Field-theoretic active matter thermodynamics

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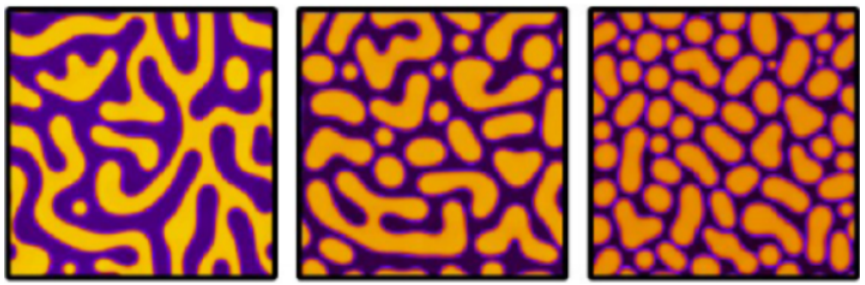
Non-Equilibrium Systems Group

What is active matter?

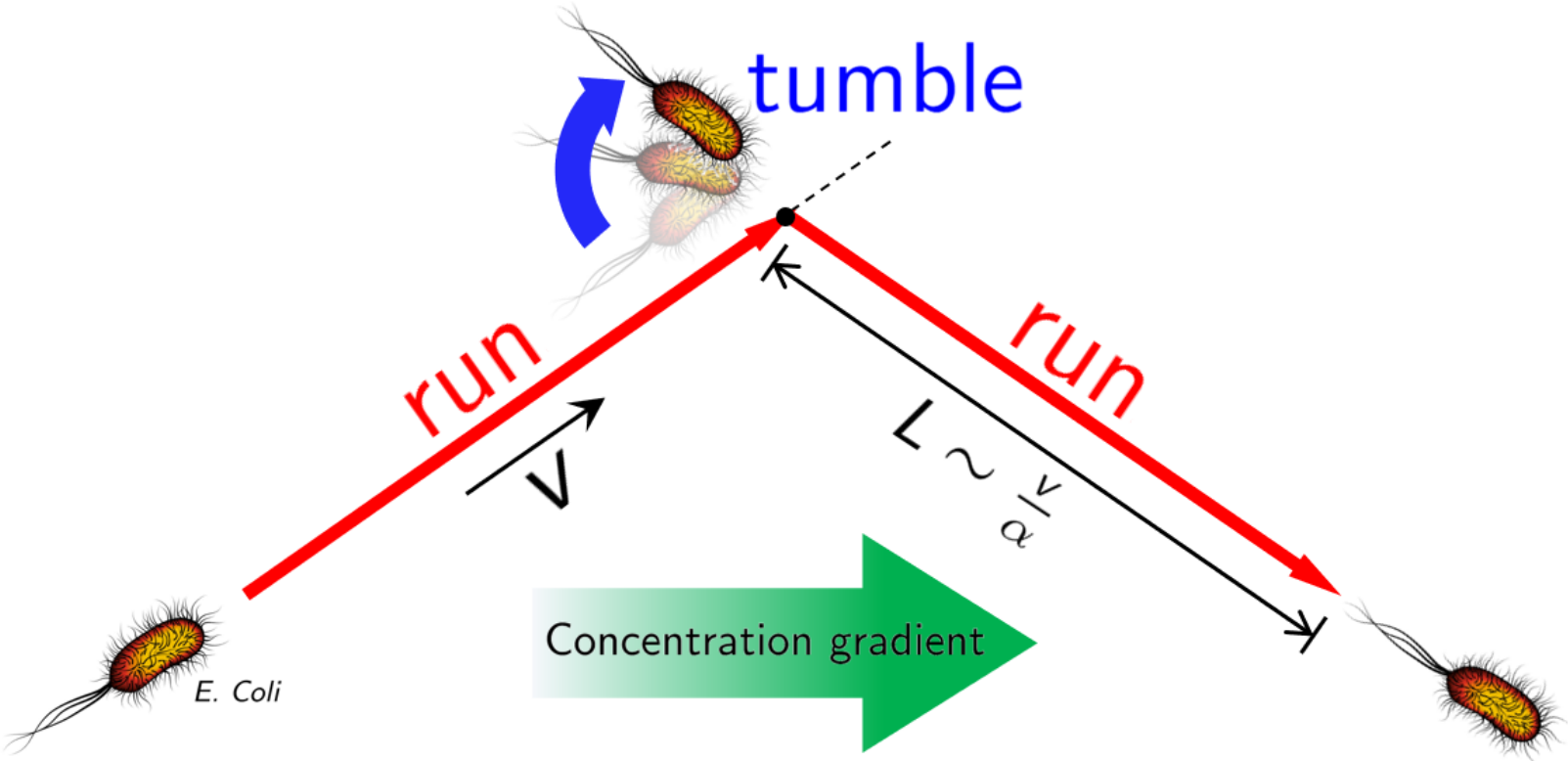
- Entities that convert energy from their environment into mechanical work, such as self-propelled motion

- Emergent collective behaviour: flocking dynamics, motility-induced phase separation



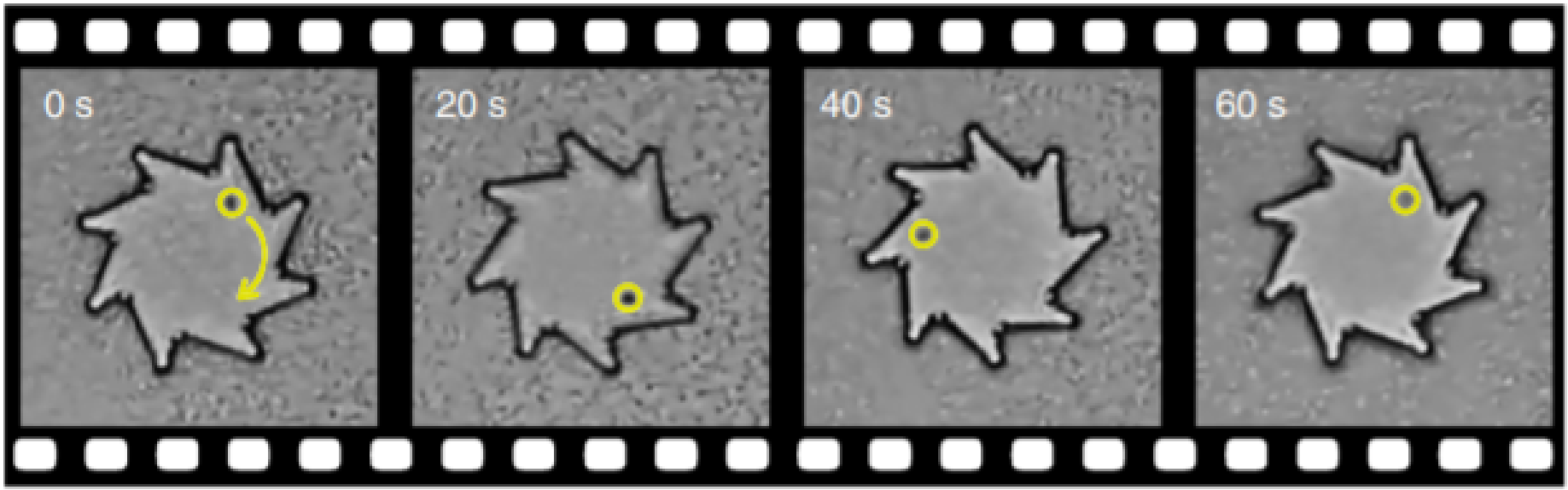
Cates, Tailleur. *Annu. Rev. Condens. Matter Phys.*, 2015

Run-and-tumble (RnT) particles



- RnT motion: straight runs, interspersed by random tumbles that change the direction of motion
- Behaviour adopted by microbial organisms, e.g. *E. Coli*

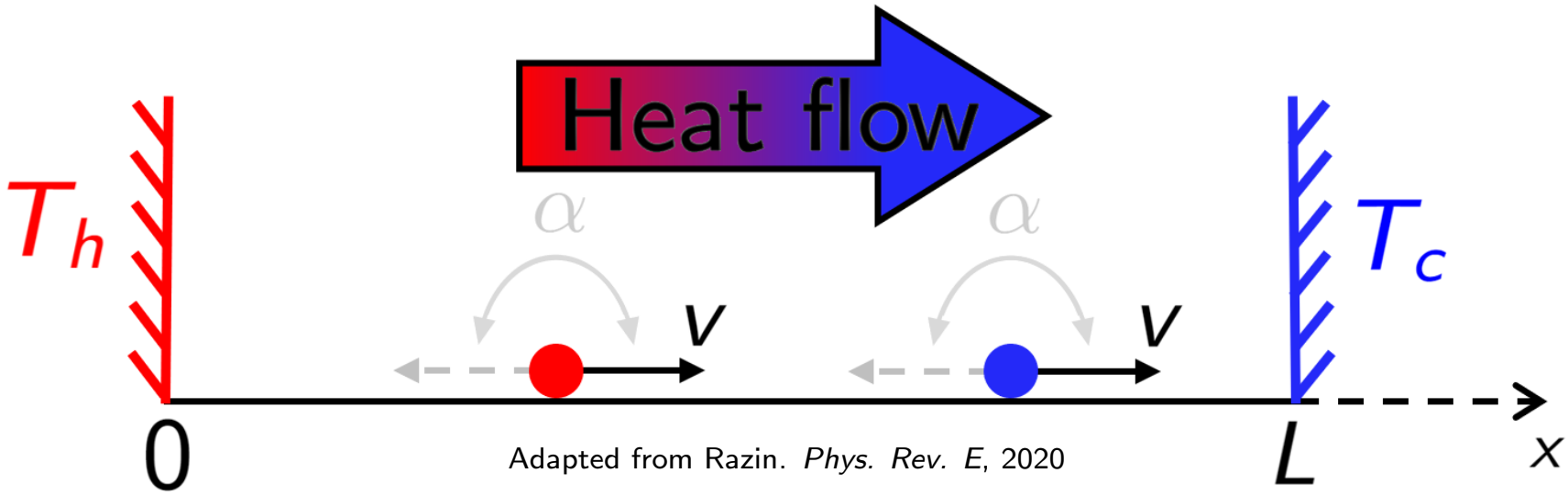
Can we make an RnT engine?



Di Leonardo *et al.* *PNAS*, 2010

- Non-equilibrium system: not 'restricted' by the second law of thermodynamics
- A gas of RnT particles could power a machine autonomously

Thermodynamics for an RnT heat engine



Adapted from Razin. *Phys. Rev. E*, 2020

- Consider a simple model of heat transfer between hot and cold objects

$$\frac{\partial P_{\text{cold},R}}{\partial t} = \underbrace{D_c \frac{\partial^2 P_{\text{cold},R}}{\partial x^2}}_{\text{Diffusion}} \underbrace{- v \frac{\partial P_{\text{cold},R}}{\partial x}}_{\text{Ballistic runs}} \underbrace{+ \frac{\alpha}{2} \left(P_{\text{cold},L} - P_{\text{cold},R} \right)}_{\text{Tumble events}} \underbrace{- \tilde{r}_c \, \delta(x) P_{\text{cold}} + \tilde{r}_h \, \delta(x - L) P_{\text{hot}}}_{\text{Heat transfer}}$$

(Plus coupled PDEs for $P_{\text{hot},L}$, $P_{\text{cold},R}$ and $P_{\text{cold},L}$)

What is field theory and how can it help?

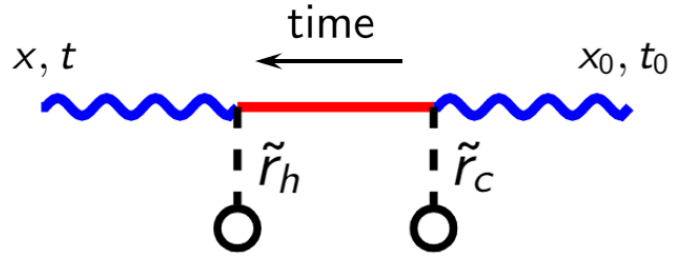
- RnTs and heat transfer treated as perturbations in a field-theoretic framework
- Particles promoted to fields

$$P_{\text{cold}}(x, t|x_0, t_0) \to \psi(x, t)\tilde{\psi}(x_0, t_0)$$

- Feynman diagrams represent terms in a perturbative expansion

$$\frac{\partial P_{\text{cold}}}{\partial t} = \cdots - \tilde{r}_c\ \delta(x) P_{\text{cold}} + \tilde{r}_h\ \delta(x - L) P_{\text{hot}}$$

$$\langle \psi(x, t)\tilde{\psi}(x_0, t_0)\mathcal{A}_{c\to h}\mathcal{A}_{h\to c}\rangle_0 =$$



Project overview

- Main aim:

  Investigate the thermodynamic properties of RnT particles using field-theoretic methods

- Steps:

  1. Field-theoretic model of heat conduction
  2. . . . with RnT particles
  3. Added realism: introduce collisions
  4. Consider in higher dimensions
  5. What other thermodynamic relations can we derive? Equation of state?

- Wider scope: is an active matter heat engine possible?

Thank you for listening!