Intelligent Portfolio Construction: Machine-Learning enabled Mean-Variance Optimization

by

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Declaration

The work contained in this thesis is my own work unless otherwise stated.

Signature and date:

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List of Mathematical Symbobls

H : X $\rightarrow$ Y	Mapping Rule between the Input and Output spaces (respectively X and Y)
$\hat{H}$: X $\rightarrow$ Y	Approximated mapping rule constructed by the learning algorithm between the Input and the Output spaces
τ	Training Set
τ_N	Set of training samples at node N
x_i	Explanatory variable (scalar or vector)
y_i	Class corresponding to x_i
$\mathcal{F}$	Set of Features, derived from the explanatory variables (In our case, the features will be technical indicators)
$\mathcal{C}$	Split Criterion
Z	Number of trees composing the random forest
$P_N(k)$	Proportion of observations belonging to the class k at node N
α	Exponential smoothing factor
m	Prediction time horizon
R_i	Return random variable for the i^{th} asset
r_i	Observation of R_i
$\overline{r_p}$	Expected return of a portfolio composed of two assets
σ_p	Volatility of a portfolio composed of two assets
$\overline{r_{p,n}}$	Expected return of a portfolio composed of n assets
$\sigma_{p,n}$	Volatility of a portfolio composed of n assets
$\boldsymbol{\mu}$	Vector in R^n of expected returns of n assets
$\boldsymbol{\Sigma}$	Covariance Matrix R^{n}xR^n
$\boldsymbol{w}$	Vector in R^n of weights allocated to n assets
$P = (P_i)_{i=1}^n$	Closing Price process
$\hat{P} = (\hat{P}_i)_{i=1}^n$	Smoothed Closing Price process
$\hat{X} = (\hat{X}_t)_{t\in\mathbb{N}}$	Inputs to our Random Forest algorithm.

Contents

1 Introduction

1.1 Motivations and Report Structure

Last decades, a considerable progress has been made in the financial mathematics field. Many subjects, such as stochastic modeling, PDEs resolutions, exotic derivatives pricing and trends predictions have been of great interest both within the academics and practitioners. The use of the increasingly powerful computational abilities helped addressing those issues in a new way, and develop new algorithms to trade, model and predict in an almost-automatized way. Precisely, many researches are currently conducted to assess the results of the use of Machine Learning in Quantitative Finance - as they are in many other fields.

Following Alpadyn in [2] (2004) , we define Machine Learning as "programming the computers to optimize a performance criterion using [training] data or past experience". It can particularly be used to optimize the construction of an investment portfolio, which is defined as an ensemble of investments in different assets aiming at earning returns in the future. Investment strategies have known a considerable progress as well, especially since the Modern Portfolio Theory, pioneered by Harry Markowitz in his paper "Portfolio Selection" (1952) [23]. In a nutshell, this theory addresses mathematically the process of selecting the investment instruments and assigning to each a part of the initial wealth. Quantitative investment strategies have also the advantages of not being impacted by the human emotions and bias given different market situations, which Keynes sees as " animal spirits— [...] spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities" in his General Theory ([17], VII).

As highlighted by Markowitz in ([23] p77), the process of selecting a portfolio is composed of two stages: the first to analyze the historical data and build an idea on the behavior of the assets in the future, and the second one uses these insights to build the portfolio. Both Machine Learning and Investment Strategies are of great interest in financial markets today. Our work attempts to combine both subjects by using machine learning to predict the stock direction in the first phase of the portfolio construction. We aim at comparing the performances of a portfolio constructed with the classic structure with one derived from a machine-learning enabled version.

Our work aims at being both theoretical and practical, this appears in the structure of this report. We first start in Section 2 by introducing the chosen Machine Learning algorithm and building its theoretical framework. In the same section, we build the investment universe: the set of assets which will be used to build our portfolios. We choose to work only on US Large

Cap[1] for generalization purposes. The data downloaded is preprocessed to be used as input to the Random Forest algorithm. In section 3 we model the volatility of the returns. The fitted model is then used to forecast the change in stock levels over the investment time horizon. Results of Sections 2 and 3 are combined to generate views on the future behavior of the stocks composing our universe. This corresponds to the first stage of portfolio construction. Finally, we build several investment strategies in Section 4 serving our goal to assess the impact of using Machine Learning on Quantitative Investment Strategies.

1.2 Forecasting of the Stock market

Our attempt to predict stock direction raises the question of the possibility of beating the market: this refers to the "Efficient Market Theory" or "The Random Walk Theory". This theory is summarized by Eugene F. Fama [13] by "the statement that security prices fully reflect all available information." Assuming this, fundamental and historical analysis shouldn't enable investors to predict the future behavior and obtain higher rate of returns.

Although, as Jensen (1978) [16] puts it, "no other proposition in economics has more solid empirical evidence supporting it than the Efficient Market Hypothesis.", researches conducted since the end of the XX^e century suggested partial predictability in the stock market. For example, Andrew W. Lo and A. Craig MacKinlay (1987) ([19]) strongly reject the hypothesis that Weekly Stock Market returns follow a Random Walk using a specification test. Researches conducted by Fama and French (1988) on equal-weighted portfolios of the NYSE provided statistical evidences on the ability of Dividend to price ratios to explain more than 25% of long-term returns.

Our goal to forecast stock market level direction is based on technical analysis: that is the use of statistical studies of trading data to forecast prices. This is addressed by Brock, Lakonishok and LeBaron (1992) in [29], where they compared buy-and-hold strategy to technical analysis based strategies on Dow Jones Index from 1897 and 1986. They provided evidence supporting the use of technical analysis to predict stock prices. The same conclusion is supported by the work of Vasilious, Eriotis and Papathanasiou (2008) on the Greek stock market, with an excess return of 13% annually in favor of prediction-based strategies. This incite us to look further into the technical-analysis based strategies. In our work, a machine learning algorithm is used to translate the technical indicators to buy or sell signals.

1.3 Literature Review

The theoretical framework for Machine Learning algorithms was mainly studied on [14]. After considering many algorithms and comparing their efficiency in our specific context, we chose Random Forests algorithms. G. Biau and Scornet (2015) [5] offers a theoretical introduction to Random

[1]Large Cap refers to companies with market capitalization higher than $5 Billions

Forest. This was completed by a practical approach in G. Louppe paper (2014) [20]. The prediction of stocks direction in Section 2.3 tries to replicate the numerical results of Khaldem et al. (2016) in [4] on our selected portfolio. We extend their work by forecasting the volatility for a more precise input to our investment engine.

The analysis of the data has been applied for portfolio construction following the methodology given in [26] by E. Qian et al. (2012). In addition to those main resources, many other papers have been used during our theoretical and practical work and will be cited specifically through the report.

2 Prediction of stock direction

We aim first at forecasting expected returns for a set of stocks. This is done in two steps : we first use a Supervised Learning algorithm to forecast the direction of the stock, we then forecast the amplitude of the move using a Garch(1,1) model to capture the volatility of the returns. We focus in this section on the theoretical framework of Random Forests algorithm and its application to predict the direction of the stock market.

2.1 Supervised Learning and Decision Trees

Supervised Learning refers to the idea of learning from examples. We provide the algorithm with two sets of data, a training set and a test set. The first set is used to build a rule mapping the inputs to the outputs. This is then assessed by testing the accuracy of this rule when applied to unlabeled inputs from the test set.

-The rule to be constructed is the best approximation $\hat{H}$ of the function H mapping X to Y, respectively the set of inputs (explanatory variables) and the set of outputs.

-The training set consists of pairs of vectors $\mathcal{T}$=(x_1,y_1),(x_2,y_2)...(x_n,y_n) where the x_i are vectors or scalars and are interpreted as the predictors or explanatory variables of the outputs y_i.

-The test set is a set (x_{n+1},x_{n+2},...x_{n+k}) of k indicators (vectors or scalars) to be labeled by the trained program.

Classification is a an example of Supervised Learning Algorithms. We give a formal definition of Classifier algorithms following ([5], 2.3 ,Page 9).

Definition 2.1. A Classifier, or classification rule $\hat{H}$ is a Borel measurable function of the feature space and $\mathcal{T}$ that attempts to estimate the label Y from an input X .

A commonly used example for Supervised Learning algorithms is Decision Tree. The idea behind Decision trees is to partition the explanatory variables space into rectangles and assign each resulting rectangle to a class. We give a formal definition following ([11], 1.4, p3).

Definition 2.2. Decision Tree (or Classification Tree) is a ”classifier expressed as a recursive partition of the instance space”. The tree has three types of nodes:
-A **root node** is one that has no incoming edges
-An **internal node** is one that has one incoming edge and at least two outgoing edges
-A **leaf node** is one which has no outgoing edges and one incoming edge.

Let $\mathcal{T}$ be the training data set and $\mathcal{F}$ a set of features. We mean by features functions of explanatory variables. In our case, the features can be for example technical trading indicators computed from the closing prices time series. Assuming that the p explanatory variables span a p-dimensional space, a decision tree divide the initial space as follows ([5], 2.2, p 202) :

Algorithm 1: *BuildTree*

Inputs:

- $\mathcal{T}$ training set of p explanatory variables with the corresponding classes
- Set of features $\mathcal{F}$
- Split Criterion $\mathcal{C}$

Output: Classification Tree

Initialization: Create node I ;

if *All the predictors correspond to the same class* ***or*** $\mathcal{T}$ *is Empty* **then**

  I is a leaf node

  return I

else

  Select feature f_i that best classifies $\mathcal{T}$;

  Select threshold a_i that best splits f_i using $\mathcal{C}$;

  $\mathcal{T}_1 \leftarrow \mathcal{T}$ **where** $f_i < a_i$;

  $\mathcal{T}_2 \leftarrow \mathcal{T}$ **where** $f_i > a_i$;

  Add BuildTree($\mathcal{T}_1$,$\mathcal{F}$, $\mathcal{C}$);

  Add BuildTree($\mathcal{T}_2$,$\mathcal{F}$, $\mathcal{C}$);

end

The choice of the threshold a_i at each split follows an optimization problem. Before introducing some of the measures which can be used to optimize the split at each node following ([14], 9.23, p308), we define the proportion of observations per class.

Definition 2.3. We define the proportion of observations belonging to class k at node N by:

$$P_N(k) = \frac{1}{card(\mathcal{T}_N)} \sum_{x_i \in \mathcal{T}_N} \mathbb{1}_{\{y_i = k\}}, \tag{2.1}$$

where $\mathbb{1}$ is the indicator function.

Definition 2.4. **Potential impurity measures to optimize the split of the feature space at node N:**

► Gini Impurity Measure:

$$G(N) = \sum_{i \neq j} P_N(i) P_N(j).$$

In the case of two classes, by symmetry in the above sum and using the fact that the sum of the proportions is equal to 1, this becomes $G(N) = 2p(1-p)$, p being the proportion of one of the two classes at node N.

► Shannon Entropy:

$$H(N) = -\sum_{k} P_N(k) log(P_N(k)).$$

In the two-classes case with proportions p and 1-p, this becomes $-plog(p) - (1-p)log(1-p)$.

▶ Misclassification Error:

$$M(N) = \frac{1}{card(\mathcal{T}_N)} \sum_{x_i \in \mathcal{T_N}} \mathbb{1}_{\{y_i \neq k\}} = 1 - P_N(k).$$

In the two-classes case, this becomes 1-max(p,1-p).

The three impurity measures can be used as target functions to optimize the split when building the Decision Tree. We plot the three measures for p $\in$ [0,1].

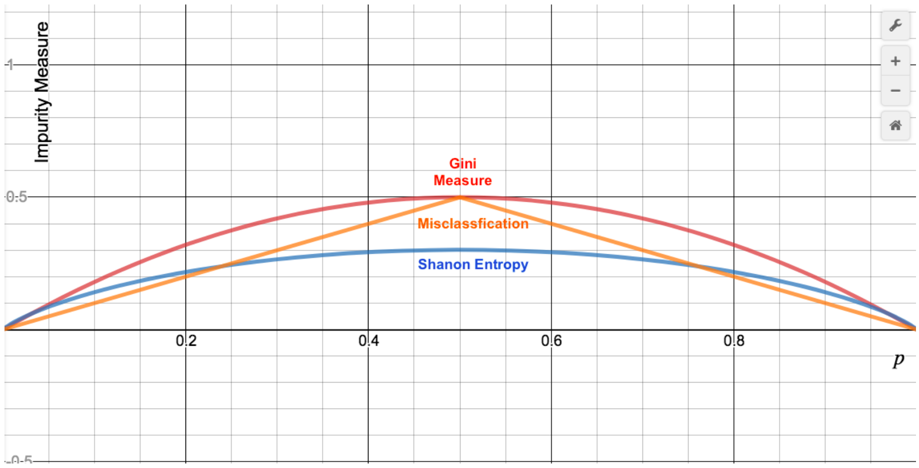


Figure 1: Plot of Impurity Measures

From the plot above, we can compare the sensitivity of the three measures with respect to little variations in p. Gini and Entropy measures are more sensitive and hence better for optimization problems. We choose Gini impurity measure in our study.

Remark 2.5. Considering $P_N(k)$ as the probability of an input of class k in Node N to be misclassified, the expected misclassification error is $\sum_{i \neq j} P_N(i)(P_N(j))$, corresponding to Gini Impurity Measure. In the two-classes case, considering a random variable equal to 1 for the right classification and 0 for a misclassification, p(1-p) can be interpreted as the variance of the right classification. This can be generalized to the k-classes case ([14], 9.23, p308).

We give below an example of few nodes extracted from the tree generated in the application of classification trees to stocks data at the end of this section.

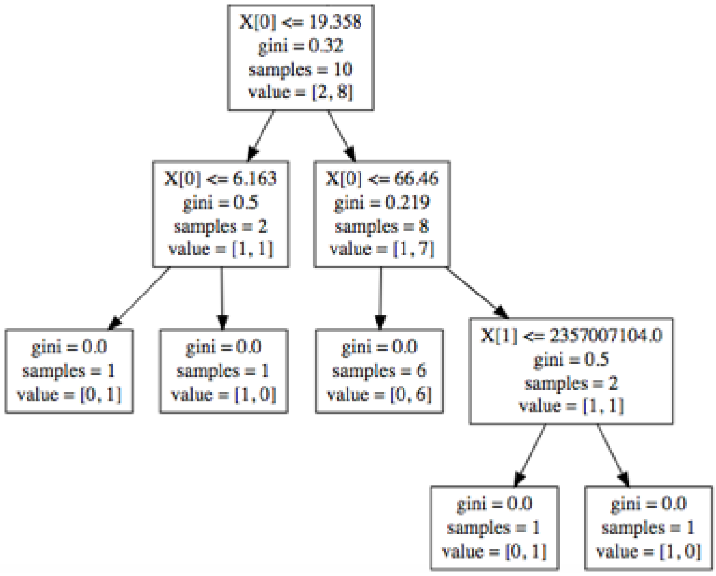


Figure 2: Tree Example using Gini Impurity. The first line of each box corresponds to the chosen condition on a feature to split the node and the split-threshold, samples gives the number of elements considered at the node and value gives the number of samples corresponding to each class, our tree being built here in the binary case.

2.2 Random Forest

The thresholds used at each node are derived from an optimization problem. Different threshold lead to different trees and different accuracy precisions as discussed in (See [14], 9.12 , p312). This high sensitivity to data may cause over-fitting and inaccuracy when applied with new sets of data. A way around this problem is the use of Random forest.

A Random Forest is as a set of N identical decision trees; the classification is done on a vote among the decision trees.

Algorithm 2: *BuildForest*

Inputs:

- $\mathcal{T}$ set of n explanatory variables with the corresponding classes
- Set of features $\mathcal{F}$
- Split Criterion $\mathcal{C}$
- Number of trees Z

Output: Forest composed of Z trees

Initialisation: TreeSet ($EmptySet$);

For i in range$(0, Z)$;

Draw a sample $\mathcal{X}$ from $\mathcal{T}$

T=BuildTree($\mathcal{X}$,$\mathcal{F}$, $\mathcal{C}$)

Add T to TreeSet

return TreeSet

End

We introduce a formal definition of the Random Forest from the algorithm given above:

Definition 2.6. A random forest is a classifier based on a set of Z decision trees ($\hat{H}_1(\mathcal{T}|\mathcal{X}_1)$, $\hat{H}_2(\mathcal{T}|\mathcal{X}_2)$,...$\hat{H}_Z(\mathcal{T}|\mathcal{X}_Z)$) where $\mathcal{X}_1, \mathcal{X}_2, ..., \mathcal{X}_Z$ are independent and identically distributed random subsets of $\mathcal{T}$ drawn before each tree is generated.

In the following, $\hat{H}_i(\mathcal{T}|\mathcal{X}_i)$ will be designed by $\hat{H}_i$ for simplicity purpose, keeping in mind the dependency to the subset randomly drawn for each tree.

Remark 2.7. In the case of two classes labeled +1 and -1, the Random Forest decision can be written:

$$\hat{H}_{Tree} = sign(\frac{1}{Z}\sum_{i\leq Z}\hat{H}_i).$$

where $\hat{H}_i$ is the label predicted by the i^{th} tree.

As expressed in ([20], 4.2, p 63) the general case is expressed by the following:

$$\hat{H}_{Tree} = \underset{y\in Y}{argmax} \sum_{1\leq i\leq Z} \mathbb{1}_{\{\hat{H}_i=y\}}$$

The trees contained in the Forest are identically distributed, but not independent. The Random Forest algorithm benefits from averaging over the trees. We write the variance of the Random Tree decision:

$$var\hat{H}_{Tree} = \frac{1}{Z^2}\sum_{1\leq i\leq Z} var\hat{H}_i + \frac{2}{Z^2}\sum_{1\leq i<j\leq Z} cov(\hat{H}_i, \hat{H}_j). \tag{2.2}$$

The second term contains $\frac{Z(Z-1)}{2}$ elements. We use the following definition to simplify it.

Definition 2.8. The random variables X_1,X_2...X_n are said to be exchangeable if their joint distribution F(X_1,X_2,...,X_n) is invariant under any permutation π. Namely, F(X_1,X_2,...,X_n)= F($X_{\pi(1)}$,$X_{\pi(2)}$,...,$X_{\pi(n)}$) for any permutation π. ([24], 2.1, p 2)

From the algorithm used to generate a random forest, one can easily check that the notion of order doesn't appear when growing the forest. Trees can hence be exchanged without any impact on the output of the forest. This implies:

$$cov(\hat{H}_i, \hat{H}_j) = cov(\hat{H}_{i'}, \hat{H}_{j'}).$$

Using this in (2.2), and naming ρ and σ respectively the pairwise correlations and each tree variance (recalling that the trees are identically distributed):

$$Var\hat{H}_{Tree} = \frac{1}{Z}\sigma + \frac{(Z-1)}{Z}\sigma\rho.$$

$$Var\hat{H}_{Tree} = \rho\sigma + \frac{1-\rho}{Z}\sigma^2. \tag{2.3}$$

The number of trees Z and the correlation ρ can have a considerable impact on the variance of the tree and hence on the reliability of its predictions. This incites us to look further into the correlation parameter.

We follow ([20], 4.2, p.67) in the definition and the interpreation of ρ :

$$\rho = \frac{Var_{\mathcal{F}}(\mathbb{E}_{\mathcal{X}|\mathcal{F}}[\hat{H}_{Tree}])}{Var_{\mathcal{F}}(\mathbb{E}_{\mathcal{X}|\mathcal{F}}[\hat{H}_{Tree}]) + \mathbb{E}_{\mathcal{F}}(Var_{\mathcal{X}|\mathcal{F}}[\hat{H}_{Tree}])}.$$

This implies that $0<\rho<1$.

Proposition 2.9. ***Law of total variance:***
Given two random variables X and Y on the same probability space and with $\mathrm{Var}[X]$ *finite:*

$$\mathrm{Var}[X] = \mathrm{E}[\mathrm{Var}[X|Y]] + \mathrm{Var}[\mathrm{E}[X|Y]]. \tag{2.4}$$

Proof.

$$\begin{aligned}
\mathrm{Var[X]} &= \mathrm{E}[X^2] - \mathrm{E}[X]^2 \\
&= \mathrm{E}[\mathrm{E}[X^2|Y]] - \mathrm{E}[\mathrm{E}[X|Y]]^2 \\
&= \mathrm{E}[[\mathrm{Var}[X|Y] + \mathrm{E}[X|Y]^2] - \mathrm{E}[\mathrm{E}[X|Y]]^2 \\
&= \mathrm{E}[\mathrm{Var}[X|Y]] + \mathrm{E}(\mathrm{E}[X|Y]^2) - \mathrm{E}[\mathrm{E}[X|Y]]^2 \\
&= \mathrm{E}[\mathrm{Var}[X|Y]] + \mathrm{Var}[\mathrm{E}[X|Y]].
\end{aligned}$$

Where the second equality is given by the tower property of conditional expectation, the third is given by the definition of the variance and the forth by linearity of the expectation.

□

Using the equality (2.4), ρ can be seen as the ratio between the variance due to the learning set and the total variance. In fact, the correlation between the trees is closely linked to the random vectors drawn before generating each tree of the random forest. When the total variance is mainly due to the learning set, the outputs of the trees are highly correlated. In this case, ρ is close to 1 and $Var\hat{H}_{Tree}$ tends to σ, which the variance of a single tree. In this case, the accuracy of the random forest doesn't benefit from the vote over the ensemble of trees. When the total variance is mainly due to the random generation of sample when building the tree, the numerator tends to 0 and $Var\hat{H}_{Tree}$ tends to $\frac{\sigma}{Z}$. The variance in this case is divided by Z.

The benefit of decreasing the correlation between the trees for variance reduction is limited by an increase of the bias. We shall not investigate this trade-off further. We refer the interested reader to [20] p[58, 67] for more details.

Following the structure of [7], we give now a theoretical framework for assessing the performance and accuracy of a random forest, and we aim at establishing an upper bound for misclassification by the random forest.

Definition 2.10. Given X, the set of explanatory variables and Y the corresponding labels, we define the margin function for a set $\hat{H} = (\hat{H})_{i=1}^{Z}$ of classifiers by:

$$mg(X, Y) = \frac{\sum_{i<Z} \mathbb{1}_{\{\hat{H}_i(X)=Y\}} - \underset{k \neq Y}{max}(\sum_{i<Z} \mathbb{1}_{\{\hat{H}_i(X)=k\}})}{Z}.$$

The margin function corresponds to the difference between the average of votes for the right label Y minus the average of votes for the most voted label different from Y. For $mg(X, Y)<0$, the voted class is wrong. The higher mg(X,Y) is, the more reliable are our classifier's predictions.

Definition 2.11. We define the generalization error ([18], 5., p 11) as the probability on the space (X,Y) of the random forest to have a negative margin function. Namely,

$$G = P_{X,Y}(mg(X, Y) < 0).$$

Theorem 2.12. *Given a set of randomly drawn vectors $\mathcal{X}=(\mathcal{X}_1, \mathcal{X}_2, ...\mathcal{X}_Z)$ to build the classifier $\hat{H}_{Tree}$*

$$mg(X, Y) \overset{as}{\to} mr(X, Y) = P_{\mathcal{X}}(\hat{H}(X) = Y) - \underset{k \neq Y}{max} P_{\mathcal{X}}(\hat{H}(X) \neq Y).$$

as the number of trees increases.

Proof. The proof is given in ([7], Appendix I, p27). □

This theorem highlights the idea that as the number of trees increases, the average vote for some class tends to the probability of the random forest to predict the right class. The over-fitting issue is limited when adding trees to the random forest. This is also confirmed by the following result, providing an upper limit to the generalization error.

Definition 2.13. We define the strength of a random tree (and more generally a set of classifiers) by :

$$s = \mathbb{E}_{X,Y} mr(X,Y).$$

Proposition 2.14. *The generalization error is bounded:*

$$G = P_{X,Y}(mr(X,Y) < 0) < \frac{Var(mr)}{s^2}.$$

Proof. Recall that given a random variable X with finite mean μ and finite variance σ and a strictly positive constant k Chebyshev inequlity holds :

$$P(|X - \mu| \geq k) \leq \frac{\sqrt{\sigma}}{k^2}. \tag{2.5}$$

The proof of 2.5 is given in appendix D.

We assume that $s > 0$, meaning that in average, the predicted class is the right one. This condition is required for a set of classifiers. If this is not verified, the set of classifiers can't be used in practice as it would underperform random predictions.

$$\begin{aligned} P_{X,Y}(mr(X,Y) < 0) &= P_{X,Y}(mr(X,Y) - s < -s) \\ &= P_{X,Y}((mr(X,Y) - s)^2 \geq s^2) \\ &= P_{X,Y}(|mr(X,Y) - s| \geq s) \leq \frac{Var(mr(X,Y))}{s^2}. \end{aligned}$$

□

In the case of two classes, the margin function can be written:

$$mr(X,Y) = 2P_{\mathcal{X}}(\hat{H}(X) = Y) - 1.$$

Hence, requiring $s > 0$ implies:

$$\mathbb{E}_{X,Y} mr(X,Y) > 0 \Rightarrow \mathbb{E}_{X,Y} P_{X,Y}(\hat{H}(X) = Y) > \frac{1}{2}.$$

That is in average, we require from our predicting set of classifiers to outperform random predictions, which have a 0.5 probability of success.

In this first part, we have set up the theoretical framework of Random Forests with an overview of its generalization abilities and an expected criteria of prediction accuracy. The rest of this section aims at applying this algorithm on stock data.

2.3 Application to the investment universe

We give first an overview of the methodology we follow to process the data and adapt it to the introduced algorithms.

Figure 3: Methodology followed to market prediction. The first step is the selection of the data (Closing prices, daily volume...). The selected data can't be directly used as input for the classifier as the considered signals are noisy. Step 2 and 3 address this issue as preprocessors. The two last steps are the direct application of the built algorithm introduced in the precedent section.

2.3.1 Data Selection

We choose to work with a universe of 8 stocks from the S&P500 with different sectors, sizes and historical volatilities for generalization purpose. The chosen stocks all have an inception date previous to 2000. We present below the chosen universe.[2]

Company	Stock	Sector	Historical 30 day volatility – 52 week High (%)	Historical 30 day volatility – 52 week Low(%)
Apple Inc.	AAPL	Information Technology	32,96%	11,34%
Amazon Inc.	AMZN	Consumer Discretionary	42,68%	11,12%
Citigroup	C	Financials	28,78%	10,31%
CVS Health	CVS	Consumer Staples	35,95%	12,29%
3M	MMM	Industrials	33,51%	8,64%
Starbucks Corp.	SBUX	Consumer Discretionary	34,88%	11,63%
Charles Schwab Corporation	SCHW	Financials	37,66%	14,86%
Exxon Mobil Corporation	XOM	Energy	29,32%	5,48%

Figure 4: Selected universe for market predictions. The historical volatility given here is derived from the variation of the prices over a 30-days time window: this corresponds to the monthly volatility. For each stock we give the highest and lowest monthly volatility over the last year (52 weeks). As we can see, the considered stocks have volatilities ranging between 8% and 42%.

The data spans the period 01/06/2000 - 25/04/2016 and is downloaded from Yahoo Finance :

[2]The data presented is taken from https://www.optionseducation.org/

https://finance.yahoo.com/. It includes:

▶ Daily Opening price

▶ Daily Closing price

▶ Daily Adjusted Closing price - which is an adjustment of the closing price taking into account dividends, stock splits and new stock offerings.

▶ Daily Traded Volume : which is the number of shares of a security traded during the day.

2.3.2 Data Smoothing.

The raw data downloaded is noisy and can't be used directly to make predictions. We use an exponential smoothing aiming at reducing effects of jumps and brusque changes in times series. This is done by averaging over the previous values with weights exponentially decreasing as the observations become older. Given a time series $P = (P_t)_{t \geq 0}$, the exponential smoothed version $\hat{P} = (\hat{P}_t)_{t \geq 0}$ is defined recursively by :

$$\hat{P}_0 = P_0, \tag{2.6}$$

$$\hat{P}_{t+1} = \alpha P_{t+1} + (1 - \alpha)\hat{P}_t. \tag{2.7}$$

$0 < \alpha < 1$ is the smoothing factor. It is the weight given to the current observation; $(1 - \alpha)$ is the weight given to the last value of the smoothed process. The smoothing effect vanishes as α becomes closer to 1.

Data smoothing is applied to the adjusted price of all the stocks. Following the recommendations of Ravinder (2013)[27] to use a smoothing factor below 0.50, we choose $\alpha = 0.20$.

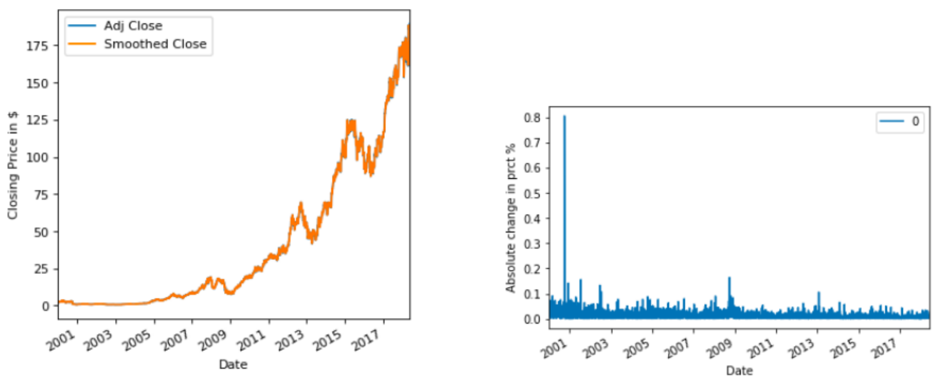


Figure 5: Absolute change in price after Smoothing AAPL. On the left hand side: the closing price between 2000 and 2018 of AAPL. On the right hand side : the plot of the absolute change in price after smoothing.

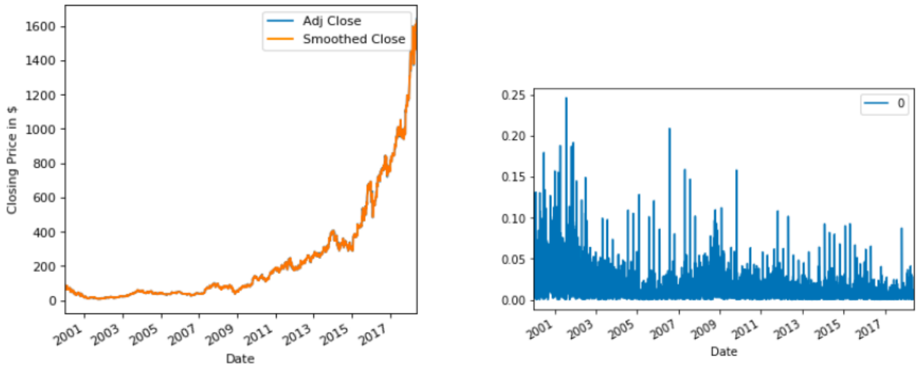


Figure 6: Absolute change in price after Smoothing AMZN. On the left hand side: the closing price between 2000 and 2018 of AMZN. On the right hand side : the plot of the absolute change in price after smoothing.

As shown above, the effect of exponential smoothing can be different from a time series to another, depending on the volatility and the jumps in the closing prices. With the same parameters, the smoothing changed the initial values by up to 25% for AMZN whereas the change in the adjusted closing prices for AAPL didn't exceed 1% over all the considered period.

2.3.3 Feature Derivation.

We aim here at extracting from the smoothed data a set of technical indicators (corresponding to the set of features in the Algorithm 2) which will be used as input to predict the direction of the stock price over a period of time.

- **On Balance Volume**

OBV is a momentum[3] indicator relating the traded volume in the stock market to the price. When the price goes up, the traded volume is accumulated; when the price goes down, the traded volume is subtracted.

$$OBV(t) = OBV(t-1) + \begin{cases} Volume(t) & if P_t > P_{t-1} \\ 0 & if\ P_t = P_{t-1} \\ -Volume(t) & if P_t < P_{t-1} \end{cases}$$

where P(t) denotes the smoothed price at time t.

As highlighted in the practitioner book ([1], p 150), the use of OBV is based on the assumption that "OBV changes precede prices changes". This is explained by the fact that "smart money[4]

[3] We denote by *Momentum* in what follows the continuance of the rise or the decline of the price of an asset. See [9] for more details on the use of this notion in trading.

[4] Investors with some expert knowledge

can be seen flowing into the security by a rising OBV" before "the public moves into the security". To capture the effective relation between prices and OBV, we compare below the adjusted closing price and OBV signals for AAPL stocks.

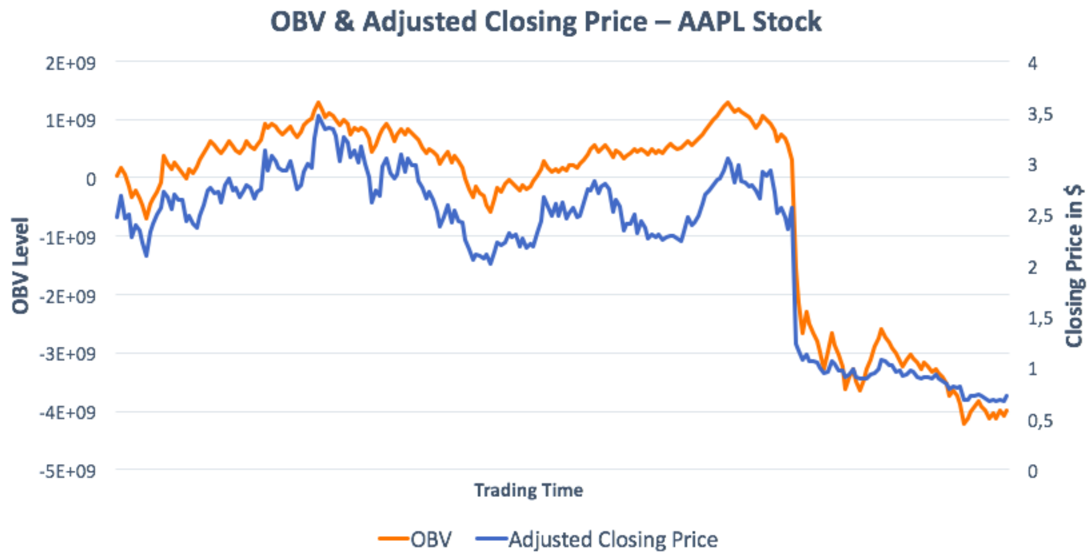


Figure 7: OBV and Adjusted Close for AAPL stock: *Given the very different scales of OBV and closing prices, we chose here to use two axis, on the left is the one giving OBV levels; on the right the one giving daily closing prices. We are mostly interested in their relative variations. The horizontal axis is in time (trading days from one to 250), the enumeration of days is hidden for clarity sakes.*

The plot above shows the OBV indicator and the adjusted closing price for 250 trading days. We can see that the OBV indicator and the price move symmetrically, with the former slightly preceding the movements.

Stochastic Oscillator %K

%K compares the closing price with a high-low range of the price over a given period of time. We will be using the default time period, which is 14 days.

$$K = 100 * \frac{P_t - Low_{14}}{High_{14} - Low_{14}},$$

Low_{14} and $High_{14}$ denoting respectively the lowest and highest price over the 14 last days period.

The stochastic oscillator ranges from 0 to 100. It is close to 0 when the current price is close to Low_{14} and it is close 100 when the asset is currently trading near $High_{14}$.

We plot below % K and the adjusted closing price for a 250 trading days period. The 80 and 20 levels corresponds respectively to an overbought and oversold asset. As highlighted in the chapter 11 of [6], those level don't imply by themselves bearish and bullish signals. However, we can notice that jumps in the oscillator value associated with crossing the 80 and 20 levels are correlated with the direction of the stock. We attempt to verify the hypothesis expressed in [6] in the following plot.

Figure 8: %K and Adjusted closing price for AAPL stock. *Given the very different scales of %K and the closing price, two axis are used in this plot as well. Again, we are mostly interested in their relative variations and the horizontal axis is in time (trading days from 1 to 250).*

We give two observations supporting the assumptions of [6] hypothesis in the following plot : in 1 the sharp increase in from below 20 level to above 80 level is followed by an increase in the closing price. In 2 the opposite happens as the stochastic oscillator decreases sharply from above 80 to below 20 and this is followed by a decrease in the closing price. This justify the use of this indicator as one the inputs of the prediction algorithm.

Moving Average Convergence Divergence:

MACD is a momentum and trend following indicator based on two moving averages:

$$MACD = EMA_{12} - EMA_{26}$$

Where EMA_n is the exponential moving average of the closing price over the last n days.

Signal MACD Defined by:

$$SignalMACD = EMA_9(MACD)$$

Both MACD and its signal are used by practitioners as indicators and interpreted according to the following [5] :

[5] This interpretation is taken from https://www.investopedia.com/terms/m/macd.asp. The same analysis can be found on this trading platform $https : //stockcharts.com/school/doku.php?id = chart\_school : technical\_indicators : moving_average\_convergence\_divergence_macd$

• Crossovers : A bearish signal is indicated when MACD falls below the signal and a bullish signal is indicated when MACD exceeds the signal.

• Divergence : A divergence of the price from the MACD indicates a change in trend.

• Dramatic Rise : As EMA_{12} rises and the EMA_{26} decreases, the indicator increases dramatically and that usually indicates an overbought stock.

We plot below the Adj close, the MACD and the MACD signal for a 250 trading days period of AAPL stock.

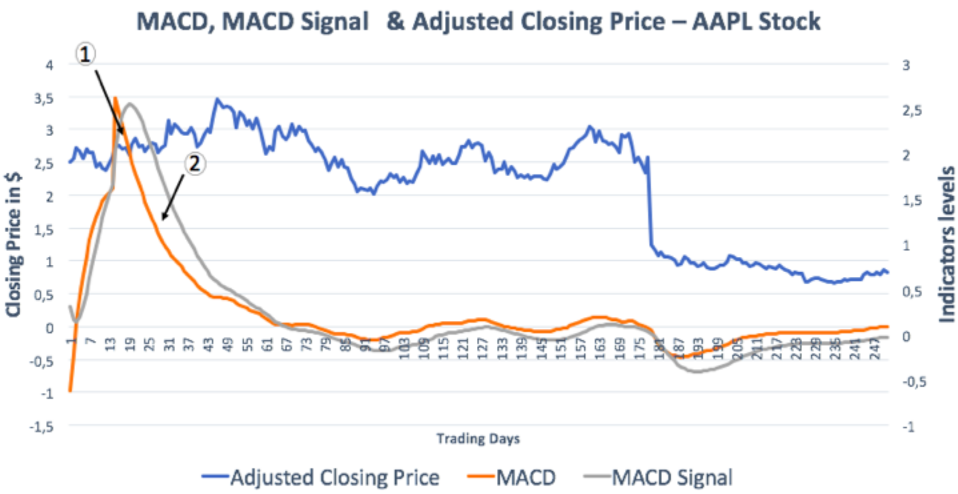


Figure 9: MACD indicators and Adjusted closing price. *Two scales are used here as well, the right axis corresponds to indicators levels, and the left one corresponds to closing prices.*

On the figure above we can interpret 1 as a case of divergence between the price and the MACD indicator. One can link this to the downtrend occurring until the 90^{th} day of trading. This is also supported by 2, as the MACD falls below MACD signal between the 13^{th} and the 61^{th} trading days.

The introduced indicators will be our main features to identify patterns from Volume and Smoothed Adjusted Closing price. We now formalize the considered classes in our classification problems corresponding to the direction of the stocks after m days.

Definition 2.15. Given a vector X of explanatory variables and a period of m trading days, the corresponding class is a scalar derived as follow:

$$Y_m(X) = \begin{cases} sign\left(\log \frac{P_{t+m}}{P_t}\right) & if P_{t+m} \neq P_t \\ 1 & if\ P_{t+m} = P_t \end{cases}$$

Remark 2.16. One can notice that the direction of the stocks could be more easily defined by the sign of the difference between the price at time $(t+m)$ and the price at time t. We choose to use the logarithm of the quotient between the two prices as this process will be used to fit the GARCH model and approximate the volatility of the returns in the next section. This choice doesn't impact the prediction algorithm.

Remark 2.17. The class chosen for the case $P_{t+m} = P_t$ is a chosen convention, this case isn't observed hasn't been observed in our work.

2.3.4 Predictions Results

The features derived in the previous subsection are used as inputs to the built Random Forest. We use the *Scikit-Learn Package* [6] and more precisely the *sklearn.ensemble.RandomForestClassifier* module [7] to generate the random forest classifier. The prediction follows the methodology below[8]:

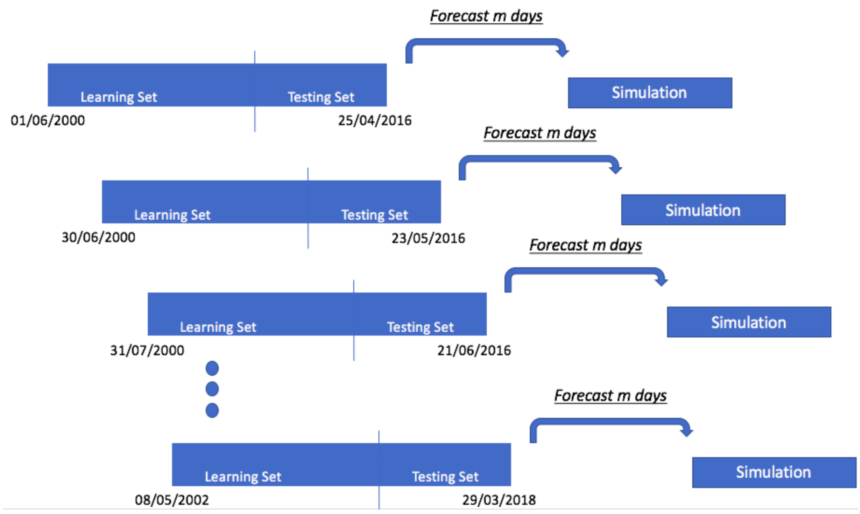


Figure 10: Methodology followed for predictions

As represented on the above scheme, it is based on a rolling window of approximately 16 years starting in 01/06/2000 and moving by m days after each prediction. The training-test sets are chosen randomly by the algorithm with a 80/20 ratio and **is done independently on each stock.** The algorithm is constructed in such a way that the inputs data used for the forecast isn't seen by the algorithm during the train-test period. Once the inputs (set of explanatory variables and their labels) fitted to the model, the fit is tested with new explanatory variables and the accuracy of prediction is assessed using the *metrics* module of *sklearn* package. An up prediction for the stock (class +1) may be an incentive to buy or to increase the weight of the asset in the portfolio, and the opposite for the down prediction. As a consequence, it is essential to assess the accuracy and the ability of the model to generalize its predictions to cases unseen in the historical

[6] See http://scikit-learn.org/stable/ for more informations

[7] http://scikit-learn.org/stable/modules/generated/sklearn.ensemble.RandomForestClassifier.html

[8] In this example, the rolling window moves by 21 trading days after each prediction.

data.

Assessing classification reliability.

We assess the reliability of the algorithm using the following metrics.

•*Accuracy:* probably the most intuitive one, it measures the proportion of right predictions among the tested set:

$$A = \frac{t_{up} + t_{down}}{t_{up} + t_{down} + f_{up} + f_{down}}$$

where:

t_{up} is the number of right up predictions,

t_{down} is the number of right down predictions,

f_{up} is the number of false up predictions - type I error,

f_{down} is the number of right up predictions type II error.

•*Precision:* gives the proportion of true predictions of a specified class among all the samples corresponding to this class.

$$P = \frac{t_Y}{t_Y + f_Y}$$

•*Recall:* which can be seen as a measure of the capacity of the algorithm to predict correctly a specified class.

$$R(Y) = \frac{t_Y}{t_Y + f_{\overline{Y}}}$$

•*F-1 score:* defined as the harmonic mean of Recall and Precision (for a binary classification problem)

$$F_1(Y) = \frac{2}{\frac{1}{P} + \frac{1}{R}} = \frac{2\,PR}{P + R}$$

In our analysis, we will consider the average of the precision and recall measures over both the +1 and -1 classes.

Choice of time period prediction.

The prediction horizon, noted m, is a very important parameter in our prediction. In fact, the accuracy of the prediction and the frequency of rebalancing depends directly on the value of m. We recall that our explanatory variables are computed over periods of 14 or 26 trading days. We fit the data with a time frame going from 2000-01-01 to 2018-06-01, using 80% for training and 20% for testing. We plot here the accuracy of the Random Forests algorithm against the prediction time horizon. We plot on the same graph the percentage of -1 label. As we can see below, the later plot has also to be taken in account into the choice of a value of m for our predictor to be significant.

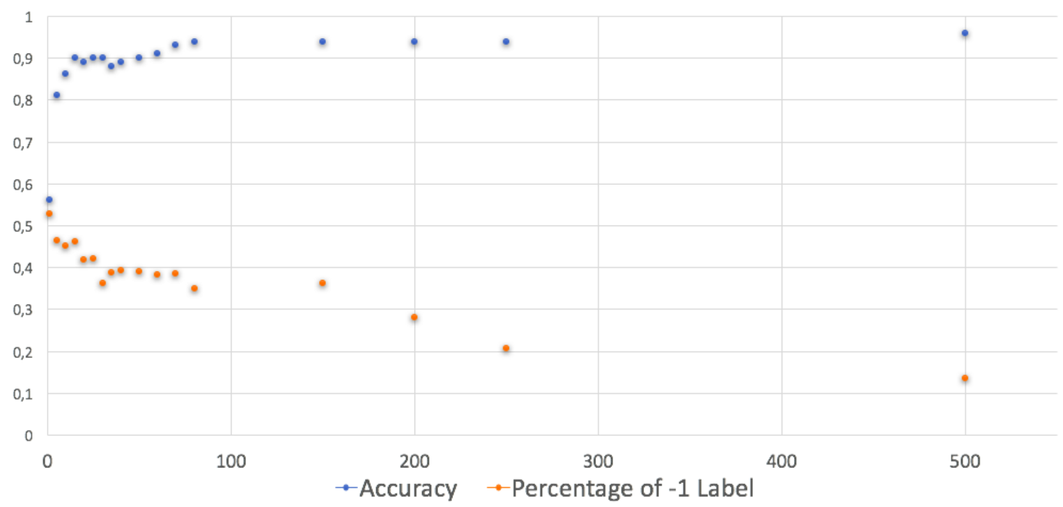


Figure 11: Accuracy with respect to predictions time horizon (Trading days)

The accuracy of predictions starts at a low level of 56% for one-day prediction and rise to reach approximately 90% for 30 days predictions. Recalling the condition expected in the proof of Proposition 2.14, the built algorithm should have an accuracy greater than 50%. We choose to use m=21 trading days prediction. This will also be the rebalancing frequency : starting from the first prediction on 25/04/2016, we will roll ahead the window by 21 trading days, re-rerun the algorithm taking into account the new incoming data and repeat the steps over again until the last prediction date. The numerical values of this plot, with the corresponding values of Recall, F-1 score and Precision are given in Appendix F. The accuracy increases with the time horizon of the prediction and reaches very high levels ($> 90\%$). This can be explained by the nature of the explanatory variables. In fact, the indicator derived above uses a 14 and 26 days of data. One can't expect to deduce accurately the direction of the stock for the next few days.

Remark 2.18. We can expect the accuracy of the prediction to reach a maximum and decrease after a certain number of days. However, one should also take into account how the proportion of each label changes as the time horizon increases. As seen on Figure 11 the percentage of samples with the label -1 decreases as the number of days ahead increases reaching 35% for 80 days time ahead and 20% for 250 days ahead. The label +1 is overrepresented and the prediction abilities of the algorithm can't be truly assessed in this case. This is confirmed by the relatively low values of the *Recall* measure (see Appendix F) for the -1 label above 50 days.

We give here the results for direction prediction using the built algorithm.

Results

. We present below the results for AMZN stock with the train-test metrics.

Train-testTime Window		Forecast	Train-test metrics				Prediction	Historical Move
Start	End	Date	Accuracy	Precision	Recall	F-1		
01/06/00	25/04/16	23/05/16	0,9	0,9	0,9	0,9	1	1
30/06/00	23/05/16	22/06/16	0,8	0,8	0,8	0,8	1	1
31/07/00	21/06/16	21/07/16	0,9	0,9	0,9	0,9	1	1
29/08/00	21/07/16	19/08/16	0,89	0,9	0,9	0,9	1	1
27/09/00	18/08/16	19/09/16	0,89	0,9	0,9	0,9	1	1
26/10/00	19/09/16	18/10/16	0,9	0,9	0,9	0,9	1	1
24/11/00	18/10/16	16/11/16	0,88	0,89	0,89	0,89	-1	-1
25/12/00	15/11/16	15/12/16	0,89	0,89	0,89	0,89	1	1
23/01/01	13/12/16	13/01/17	0,9	0,9	0,9	0,9	1	1
21/02/01	12/01/17	13/02/17	0,89	0,9	0,9	0,9	1	1
22/03/01	10/02/17	14/03/17	0,89	0,9	0,9	0,9	1	1
20/04/01	14/03/17	12/04/17	0,88	0,89	0,89	0,89	1	1
21/05/01	11/04/17	11/05/17	0,88	0,89	0,89	0,89	1	1
19/06/01	10/05/17	09/06/17	0,89	0,89	0,89	0,89	1	1
18/07/01	08/06/17	10/07/17	0,9	0,9	0,9	0,9	-1	-1
16/08/01	10/07/17	08/08/17	0,89	0,9	0,9	0,9	1	1
14/09/01	07/08/17	06/09/17	0,89	0,9	0,9	0,9	-1	-1
15/10/01	06/09/17	05/10/17	0,9	0,9	0,9	0,9	1	1
13/11/01	05/10/17	03/11/17	0,87	0,88	0,88	0,88	1	1
12/12/01	02/11/17	04/12/17	0,88	0,88	0,88	0,88	1	1
10/01/02	30/11/17	02/01/18	0,88	0,88	0,88	0,88	-1	1
08/02/02	29/12/17	31/01/18	0,89	0,9	0,9	0,9	1	1
11/03/02	30/01/18	01/03/18	0,89	0,9	0,9	0,9	1	1
09/04/02	28/02/18	29/03/18	0,89	0,9	0,9	0,9	-1	-1
08/05/02	29/03/18	30/04/18	0,89	0,9	0,9	0,9	1	1

Figure 12: Random Forest Predictions - AMZN Stock

As expected when choosing the prediction horizon, the accuracy metrics are close to 90%. This is confirmed by the backtest, as one prediction is wrong (highlighted in red).

We also give below the results for SCHW (Schwab Corporation) stock and MMM (3M) stock below. The results of the other stocks are given in Appendix.

Train-Test Time Window				
Rolling Window Start	End rolling window	Forecast Date	Prediction	Historical Move
01/06/00	25/04/16	23/05/16	-1	-1
30/06/00	23/05/16	22/06/16	1	1
31/07/00	21/06/16	21/07/16	1	1
29/08/00	21/07/16	19/08/16	-1	-1
27/09/00	18/08/16	19/09/16	-1	-1
26/10/00	19/09/16	18/10/16	-1	-1
24/11/00	18/10/16	16/11/16	1	1
25/12/00	15/11/16	15/12/16	1	1
23/01/01	13/12/16	13/01/17	-1	-1
21/02/01	12/01/17	13/02/17	1	1
22/03/01	10/02/17	14/03/17	1	1
20/04/01	14/03/17	12/04/17	-1	-1
21/05/01	11/04/17	11/05/17	1	1
19/06/01	10/05/17	09/06/17	1	1
18/07/01	08/06/17	10/07/17	1	1
16/08/01	10/07/17	08/08/17	-1	-1
14/09/01	07/08/17	06/09/17	1	-1
15/10/01	06/09/17	05/10/17	1	1
13/11/01	05/10/17	03/11/17	1	1
12/12/01	02/11/17	04/12/17	1	1
10/01/02	30/11/17	02/01/18	-1	-1
08/02/02	29/12/17	31/01/18	1	-1
11/03/02	30/01/18	01/03/18	-1	-1
09/04/02	28/02/18	29/03/18	-1	-1
08/05/02	29/03/18	30/04/18	-1	-1

Figure 13: Random Forest Predictions - MMM Stock

There are two wrong predictions for the MMM stock over 25 forecasts (06/09 and 31/01), which corresponds to 92% accuracy. This confirms the metrics measured during the train-test period. The same performance is observed for the prediction of XOM stock below.

Train-Test Time Window				
Rolling Window Start	End rolling window	Forecast Date	Prediction	Historical Move
01/06/00	25/04/16	23/05/16	1	1
30/06/00	23/05/16	22/06/16	1	1
31/07/00	21/06/16	21/07/16	1	1
29/08/00	21/07/16	19/08/16	-1	-1
27/09/00	18/08/16	19/09/16	1	-1
26/10/00	19/09/16	18/10/16	1	1
24/11/00	18/10/16	16/11/16	1	1
25/12/00	15/11/16	15/12/16	1	1
23/01/01	13/12/16	13/01/17	-1	-1
21/02/01	12/01/17	13/02/17	-1	-1
22/03/01	10/02/17	14/03/17	-1	-1
20/04/01	14/03/17	12/04/17	1	1
21/05/01	11/04/17	11/05/17	-1	-1
19/06/01	10/05/17	09/06/17	-1	-1
18/07/01	08/06/17	10/07/17	-1	-1
16/08/01	10/07/17	08/08/17	1	-1
14/09/01	07/08/17	06/09/17	-1	-1
15/10/01	06/09/17	05/10/17	1	1
13/11/01	05/10/17	03/11/17	1	1
12/12/01	02/11/17	04/12/17	1	1
10/01/02	30/11/17	02/01/18	1	1
08/02/02	29/12/17	31/01/18	1	1
11/03/02	30/01/18	01/03/18	-1	-1
09/04/02	28/02/18	29/03/18	-1	-1
08/05/02	29/03/18	30/04/18	1	1

Figure 14: Random Forest Predictions - XOM Stock

The constructed algorithm can predict the direction of the stock with high accuracy. To estimate the expected return, we need now a model predicting the amplitude of stocks movement, which is approximated by a prediction of the monthly volatility of the stock. This is done using a GARCH(1,1)-model.

3 Volatility Modeling and Forecast

3.1 Statistical Introductory Analysis

We start this section by examining the time series of log returns and assessing their normality. This assumption will be used in the next section when deriving the optimal portfolio. We plot below the daily and monthly log returns.

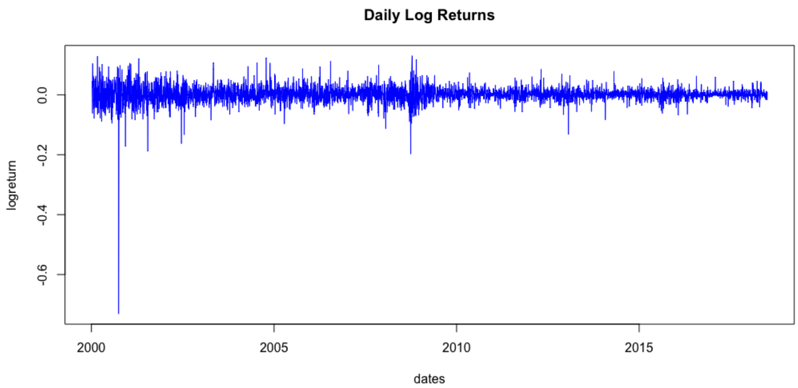


Figure 15: Daily Log Returns AAPL stocks

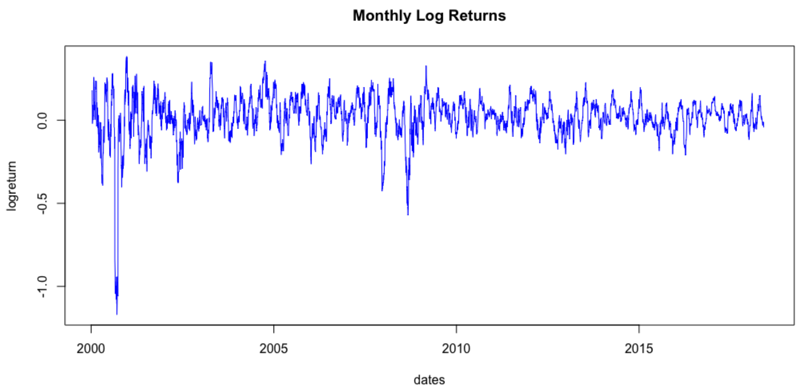


Figure 16: Monthly Log Returns AAPL stocks

Log returns seem to fluctuate around 0 with a major breakdown in 2001. In our attempt to model the log returns of the stocks, one may start by trying to fit a well know distribution, the Normal Distribution $\mathcal{N}(\mu, \sigma)$. This is assessed in what follows using the statistical parameters, the plotted histogram and the Q-Q plots.[9]

[9]This is the plot of the quantiles of the data against a given distribution. We will use the normal distribution in our case.

Data	Mean	Std. Deviation	Skew	Kurtosis
Daily Log Returns	0.000908867	0.02700756	-4.400373	121.8109
Monthly Log Returns	0.01960214	0.1286435	-2.689564	22.01792

Table 1: Statistical Parameters of Daily and Monthly Log Returns

Using the estimated mean and standard deviation, we compare the distribution of our data to the corresponding normal distribution.

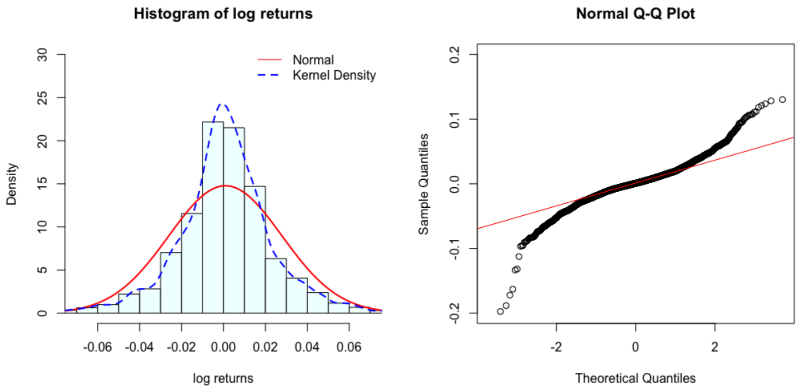


Figure 17: Daily Log Returns AAPL stocks

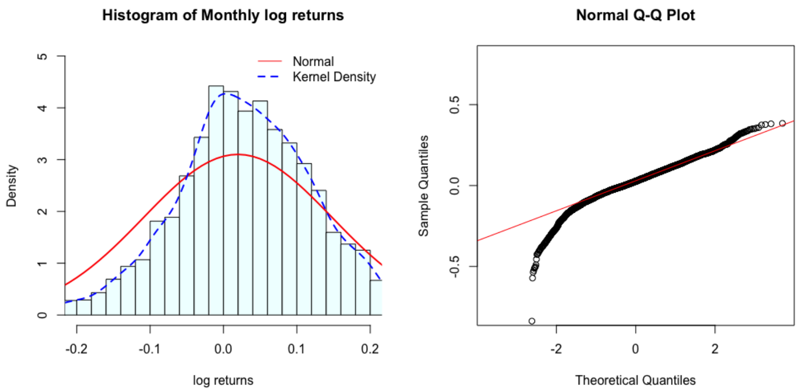


Figure 18: Monthly Log Returns AAPL stocks

The daily log returns distribution seem to be more peaky (given by Kurtosis) and to have heavier tails than the normal distribution. We recall that a normal distribution has a kurtosis equal to 3. As the log returns are extended to a longer period (monthly log returns), their distribution become closer to the Normal Distribution. The data distribution still has a heavier left tail, corresponding to losses.

Those two observations are among stylized facts of returns. More on stylized facts and statistical properties of returns can be found in [10] (Aggregational Gaussianity, p224).

The Aggregational Gaussianity fact justifies the normality assumption used in the next section when building the portfolio. One should keep in mind that the left tail (associated with losses) of monthly log returns are heavier than the normal distribution's one, which raises risk tails issues: risk associated with extreme losses happening with small probabilities. As the normal distribution has light tails, this is not considered when building the portfolio using normality assumption. Including the potential extreme losses in our model can be done using Extreme Values theory. This is out of the scope of our study, we refer to ([10], 4.4, p227) for an introduction to the subject.

3.2 Introduction to GARCH Models.

We consider a probability space probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and the time series of inputs to the Random Forest $(\hat{X}_1, \hat{X}_2, ..\hat{X}_n)$. $\Omega, \mathcal{F}$ and $\mathbb{P}$ respectively denote the set of all possible outcomes (samples space), the set of events and a probability measure. Detailed definition ans properties of these mathematical objects can be found in [3] in Chapter 1&2.

Definition 3.1. A process Z=$(Z_t)_{t\in\mathbb{N}}$ is a strict white noise if it is square integrable, i.e $\mathbb{E}[Z^2] < \infty$, and consisting of independent and identically distributed random variables.

Definition 3.2. A process $\hat{X} = (\hat{X}_t)_{t\in\mathbb{N}}$ is said to be strictly stationary if for any set $(t_1, t_2, .., t_n) \in \mathbb{N}^n$ and k $\in \mathbb{Z}$:

$$(\hat{X}_{t_1}, \hat{X}_{t_2}, ..\hat{X}_{t_n}) \stackrel{d}{=} (\hat{X}_{t_1+k}+, \hat{X}_{t_2+k}, ..\hat{X}_{t_n+k}),$$

where $\stackrel{d}{=}$ denotes equality in distribution.

Definition 3.3. A strictly stationary process $\hat{X} = (\hat{X}_t)_{t\in\mathbb{N}}$ is a Generalized Autoregressive Conditional Heteroskedasticity (p,q) model GARCH(p,q), with p,q $\in \mathbb{N}$ if for some strict noise $(Z_t)_{t\in\mathbb{N}}$ with mean 0 and variance 1:

$$\begin{aligned}\hat{X}_t &= \sigma_t Z_t,\\ {\sigma_t}^2 &= \alpha_0 + \sum_{i=1}^{p} \alpha_i \hat{X}_{t-i}^2 + \sum_{j=1}^{q}, \beta_i \sigma_{t-j}^2\end{aligned}$$

where $(\sigma_t)_{t\in\mathbb{N}}$ is strictly stationary and positive-valued.

$\sigma = (\sigma_t)_{t\in\mathbb{Z}}$ can be interpreted in the definition above as the volatility of the process. One can easily see the dependence of the volatility at time t on the historical volatilities and the historical values of the process from the definition. This model captures volatility clustering with the parameter q and volatility persistence with the parameter p defined in ([21], 418, VII) by the fact that "large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes". This fact can be seen quantitatively as daily log returns are uncorrelated while their absolute values display some correlation, as shown below.

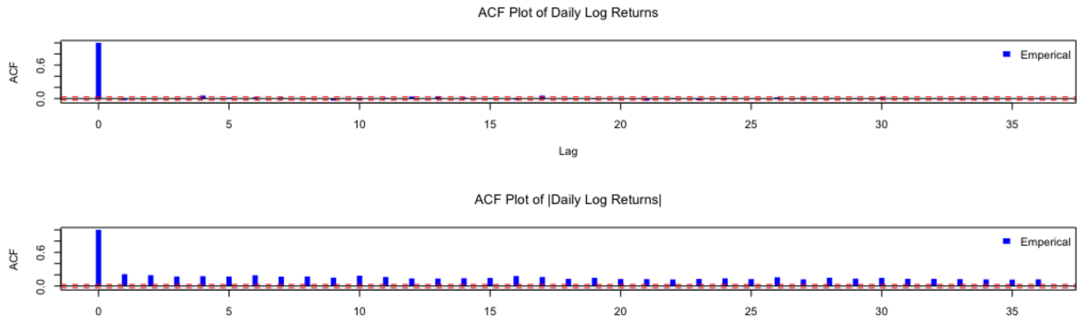


Figure 19: Volatility Clustering Displayed

Remark 3.4. Another process called ARMA(p,q) - Autoregressive Moving Average process - can be fitted as well to our data. This would capture the moving average part and can be combined to the GARCH model to capture both the volatility dependence and the time-dependent average of the process. In our study, as we are interested in forecasting the volatility only, we focus on GARCH model.

3.3 Fit to GARCH model and results.

We follow the same methodology introduced in the section above, i.e. we use a rolling window to fit the data (monthly log returns) to the model and we use the obtained process to forecast the volatility 21 working days ahead (corresponding to 1 calendar month). This is done on *Python* using the *arch_model* package.[10]. The GARCH(1,1) model is fitted to the monthly log returns. We give below the parameters obtained for the the first five periods.

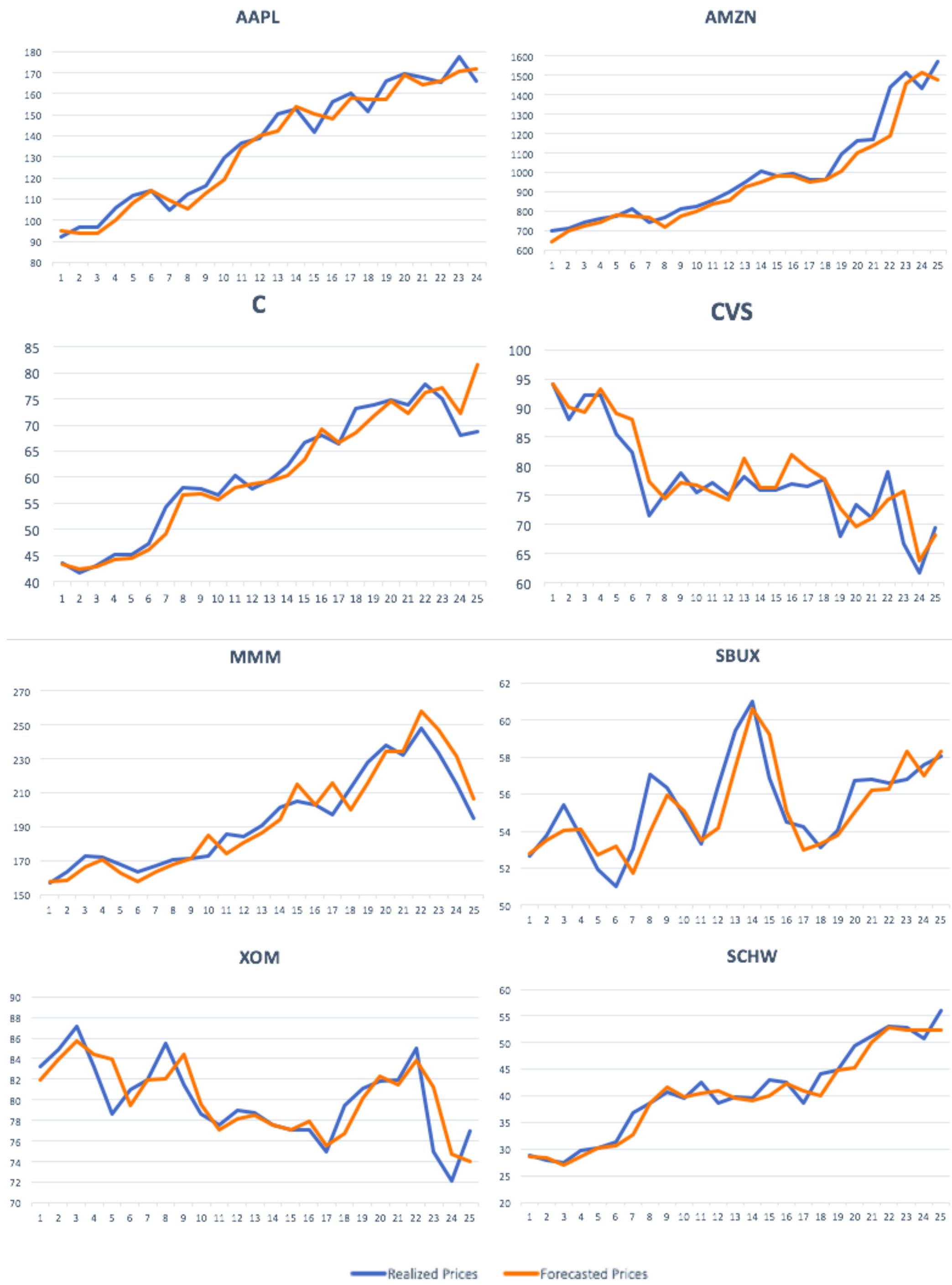
[10] See *https : arch.readthedocs.io/en/latest/univariate/introduction.html* for more details on the fit and forecast functions

Volatility Model					
	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	7.5547e-05	1.819e-05	4.152	3.289e-05	[3.989e-05,1.112e-04]
alpha[1]	0.2000	9.751e-03	20.512	1.698e-93	[0.181, 0.219]
beta[1]	0.7800	8.956e-03	87.090	0.000	[0.762, 0.798]
omega	7.5243e-05	1.847e-05	4.073	4.639e-05	[3.904e-05,1.114e-04]
alpha[1]	0.2000	9.823e-03	20.361	3.742e-92	[0.181, 0.219]
beta[1]	0.7800	8.973e-03	86.924	0.000	[0.762, 0.798]
omega	7.4310e-05	1.966e-05	3.779	1.574e-04	[3.577e-05,1.129e-04]
alpha[1]	0.2000	9.767e-03	20.478	3.402e-93	[0.181, 0.219]
beta[1]	0.7800	9.311e-03	83.773	0.000	[0.762, 0.798]
omega	7.2588e-05	2.332e-05	3.112	1.857e-03	[2.688e-05,1.183e-04]
alpha[1]	0.2000	9.730e-03	20.556	6.845e-94	[0.181, 0.219]
beta[1]	0.7800	1.062e-02	73.425	0.000	[0.759, 0.801]
omega	7.1494e-05	2.710e-05	2.638	8.339e-03	[1.838e-05,1.246e-04]
alpha[1]	0.2000	9.688e-03	20.643	1.120e-94	[0.181, 0.219]
beta[1]	0.7800	1.250e-02	62.418	0.000	[0.756, 0.804]

Figure 20: Fitted GARCH Model parameters - AMZN Stock

As we can see, the coefficients all belong to the 95% confidence intervals. Moreover, $P > |t|$, which corresponds to the significance level (i.e. probability that those results would have occurred by chance) is less than 0.05 (value conventionally used) for all the given parameters. We can conclude from this that we fail to reject the null hypothesis, and that the data doesn't not follow the Garch(1,1) model.

From the fitted model, we forecast at the end of each period the volatility one month ahead. This approximates the magnitude of change in the stock price. Multiplying this by the sign of the movement of the stock predicted before, we obtain a forecast of the price 21 trading days ahead. We deduce from this the expected returns. We plot the realized monthly closing prices and the predicted prices for the 8 stocks composing our universe : AAPL-AMZN-C-CVS-MMM-SBUX-SCHW-XOM during all the prediction period. For clarity sakes, the dates aren't displayed on the graph. We give in appendix E the correspondence between the number of the period and the dates.



Remark 3.5. *One should notice that the y-axes don't start at 0. The chosen scale covers the prices ranges for each stock. The x-axis denotes the investment periods.*

Overall, the predictive model follow the trend of the price closely. Volatility seems to be underestimated by the GARCH-model, but the predicted prices are close to the realized ones. From the predicted prices, we can compute the predicted returns, that we expect to be close to the realized one. We give now a brief introduction to the Modern Portfolio Theory before applying it to our investment universe.

4 Modern Portfolio Theory

4.1 Introduction to Portfolio Construction.

Given a set of assets composing a universe, constructing a portfolio aims at choosing the weights to assign to each asset according to performances goals criteria. As the parameters of the market evolve in time, the portfolio is rebalanced : the weights are re-derived taking into account the new market conditions and the incoming data. Constructing the optimal portfolio results from an optimization problem where many unknown parameters, such as expected returns and correlation, have to be estimated implying a high sensitivity to the accuracy of the estimation methodologies. This subject has been extensively tackled by academic researches since the introduction of the Modern Portfolio Theory by Markowitz in 1952. The first layers of the Mean-Variance optimization was introduced in [22] and [23], the main ideas being that risk and return should be thought of together, not separately, and that a portfolio should be diversified, as the old saying highlights - "don't put all your eggs in one basket." This is explained by the fact that when adding negatively correlated assets to a portfolio, the losses incurred by one may be offset by the gains of the others. The Mean-Variance optimization was extended during the last decades by practitioners and academics to address its main limitation, including the high sensitivity to historical data and the impossibility to include investors views. One of these extensions is the Black-Litterman model, developed in 1990 by Fisher Black and Robert Litterman at Goldman Sachs. This framework is out of the scope of our study, an introduction to the subject can be found in [15].

4.2 Mean-Variance Optimization.

We can see the mean as an approximation of the returns and the variance as an approximation of risk. From the mean-variance trade-off introduced by Markowitz [23] , one should either maximize his returns for a given level of risk, or minimize his risk for a given level of returns.

Definition 4.1. *Given an asset with price at time t denoted by p_t and two investments periods t_1 and t_2, we define the return over the $t_1 - t_2$ in percentage by :*

$$r\% = 100 * \frac{p_{t_2} - p_{t_1}}{p_{t_1}}$$

This is an unknown parameter when building the portfolio, it is modeled by a random variable R on a probability space $(\Omega, \mathcal{F}, P)$.

In the basic mean-variance optimization framework, we assume an investment done by a risk-averse investor on a single time period. We also assume normally distributed returns for the risky assets. A risk-averse investor is one who, given two assets with the same returns, would choose the less risky one. The investor only takes into account means, variances and correlation between the assets in choosing his portfolio given the normal distribution assumption[11].

We consider here the basic case of two risky assets 1 and 2 with returns respectively modeled by R_1 and R_2 with $R_i \sim \mathcal{N}(\mu_i, \sigma_i)$, i denoting 1 or 2. A realization of R_i will be denoted r_i. Both risky assets will contribute to the portfolio with weights w_1 and w_2 with the initial wealth fully invested ($w_1 + w_2 = 1$).

The correlation between the assets is given by:

$$\rho = \frac{\mathbb{E}[(r_1 - \mu_1)(r_2 - \mu_2)]}{\sqrt{\sigma_1 \sigma_2}}$$

By linearity of the expectation, the expected portfolio return is the weighted average of the expected returns of the assets, namely:

$$\overline{r_p} = \mathbb{E}(r_p) = \mathbb{E}[w_1 r_1 + w_2 r_2] = w_1 \mu_1 + w_2 \mu_2.$$

The risk of the portfolio is defined by the variance of portfolio returns:

$$\begin{aligned} \sigma_p &= Var(r_p) = \mathbb{E}[(r_p - \overline{r_p})^2] \\ &= \mathbb{E}[(w_1(r_1 - \mu_1) + w_2(r_2 - \mu_2)^2] \\ &= w_1^2 \sigma_1 + w_2^2 \sigma_2 + 2\rho w_1 w_2 \sqrt{\sigma_1 \sigma_2}. \end{aligned}$$

The above expressions can be extended to the n-dimensional case (n-risky assets). Using the convention that bold letters denote n-dimensional vectors, the expected returns of a portfolio composed of n assets is:

$$\begin{aligned} \overline{r_{p,n}} &= \mathbb{E}[\sum_{i=1}^{n} w_i r_i] \\ &= \sum_{i=1}^{n} w_i \mu_i = \mathbf{w} \boldsymbol{\mu}^{\boldsymbol{T}}. \end{aligned}$$

With $\rho_{i,j}$ denoting the correlation between the assets i and j, portfolio's variance is:

$$\begin{aligned} \sigma_{p,n} &= Var(\overline{r_p^n}) = E[(r_i^n - \mathbf{w}\boldsymbol{\mu}^{\boldsymbol{T}})^2] \\ &= \textstyle\sum_{i=1}^{n} \sum_{j=1}^{n} \sqrt{\sigma_i \sigma_j} w_i w_j \rho_{i,j}. \end{aligned} \tag{4.1}$$

We define the covariance matrix (symmetric and positive definite) by $\Sigma_{i,j} = \rho_{i,j}\sigma_i\sigma_j$. Equation (4.1) becomes

$$\sigma_{p,n} = \boldsymbol{w}^{\boldsymbol{T}} \boldsymbol{\Sigma} \boldsymbol{w}. \tag{4.2}$$

Given the introduced parameters, one can define different optimization problems. We first start by considering risk as the target function.

[11] A normally distributed random variable is completely defined by its mean and variance.

•***Minimize Risk for a given level of returns*** r_0***:***

The optimization problem is the following:

$$\begin{aligned} &\min \frac{1}{2}\boldsymbol{w}^T\boldsymbol{\Sigma}\boldsymbol{w} \\ \text{subject to} \quad &\boldsymbol{w}\boldsymbol{\mu}^{\boldsymbol{T}} = r_0 \\ &\boldsymbol{w}\boldsymbol{1}^{\boldsymbol{T}} = 1 \end{aligned}$$

where $\mathbf{1} = (1, ...1)^T \in \mathbb{R}^n$.

We use the Lagrangian Method to define the set of optimal portfolios and introduce the *Efficient Frontier*.

The Lagrangian is given by:

$$L(\boldsymbol{w}, \alpha_1, \alpha_2) = \frac{1}{2}\boldsymbol{w}^{\boldsymbol{T}}\boldsymbol{\Sigma}\boldsymbol{w} - \alpha_1(\boldsymbol{w}\boldsymbol{\mu}^{\boldsymbol{T}} - r_0) - \alpha_2(\boldsymbol{w}\boldsymbol{1}^{\boldsymbol{T}} - 1)$$

We rewrite the above expression using (4.1):

$$L(\boldsymbol{w}, \alpha_1, \alpha_2) = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_i\sigma_j w_i w_j \rho_{i,j} - \alpha_1(\sum_{i=1}^{n} w_i\mu_i - r_0) - \alpha_2(\sum_{i=1}^{n} w_i - 1)$$

We compute the first derivative with respect to w_i, α_1 and α_2 and we set them to 0:

$$\frac{\partial L}{\partial w_i} = \sum_{j=1}^{n} w_j\rho_{i,j}\sigma_i\sigma_j - \alpha_1\mu_i - \alpha_2 = 0 \tag{4.3}$$

$$\frac{\partial L}{\partial \alpha_1} = -\sum_{i=1}^{n} w_i\mu_i + r_0 = -\boldsymbol{\mu}^T\boldsymbol{w} + r_0 = 0 \tag{4.4}$$

$$\frac{\partial L}{\partial \alpha_2} = -\sum_{i=1}^{n} w_i + 1 = -\mathbf{1}^T\boldsymbol{w} + 1 = 0. \tag{4.5}$$

The above system of 3 equations can be written in the following matrix form:

$$\boldsymbol{\Sigma}\boldsymbol{w} - \alpha_1\boldsymbol{\mu} - \alpha_2\mathbf{1} = 0. \tag{4.6}$$

From the Spectral Theorem, $\boldsymbol{\Sigma}$ is an invertible matrix as it symmetric.[12]. Equation (4.6) is rearranged as follows:

$$\boldsymbol{w} = \boldsymbol{\Sigma}^{-1}(\alpha_1\boldsymbol{\mu} + \alpha_2\mathbf{1}) \tag{4.7}$$

Re-Writing (4.4) and (4.5) using the formula of $\boldsymbol{w}$ derived in (4.7) :

$$\begin{aligned} \alpha_1\mathbf{1}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + \alpha_2\mathbf{1}^T\boldsymbol{\Sigma}^{-1}\mathbf{1} &= 1 \\ \alpha_1\boldsymbol{\mu}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} + \alpha_2\boldsymbol{\mu}^T\boldsymbol{\Sigma}^{-1}\mathbf{1} &= r_0. \end{aligned} \tag{4.8}$$

Let A, B and C three scalars denoting respectively $\mathbf{1}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}$, $\mathbf{1}^T\boldsymbol{\Sigma}^{-1}\mathbf{1}$ and $\boldsymbol{\mu}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}$. Equation (4.8) becomes:

$$A\alpha_1 + B\alpha_2 = 1 \tag{4.9}$$

$$C\alpha_1 + A\alpha_2 = r_0. \tag{4.10}$$

[12]More on the spectral theorem and its proof can be found in [28], Theorem 1, p2

Re-writing this in a matrix form :

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} \begin{pmatrix} A & B \\ C & A \end{pmatrix} = \begin{pmatrix} 1 \\ r_0 \end{pmatrix} \tag{4.11}$$

This system admits a solution if:

$$\begin{vmatrix} A & B \\ C & A \end{vmatrix} = A^2 - BC \neq 0. \tag{4.12}$$

This is true when μ isn't proportional to $\mathbf{1}$. Assuming this - namely that assets returns aren't all equal- we solve the second order system equations in α_1 and α_2 and we obtain:

$$\alpha_1 = \frac{A - Br_0}{A^2 - BC}$$
$$\alpha_2 = \frac{Ar_0 - C}{A^2 - BC}.$$

The variance of the mean-variance optimized portfolio is:

$$\begin{aligned} \sigma_{p,n} &= \boldsymbol{w}^T \boldsymbol{\Sigma} \boldsymbol{w} \\ &= \boldsymbol{w}^T \boldsymbol{\Sigma} \boldsymbol{\Sigma}^{-1} (\alpha_1 \boldsymbol{\mu} + \alpha_2 \mathbf{1}) \\ &= \alpha_1 \boldsymbol{w}^T \boldsymbol{\mu} + \alpha_2 \boldsymbol{w}^{\boldsymbol{T}} \mathbf{1} \end{aligned}$$

Using (4.4) and (4.5) the variance of the portfolio becomes:

$$\sigma_{p,n} = \alpha_1 r_0 + \alpha_2 = \frac{Br_0^2 - 2Ar_0 + C}{BC - A^2} \tag{4.13}$$

where the second equality is derived using the derived expressions of α_1 and α_2.

The derived variance is positive for all the values of r_0, this is checked in Appendix A. The set of optimal portfolios defines the Efficient Frontier, which is an hyperbola when plotting Expected Returns against Portfolio's Variance. To verify this, we use the predictions made for the first period (See Appendix E) of time using the built algorithm to generate expected returns of the assets. We then build mean-variance optimized portfolios and plot returns against risk.

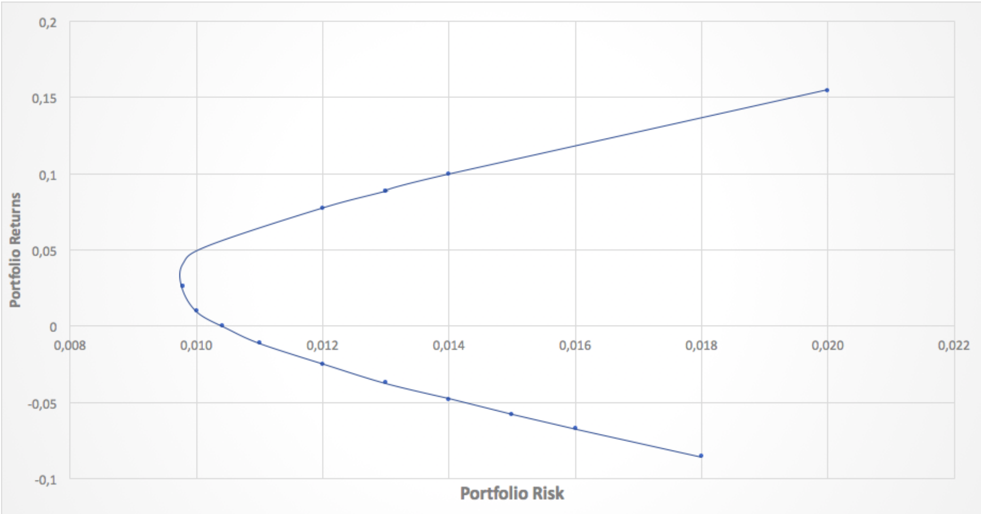


Figure 21: Portfolio Efficient Frontier

The optimal portfolios -from the mean-variance perspective- are located on the superior frontier. Given a level of risk, every portfolio under the line is suboptimal. One should also note that the frontier is convex, implying that every portfolio between two given optimal portfolios is optimal.

We also give an overview of alternative optimization problems.

•***Maximize returns for a given level of risk*** σ_0:

$$\begin{aligned} &\max \boldsymbol{w}\boldsymbol{\mu}^{\boldsymbol{T}} \\ \text{subject to} \quad &\boldsymbol{w}\boldsymbol{\Sigma}^{\boldsymbol{T}}\boldsymbol{w} = \sigma_0, \\ &\boldsymbol{w}\mathbf{1}^{\boldsymbol{T}} = 1, \end{aligned}$$

where $\mathbf{1} = (1, ...1)^T \in \mathbb{R}^n$.

The resulting portfolios are also on the efficient frontier plotted above.

•***Maximize Sharp Ratio***.

Sharp ratio is defined as the unit of excess returns obtained by unit of risk taken, namely :

$$S = \frac{r_p - r_{riskfree}}{\sqrt{\sigma_p}},$$

$r_{riskfree}$ being the risk free rate.

Assuming the interest rates are equal to 0, which is the case currently in Europe, this is corresponds to:

$$S = \frac{r_p}{\sqrt{\sigma_p}},$$

where $\sqrt{\sigma_p}$ denotes the standard deviation.

The optimization problems is :

$$\begin{aligned} & \max \frac{\boldsymbol{w\mu^T}}{\boldsymbol{w^T\Sigma w}} \\ \text{subject to} \quad & \boldsymbol{w\Sigma^T w} = \sigma_0 \\ & \boldsymbol{w1^T} = 1. \end{aligned}$$

4.3 Investment strategies performances

We now consider the following situation : five risk-averse investors aim at maximizing their returns with an exposure of less than 10% in risk (variance of returns) by investing in the considered universe. All the investors starts with a wealth of 100 units. Before introducing the performance of the strategies, we start by giving the main assumptions used in building the investment strategies and assessing their results.

Assumptions and Practical Considerations.

• *Portfolios are all self-financing and not leveraged.* Starting with an initial wealth π_0, no cash is added or extracted from the portfolio during the whole investment period.

Definition 4.2. A portfolio worth π_t at time t and composed of n assets with prices S_t^i at time t and weighted w_t^i is self-financing if at every time t:

$$\pi_t = \pi_{t-1} + \sum_{i=1}^{n} w_t^i (S_t^i - S_{t-1}^i)$$

This means that the change in value of the portfolio comes only from the change in the price of the assets.

• *Volatility considered as measure of risk.* We also highlight here the fact that we are considering the ex-ante volatility as the optimization is based on the historical volatility for all the strategies. The realized volatility, ex-post volatility, can possibly expose the investor to higher risks. In fact, the practitioners usually adjust this using models based on the observed gap between the ex-ante and ex-post volatility . This adjustment is out of the scope of our report, but more details on volatility targeting strategies can be found in [25]. We assume that the volatility is piecewise constant : this meaning that it is constant between two rebalancing dates and equal to the ex-ante volatility observed at the last day of the rolling window.

• *Drift effect on weights*: Between two consecutive rebalancing, as the closing prices of the assets change, the actual weights drift from their initial weights. This may be a considerable issue for the investor in the case where the drift increases considerably the weight for a few asset and change the initial wanted exposure. In our case, as the rebalancing is done monthly, we can neglect the effect of the drift on our portfolio.

• *Short-selling allowed with no costs*: We consider long-only and long-short strategies and we assume that one can short with no additional cost. In practice, short-selling involves considerable costs which should be taken into account when building the portfolio.

• *Constraints on weights*: We choose to add constraints on individual assets weights in the portfolios : $0\% \leq w_i \leq 20\%$ when short selling is not allowed and $-20\% \leq w_i < 20\%$ when it is. This has a considerable impact on the construction of the portfolio and its risk. In fact, when the algorithm predicts high returns for an individual asset, maximizing returns imply assigning to it a major weight and hence being strongly correlated to its performance. Concentrated portfolios go against the diversification principle suggested by the Modern Portfolio Theory. The choice of the value of the constraint is subjective and may vary according to the risk aversion of the investor.

As these constraints are applied to all the considered strategies, they actually have a limited impact on our goal of capturing the impact of the Random Forest algorithm/GARCH Model on the performance with respect to the historical data based mean-variance optimization.

The strategies considered are summarized in the following table:

Investment Strategy	Optimization Target	Constraints	Inputs
Equally Distributed Weights	No optimization is done for this strategy. The weights are equal to 12,5% for each asset and are rebalanced at the beginning of each	N/A	N/A
Prediction Enabled Mean Variance Optimization - Short Selling Not Allowed	Maximizing Returns	• Constraint on risk < 10% • Assets weights are positive are less than 20%	•Expected Returns are derived from the Random Forest Algorithm and the GARCH model fit
Prediction Enabled Mean Variance Optimization - Short Selling Allowed	Maximizing Returns	• Constraint on risk < 10% • Weights range between -20% and 20%	•Expected Returns are derived from the Random Forest Algorithm and the GARCH model fit
Classic Mean-Variance Optimization - Short Selling Allowed	Maximizing Returns	• Constraint on risk < 10% • Weights range between -20% and 20%	•Expected Returns are derived from historical returns
Classic Mean-Variance Optimization- Short Selling Allowed	Maximizing Returns	• Constraint on risk < 10% • Weights range between -20% and 20%	•Expected Returns are derived from historical returns

Figure 22: Investment Strategies considered

We start first by comparing the performances in absolute returns, without considering the risk taken. We give the numerical results and methodology for the first three periods. The remaining results follow the same methodology and are given in appendix G . Numerical data includes derived returns, weights and volatilities.

Period 1

Assets	µ Predicted(%)	µ Historical (%)	µ Realized(%)	σ	µ/σ
AAPL	-6,66	2,78	-9,32	0,05	-1,24
AMZN	3,77	2,46	13,23	0,06	0,60
C	-4,95	-0,24	-1,99	0,06	-0,80
CVS	-1,89	1,26	-1,95	0,03	-0,58
MMM	-1,36	1,15	-1,50	0,03	-0,52
SBUX	-4,70	1,79	-4,98	0,05	-1,02
SCHW	-1,86	0,57	-0,20	0,04	-0,50
XOM	1,69	0,78	3,39	0,02	0,71

Correlation Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	1,00	0,94	-0,70	0,96	0,95	0,94	0,70	0,80
AMZN	0,94	1,00	-0,69	0,93	0,93	0,96	0,68	0,76
C	-0,70	-0,69	1,00	-0,60	-0,61	-0,55	-0,35	-0,73
CVS	0,96	0,93	-0,60	1,00	0,96	0,96	0,78	0,80
MMM	0,95	0,93	-0,61	0,96	1,00	0,96	0,67	0,80
SBUX	0,94	0,96	-0,55	0,96	0,96	1,00	0,68	0,74
SCHW	0,70	0,68	-0,35	0,78	0,67	0,68	1,00	0,58
XOM	0,80	0,76	-0,73	0,80	0,80	0,74	0,58	1,00

Covariance Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	0,34%	0,32%	-0,23%	0,17%	0,13%	0,23%	0,14%	0,10%
AMZN	0,32%	0,39%	-0,26%	0,19%	0,15%	0,27%	0,16%	0,11%
C	-0,23%	-0,26%	0,38%	-0,12%	-0,10%	-0,15%	-0,08%	-0,11%
CVS	0,17%	0,19%	-0,12%	0,11%	0,08%	0,14%	0,09%	0,06%
MMM	0,13%	0,15%	-0,10%	0,08%	0,07%	0,11%	0,06%	0,05%
SBUX	0,23%	0,27%	-0,15%	0,14%	0,11%	0,21%	0,12%	0,08%
SCHW	0,14%	0,16%	-0,08%	0,09%	0,06%	0,12%	0,14%	0,05%
XOM	0,10%	0,11%	-0,11%	0,06%	0,05%	0,08%	0,05%	0,06%

			No shorting		Shorting Allowed	
	Stocks	Equally distributed Portfolio	Prediction Enabled MVO	Classic MVO	Prediction Enabled MVO	Classic MVO
	AAPL	12,5%	0,0%	20,0%	-20,0%	20,0%
	AMZN	12,5%	20,0%	20,0%	20,0%	20,0%
	C	12,5%	0,0%	0,0%	0,0%	-20,0%
	CVS	12,5%	20,0%	20,0%	20,0%	20,0%
	MMM	12,5%	20,0%	20,0%	20,0%	20,0%
	SBUX	12,5%	0,0%	20,0%	20,0%	20,0%
	SCHW	12,5%	20,0%	0,0%	20,0%	0,0%
	XOM	12,5%	20,0%	0,0%	20,0%	20,0%
	SUM Of Weights	100%	100%	100%	100%	100%
Predicted	Volatility		0,1%	0,2%	0,1%	0,3%
	Portfolio Returns (%)		0,07	1,89	0,46	2,09
	Sharpe Ratio		0,02	0,44	0,14	0,37
Realized	Volatility	0,1%	0,1%	4,3%	0,1%	0,3%
	Portfolio Ret	-0,41	2,59	-0,90	3,46	0,17
	Sharpe Ratio	-0,15	0,78	-0,04	1,08	0,03

Period 2

Assets	µ Predicted(%)	µ Historical (%)	µ Realized(%)	σ	µ/σ
AAPL	2,13	2,74	4,94	0,05	0,40
AMZN	2,15	2,59	1,86	0,06	0,35
C	-2,83	-0,26	-6,25	0,06	-0,46
CVS	-4,12	1,18	-1,93	0,03	-1,27
MMM	1,01	1,13	3,84	0,03	0,39
SBUX	1,66	1,73	2,17	0,04	0,45
SCHW	-2,14	0,53	-1,04	0,05	-0,47
XOM	0,79	0,81	1,99	0,02	0,34

Correlation Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	1,00	0,93	-0,70	0,96	0,95	0,94	0,72	0,80
AMZN	0,93	1,00	-0,68	0,93	0,93	0,96	0,69	0,76
C	-0,70	-0,68	1,00	-0,60	-0,60	-0,55	-0,37	-0,73
CVS	0,96	0,93	-0,60	1,00	0,96	0,96	0,80	0,80
MMM	0,95	0,93	-0,60	0,96	1,00	0,96	0,70	0,80
SBUX	0,94	0,96	-0,55	0,96	0,96	1,00	0,70	0,74
SCHW	0,72	0,69	-0,37	0,80	0,70	0,70	1,00	0,60
XOM	0,80	0,76	-0,73	0,80	0,80	0,74	0,60	1,00

Covariance Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	0,33%	0,31%	-0,23%	0,17%	0,13%	0,18%	0,17%	0,10%
AMZN	0,31%	0,38%	-0,26%	0,19%	0,15%	0,22%	0,20%	0,11%
C	-0,23%	-0,26%	0,38%	-0,12%	-0,10%	-0,12%	-0,10%	-0,11%
CVS	0,17%	0,19%	-0,12%	0,11%	0,08%	0,11%	0,12%	0,06%
MMM	0,13%	0,15%	-0,10%	0,08%	0,07%	0,09%	0,08%	0,05%
SBUX	0,18%	0,22%	-0,12%	0,11%	0,09%	0,14%	0,12%	0,06%
SCHW	0,17%	0,20%	-0,10%	0,12%	0,08%	0,12%	0,21%	0,07%
XOM	0,10%	0,11%	-0,11%	0,06%	0,05%	0,06%	0,07%	0,06%

			No shorting		Shorting Allowed	
	Stocks	Equally distributed Portfolio	Prediction Enabled MVO	Classic MVO	Prediction Enabled MVO	Classic MVO
	AAPL	12,5%	20,0%	20,0%	20,0%	20,0%
	AMZN	12,5%	20,0%	20,0%	20,0%	20,0%
	C	12,5%	0,0%	0,0%	0,0%	-20,0%
	CVS	12,5%	0,0%	20,0%	-20,0%	20,0%
	MMM	12,5%	20,0%	20,0%	20,0%	20,0%
	SBUX	12,5%	20,0%	20,0%	20,0%	20,0%
	SCHW	12,5%	0,0%	0,0%	20,0%	0,0%
	XOM	12,5%	20,0%	0,0%	20,0%	20,0%
	SUM Of Weights	100%	100%	100%	100%	100%
Predicted	Volatility		0,1%	0,2%	0,2%	0,3%
	Portfolio Returns (%)		1,55	1,87	1,95	2,09
	Sharpe Ratio		0,40	0,46	0,49	0,39
Realized	Volatility	0,1%	0,1%	4,1%	0,2%	0,3%
	Portfolio Ret	0,70	2,96	2,18	3,14	3,82
	Sharpe Ratio	0,25	0,77	0,11	0,80	0,68

Period 3

Assets	µ Predicted(%)	µ Historical (%)	µ Realized(%)	σ	µ/σ
AAPL	2,17	2,76	5,08	0,05	0,43
AMZN	1,94	2,63	4,43	0,06	0,31
C	2,93	-0,34	4,50	0,06	0,48
CVS	1,32	1,23	1,61	0,03	0,41
MMM	2,03	1,12	5,94	0,03	0,78
SBUX	1,19	1,74	3,89	0,04	0,32
SCHW	-4,08	0,44	-0,71	0,05	-0,89
XOM	1,33	0,80	3,07	0,02	0,56

Correlation Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	1,00	0,92	-0,70	0,96	0,95	0,94	0,76	0,81
AMZN	0,92	1,00	-0,67	0,93	0,94	0,95	0,73	0,76
C	-0,70	-0,67	1,00	-0,60	-0,60	-0,55	-0,42	-0,73
CVS	0,96	0,93	-0,60	1,00	0,96	0,96	0,84	0,81
MMM	0,95	0,94	-0,60	0,96	1,00	0,96	0,76	0,80
SBUX	0,94	0,95	-0,55	0,96	0,96	1,00	0,74	0,74
SCHW	0,76	0,73	-0,42	0,84	0,76	0,74	1,00	0,66
XOM	0,81	0,76	-0,73	0,81	0,80	0,74	0,66	1,00

Covariance Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	0,32%	0,29%	-0,22%	0,16%	0,13%	0,18%	0,18%	0,10%
AMZN	0,29%	0,38%	-0,25%	0,18%	0,15%	0,22%	0,21%	0,11%
C	-0,22%	-0,25%	0,38%	-0,12%	-0,10%	-0,12%	-0,12%	-0,11%
CVS	0,16%	0,18%	-0,12%	0,10%	0,08%	0,11%	0,12%	0,06%
MMM	0,13%	0,15%	-0,10%	0,08%	0,07%	0,09%	0,09%	0,05%
SBUX	0,18%	0,22%	-0,12%	0,11%	0,09%	0,14%	0,13%	0,06%
SCHW	0,18%	0,21%	-0,12%	0,12%	0,09%	0,13%	0,21%	0,07%
XOM	0,10%	0,11%	-0,11%	0,06%	0,05%	0,06%	0,07%	0,06%

			No shorting		Shorting Allowed	
	Stocks	Equally distributed Portfolio	Prediction Enabled MVO	Classic MVO	Prediction Enabled MVO	Classic MVO
	AAPL	12,5%	20,0%	20,0%	20,0%	20,0%
	AMZN	12,5%	20,0%	20,0%	20,0%	20,0%
	C	12,5%	20,0%	0,0%	20,0%	-20,0%
	CVS	12,5%	0,0%	20,0%	20,0%	20,0%
	MMM	12,5%	20,0%	20,0%	20,0%	20,0%
	SBUX	12,5%	0,0%	20,0%	0,0%	20,0%
	SCHW	12,5%	0,0%	0,0%	-20,0%	0,0%
	XOM	12,5%	20,0%	0,0%	20,0%	20,0%
	SUM Of Weights	100%	100%	100%	100%	100%
Predicted	Volatility		0,1%	0,2%	0,1%	0,3%
	Portfolio Returns (%)		2,08	1,90	3,16	2,12
	Sharpe Ratio		0,86	0,47	1,34	0,40
Realized	Volatility	0,1%	0,1%	4,1%	0,1%	0,3%
	Portfolio Ret	3,48	4,61	4,19	5,07	3,91
	Sharpe Ratio	1,24	1,91	0,21	2,15	0,73

Figure 23: Numerical Parameters and results for the three first investment periods.

For a given period, **the table on the left** gives for each asset :

▶ μ predicted using Random Forests + GARCH model. This is used to build the Prediction enabled MVO strategies.

▶ μ historical. This is used to build the classic MVO strategies.

▶ μ realized. This is used to compute the realized performance in returns monthly.

▶ σ is the volatility of the returns.

▶ $\frac{\mu}{\sigma}$ is the volatility of the returns.

The **matrices in the middle** give respectively correlation and covariance matrices derived from historical data. The correlation matrix is computed on Python using the *DataFrame.corr* function of the *Pandas* package, and the covariance matrix is deduced using the variances of each asset.

The table on the right gives the optimal weights computed using the *Solver* tool on Excel with the corresponding constraints for each strategy. Using those derived weights we compute: the Portfolio volatility, the Portfolio Return (in percentage) and the Portfolio Sharp ratio from two-time perspectives:

▶ Predicted: this is the one expected when constructing the portfolio at the end of the rolling window

▶ Realized: this is the one obtained at the end of the investment period.

Results.

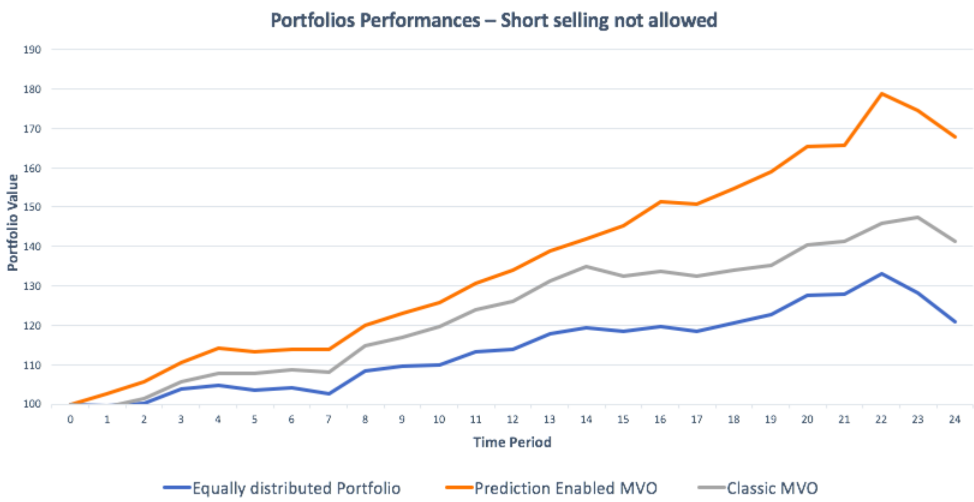


Figure 24: Investment Strategies comparison - Absolute Returns

We can first notice that the equally weighted portfolio underperforms the mean-variance optimized strategies. In fact, this portfolio doesn't take in account any particular features of the investment portfolio and historical behavior. The performance is improved for the classic MVO portfolio as an optimization is done considering the particular behavior of each assets during the rolling window. The machine learning enabled optimization outperforms in returns by over 25%.

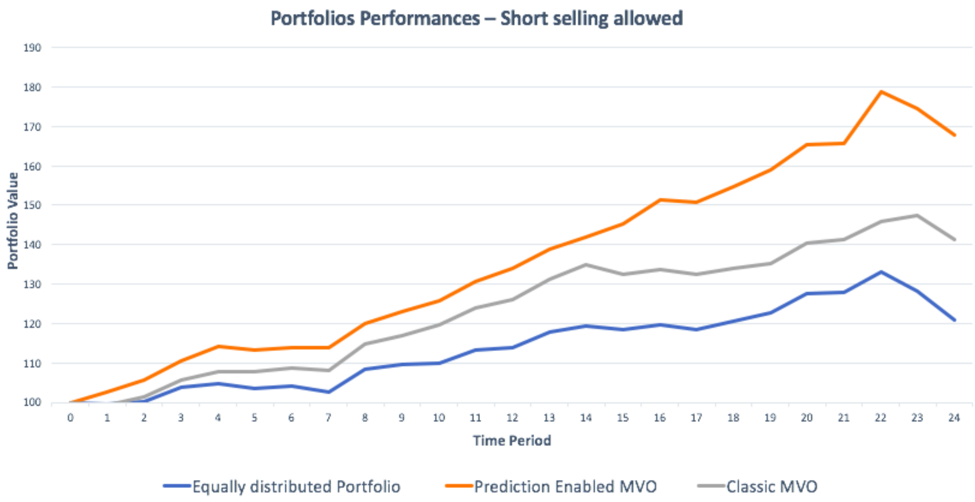


Figure 25: Investment Strategies comparison - Absolute Returns

We have assessed up to now the performance of our portfolios from the absolute returns perspective, without taking into account the risk taken by the investor. As highlighted by Harry Markowitz [23], returns and risk should be considered together and not separately. The more risk an investor takes, the more compensated he expects to be. We compare now the sharp ratios of both the Prediction-enabled and the classic MVO portfolios when shorting is allowed and when it is not.

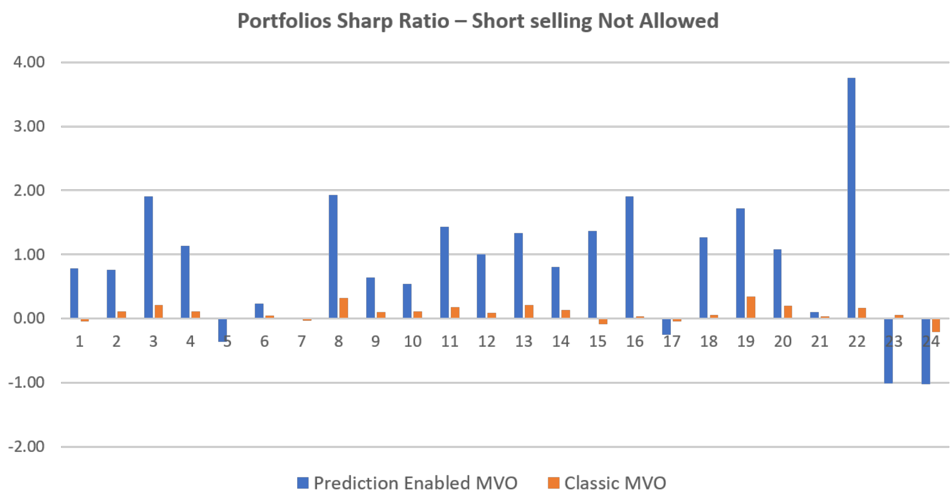


Figure 26: Investment Strategies comparison - Sharp Ratios. *The x-axis corresponds to the investment periods.*

The prediction enabled MVO seems to outperform the classic MVO by far in terms of Sharp Ratio when short selling is not allowed.

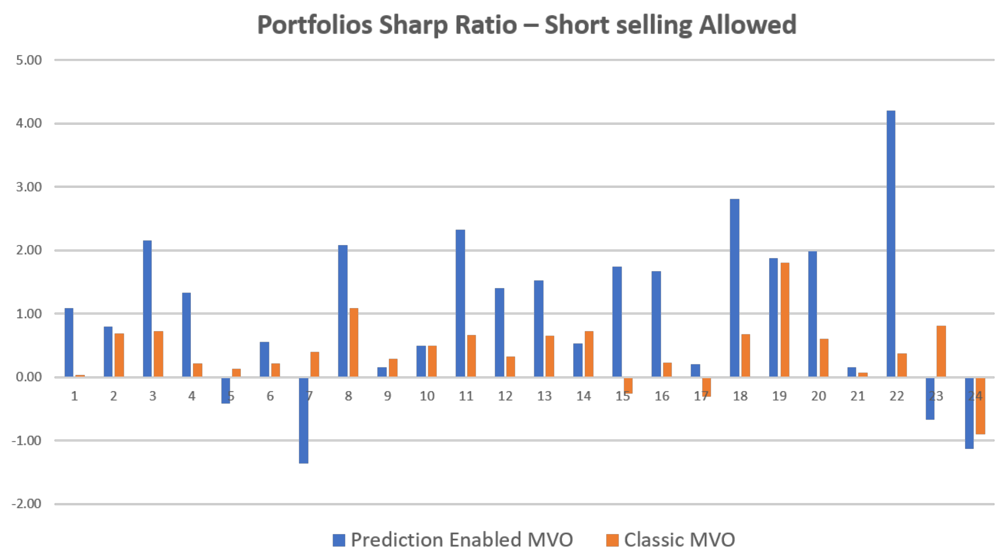


Figure 27: Investment Strategies comparison - Sharp Ratios. *The x-axis corresponds to the investment periods.*

Risk is better compensated by the Prediction-enabled strategy when shorting is allowed as well. However, comparing with Figure B, we can notice that the gap between both strategies is decreased overall when short selling is allowed.

5 Conclusion

We have compared a classic mean-variance optimized portfolio to an extended version where Random Forest and Garch(1,1) are used to derive the expected returns. The results support the idea that Machine Learning can improve the performances of an investment portfolio. The impact of predictions (+20% in absolute returns over two years) are a great incentive to further develop this model and extend the role played by Machine Learning in portfolio construction and monitoring.

Appendices

A Investment periods to dates correspondence

Period	Train-Test Time Window: *Rolling Window Start*	Train-Test Time Window: *End rolling window*	Forecast Date
Period 1	01/06/00	25/04/16	23/05/16
Period 2	30/06/00	23/05/16	22/06/16
Period 3	31/07/00	21/06/16	21/07/16
Period 4	29/08/00	21/07/16	19/08/16
Period 5	27/09/00	18/08/16	19/09/16
Period 6	26/10/00	19/09/16	18/10/16
Period 7	24/11/00	18/10/16	16/11/16
Period 8	25/12/00	15/11/16	15/12/16
Period 9	23/01/01	13/12/16	13/01/17
Period 10	21/02/01	12/01/17	13/02/17
Period 11	22/03/01	10/02/17	14/03/17
Period 12	20/04/01	14/03/17	12/04/17
Period 13	21/05/01	11/04/17	11/05/17
Period 14	19/06/01	10/05/17	09/06/17
Period 15	18/07/01	08/06/17	10/07/17
Period 16	16/08/01	10/07/17	08/08/17
Period 17	14/09/01	07/08/17	06/09/17
Period 18	15/10/01	06/09/17	05/10/17
Period 19	13/11/01	05/10/17	03/11/17
Period 20	12/12/01	02/11/17	04/12/17
Period 21	10/01/02	30/11/17	02/01/18
Period 22	08/02/02	29/12/17	31/01/18
Period 23	11/03/02	30/01/18	01/03/18
Period 24	09/04/02	28/02/18	29/03/18
Period 25	08/05/02	29/03/18	30/04/18

Figure 28: On the plots appearing in our reports, the periods numbers corresponds to the following dates.

B Direction forecast using Random Forests - Numerical Results

	AAPL	C	CVS	SBUX	SCWH
Period 1	-1	-1	-1	-1	-1
Period 2	1	-1	-1	1	-1
Period 3	1	1	1	1	-1
Period 4	1	1	1	-1	1
Period 5	1	-1	-1	-1	1
Period 6	1	1	1	1	1
Period 7	-1	1	-1	1	1
Period 8	1	1	1	1	1
Period 9	1	-1	1	-1	1
Period 10	1	-1	-1	-1	-1
Period 11	1	1	1	-1	1
Period 12	1	-1	-1	1	-1
Period 13	1	1	1	1	1
Period 14	1	1	-1	1	-1
Period 15	-1	1	1	-1	1
Period 16	1	1	1	-1	-1
Period 17	1	-1	1	-1	-1
Period 18	-1	1	1	-1	1
Period 19	1	-1	-1	1	1
Period 20	1	1	1	1	1
Period 21	-1	-1	-1	-1	1
Period 22	-1	1	1	-1	1
Period 23	1	-1	-1	1	-1
Period 24	-1	-1	-1	1	-1
Period 25	-1	1	1	1	1

Figure 29: Outputs of the Random Forests algorithm for the 24 investment periods. We give here the numerical results for direction prediction for the remaining stocks of the universe.

C Minimum Variance Portfolio - Variance Positivity

We check here that the derived variance 4.13 is positive. In fact, writing $A^2 - BC$ in a matrix format we obtain:

$$(\mathbf{1^T \Sigma^{-1} \boldsymbol{\mu}})^2 - \mathbf{1^T \Sigma^{-1} 1 \boldsymbol{\mu}^T \Sigma^{-1} \boldsymbol{\mu}} \tag{C.1}$$

We define $\psi(\boldsymbol{x}, \boldsymbol{y}) = \boldsymbol{x^T \Sigma^{-1} y}$ as the symmetric bilinear form associated with the quadratic form $q(x) = \boldsymbol{x^T \Sigma^{-1} x}$. Using Cauchy Scwharz inequality:

$$\psi(\mathbf{1}, \boldsymbol{\mu})^2 \leq q(\mathbf{1}) q(\boldsymbol{\mu}) \iff A^2 - BC \leq 0 \tag{C.2}$$

Hence, $A^2 - BC \leq 0$ with the equality case occurring when μ is proportional to the $\mathbf{1}$. Recalling the formula of the variance 4.13:

$$\sigma_{p,n} = \frac{Br_0^2 - 2Ar_0 + C}{BC - A^2} \tag{C.3}$$

The discriminant of the numerator of the fraction above is $\Delta = 4(A^2 - BC) < 0$. The numerator is of the sign of B, which is positive, for all the values of returns r_0. The denominator is positive. Hence, the variance is positive indeed.

D Proof of Chebyshev's Inequality

Proof.

$$\begin{aligned} P(|X-\mu| \geq k\sigma) &= \mathrm{E}[\mathbb{1}_{\{|X-\mu|\geq k\sigma\}}] \\ &= \mathrm{E}[\mathbb{1}_{\{\frac{(X-\mu)^2}{(k\mu)^2}\geq 1\}}] \\ &\leq \mathrm{E}[(\frac{X-\mu}{k\sigma})^2)] \\ &= \frac{1}{k^2}\frac{\mathrm{E}((X-\mu)^2)}{\sigma^2} \\ &= \frac{1}{k^2} \end{aligned}$$

The first equality comes from the definition of the expectation combined with the fact that the indicator function is equal to 1 if $\mathbb{1}_{\{|X-\sigma|\geq k\sigma\}}$ and 0 otherwise. The inequality comes from the fact that when the event inside the indicator function is true, then both quantities are equal and when it's not, the second is positive where as the first is equal to 0.

□

E Market Indicator

• **Relative Strength Index:** RSI is an momentum oscillator measuring the speed and magnitude of price movements and indicating strength and weakness of a the asset over a certain period of time. We will be using the default time period, which is 14 days.

$$RSI = 100 - \frac{100}{1+RS} \tag{E.1}$$

$$RS = \frac{Average\ Gain\ over\ 14\ days}{Average\ Loss\ over\ 14\ days} \tag{E.2}$$

Notice that RS $\in \mathbb{R}^+$ and that RSI ranges from 0 to 100.

F Accuracy/Time horizon data

Prediction Time Horizon	Accuracy	Precision	Recall	F1-score	Support	Prct of -1 Label
1	0,56	0,6	0,52	0,56	423	53%
		0,53	0,61	0,57	378	
5	0,8114	0,82	0,76	0,79	371	46%
		0,81	0,85	0,83	430	
10	0,8625	0,85	0,85	0,85	362	45%
		0,87	0,87	0,87	438	
15	0,9	0,92	0,88	0,9	368	46%
		0,9	0,93	0,92	431	
20	0,89	0,87	0,86	0,87	334	42%
		0,9	0,91	0,9	464	
25	0,9	0,89	0,86	0,87	335	42%
		0,9	0,92	0,91	462	
30	0,9	0,87	0,85	0,86	289	36%
		0,88	0,93	0,92	507	
35	0,88	0,88	0,81	0,85	308	39%
		0,89	0,93	0,91	487	
40	0,89	0,91	0,83	0,87	312	39%
		0,89	0,95	0,92	482	
50	0,9	0,92	0,85	0,89	309	39%
		0,91	0,95	0,93	483	
60	0,91	0,92	0,91	0,92	303	38%
		0,94	0,95	0,95	487	
70	0,93	0,93	0,88	0,9	303	38%
		0,93	0,96	0,94	485	
80	0,94	0,91	0,89	0,9	274	35%
		0,94	0,95	0,95	512	
150	0,94	0,94	0,89	0,91	280	36%
		0,94	0,97	0,95	492	
200	0,94	0,93	0,87	0,9	213	28%
		0,95	0,97	0,96	549	
250	0,94	0,94	0,85	0,89	156	21%
		0,96	0,98	0,97	596	
500	0,96	0,89	0,86	0,88	94	13%
		0,9	0,98	0,98	608	

Figure 30: Random Forest Predictions - XOM Stock

G Portfolios performances - Numerical Data

We give below the numerical results of the expected return prediction and the performance of the built strategies from periods 4 to 24.

Period 4

Assets	µ Predicted(%)	µ Historical (%)	µ Realized(%)	σ	µ/σ
AAPL	3,44	2,72	9,73	0,05	0,69
AMZN	1,87	2,54	2,50	0,06	0,31
C	2,74	-0,31	0,18	0,06	0,45
CVS	1,18	1,18	2,14	0,03	0,37
MMM	-1,20	1,14	-0,60	0,03	-0,46
SBUX	-2,36	1,76	-3,18	0,04	-0,64
SCHW	3,94	0,53	2,30	0,05	0,86
XOM	-3,13	0,75	-4,56	0,02	-1,32

Correlation Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	1,00	0,92	-0,70	0,97	0,95	0,95	0,79	0,81
AMZN	0,92	1,00	-0,66	0,92	0,94	0,95	0,75	0,75
C	-0,70	-0,66	1,00	-0,60	-0,60	-0,55	-0,44	-0,73
CVS	0,97	0,92	-0,60	1,00	0,96	0,96	0,87	0,81
MMM	0,95	0,94	-0,60	0,96	1,00	0,96	0,79	0,80
SBUX	0,95	0,95	-0,55	0,96	0,96	1,00	0,77	0,74
SCHW	0,79	0,75	-0,44	0,87	0,79	0,77	1,00	0,69
XOM	0,81	0,75	-0,73	0,81	0,80	0,74	0,69	1,00

Covariance Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	0,31%	0,28%	-0,21%	0,15%	0,12%	0,17%	0,18%	0,10%
AMZN	0,28%	0,38%	-0,25%	0,18%	0,15%	0,22%	0,21%	0,11%
C	-0,21%	-0,25%	0,38%	-0,12%	-0,09%	-0,12%	-0,13%	-0,11%
CVS	0,15%	0,18%	-0,12%	0,10%	0,08%	0,11%	0,13%	0,06%
MMM	0,12%	0,15%	-0,09%	0,08%	0,07%	0,09%	0,09%	0,05%
SBUX	0,17%	0,22%	-0,12%	0,11%	0,09%	0,14%	0,13%	0,06%
SCHW	0,18%	0,21%	-0,13%	0,13%	0,09%	0,13%	0,21%	0,08%
XOM	0,10%	0,11%	-0,11%	0,06%	0,05%	0,06%	0,08%	0,06%

			No shorting		Shorting Allowed	
	Stocks	Equally distributed Portfolio	Prediction Enabled MVO	Classic MVO	Prediction Enabled MVO	Classic MVO
	AAPL	12,5%	20,0%	20,0%	20,0%	20,0%
	AMZN	12,5%	20,0%	20,0%	20,0%	20,0%
	C	12,5%	20,0%	0,0%	20,0%	-20,0%
	CVS	12,5%	20,0%	20,0%	20,0%	20,0%
	MMM	12,5%	0,0%	20,0%	20,0%	20,0%
	SBUX	12,5%	0,0%	20,0%	0,0%	20,0%
	SCHW	12,5%	20,0%	0,0%	20,0%	0,0%
	XOM	12,5%	0,0%	0,0%	-20,0%	20,0%
	SUM Of Weights	100%	100%	100%	100%	100%
Predicted	Volatility		0,1%	0,2%	0,1%	0,3%
	Portfolio Returns (%)		2,64	1,87	3,02	2,08
	Sharpe Ratio		0,89	0,46	0,97	0,39
Realized	Volatility	0,1%	0,1%	4,0%	0,1%	0,3%
	Portfolio Ret	1,06	3,37	2,12	4,16	1,17
	Sharpe Ratio	0,38	1,14	0,11	1,33	0,22

Period 5

Assets	µ Predicted(%)	µ Historical (%)	µ Realized(%)	σ	µ/σ
AAPL	2,05	3,03	5,20	0,05	0,41
AMZN	2,15	2,68	1,81	0,06	0,36
C	-2,22	-0,25	-7,63	0,06	-0,36
CVS	-3,82	1,10	-0,21	0,03	-1,20
MMM	-5,66	1,13	-2,65	0,03	-2,20
SBUX	-2,28	1,72	-3,71	0,04	-0,62
SCHW	1,77	0,62	0,42	0,05	0,39
XOM	1,83	0,72	-4,65	0,02	0,77

Correlation Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	1,00	0,92	-0,69	0,97	0,96	0,95	0,80	0,81
AMZN	0,92	1,00	-0,65	0,92	0,94	0,95	0,77	0,75
C	-0,69	-0,65	1,00	-0,60	-0,59	-0,55	-0,46	-0,73
CVS	0,97	0,92	-0,60	1,00	0,96	0,96	0,88	0,81
MMM	0,96	0,94	-0,59	0,96	1,00	0,96	0,81	0,80
SBUX	0,95	0,95	-0,55	0,96	0,96	1,00	0,79	0,75
SCHW	0,80	0,77	-0,46	0,88	0,81	0,79	1,00	0,71
XOM	0,81	0,75	-0,73	0,81	0,80	0,75	0,71	1,00

Covariance Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	0,31%	0,27%	-0,21%	0,15%	0,12%	0,17%	0,18%	0,09%
AMZN	0,27%	0,36%	-0,24%	0,18%	0,15%	0,21%	0,21%	0,11%
C	-0,21%	-0,24%	0,38%	-0,12%	-0,09%	-0,12%	-0,13%	-0,11%
CVS	0,15%	0,18%	-0,12%	0,10%	0,08%	0,11%	0,13%	0,06%
MMM	0,12%	0,15%	-0,09%	0,08%	0,07%	0,09%	0,10%	0,05%
SBUX	0,17%	0,21%	-0,12%	0,11%	0,09%	0,13%	0,13%	0,06%
SCHW	0,18%	0,21%	-0,13%	0,13%	0,10%	0,13%	0,21%	0,08%
XOM	0,09%	0,11%	-0,11%	0,06%	0,05%	0,06%	0,08%	0,06%

			No shorting		Shorting Allowed	
	Stocks	Equally distributed Portfolio	Prediction Enabled MVO	Classic MVO	Prediction Enabled MVO	Classic MVO
	AAPL	12,5%	20,0%	20,0%	20,0%	20,0%
	AMZN	12,5%	20,0%	20,0%	20,0%	20,0%
	C	12,5%	20,0%	0,0%	20,0%	-20,0%
	CVS	12,5%	0,0%	20,0%	0,0%	20,0%
	MMM	12,5%	0,0%	20,0%	-20,0%	20,0%
	SBUX	12,5%	0,0%	20,0%	20,0%	20,0%
	SCHW	12,5%	20,0%	0,0%	20,0%	0,0%
	XOM	12,5%	20,0%	0,0%	20,0%	20,0%
	SUM Of Weights	100%	100%	100%	100%	100%
Predicted	Volatility		0,1%	0,2%	0,1%	0,3%
	Portfolio Returns (%)		1,11	1,93	1,79	2,13
	Sharpe Ratio		0,42	0,48	0,62	0,40
Realized	Volatility	0,1%	0,1%	4,0%	0,1%	0,3%
	Portfolio Ret	-1,43	-0,97	0,09	-1,18	0,68
	Sharpe Ratio	-0,51	-0,36	0,00	-0,41	0,13

Period 6

Assets	µ Predicted(%)	µ Historical (%)	µ Realized(%)	σ	µ/σ
AAPL	2,14	3,14	2,31	0,05	0,43
AMZN	2,52	2,69	4,44	0,06	0,42
C	2,09	-0,24	-3,81	0,06	0,34
CVS	2,90	0,99	2,14	0,03	0,92
MMM	-5,81	1,09	-2,79	0,03	-2,26
SBUX	2,40	1,63	-1,80	0,04	0,66
SCHW	1,62	0,62	1,14	0,05	0,35
XOM	1,07	0,71	3,00	0,02	0,45

Correlation Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	1,00	0,92	-0,69	0,97	0,96	0,95	0,82	0,81
AMZN	0,92	1,00	-0,65	0,91	0,94	0,95	0,79	0,74
C	-0,69	-0,65	1,00	-0,61	-0,59	-0,55	-0,48	-0,73
CVS	0,97	0,91	-0,61	1,00	0,96	0,96	0,89	0,81
MMM	0,96	0,94	-0,59	0,96	1,00	0,96	0,84	0,80
SBUX	0,95	0,95	-0,55	0,96	0,96	1,00	0,81	0,75
SCHW	0,82	0,79	-0,48	0,89	0,84	0,81	1,00	0,74
XOM	0,81	0,74	-0,73	0,81	0,80	0,75	0,74	1,00

Covariance Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	0,31%	0,27%	-0,21%	0,15%	0,12%	0,17%	0,19%	0,09%
AMZN	0,27%	0,36%	-0,24%	0,17%	0,15%	0,21%	0,22%	0,11%
C	-0,21%	-0,24%	0,38%	-0,12%	-0,09%	-0,12%	-0,13%	-0,11%
CVS	0,15%	0,17%	-0,12%	0,10%	0,08%	0,11%	0,13%	0,06%
MMM	0,12%	0,15%	-0,09%	0,08%	0,07%	0,09%	0,10%	0,05%
SBUX	0,17%	0,21%	-0,12%	0,11%	0,09%	0,13%	0,13%	0,06%
SCHW	0,19%	0,22%	-0,13%	0,13%	0,10%	0,13%	0,21%	0,08%
XOM	0,09%	0,11%	-0,11%	0,06%	0,05%	0,06%	0,08%	0,06%

			No shorting		Shorting Allowed	
	Stocks	Equally distributed Portfolio	Prediction Enabled MVO	Classic MVO	Prediction Enabled MVO	Classic MVO
	AAPL	12,5%	20,0%	20,0%	20,0%	20,0%
	AMZN	12,5%	20,0%	20,0%	20,0%	20,0%
	C	12,5%	20,0%	0,0%	20,0%	0,0%
	CVS	12,5%	20,0%	20,0%	0,0%	20,0%
	MMM	12,5%	0,0%	20,0%	-20,0%	20,0%
	SBUX	12,5%	20,0%	20,0%	20,0%	20,0%
	SCHW	12,5%	0,0%	0,0%	20,0%	0,0%
	XOM	12,5%	0,0%	0,0%	20,0%	0,0%
	SUM Of Weights	100%	100%	100%	100%	100%
Predicted	Volatility		0,1%	0,2%	0,1%	0,2%
	Portfolio Returns (%)		2,41	1,91	3,53	1,91
	Sharpe Ratio		0,85	0,48	1,22	0,48
Realized	Volatility	0,1%	0,1%	4,0%	0,1%	0,2%
	Portfolio Ret	0,58	0,65	0,86	1,61	0,86
	Sharpe Ratio	0,21	0,23	0,04	0,56	0,22

Figure 31:

Period 7

Assets	µ Predicted(%)	µ Historical (%)	µ Realized(%)	σ	µ/σ
AAPL	-4,29	3,23	-8,37	0,05	-0,88
AMZN	-5,68	2,82	-8,58	0,06	-0,96
C	3,69	-0,19	-13,00	0,06	0,60
CVS	-6,10	0,93	6,83	0,03	-1,94
MMM	3,52	1,00	2,36	0,03	1,39
SBUX	1,36	1,64	3,98	0,04	0,37
SCHW	4,43	0,69	5,56	0,05	0,97
XOM	1,21	0,71	1,18	0,02	0,51

Correlation Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	1,00	0,92	-0,69	0,97	0,96	0,95	0,84	0,81
AMZN	0,92	1,00	-0,64	0,91	0,94	0,95	0,81	0,74
C	-0,69	-0,64	1,00	-0,61	-0,59	-0,55	-0,49	-0,73
CVS	0,97	0,91	-0,61	1,00	0,96	0,96	0,90	0,81
MMM	0,96	0,94	-0,59	0,96	1,00	0,96	0,85	0,80
SBUX	0,95	0,95	-0,55	0,96	0,96	1,00	0,82	0,75
SCHW	0,84	0,81	-0,49	0,90	0,85	0,82	1,00	0,75
XOM	0,81	0,74	-0,73	0,81	0,80	0,75	0,75	1,00

Covariance Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	0,31%	0,27%	-0,21%	0,15%	0,12%	0,17%	0,19%	0,09%
AMZN	0,27%	0,35%	-0,23%	0,17%	0,14%	0,20%	0,22%	0,10%
C	-0,21%	-0,23%	0,38%	-0,12%	-0,09%	-0,12%	-0,14%	-0,11%
CVS	0,15%	0,17%	-0,12%	0,10%	0,08%	0,11%	0,13%	0,06%
MMM	0,12%	0,14%	-0,09%	0,08%	0,06%	0,09%	0,10%	0,05%
SBUX	0,17%	0,20%	-0,12%	0,11%	0,09%	0,13%	0,14%	0,06%
SCHW	0,19%	0,22%	-0,14%	0,13%	0,10%	0,14%	0,21%	0,08%
XOM	0,09%	0,10%	-0,11%	0,06%	0,05%	0,06%	0,08%	0,06%

			No shorting		Shorting Allowed	
	Stocks	Equally distributed Portfolio	Prediction Enabled MVO	Classic MVO	Prediction Enabled MVO	Classic MVO
	AAPL	12,5%	0,0%	20,0%	20,0%	20,0%
	AMZN	12,5%	0,0%	20,0%	0,0%	20,0%
	C	12,5%	20,0%	0,0%	20,0%	-20,0%
	CVS	12,5%	0,0%	20,0%	-20,0%	20,0%
	MMM	12,5%	20,0%	20,0%	20,0%	20,0%
	SBUX	12,5%	20,0%	20,0%	20,0%	20,0%
	SCHW	12,5%	20,0%	0,0%	20,0%	0,0%
	XOM	12,5%	20,0%	0,0%	20,0%	20,0%
	SUM Of Weights	100%	100%	100%	100%	100%
Predicted	Volatility		0,0%	0,2%	0,0%	0,3%
	Portfolio Returns (%)		2,84	1,92	3,20	2,10
	Sharpe Ratio		1,45	0,49	1,45	0,40
Realized	Volatility	0,1%	0,0%	3,9%	0,0%	0,3%
	Portfolio Ret	-1,25	0,02	-0,76	-3,02	2,08
	Sharpe Ratio	-0,45	0,01	-0,04	-1,36	0,40

Period 8

Assets	µ Predicted(%)	µ Historical (%)	µ Realized(%)	σ	µ/σ
AAPL	2,14	3,17	8,95	0,05	0,44
AMZN	2,50	2,91	6,83	0,06	0,43
C	5,92	-0,16	3,90	0,06	0,96
CVS	2,69	0,90	4,66	0,03	0,85
MMM	1,40	0,99	2,77	0,03	0,56
SBUX	2,89	1,69	8,80	0,04	0,79
SCHW	6,12	0,79	2,21	0,05	1,34
XOM	1,94	0,76	6,19	0,02	0,83

Correlation Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	1,00	0,92	-0,69	0,96	0,96	0,95	0,85	0,81
AMZN	0,92	1,00	-0,64	0,91	0,94	0,95	0,82	0,74
C	-0,69	-0,64	1,00	-0,61	-0,59	-0,54	-0,50	-0,73
CVS	0,96	0,91	-0,61	1,00	0,96	0,96	0,90	0,82
MMM	0,96	0,94	-0,59	0,96	1,00	0,97	0,87	0,80
SBUX	0,95	0,95	-0,54	0,96	0,97	1,00	0,83	0,75
SCHW	0,85	0,82	-0,50	0,90	0,87	0,83	1,00	0,76
XOM	0,81	0,74	-0,73	0,82	0,80	0,75	0,76	1,00

Covariance Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	0,31%	0,26%	-0,21%	0,15%	0,12%	0,17%	0,19%	0,09%
AMZN	0,26%	0,33%	-0,23%	0,17%	0,14%	0,20%	0,22%	0,10%
C	-0,21%	-0,23%	0,38%	-0,12%	-0,09%	-0,12%	-0,14%	-0,11%
CVS	0,15%	0,17%	-0,12%	0,10%	0,08%	0,11%	0,13%	0,06%
MMM	0,12%	0,14%	-0,09%	0,08%	0,06%	0,09%	0,10%	0,05%
SBUX	0,17%	0,20%	-0,12%	0,11%	0,09%	0,13%	0,14%	0,06%
SCHW	0,19%	0,22%	-0,14%	0,13%	0,10%	0,14%	0,21%	0,08%
XOM	0,09%	0,10%	-0,11%	0,06%	0,05%	0,06%	0,08%	0,06%

			No shorting		Shorting Allowed	
	Stocks	Equally distributed Portfolio	Prediction Enabled MVO	Classic MVO	Prediction Enabled MVO	Classic MVO
	AAPL	12,5%	0,0%	20,0%	20,0%	20,0%
	AMZN	12,5%	20,0%	20,0%	20,0%	20,0%
	C	12,5%	20,0%	0,0%	20,0%	-20,0%
	CVS	12,5%	20,0%	20,0%	20,0%	20,0%
	MMM	12,5%	0,0%	20,0%	-20,0%	20,0%
	SBUX	12,5%	20,0%	20,0%	20,0%	20,0%
	SCHW	12,5%	20,0%	0,0%	20,0%	20,0%
	XOM	12,5%	0,0%	0,0%	0,0%	0,0%
	SUM Of Weights	100%	100%	100%	100%	100%
Predicted	Volatility		0,1%	0,2%	0,1%	0,3%
	Portfolio Returns (%)		4,03	1,93	4,17	2,12
	Sharpe Ratio		1,47	0,49	1,33	0,38
Realized	Volatility	0,1%	0,1%	3,9%	0,1%	0,3%
	Portfolio Ret	5,54	5,28	6,40	6,52	6,07
	Sharpe Ratio	2,00	1,93	0,32	2,08	1,09

Period 9

Assets	µ Predicted(%)	µ Historical (%)	µ Realized(%)	σ	µ/σ
AAPL	1,84	3,10	5,22	0,05	0,38
AMZN	1,94	2,95	7,01	0,06	0,34
C	-2,40	-0,15	3,58	0,06	-0,39
CVS	1,33	0,89	-0,31	0,03	0,42
MMM	-1,14	1,01	-1,22	0,03	-0,45
SBUX	-2,02	1,64	-1,28	0,04	-0,56
SCHW	7,71	0,86	2,21	0,05	1,69
XOM	-1,79	0,76	-5,13	0,02	-0,76

Correlation Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	1,00	0,92	-0,69	0,96	0,96	0,95	0,86	0,81
AMZN	0,92	1,00	-0,63	0,90	0,94	0,95	0,84	0,74
C	-0,69	-0,63	1,00	-0,61	-0,58	-0,54	-0,51	-0,73
CVS	0,96	0,90	-0,61	1,00	0,96	0,96	0,90	0,82
MMM	0,96	0,94	-0,58	0,96	1,00	0,97	0,88	0,79
SBUX	0,95	0,95	-0,54	0,96	0,97	1,00	0,84	0,75
SCHW	0,86	0,84	-0,51	0,90	0,88	0,84	1,00	0,77
XOM	0,81	0,74	-0,73	0,82	0,79	0,75	0,77	1,00

Covariance Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	0,30%	0,26%	-0,21%	0,15%	0,12%	0,17%	0,19%	0,09%
AMZN	0,26%	0,33%	-0,22%	0,16%	0,14%	0,20%	0,22%	0,10%
C	-0,21%	-0,22%	0,38%	-0,12%	-0,09%	-0,12%	-0,14%	-0,11%
CVS	0,15%	0,16%	-0,12%	0,10%	0,08%	0,11%	0,13%	0,06%
MMM	0,12%	0,14%	-0,09%	0,08%	0,06%	0,09%	0,10%	0,05%
SBUX	0,17%	0,20%	-0,12%	0,11%	0,09%	0,13%	0,14%	0,06%
SCHW	0,19%	0,22%	-0,14%	0,13%	0,10%	0,14%	0,21%	0,08%
XOM	0,09%	0,10%	-0,11%	0,06%	0,05%	0,06%	0,08%	0,06%

			No shorting		Shorting Allowed	
	Stocks	Equally distributed Portfolio	Prediction Enabled MVO	Classic MVO	Prediction Enabled MVO	Classic MVO
	AAPL	12,5%	20,0%	20,0%	20,0%	20,0%
	AMZN	12,5%	20,0%	20,0%	20,0%	20,0%
	C	12,5%	0,0%	0,0%	-20,0%	-20,0%
	CVS	12,5%	20,0%	20,0%	20,0%	20,0%
	MMM	12,5%	20,0%	20,0%	20,0%	20,0%
	SBUX	12,5%	0,0%	20,0%	0,0%	20,0%
	SCHW	12,5%	20,0%	0,0%	20,0%	20,0%
	XOM	12,5%	0,0%	0,0%	20,0%	0,0%
	SUM Of Weights	100%	100%	100%	100%	100%
Predicted	Volatility		0,2%	0,2%	0,3%	0,3%
	Portfolio Returns (%)		2,34	1,92	2,46	2,12
	Sharpe Ratio		0,58	0,49	0,46	0,38
Realized	Volatility	0,1%	0,2%	3,9%	0,3%	0,3%
	Portfolio Ret	1,26	2,58	1,88	0,84	1,61
	Sharpe Ratio	0,46	0,64	0,10	0,16	0,29

Figure 32:

Period 10

Assets	μ Predicted(%)	μ Historical (%)	μ Realized(%)	σ	μ/σ
AAPL	3,28	3,15	10,83	0,05	0,69
AMZN	2,07	3,13	3,55	0,06	0,37
C	-4,78	-0,11	-4,54	0,06	-0,78
CVS	-2,86	0,89	-2,08	0,03	-0,91
MMM	7,94	1,01	0,65	0,03	3,17
SBUX	-2,41	1,65	-2,80	0,04	-0,66
SCHW	-2,50	1,00	-1,27	0,04	-0,56
XOM	-2,91	0,73	-4,05	0,02	-1,24

Correlation Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	1,00	0,92	-0,69	0,96	0,96	0,95	0,86	0,81
AMZN	0,92	1,00	-0,63	0,90	0,94	0,95	0,84	0,74
C	-0,69	-0,63	1,00	-0,61	-0,58	-0,54	-0,51	-0,73
CVS	0,96	0,90	-0,61	1,00	0,96	0,96	0,90	0,82
MMM	0,96	0,94	-0,58	0,96	1,00	0,97	0,88	0,79
SBUX	0,95	0,95	-0,54	0,96	0,97	1,00	0,84	0,75
SCHW	0,86	0,84	-0,51	0,90	0,88	0,84	1,00	0,77
XOM	0,81	0,74	-0,73	0,82	0,79	0,75	0,77	1,00

Covariance Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	0,30%	0,24%	-0,20%	0,14%	0,11%	0,16%	0,18%	0,09%
AMZN	0,24%	0,31%	-0,22%	0,16%	0,13%	0,19%	0,21%	0,10%
C	-0,20%	-0,22%	0,38%	-0,12%	-0,09%	-0,12%	-0,14%	-0,11%
CVS	0,14%	0,16%	-0,12%	0,10%	0,08%	0,11%	0,13%	0,06%
MMM	0,11%	0,13%	-0,09%	0,08%	0,06%	0,09%	0,10%	0,05%
SBUX	0,16%	0,19%	-0,12%	0,11%	0,09%	0,13%	0,14%	0,06%
SCHW	0,18%	0,21%	-0,14%	0,13%	0,10%	0,14%	0,20%	0,08%
XOM	0,09%	0,10%	-0,11%	0,06%	0,05%	0,06%	0,08%	0,06%

			No shorting		Shorting Allowed	
	Stocks	Equally distributed Portfolio	Prediction Enabled MVO	Classic MVO	Prediction Enabled MVO	Classic MVO
	AAPL	12,5%	20,0%	20,0%	20,0%	20,0%
	AMZN	12,5%	20,0%	20,0%	20,0%	20,0%
	C	12,5%	0,0%	0,0%	-20,0%	-20,0%
	CVS	12,5%	0,0%	0,0%	20,0%	20,0%
	MMM	12,5%	20,0%	20,0%	20,0%	20,0%
	SBUX	12,5%	20,0%	20,0%	20,0%	20,0%
	SCHW	12,5%	20,0%	20,0%	20,0%	20,0%
	XOM	12,5%	0,0%	0,0%	0,0%	0,0%
	SUM Of Weights	100%	100%	100%	100%	100%
Predicted	Volatility		0,2%	0,2%	0,3%	0,3%
	Portfolio Returns (%)		1,68	1,99	2,06	2,19
	Sharpe Ratio		0,41	0,49	0,37	0,40
Realized	Volatility	0,1%	0,2%	4,0%	0,3%	0,3%
	Portfolio Ret	0,04	2,19	2,19	2,68	2,68
	Sharpe Ratio	0,01	0,54	0,11	0,49	0,49

Period 11

Assets	μ Predicted(%)	μ Historical (%)	μ Realized(%)	σ	μ/σ
AAPL	2,61	3,15	5,12	0,05	0,56
AMZN	1,73	3,14	4,03	0,06	0,31
C	3,22	-0,05	4,01	0,06	0,52
CVS	1,63	0,92	4,18	0,03	0,52
MMM	1,26	1,06	8,17	0,02	0,51
SBUX	-1,74	1,71	-2,14	0,04	-0,48
SCHW	2,13	1,09	2,87	0,04	0,49
XOM	-1,15	0,73	-0,54	0,02	-0,49

Correlation Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	1,00	0,92	-0,68	0,96	0,96	0,95	0,87	0,81
AMZN	0,92	1,00	-0,62	0,89	0,95	0,95	0,86	0,73
C	-0,68	-0,62	1,00	-0,61	-0,58	-0,54	-0,50	-0,72
CVS	0,96	0,89	-0,61	1,00	0,96	0,96	0,89	0,82
MMM	0,96	0,95	-0,58	0,96	1,00	0,97	0,89	0,79
SBUX	0,95	0,95	-0,54	0,96	0,97	1,00	0,85	0,75
SCHW	0,87	0,86	-0,50	0,89	0,89	0,85	1,00	0,77
XOM	0,81	0,73	-0,72	0,82	0,79	0,75	0,77	1,00

Covariance Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	0,29%	0,24%	-0,20%	0,14%	0,11%	0,16%	0,18%	0,09%
AMZN	0,24%	0,31%	-0,21%	0,16%	0,13%	0,19%	0,21%	0,10%
C	-0,20%	-0,21%	0,38%	-0,12%	-0,09%	-0,12%	-0,14%	-0,10%
CVS	0,14%	0,16%	-0,12%	0,10%	0,08%	0,11%	0,12%	0,06%
MMM	0,11%	0,13%	-0,09%	0,08%	0,06%	0,09%	0,10%	0,05%
SBUX	0,16%	0,19%	-0,12%	0,11%	0,09%	0,13%	0,13%	0,06%
SCHW	0,18%	0,21%	-0,14%	0,12%	0,10%	0,13%	0,19%	0,08%
XOM	0,09%	0,10%	-0,10%	0,06%	0,05%	0,06%	0,08%	0,06%

			No shorting		Shorting Allowed	
	Stocks	Equally distributed Portfolio	Prediction Enabled MVO	Classic MVO	Prediction Enabled MVO	Classic MVO
	AAPL	12,5%	20,0%	20,0%	20,0%	20,0%
	AMZN	12,5%	20,0%	20,0%	20,0%	20,0%
	C	12,5%	20,0%	0,0%	20,0%	-20,0%
	CVS	12,5%	20,0%	0,0%	20,0%	20,0%
	MMM	12,5%	0,0%	20,0%	20,0%	20,0%
	SBUX	12,5%	0,0%	20,0%	-20,0%	20,0%
	SCHW	12,5%	20,0%	20,0%	20,0%	20,0%
	XOM	12,5%	0,0%	0,0%	0,0%	0,0%
	SUM Of Weights	100%	100%	100%	100%	100%
Predicted	Volatility		0,1%	0,2%	0,1%	0,3%
	Portfolio Returns (%)		2,26	2,03	2,86	2,22
	Sharpe Ratio		0,80	0,51	1,09	0,41
Realized	Volatility	0,1%	0,1%	4,0%	0,1%	0,3%
	Portfolio Ret	3,21	4,04	3,61	6,10	3,64
	Sharpe Ratio	1,19	1,43	0,18	2,32	0,67

Period 12

Assets	μ Predicted(%)	μ Historical (%)	μ Realized(%)	σ	μ/σ
AAPL	1,42	3,09	1,75	0,05	0,30
AMZN	1,95	2,92	5,58	0,05	0,36
C	-2,47	-0,12	-2,81	0,06	-0,40
CVS	-3,80	0,91	-2,46	0,03	-1,21
MMM	-2,72	1,01	-0,75	0,02	-1,10
SBUX	1,58	1,74	5,95	0,04	0,44
SCHW	-3,90	0,95	-4,02	0,04	-0,90
XOM	0,84	0,68	1,76	0,02	0,36

Correlation Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	1,00	0,93	-0,68	0,96	0,96	0,95	0,87	0,80
AMZN	0,93	1,00	-0,61	0,88	0,95	0,95	0,86	0,73
C	-0,68	-0,61	1,00	-0,61	-0,57	-0,54	-0,51	-0,72
CVS	0,96	0,88	-0,61	1,00	0,95	0,95	0,89	0,82
MMM	0,96	0,95	-0,57	0,95	1,00	0,97	0,90	0,78
SBUX	0,95	0,95	-0,54	0,95	0,97	1,00	0,86	0,75
SCHW	0,87	0,86	-0,51	0,89	0,90	0,86	1,00	0,77
XOM	0,80	0,73	-0,72	0,82	0,78	0,75	0,77	1,00

Covariance Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	0,29%	0,23%	-0,19%	0,14%	0,11%	0,16%	0,18%	0,09%
AMZN	0,23%	0,29%	-0,20%	0,15%	0,13%	0,18%	0,20%	0,09%
C	-0,19%	-0,20%	0,38%	-0,12%	-0,09%	-0,12%	-0,13%	-0,10%
CVS	0,14%	0,15%	-0,12%	0,10%	0,07%	0,11%	0,12%	0,06%
MMM	0,11%	0,13%	-0,09%	0,07%	0,06%	0,09%	0,10%	0,05%
SBUX	0,16%	0,18%	-0,12%	0,11%	0,09%	0,13%	0,13%	0,06%
SCHW	0,18%	0,20%	-0,13%	0,12%	0,10%	0,13%	0,19%	0,08%
XOM	0,09%	0,09%	-0,10%	0,06%	0,05%	0,06%	0,08%	0,05%

			No shorting		Shorting Allowed	
	Stocks	Equally distributed Portfolio	Prediction Enabled MVO	Classic MVO	Prediction Enabled MVO	Classic MVO
	AAPL	12,5%	20,0%	20,0%	20,0%	20,0%
	AMZN	12,5%	20,0%	20,0%	20,0%	20,0%
	C	12,5%	20,0%	0,0%	20,0%	-20,0%
	CVS	12,5%	0,0%	0,0%	0,0%	20,0%
	MMM	12,5%	0,0%	20,0%	20,0%	20,0%
	SBUX	12,5%	20,0%	20,0%	20,0%	20,0%
	SCHW	12,5%	0,0%	20,0%	-20,0%	20,0%
	XOM	12,5%	20,0%	0,0%	20,0%	0,0%
	SUM Of Weights	100%	100%	100%	100%	100%
Predicted	Volatility		0,1%	0,2%	0,0%	0,3%
	Portfolio Returns (%)		0,66	1,94	0,90	2,15
	Sharpe Ratio		0,27	0,49	0,41	0,40
Realized	Volatility	0,1%	0,1%	4,0%	0,0%	0,3%
	Portfolio Ret	0,63	2,45	1,70	3,10	1,78
	Sharpe Ratio	0,23	1,00	0,09	1,41	0,33

Figure 33:

Period 13

Assets	µ Predicted(%)	µ Historical (%)	µ Realized(%)	σ	µ/σ
AAPL	1,84	3,18	7,08	0,05	0,40
AMZN	1,89	2,96	4,63	0,05	0,35
C	1,92	-0,13	4,71	0,06	0,31
CVS	8,75	0,94	1,24	0,03	2,78
MMM	1,12	1,01	3,46	0,02	0,45
SBUX	1,56	1,76	5,10	0,04	0,43
SCHW	1,85	0,98	0,93	0,04	0,43
XOM	-0,93	0,67	-0,52	0,02	-0,40

Correlation Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	1,00	0,93	-0,67	0,95	0,96	0,95	0,88	0,80
AMZN	0,93	1,00	-0,60	0,88	0,95	0,95	0,87	0,72
C	-0,67	-0,60	1,00	-0,61	-0,57	-0,54	-0,51	-0,72
CVS	0,95	0,88	-0,61	1,00	0,95	0,95	0,89	0,82
MMM	0,96	0,95	-0,57	0,95	1,00	0,97	0,90	0,78
SBUX	0,95	0,95	-0,54	0,95	0,97	1,00	0,87	0,75
SCHW	0,88	0,87	-0,51	0,89	0,90	0,87	1,00	0,77
XOM	0,80	0,72	-0,72	0,82	0,78	0,75	0,77	1,00

Covariance Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	0,29%	0,23%	-0,19%	0,14%	0,11%	0,16%	0,18%	0,09%
AMZN	0,23%	0,29%	-0,20%	0,15%	0,13%	0,18%	0,20%	0,09%
C	-0,19%	-0,20%	0,38%	-0,12%	-0,09%	-0,12%	-0,13%	-0,10%
CVS	0,14%	0,15%	-0,12%	0,10%	0,07%	0,11%	0,12%	0,06%
MMM	0,11%	0,13%	-0,09%	0,07%	0,06%	0,09%	0,10%	0,05%
SBUX	0,16%	0,18%	-0,12%	0,11%	0,09%	0,13%	0,13%	0,06%
SCHW	0,18%	0,20%	-0,13%	0,12%	0,10%	0,13%	0,19%	0,08%
XOM	0,09%	0,09%	-0,10%	0,06%	0,05%	0,06%	0,08%	0,05%

		No shorting		Shorting Allowed	
Stocks	Equally distributed Portfolio	Prediction Enabled MVO	Classic MVO	Prediction Enabled MVO	Classic MVO
AAPL	12,5%	20,0%	20,0%	20,0%	20,0%
AMZN	12,5%	20,0%	20,0%	20,0%	20,0%
C	12,5%	20,0%	0,0%	20,0%	-20,0%
CVS	12,5%	20,0%	0,0%	20,0%	20,0%
MMM	12,5%	0,0%	20,0%	0,0%	20,0%
SBUX	12,5%	0,0%	20,0%	20,0%	20,0%
SCHW	12,5%	20,0%	20,0%	20,0%	20,0%
XOM	12,5%	0,0%	0,0%	-20,0%	0,0%
SUM Of Weights	100%	100%	100%	100%	100%

Predicted	Volatility		0,1%	0,2%	0,1%	0,3%
	Portfolio Returns (%)		3,25	1,98	3,75	2,19
	Sharpe Ratio		1,16	0,50	1,18	0,41
Realized	Volatility	0,1%	0,1%	4,0%	0,1%	0,3%
	Portfolio Ret	3,33	3,72	4,24	4,84	3,54
	Sharpe Ratio	1,24	1,33	0,21	1,53	0,66

Period 14

Assets	µ Predicted(%)	µ Historical (%)	µ Realized(%)	σ	µ/σ
AAPL	1,69	3,16	1,00	0,05	0,37
AMZN	2,10	3,02	5,98	0,05	0,39
C	2,10	-0,11	-3,26	0,06	0,34
CVS	-2,66	1,05	2,95	0,03	-0,89
MMM	1,16	1,05	4,67	0,02	0,47
SBUX	1,51	1,74	2,07	0,04	0,42
SCHW	-2,31	1,06	-0,62	0,04	-0,54
XOM	-1,10	0,69	-1,10	0,02	-0,47

Correlation Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	1,00	0,93	-0,67	0,95	0,97	0,96	0,88	0,80
AMZN	0,93	1,00	-0,59	0,87	0,95	0,94	0,87	0,71
C	-0,67	-0,59	1,00	-0,61	-0,56	-0,54	-0,51	-0,72
CVS	0,95	0,87	-0,61	1,00	0,94	0,95	0,88	0,83
MMM	0,97	0,95	-0,56	0,94	1,00	0,97	0,91	0,77
SBUX	0,96	0,94	-0,54	0,95	0,97	1,00	0,87	0,75
SCHW	0,88	0,87	-0,51	0,88	0,91	0,87	1,00	0,77
XOM	0,80	0,71	-0,72	0,83	0,77	0,75	0,77	1,00

Covariance Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	0,29%	0,23%	-0,19%	0,13%	0,11%	0,16%	0,17%	0,09%
AMZN	0,23%	0,29%	-0,20%	0,14%	0,13%	0,18%	0,20%	0,09%
C	-0,19%	-0,20%	0,38%	-0,11%	-0,09%	-0,12%	-0,13%	-0,10%
CVS	0,13%	0,14%	-0,11%	0,09%	0,07%	0,10%	0,11%	0,06%
MMM	0,11%	0,13%	-0,09%	0,07%	0,06%	0,09%	0,10%	0,04%
SBUX	0,16%	0,18%	-0,12%	0,10%	0,09%	0,13%	0,13%	0,06%
SCHW	0,17%	0,20%	-0,13%	0,11%	0,10%	0,13%	0,18%	0,08%
XOM	0,09%	0,09%	-0,10%	0,06%	0,04%	0,06%	0,08%	0,05%

		No shorting		Shorting Allowed	
Stocks	Equally distributed Portfolio	Prediction Enabled MVO	Classic MVO	Prediction Enabled MVO	Classic MVO
AAPL	12,5%	20,0%	20,0%	20,0%	20,0%
AMZN	12,5%	20,0%	20,0%	20,0%	20,0%
C	12,5%	20,0%	0,0%	20,0%	-20,0%
CVS	12,5%	0,0%	0,0%	-20,0%	20,0%
MMM	12,5%	20,0%	20,0%	20,0%	20,0%
SBUX	12,5%	20,0%	20,0%	20,0%	20,0%
SCHW	12,5%	0,0%	20,0%	0,0%	20,0%
XOM	12,5%	0,0%	0,0%	20,0%	0,0%
SUM Of Weights	100%	100%	100%	100%	100%

Predicted	Volatility		0,1%	0,2%	0,1%	0,3%
	Portfolio Returns (%)		1,71	2,01	2,02	2,24
	Sharpe Ratio		0,65	0,51	0,84	0,42
Realized	Volatility	0,1%	0,1%	3,9%	0,1%	0,3%
	Portfolio Ret	1,46	2,09	2,62	1,28	3,86
	Sharpe Ratio	0,55	0,80	0,13	0,54	0,72

Period 15

Assets	µ Predicted(%)	µ Historical (%)	µ Realized(%)	σ	µ/σ
AAPL	-4,31	3,21	-7,19	0,05	-0,94
AMZN	-3,02	3,07	-3,07	0,05	-0,57
C	4,34	-0,06	0,60	0,06	0,71
CVS	1,14	1,10	5,90	0,03	0,38
MMM	7,50	1,09	2,23	0,02	3,04
SBUX	-4,80	1,78	-8,57	0,04	-1,35
SCHW	3,62	1,13	4,41	0,04	0,85
XOM	-0,97	0,71	-0,85	0,02	-0,42

Correlation Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	1,00	0,93	-0,66	0,94	0,97	0,96	0,89	0,80
AMZN	0,93	1,00	-0,59	0,86	0,95	0,94	0,88	0,71
C	-0,66	-0,59	1,00	-0,61	-0,56	-0,54	-0,50	-0,72
CVS	0,94	0,86	-0,61	1,00	0,94	0,95	0,88	0,83
MMM	0,97	0,95	-0,56	0,94	1,00	0,97	0,91	0,77
SBUX	0,96	0,94	-0,54	0,95	0,97	1,00	0,87	0,75
SCHW	0,89	0,88	-0,50	0,88	0,91	0,87	1,00	0,76
XOM	0,80	0,71	-0,72	0,83	0,77	0,75	0,76	1,00

Covariance Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	0,29%	0,23%	-0,19%	0,13%	0,11%	0,16%	0,18%	0,09%
AMZN	0,23%	0,28%	-0,19%	0,14%	0,12%	0,18%	0,20%	0,09%
C	-0,19%	-0,19%	0,38%	-0,11%	-0,08%	-0,12%	-0,13%	-0,10%
CVS	0,13%	0,14%	-0,11%	0,09%	0,07%	0,10%	0,11%	0,06%
MMM	0,11%	0,12%	-0,08%	0,07%	0,06%	0,09%	0,10%	0,04%
SBUX	0,16%	0,18%	-0,12%	0,10%	0,09%	0,13%	0,13%	0,06%
SCHW	0,18%	0,20%	-0,13%	0,11%	0,10%	0,13%	0,18%	0,08%
XOM	0,09%	0,09%	-0,10%	0,06%	0,04%	0,06%	0,08%	0,05%

		No shorting		Shorting Allowed	
Stocks	Equally distributed Portfolio	Prediction Enabled MVO	Classic MVO	Prediction Enabled MVO	Classic MVO
AAPL	12,5%	0,0%	20,0%	0,0%	20,0%
AMZN	12,5%	0,0%	20,0%	20,0%	20,0%
C	12,5%	20,0%	0,0%	20,0%	-20,0%
CVS	12,5%	20,0%	20,0%	20,0%	20,0%
MMM	12,5%	20,0%	0,0%	20,0%	20,0%
SBUX	12,5%	0,0%	20,0%	-20,0%	20,0%
SCHW	12,5%	20,0%	20,0%	20,0%	20,0%
XOM	12,5%	20,0%	0,0%	20,0%	0,0%
SUM Of Weights	100%	100%	100%	100%	100%

Predicted	Volatility		0,0%	0,2%	0,0%	0,3%
	Portfolio Returns (%)		3,13	2,06	3,48	2,29
	Sharpe Ratio		1,74	0,51	1,70	0,43
Realized	Volatility	0,1%	0,0%	4,0%	0,0%	0,3%
	Portfolio Ret	-0,82	2,46	-1,71	3,56	-1,38
	Sharpe Ratio	-0,31	1,36	-0,09	1,74	-0,26

Figure 34:

Period 16

Assets	μ Predicted(%)	μ Historical (%)	μ Realized(%)	σ	μ/σ
AAPL	2,42	3,27	10,15	0,05	0,53
AMZN	1,68	3,23	1,34	0,05	0,33
C	3,77	0,01	1,26	0,06	0,62
CVS	7,90	1,11	1,43	0,03	2,66
MMM	-0,96	1,11	-1,04	0,02	-0,39
SBUX	-3,17	1,80	-4,17	0,04	-0,89
SCHW	-1,81	1,22	-0,45	0,04	-0,43
XOM	1,01	0,71	-0,08	0,02	0,43

Correlation Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	1,00	0,93	-0,66	0,94	0,97	0,95	0,89	0,79
AMZN	0,93	1,00	-0,58	0,86	0,96	0,94	0,88	0,70
C	-0,66	-0,58	1,00	-0,61	-0,56	-0,54	-0,50	-0,72
CVS	0,94	0,86	-0,61	1,00	0,93	0,95	0,88	0,83
MMM	0,97	0,96	-0,56	0,93	1,00	0,97	0,92	0,76
SBUX	0,95	0,94	-0,54	0,95	0,97	1,00	0,87	0,75
SCHW	0,89	0,88	-0,50	0,88	0,92	0,87	1,00	0,76
XOM	0,79	0,70	-0,72	0,83	0,76	0,75	0,76	1,00

Covariance Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	0,29%	0,22%	-0,18%	0,13%	0,11%	0,16%	0,17%	0,08%
AMZN	0,22%	0,27%	-0,18%	0,13%	0,12%	0,17%	0,19%	0,08%
C	-0,18%	-0,18%	0,38%	-0,11%	-0,08%	-0,12%	-0,13%	-0,10%
CVS	0,13%	0,13%	-0,11%	0,09%	0,07%	0,10%	0,11%	0,06%
MMM	0,11%	0,12%	-0,08%	0,07%	0,06%	0,08%	0,10%	0,04%
SBUX	0,16%	0,17%	-0,12%	0,10%	0,08%	0,13%	0,13%	0,06%
SCHW	0,17%	0,19%	-0,13%	0,11%	0,10%	0,13%	0,18%	0,07%
XOM	0,08%	0,08%	-0,10%	0,06%	0,04%	0,06%	0,07%	0,05%

		No shorting		Shorting Allowed	
Stocks	Equally distributed Portfolio	Prediction Enabled MVO	Classic MVO	Prediction Enabled MVO	Classic MVO
AAPL	12,5%	20,0%	20,0%	20,0%	20,0%
AMZN	12,5%	20,0%	20,0%	20,0%	20,0%
C	12,5%	20,0%	-40,0%	20,0%	-20,0%
CVS	12,5%	20,0%	20,0%	20,0%	20,0%
MMM	12,5%	20,0%	20,0%	20,0%	20,0%
SBUX	12,5%	-40,0%	20,0%	-20,0%	20,0%
SCHW	12,5%	20,0%	20,0%	0,0%	20,0%
XOM	12,5%	20,0%	20,0%	20,0%	0,0%
SUM Of Weights	100%	100%	100%	100%	100%

		Equally distributed Portfolio	Prediction Enabled MVO (No shorting)	Classic MVO (No shorting)	Prediction Enabled MVO (Shorting Allowed)	Classic MVO (Shorting Allowed)
Predicted	Volatility		0,0%	0,4%	0,0%	0,3%
	Portfolio Returns (%)		4,07	2,49	3,80	2,35
	Sharpe Ratio		1,85	0,38	1,84	0,45
Realized	Volatility	0,1%	0,0%	6,6%	0,0%	0,3%
	Portfolio Ret	1,06	4,19	0,93	3,45	1,20
	Sharpe Ratio	0,40	1,90	0,04	1,67	0,23

Period 17

Assets	μ Predicted(%)	μ Historical (%)	μ Realized(%)	σ	μ/σ
AAPL	1,94	3,27	4,01	0,05	0,43
AMZN	-3,46	3,27	-2,26	0,05	-0,68
C	-2,09	0,00	0,92	0,06	-0,34
CVS	5,05	1,25	-1,79	0,03	1,71
MMM	6,40	1,10	-2,64	0,02	2,63
SBUX	-2,45	1,75	-0,14	0,04	-0,70
SCHW	-4,43	1,13	-4,23	0,04	-1,06
XOM	-2,00	0,72	-2,81	0,02	-0,86

Correlation Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	1,00	0,94	-0,65	0,94	0,97	0,95	0,90	0,79
AMZN	0,94	1,00	-0,58	0,86	0,96	0,94	0,89	0,70
C	-0,65	-0,58	1,00	-0,61	-0,55	-0,53	-0,50	-0,72
CVS	0,94	0,86	-0,61	1,00	0,93	0,95	0,88	0,83
MMM	0,97	0,96	-0,55	0,93	1,00	0,96	0,92	0,76
SBUX	0,95	0,94	-0,53	0,95	0,96	1,00	0,87	0,75
SCHW	0,90	0,89	-0,50	0,88	0,92	0,87	1,00	0,75
XOM	0,79	0,70	-0,72	0,83	0,76	0,75	0,75	1,00

Covariance Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	0,28%	0,21%	-0,18%	0,13%	0,11%	0,15%	0,17%	0,08%
AMZN	0,21%	0,25%	-0,18%	0,13%	0,12%	0,17%	0,19%	0,08%
C	-0,18%	-0,18%	0,37%	-0,11%	-0,08%	-0,12%	-0,13%	-0,10%
CVS	0,13%	0,13%	-0,11%	0,09%	0,07%	0,10%	0,11%	0,06%
MMM	0,11%	0,12%	-0,08%	0,07%	0,06%	0,08%	0,09%	0,04%
SBUX	0,15%	0,17%	-0,12%	0,10%	0,08%	0,12%	0,13%	0,06%
SCHW	0,17%	0,19%	-0,13%	0,11%	0,09%	0,13%	0,18%	0,07%
XOM	0,08%	0,08%	-0,10%	0,06%	0,04%	0,06%	0,07%	0,05%

		No shorting		Shorting Allowed	
Stocks	Equally distributed Portfolio	Prediction Enabled MVO	Classic MVO	Prediction Enabled MVO	Classic MVO
AAPL	12,5%	20,0%	20,0%	20,0%	20,0%
AMZN	12,5%	0,0%	20,0%	0,0%	20,0%
C	12,5%	20,0%	0,0%	20,0%	-20,0%
CVS	12,5%	20,0%	20,0%	20,0%	20,0%
MMM	12,5%	20,0%	0,0%	20,0%	20,0%
SBUX	12,5%	0,0%	20,0%	20,0%	20,0%
SCHW	12,5%	0,0%	20,0%	-20,0%	20,0%
XOM	12,5%	20,0%	0,0%	20,0%	0,0%
SUM Of Weights	100%	100%	100%	100%	100%

		Equally distributed Portfolio	Prediction Enabled MVO (No shorting)	Classic MVO (No shorting)	Prediction Enabled MVO (Shorting Allowed)	Classic MVO (Shorting Allowed)
Predicted	Volatility		0,0%	0,2%	0,0%	0,3%
	Portfolio Returns (%)		1,86	2,14	2,26	2,35
	Sharpe Ratio		1,02	0,54	1,27	0,45
Realized	Volatility	0,1%	0,0%	3,9%	0,0%	0,3%
	Portfolio Ret	-1,12	-0,46	-0,88	0,36	-1,60
	Sharpe Ratio	-0,43	-0,25	-0,04	0,20	-0,31

Period 18

Assets	μ Predicted(%)	μ Historical (%)	μ Realized(%)	σ	μ/σ
AAPL	-3,71	3,21	-5,28	0,05	-0,82
AMZN	2,25	3,10	0,04	0,05	0,45
C	3,30	0,00	-4,65	0,06	0,54
CVS	1,57	1,23	6,84	0,03	0,55
MMM	1,45	1,09	7,72	0,02	0,60
SBUX	-1,77	1,74	-2,13	0,03	-0,51
SCHW	3,62	1,11	5,49	0,04	0,88
XOM	2,32	0,76	5,94	0,02	1,00

Correlation Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	1,00	0,94	-0,65	0,93	0,97	0,95	0,90	0,79
AMZN	0,94	1,00	-0,57	0,85	0,96	0,93	0,89	0,70
C	-0,65	-0,57	1,00	-0,61	-0,54	-0,53	-0,49	-0,72
CVS	0,93	0,85	-0,61	1,00	0,92	0,95	0,87	0,83
MMM	0,97	0,96	-0,54	0,92	1,00	0,96	0,92	0,76
SBUX	0,95	0,93	-0,53	0,95	0,96	1,00	0,87	0,75
SCHW	0,90	0,89	-0,49	0,87	0,92	0,87	1,00	0,75
XOM	0,79	0,70	-0,72	0,83	0,76	0,75	0,75	1,00

Covariance Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	0,28%	0,21%	-0,18%	0,12%	0,11%	0,15%	0,17%	0,08%
AMZN	0,21%	0,25%	-0,18%	0,12%	0,12%	0,16%	0,19%	0,08%
C	-0,18%	-0,18%	0,37%	-0,11%	-0,08%	-0,11%	-0,12%	-0,10%
CVS	0,12%	0,12%	-0,11%	0,08%	0,06%	0,10%	0,10%	0,05%
MMM	0,11%	0,12%	-0,08%	0,06%	0,06%	0,08%	0,09%	0,04%
SBUX	0,15%	0,16%	-0,11%	0,10%	0,08%	0,12%	0,13%	0,06%
SCHW	0,17%	0,19%	-0,12%	0,10%	0,09%	0,13%	0,17%	0,07%
XOM	0,08%	0,08%	-0,10%	0,05%	0,04%	0,06%	0,07%	0,05%

		No shorting		Shorting Allowed	
Stocks	Equally distributed Portfolio	Prediction Enabled MVO	Classic MVO	Prediction Enabled MVO	Classic MVO
AAPL	12,5%	0,0%	20,0%	-20,0%	20,0%
AMZN	12,5%	20,0%	20,0%	20,0%	20,0%
C	12,5%	20,0%	0,0%	20,0%	-20,0%
CVS	12,5%	20,0%	20,0%	20,0%	20,0%
MMM	12,5%	0,0%	0,0%	20,0%	20,0%
SBUX	12,5%	0,0%	20,0%	0,0%	20,0%
SCHW	12,5%	20,0%	20,0%	20,0%	20,0%
XOM	12,5%	20,0%	0,0%	20,0%	0,0%
SUM Of Weights	100%	100%	100%	100%	100%

		Equally distributed Portfolio	Prediction Enabled MVO (No shorting)	Classic MVO (No shorting)	Prediction Enabled MVO (Shorting Allowed)	Classic MVO (Shorting Allowed)
Predicted	Volatility		0,0%	0,2%	0,0%	0,3%
	Portfolio Returns (%)		2,61	2,08	3,64	2,30
	Sharpe Ratio		1,22	0,54	1,92	0,44
Realized	Volatility	0,1%	0,0%	3,9%	0,0%	0,3%
	Portfolio Ret	1,75	2,73	0,99	5,33	3,47
	Sharpe Ratio	0,68	1,27	0,05	2,80	0,67

Figure 35:

Period 19

Assets	µ Predicted(%)	µ Historical (%)	µ Realized(%)	σ	µ/σ
AAPL	2,39	3,23	9,56	0,04	0,53
AMZN	3,99	3,11	13,34	0,05	0,81
C	-1,99	-0,01	-12,80	0,06	-0,33
CVS	-6,49	1,16	0,64	0,03	-2,30
MMM	1,57	1,10	7,26	0,02	0,65
SBUX	1,26	1,71	1,76	0,03	0,36
SCHW	1,66	1,09	0,76	0,04	0,40
XOM	0,97	0,75	2,11	0,02	0,42

Correlation Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	1,00	0,94	-0,64	0,93	0,97	0,95	0,90	0,78
AMZN	0,94	1,00	-0,56	0,84	0,96	0,93	0,90	0,69
C	-0,64	-0,56	1,00	-0,61	-0,54	-0,53	-0,49	-0,71
CVS	0,93	0,84	-0,61	1,00	0,91	0,95	0,86	0,83
MMM	0,97	0,96	-0,54	0,91	1,00	0,96	0,93	0,75
SBUX	0,95	0,93	-0,53	0,95	0,96	1,00	0,87	0,75
SCHW	0,90	0,90	-0,49	0,86	0,93	0,87	1,00	0,75
XOM	0,78	0,69	-0,71	0,83	0,75	0,75	0,75	1,00

Covariance Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	0,28%	0,21%	-0,18%	0,12%	0,10%	0,15%	0,17%	0,08%
AMZN	0,21%	0,24%	-0,17%	0,12%	0,11%	0,16%	0,18%	0,08%
C	-0,18%	-0,17%	0,37%	-0,10%	-0,08%	-0,11%	-0,12%	-0,10%
CVS	0,12%	0,12%	-0,10%	0,08%	0,06%	0,09%	0,10%	0,05%
MMM	0,10%	0,11%	-0,08%	0,06%	0,06%	0,08%	0,09%	0,04%
SBUX	0,15%	0,16%	-0,11%	0,09%	0,08%	0,12%	0,13%	0,06%
SCHW	0,17%	0,18%	-0,12%	0,10%	0,09%	0,13%	0,17%	0,07%
XOM	0,08%	0,08%	-0,10%	0,05%	0,04%	0,06%	0,07%	0,05%

			No shorting		Shorting Allowed	
	Stocks	Equally distributed Portfolio	Prediction Enabled MVO	Classic MVO	Prediction Enabled MVO	Classic MVO
	AAPL	12,5%	20,0%	20,0%	20,0%	20,0%
	AMZN	12,5%	20,0%	20,0%	20,0%	20,0%
	C	12,5%	0,0%	0,0%	0,0%	-20,0%
	CVS	12,5%	0,0%	20,0%	-20,0%	20,0%
	MMM	12,5%	20,0%	20,0%	20,0%	20,0%
	SBUX	12,5%	20,0%	20,0%	20,0%	20,0%
	SCHW	12,5%	20,0%	0,0%	20,0%	20,0%
	XOM	12,5%	0,0%	0,0%	20,0%	0,0%
	SUM Of Weights	100%	100%	100%	100%	100%
Predicted	Volatility		0,1%	0,1%	0,1%	0,3%
	Portfolio Returns (%)		2,17	2,06	3,66	2,28
	Sharpe Ratio		0,57	0,58	1,01	0,45
Realized	Volatility	0,1%	0,1%	3,5%	0,1%	0,3%
	Portfolio Ret	2,83	6,53	6,51	6,83	9,22
	Sharpe Ratio	1,11	1,72	0,35	1,88	1,80

Period 20

Assets	µ Predicted(%)	µ Historical (%)	µ Realized(%)	σ	µ/σ
AAPL	2,32	3,20	2,83	0,04	0,52
AMZN	3,88	3,07	5,29	0,05	0,79
C	2,05	0,02	8,77	0,06	0,34
CVS	3,06	1,19	1,80	0,03	1,09
MMM	2,80	1,15	4,35	0,02	1,16
SBUX	1,47	1,66	4,58	0,03	0,42
SCHW	2,63	1,19	5,34	0,04	0,64
XOM	1,02	0,76	0,44	0,02	0,45

Correlation Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	1,00	0,94	-0,64	0,92	0,97	0,95	0,91	0,78
AMZN	0,94	1,00	-0,55	0,83	0,96	0,93	0,91	0,69
C	-0,64	-0,55	1,00	-0,60	-0,53	-0,53	-0,49	-0,71
CVS	0,92	0,83	-0,60	1,00	0,90	0,95	0,85	0,83
MMM	0,97	0,96	-0,53	0,90	1,00	0,96	0,93	0,75
SBUX	0,95	0,93	-0,53	0,95	0,96	1,00	0,87	0,75
SCHW	0,91	0,91	-0,49	0,85	0,93	0,87	1,00	0,75
XOM	0,78	0,69	-0,71	0,83	0,75	0,75	0,75	1,00

Covariance Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	0,28%	0,21%	-0,17%	0,12%	0,10%	0,15%	0,17%	0,08%
AMZN	0,21%	0,24%	-0,16%	0,11%	0,11%	0,16%	0,18%	0,08%
C	-0,17%	-0,16%	0,37%	-0,10%	-0,08%	-0,11%	-0,12%	-0,10%
CVS	0,12%	0,11%	-0,10%	0,08%	0,06%	0,09%	0,10%	0,05%
MMM	0,10%	0,11%	-0,08%	0,06%	0,06%	0,08%	0,09%	0,04%
SBUX	0,15%	0,16%	-0,11%	0,09%	0,08%	0,12%	0,12%	0,06%
SCHW	0,17%	0,18%	-0,12%	0,10%	0,09%	0,12%	0,17%	0,07%
XOM	0,08%	0,08%	-0,10%	0,05%	0,04%	0,06%	0,07%	0,05%

			No shorting		Shorting Allowed	
	Stocks	Equally distributed Portfolio	Prediction Enabled MVO	Classic MVO	Prediction Enabled MVO	Classic MVO
	AAPL	12,5%	20,0%	20,0%	20,0%	20,0%
	AMZN	12,5%	20,0%	20,0%	20,0%	20,0%
	C	12,5%	0,0%	0,0%	20,0%	-20,0%
	CVS	12,5%	20,0%	20,0%	20,0%	20,0%
	MMM	12,5%	20,0%	0,0%	20,0%	20,0%
	SBUX	12,5%	0,0%	20,0%	0,0%	20,0%
	SCHW	12,5%	20,0%	20,0%	20,0%	20,0%
	XOM	12,5%	0,0%	0,0%	-20,0%	0,0%
	SUM Of Weights	100%	100%	100%	100%	100%
Predicted	Volatility		0,1%	0,1%	0,1%	0,3%
	Portfolio Returns (%)		2,94	2,06	3,14	2,29
	Sharpe Ratio		0,81	0,54	1,11	0,45
Realized	Volatility	0,1%	0,1%	3,8%	0,1%	0,3%
	Portfolio Ret	4,18	3,92	3,97	5,59	3,08
	Sharpe Ratio	1,65	1,08	0,20	1,98	0,60

Period 21

Assets	µ Predicted(%)	µ Historical (%)	µ Realized(%)	σ	µ/σ
AAPL	-2,02	3,19	-0,14	0,04	-0,45
AMZN	-1,93	3,04	0,71	0,05	-0,40
C	-2,91	0,07	-1,28	0,06	-0,48
CVS	-1,27	1,23	-0,63	0,03	-0,45
MMM	-1,44	1,12	-2,38	0,02	-0,60
SBUX	-1,25	1,67	-0,14	0,03	-0,36
SCHW	3,96	1,23	2,93	0,04	0,97
XOM	1,05	0,76	1,67	0,02	0,46

Correlation Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	1,00	0,94	-0,63	0,92	0,97	0,95	0,91	0,78
AMZN	0,94	1,00	-0,54	0,82	0,96	0,92	0,91	0,68
C	-0,63	-0,54	1,00	-0,60	-0,52	-0,53	-0,48	-0,71
CVS	0,92	0,82	-0,60	1,00	0,90	0,95	0,85	0,83
MMM	0,97	0,96	-0,52	0,90	1,00	0,95	0,93	0,74
SBUX	0,95	0,92	-0,53	0,95	0,95	1,00	0,87	0,75
SCHW	0,91	0,91	-0,48	0,85	0,93	0,87	1,00	0,74
XOM	0,78	0,68	-0,71	0,83	0,74	0,75	0,74	1,00

Covariance Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	0,28%	0,20%	-0,17%	0,11%	0,10%	0,15%	0,17%	0,08%
AMZN	0,20%	0,24%	-0,16%	0,11%	0,11%	0,15%	0,18%	0,08%
C	-0,17%	-0,16%	0,37%	-0,10%	-0,08%	-0,11%	-0,12%	-0,10%
CVS	0,11%	0,11%	-0,10%	0,08%	0,06%	0,09%	0,10%	0,05%
MMM	0,10%	0,11%	-0,08%	0,06%	0,06%	0,08%	0,09%	0,04%
SBUX	0,15%	0,15%	-0,11%	0,09%	0,08%	0,12%	0,12%	0,06%
SCHW	0,17%	0,18%	-0,12%	0,10%	0,09%	0,12%	0,17%	0,07%
XOM	0,08%	0,08%	-0,10%	0,05%	0,04%	0,06%	0,07%	0,05%

			No shorting		Shorting Allowed	
	Stocks	Equally distributed Portfolio	Prediction Enabled MVO	Classic MVO	Prediction Enabled MVO	Classic MVO
	AAPL	12,5%	0,0%	20,0%	0,0%	20,0%
	AMZN	12,5%	0,0%	20,0%	20,0%	20,0%
	C	12,5%	0,0%	0,0%	-20,0%	-20,0%
	CVS	12,5%	20,0%	20,0%	20,0%	20,0%
	MMM	12,5%	20,0%	0,0%	20,0%	20,0%
	SBUX	12,5%	20,0%	20,0%	20,0%	20,0%
	SCHW	12,5%	20,0%	20,0%	20,0%	20,0%
	XOM	12,5%	20,0%	0,0%	20,0%	0,0%
	SUM Of Weights	100%	100%	100%	100%	100%
Predicted	Volatility		0,1%	0,1%	0,2%	0,3%
	Portfolio Returns (%)		0,21	2,07	0,41	2,28
	Sharpe Ratio		0,07	0,54	0,09	0,45
Realized	Volatility	0,1%	0,1%	3,8%	0,2%	0,3%
	Portfolio Ret	0,09	0,29	0,55	0,69	0,33
	Sharpe Ratio	0,04	0,10	0,03	0,15	0,06

Figure 36:

Period 22

Assets	µ Predicted(%)	µ Historical (%)	µ Realized(%)	σ	µ/σ
AAPL	-2,41	3,16	-2,38	0,04	-0,54
AMZN	5,57	3,07	21,27	0,05	1,15
C	2,53	0,01	10,43	0,06	0,41
CVS	3,81	1,07	3,52	0,03	1,36
MMM	1,07	1,10	-2,73	0,02	0,45
SBUX	-1,58	1,61	-1,07	0,03	-0,46
SCHW	1,80	1,24	1,17	0,04	0,44
XOM	1,84	0,67	3,30	0,02	0,80

Correlation Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	1,00	0,94	-0,62	0,91	0,97	0,95	0,91	0,78
AMZN	0,94	1,00	-0,53	0,80	0,96	0,91	0,92	0,67
C	-0,62	-0,53	1,00	-0,60	-0,52	-0,52	-0,47	-0,71
CVS	0,91	0,80	-0,60	1,00	0,89	0,94	0,84	0,83
MMM	0,97	0,96	-0,52	0,89	1,00	0,95	0,94	0,74
SBUX	0,95	0,91	-0,52	0,94	0,95	1,00	0,87	0,75
SCHW	0,91	0,92	-0,47	0,84	0,94	0,87	1,00	0,74
XOM	0,78	0,67	-0,71	0,83	0,74	0,75	0,74	1,00

Covariance Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	0,28%	0,20%	-0,17%	0,11%	0,10%	0,15%	0,17%	0,08%
AMZN	0,20%	0,23%	-0,16%	0,11%	0,11%	0,15%	0,18%	0,07%
C	-0,17%	-0,16%	0,37%	-0,10%	-0,08%	-0,11%	-0,12%	-0,10%
CVS	0,11%	0,11%	-0,10%	0,08%	0,06%	0,09%	0,10%	0,05%
MMM	0,10%	0,11%	-0,08%	0,06%	0,06%	0,08%	0,09%	0,04%
SBUX	0,15%	0,15%	-0,11%	0,09%	0,08%	0,12%	0,12%	0,06%
SCHW	0,17%	0,18%	-0,12%	0,10%	0,09%	0,12%	0,17%	0,07%
XOM	0,08%	0,07%	-0,10%	0,05%	0,04%	0,06%	0,07%	0,05%

			No shorting		Shorting Allowed	
	Stocks	Equally distributed Portfolio	Prediction Enabled MVO	Classic MVO	Prediction Enabled MVO	Classic MVO
	AAPL	12,5%	0,0%	20,0%	-20,0%	20,0%
	AMZN	12,5%	20,0%	20,0%	20,0%	20,0%
	C	12,5%	20,0%	0,0%	20,0%	-20,0%
	CVS	12,5%	20,0%	0,0%	20,0%	20,0%
	MMM	12,5%	0,0%	20,0%	20,0%	20,0%
	SBUX	12,5%	0,0%	20,0%	0,0%	20,0%
	SCHW	12,5%	20,0%	20,0%	20,0%	20,0%
	XOM	12,5%	20,0%	0,0%	20,0%	0,0%
	SUM Of Weights	100%	100%	100%	100%	100%
Predicted	Volatility	#VALUE!	0,0%	0,1%	0,0%	0,3%
	Portfolio Ret	#VALUE!	3,11	2,03	3,81	2,25
	Sharpe Ratio	#VALUE!	1,47	0,54	2,03	0,44
Realized	Volatility	0,1%	0,0%	3,8%	0,0%	0,3%
	Portfolio Ret	4,19	7,94	3,25	7,87	1,87
	Sharpe Ratio	1,66	3,76	0,17	4,20	0,37

Period 23

Assets	µ Predicted(%)	µ Historical (%)	µ Realized(%)	σ	µ/σ
AAPL	3,31	3,18	6,46	0,04	0,75
AMZN	2,93	3,09	6,63	0,05	0,61
C	-2,61	0,03	-18,96	0,06	-0,43
CVS	-7,96	1,03	-4,14	0,03	-2,87
MMM	-2,14	1,06	-7,47	0,02	-0,89
SBUX	3,39	1,61	0,65	0,03	0,98
SCHW	-2,76	1,33	-0,93	0,04	-0,68
XOM	-5,75	0,67	-13,05	0,02	-2,48

Correlation Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	1,00	0,94	-0,62	0,90	0,97	0,95	0,92	0,77
AMZN	0,94	1,00	-0,52	0,78	0,96	0,90	0,92	0,66
C	-0,62	-0,52	1,00	-0,60	-0,51	-0,52	-0,47	-0,71
CVS	0,90	0,78	-0,60	1,00	0,88	0,94	0,82	0,83
MMM	0,97	0,96	-0,51	0,88	1,00	0,95	0,94	0,73
SBUX	0,95	0,90	-0,52	0,94	0,95	1,00	0,87	0,75
SCHW	0,92	0,92	-0,47	0,82	0,94	0,87	1,00	0,73
XOM	0,77	0,66	-0,71	0,83	0,73	0,75	0,73	1,00

Covariance Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	0,28%	0,20%	-0,17%	0,11%	0,10%	0,15%	0,16%	0,08%
AMZN	0,20%	0,23%	-0,15%	0,10%	0,11%	0,15%	0,18%	0,07%
C	-0,17%	-0,15%	0,37%	-0,10%	-0,07%	-0,11%	-0,12%	-0,10%
CVS	0,11%	0,10%	-0,10%	0,08%	0,06%	0,09%	0,09%	0,05%
MMM	0,10%	0,11%	-0,07%	0,06%	0,06%	0,08%	0,09%	0,04%
SBUX	0,15%	0,15%	-0,11%	0,09%	0,08%	0,12%	0,12%	0,06%
SCHW	0,16%	0,18%	-0,12%	0,09%	0,09%	0,12%	0,17%	0,07%
XOM	0,08%	0,07%	-0,10%	0,05%	0,04%	0,06%	0,07%	0,05%

			No shorting		Shorting Allowed	
	Stocks	Equally distributed Portfolio	Prediction Enabled MVO	Classic MVO	Prediction Enabled MVO	Classic MVO
	AAPL	12,5%	20,0%	20,0%	20,0%	20,0%
	AMZN	12,5%	20,0%	20,0%	20,0%	20,0%
	C	12,5%	20,0%	0,0%	20,0%	-20,0%
	CVS	12,5%	0,0%	0,0%	-20,0%	20,0%
	MMM	12,5%	20,0%	20,0%	20,0%	20,0%
	SBUX	12,5%	20,0%	20,0%	20,0%	20,0%
	SCHW	12,5%	0,0%	20,0%	20,0%	20,0%
	XOM	12,5%	0,0%	0,0%	0,0%	0,0%
	SUM Of Weights					
Predicted	Volatility	#VALUE!	0,1%	0,1%	0,1%	0,3%
	Portfolio Ret	#VALUE!	0,98	2,05	2,01	2,25
	Sharpe Ratio	#VALUE!	0,39	0,55	0,71	0,45
Realized	Volatility	0,1%	0,1%	3,7%	0,1%	0,3%
	Portfolio Ret	-3,85	-2,54	1,07	-1,90	4,04
	Sharpe Ratio	-1,53	-1,01	0,06	-0,67	0,80

Period 24

Assets	µ Predicted(%)	µ Historical (%)	µ Realized(%)	σ	µ/σ
AAPL	-2,68	3,17	-6,66	0,04	-0,61
AMZN	-2,84	2,94	-5,31	0,05	-0,59
C	-4,84	0,02	-8,12	0,06	-0,80
CVS	-5,19	1,04	-8,08	0,03	-1,86
MMM	-3,15	1,01	-10,01	0,02	-1,32
SBUX	1,32	1,64	2,33	0,03	0,38
SCHW	-2,66	1,31	-2,86	0,04	-0,66
XOM	-2,55	0,71	-5,97	0,02	-1,10

Correlation Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	1,00	0,94	-0,61	0,90	0,97	0,95	0,92	0,77
AMZN	0,94	1,00	-0,51	0,77	0,96	0,90	0,92	0,65
C	-0,61	-0,51	1,00	-0,60	-0,51	-0,52	-0,46	-0,71
CVS	0,90	0,77	-0,60	1,00	0,88	0,94	0,81	0,83
MMM	0,97	0,96	-0,51	0,88	1,00	0,95	0,94	0,73
SBUX	0,95	0,90	-0,52	0,94	0,95	1,00	0,87	0,75
SCHW	0,92	0,92	-0,46	0,81	0,94	0,87	1,00	0,72
XOM	0,77	0,65	-0,71	0,83	0,73	0,75	0,72	1,00

Covariance Matrix

	AAPL	AMZN	C	CVS	MMM	SBUX	SCHW	XOM
AAPL	0,27%	0,20%	-0,16%	0,11%	0,10%	0,14%	0,16%	0,08%
AMZN	0,20%	0,23%	-0,15%	0,10%	0,11%	0,15%	0,18%	0,07%
C	-0,16%	-0,15%	0,37%	-0,10%	-0,07%	-0,11%	-0,11%	-0,10%
CVS	0,11%	0,10%	-0,10%	0,08%	0,06%	0,09%	0,09%	0,05%
MMM	0,10%	0,11%	-0,07%	0,06%	0,06%	0,08%	0,09%	0,04%
SBUX	0,14%	0,15%	-0,11%	0,09%	0,08%	0,12%	0,12%	0,06%
SCHW	0,16%	0,18%	-0,11%	0,09%	0,09%	0,12%	0,16%	0,07%
XOM	0,08%	0,07%	-0,10%	0,05%	0,04%	0,06%	0,07%	0,05%

			No shorting		Shorting Allowed	
	Stocks	Equally distributed Portfolio	Prediction Enabled MVO	Classic MVO	Prediction Enabled MVO	Classic MVO
	AAPL	12,5%	20,0%	20,0%	20,0%	20,0%
	AMZN	12,5%	20,0%	20,0%	20,0%	20,0%
	C	12,5%	0,0%	0,0%	0,0%	-20,0%
	CVS	12,5%	0,0%	20,0%	-20,0%	20,0%
	MMM	12,5%	0,0%	0,0%	20,0%	20,0%
	SBUX	12,5%	20,0%	20,0%	20,0%	20,0%
	SCHW	12,5%	20,0%	20,0%	20,0%	20,0%
	XOM	12,5%	20,0%	0,0%	20,0%	0,0%
	SUM Of Weights	100%	100%	100%	100%	100%
Predicted	Volatility		0,1%	0,1%	0,1%	0,3%
	Portfolio Returns (%)		-1,88	2,02	-1,47	2,22
	Sharpe Ratio		-0,52	0,54	-0,41	0,44
Realized	Volatility	0,1%	0,1%	3,8%	0,1%	0,3%
	Portfolio Ret	-5,58	-3,69	-4,11	-4,08	-4,49
	Sharpe Ratio	-2,23	-1,02	-0,21	-1,13	-0,90

Figure 37:

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