PHONONIC BOX CRYSTALS

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Motivation: Study acoustic metamaterials formed by arrays of strongly coupled Helmholtz resonators and characterise these materials with a versatile asymptotic analysis approach, allowing to model and easily tune arbitrary lattice and inclusion geometries [1].

CONTINUOUS PHONONIC CRYSTALS

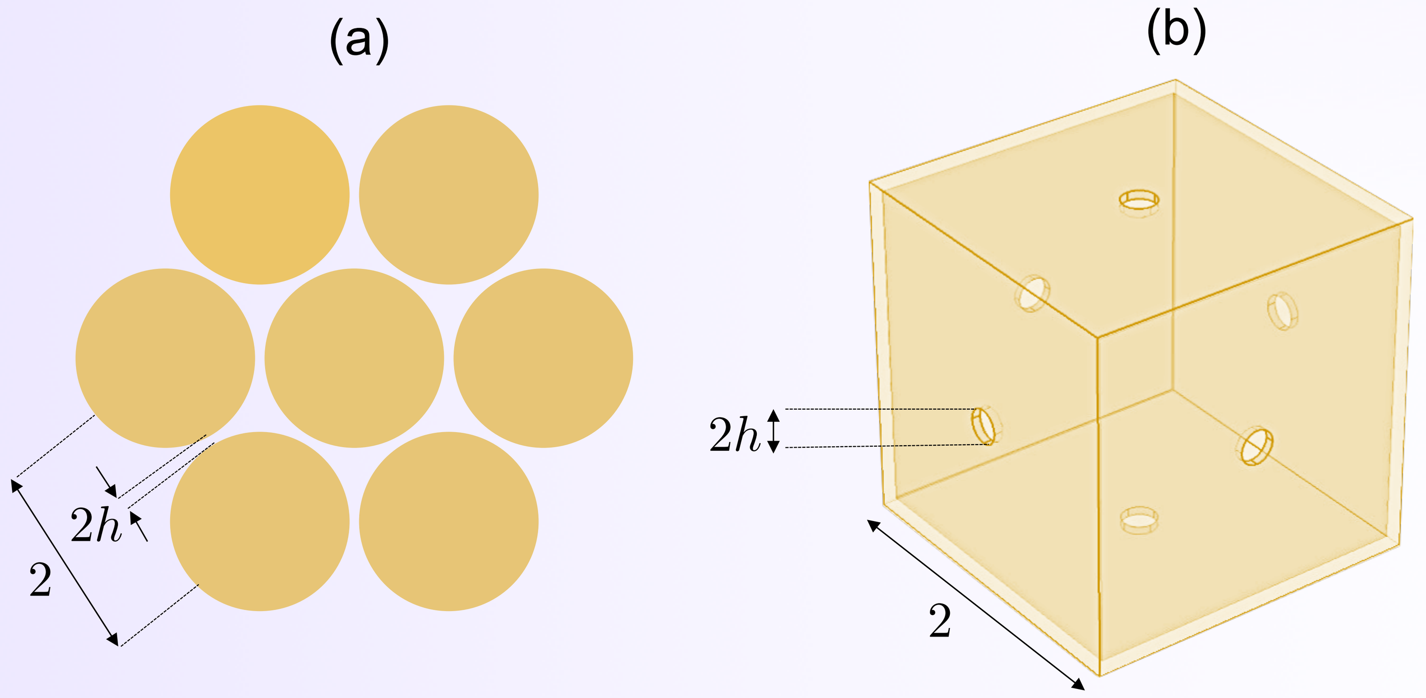


Fig. 1: Examples in 2D and 3D: circular rigid inclusions on a hexagonal lattice (a) and perforated thin boxes in a cubic arrangement (b).

$$\begin{cases} \nabla_{\mathbf{x}}^2 u + h\Omega^2 u = f(\mathbf{x}) \\ \dfrac{\partial u}{\partial n} = 0 \end{cases}$$

Solving the forced problem numerically in 3D for thousands of resonating cells containing singular openings is essentially impossible.

Limit as $h \to 0$

Recipe to turn a numerical weakness into an analytical strength

Systems become subwavelength metamaterials

ASYMPTOTIC NETWORK MODEL

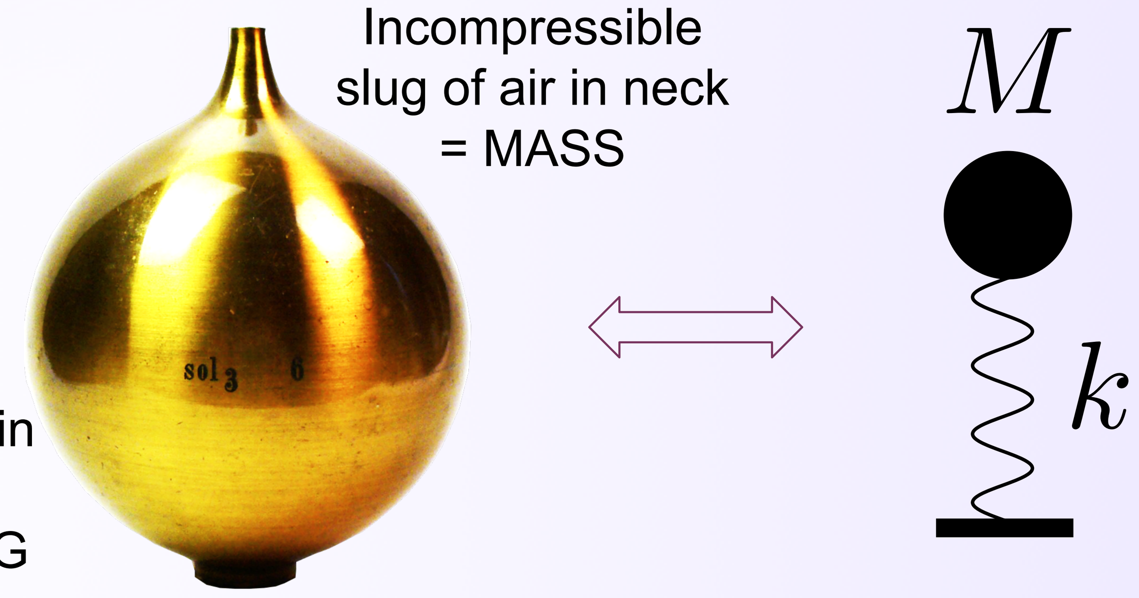


Fig. 3: Helmholtz resonator bottle [2] and its physical interpretation

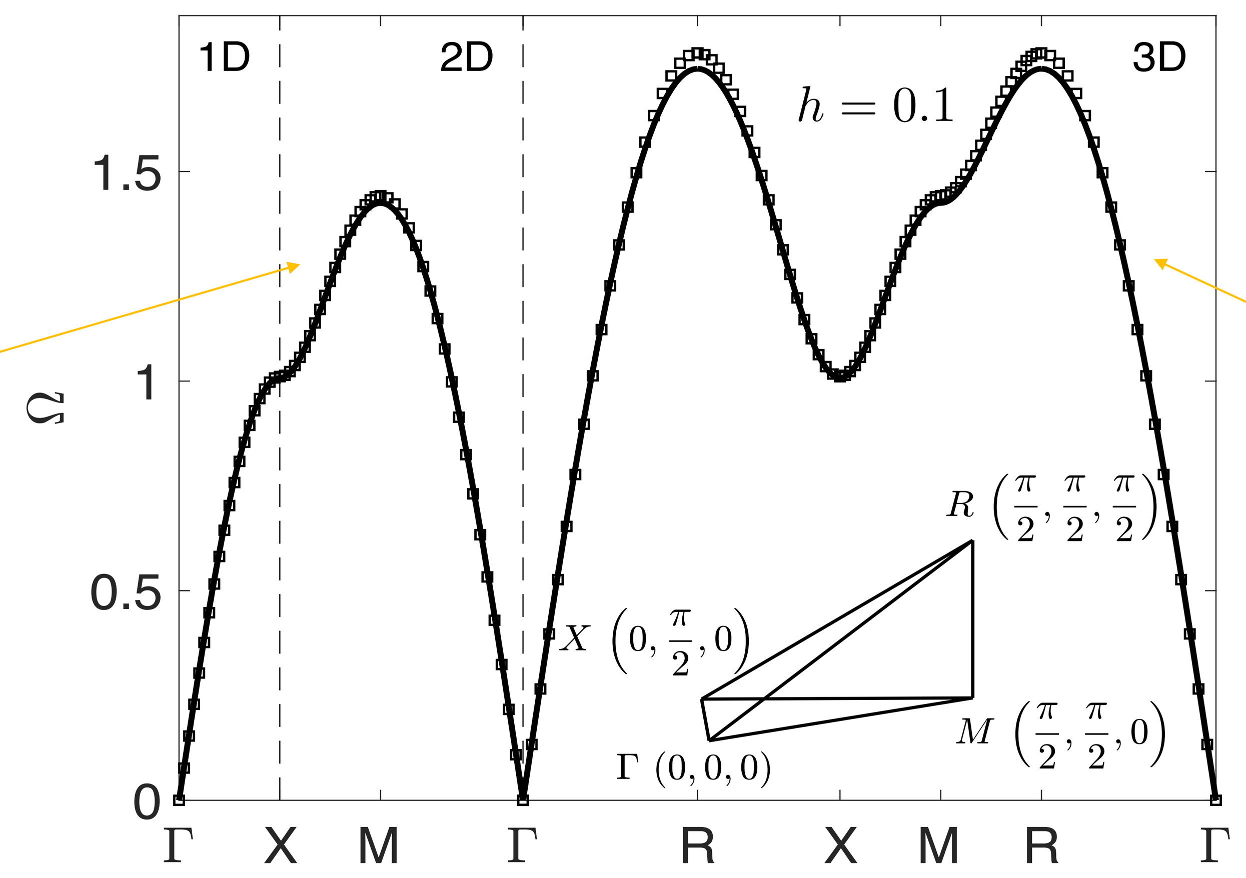
Use the method of Matched Asymptotic Expansions (MAE) between the opening and the void to derive asymptotic, discrete, wave equations governing the void pressures:

$$\sum_{\mathbf{m}} u^{\mathbf{n}+\mathbf{m}} + \left(\frac{V}{2}\Omega^2 - 2d\right) u^{\mathbf{n}} = F(\mathbf{n})$$

Yields an explicit dispersion relation for the entire acoustic branch and a closed form formula for the refractive index.

I. Rayleigh Bloch Wave problem : $f(\mathbf{x}) = 0$

Symbols = Finite Elements Methods



$F(\mathbf{n}) = 0$

Line = asymptotic discrete model

Fig. 2: Band diagram: free wave propagation in the system for the box.

II. Source problem : $f(\mathbf{x}) \neq 0$ in one box

Challenge: Transform microscopic forcing $f(\mathbf{x})$ onto the macroscale to calculate $F(\mathbf{n})$.

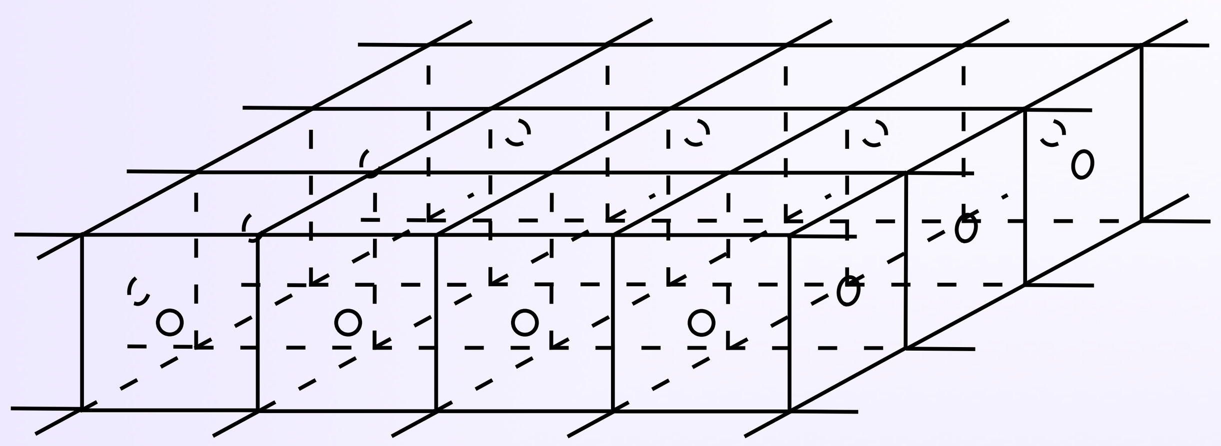
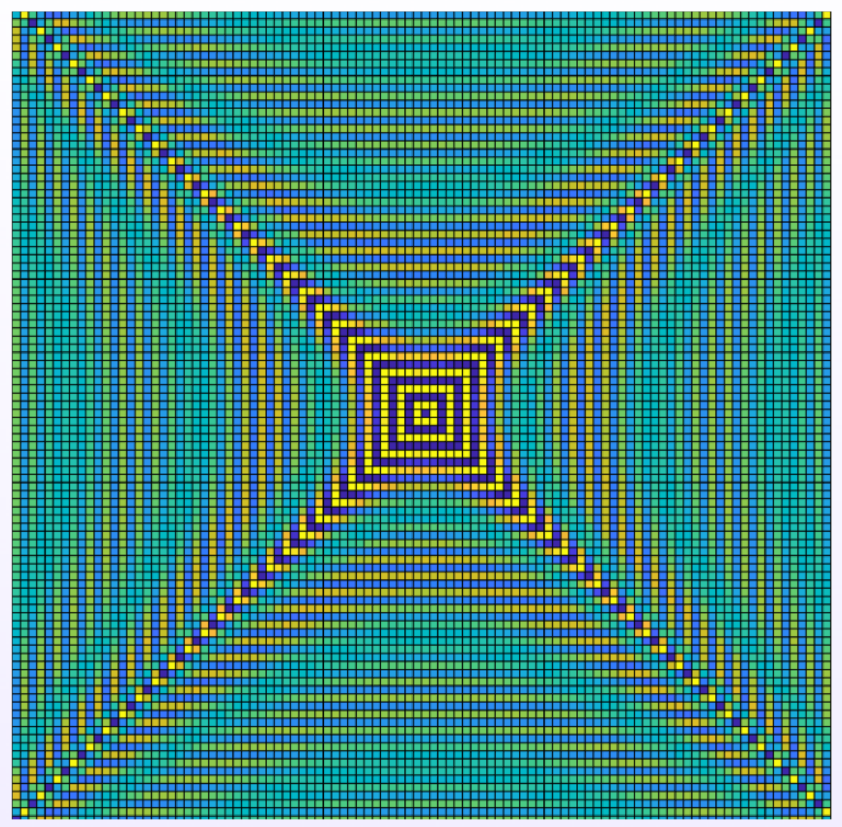


Fig. 4: Phonic box crystal: 2D plane array forced in the centre at $\Omega = 1$.

1. Monopole forcing

$$f(\mathbf{x}) = \delta(\mathbf{x})$$

$F(\mathbf{n})$ is simply the integral of $f(\mathbf{x})$ over the unit-cell, which yields a Kronecker delta function.



2. Dipole forcing

$$f(\mathbf{x}) = \mathbf{k} \cdot \nabla_{\mathbf{x}} \delta(\mathbf{x})$$

$F(\mathbf{n})$ is found by solving a canonical Green's function in the zeroth box, which requires matched asymptotic expansions and numerical simulations.

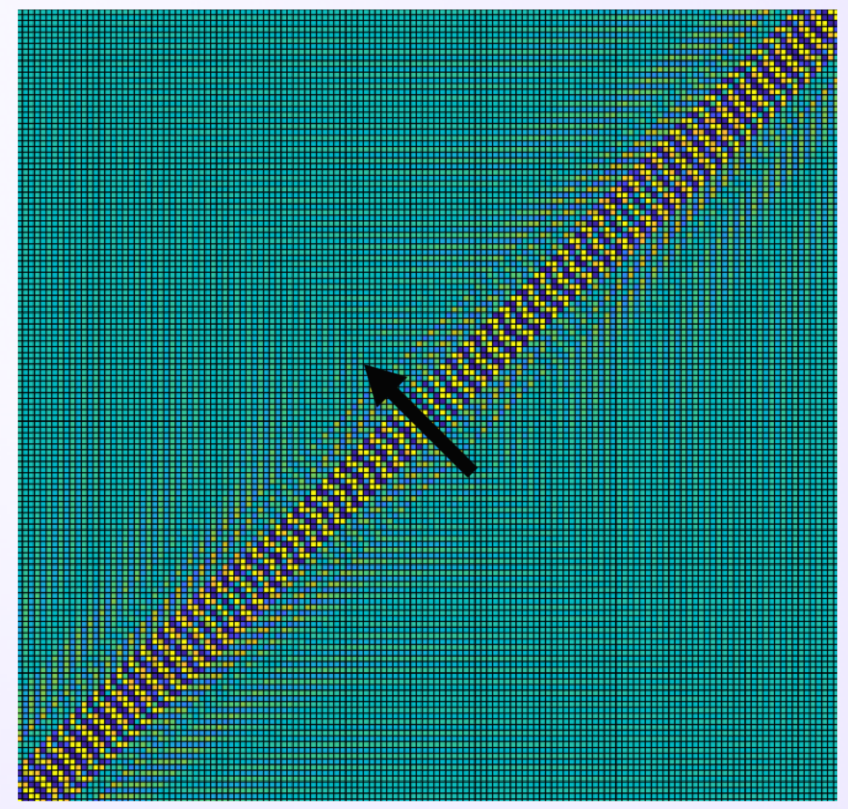


Fig. 5: Examples of responses for the two-dimensional slab array of boxes forced with a monopole/dipole in the zeroth box at anisotropy point X.

References:
[1] A. L. Vanel, O. Schnitzer and R. V. Craster. Asymptotic network models of subwavelength metamaterials formed by closely packed photonic and phononic crystals, *EPL*, 2017
[2] Brass spherical Helmholtz resonator, Max Kohl.

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