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Surface polaritons born by inter-element coupling in magnetic metamaterials

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Abstract – **We studied theoretically electromagnetic surface waves that propagate along an interface between air and magnetic metamaterials. The metamaterials comprise coupled split-ring resonators. We found two types of waves. Waves of the first type propagate due to interaction of the electromagnetic wave with individual rings, which leads to negative magnetic permeability. They are analogous to the well-known surface plasmons. Waves of the second type, however, are of a different nature and propagate when near-field coupling between the rings exists in the transverse direction. Interaction between the two types leads to complex dispersion characteristics, whose properties can be tailored by changing the inter-element coupling.**

I. INTRODUCTION

Surface waves are responsible for many physical phenomena and form a basis for a number of devices. In the context of plasmonics, most attention has been paid to surface plasmons that propagate on interfaces between dielectrics and metals with negative dielectric permittivity [1, 2]. The advent of metamaterials, whose electric and magnetic properties can be controlled, has expanded the range of structures capable of supporting surface waves. One example is a surface wave at the interface of two metamaterials [3] both supporting magnetoinductive waves. Another example, closer to the classical definition, is a surface wave at the interface between air and an anisotropic magnetic metamaterial made up by split-ring resonators. Such surface waves, existing mainly due to a negative permeability, has recently been shown by Shamonina [4]. The elements were coupled to each other in the propagation direction. Inter-element coupling can, however, exist also in the transverse direction, leading, as this paper will show, to yet another type of surface polaritons. The paper starts by presenting a theoretical model and proceeds by discussing various metamaterial structures where one or both types of waves can propagate.

II. PROPERTIES OF THE SURFACE POLARITONS

We consider a two-dimensional metamaterial comprising two sub-lattices of split-ring resonators, see Fig. 1. Each split ring is represented by an LC-circuit with the inductance L and the capacitance C. The mutual inductance M between the neighbouring rings will depend on their relative orientation and the distance between them. It is positive for elements in the axial configuration, negative for elements in the planar configuration, and zero when the elements' planes are perpendicular to each other. Taking into account only the coupling between the nearest neighbours in a sub-lattice and between the sub-lattices, four nonzero coupling constants $\kappa = 2M/L$ will characterise the metamaterial: two positive axial constants $\kappa_{\mathrm{a}x}$ and $\kappa_{\mathrm{a}z}$ as well as two negative planar constants $\kappa_{\mathrm{p}x}$ and $\kappa_{\mathrm{p}z}$, see Fig. 1.

Following Syms *et al.* [5] and Shamonina [4], we use a circuit model to describe the interaction of electromagnetic waves with the metamaterial. The electromagnetic wave propagates in the z-direction and has the magnetic field components H_x and H_z. The wave propagates along an LC transmission line. The coupling between the transmission line and the split rings is described by the filling factor $q^2_{x(z)}$ [4, 5].

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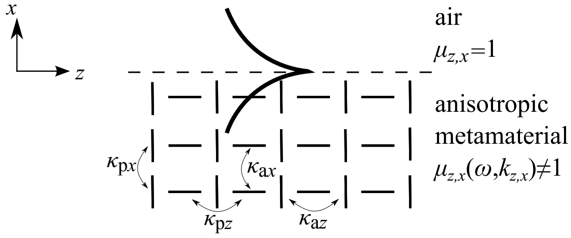

Fig. 1: The interface between a magnetic metamaterial and air can support electromagnetic surface waves.

Using Kirchoff's voltage and current laws and assuming an effective medium, the effective permeabilities of the metamaterial in the x and z directions can be written as [4]

$$\mu_{z(x)} = 1 - \frac{q^2_{x(z)}}{1 - \dfrac{\omega_0^2}{\omega^2} + \kappa_{\mathrm{a}z(x)} \cos(k_{z(x)} d) + \kappa_{\mathrm{p}x(z)} \cos(k_{x(z)} d)}, \tag{1}$$

where ω is the angular frequency, ω_0 is the resonant frequency of the rings, $k_{z(x)}$ is the longitudinal (transverse) wavenumber, d is the period. In our calculations $q^2_{x(z)} = 0.2$, $d = 19$ cm, $\omega_0 = 2\pi \times 100$ MHz. The permeabilities depend both on the frequency and on the wavenumbers. Denoting the air by subscript 1 and the metamaterial by subscript 2, the dispersion relations in both media can be written as

$$\begin{aligned} k_z^2 + k_{x1}^2 &= k_0^2, \\ \mu_z k_z^2 + \mu_x k_{x2}^2 &= \mu_z \mu_x k_0^2. \end{aligned} \tag{2}$$

At the interface $x = 0$ the boundary conditions for continuity of the tangential components of electric and magnetic fields should be satisfied, leading to the dispersion relation of the surface polaritons in the form

$$k_z^2 = k_0^2 \frac{\mu_x (1 - \mu_z)}{1 - \mu_x \mu_z}, \tag{3}$$

where $k_0 = \omega / c$ with the vacuum light velocity c.

The properties of the surface polaritons will depend on the coupling coefficients. The coefficients can be controlled by changing the metamaterial structure. This paper considers three structures: the "brick wall", Fig. 2(a), (also studied in [4]), the "rotated brick wall", Fig. 2(b), and the "fence", Fig. 2(c). The first has non-zero inter-element couplings only in the longitudinal z-direction ($\kappa_{\mathrm{a}z}$, $\kappa_{\mathrm{p}z} \neq 0$; $\kappa_{\mathrm{a}x}$, $\kappa_{\mathrm{p}x} = 0$). The second structure, obtained by rotating the first by 90°, has non-zero inter-element couplings only in the transverse x-direction. The coupling constants are also smaller than for the brick wall. For the third structure coupling exists in both directions.

The brick-wall structure supports a single branch of surface polaritons, see Fig. 2(d) and [4]. These waves propagate around the resonant frequency to the right of the light line (dashed line in Fig. 2). The permissible wavenumbers extend up to the edge of the Brillouin zone. The dispersion diagram for the rotated brick-wall structure, see Fig. 2(e), is radically different: another branch of waves appears, and the region where the waves exist shrinks both in frequency and in wavenumbers. The dispersion diagram is similar for the fence structure, see Fig. 2(f). Two branches exist but in wider ranges of frequencies and wavenumbers. Since the brick wall, in contrast to the other two structures, has no inter-element coupling in the x-direction, this coupling should be responsible for the second branch of the surface polaritons in Fig. 2(e) and (f). Indeed, the coupling can lead to propagation of magnetoinductive waves in the transverse direction. Away from the pass band, the waves will decay inside the metamaterial. Their coupling to electromagnetic waves could create surface waves.

Summarising, three different physical mechanisms exist that affect surface polaritons. The first is the classical case: the interaction of the electromagnetic waves with the resonators is responsible for the surface wave by creating a negative magnetic permeability. The second is inter-element coupling in the propagation direction that modifies the dispersion. The third is inter-element coupling in the transverse direction that results in a second branch of the dispersion diagram.

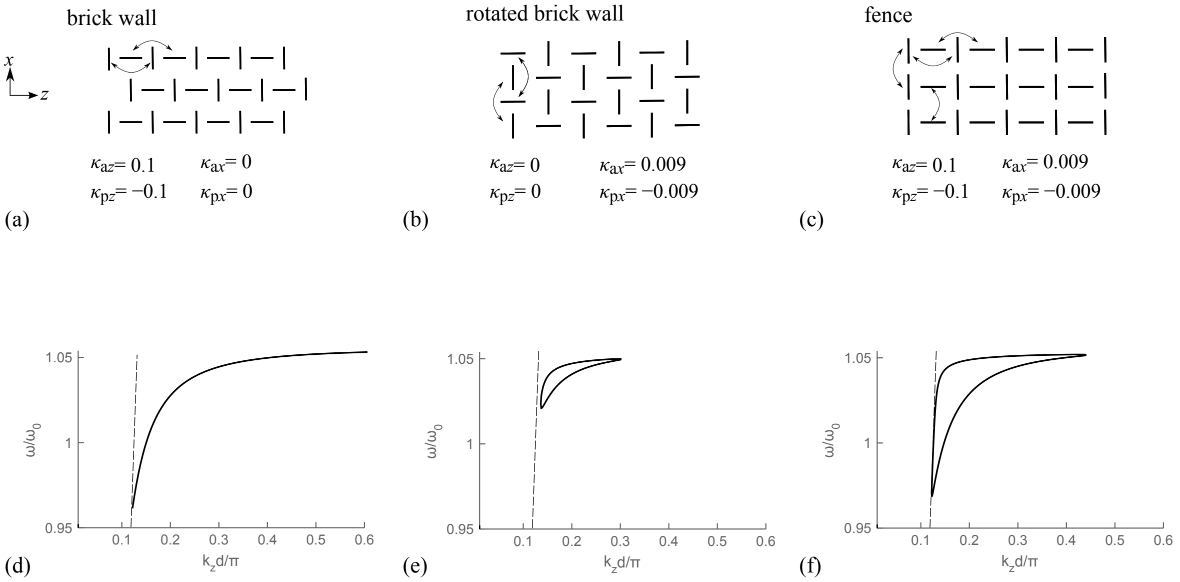

Fig. 2: The surface polaritons supported by a metamaterial structure depend on the inter-element coupling coefficients. The structures with non-zero transverse coupling, (b) and (c), can simultaneously support two waves, (e) and (f).

III. CONCLUSION

As the paper showed, two types of surface polaritons can propagate along a metamaterial-air interface. One of them is born by the inter-element coupling between the split rings constituting the metamaterial. The interaction between the waves leads to multi-branch dispersion diagrams that can be controlled by changing the inter-element coupling. It suggests possibilities to tailor the metamaterial properties.

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