Overall Outline

- **Lecture I: Observations and planetary flow theory (GFD[⌘])**

➔**Lecture II: Atmospheric LFV[*] & LRF[**]**

- **Lecture III: EBMs[+], paleoclimate & "tipping points"**
- **Lecture IV: The wind-driven ocean circulation**
- **Lecture V: Advanced spectral methods–SSA[±] *et al.***
- **Lecture VI: Nonlinear & stochastic models—RDS[❖]**

(⌘) GFD = Geophysical fluid dynamics
(*) LFV = Low-frequency variability
(**) LRF = Long-range forecasting
(+) EBM = Energy balance model
(±) SSA = Singular-spectrum analysis
(❖) RDS = Random dynamical system

Lecture II: Atmospheric Low-Frequency Variability (LFV) & Long-Range Forecasting (LRF)

Outline

1. Observations of persistent anomalies
   - Blocked & zonal flows
   - Characteristics of persistent anomalies
2. The deterministic chaos paradigm
   - Forced dissipative systems
   - Succesive bifurcations
   - Predictability and prediction
3. "Waves" vs. "particles"
   - Multiple regimes & Markov chains
   - Oscillatory modes & broad spectral peaks
   - Which one is it & how does that help?

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Transitions Between Blocked and Zonal Flows
in a Barotropic Rotating Annulus with Topography

Zonal Flow
13–22 Dec. 1978

Blocked Flow
10–19 Jan. 1963

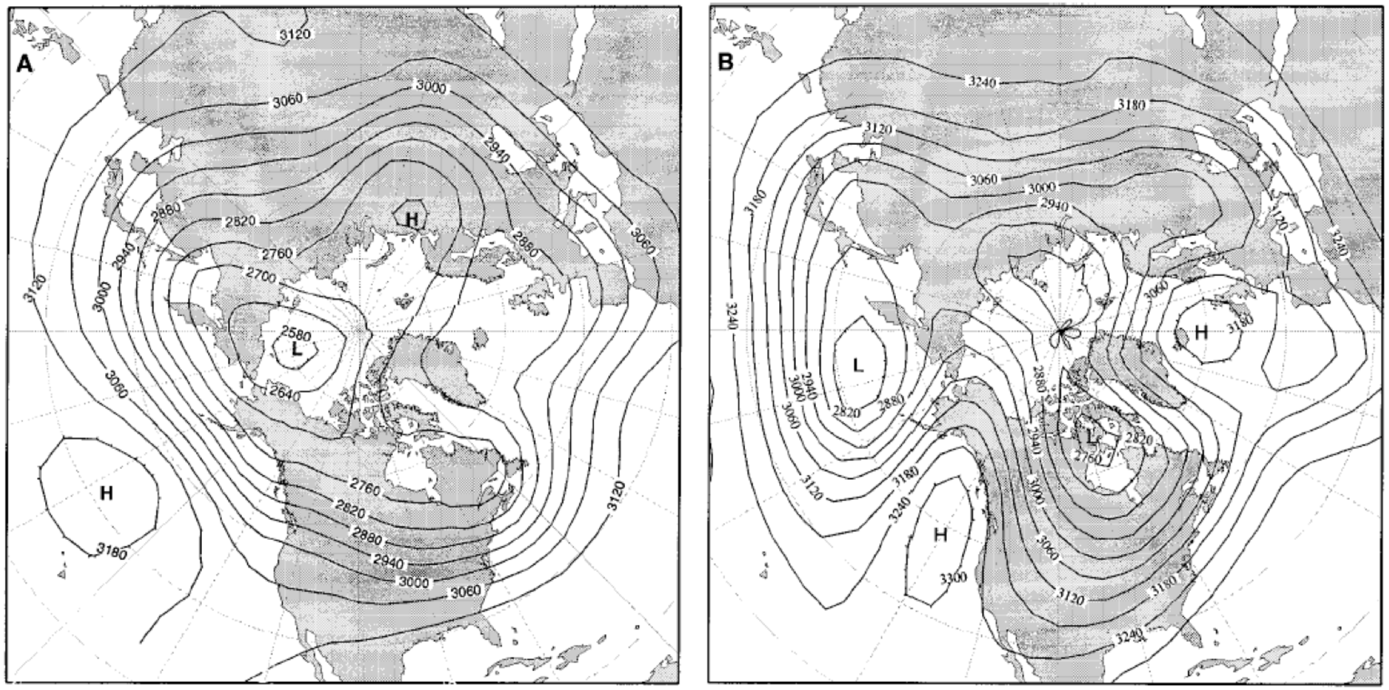

Fig. 1. Atmospheric pictures of (**A**) zonal and (**B**) blocked flow, showing contour plots of the height (m) of the 700-hPa (700 mbar) surface, with a contour interval of 60 m for both panels. The plots were obtained by averaging 10 days of twice-daily data for (A) 13 to 22 December 1978 and (B) 10 to 19 January 1963; the data are from the National Oceanic and Atmospheric Administration's Climate Analysis Center. The nearly zonal flow of (A) includes quasi-stationary, small-amplitude waves (32). Blocked flow advects cold Arctic air southward over eastern North America or Europe, while decreasing precipitation in the continent's western part (26).

Weeks, Tian, Urbach, Ide, Swinney, & Ghil (*Science*, 1997)

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Characteristics of intraseasonal variability (~ atmospheric LFV)

1. **Geographically fixed appearance and regional character** [*]
   ("teleconnections" – Wallace & Gutzler, 1981)
2. **Persistence**
   (*persistent anomalies* – Dole, 1982, 1986; Horel, 1985)
3. **Recurrence**
   (*multiple regimes* – Mo & Ghil, 1987, 1988; Kimoto & Ghil, 1993a,b)
4. **Barotropic structure**
   (barotropic, or 3rd, adjustment; see next page)

[*] **but Branstator (1987) & Kushnir (1987), 25-day hemispheric wave;**
Benzi et al., 1984 +, hemispheric bimodality;
Wallace, Thompson & co. – Arctic Oscillation.

Barotropization
– barotropic (3rd) adjustment(*)

(a) statistical theory of turbulence
(Charney, 1971; Rhines, 1979; Salmon, 1980)

(b) evolution of baroclinic eddies & "wave maker"
(Hoskins & Simmons, 1978; Green-Illari-Shutts)

(c) external Rossby wave, & its instability
(Held-Panetta-Pierrehumbert, 1985–87)

(*)After hydrostatic (1st) and baroclinic (2nd) adjustment.

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Forced dissipative systems

Most fluid dynamical problems — and many other problems
in biology, chemistry, and continuum physics—
lead to ODEs (or equivalent PDEs) of the form

$$\dot{x}_i = a_{ijk} x_j x_k - b_{ij} x_j + c_i \,, \quad i = 1, 2, \ldots, N \,. \qquad \text{(FD)}$$

Here we used the summation convention for repeated indices. In fluid-flow problems, the quadratic terms in (FD) above represent the nonlinear advection term $\vec{u} \cdot \nabla \vec{u}$. This term is associated with the Jacobian in the QG equation.

The above equation is *autonomous* and it has unique solutions for all initial data (ID) $x(0) = x_0$; these solutions depend continuously on the ID, $x = x(t; x_0)$. When the solutions exist for all times, $-\infty < t < \infty$ (*), then Eqs. (FD) define a *differentiable dynamical system* (DDS). In particular, we shall assume that this system is *forced*, $c_i c_i \neq 0$, and *dissipative*, $b_{ij} x_i x_j > 0$.

N.B. The quadratic terms are necessarily *energy conserving* if $a_{ijk} = -a_{ikj}$. and the orbits of (FD) describe a flow in the phase space of $\{x_i,\ i = 1, \ldots, N\}$..

(*) *Counterexample*. The solutions of $\dot{x} = x^2$ are unique and depend continuously on x_0 but they blow up at $t = 1$!

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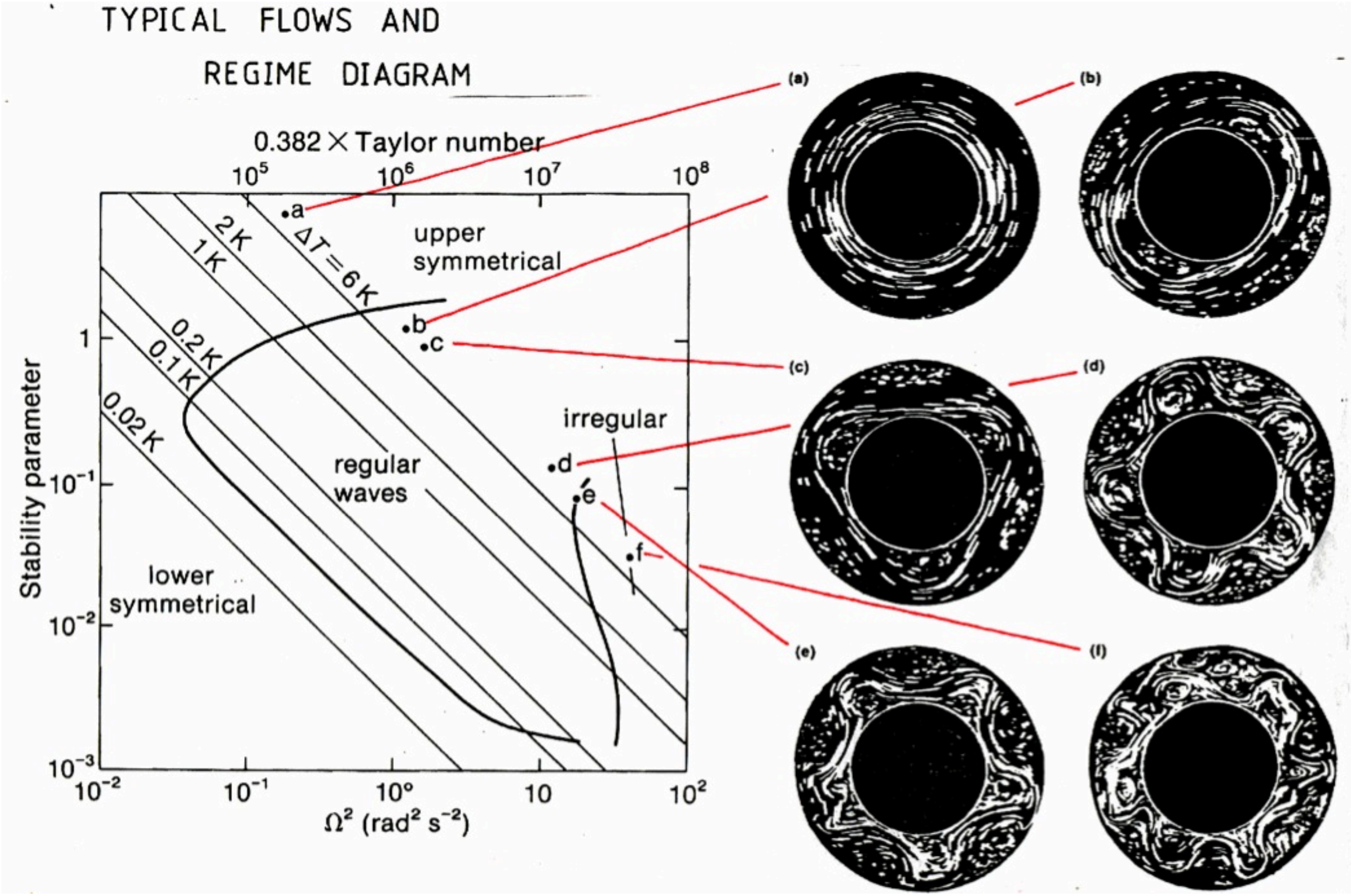
Rotating Convection: An Illustration



M. Ghil, P.L. Read & L.A. Smith (*Astron. Geophys., 2010)*

GFD, bifurcations and chaos

Problem 3: Read the paper listed below and report to the class on its contents.

Ghil, M., P. L. Read and L. A. Smith, 2010: Geophysical flows as dynamical systems: The influence of Hide's experiments, *Astron. Geophys*., **51**(4), 4.28–4.35

General idea

As we push the system harder, it responds by coming up with more complex responses, i.e., it loses symmetry in both time & space. ***In time***, it may go from being in steady state to being periodic and then chaotic; ***in space***, it often goes from being homogeneous to periodic and then to irregular. thus, the two kinds of symmetry loss are interrelated.

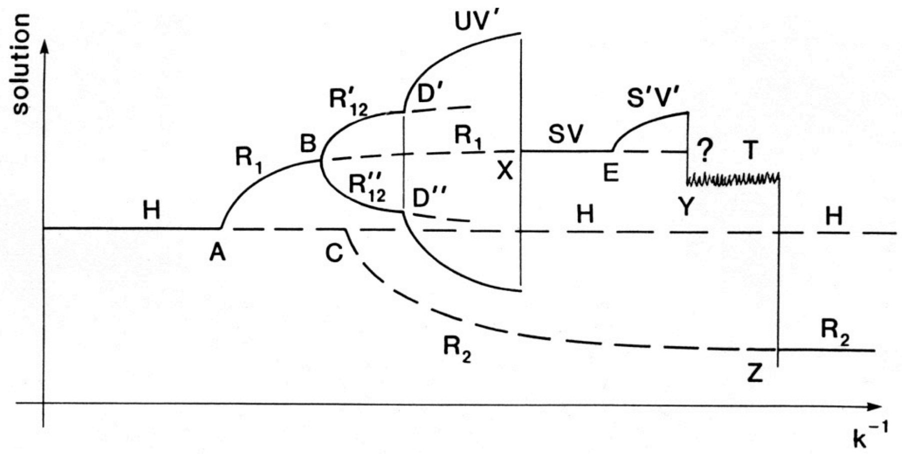
Bifurcation diagram

General situation

$u_t = N(u;\mu)$

$N(u_o;\mu_o) = 0.$

1) If $L_o = N/\partial u$ at $(u_o;\mu_o)$ is nonsingular, then a unique branch of solutions $u = u(\mu)$ through it exists and is given by $u \cong u_o + (\partial u/\partial \mu)\big|_{\mu=\mu_o} u = u_o$.



2) The points at which $det\ L_o = 0$ (i.e., where the Implicit Function Theorem fails) are called bifurcation points, and they are in general isolated. Near such points, the behavior of (2 or more) solutions is parabolic: $u - u_o \sim (\mu - \mu_o)^{1/2}$

Calm in the face of chaos …

But just wait till we bring

in randomness, too!

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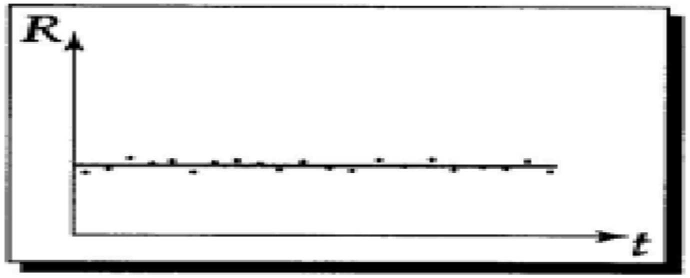
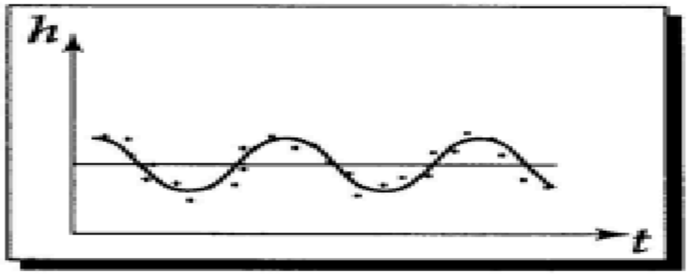
Predictability & prediction

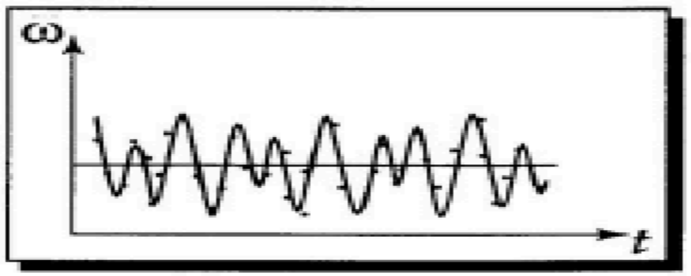
1. Easiest to predict: a constant state, e.g., Earth's radius *R* ➔ one needs only one number.



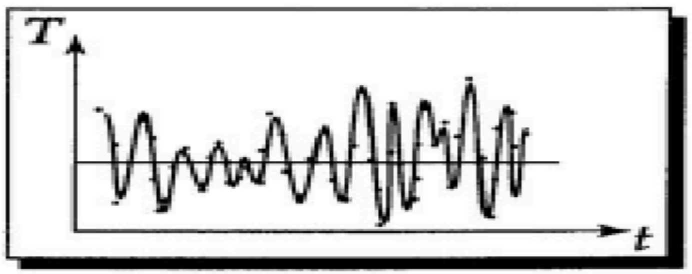
2. A little harder: periodic phenomena, e.g., sunrise, the tides ➔ this requires 3 numbers — the period *p*, the amplitude *A* & the phase ϕ, in this order.



3. Even harder: quasi-periodic phenomena, e.g. the planetary orbits in celestial mechanics ➔ we need 3*n* numbers, where *n* is the numbers of periodic orbits, which may be large but finite.

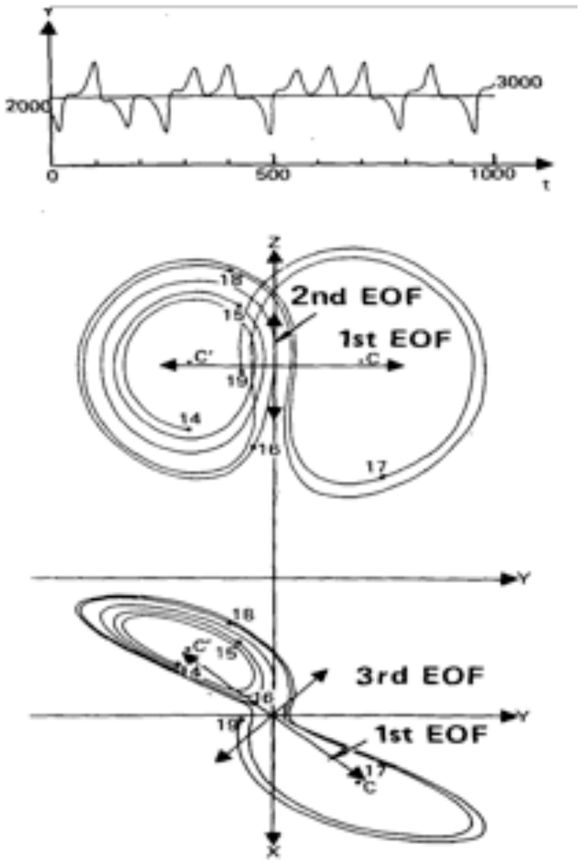


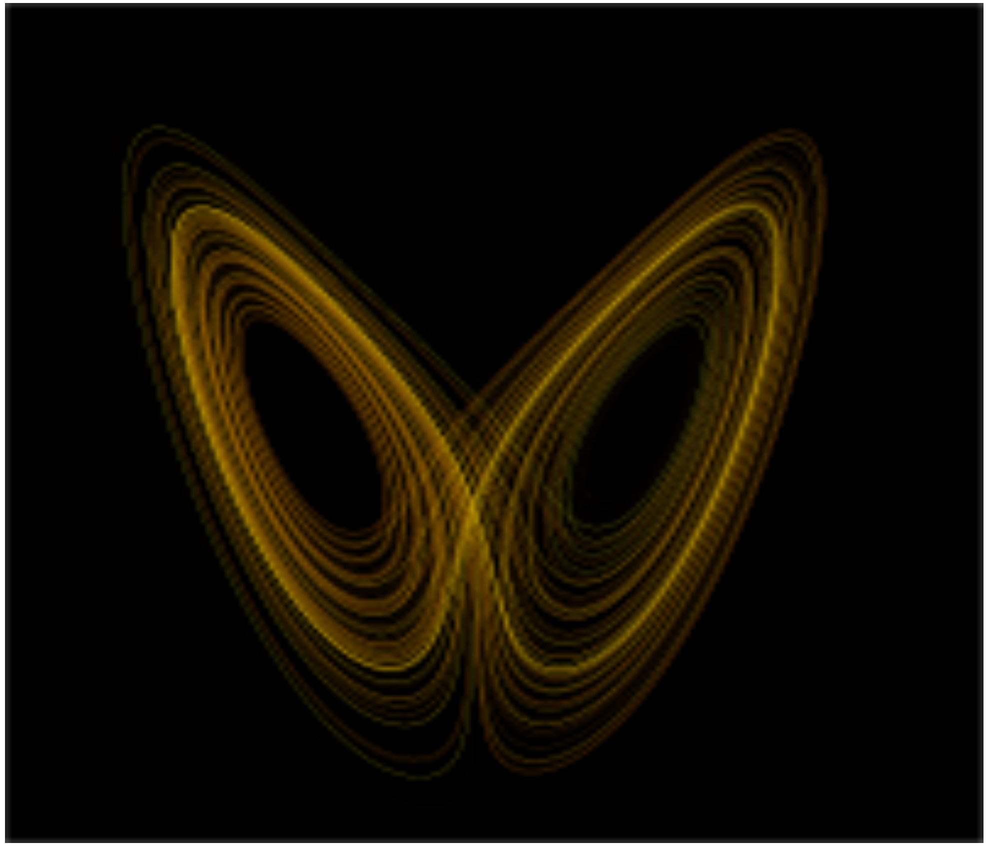
4. And so how about some real stuff, like thermal convection, weather, the markets ➔ one needs an infinity of numbers.



The more complex the phenomenon, the harder it is to predict.

The Lorenz convection (1963a) model
– some numerical solutions





Plot of *Y* = *Y*(*t*) + projections onto the (*X*, *Y*) & (*Y*, *Z*) planes

Trajectory in phase space

Both for the canonical “chaotic” values $\rho = 28$, $\sigma = 10$, $\beta = 8/3$.

The Lorenz (1963a) convection model

Problem 4: Find the appropriate software to compute the statistics
of the Lorenz "butterfly" – e.g., pdf, EOFs – and use it to do so.

Glossary
pdf = probability density function
EOF = empirical orthogonal function

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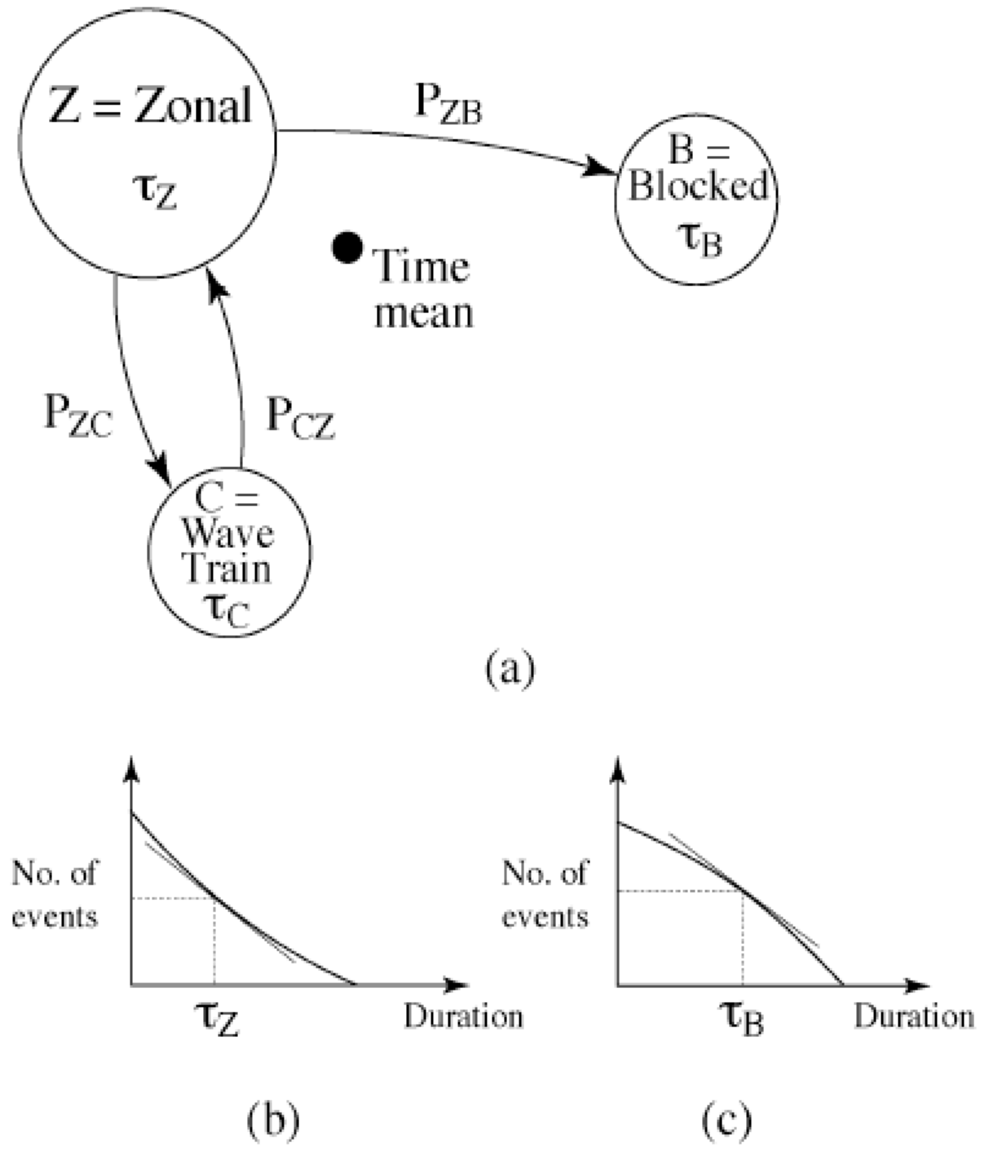
Coarse-graining *Markov-chain description of LFV*

1. Reduce the number of degrees of freedom to the most important ones – highest variance.
2. Describe the dynamics in this reduced subspace.



Multiple Flow Regimes

A. Classification schemes

1) By position

(i) *Cluster analysis*

– categorical

– NH, Mo & Ghil (1988, *JGR*) – fuzzy

– NH + sectorial, Michelangeli et al. (1995, *JAS*) – hard (*K*–means)

– hierarchical

– NH + sectorial, Cheng & Wallace (1993, *JAS*)

(ii) *PDF estimation*

– univariate

– NH, Benzi *et al*. (1986, *QJRMS*); Hansen & Sutera (1995, *JAS*)

– multivariate

– NH, Molteni *et al*. (1990, *QJRMS*); Kimoto & Ghil (1993a, *JAS*)

– NH + sectorial, Kimoto & Ghil (1993b, *JAS*); Smyth *et al*. (1999, *JAS*)

After Ghil & Robertson (2002, *PNAS*)

Multiple Flow Regimes **(continued)**

A. Classification schemes **(continued)**

2) By persistence

(iii) *Pattern correlations*

– NH, Horel (1985, *MWR*)

– SH, Mo & Ghil (1987, *JAS*)

(iv) *Minima of tendencies*

– Models: Legras & Ghil (1985, *JAS*); Mukougawa (1988, JAS);
Vautard & Legras (1988, *JAS*)

– Atlantic- European sector : Vautard (1990, *MWR*)

B. Transition probabilities

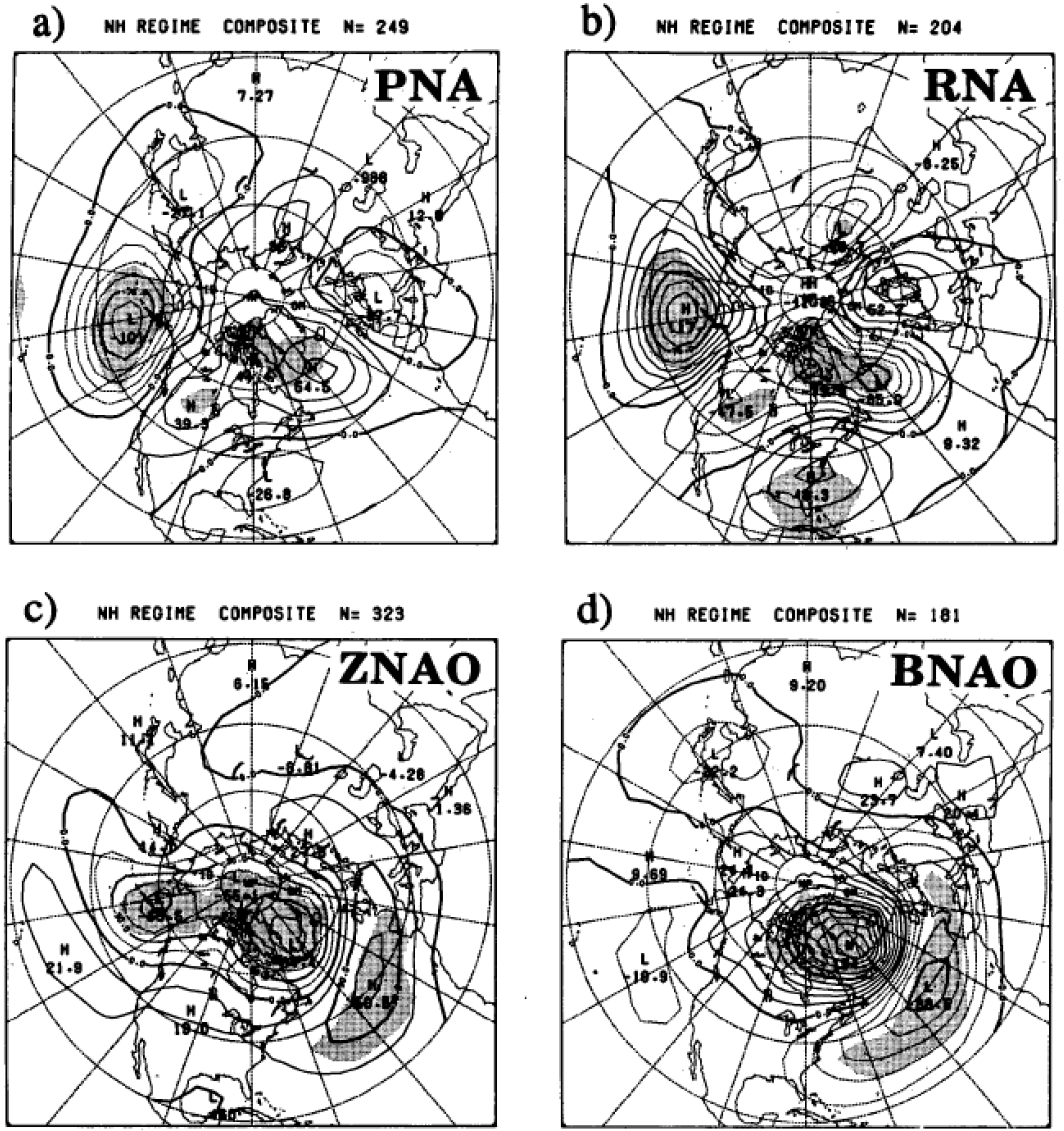
(i) Model & NH – counts (Mo & Ghil, 1988, *JGR*)

(ii) NH & SH – Monte Carlo (Vautard *et al*., 1990, *JAS*)

After Ghil & Robertson (2002, *PNAS*)

Multiple Flow Regimes
– lowest common denominator, I

Four regimes:
blocked vs. zonal,
in the Pacific–North-American (PNA) & the Atlantic-European sector, respectively
(Kimoto & Ghil, *JAS*, 1993a)



Multiple Flow Regimes
– lowest common denominator, II

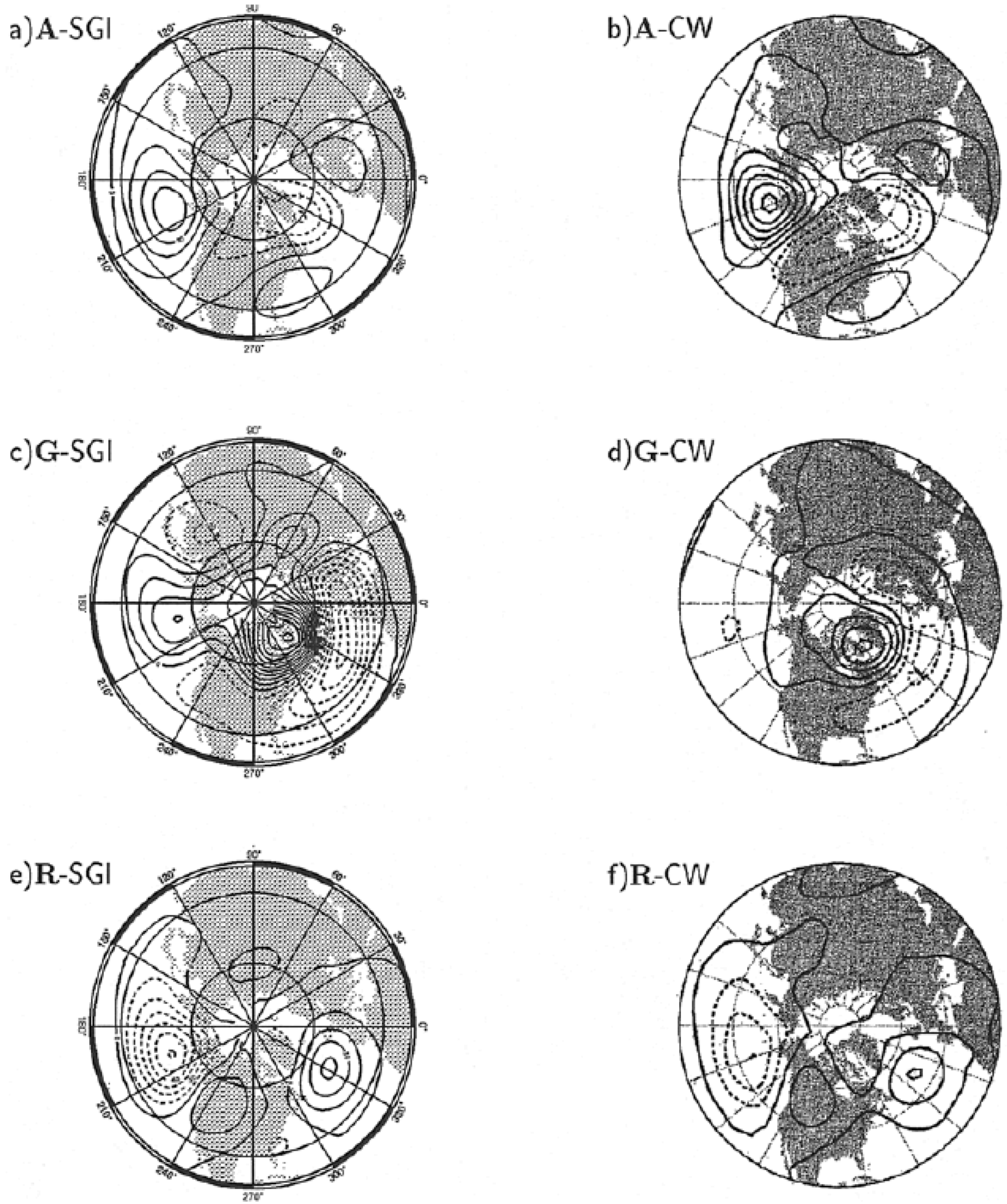
Cheng & Wallace (*JAS*, 1993; **CW**) &. Smyth, Ghil & Ide (*JAS*, 1997; **SGI**) agree well on 3 of the 4 regimes in Kimoto & Ghil (*JAS*, 1993a; **KG**):

A – Gulf of Alaska ridge ~ KG's **RNA**

G – high over Greenland ~ KG's **PNA**

R – enhanced ridge over Rockies ~ **BNAO**

SGI's sectorial analyses also find KG's **ZNAO** to be quite robust.



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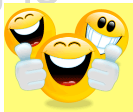
Waves vs. Particles: A Pathway to Prediction?

Is predicting as hard as it is claimed to be?

No, it's actually quite easy: Just flip a coin or roll a die!

What's difficult, though, is trusting the prediction

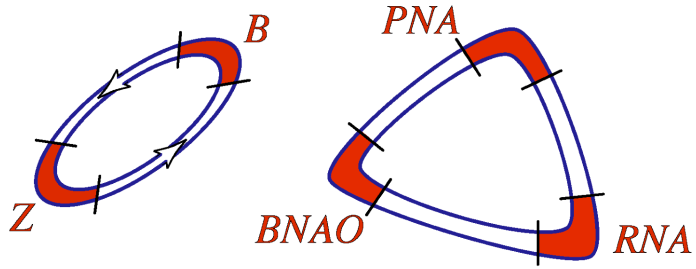


That's where a little understanding of what we're trying to predict helps!

Based on Ghil & Robertson (2002)

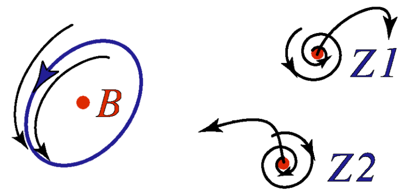
"Waves vs. Particles"
in Atmospheric Low-Frequency Variability

1. Are the regimes but slow phases of the oscillations?



Kimoto & Ghil (*JAS*, 1993a, b)

2. Are the oscillations but instabilities of particular equilibria?

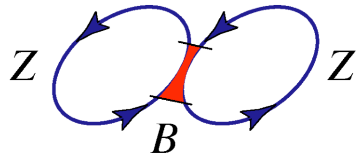


Legras & Ghil (*JAS*, 1985)

3. How about both: "chaotic itinerancy" (Itoh & Kimoto, *JAS*, 1999)

4. How about neither? Null hypotheses:

a) It's all due to interference of linear waves, *e.g.*, neutrally stable Rossby waves;



Lindzen *et al.* (*JAS*, 1982)

b) It's all due to red noise — Hasselmann (*Tellus*, 1976), Mitchell (*Quatern. Res.*, 1976), Penland & co. (Magorian, Sardeshmukh, 1990s).

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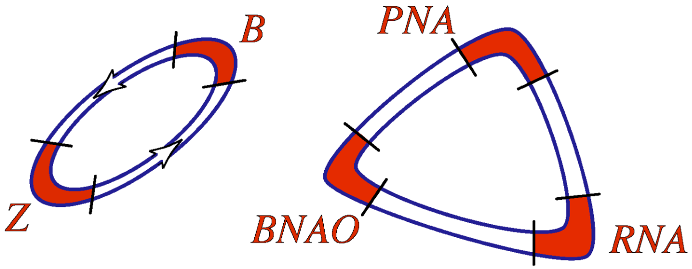

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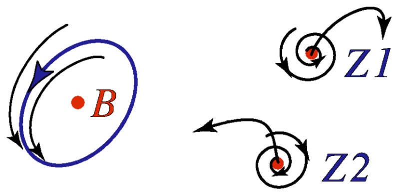

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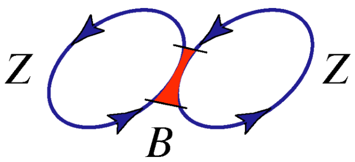

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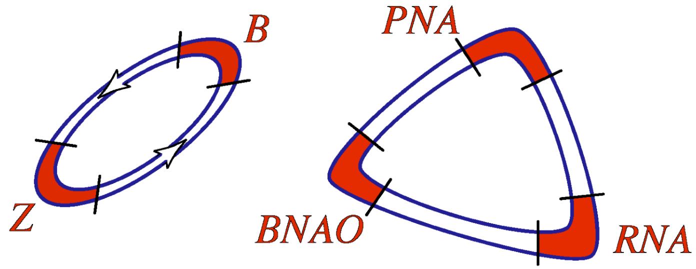

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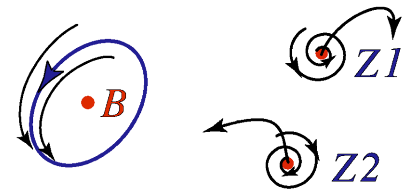

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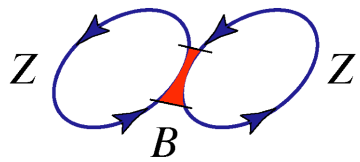

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Reserve slides

Lecture I: Outline

1. General introduction and bibliography
   - Scale dependence of atmospheric & oceanic flows
   - Turbulence & predictability
2. Basic facts of large-scale atmospheric life
   - The atmospheric heat engine
   - Shallowness
   - Rotation
3. Flow regimes, bifurcations & symmetry breaking
   - The rotating, differentially heated annulus
   - Regime diagram & transitions

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