Measures of Systemic Risk: Analysing CoVaR

Eric Schaanning[1] *and Georg Mainik*[2]

[1]*Imperial College London, UK;* [2]*ETH Zurich, Switzerland.*

Imperial College London

Systemic Risk and Bank Capital Regulation

- Motivation: Account for the dependence structure between banks in the computation of their respective regulatory capitals;
- In 2008, Adrian and Brunnermeier introduced CoVaR – Conditional-Value-at-Risk – as a dependence adjusted version of VaR. Girardi and Ergün introduced a modified definition of CoVaR in an M-GARCH setting;
- Goal of this study: Compare the performance of these two different definitions in measuring systemic risk.

From VaR to CoVaR

- Let $L \sim F_L$, continuous. $\mathrm{VaR}_\alpha(L) := F_L^{-1}(\alpha)$.

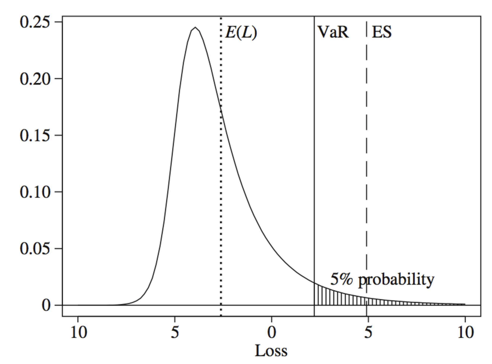


Figure 1: *Loss distribution of a univariate random variable L. Losses are given by positive numbers, gains by negative ones. The 95%-VaR-level,* $\mathrm{VaR}_{0.95}(L)$, *is the loss, such that on average only 5% of the losses will be bigger than this. Credits McNeil et al. [2005].*

Framework and Analysis

- Stochastic framework: Bivariate model $(X,Y) \sim F_{XY}(x,y)$, where the random variables X, Y model the losses of two respective financial institutions.
- The main goal of CoVaR is to quantify: *What happens to Y, given that X is "in a bad state", i.e. under financial stress.*

Original definition:

$$\mathrm{CoVaR}^{=}_{\alpha,\beta} := F^{-1}_{Y|X=\mathrm{VaR}_\alpha(X)}(\beta).$$

Modified definition:

$$\mathrm{CoVaR}_{\alpha,\beta} := F^{-1}_{Y|X\geq\mathrm{VaR}_\alpha(X)}(\beta).$$

- How much of a difference does conditioning on the bad state "$X \geq \mathrm{VaR}_\alpha(X)$" instead of "$X = \mathrm{VaR}_\alpha(X)$" make?

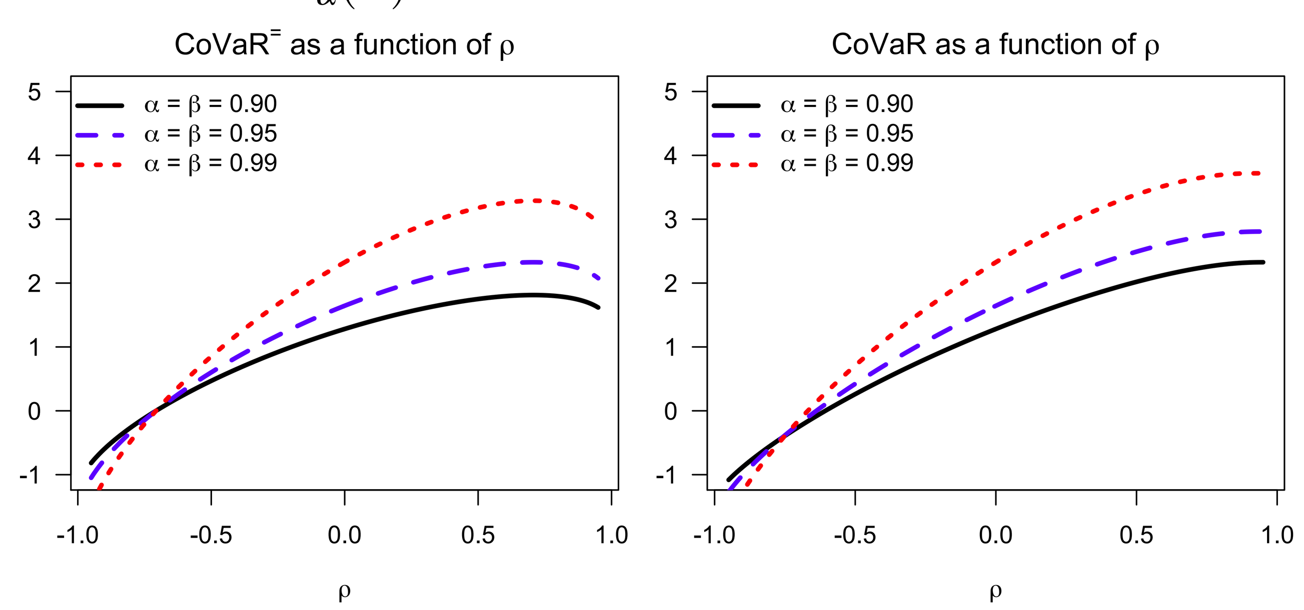


Figure 2: $\mathrm{CoVaR}^{=}$ *seems, as opposed to* CoVaR, *not to be a monotonically increasing function of the dependence parameter. Capital requirements linked to this could lead to regulatory arbitrage.*

Proofs

Non-monotonicity of $\mathrm{CoVaR}^{=}$

- Combining a few well known explicit formulas for the Gaussian distribution one can compute that:

$$\mathrm{CoVaR}^{=}_{\alpha,\beta}(Y|X) = \mu_Y + \sigma_Y\left(\rho\Phi^{-1}(\alpha) + \Phi^{-1}(\beta)\sqrt{1-\rho^2}\right).$$

- Differentiate with respect to ρ and non-monotonicity follows immediately:

$$\partial\rho\,\mathrm{CoVaR}^{=}_{\alpha,\beta}(Y|X) = \sigma_Y\left(\Phi^{-1}(\alpha) - \frac{\rho\Phi^{-1}(\beta)}{\sqrt{1-\rho^2}}\right).$$

Monotonicity of CoVaR

A bivariate random vector (X,Y) is *elliptically distributed* $\mathcal{E}(\mu,\Sigma,R)$ if

$$(X,Y)^\top \stackrel{d}{=} \mu^\top + RAW^\top, \text{where}$$

- $W = (W_1, W_2)$ is a uniformly distributed r.v. on the unit sphere $\{x \in \mathbb{R}^2 : \|x\|_2 = 1\}$;
- R is a non-negative random variable independent of W, called the *radial* part;
- The covariance matrix of (X,Y) is defined if and only if $\mathbb{E}R^2 < \infty$ and can always be written as:

$$\Sigma' = \begin{pmatrix} \sigma_X^2 & \sigma_X\sigma_Y\rho' \\ \sigma_X\sigma_Y\rho' & \sigma_Y^2 \end{pmatrix}.$$

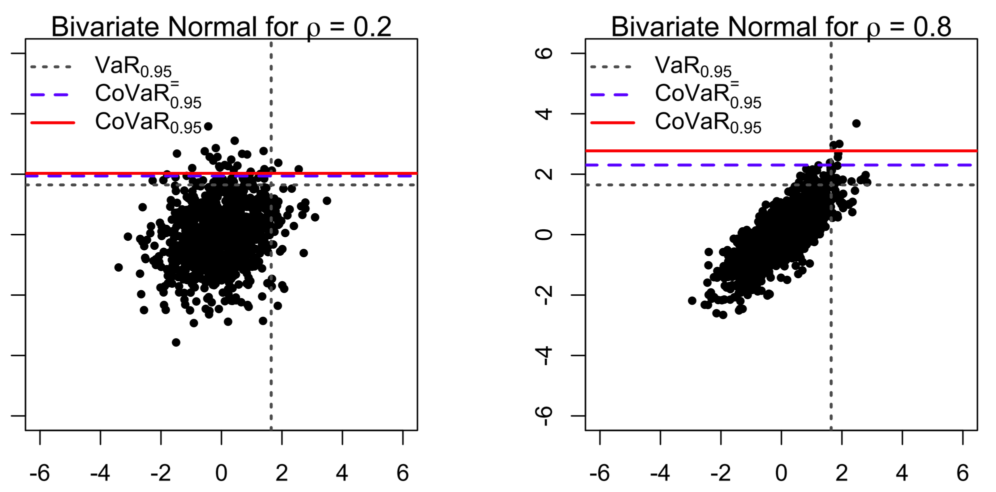
Proposition [M. and S., 2012]. Let F_{XY} and $F_{X'Y'}$ have elliptical copulas with equal radial parts and dependence parameters ρ and ρ', respectively. If F_X and $F_{X'}$ are continuous and $F_Y(y) \geq F_{Y'}(y)$ for all $y \in \mathbb{R}$, then $\rho \leq \rho'$ implies

$$\forall \alpha,\beta \in (0,1) \quad \mathrm{CoVaR}_{\alpha,\beta}(Y|X) \leq \mathrm{CoVaR}_{\alpha,\beta}(Y'|X').$$

The modified definition of CoVaR is hence consistent with the so-called *concordance ordering* of the underlying distributions and well-known results can readily be applied to a multitude of different distributions. Hence, at all confidence levels, CoVaR is an increasing function of the dependence between the components.

Results

Figure 3: *The probability of observing joint extremes, i.e. points falling into the upper right-hand corner, increases with stronger dependence (higher ρ here).*



Backtesting

Bound	$\rho=0$	$\rho=0.2$	$\rho=0.5$	$\rho=0.7$	$\rho=0.9$
$\mathrm{CoVaR}^{=}_{0.95,0.95}(Y\|X)$	0.0503	0.0601	0.0857	0.1229	0.2520
$\mathrm{CoVaR}_{0.95,0.95}(Y\|X)$	0.0503	0.0500	0.0503	0.0495	0.0499
$\mathrm{CoVaR}^{=}_{0.99,0.99}(Y\|X)$	0.0099	0.0124	0.0189	0.0292	0.0875
$\mathrm{CoVaR}_{0.99,0.99}(Y\|X)$	0.0099	0.0101	0.0104	0.0099	0.0098

Table 1: *Violation rates in the bivariate normal case. Monte Carlo backtesting with $n = 10^7$ and $\alpha,\beta \in \{0.95, 0.99\}$.*

- $\mathrm{CoVaR}^{=}$ fails to pick up risk when it is most pronounced and achieves a too high violation rate.
- $\mathrm{CoVaR}^{=}$'s confidence level β can be misleading: For high-dependence scenarios, e.g. $\rho = 0.9, \beta = 0.95$, the level is exceeded over 25% of the instances, whereas one would expect a violation rate of $1-\beta \approx 5\%$.
- By construction, CoVaR keeps a violation rate of $1-\beta$.

Conclusions

- $\mathrm{CoVaR}^{=}$ (and extensions of it using $\mathrm{CoVaR}^{=}$ as building block) can lead to regulatory arbitrage, as they provide explicit incentives for banks to become more dependent on each other. In terms of regulatory capital one would conclude:
  - $\mathrm{CoVaR}^{=}$: The more Y depends on X, the less capital Y requires.
  - CoVaR: The more Y depends on X, the more capital Y requires.
- The results are even worse for non-Gaussian distributions.
- In general, dependence consistency seems to be a reasonable property to expect from systemic risk measures, and multivariate stochastic orders seem to provide a natural framework in which to analyse the behaviour of these.
- Open question: Is systemic risk measurable from market data at all? This is an important underlying assumption of the CoVaR approach.

References

T. Adrian and M.K. Brunnermeier. CoVaR. *Preprint*, 2010. URL `http://www.princeton.edu/~markus/research/papers/CoVaR`.

G. Girardi and T. Ergün. Systemic risk measurement: Multivariate garch estimation of covar. *Preprint*, 2012. URL `http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1783958`.

G. Mainik and E. Schaanning. On dependence consistency of CoVaR and some other systemic risk measures. *To appear: Statistics and Risk Modeling*, 2013.

A.J. McNeil, R. Frey, and P. Embrechts. *Quantitative Risk Management: Concepts, Techniques and Tools*. Princeton University Press, 2005.