Terahertz instability of optical phonons interacting with plasmons in two-dimensional electron channels

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A terahertz (THz) instability can occur when optical phonons interact with drifting solid-state plasmas. We developed a theoretical model for the optical-phonon instability in two-dimensional electron channels. The paper derives the dispersion relation and analyzes the instability using parameters measured in InSb. As the calculations show, strong instability occurs around the longitudinal optical-phonon frequency, and both the growth rate and the unstable frequency band are larger for higher electron densities and lower drift velocities. The results demonstrate the potential of the optical-phonon instability for active THz devices. © *2010 American Institute of Physics*. [doi:10.1063/1.3479416]

Driven by potential applications in security, spectroscopy, and imaging, terahertz (THz) technology has been rapidly developing in recent years. The progress has been impeded by the lack of efficient sources and the search for ways to generate THz radiation continues. The interest to using solid-state plasmas for THz generation was intensified by a seminal paper by Dyakonov and Shur.[1] They showed that plasma waves (often also referred to as plasmons or plasmon polaritons) in a field-effect transistor can become unstable when reflecting from its source and drain. Plasma effects in transistorlike structures have since been extensively studied both theoretically and experimentally.[2]

An alternative for employing drifting solid-state plasmas in oscillators and amplifiers is to couple them to slow electromagnetic waves, thus realizing solid-state versions of vacuum traveling-wave devices.[3,4] Promising candidates for the slow circuit wave are optical phonons whose frequencies lie in the THz range for most diatomic solids. Previous theoretical studies of the interaction between optical phonons and drifting plasmons concentrated on bulk materials.[5,6] As they showed, this interaction can become unstable at high electron drift velocities and high electron densities. High drift velocities are needed for plasmons to actively interact with optical phonons, whereas high electron densities are needed to increase the plasma frequency and, thus, the gain.

In bulk materials, however, the requirements for large electron densities and large drift velocities often contradict each other: high electron densities are realized by doping, which, introducing impurities, decreases the electron velocity. This conflict can be resolved in two-dimensional (2D) electron channels, where high densities can be achieved without degrading mobility. For example, mobilities as high as 250 000 $cm^2/V\,s$ for the densities of 4×10^{11} cm^{-2} have been recently reported in InSb channels at cryogenic temperatures.[7]

This paper considers the instability that arises when optical phonons interact with 2D drifting plasmons. In our configuration, a 2D electron channel is surrounded by a diatomic dielectric, Fig. 1. The channel, occupying the plane $x=0$, is assumed to be infinitely thin.

First, we derive the dispersion relation for the phonon–plasmon interaction. To describe the channel, we present the electron movement as a flow of a charged liquid and we take into account effects of electron collisions and diffusion. For optical phonons, we assume permittivity that depends on the optical-phonon frequencies and the wave number. Then, we analyze the dispersion relation for representative examples where the parameters correspond to values reported for InSb. For these parameters, the coupled phonon-plasmon modes become unstable around the longitudinal optical-phonon frequency, thus showing the potential of the phonon–plasmon interaction for generating and amplifying THz radiation.

To derive the dispersion relation, we assume TM waves with the electric-field components E_x and E_z and the magnetic-field component H_y. The time variation in the fields is in the form $\exp(j\omega t)$, where ω is the frequency, and the spatial variation is in the form $\exp(-jk_z z-\kappa_x x)$ for $x>0$, where $k_z>0$ and $\kappa_x>0$ are the longitudinal and the transverse wave numbers, respectively. The fields decay away from the electron channel, and the corresponding dispersion relation is $k_z^2-\kappa_x^2=k_0^2\varepsilon_{\mathrm{phon}}$, where $k_0=\omega/c$ (c is the light velocity) and $\varepsilon_{\mathrm{phon}}$ is the permittivity of the dielectric. As shown later, the instability occurs at large wave numbers for which $k_z\gg k_0$. Then, the right-hand side on this dispersion relation can be set to zero and $\kappa_x\approx k_z$.

For the permittivity in the presence of optical phonons, $\varepsilon_{\mathrm{phon}}$, we assume the following expression[8]

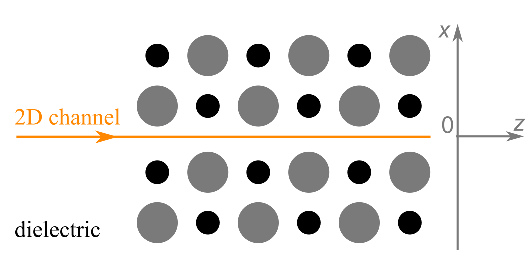

FIG. 1. (Color online) A 2D electron channel is surrounded by a diatomic material supporting optical phonons. Interaction between the electrons drifting in the channel and the optical phonons can lead to a THz instability.

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TABLE I. Parameters used in numerical calculations correspond to InSb.

$f_L=\omega_L/(2\pi)$	5.71 THz
$f_T=\omega_T/(2\pi)$	5.37 THz
ε_∞	15.7
v_s	3.7×10^5 cm/s
m^*	$0.014m_0$
γ_{pl}	0.5 THz
γ_{ph}	0.09 THz

$$\varepsilon_{phon}=\varepsilon_\infty\left[1-\frac{\omega_L^2-\omega_T^2}{\omega^2-\omega_T^2+(k_zv_s)^2-j\omega\gamma_{ph}}\right], \quad (1)$$

where ω_L and ω_T are the longitudinal and transverse optical-phonon frequencies, ε_∞ is the high-frequency permittivity, v_s is the sound velocity, and γ_{ph} characterizes phonon loss.

Further, we linearize the expression for the current density and the equation of motion. To do so, we assume that the electron velocity and density have a constant part and a small time-varying part and neglect products of small terms. The linearized expression for the ac 2D current density, J, is then $J=en_0v+env_0$, where n_0 and n are the dc and the ac 2D electron densities, respectively, and v_0 and v are the dc and ac electron velocities. The linearized equation of motion is[3]

$$\frac{\partial v}{\partial t}+v_0\frac{\partial v}{\partial z}+\gamma_{pl}v+\frac{v_T^2}{n_0}\frac{\partial n}{\partial z}=\frac{e}{m}E_z|_{x=0}, \quad (2)$$

where γ_{pl} is the collision frequency and v_T is the thermal velocity, which characterizes electron diffusion. For the Maxwell–Boltzmann distribution, the thermal velocity is of the order of k_BT, where k_B is the Boltzmann constant and T is the temperature; for the Fermi distribution, it is of the order of the Fermi velocity, v_F. Below, we assume the Fermi distribution and take[9] $v_T=v_F/\sqrt{2}$.

Finally, assuming for the ac density and velocity the variation $\exp[j(\omega t-k_zz)]$ and using the standard boundary conditions at $x=0$ to relate the density and velocity to the fields, we obtain the following dispersion equation:

$$[(\omega-k_zv_0)(\omega-k_zv_0-j\gamma_{pl})-k_z^2v_T^2-\Omega_p^2k_z]\times[\omega^2-\omega_L^2+k_z^2v_s^2-j\omega\gamma_{ph}]=\Omega_p^2k_z(\omega_L^2-\omega_T^2), \quad (3)$$

with $\Omega_p^2=(e^2n_0)/(2m^*\varepsilon_0\varepsilon_\infty)$, where ε_0 is the free-space permittivity.

The dispersion equation Eq. (3) is written in the form standard for coupled–wave interactions. The term on the left-hand side between the square brackets describes the 2D plasmons in the presence of collisions and diffusion; the second term describes the longitudinal optical phonons propagating in the diatomic material. The term on the right-hand side is the coupling coefficient.

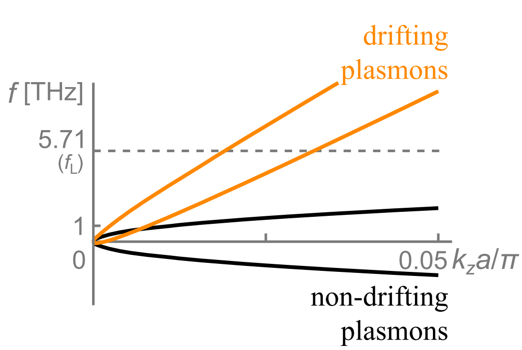

FIG. 2. (Color online) The dispersion curves of the drifting 2D plasmons cross the dispersion curves of the optical phonons leading to strong interaction between them. (The wave number is normalized to the lattice constant $a=0.648$ nm.)

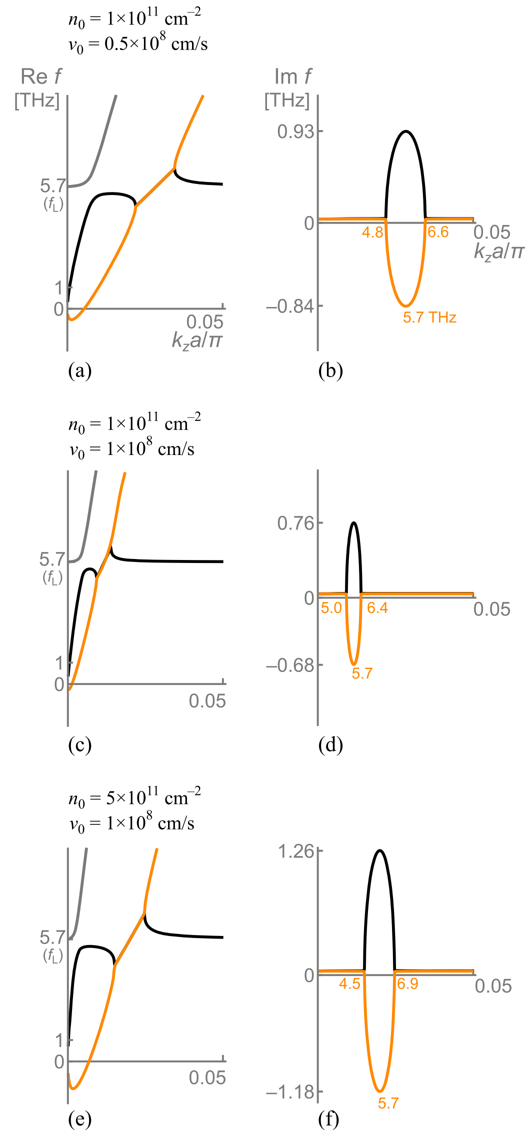

FIG. 3. (Color online) Interaction between the drifting 2D plasmons and the optical phonons is unstable as indicated by Im $f<0$ on the dispersion diagrams (b), (d), and (f). The instability is stronger for lower drift velocities, compare (a) and (b) with (c) and (d), and for higher electron densities, compare (c) and (d) with (e) and (f). For all parameters, the instability is the strongest near the longitudinal optical-phonon frequency.

By solving the dispersion relation for ω assuming real k_z, we are able to determine whether the system can become unstable. The criterion for an instability, whether it is convective or absolute, is Im $\omega<0$; the magnitude |Im ω| characterizes the temporal growth rate. Below, we analyze the dispersion equation, Eq. (3), and determine when the optical-phonon instability occurs. For the instability, the most important parameters are the drift velocity, v_0, and the electron density, n_0; we will vary them in the calculations that follow. For the remaining parameters, we chose the values corresponding to InSb, see Table I.

To study the dispersion relation, we first consider uncoupled plasmons and optical phonons by setting the coupling coefficient in Eq. (3) to zero and ignoring losses and diffusion. The plasmon dispersion relation in the absence of drift, $v_0=0$ is[9] $\omega=\Omega_p\sqrt{k_z}$. The corresponding dispersion curves are plotted in Fig. 2 by black lines for $n_0=10^{10}$ cm^{-2}. When the electrons drift, the plasmon dispersion curves rotate counter clock-wise (orange lines) and cross the optical-phonon dispersion curve (gray lines) at two points, as shown in Fig. 2 for $v_0=c/1000=3\times10^7$ cm/s.

The synchronism between the drifting plasmons and the optical phonons leads to strong instability. The presence of the instability is seen in the dispersion diagrams of Figs. 3(a) and 3(b) plotted for $n_0=10^{11}$ cm^{-2} and $v_0=c/600=0.5\times10^8$ cm/s. For the unstable branch (orange lines), Im $f<0$ between approximately 4.8 and 6.6 THz. The instability is the strongest close to the longitudinal optical-phonon frequency, $f_L=5.71$ THz, where the temporal growth rate is $|\text{Im}\, f|=0.84$ THz. This value of the growth rate is smaller then the corresponding value of loss of the other branch (0.93 THz) due to detrimental effects of losses and diffusion.

The coupling coefficient in Eq. (3) is proportional to the wave number, k_z. Because the wave number at which the instability occurs decreases with increase in the drift velocity, higher drift velocities lead to weaker instabilities. It is demonstrated in Figs. 3(c) and 3(d), where the electron density is the same as in the previous example but the drift velocity was increased to[10] $v_0=c/300=10^8$ cm/s. Compared to the previous example, Figs. 3(a) and 3(b), the instability bandwidth shrunk from 1.8 to 1.4 THz, and the maximal growth rate is lower. Such a behavior is not observed for the instability in bulk materials where the coupling coefficient is constant.[6]

On the other hand, larger electron densities lead to stronger coupling between the waves and, thus, to stronger instabilities. This effect is demonstrated in Figs. 3(e) and 3(f), where the value of the drift velocity is the same as in Figs. 3(c) and 3(d), but the density is higher, $n_0=5\times10^{11}$ cm^{-2}. The instability bandwidth of 2.4 THz and the maximum temporal growth rate of 1.18 THz are higher than those of the previous example.

The linear theory used here cannot predict the output power but we can estimate it from the dc power assuming certain conversion efficiency. For the parameters of Fig. 3(a) and the conversion efficiency of 1%, we estimate the output power as 20 nW/μm^2 of the channel area. To achieve high drift velocities, the channel might need to be cooled down to cryogenic temperatures. For the thickness of InSb above and below the channel (see Fig. 1) of 300 μm, we estimate the temperature in the channel to rise by about 15 K.

The choice of materials where the instability could be observed is not limited to InSb. One of the alternative materials for the 2D channel could be graphene. For the electron density of 5×10^{11} cm^{-2}, the drift velocity in graphene could be as high as[11] 0.45×10^8 cm/s. These values are close to those used in our calculations, suggesting similar properties of the graphene and semiconductor configurations.

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