Mathematical Problems in Climate Dynamics, Nelder Fellow Lectures

17 February–3 March 2014
Math Dept., Imperial College

Basic Facts of GFD + Atmospheric LFV, Wind-driven Oceans, Paleoclimate & "Tipping Points"

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Please visit these sites for more info.
http://www.atmos.ucla.edu/tcd/
http://www.environnement.ens.fr/

Overall Outline

- **Lecture I: Observations and planetary flow theory (GFD(⌘))**
- **Lecture II: Atmospheric LFV(*) & LRF(**)**
- **Lecture III: EBMs(+), paleoclimate & "tipping points"**
- **Lecture IV: The wind-driven ocean circulation**
- **Lecture V: Advanced spectral methods–SSA(±) *et al*.**
- **Lecture VI: Nonlinear & stochastic models—RDS(❖)**

(⌘) GFD = Geophysical fluid dynamics
(*) LFV = Low-frequency variability
(**) LRF = Long-range forecasting
(+) EBM = Energy balance model
(±) SSA = Singular-spectrum analysis
(❖) RDS = Random dynamical system

Lecture I: Observations and Basic Planetary Flow Theory

Outline

1. General introduction and motivation
   - Scale dependence of atmospheric & oceanic flows
   - Turbulence & predictability
2. Basic facts of large-scale atmospheric life
   - The atmospheric heat engine
   - Shallowness
   - Rotation
3. Flow regimes, bifurcations & symmetry breaking
   - The rotating, differentially heated annulus
   - Regime diagram & transitions

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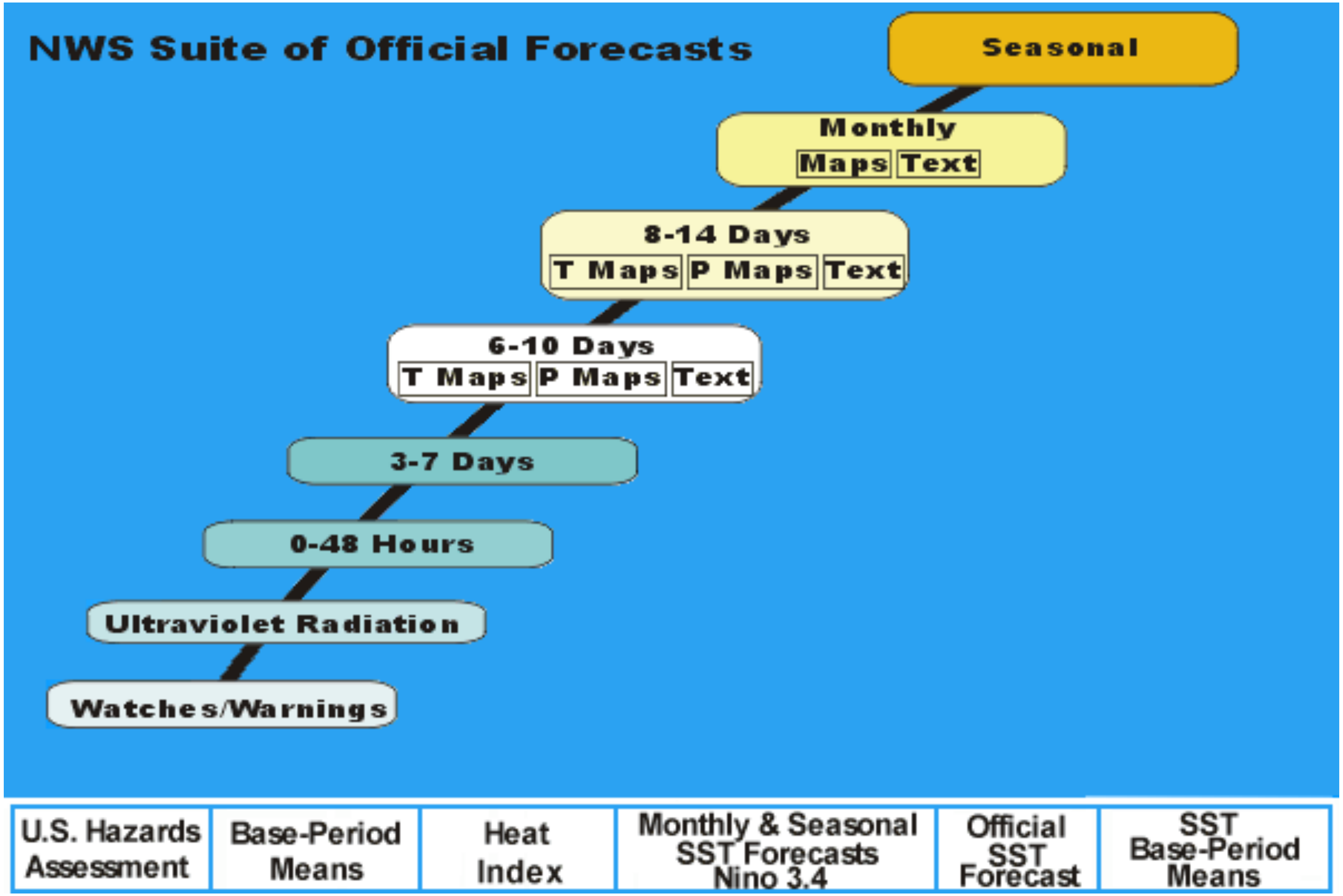
Weather & climate: variability and prediction, I

U.S. National Weather Service (NWS): Forecast suite



Weather & climate: variability and prediction

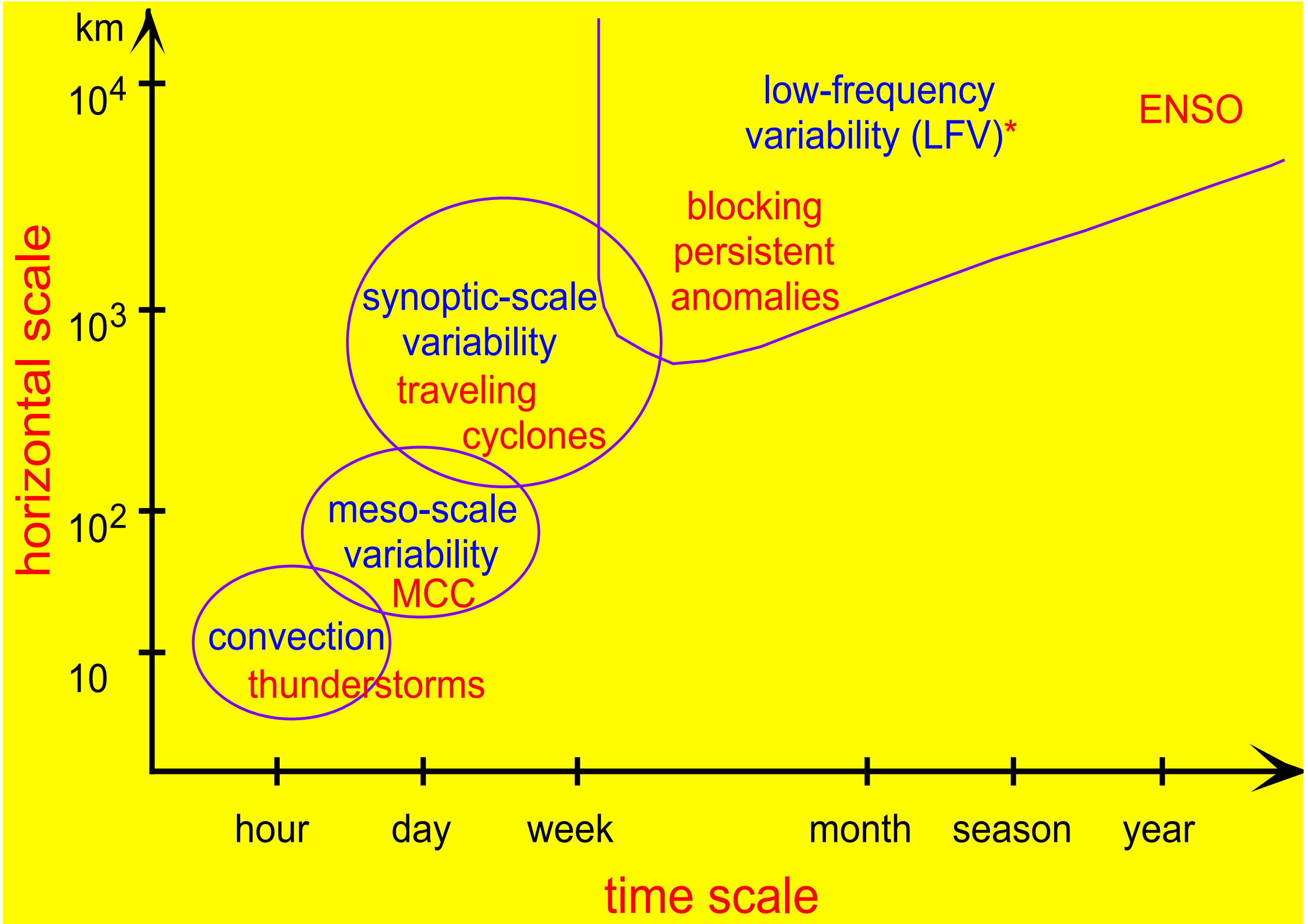
Problem 1: Find the comparable forecast suites on the
web sites of the UK Met Office & the ECMWF

Weather & climate: Observations, II

Space & time scales, $k \sim \omega^{(*)}$

Atmospheric LFV ≈ 10–100 days (intraseasonal)

Oceanic LFV ≈ 3–300 years (interannual–interdecadal)



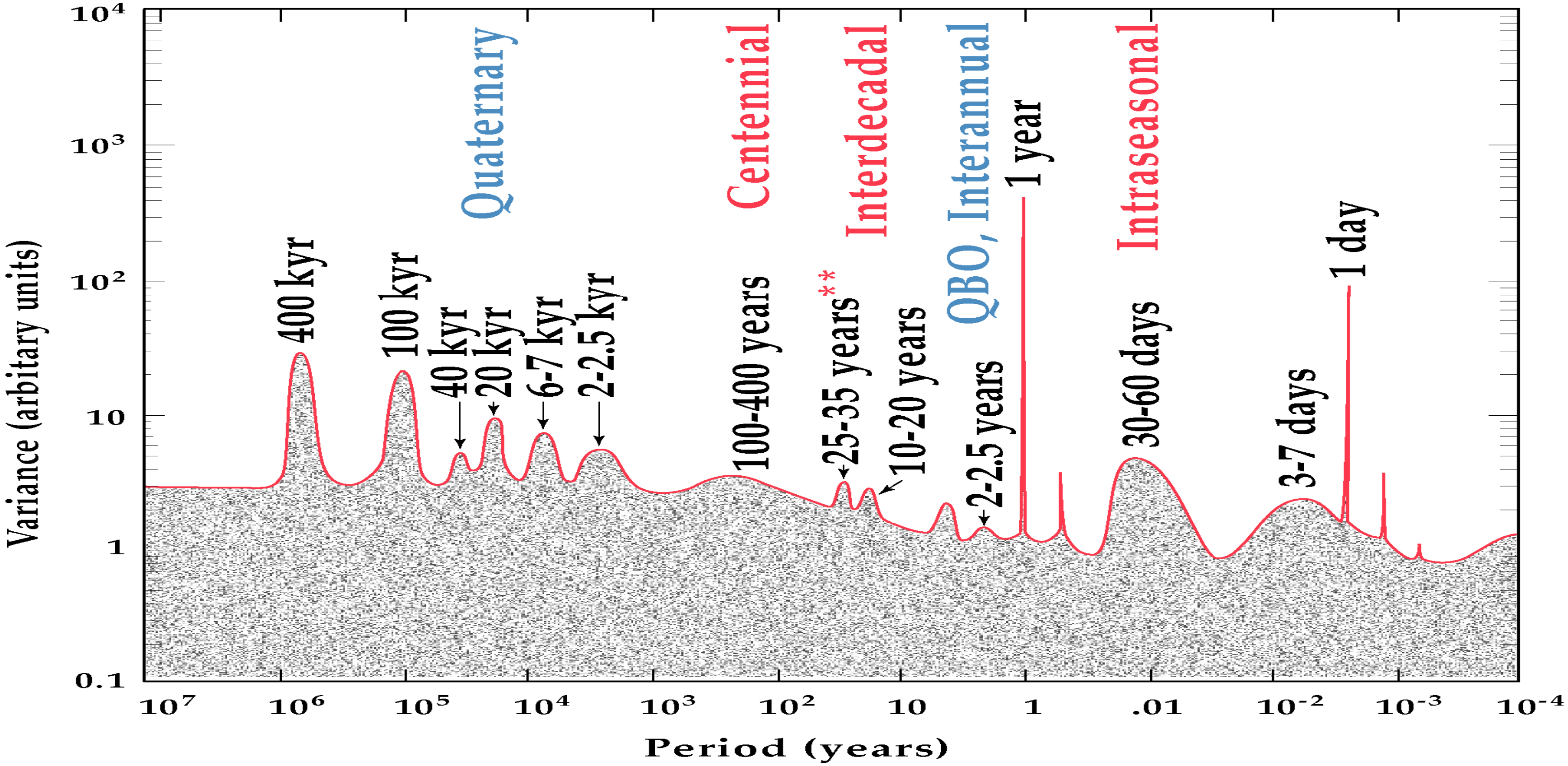
(*) A high-variability ridge lies close to the diagonal of the plot (cf. also Fraedrich & Böttger, *JAS*, 1978)

Composite spectrum of climate variability

Standard treatement of frequency bands:

1. High frequencies – white noise (or "colored")

2. Low frequencies – slow evolution of parameters

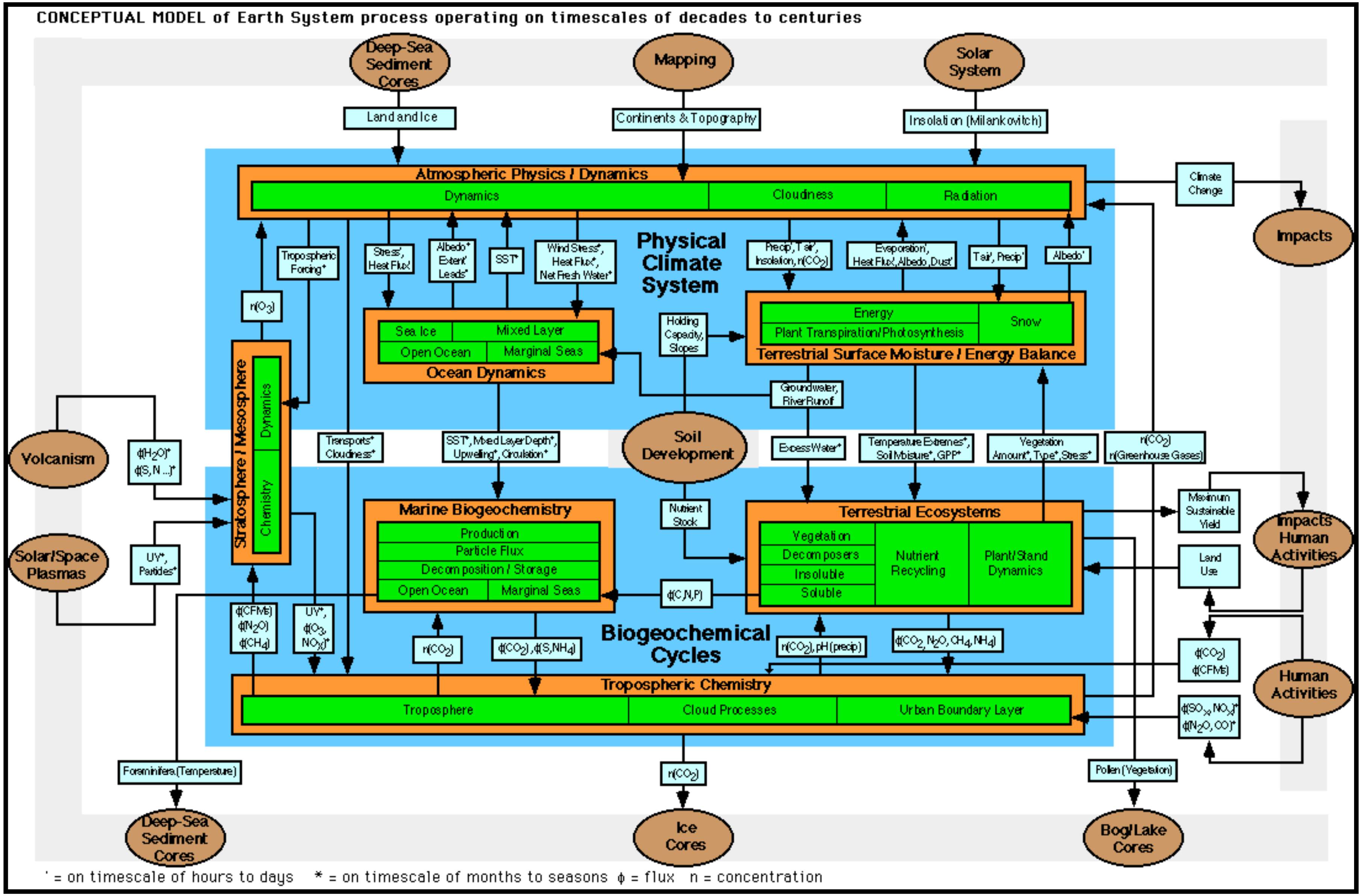


From **Ghil (2001, *EGEC*)**, after **Mitchell* (1976)**

*** "No known source of deterministic internal variability"**

**** 27 years – Brier (1968, *Rev. Geophys.*)**

F. Bretherton's "horrendogram" of Earth System Science

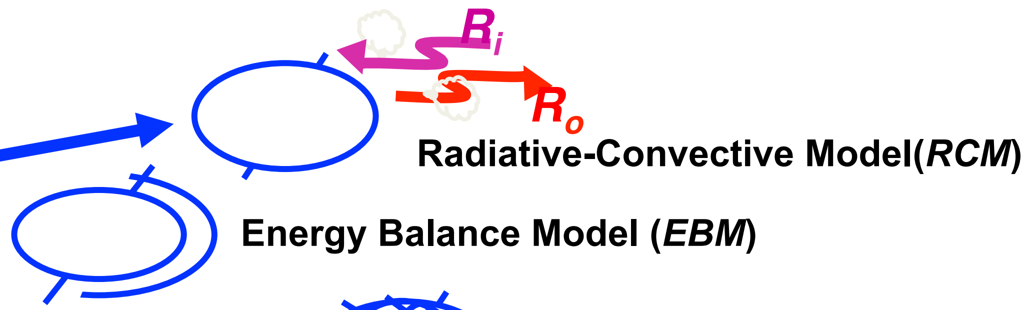


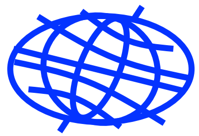
Earth System Science Overview, *NASA Advisory Council*, 1986

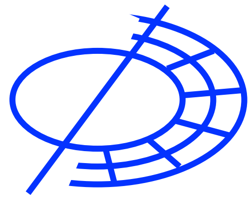
Climate models (atmospheric & coupled) : A classification

- *Temporal*
  - stationary, (quasi-)equilibrium
  - transient, climate variability

- *Space*
  - 0-D (dimension 0)
  - 1-D
    - vertical
    - latitudinal
  - 2-D
    - horizontal
    - meridional plane
  - 3-D, GCMs (Général Circulation Model)
    - horizontal
    - meridional plane
  - Simple and intermediate 2-D & 3-D models







- *Coupling*
  - Partial
    - unidirectional
    - asynchronous, hybrid
  - Full

Hierarchy: from the simplest to the most elaborate,
iterative comparison with the observational data

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ITALIAN PHYSICAL SOCIETY

PROCEEDINGS
OF THE
INTERNATIONAL SCHOOL OF PHYSICS
«ENRICO FERMI»

COURSE LXXXVIII

edited by M. GHIL
Director of the Course
and by R. BENZI and G. PARISI
VARENNA ON LAKE COMO
VILLA MONASTERO
14 - 24 June 1983

Turbulence and Predictability in Geophysical Fluid Dynamics and Climate Dynamics

1985

NORTH-HOLLAND
AMSTERDAM - OXFORD - NEW YORK - TOKYO

The Lorenz model (1963a): a concrete example of a strange attractor[*]

- ***The model equations: 3 coupled, nonlinear ODEs***

$$
\begin{aligned}
\dot{X} &= & -\sigma X & & +\sigma Y & \quad (1)\\
\dot{Y} &= -XZ & +rX & & -Y & \quad (2)\\
\dot{Z} &= XY & -bZ & & & \quad (3)
\end{aligned}
$$

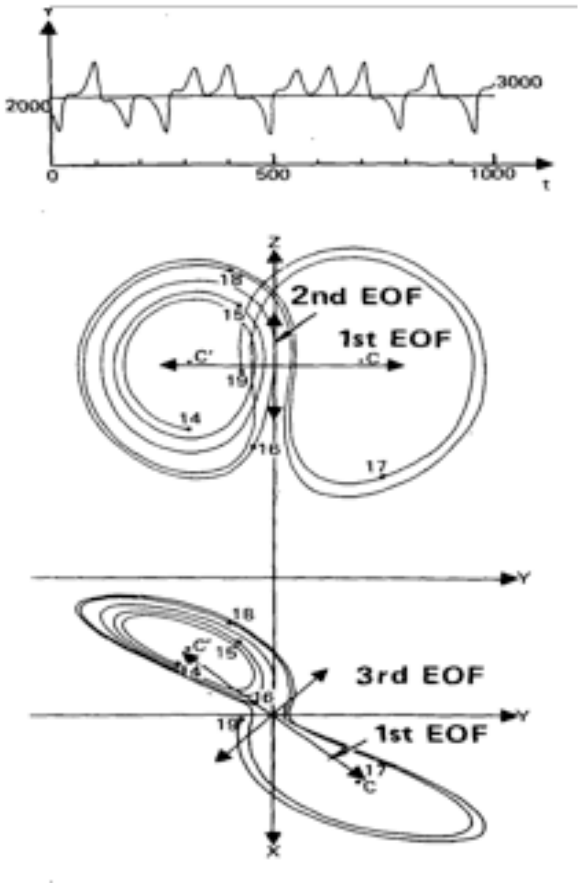
- ***Physics: a model of thermal convection in 2-D***

The variables *X* and *Y* represent the intensity of the velocity field in a 2-D space, *Z* is the deviation of the vertical temperature profile from pure conduction (no motion), and (*X, Y, Z*)$^{\bullet}$ is their rate of change.

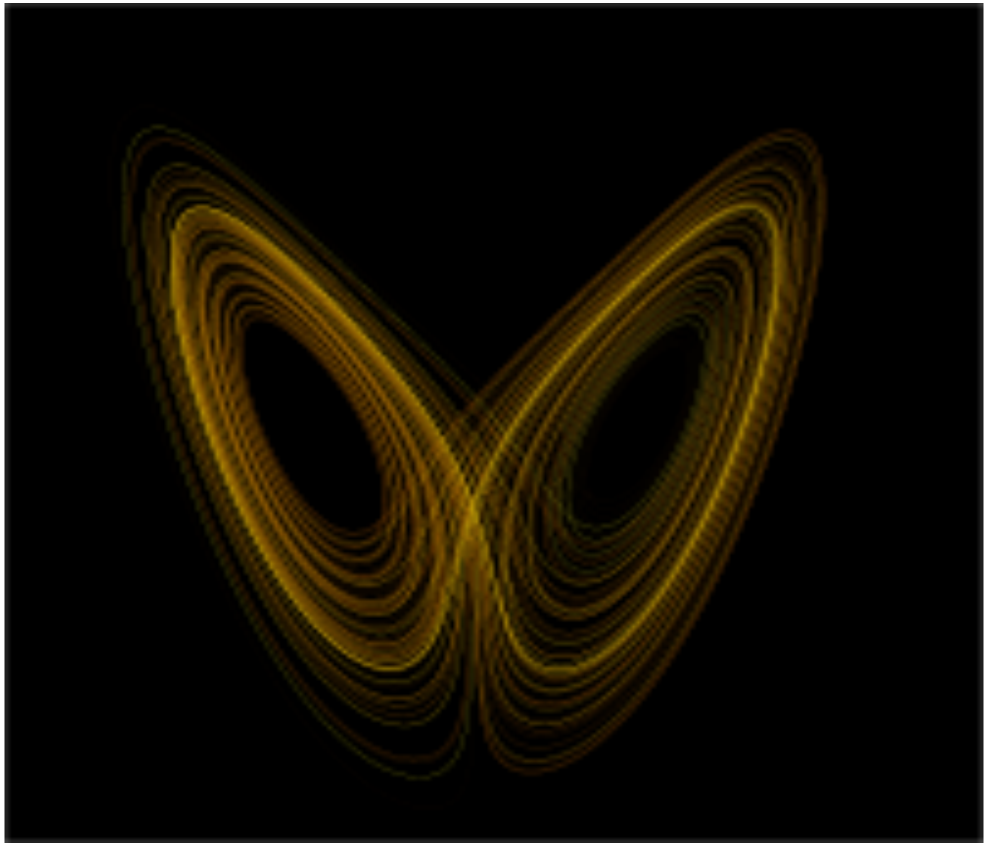
The parameters are the Rayleigh number *ρ* (intensity of the thermal forcing), the Prandtl number σ (the fluid's dissipative properties) and *β* caracterizes the wave length of the perturbation from pure conduction.

[*] Mommy, what's a strange attractor, please?

The Lorenz convection (1963a) model
– some numerical solutions



Plot of $Y = Y(t)$ + projections onto the (X, Y) & (Y, Z) planes



Trajectory in phase space

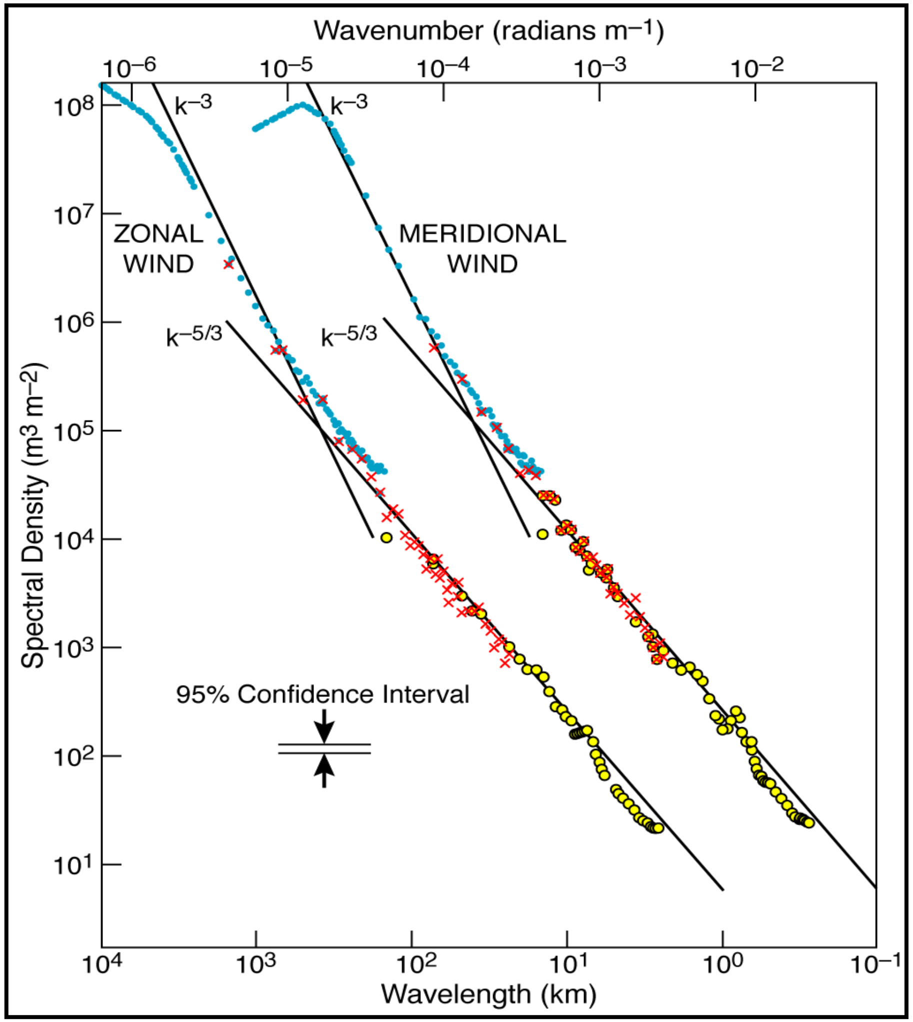
Both for the canonical “chaotic” values $\rho = 28$, $\sigma = 10$, $\beta = 8/3$.

The Lorenz (1963a) convection model

Problem 2: Find the appropriate software to compute the
Lorenz "butterfly" and use it to do so.

But chaos doesn't explain everything: there are many other sources of irregularity!

- Indeed, the atmosphere's & oceans' energy spectrum is "full" – all the time & space scales are active, and contribute to prediction errors.
- Still, one can imagine that the longest, slowest & most energetic modes play a key role.
- "One person's signal is another person's noise."



After Nastrom & Gage (*JAS*, 1985)

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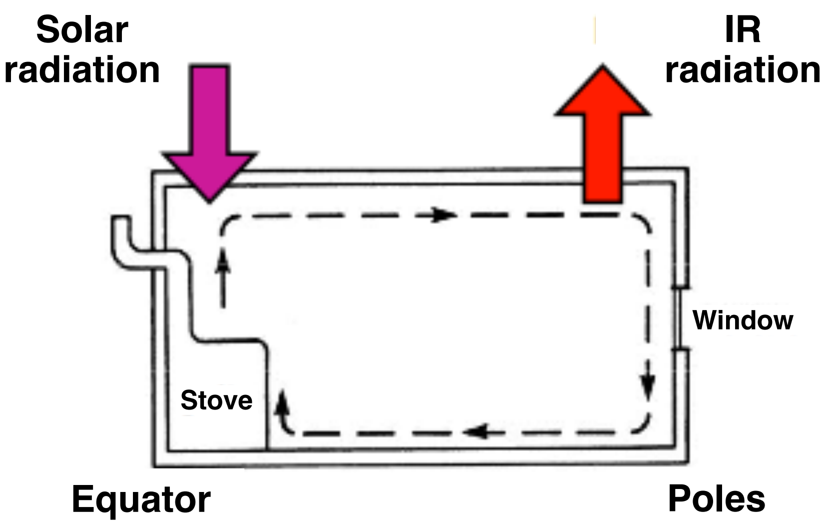
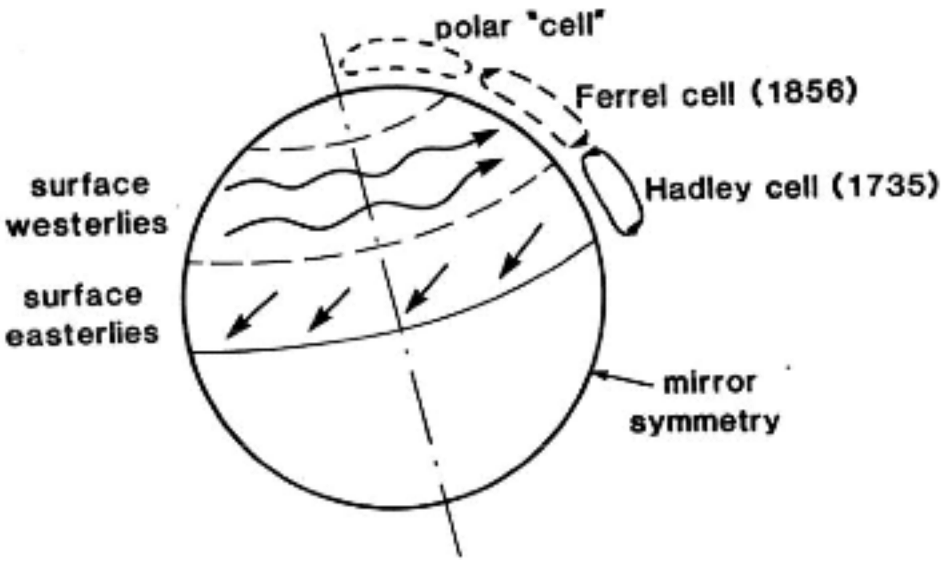
The mean atmospheric circulation

Direct Hadley circulation



Idealized view of the atmosphere's global circulation.*

Observed circulation



Schematic diagram of the atmospheric global circulation.*

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*From **Ghil and Childress (1987), Ch. 4**

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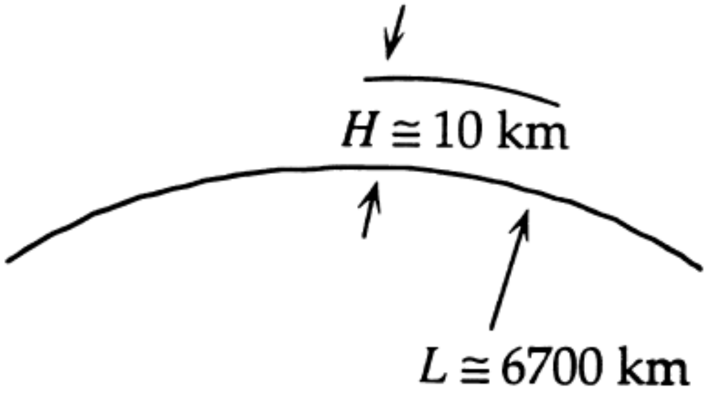
Basic facts of large-scale atmospheric life, or how to read weather maps – I

1. *Shallowness, I*

$$\delta = H/L \ll 1 \qquad \Rightarrow$$



The flow is approximately 2-D (``barotropic") & hence, to a good approximation, it is governed by shallow-water equations (SWE):

$$u_t + uu_x + vu_y = -\phi_x + fv,$$
$$v_t + uv_x + vv_y = -\phi_y - fu,$$
$$\phi_t + (u\phi)_x + (v\phi)_y = 0.$$

Here *h* is the height of the "free surface," which is of order *H*, while $\phi = gh$ is the *geopotential*.

Basic facts of large-scale atmospheric life, or how to read weather maps - II

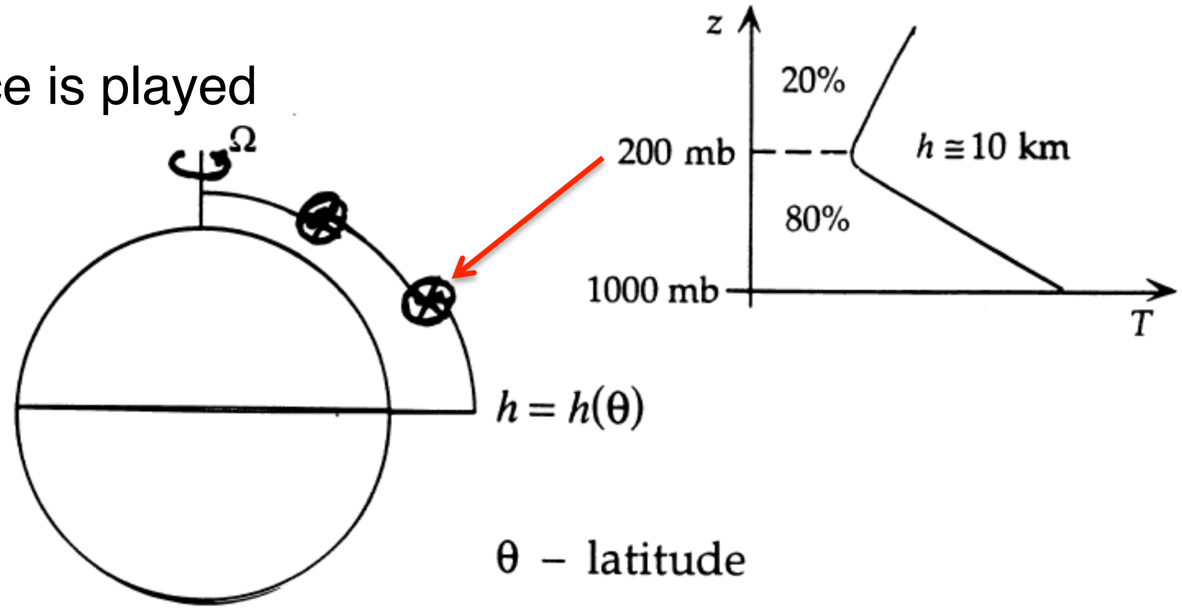
1. *Shallowness, II*

$\delta = H/L \ll 1$ also implies that the flow is approximately

hydrostatic, $p_z = -\rho g < 0\,;$ hence "pressure coordinates":

$$z_p = -\frac{1}{gp} \text{ or } \phi_p = -\frac{RT(p)}{p}.$$

The role of the free surface is played by the *tropopause*.
The atmospheric jets coincide roughly with the "tropopause gaps."



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Basic facts of large-scale atmospheric life, or how to read weather maps – III

2. *Rotation & geostrophy*

$f = 2\Omega \sin\theta$ is the *planetary vorticity*, or the *Coriolis parameter*.

The Rossby number $\epsilon = U/fL$ measures the importance of rotation:

It's important if ε is small: $\epsilon \ll 1.$

In *geostrophic* f*low*, $\epsilon \to 0$ and thus the SWE are reduced to

$$u = -(1/f)\phi_y, \quad v = (1/f)\phi_x.$$

The flow is parallel to isobaric contours, rather than perpendicular, and thus

$\psi = (g/f)h$ is a stream function.

In the quasi-geostrophic approximation, $0 < \epsilon \ll 1$

allows for small, slow deviations from exact geostrophy.

Basic facts of large-scale atmospheric life – IV

3. *Rotation + shallowness* ➔ The quasi-geostrophic, equivalent-barotropic potential vorticity equation with topography

$$(\Delta - \lambda^{-2})\eta_t + J(\eta, \Delta - \lambda^{-2}\eta + h_0) = 0\,;$$

here Δ is the *Laplacian*, is the *Jacobian*,

$$J(\eta, Q) = \frac{\partial\eta}{\partial x}\frac{\partial Q}{\partial y} - \frac{\partial\eta}{\partial y}\frac{\partial Q}{\partial x} = (u, v)\cdot\nabla Q$$

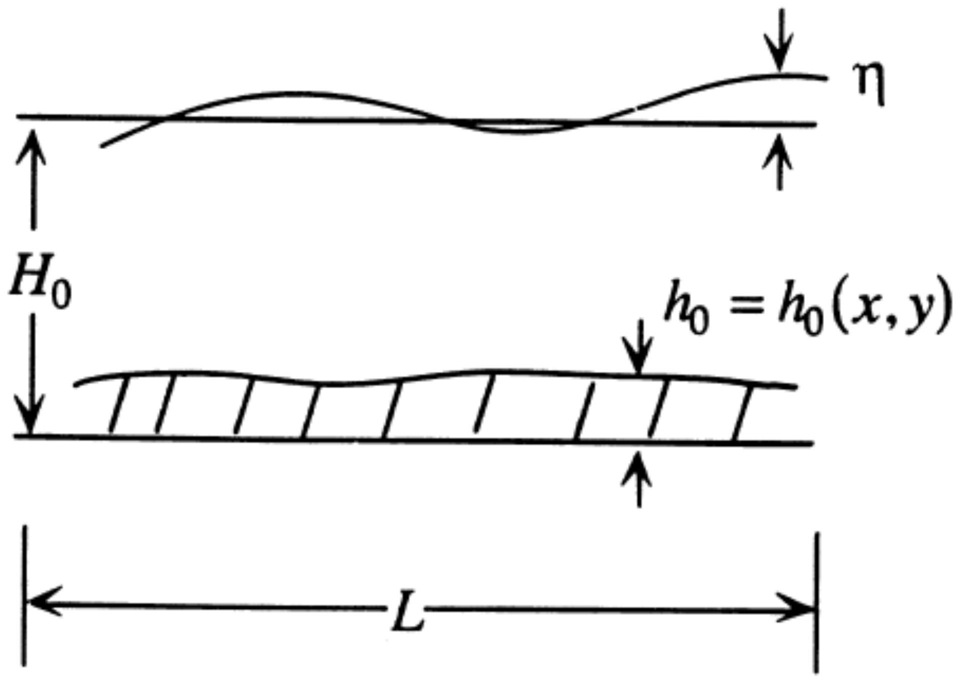
$$h = H_0(1 + \epsilon\lambda^{-2}\eta), \quad h_0 = H_0\epsilon h_0^*.$$

The *potential vorticity* Q equals

$$Q = (\Delta - \lambda^{-2})\eta + h_0,$$

and the *Rossby deformation radius* L_R plays a key role in it,

$$\lambda = L/L_R, \quad L_R = (gH_0)^{1/2}/f_0.$$



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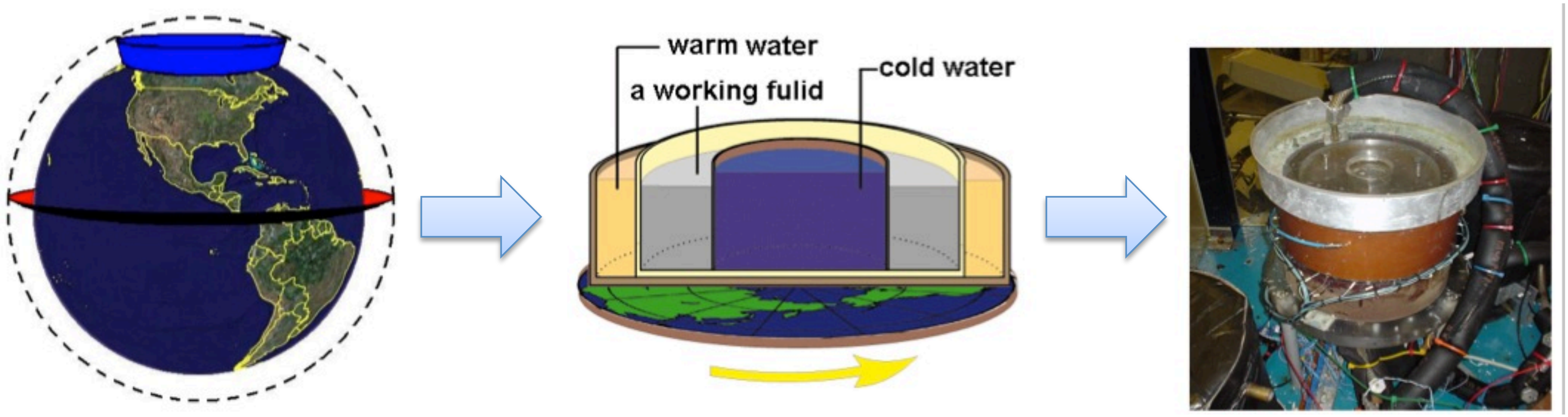
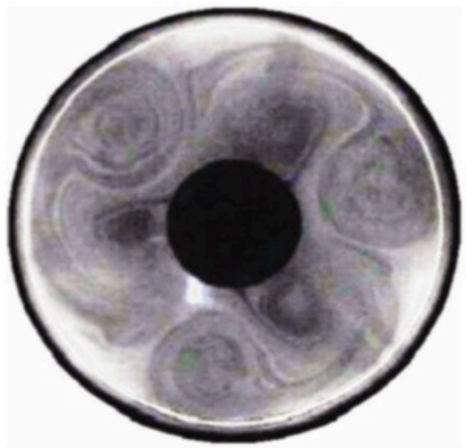
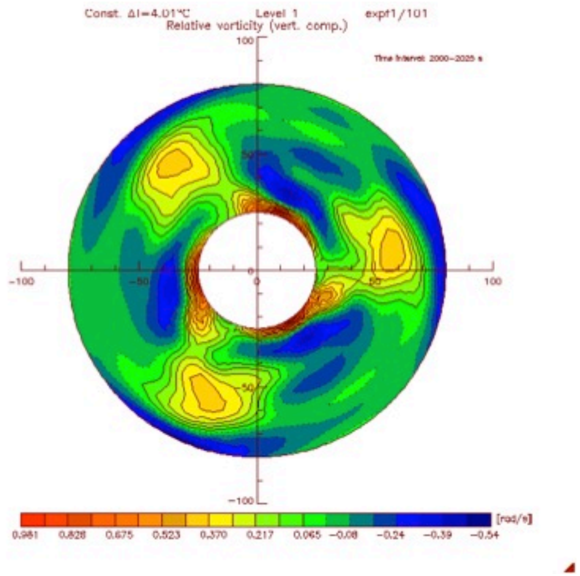
Laboratory Analogues of Planetary Atmospheric Circulation Systems



- *Baroclinic instability*:

- A potential energy releasing instability in the atmosphere and the oceans





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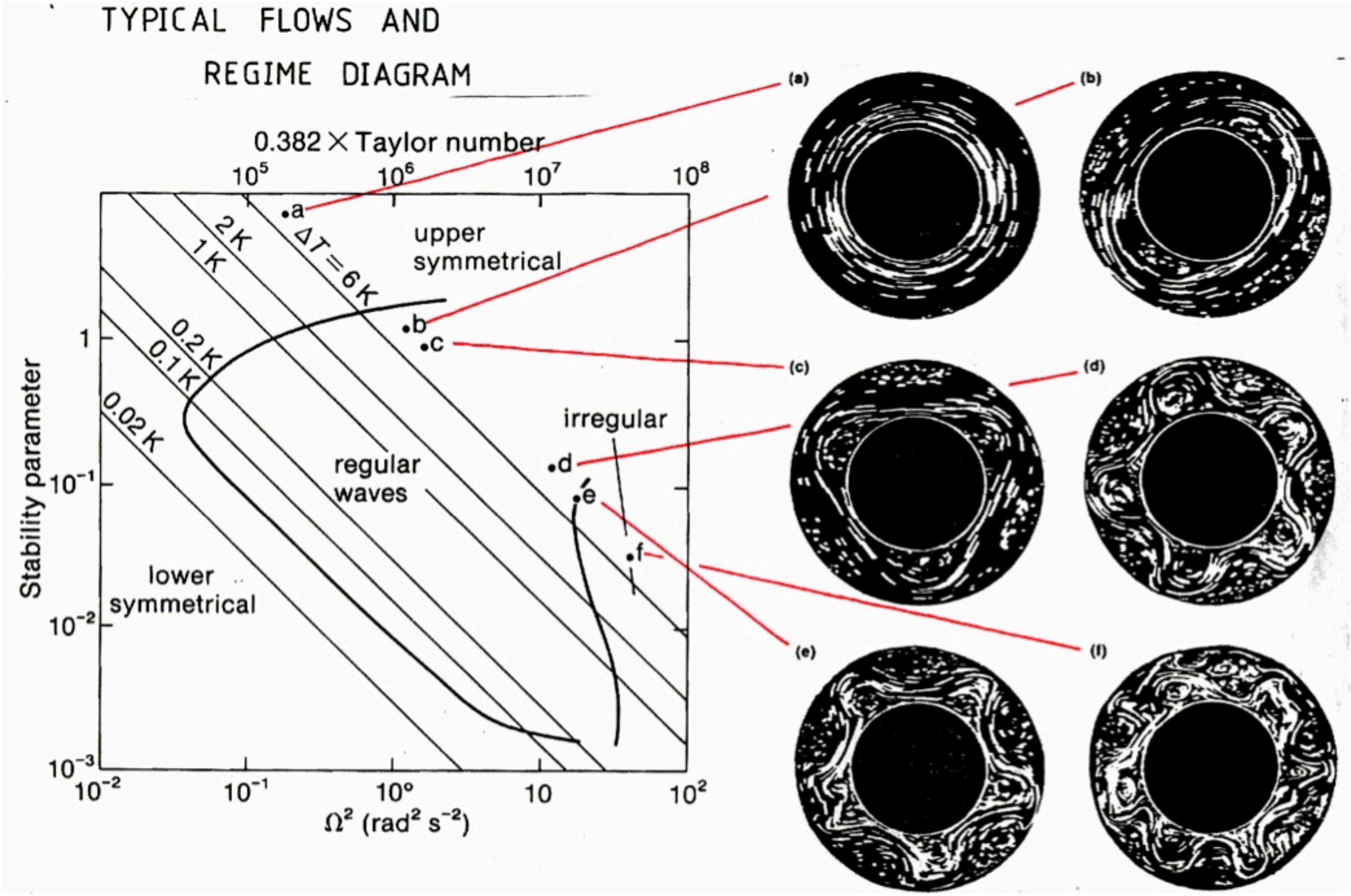
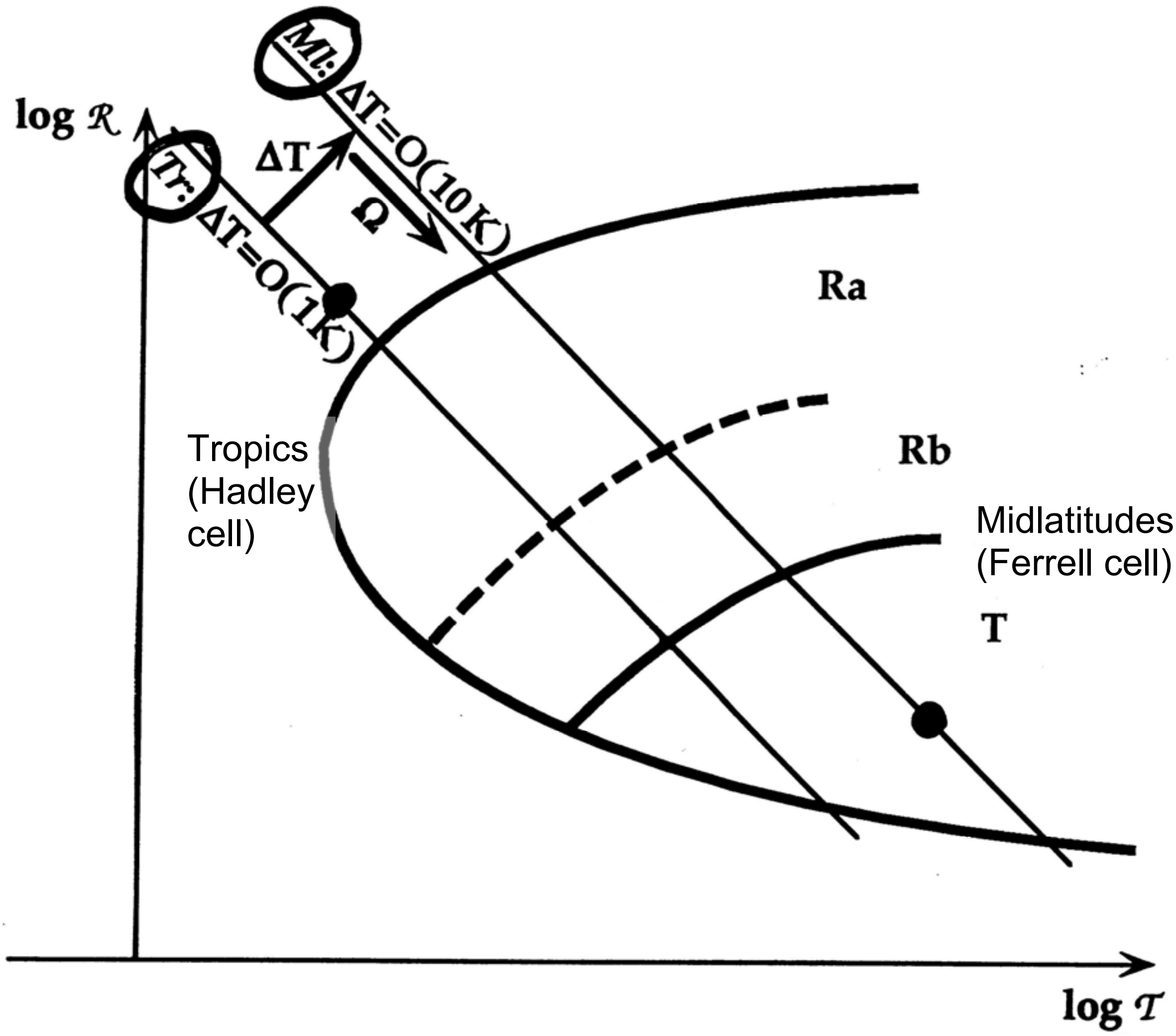
Rotating Convection: An Illustration

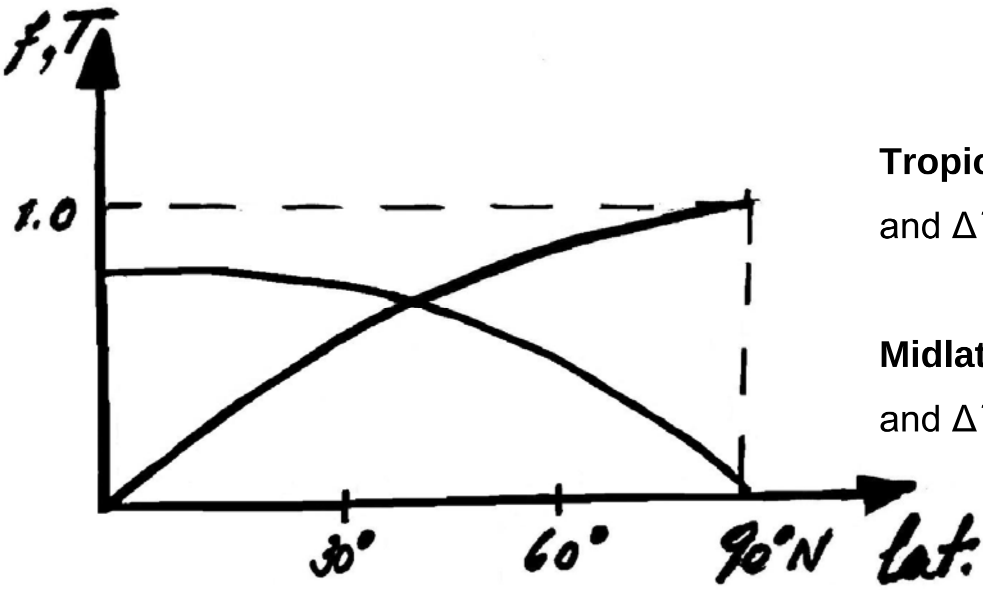


M. Ghil, P.L. Read & L.A. Smith (*Astron. Geophys., 2010)*

Rotating annulus & Earth's atmosphere



Or why doesn't the Hadley cell on Earth extend to the poles, like on Venus ?



Tropics : both f (i.e., Ω) and ΔT small

Midlatitudes : both Ω and ΔT large

Some general references

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