Imperial College London

MATHS TEACHING RESOURCES

For teachers of high-achieving students in KS2

2 Linear Equations

Welcome

These resources have been put together with you, the primary teacher, at the forefront of our thinking. At Imperial College London we recognise the importance of keeping high-attaining students engaged with maths and want to do everything we can to help teachers provide for all of their students. Maths is critical for us as a facilitating subject for further eventual study in engineering or science, as well as being fascinating in its own right.

The content in this booklet was developed by teachers at Colchester Royal Grammar School in conjunction with Imperial. Based on initial feedback from primary teachers who trialled it, we have included specific guidance and examples to help teachers develop their own understanding of the material but we would welcome any further suggestions for improvement.

The material for the first three booklets (Percentages, Linear Equations and Cubic Equations) has been loosely based on the previous 'Level 6' curriculum content and we are currently looking into developing some further assessment materials to sit alongside these in case schools wish to offer an informal certification.

I hope that you and your students find these resources useful and most of all enjoyable.

George Constantinides

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Linear Equations

Often written in the form $y = mx + c$, linear equations consist of either just a constant or the product of a constant and a single variable. When plotted on a graph, linear equations produce a straight line.

This document begins by outlining how to formulate and solve linear equations, then looks at graphical representations of linear equations and finally discusses various methods for plotting such equations.

$$y = mx + c$$

Formulating and solving linear equations

Definitions

VARIABLE: A symbol used to represent an unknown number in an expression or an equation.

The value of this number can vary depending on the mathematical problem.

$4 + x$

CONSTANT: A fixed value that does not change.

Key concept

EXPRESSION

$$4x - 7$$

$$4x - 7 = 5$$

EQUATION

Expressions

Mathematical expressions use a string of symbols such as numbers and letters grouped together with mathematical operations such as addition, subtraction, or multiplication to show the value of something. They do not contain an equals sign, and as such can only be simplified.

EXAMPLE

Simplify the following:

$3x + 5x$

$5\ (x - 2)$

$3x + 5x \rightarrow 8x$
(Add together the 'x terms')

$5\ (x - 2) \rightarrow 5x - 10$
(Expand the brackets out)

Equations

Equations are similar to mathematical expressions except that they also contain an equals sign. Given a mathematical problem, equations can be used to find the value of an unknown. The following steps explain how to approach a problem:

- Read through the problem carefully
- Read through again picking out the key points
- Decide on a variable to represent the unknown
- Formulate an equation
- Solve the equation using the balance method — reversing the operations

EXAMPLE 1

If I treble the length of my rope, and take this away from 100, I am left with 34cm.

→ Calculate the length of my rope.

Let x be the length of the rope, therefore three times this is $3x$. This can be taken away from 100 and equated to 34:

$100 - 3x = 34$		(it is always good to make the x term positive so add $3x$ to both sides)
$100 = 3x + 34$		(subtract 34 from both sides)
$66 = 3x$		(divide by 3)
$x = 22$		

Answer → The length of rope is therefore 22cm.

EXAMPLE 2

The Hill family is planning a holiday. Twice the price of a child's holiday minus £50 is the same as an adult's holiday.

→ How much would it cost for Mr and Mrs Hill and their three children to go on holiday knowing that the cost for each adult is £500?

Let the cost of a child's holiday be x, the price of an adult will be $2x - 50$. This is equal to £500 and our equation is:

$2x - 50 = £500$	(add 50 to both sides)
$2x = £550$	(divide by 2)
$x = £275$	

Answer → The cost of two adults and three children would be: $(2 \times £500) + (3 \times £275) = £1,825$

EXAMPLE 3

In five years time Sally will be double the age she was 3 years ago.

→ How old is she now?

Let x be Sally's current age. In 5 years times she will therefore be $x + 5$. Three years ago she was $x - 3$. In five years time she will be double the age she was 3 years ago.This implies that the equation we need to solve is:

$2(x - 3) = x + 5$	(expand the brackets)
$2x - 6 = x + 5$	(subtract x from both sides)
$x - 6 = 5$	(add 6 to both sides)
$x = 11$	

Answer → Sally is now 11 years old.

EXCERCISES / LINEAR EQUATIONS

Write equations for the following and solve:

1. If I double a number and add 10, I get 44. What is the number?
2. I multiply a number by 22 and then subtract 50 to get 16. Find the value of the number.
3. The sum of three consecutive whole numbers is 33. Let the first number be x. What are the values of the three numbers? **Note ▸** Consecutive numbers are numbers that follow one after another e.g. 1, 2, 3, 4 or 35, 36, 37, 38.
4. The sum of the ages of Alice, Bob and Charlie is 40 years. Bob is three times as old as Alice and Charlie is five years older than Alice. How old are the children?
5. The length of a rectangle is 3 times its width. If the perimeter is 24cm find:
   a) the width of the rectangle
   b) the length of the rectangle
   c) the area of the rectangle
6. The sum of three consecutive even numbers is 42. Find the numbers.

★ USEFUL TIP
When solving equations you should always check your answer by substituting back into the original equation to confirm it is correct.

HINT ▸ An even number can be expressed as $2n$ and an odd number can be expressed as $2n + 1$.

Probing question:

On question 3, if you made x the second number, what would your solution look like?

HOW TO CHECK AN ANSWER, USING EXAMPLE 3 ON PAGE 5

$2(x - 3) = x + 5$ is the equation
$x = 11$ is the answer
now substitute in $x = 11$ into original equation
$2(11 - 3) = 11 + 5$
$2(8) = 16$
$16 = 16$

$x = 11$ is the correct answer

SOLUTIONS / LINEAR EQUATIONS

1

$2x + 10 = 44$
$2x = 34$
$x = 17$

Check ▸
$2x + 10 = 44$
$2(17) + 10 = 44$
$34 + 10 = 44$
$44 = 44$

2

$22x - 50 = 16$
$22x = 66$
$x = 3$

Check ▸
$22x - 50 = 16$
$22(3) - 50 = 16$
$66 - 50 = 16$
$16 = 16$

3

$x + (x + 1) + (x + 2) = 33$
$3x + 3 = 33$
$3x = 30$
$x = 10$
So the numbers are 10, 11 and 12

Check ▸
$x + (x + 1) + (x + 2) = 33$
$10 + (10 + 1) + (10 + 2) = 33$
$10 + 11 + 12 = 33$
$33 = 33$

Probing question:
$(x - 1) + x + (x + 1) = 33$
$3x = 33$
$x = 11$
$10 + 11 + 12 = 33$ (same as previous solution)

4

$a + b + c = 40$
$b = 3a$
$c = a + 5$
$a + 3a + (a + 5) = 40$
$5a + 5 = 40$
$a = 7$
$b = 3a$
$b = 21$
$c = a + 5$
$c = 12$
So Alice is 7 years, Bob is 21 years and Charlie is 12 years.

Check ▸
$a + b + c = 40$
$12 + 7 + 21 = 40$
$40 = 40$

5

l = 3w

a. 24 = w + w + l + l
24 = 2w + 2l
24 = 2w + 2(3w)
24 = 8w
w = 3cm

b. l = 3w
l = 9cm

Check ▸
24 = w + w + l + l
24 = 3 + 3 + 9 + 9
24 = 24

c. A = l × w
A = 3 × 9
A = 27cm²

TEACHER'S NOTE ▸ A function machine can be used to illustrate the equation.

6

$x + (x + 2) + (x + 4) = 42$
$3x + 6 = 42$
$x = 12$
So the numbers are 12, 14 and 16.

Check ▸
$x + (x + 2) + (x + 4) = 42$
$12 + (12 + 2) + (12 + 4) = 42$
$12 + 14 + 16 = 42$
$42 = 42$

OR ▸ This is the solution that goes with the hint in the questions — it is a different way to find the solution and is a slightly more changing method.

$x + (x + 1) + (x + 2)$
(three consecutive numbers)

Remember ▸ An even number can be represented as $2n$ so let the first even number be $2n$ instead of x:

$2n + 2(n + 1) + 2(n + 2) = 42$
(three consecutive even numbers) $6n + 6 = 42$

$6n = 36$
$n = 6$
$x = 2n = 2 \times 6 = 12$
(remember x is the first even number)

So the numbers are 12, 14 and 16.

Check ▸
$2n + 2(n + 1) + 2(n + 2) = 42$
$2(6) + 2(6 + 1) + 2(6 + 2) =$
42
$12 + 14 + 16 = 42$
$42 = 42$

Graphs of straight lines

Constant Functions

In mathematics, a constant function is a function for which its output value is the same for every input value. When these functions are plotted on a graph they produce lines that are either horizontal or vertical (i.e. parallel to the x-axis or y-axis).

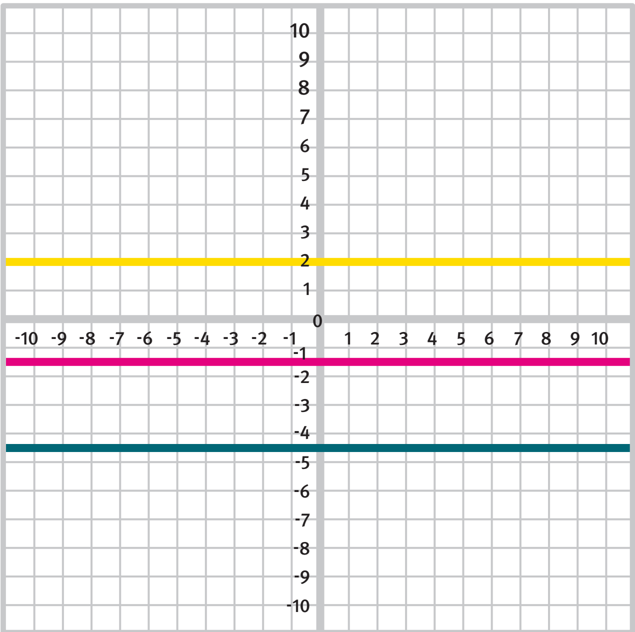
Horizontal Lines

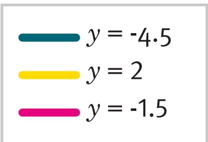
These lines are parallel to the x-axis and take the form $y = c$ where c is a constant.

For example ▸

$y = 4$, $y = -2$, $y = 5.6$, $y = 0$ (the x-axis)

This type of function can be plotted to produce horizontal lines and examples are shown in the graph below.





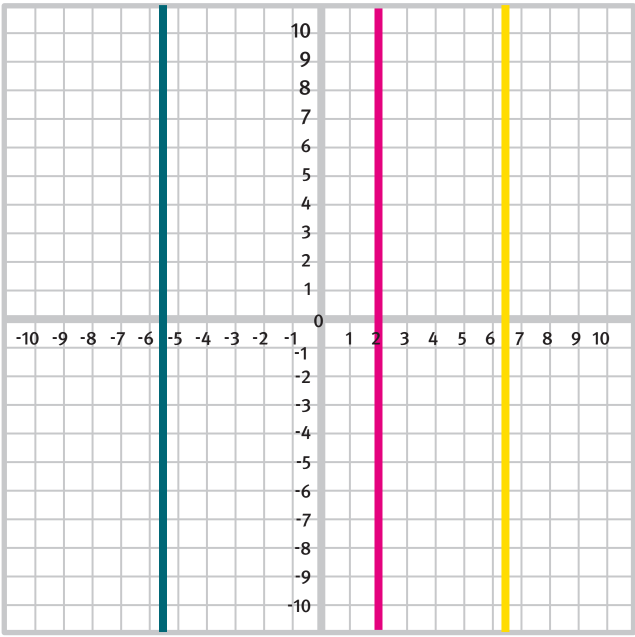
Vertical Lines

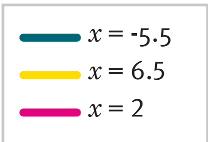
These lines are parallel to the y-axis and take the form $x = c$ where c is a constant.

For example ▸

$x = 3$, $x = -5$, $x = 25.7$, $x = 0$ (the y-axis)

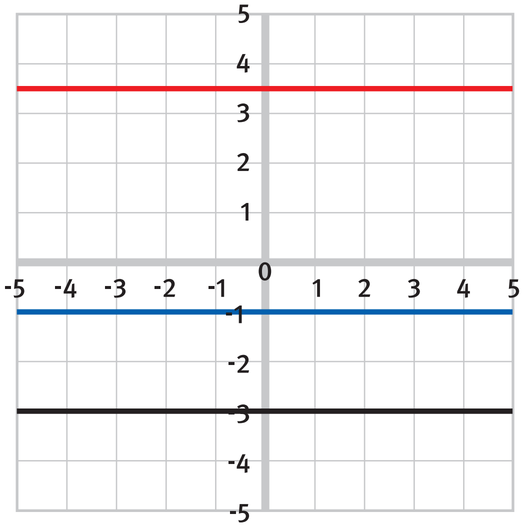
This type of function can be plotted to produce vertical lines and examples are shown in the graph below.



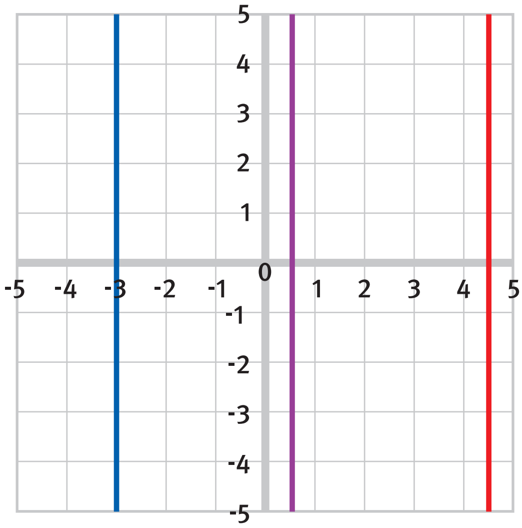


EXCERCISES / STRAIGHT LINES

1. The graph below shows three horizontal lines.
   a) State the coordinates for any four points on each of the lines: red, blue and black.
   b) State the equation of the red, blue and black lines.



2. The graph below shows three vertical lines.
   a) State the coordinates for any four points on each of the lines: blue, pink and red.
   b) State the equation of the blue, purple and red lines.



3. These points all lie on the same line:
   (0, 5) (5, 5) (-4, 5) (100, 5)
   Write down the equation of the line.

4. These points all lie on the same line:
   (-4, 10) (-4, 75) (-4, 3) (-4, -7)
   Write down the equation of the line.

5. Write down four points that lie on the line $y = 7$.

6. Write down four points that lie on the line $x = 10$.

7. Write down the point that lies both on the line $y = 2$ and on the line $x = 5$.

8. Write down the point that lies both on the line $y = -3$ and on the line $x = -5.7$.

9. Write down the equation of the x-axis.

10. Write down the equation of the y-axis.

11. Write down the coordinate of the point that lies on both the line $x = 0$ and the line $y = 0$.

SOLUTIONS / STRAIGHT LINES

1. a) Any coordinates of the form: Red (? , 3.5), blue (?, -1), black (?, -3)
   b) Red $y = 3.5$, blue $y = -1$, black $y = -3$

2. a) Any coordinates of the form: Red (4.5, ?), purple (-0.5, ?), blue (-3, ?),
   b) Red $x = 4.5$, purple $x = 0.5$, blue $x = -3$

3. By looking at the coordinates it can be seen that all the y-values are 5.
   If this is a straight line, then this means that the equation of the line must be $y = 5$.

4. By looking at the coordinates it can be seen that all the x-values are -4.
   If this is a straight line then this means that the equation of the line must be $x = -4$.

5. Any coordinates of the form (?, 7)

6. Any coordinates of the form (10, ?)

7. (5, 2)

8. (-5.7, -3)

9. $y = 0$

10. $x = 0$

11. (0, 0)

Non-constant Functions

Non-constant functions produce lines that are neither horizontal nor vertical.
They are expressed in terms of a variable as well as a constant.

For example ▸

$y = 2x - 3$, $y = -x + 10$, $y = 0.5x + 25$

These are written in the form: $\boxed{y = mx + c}$

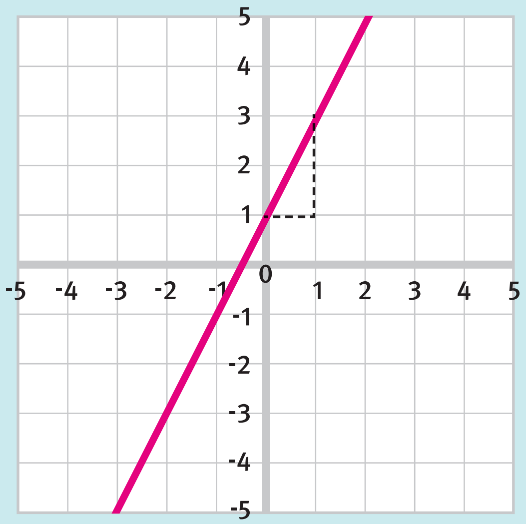
The ‘m’ tells us how steep the line is and is called the **gradient.**

The ‘c’ tells us where the line will intersect the y-axis and is known as the **y-intercept**.

EXAMPLE 1

The equation $y = 2x + 1$ has a gradient of 2 and a y-intercept of 1. This is shown in the plot below.

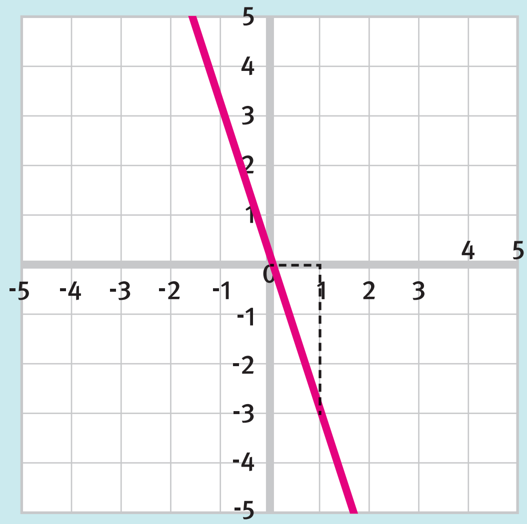
If a line has a gradient of 2, this means that for every one unit moved across to the right there are two units moved upwards on the line.



EXAMPLE 2

The equation $y = -3x$ has a gradient of -3 and a y-intercept of 0. This is shown in the plot below.

The gradient is -3 and means that for every one unit moved across to the right there are 3 units moved downwards on the line. The line crosses the y-axis at 0 as the constant is 0.



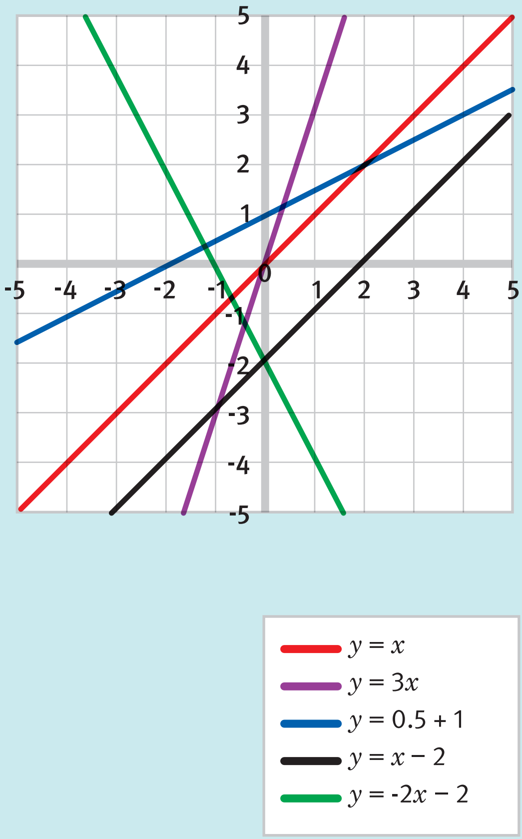
These examples illustrate two important points ▸

a) The larger the absolute value of the gradient, the steeper the slope of the line.

b) That the sign of the gradient is significant. A positive value gives a forward sloping diagonal line, forward slash ‘/’ shape and a negative value gives a backward sloping diagonal line, backslash ‘\’ shape.

EXAMPLE 3

Using the graph to the right:



1. **Find the lines that have the same gradient.**
   Gradient is defined by the slope of a line. Therefore, if two lines have the same gradient, they have the same slope, and are parallel lines. The red and black lines have the same slope and so have the same gradient.

2. **Find the lines that have the same y-intercept.**
   The y-intercept is defined as the point at which a line crosses the y-axis. The green and black lines both cross the y-axis at -2 and so have the same intercept. Also the red and the purple lines both cross the y-axis at 0 and so also have the same intercept.

3. **Find the line that has the largest gradient.**
   The line which has the largest gradient will have the greatest slope. The steepest line is the purple line and therefore it has the largest gradient.

4. **Find the line that has a negative gradient.**
   A line that has a negative gradient describes a line that is trending downward from left to right. An increase in the x-value results in a decrease in the y-value. This is the green line.

5. **Write the gradient, m, and y-intercept, c, for each line**
   Red line: $m = 1$, $c = 0$
   Purple line: $m = 3$, $c = 0$
   Blue line: $m = 0.5$, $c = 1$
   Black line: $m = 1$, $c = -2$
   Green line: $m = -2$, $c = -2$

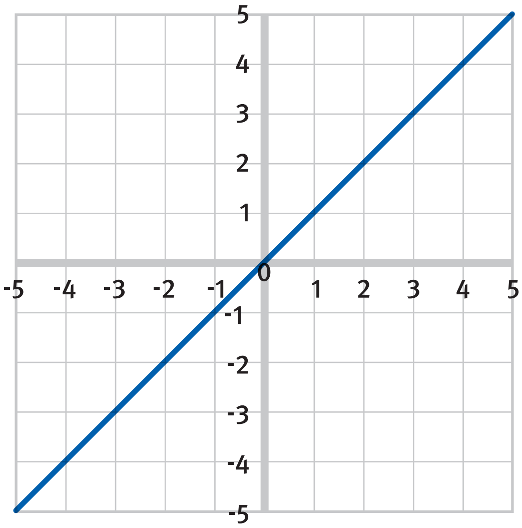
EXERCISES / NON-CONSTANT FUNCTIONS

1 For the line in the graph below, write down:

a) The coordinates of four points that are on the line

b) The gradient and the y-intercept

c) The equation of the line

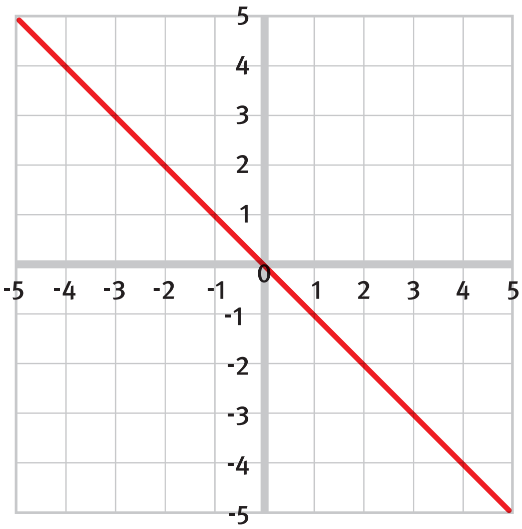


2 For the line in the graph below, write down:

a) The coordinates of four points that are on the line

b) The gradient and the y-intercept

c) The equation of the line



3 These points lie on the same line (0, 0) (1, 2) (2, 4) (3, 6) (4, 8). State the gradient, y-intercept, and the equation of the line.

4 These points lie on the same line (0, 1) (1, 3) (2, 5) (3, 7) (4, 9). State the gradient, y-intercept, and the equation of the line.

5 From the graph below find:

a) The two lines that have the same gradient

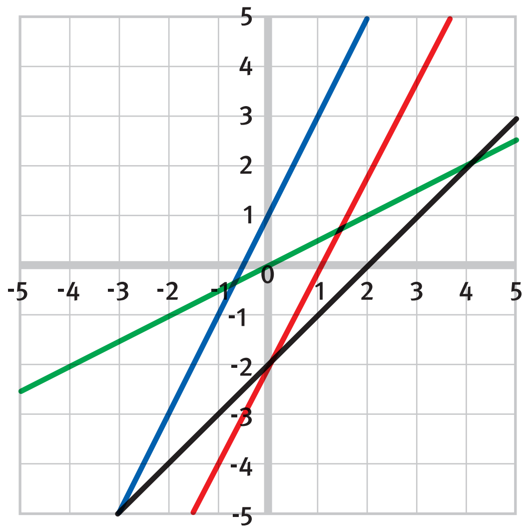
b) The two lines that have the same y-intercept

c) The line that has the smallest gradient

d) The equation of each line

e) The coordinate of the intersection of the green line and the black line

f) The coordinate of the intersection of the red line and the black line.



1 a) Example: (2, 2), (-3, -3), (1.5, 1.5), (-7, -7)
b) $m = 1$, $c = 0$
c) $y = x$

2 a) Example: (2, -2), (-3, 3), (1.5, -1.5), (-7, 7)
b) $m = -1$, $c = 0$
c) $y = -x$

3 By looking at the coordinates it can be seen that for every increase of one unit in the x-axis there is an increase of two units in the y-axis. This means that the gradient must be 2. The line passes through the point (0, 0) therefore the y-intercept is 0.
a) $m = 2$
b) $c = 0$
c) $y = 2x$

4 By looking at the coordinates it can be seen that for every increase of one unit in the x-axis there is an increase of two units in the y-axis. This means that the gradient must be 2. The line passes through the point (0, 1) therefore the y-intercept is 1.
a) $m = 2$
b) $c = 1$
c) $y = 2x + 1$

★ TEACHER'S NOTE
Linear equations may be written in other forms such as
$2y = 4x + 6$, $y - x + 4 = 0$,
$-3y = -5x + 10$

They need to be rearranged to the $y = mx + c$ format to identify the gradient and y-intercept.

Gradient, often called **slope**, is the change in y over the change in x ($\Delta y/\Delta x$) often referred to as rise / run.

5 a) Blue/Red
b) Red/Black
c) Green
d) Green: $y = 0.5x$
Blue: $y = 2x + 1$
Red: $y = 2x - 2$
Black: $y = x - 2$
e) (4, 2)
f) (0, -2)

★ TEACHER'S NOTE
Q5 parts **e** and **f** can be found using algebra – solving simultaneous equations.

Note ▸ To solve part (e) and (f) using simultaneous equations.

(e) Equation of the green line is $y = 0.5x$, and of the black line is $y = x - 2$

Using simultaneous equations:
$0.5x = x - 2$
$x = 2x - 4$
$x = 4$

Substitute back into one of the original equations to find y:
$y = 0.5x = 0.5 \times 4 = 2$
or
$y = x - 2 = 4 - 2 = 2$

Therefore the intersect point is (4, 2)

(f) Equation of the red line is $y = 2x - 2$, and of the black line $y = x - 2$

Using simultaneous equations:
$2x - 2 = x - 2$
$x = 0$

Substitute back into one of the original equations to find y:

$y = 2x - 2 = (2 \times 0) - 2 = -2$
or
$y = x - 2 = 0 - 2 = -2$

Therefore the intersect point is (0, -2)

★ TEACHER'S NOTE
Absolute value is the magnitude of a real number without regard to its sign. Therefore a gradient of -3, absolute value 3, is steeper than a gradient of 2 as seen in the examples.

Drawing a straight line

Using a table of values

This method involves using a function to calculate a set of y values that correspond to a set of x values. The workings for these calculations are set out in a table.

EXAMPLE 1

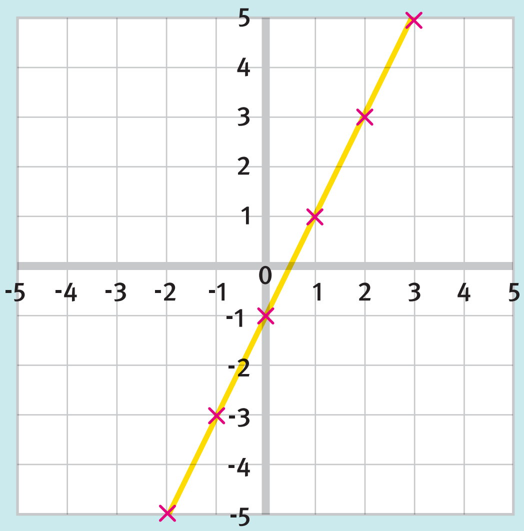
Draw a graph of $y = 2x - 1$ using a range of x-values from -4 to 4.

For each value of x, work out the corresponding value of y.

This is usually shown in a table:

x	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**
$2x$	-8	-6	-4	-2	0	2	4	6	8
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
y	-9	-7	-5	-3	-1	1	3	5	7

The x and y coordinates from the table can then be plotted and joined with a straight line: (-4, -9), (-3, -7), (-2, -5), (-1, -3), (0, -1), (1, 1), (2, 3), (3, 5), (4, 7)



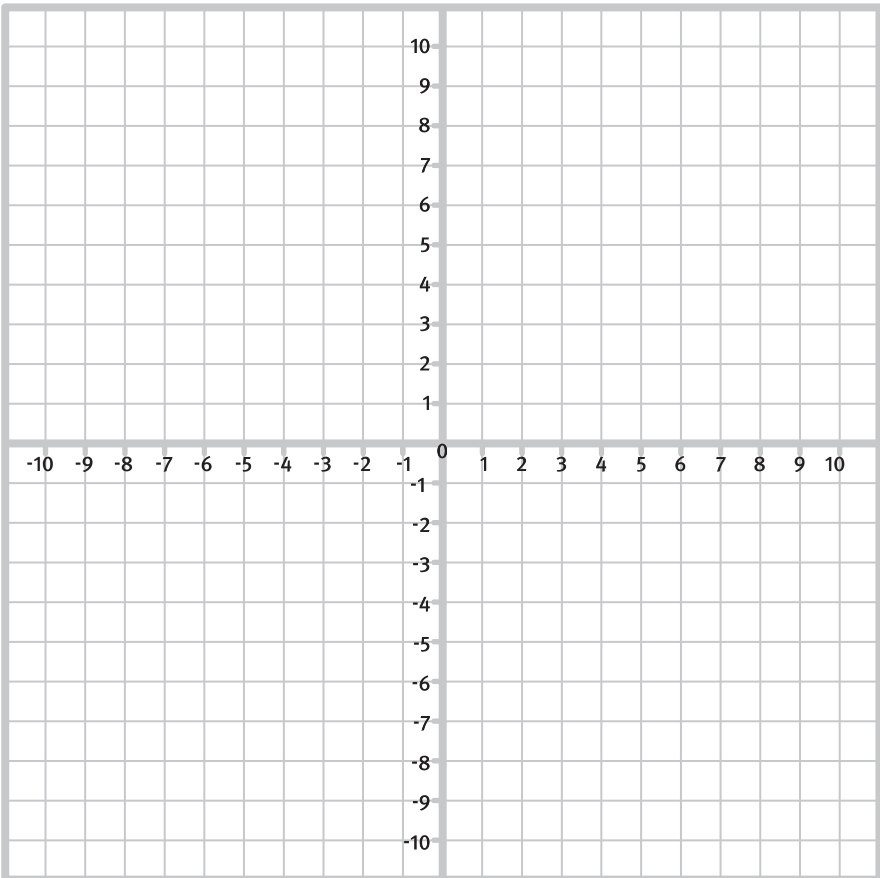
Note ▸ This is not the complete line requested in the question. Why not draw it yourself on a larger graph?

EXERCISES / STRAIGHT LINES

Showing your working and taking your x values from -4 to 4, draw the following lines:

1. $y = x + 3$
2. $y = 3x - 2$
3. $y = -2x + 2$ **(be careful)**

★ **USEFUL TIP**
Use a table to help find your coordinates.



SOLUTIONS / STRAIGHT LINES

1 $y = x + 3$

x	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**
+3	+3	+3	+3	+3	+3	+3	+3	+3	+3
y	**-1**	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**

All of the following coordinates should now be plotted on a graph and connected by a straight line.

(-4, -1) (-3, 0) (-2, 1) (-1, 2) (0, 3) (1, 4) (2, 5) (3, 6) (4, 7)

★ **TEACHER'S NOTE** ▸
Lines need to extend past the plotted points or it is a 'line segment' not a line.

2 $y = 3x - 2$

x	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**
$3x$	-12	-9	-6	-3	0	3	6	9	12
-2	-2	-2	-2	-2	-2	-2	-2	-2	-2
y	**-14**	**-11**	**-8**	**-5**	**-2**	**1**	**4**	**7**	**10**

All of the following coordinates should now be plotted on a graph and connected by a straight line.

(-4, -14) (-3, -11) (-2, -8) (-1, -5) (0, -2) (1, 1) (2, 4) (3, 7) (4, 10)

3 $y = -2x + 2$

x	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**
$-2x$	8	6	4	2	0	-2	-4	-6	-8
+2	+2	+2	+2	+2	+2	+2	+2	+2	+2
y	**10**	**8**	**6**	**4**	**2**	**0**	**-2**	**-4**	**-6**

All of the following coordinates should now be plotted on a graph and connected by a straight line.

(-4, 10) (-3, 8) (-2, 6) (-1, 4) (0, 2) (1, 0) (2, -2) (3, -4) (4, -6)

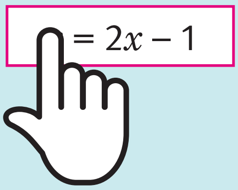
Cover-up method for drawing graphs

EXAMPLE 1

Draw the graph of $y = 2x - 1$

What is x when $y = 0$?

If you cover the 'y' term with your finger



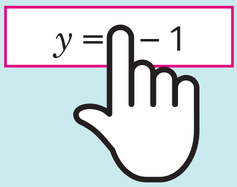
You are left with '$= 2x - 1$'.
Set this equal to zero and solve.

$0 = 2x - 1$
$1 = 2x$
$½ = x$

Therefore, the coordinates for one point is (½, 0)

What is y when $x = 0$?

If you cover the 'x' term with your finger



You are left with '$y = -1$'

Therefore, the coordinate of the other point is (0, -1)

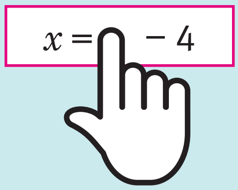
You could now plot these two points and join them with a line. Be sure to extend your line beyond each point.

EXAMPLE 2

Draw the graph of $x = 2y - 4$

What is x when $y = 0$?

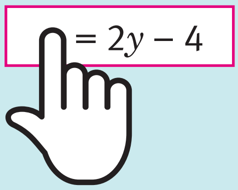
If you cover the 'y' term with your finger



You are left with '$x = -4$'
Therefore, the coordinate for one point is (4, 0)

What is y when $x = 0$?

If you cover the 'x' term with your finger



You are left with '$= 2y - 4$'.
Set this equal to zero and solve.

$0 = 2y - 4$
$4 = 2y$
$2 = y$

Therefore, the coordinate for the other point is (2, 0)

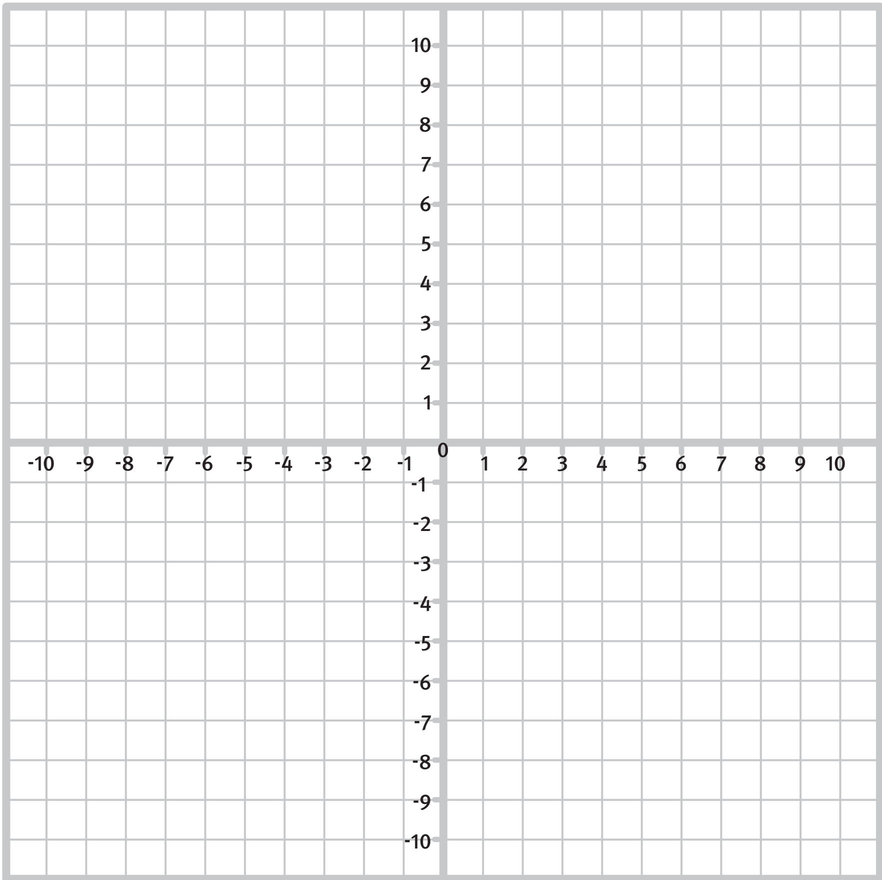
You could now plot these two points and join them with a line. Be sure to extend your line beyond each point.

EXERCISES / COVER-UP METHOD

Draw the following graphs using the cover-up method:

Plot these coordinates and join up the points extending the line.

1. $y = 3x + 9$
2. $y - 10 = 2x$
3. $y = \frac{1}{2}x + 4$
4. $y + 2 = x$



Probing Question:

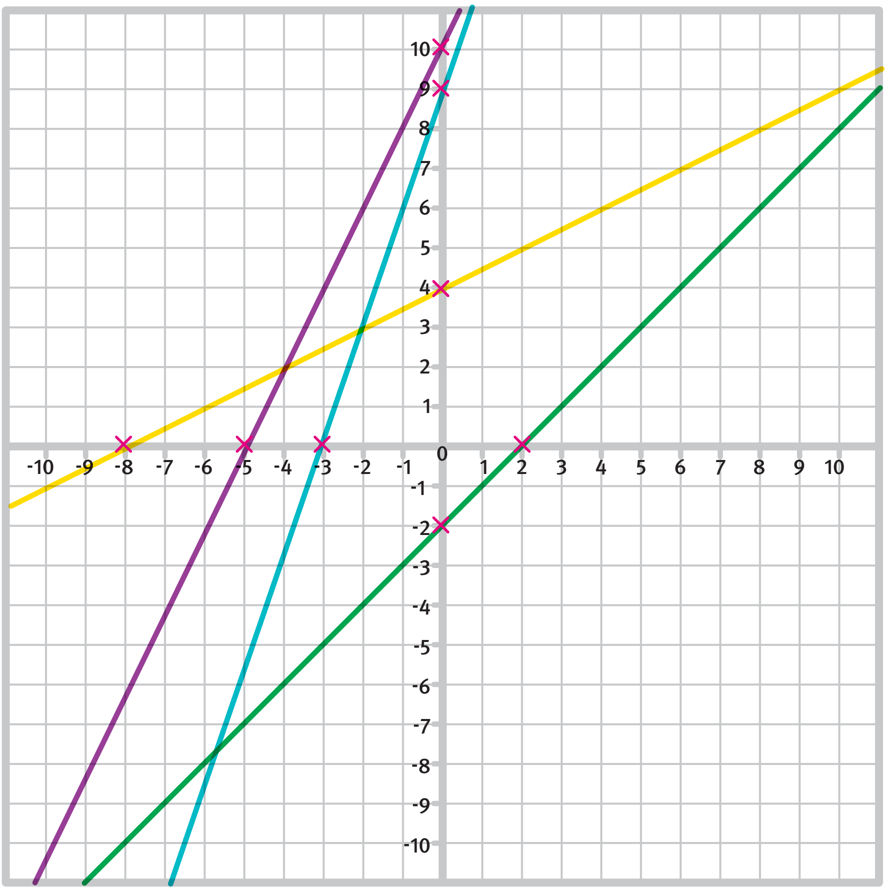
What would happen if $c = 0$? Example: $y = 5x$

SOLUTIONS / COVER-UP METHOD

TEACHER'S NOTE
This method is simulating setting x and then y to zero in order to obtain two coordinates on the line.

TEACHER'S NOTE
This method works for linear equations not written in the form $y = mx + c$.

1. (-3, 0) (0, 9)
2. (0, 10) (-5, 0)
3. (-8, 0) (0, 4)
4. (2, 0) (0, -2)



Probing Question:

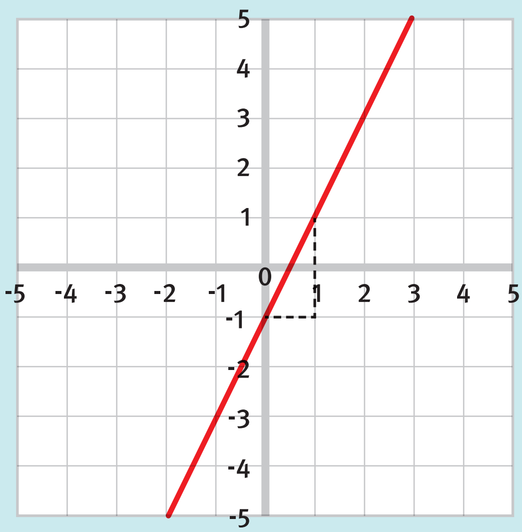
When $c = 0$ we do not have enough information to obtain two points on the graph, we can only get (0, 0). This means you cannot draw the line with this method.

Gradient-intercept method

EXAMPLE 1

Draw the graph of $y = 2x - 1$

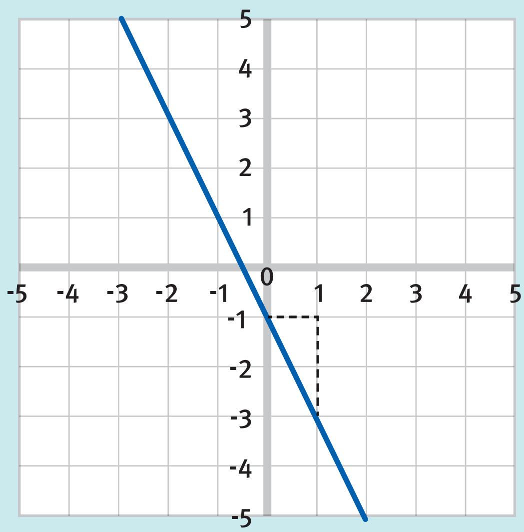
This function is in the form $y = mx + c$.
The fact that the constant term, $c = -1$, means that the line goes through the y-axis at -1. This can be used as the first point to plot (0, -1).
As the gradient is 2, for every one unit in the x-direction, you will move 2 in the y-direction. Starting at -1 on the y-axis, move one square to the right and two squares up. This will give a second point on the line, (1, 1). These points can then be plotted and joined with a line extending through them.



EXAMPLE 2

Draw the graph of $y = -2x - 1$

This is exactly the same line except the gradient is negative. For a gradient of -2, moving one square across to the right results in moving two squares downwards instead of upwards as in the previous graph. Starting at (0, -1) the next point would be (1, -3).



EXERCISES / GRADIENT-INTERCEPT METHOD

1. Draw these lines, using the gradient-intercept method:

   a) $y = 2x + 1$
   b) $y = x + 3$
   c) $y = 3x - 4$
   d) $y = -2x$
   e) $y = -x - 2$
   f) $y = 0.5x + 2$

Note ▸ Gradient of 0.5 is the same as a gradient of ½ this means move across 1 square and up ½ square.

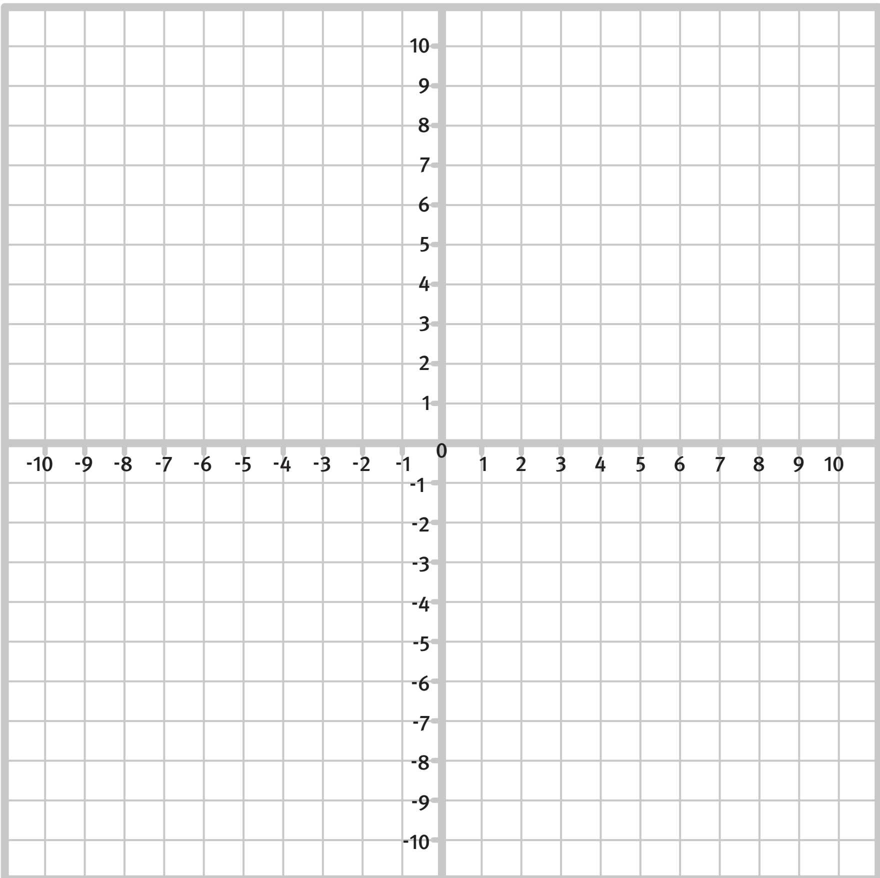
2. Draw the lines $y = 3x + 1$ and $y = 2x + 3$ on the same grid.

   Where do the lines cross?

3. Draw the lines $y = 0.5x$ and $y = 2x - 3$ on the same grid.

   Where do the lines cross?

The grid below can be used to draw the graphs. Alternatively, use square paper and ensure that both axes go from -10 to 10.



SOLUTIONS / GRADIENT-INTERCEPT METHOD

1

a) $y = 2x + 1$
$c = 1 \rightarrow (0, 1)$
$m = 2 \rightarrow (1, 3)$

b) $y = x + 3$
$c = 3 \rightarrow (0, 3)$
$m = 1 \rightarrow (1, 4)$

c) $y = 3x - 4$
$c = -4 \rightarrow (0, -4)$
$m = 3 \rightarrow (1, -1)$

d) $y = -2x$
$c = 0 \rightarrow (0, 0)$
$m = -2 \rightarrow (1, -2)$

e) $y = -x - 2$
$c = -2 \rightarrow (0, -2)$
$m = -1 \rightarrow (1, -3)$

f) $y = 0.5x + 2$
$c = 2 \rightarrow (0, 2)$
$m = 0.5 \rightarrow (1, 2.5)$

★ **TEACHER'S NOTE** ▸
Best practice is for students to plot at least three points before drawing the line with a ruler. Remember the line must extend past the plotted points.

2 The two lines cross (intersect) at (2, 7)

Gradient method

$y = 3x + 1 : c = 1, m = 3$ gives points (0, 1) and (1, 4)
$y = 2x + 3 : c = 3, m = 2$ gives points (0, 3) and (1, 5)
Plot both lines to find the point where they intersect

Simulations Equations method

$3x + 1 = 2x + 3$
$x = 2$

Substitute x back into one of the original equations
$y = 3x + 1$
$y = (3 \times 2) + 3$
$y = 7$

Gives the coordinate (2, 7) as the point where the two lines intersect

★ **TEACHER'S NOTE** ▸
Questions 2 and 3 can be solved using simultaneous equations.

3 The two lines cross (intersect) at (2, 1)

Gradient method

$y = 0.5x : c = 0, m = 0.5$ gives points (0, 0) and (1, 0.5)
$y = 2x - 3 : c = -3, m = 2$ gives points (0, -3) and (1, 1)
Plot both lines to find the point where they intersect

Simulations Equations method

$0.5x = 2x - 3$
$x = 2$

Substitute x back into one of the original equations
$y = 0.5x$
$y = (0.5 \times 2)$
$y = 1$

Gives the coordinate (2, 1) as the point where the two lines intersect

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With thanks to Susan Gill and Mark Walsh at Colchester Royal Grammar School, as well as those primary teachers who trialled these resources and gave valuable feedback.

Should you have any thoughts as to how we might improve future versions, or if there are other topics you would like us to cover, please email:
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