*Special Issue on Metamaterials LHM*

# Experimental study of a bi-periodic magnetoinductive waveguide: comparison with theory

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**Abstract:** Magnetoinductive waves propagating along a line consisting of two kinds of metamaterial elements are studied. Both elements are made up by the same metallic loop but are loaded by different capacitors resulting in different resonant frequencies. The dispersion characteristics are derived from phase and amplitude measurements for the cases when the line consists of (i) identical elements and (ii) of alternating elements. Both planar (elements in the same plane as the axis of the line) and axial (elements perpendicular to the axis of the line) configurations are investigated. It is shown that in the bi-periodic arrangement of the elements, the dispersion curves have a forward wave in the lower frequency branch and a backward wave in the upper frequency branch independent of the configuration whether it is planar or axial. Comparisons between theoretical and experimental results show good agreement.

## 1 Introduction

A by-product of the recent studies on negative refraction and metamaterials [1, 2] is the surge of interest in the properties of magnetoinductive (MI) waves. First proposed about four years ago [3, 4] they have been considered for various applications as waveguide components [5, 6], delay lines [7], phase shifters [8] and sub-wavelength imagers [9]. In addition, their interaction with electromagnetic waves [10] and their refraction (both positive and negative) in two dimensions [11] have been investigated.

In analogy with studies on acoustic waves [12], it has recently been shown [13] that the MI wave dispersion curve for a linear bi-periodic array has two branches. For an array composed of identical elements, MI waves are forward waves for the axial configuration (plane of elements perpendicular to the axis) and backward waves for the planar configuration (all elements in the same plane). One of the interesting conclusions drawn in the work of Sydoruk *et al.* [13] was that for the bi-periodic case the lower one of the two branches is always a forward wave and the upper one is always a backward wave independent of the orientation of the elements whether they are planar or axial. The aim of the present paper is to compare this and other theoretical predictions with the measured results and to demonstrate that the dispersion characteristics of MI waves can be tailored to specified requirements enabling nonlinear processes such as parametric amplification [13]. Our experiments were carried out in the tens of MHz region but the conclusions are equally valid in all frequency bands in which magnetically coupled resonant elements can be produced. These were done for example at microwaves [7, 9] and in the THz region [14].

## 2 Elements

The elements are identical singly split single rings (so-called split-pipes [15]) loaded with two different sets of capacitors so that alternating elements in the one-dimensional array are tuned to different resonant frequencies as shown in Figs. 1*a* and *b* for the planar and axial configurations, respectively. The ring dimensions (Fig. 1*c*) are $r = 10$ mm, $w = 1$ mm, $h = 5$ mm and $g = 1$ mm. $C_A$ and $C_B$, the capacitances inserted into elements A and B, were 330 pF and 680 pF, respectively, resulting in resonant frequencies of $\omega_{0A}/(2\pi) = 46.21$ MHz and $\omega_{0B}/(2\pi) = 32.46$ MHz. Careful selection of the capacitors ensured that the resonant frequencies varied by no more than 0.3%.

## 3 Measurements

First the $Q$ factor was determined by placing one of the elements on a thin piece of balsawood. The measuring coils were made of wires connecting the inner and outer conductors of a coaxial cable. The sample was placed half-way between the transmitter and receiver coils, both at a distance of 2 mm from the sample edge. Increasing the distance any further had no effect upon the resonance frequency. The quality factor extracted from the measured resonance curve was found to be between 100 and 110 for both kinds of elements. Apparently, the capacitor in element A was somewhat lossier than that in element B.

For the planar array measurements, the elements were placed again on a balsawood. The first element was excited by the transmitter coil placed below the element and the current in the other elements was probed by a



doi:10.1049/iet-map:20050289

Paper first received 22nd November 2005 and in revised form 20th April 2006

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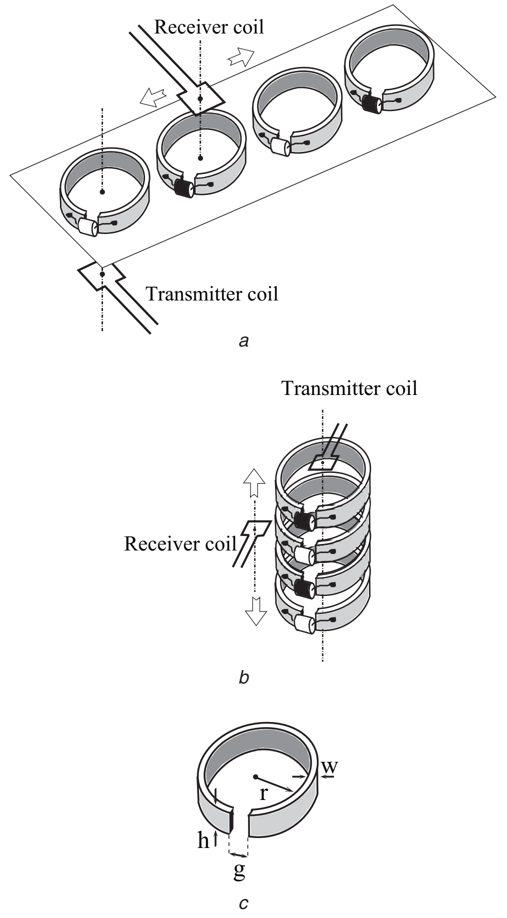


**Fig. 1** *Schematic presentations*

*a* Planar configurations with measuring coils
*b* Axial configurations with measuring coils
*c* Dimensions of the split ring

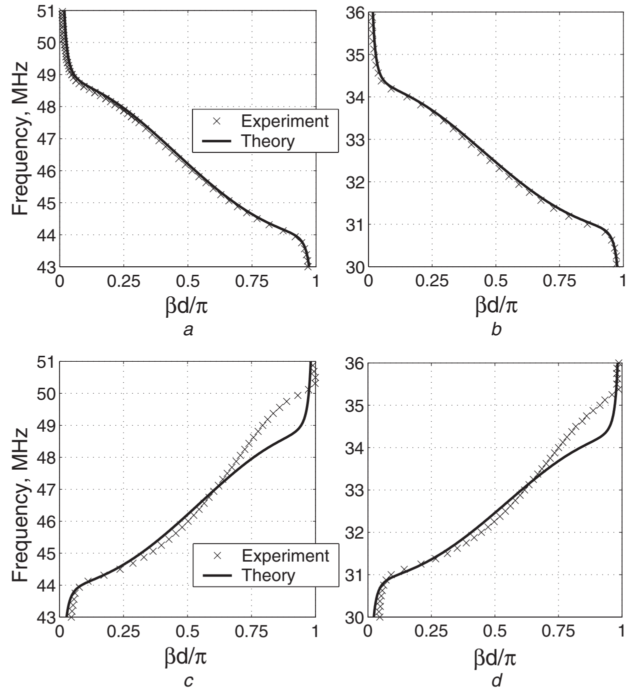


**Fig. 2** *Dispersion characteristics of arrays consisting of identical elements*

*a* and *b* Planar configuration
*c* and *d* Axial configuration

receiver coil moved along the line (see Fig. 1*a*) and placed each time above the centre of the element.

For the axial array, the measurements were performed in the same ranges of frequency but the measurement set-up (Fig. 1*b*) was somewhat different. The transmitter coil was placed concentrically very close to the first element but for probing the current in subsequent elements, the receiver coil moved parallel with the elements slightly above them, taking measurements at positions where the coil and the element were aligned.

## 4 Results

The dispersion characteristics for an array of identical elements for capacitively loaded loops and Swiss Rolls have already been reported in previous publications [16, 17] and compared with theoretical results. Since those measurements were performed for different kind of elements, we decided to repeat them at least for the phase measurements. By using the same technique [16, 17], we measured the phase variation for two-element arrays in AA and BB configurations. The measurements were performed at 1601 discrete values of frequency between 40 and 55 MHz for AA and between 25 and 40 MHz for BB configurations with the aid of a network analyser of the type HP8753C. The dispersion curves deduced from the experiments are shown in Fig. 2 by crosses. As expected the waves are backward (Figs. 2*a* and *b*) for the planar case and forward (Figs. 2*c* and *d*) for the axial case. The pass band lies in a narrow range around the resonant frequencies. Figs. 2*a* and *b* as well as *c* and *d* look very similar: the only difference between them is a frequency shift due to the different resonant frequencies.

The theoretical dispersion curve for nearest neighbour interaction is given by Shamonina *et al.* [3, 4] in the form

$$\cos kd = -\frac{1}{\kappa}\left(1 - \frac{\omega_0^2}{\omega^2} - \frac{\mathrm{j}}{Q}\right) \tag{1}$$

where $\omega$ is the frequency, $\omega_0 = 1/\sqrt{(LC)}$ is the resonant frequency, $\kappa = 2M/L$ is the coupling coefficient, $M$ is the mutual inductance between nearest neighbours, $L$ is the inductance of the element, $d$ is the distance between the elements, $k = \beta - \mathrm{j}\alpha$, $\alpha$ is the attenuation and $\beta$ is the propagation constant. Note that, once the frequency is given, $d$ is chosen and the phases and amplitudes are measured, the only free parameter in the dispersion curve is the coupling coefficient, $\kappa$. For the same orientation and the same distance between the elements, the mutual inductance is practically the same in arrays A and B but the resonant frequencies are of course different. For the theoretical curves, we have taken $\kappa = 0.1$ and $\kappa = -0.1$ for the axial and planar cases, respectively. The absolute values of the coupling coefficients were made the same by choosing correctly the relative spacings: $d = 20$ mm for the axial and $d = 24$ mm for the planar configuration. As may be seen in Fig. 2, we obtain remarkably good agreement between theory (denoted by continuous lines) and experiment.

In our next set of experiments, we looked at bi-periodic lines consisting of altogether 16 elements, in which elements A and B alternate. The spacing between the elements was the same as in the identical-element measurements. The first element was excited. The phase and amplitude at each element was measured for 1601 frequency points in the range 25–55 MHz for the planar and 25–65 MHz for the axial arrangement. The dispersion curve is extracted from

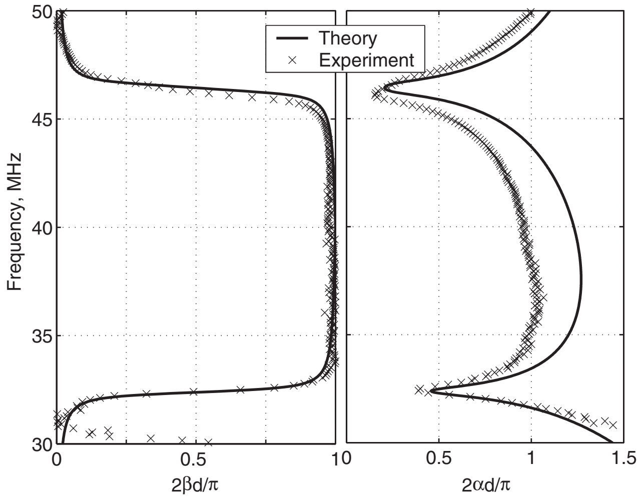


**Fig. 3** *Dispersion characteristics for a bi-periodic planar array*

*a* Propagation constant
*b* Attenuation constant

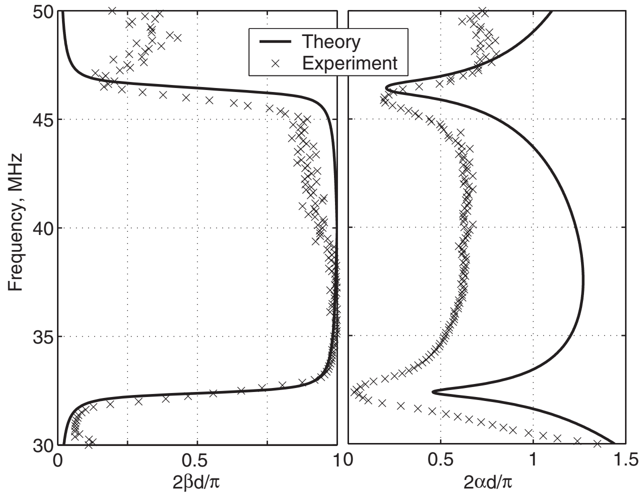


**Fig. 4** *Dispersion characteristics for a bi-periodic axial array*

*a* Propagation constant
*b* Attenuation constant

the phase and amplitude measurements at the first five elements. The experimentally obtained values, phase change and attenuation, are shown by crosses in Figs. 3*a* and *b* for the planar and in Figs. 4*a* and *b* for the axial configuration.

For the theory, there are no longer any free parameters. The coupling coefficients have already been determined from the identical-element measurements. The theoretical dispersion equation for a bi-periodic line is given by Sydoruk *et al.* [13] in the form

$$\cos 2kd = \pm \frac{1}{\kappa}\sqrt{\left(1 - \frac{\omega_{0A}^2}{\omega^2} - \frac{j}{Q_1}\right)\left(1 - \frac{\omega_{0B}^2}{\omega^2} - \frac{j}{Q_2}\right)} \quad (2)$$

Substituting the parameters of the experiment into (2), we obtain the phase change and the attenuation for both configurations. They are shown by continuous lines in Fig. 3 for the planar and in Fig. 4 for the axial case. It is gratifying to note that the principal prediction of the theory, that the bi-periodic dispersion curves look the same for the planar and axial configurations, is indeed borne out by the experimental results. The detailed agreement is excellent for the phase changes. For the attenuation, there is only qualitative agreement: it can be seen that the theoretically predicted attenuation is larger than the measured one. The likely reason is higher-order interactions which will increase the amplitude measured.

We have also measured the relative phases in neighbouring elements for low values of $\beta d$ in order to check the theoretical predictions [13]. For the planar case, they are in anti-phase in the lower and in-phase in the upper branch in agreement with the theory. In analogy with phonon dispersion, one may call then the lower branch 'optical' and the upper one 'acoustic'. For the axial configuration, the experimental phases behave in the opposite manner: they are in-phase for the lower ('acoustic') and in anti-phase for the upper ('optical') branch.

## 5 Conclusions

MI waves have been studied under geometrical arrangements when elements of different resonant frequencies alternate in a line. It has been shown that in contrast to what happens in identical arrays, which exhibit forward waves for the axial and backward waves for the planar configuration, the bi-periodic arrangement leads always to a backward wave in the upper branch and to a forward wave in the lower branch. Comparisons between the theoretical and experimental dispersion characteristics show good agreement. The measurements have also shown that there is a certain duality between the planar and axial configurations concerning the relative phases of neighbouring elements for low values of $\beta d$. When, in a given branch of the dispersion characteristics, one configuration leads to identical phases, the other configuration will lead to opposite phases. This study has confirmed that the dispersion of MI waves can be tailored to specific requirements, which opens the road for the investigation of nonlinear interactions.

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