normal to $\partial\Omega$.

Results of the simulations



References

[1] Z. Li, H. Zhao, H. Gao (1999), *A numerical study of electro-migration voiding by evolving level set functions on a fixed cartesian grid*, Journal of Computational Physics, Vol. 152, pp. 281–304.

[2] E. Bänsch, A. Schmidt (2000), *Simulation of dendritic crystal growth with thermal convection*, Interfaces and Free Boundaries, Vol. 2, pp. 95–115.

[3] J. W. Barrett, R. Nürnberg, V. Styles (2004), *Finite element approximation of a phase field model for a void electromigration*, SIAM Journal of Numerical Analysis, Vol. 42, No. 2, pp. 738–772.

[4] J. W. Barrett, H. Garcke, R. Nürnberg (2008), *On the parametric finite element approximation of evolving hypersurfaces in* $\mathbb{R}^3$, Journal of Computational Physics, Vol. 227, pp. 4281–4307.