How to Make a Splash

Droplet Impact and Liquid Film Applications in Aerodynamics

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Motivation

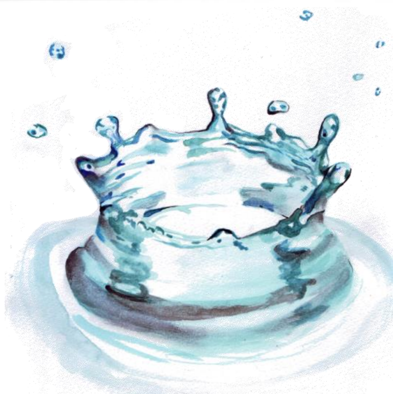
It all starts from a few innocent drops of rain. Travelling through clouds ensures exposure to a sufficiently high quantity of liquid to drastically affect flight stability. Below freezing temperatures provide the final ingredient in the recipe for disaster. Ice formation on aircraft remains to this day one of the most treacherous challenges[1] in flight safety.

Motivated by far reaching practical implications

- flight control and reliability
- de-icing procedure design
- efficient fuel consumption
- aircraft certification

we turn to theoretical and numerical techniques in multi-phase fluid dynamics to weather the storm.

Mathematical Model



We consider the Navier-Stokes equations, as well as the continuity equation, in each of the modelled fluids:

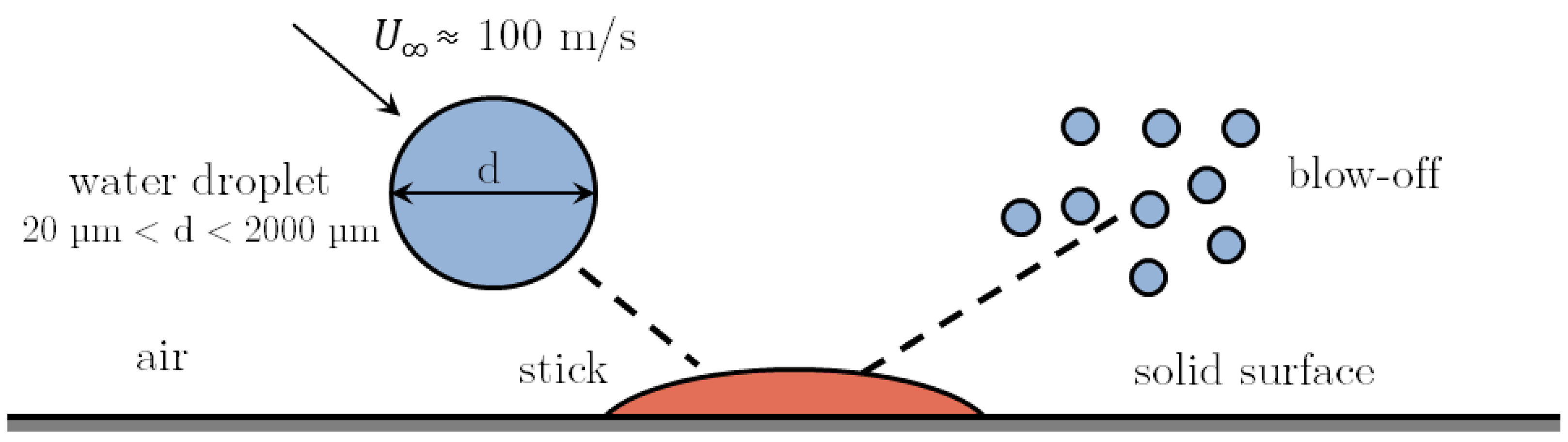
$$\begin{aligned}\rho_l(\partial_t \tilde{\mathbf{u}}_l + (\tilde{\mathbf{u}}_l\cdot\nabla)\tilde{\mathbf{u}}_l) &= -\nabla\tilde{p}_l + \mu_l\Delta\tilde{\mathbf{u}}_l - \rho_l g\mathbf{j},\\ \rho_a(\partial_t \tilde{\mathbf{u}}_a + (\tilde{\mathbf{u}}_a\cdot\nabla)\tilde{\mathbf{u}}_a) &= -\nabla\tilde{p}_a + \mu_a\Delta\tilde{\mathbf{u}}_a - \rho_a g\mathbf{j},\\ \nabla\cdot\tilde{\mathbf{u}}_{l,a} &= 0.\end{aligned}$$

In a standard water-air configuration, we underline the strong contrast in both density and viscosity between the two fluids to be

$$r = \rho_{\text{liquid}}/\rho_{\text{air}} \approx 787.32, \quad m = \mu_{\text{liquid}}/\mu_{\text{air}} \approx 79.54.$$

The main dimensionless groups are the Reynolds number $\text{Re} = \rho_a U_\infty L/\mu_a$, the Weber number $\text{We} = \rho_a U_\infty^2 L/\sigma$, as well as the splashing parameter $\mathcal{K} = \text{We}\sqrt{\text{Re}}$.

This regime produces violent flows, prone to strong topological changes and intricate multi-scale dynamics. The nature of the impact in such conditions is outside the reach of purely analytical treatments.



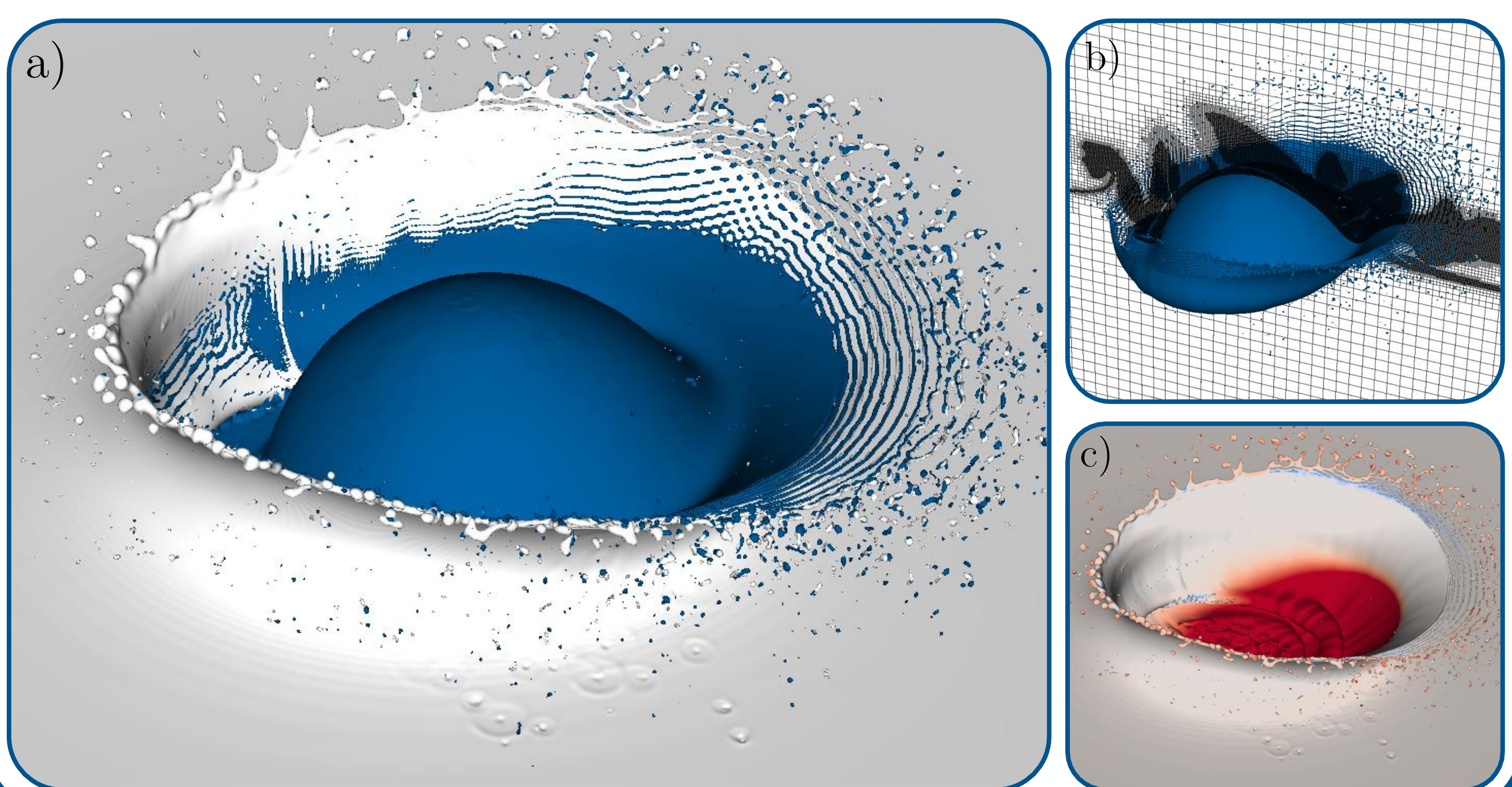
A versatile numerical platform based on the volume-of-fluid method is the most promising tool for this investigation. In recent years the open-source package Gerris[2] has enjoyed tremendous success in joint numerical and experimental work in related fields[3,4].

Our primary aim is to provide an accurate description of the fluid quantity remaining on the solid surface and prone to form a liquid film which may severely alter the global properties of the flow.

Exploring the Flow Anatomy

The fundamental problem of droplet impact onto a solid surface offers useful information regarding water retention and liquid film formation. Once a mass of fluid takes shape however, all subsequent droplets will impact a wet surface (a).

Even when benefiting from adaptive mesh refinement (b) and efficient parallelisation, such computations require the support of HPC structures in order to reach sub-micron scales. The flow can be examined beyond the scope of traditional methods, for example, by isolating the droplet from the full system and studying quantities of interest such as pressure distribution on impact (c).



Splash Dynamics

In typical flight conditions and geometries, the disparity in scales encourages the modelling of a local angle of incidence. Hence, we consider an oblique stagnation point flow[5] as an initial condition in a region sufficiently close to the solid body of interest.

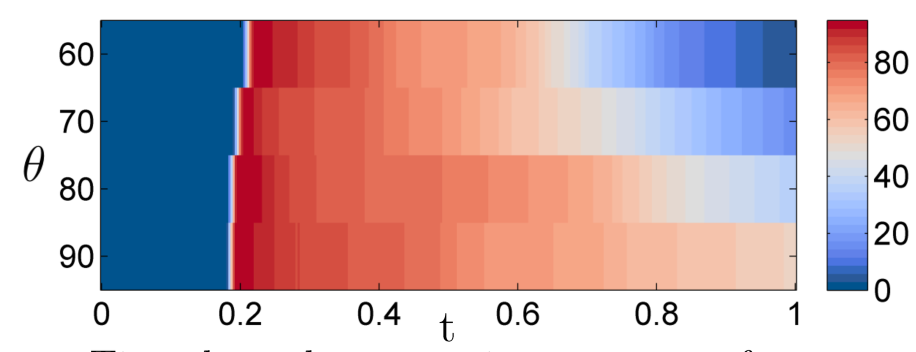
Assuming a similarity solution of type $\psi(x,y) = g(y) + xf(y)$, we obtain

$$f'''(y) + f(y)f''(y) - f'(y)^2 + \sin^2\theta = 0, \quad f(0) = f'(0) = 0, f'(\infty) = \sin\theta,$$

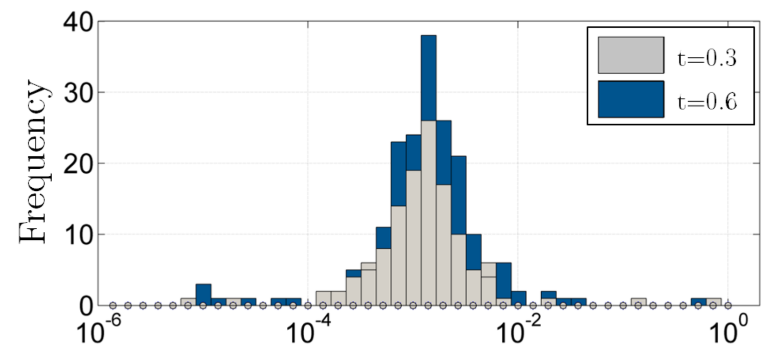
$$g^{iv}(y) + f(y)g'''(y) - f''(y)g'(y) = 0, \quad g(0) = g'(0) = 0, g''(\infty) = \cos\theta,$$

as the ordinary differential equations describing the flow, $0 \le \theta \le \pi/2$.

By seeding the domain with typical droplet configurations, the numerical experiments reveal interesting new features, such as an approximately normal distribution of satellite droplet volumes in the system[6].



Time-dependent retention rate on surface



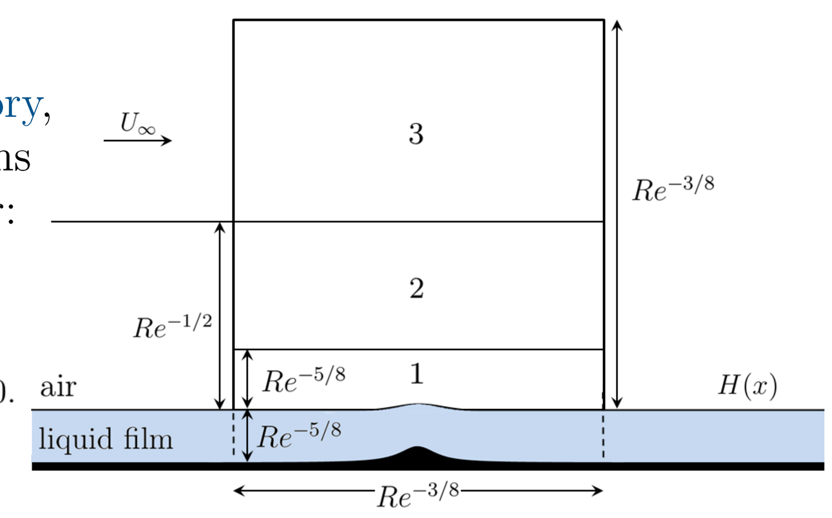
Normalised droplet volume distribution

Multiple Deck Theory

The liquid film interaction with the high speed flow is an ideal candidate for the asymptotic framework of multiple deck theory, resulting in the following governing equations for the water layer and the lower deck in air:

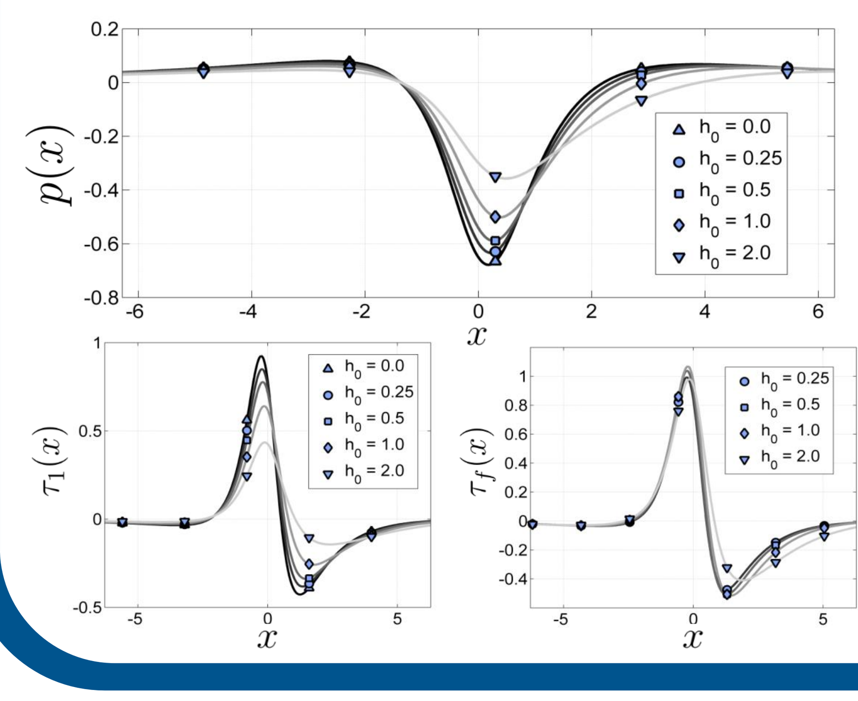
$$\begin{aligned}\frac{r}{m^2}\left(u_l\frac{\partial u_l}{\partial x} + v_l\frac{\partial u_l}{\partial y}\right) &= -\frac{dp}{dx} + \frac{\partial^2 u_l}{\partial y^2}, \quad \frac{\partial u_l}{\partial x} + \frac{\partial v_l}{\partial y} = 0,\\ u_1\frac{\partial u_1}{\partial x} + v_1\frac{\partial u_1}{\partial y} &= -\frac{dp}{dx} + \frac{\partial^2 u_1}{\partial y^2}, \quad \frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0.\end{aligned}$$

accompanied by a set of interfacial conditions.



The sketch above illustrates the flow schematic for the evolution of the liquid film around a surface roughness, modelled by the well known Witch of Agnesi.

By transferring the problem into Fourier space we obtain exact analytical solutions of the linearised flow.



Pressure and skin friction (left) in both liquid and air are the most important flow quantities we are interested in.

Our main objective is a rigorous study of the flow separation in the presence of typical obstacles (such as the case of flaps). The nonlinear viscous-inviscid interaction is presently under construction by extending state-of-the-art algorithms[7] to allow further insight into the impact of the liquid film.

Conclusions

- We characterise aspects of the complex morphology of high speed liquid impact.
- Deformable droplet behaviour and splashing dynamics are studied in detail.
- The effect of the resulting liquid film is described employing analytical tools.
- We obtain exciting results in a context of practical significance in the aeronautics industry.

The interplay between theoretical insight and novel numerical methods provides an ideal balance to target real world applications. We aim to extend the constructed machinery and enhance knowledge in the vibrant area of high speed multi-phase systems.

References

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Flow visualisations

Artwork

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