9th International Congress on Advanced Electromagnetic Materials in Microwaves and Optics - Metamaterials 2015
Oxford, United Kingdom, 7-12 September 2015

Line-defect magneto-inductive waveguides and waveguide components

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Abstract **– We study theoretically magneto-inductive waveguides made by defect lines in metamaterial surfaces that comprise coupled split-ring resonators. The defects are introduced by changing the resonant frequency of the rings. We derive the wave dispersion relation and show how its properties depend on the sign and the strength of the inter-element coupling and the relationship between the resonant frequencies. We also demonstrate wave propagation along a 90-degree waveguide bend. The results show potential for novel planar magneto-inductive components.**

I. INTRODUCTION

Like photonic crystals and arrays of coupled optical resonators [1], metamaterials with line defects can guide waves. Radkovskaya *et al.* [2] studied magneto-inductive waves [3, 4] that propagate along the interface between two metamaterial surfaces consisting of coupled split-ring resonators. They could control the wave properties by varying the distance between the surfaces and thus the coupling strength between the interface elements. Here, we discuss an alternative way to realise line-defect magnetoinductive waveguides and components by varing the resonant frequency of the elements. We first describe the dispersion of the waves and then demonstrate a 90° waveguide bend.

II. WAVE DISPERSION

We consider a two-dimensional periodic array of identical coupled metamaterial elements, see Fig. 1(a). The period is d; the resonant frequency is $\omega_0 = 1/\sqrt{LC}$, where L and C are the self-inductance and self-capacitance respectively. The coupling constants, $\kappa = 2M/L$ (where M is the mutual inductance), depend on the relative orientation and distance between the elements. In our geometry, the elements are in the axial configuration (with positive values of κ) along the columns and in the planar configuration (with negative values of κ) along the rows. We take only nearest-neighbour coupling into account and denote the vertical and horizontal coupling constants by $\kappa_{\mathrm{a}x}$ and $\kappa_{\mathrm{p}z}$, respectively. We then introduce line defects by changing the capacitance C', and hence the resonant frequency ω_0', of the elements either along a row or along a column.

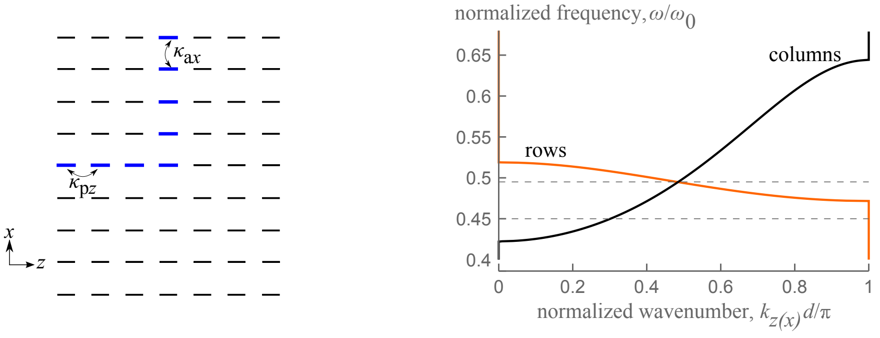
We assume a wave of currents with the wavenumbers k_z in the horizontal z- and k_x in the vertical x-direction. The wave obeys the known [4, 5] bulk dispersion for two-dimensional lattices in the form

$$1 - \frac{\omega_0^2}{\omega^2} - \kappa_{\mathrm{a}x}\cos(k_x d) - \kappa_{\mathrm{p}z}\cos(k_z d) = 0 \tag{1}$$

where ω is the angular frequency. Writing down Kirchoff's equations for the line defect, we also obtain

$$\frac{\omega_0 - \omega_0'}{\omega^2} - j\kappa_{\mathrm{a}x(\mathrm{p}z)}\sin(k_{x(z)}d) = 0 \tag{2}$$

Equations (2) and (1) form a system that can be solved for the wavenumbers k_x and k_z at a frequency ω. We are interested in waves that propagate along the defect but decay in the perpendicular directions, and so demand real k_z and imaginary k_x for waves along the horizontal defects. For vertical defects, k_x is real and k_z is imaginary.



(a) (b)

Fig. 1: Line defects in metamaterial surfaces (a) can support waves. For the planar-axial configuration, forward waves propagate along the columns (vertically) and backward waves propagate along the rows (horizontally)

For a numerical example, we took ω_0= 46 MHz, $\omega_0'/\omega_0 = 0.5$, $\kappa_{ax} = 0.4$ and $\kappa_{pz} = -0.1$. Figure 1(b) shows the corresponding dispersion curves for the waves along the rows (orange line) and along the columns (black line). Both waves exist in a frequency band centred at the resonant frequency of the defects. Similarly to the known magneto-inductive waves supported by one-dimensional arrays, forward waves now propagate when the inter-element coupling constant in the propagation direction is positive. Backward waves propagate when the coupling constant is negative. The band for the waves propagating vertically is wider, due to the higher coupling strength along the elements in the columns.

III. WAVEGUIDES AND COMPONENTS

To show how waves can propagate in both directions, we consider a 90° bend, see Fig. 1(a). The metamaterial surface is 51 by 60 elements large, and the defects are placed in the middle row and column. We then excite either the first horizontal or the first vertical element by a voltage source. The currents in all elements can be calculated from the Kirchoff's equation in the matrix form [3, 4]

$$\mathbf{V} = \mathbf{Z}\mathbf{I} \tag{3}$$

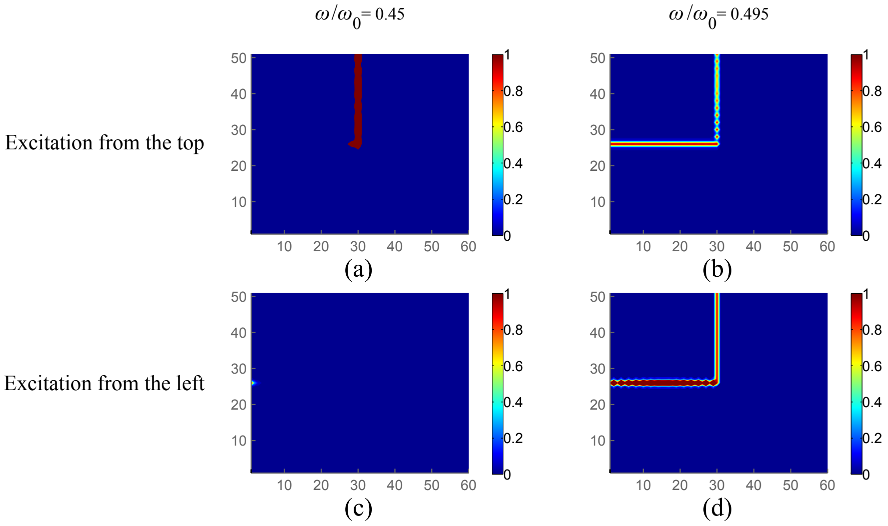
where **V**=$(V_1, V_2, V_3, ..., V_N)$ is the column-vector of voltages, **I**=$(I_1, I_2, I_3, ..., I_N)$ is the column vector of currents, and **Z** is a $N \times N$ impedance matrix. The elements at the perimeter of the surface (see Fig. 1(a)) are missing one coupled neighbour, which can lead to wave reflection. To minimise reflections, we inserted absorbing terminations

$$Z_{\mathrm{T}p(a)} = Z_0' + j\omega M_{\mathrm{p}z(\mathrm{a}x)} e^{jk_{z(x)}d} \tag{4}$$

where $Z_0' = j\omega L + 1/(j\omega C')$. The impedance $Z_{\mathrm{T}p}$ is placed in the far left element of the row and $Z_{\mathrm{T}a}$ is the terminal impedance placed at the top element of the column.

Figure 2 shows the current distributions at two frequencies $\omega/\omega_0 = 0.45$ and $\omega/\omega_0 = 0.495$, shown by dashed lines in Fig. 1(b). The excitation can be placed either at the top element of the vertical part (see Fig. 2(a) and (b)) or at the first element from the left of the horizontal part (see Fig. 2(c) and (d)). At $\omega/\omega_0 = 0.45$, only the vertical wave can propagate as can be seen from the dispersion diagram. Therefore, with excitation from the top, a wave propagated along a column but it cannot continue to the defect row (see Fig. 2(a)). With excitation from the left, no wave can be excited in the row and anywhere on the surface (see Fig. 2(c)).

At $\omega/\omega_0 = 0.495$, where the two dispersion lines intersect, a wave can propagate in both directions independent of the excitation position. For the top excitation, no reflections occur for the wave along the row (see Fig. 2(b)), whilst an excitation from the left gives no reflections in the column (see Fig. 2(d)).



(a) (b) (c) (d)

Fig. 2: Current distributions for the 90° bend when the excitation is placed at the top element of the defect column (a) and (b), and when it is placed at the far left element of defect row, (c) and (d). The currents are plotted for $\omega/\omega_0 = 0.45$ in (a) and (c), where only the column can support a surface wave. The currents are plotted for $\omega/\omega_0 = 0.495$ in (b) and (d) where both the column and the row support waves.

IV. CONCLUSION

We have theoretically shown that waves can propagate along a defect line between homogeneous metamaterial lattices consisting of coupled split-ring resonators. The defect-line elements have resonant frequency that differ from that of the surrounding surface. We also studied a 90° bend formed in a planar-axial lattice, and discussed how the waves propagate along the bend at different frequencies. The approach shows potential also for other planar components, such as couplers and dividers.

ACKNOWLEDGEMENT

The authors thank R. R. A. Syms and E. Shamonina for useful discussions. K. H. and S. A. M. gratefully acknowledge financial support from the Leverhulme Trust.

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