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Seed function combination selection for chained function filters

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Abstract: An algorithm is presented for seed function combination selection for chained function filters on the basis of pre-defined manufacturing limitations. To this end, and for the first time, combinatorial analysis methods have been applied to computer-aided filter design routines. A fundamental Monte-Carlo margin has been discovered that represents an upper-bound on the worst-case pass-band return-loss responses that cannot be exceeded. This can be exploited to extend the state-of-the art in tuningless high-performance filter implementations towards higher frequencies and smaller fractional bandwidths or, alternatively, to lower the accuracy and manufacturing cost for a given set of filter specifications.

1 Introduction

Manufacturers are under constant pressure to reduce the development time and cost of front-end microwave and millimetre-wave hardware. One of the most expensive components are high-performance filters – due to the associated costs of high quality (Q)-factor materials, any precision assembly and, more significantly, any post-manufacturing tuning. In the context of such filters, this effort has resulted in considerable progress in the area of advanced electromagnetic (EM) simulation tools. However, the simulation accuracy that is now available has diverted attention to the manufacturing process. This is because, in order to draw the full benefits from the increased simulation accuracy, hardware must be manufactured with low tolerances. This, in turn, can significantly increase the final filter costs.

With current practice, the available filter computer-aided design (CAD) techniques do not take into consideration the limitations of the manufacturing process at the initial filter approximation stage. As a result, engineers must first design a filter and then find the most appropriate manufacturing technology for practical implementation. This approach often requires the use of computationally intensive EM codes, together with fabrication process optimisation and/or post-manufacturing tuning.

Through simulations only, one recent attempt to address the issue of reduced fabrication sensitivity, by changing the filter functions, was reported by Jayyousi *et al.* [1]. The algorithm given here is an alternative to this approach, by using chained function (CF) filters to address the limitations of the available manufacturing technology at the initial approximation stage [2, 3]. This can reduce, or even eliminate, the need for lengthy fabrication process optimisation and post-manufacture tuning. Indeed, a practical measured demonstration of our proposed methodology has already been published [4], but is not reproduced here. The key feature in the proposed algorithm is that it changes the existing filter CAD approach to select the most suited filter transfer function to satisfy the electrical requirements within the limitations imposed by the available manufacturing technology.

The design of a filter usually starts with the selection of a suitable transfer function that will satisfy a set of given electrical specifications. The next step is to translate the computed transfer function into an ideal electrical network representation of the filter, involving lengths of transmission lines (if the filter is to be implemented using distributed components) and impedance inverters. Finally, the ideal network elements are implemented by real components that, when assembled together, exhibit the required electrical behaviour. It is in this final stage that the manufacturing tolerances are usually taken into account using, for instance, accurate EM simulators. In general, however, manufacturing errors will change the electrical performance of the real components, so that additional adjustments are required using either fabrication process optimisation and/or post-manufacture tuning [5].

The most common form of amplitude approximation for microwave filters is the generalised Chebyshev approximation. Using Chebyshev transfer functions, however, once the appropriate function has been identified, it will require a specific manufacturing tolerance and a specific unloaded Q for the individual resonators that cannot be changed or modified. In particular, an important consideration for achieving a first-pass result with this family of filters is the relative frequency separation of the pass-band return-loss (RL) zeros. It is known that the smaller the separation of the RL zeros the higher the sensitivity to any structural parameter variation.

One way to overcome this problem is to address the sensitivity to manufacturing tolerance at the filter's approximation stage using, for instance, a CF transfer function [2, 3]. As already shown both in theory [2] and practice [3], the chained filter transfer function concept provides a variety of transfer functions that can meet all the target filter specifications. With the same filter order, different frequency-domain, time-domain and implementation characteristics can be achieved. Also, unlike the conventional Chebyshev (CC) filter, an optimal filter can be designed having a reduction in sensitivity to manufacturing errors, resonator unloaded Q factor and filter losses [3]. This feature, however, comes at a price. When emulating conventional high-order Chebyshev filters, with CFs, to achieve the same level of out-of-band rejection, the total order of the CF needs to be increased. It can be shown that increasing the total filter order by one is sufficient, without incurring a loss penalty [3, 6]. Indeed, contrary to expectations, when compared to an n^{th}-order Chebyshev filter, the pass-band insertion loss can be less with an $(n+1)^{\text{th}}$-order CF filter. For example, a conventional ninth-order all-pole (i.e. a filter with no finite transmission zeros) Chebyshev filter can be emulated with a tenth-order all-pole CF filter, formed by the product of low-order (having much lower sensitivity) seed functions. There are, however, many different combinations of seed functions that can be used to emulate the given filter order. In theory, for a tenth-order filter, there are 42 different seed function combinations, with each having different characteristics. The objective of this paper is to present an algorithm for seed function combination selection. For the first time, combinatorial analysis is used for the selection of the most appropriate seed function combination, based on pre-defined manufacturing limitations.

2 Algorithm description

The flow diagram for the complete design procedure for CF filters is shown in Fig. 1. The proposed algorithm is enclosed within the dotted border. As with standard filter design algorithms, the input parameters are a set of electrical specifications (e.g. total filter order, number of seed functions, multiplicity of each seed function, number of

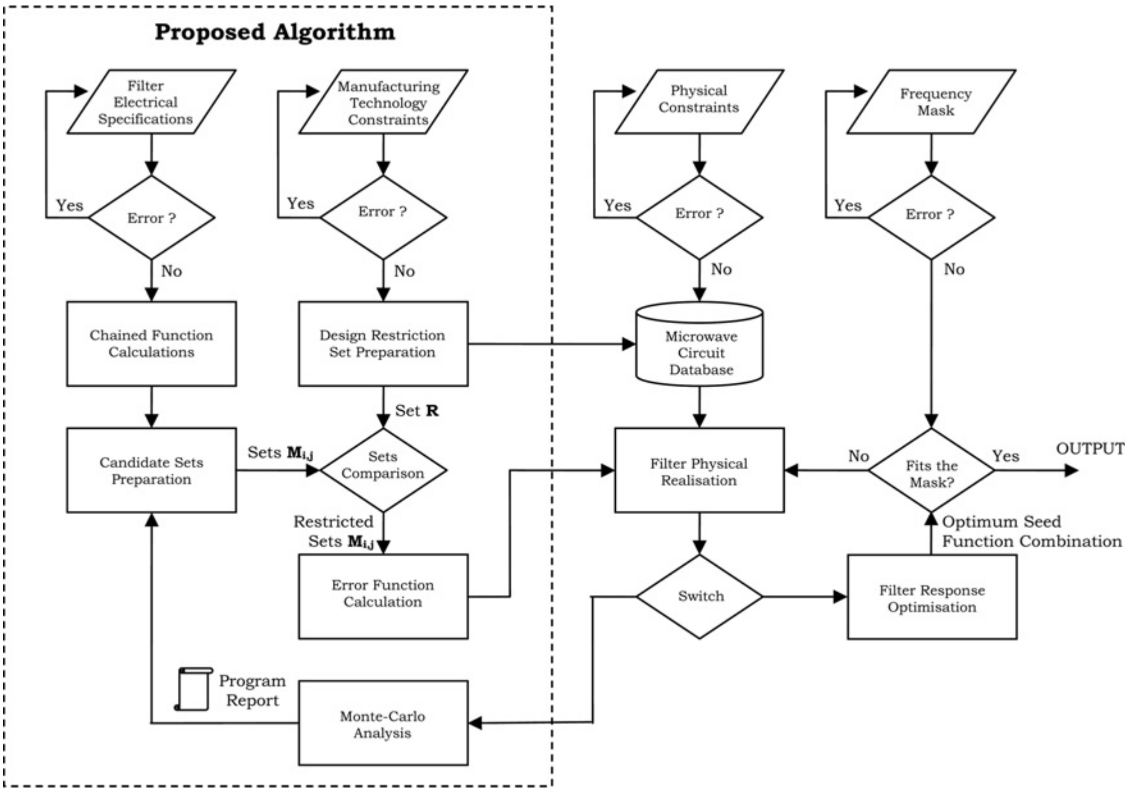

Figure 1 *Complete flow diagram for designing CF filters*

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transmission zeros (or attenuation poles), number of group delay equalisation zeros, worst-case pass-band RL levels, pass-band slope, etc.), the given manufacturing technology restrictions (e.g. available Q-factor, tolerance, etc.) and physical constraints (e.g. size and aspect ratio restrictions, monolithic or hybrid filter integration, etc.).

A physical structure that is most likely to satisfy the given combination of electrical and physical constraints is chosen. This choice is made from what is effectively a database of circuit structures, available to the filter designer. It should be mentioned that more than one structure can be selected at this stage, so the designer is able to check the available manufacturing technology effects on different filter structures.

The process of finding the optimum seed function combination can then be initiated. From the restricted sets of possible seed function combinations, the physical filter parameters (e.g. line widths, lengths, separations, etc.) can be evaluated. The required physical structure(s) can then be applied to Monte-Carlo analysis. As soon as the optimal seed function combination has been identified, the final response optimisation (to the filter poles-zeros) can be initiated to fit the filter response to a given frequency mask [6].

3 Seed function combinations

The first step in developing this algorithm is to find the total number of possible seed function combinations, $\mathrm{SFC}_{(n_\mathrm{T})}$, for a given total CF filter order n_T. As shown in [3], both the total filter order n_T and the seed function orders, $n_{\mathrm{s}(k)}$, are integers; thus, the number of combinations can be defined as

$$\mathrm{SFC}_{(n_\mathrm{T})} = \text{number of ways of partitioning } n_\mathrm{T} \text{ into } n_{\mathrm{s}(k)} \tag{1}$$

This equivalence can be expressed mathematically by applying combinatorial analysis techniques and using partition functions [7–12]. In general, a partition is a way of writing an integer κ as a sum of positive integers without regard to order, possibly subject to one or more additional constraints. There are three types of partition functions used in the proposed algorithm. The first is the partition function $P(\kappa)$, which gives the number of unrestricted decompositions of an integer number κ (i.e. the CF total order) as a sum of smaller integers (i.e. seed function orders) without regard to the order. This function can be used to generate all possible seed function combinations. The second is the restricted partition function $P_\mathrm{R}(\kappa)$, which gives the number of ways of writing the integer κ as a sum of integers without regard to the order and with the constraint that all integers in the sum are distinct. This function can be used to restrict the seed function combinations to either distinct sums or to powered solutions (i.e. solutions such as cubed second-order, etc.). This will be made clear later in this paper. The third is the unordered partitions of κ into s parts, $P(\kappa,s)$ (i.e. assuming that one wishes to use three seed functions only and the designer needs to know how many combinations exist), which can be used to assist in the enumeration process.

3.1 Partitioning the CF total order

The number of unrestricted partitions of an integer κ appears in the expansion of a generating function given by Euler. Consider the q-series, involving coefficients of the form [7–12]

$$(q)_\infty \equiv \prod_{m=1}^{+\infty}(1-q^m) = \sum_{\lambda=-\infty}^{+\infty}(-1)^\lambda q^{\lambda(3\lambda+1)/2} \tag{2}$$

One can define a function, $F_1(q)$, as $F_1(q) = 1/(q)_\infty$, and obtain its Taylor polynomial approximation about the point $q = 0$. Finally, the number of unrestricted partitions of the integer κ appears as the coefficient of q^κ, and thus [7–12]

$$P(\kappa) = \frac{1}{\kappa!}\frac{\partial^\kappa F_1(q)}{\partial q^\kappa}\bigg|_{q=0} \tag{3}$$

A useful recurrence relation that exists in a form suitable for computer implementation has the form [7–12]

$$P(\kappa) = \frac{1}{\kappa}\sum_{m=0}^{\kappa-1}\sigma_1(\kappa-m)P(m) \tag{4}$$

where $P(0) = 1$, and $\sigma_1(\kappa)$ is the divisor function defined as [7–12]

$$\sigma_1(\kappa) = \sum_{i=1}^{M} d_i \tag{5}$$

where d_i are the divisors of κ and M is the total number of divisors. Similarly, the generating function $F_2(g)$ for the restricted partitions of an integer κ, $P_\mathrm{R}(\kappa)$, is [7–12]

$$F_2(g) = \prod_{m=1}^{\infty}(1+g^m) = \prod_{m=1}^{\infty}\frac{1-g^{2m}}{1-g^m} \tag{6}$$

The number of restricted partitions appear as the coefficient of g^κ in the Taylor polynomial approximation of $F_2(g)$. A recurrence relationship suitable for computer implementation is given as [7–12]

$$P_\mathrm{R}(\kappa) = \frac{1}{\kappa}\sum_{m=1}^{\kappa}\sigma_1^{(\mathrm{o})}(m)P_\mathrm{R}(\kappa-m) \tag{7}$$

where $P_\mathrm{R}(0) = P_\mathrm{R}(1) = 1$ and $\sigma_1^{(\mathrm{o})}(m)$ is the odd divisor function, defined as the sum of powers of odd divisors of a number such that [7–12]

$$\sigma_1^{(\mathrm{o})}(m) = \begin{cases}\sigma_1(m) - 2\sigma_1(m/2), & \text{if } m \text{ is even}\\ \sigma_1(m), & \text{if } m \text{ is odd}\end{cases} \tag{8}$$

Here $\sigma_1(m)$ is the divisor function as defined in (5). Finally, the unordered partitions of κ into s parts, $P(\kappa,\ s)$ must be considered. The proposed recursive formula has the form [7–12]

$$P(\kappa, s) = P(\kappa - 1, s - 1) + P(\kappa - s, s) \tag{9}$$

where $P(\kappa, s) = 0$, for $\kappa < s$, $P(\kappa, \kappa) = 1$ and $P(\kappa, 0) = 0$. Equation (9) can be given explicitly for the first few values of s in the simple forms of

$$\begin{aligned} P(\kappa, 1) &= 1 \\ P(\kappa, 2) &= \left\lfloor \frac{\kappa}{2} \right\rfloor \\ P(\kappa, 3) &= \left\lfloor \frac{\kappa^2}{12} \right\rfloor \end{aligned} \tag{10}$$

Table 1 shows the calculated number of unrestricted, $P(n_T)$, and restricted, $P_R(n_T)$, seed function combinations for different CF total order, n_T.

3.2 Enumeration of CF partitions

One way of enumerating all possible seed function combinations is to generate the combinations using a fixed number of n_T seed functions, then reduce this number by one (i.e. $n_T - 1$) and enumerate all the possible combinations, and so on. Eventually, this process will end when one enumerates the combinations using only one seed function. The starting point of the enumeration algorithm is the solution $n_T = 1 + 1 + \cdots + 1$. Here, a set is defined as follows

$$\boldsymbol{M}_{n_T,1} = \Big\{\underbrace{1,\ 1,\ \ldots,\ 1}_{n_T}\Big\} \tag{11}$$

Alternatively, any combination of seed functions of orders first, second, third, …, n_T could be a possible solution to the algorithm. This can be seen by bracketing the different terms together in the set $\boldsymbol{M}_{n_T,1}$. To do this, one needs to define the consequent sets as $\boldsymbol{M}_{i,j}$, where i is the total number of seed functions whose orders will sum up to n_T. For example, if $i = 3$ then there will be three seed functions and when the orders of these three seed functions are summated they will be equal to n_T. Thus, $i = 1, 2, \ldots, n_T$, while j is the corresponding combination index with $j = 1, 2, \ldots, P(n_{T,i})$. Therefore the next sets, in sequence, will consist of the combinations of two seed functions formed as:

$$\begin{aligned}
\boldsymbol{M}_{2,1} &= \Big\{\underbrace{1,\ 1,\ \ldots,\ 1}_{n_T-1},\ \underbrace{1}_{1^{\text{st}}}\Big\} = \{(n_T - 1), 1\} \\
\boldsymbol{M}_{2,2} &= \Big\{\underbrace{1,\ 1,\ \ldots,\ 1}_{n_T-2},\ \underbrace{1,\ 1}_{2^{\text{nd}}}\Big\} = \{(n_T - 2), 2\} \\
&\ \vdots \\
\boldsymbol{M}_{2,n_T/2} &= \Big\{\underbrace{1,\ 1,\ \ldots,\ 1}_{n_T/2},\ \underbrace{1,\ 1,\ \ldots,\ 1}_{n_T/2}\Big\} \\
&= \left\{\frac{n_T}{2}, \frac{n_T}{2}\right\} \quad \text{if } n_T \text{ is even} \\
\boldsymbol{M}_{2,\frac{(n_T+1)}{2}} &= \Big\{\underbrace{1,\ 1,\ \ldots,\ 1}_{\frac{n_T+1}{2}},\ \underbrace{1,\ 1,\ \ldots,\ 1}_{\frac{n_T-1}{2}}\Big\} \\
&= \left\{\frac{n_T+1}{2}, \frac{n_T-1}{2}\right\} \quad \text{if } n_T \text{ is odd}
\end{aligned} \tag{12}$$

In a similar manner, one can form the sets for three, four or more seed function combinations as

$$\begin{aligned}
\boldsymbol{M}_{3,1} &= \Big\{\underbrace{1,\ 1,\ \ldots,\ 1}_{n_T-2},\ 1,\ 1\Big\} = \{(n_T - 2), 1, 1\} \\
\boldsymbol{M}_{3,2} &= \Big\{\underbrace{1,\ 1,\ \ldots,\ 1}_{n_T-3},\ \underbrace{1,\ 1}_{2^{\text{nd}}},\ 1\Big\} = \{(n_T - 3), 2, 1\} \\
&\ \vdots \\
\boldsymbol{M}_{4,1} &= \Big\{\underbrace{1,\ 1,\ \ldots,\ 1}_{n_T-3},\ 1,\ 1,\ 1\} = \{(n_T - 3), 1, 1, 1\Big\} \\
\boldsymbol{M}_{4,2} &= \Big\{\underbrace{1,\ 1,\ \ldots,\ 1}_{n_T-4},\ \underbrace{1,\ 1}_{2^{\text{nd}}},\ 1,\ 1\Big\} = \{(n_T - 4), 2, 1, 1\} \\
&\ \vdots
\end{aligned} \tag{13}$$

The set $\boldsymbol{M}_{1,1} = \{n_T\}$ completes the calculation cycle of the sets $\boldsymbol{M}_{i,j}$. Table 2 shows the calculated seed function combinations for $n_T = 8$. It should be noted, from Table 2, that not all decompositions are possible solutions to the approximation problem, since CFs need to be formed by

Table 1 Number of unrestricted $P(n_T)$ and restricted $P_R(n_T)$ seed function combinations

n_T	$P(n_T)$	$P_R(n_T)$	n_T	$P(n_T)$	$P_R(n_T)$
4	5	2	13	101	18
5	7	3	14	135	22
6	11	4	15	176	27
7	15	5	16	231	32
8	22	6	17	297	38
9	30	8	18	385	46
10	42	10	19	490	54
11	56	12	20	627	64
12	77	15	21	792	76

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Table 2 Seed function combinations for $n_T = 8$

Seed function	Seed function orders
2	{4, 4}, {3, 5}, {2,6}
3	{2, 3, 3}, {2, 2, 4}, {1, 3, 4}, {1, 2, 5}, {1, 1, 6}
4	{2, 2, 2, 2}, {1, 2, 2, 3}, {1, 1, 3, 3}, {1, 1, 2, 4}, {1, 1, 1, 5}
5	{1, 1, 2, 2, 2}, {1, 1, 1, 2, 3 }, {1, 1, 1, 1, 4}
6	{1, 1, 1, 1, 2, 2}, {1, 1, 1, 1, 1, 3}
7	{1, 1, 1, 1, 1, 1, 2}
8	{1, 1, 1, 1, 1, 1, 1}

the product of lower order (than the emulated filter) seed functions. Bearing in mind that the CF's total order is greater than the emulated filter's order by one, some solutions that consist of seed functions of orders equal to, or greater than, n need to be rejected. Thus, the decomposition for n_T into a single integer will give a single seed function having an order n_T, which needs to be rejected from the possible solutions set. Moreover, the decomposition of n_T into two seed functions will have the form

$$n_T = (n_T - 1) + 1 = (n_T - 2) + 2 = \cdots \tag{14}$$

The first solution, in (14), will give two seed functions; one of which will have an order of $(n_T - 1) = n$ and, thus, needs to be rejected. The decomposition of the total filter order n_T into combinations of three (or more) seed functions needs no further reduction at this stage. The reason for this is that these solutions will have the form

$$n_T = (n_T - 2) + 1 + 1 = (n_T - 3) + 2 + 1 = \cdots \tag{15}$$

Therefore this ensures that the seed function orders will be smaller, by at least one, compared to the order of the filter being approximated. The number of all possible seed function combinations, δ_{n_T}, available to the filter designer can be expressed as

$$\delta_{n_T} = P(n_T) - 2 \quad \text{or} \quad P_R(n_T) - 2 \tag{16}$$

4 Seed function parameter evaluation

Since the out-of-band response is of great importance, one can define a frequency interval where the two responses (i.e. the CC and the CF) must have minimum deviation. It can be shown that the filter emulated using CFs will have a slightly slower roll-off rate just above the cut-off frequency, Ω_c [3]. However, after a frequency Ω_χ (where $\Omega_\chi > \Omega_c$), it will have a faster roll-off rate than the conventional filter (since $n_T > n$). The frequency range from Ω_c up to Ω_χ is the frequency interval for which the two responses must have minimum deviation. Now, Ω_χ is different for every unique seed function combinations and, thus, it needs to be evaluated for each combination. It should be noted that some seed function combinations will provide Ω_χ very close to Ω_c, whereas others will provide Ω_χ far away from Ω_c. The longest distance, as expected, will be provided by the combination consisting of first-order seed functions only (i.e. the Butterworth amplitude approximation). A function $A^{TF}_{min}(\Omega)$ can be defined as the minimum out-of-band rejection for the CC filter, such that

$$A^{TF}_{min}(\Omega) = 10\log_{10}[1 + \varepsilon^2 T_n^2(\Omega)] \tag{17}$$

where $T_n(\Omega)$ is the Chebyshev polynomial of the first kind. The corresponding CF minimum out-of-band rejection is calculated for every candidate function set $M_{i,j}$ as

$$A^{i,j}_{min}(\Omega) = 10\log_{10}[1 + \varepsilon^2 G^2_{i,j}(\Omega)] \tag{18}$$

where $G^2_{i,j}(\Omega)$ can be formed as shown in [3]. Since different seed function combinations will result in different Ω_χ, a numerical method can be applied to solve (17) and (18) for Ω as

$$\begin{aligned} &A^{TF}_{min}(\Omega) = A^{i,j}_{min}(\Omega) \\ &10\log_{10}[1 + \varepsilon^2 T_n^2(\Omega)] = 10\log_{10}[1 + \varepsilon^2 G^2_{i,j}(\Omega)] \\ &T_n^2(\Omega) - G^2_{i,j}(\Omega) = 0 \end{aligned} \tag{19}$$

The Newton–Raphson iteration method seems to be ideal for solving (19), to find $\Omega^{i,j}_\chi$ for each seed function combination as

$$\Omega^{i,j}_{\chi,m+1} = \Omega^{i,j}_{\chi,m} - \frac{T_n^2(\Omega^{i,j}_{\chi,m}) - G^2_{i,j}(\Omega^{i,j}_{\chi,m})}{\left.\dfrac{\partial\{T_n^2(\Omega) - G^2_{i,j}(\Omega)\}}{\partial\Omega}\right|_{\Omega=\Omega^{i,j}_{\chi,m}}} \tag{20}$$

where $\Omega^{i,j}_{\chi,m}$ is the initial guess value for $\Omega^{i,j}_\chi$. Both functions, $T_n^2(\Omega)$ and $G^2_{i,j}(\Omega)$, are well-behaved polynomials and their derivatives, required by the Newton–Raphson iteration method, can be easily evaluated. Additional parameters can also be used depending upon the application [4]. For example, one may be interested in the maximum and minimum even and odd-mode impedances for each seed function combination. This is particularly useful when a coplanar waveguide (CPW) implementation is required. It is well known that CPW circuits have a limited impedance range when compared with microstrip. Here, the CC filter will require a specific range of even and odd-mode impedances, whereas CF filters are able to provide a different range of even and odd-mode impedances for different seed function combinations. Even when the filter implementation technique allows for an impedance scaling factor, it is clear that the variety of different seed

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combinations resulting in different even and odd-mode impedances is an extra advantage when compared to the CC design. Moreover, resonator unloaded Q-factor, RL zero-frequency distribution, group-delay restrictions, time-domain restrictions (e.g. rise-time, ringing), and so on, may also be inserted into the $\boldsymbol{M}_{i,j}$ candidate function sets, giving the final form, for example, as [6]

$$\boldsymbol{M}_{i,j} = \left[\{n_{s_1}^{i,j}, \ \ldots, \ n_{s_i}^{i,j}\}, \{Q^{i,j}\}, \{\delta\omega_{\min}^{i,j}\}, \{\Omega_\chi^{i,j}\}, \left\{\sum\nolimits_g^{i,j}\right\}, \left\{\frac{g_{\max}^{i,j}}{g_{\min}^{i,j}}\right\}, R_o^{i,j}\right] \quad (21)$$

where $n_{s_1}^{i,j}, \ldots, n_{s_i}^{i,j}$ are the corresponding constituent seed function orders, $Q^{i,j}$ is the quality factor of the corresponding pole closest to the imaginary axis, $\delta\omega_{\min}^{i,j}$ is the corresponding minimum frequency separation of the RL zeros, $\Sigma_g^{i,j}$ is the sum of the corresponding filter elements, $g_{\max}^{i,j}/g_{\min}^{i,j}$ is the element maximum to minimum value ratio and $R_o^{i,j}$ is the filter termination ratio (i.e. $R_{\text{out}}^{i,j}/R_{\text{in}}^{i,j}$).

5 Design restrictions

When all possible solution sets have been enumerated, the design restrictions can be introduced. Here, a restriction set, R, is created to contain the imposed design restrictions, having the form [6]

$$R = \left[\{n_{\min}^{r}, \ \ldots, \ n_{\max}^{r}, \ w_1\}, \{Q_{\max}^{r}, \ w_2\}, \{\delta\omega_{\min}^{r}, \ w_3\}, \left\{\sum\nolimits_g^{r}, \ w_4\right\}, \left\{\frac{g_{\max}^{r}}{g_{\min}^{r}}, \ w_5\right\}, \{R_o^{r}, \ w_6\}\right] \quad (22)$$

where $n_{\min}^{r}$ and $n_{\max}^{r}$ are the required minimum and maximum seed function orders, respectively, $Q_{\max}^{r}$ is the required maximum CF Q, $\delta\omega_{\min}^{r}$ is the required minimum RL zero relative frequency separation, and so on. The variables $w_1, \ w_2, \ldots$ are the corresponding restriction weightings that can be used to emphasise the significance of each parameter. These weightings are integer numbers, such as $1 \le w_1 \le w_2 \le \ldots \le w_k$. Here, w_k is the most critical parameter and w_1 is the least critical parameter. The value 0 can be reserved for use when one does not want to restrict the specific parameter. Other definitions for the weightings are also possible (e.g. in percentage terms, etc.) depending upon the application.

This restriction set is then compared against all possible $\boldsymbol{M}_{i,j}$, by forming the difference set $\boldsymbol{D}_{i,j}$ (i.e. $\boldsymbol{M}_{i,j}$ without $\boldsymbol{R}$), as

$$\boldsymbol{D}_{i,j} = \boldsymbol{M}_{i,j} \backslash \boldsymbol{R} \quad \text{or} \quad \{\varsigma | \varsigma \in \boldsymbol{M}_{i,j} \quad \text{and} \quad \varsigma \notin \boldsymbol{R}\} \quad (23)$$

where ς is the element of each set forming the difference. If the resulting $\boldsymbol{D}_{i,j} = 0$ (i.e. empty set) then the corresponding $\boldsymbol{M}_{i,j}$ satisfies the restrictions. It should be noted that the parameter $\Omega_\chi^{i,j}$ in the set $\boldsymbol{M}_{i,j}$ is not used during this stage, as will be made clear in the next section. When all restrictions have been identified from the possible solution set, $\delta_{n_T}^R$ possible combinations will be left, where $\delta_{n_T}^R < \delta_{n_T}$, for which the error function needs to be formulated and finally compared in order to identify the best combination.

6 Error function formulation

When all $\delta_{n_T}^R$ possible solutions have been collected, a normalised (with respect to the frequency interval) error function can be derived for every solution. For example, with reference to (24) and with (17) and (18), the error to be reduced is that between transfer function minimum out-of-band rejection values, that is between the CC filter (as one example for the reference) and the CF filter calculated for every function set. Here, two sets can be defined, $\boldsymbol{A}_{i,j}(\Omega)$ and $\boldsymbol{B}_{i,j}(\Omega)$, such that

$$\begin{aligned} \boldsymbol{A}_{i,j}(\Omega) &\mapsto A_{\min}^{\text{TF}}(\Omega) \quad \forall\Omega \in [1, \ \Omega_\chi^{i,j}] \\ \boldsymbol{B}_{i,j}(\Omega) &\mapsto A_{\min}^{i,j}(\Omega) \quad \forall\Omega \in [1, \ \Omega_\chi^{i,j}] \end{aligned} \quad (24)$$

and the corresponding error function term can be formulated as

$$\boldsymbol{E}_{i,j}^2(\Omega) = |\boldsymbol{A}_{i,j}(\Omega)|^2 - |\boldsymbol{B}_{i,j}(\Omega)|^2 \quad (25)$$

The smallness of the error, $\boldsymbol{E}_{i,j}(\Omega)$, can be defined by the least-mean-square-error criterion. The $\boldsymbol{E}_{i,j}(\Omega)$ function is the best in the least-mean-square-error sense if the integral

$$\int_1^{\Omega_\chi^{i,j}} \boldsymbol{E}_{i,j}^2(\Omega)\,\mathrm{d}\Omega = \int_1^{\Omega_\chi^{i,j}} \{|\boldsymbol{A}_{i,j}(\Omega)|^2 - |\boldsymbol{B}_{i,j}(\Omega)|^2\}\mathrm{d}\Omega \quad (26)$$

is minimal in the interval of interest. Thus, the objective of the overall methodology presented in this paper is to identify the index combination (i, j) for which the integral square error is minimal. To solve (26), numerical integration techniques can be applied. For example, by dividing the integration interval into a number, u, of equal width, δu, segments as

$$\delta u = \frac{\Omega_\chi^{i,j}}{u} \quad (27)$$

and by using the multiple application of Simpson's 1/3 rule,

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one can finally obtain

$$\int_1^{\Omega_\chi^{i,j}} \boldsymbol{E}_{i,j}^2(\Omega)\,\mathrm{d}\Omega \simeq (\Omega_\chi^{i,j}-1)\frac{1}{3u}\cdot\Bigg\{\{|\boldsymbol{A}_{i,j}(1)|^2-|\boldsymbol{B}_{i,j}(1)|^2\} + 4\sum_{\lambda=1,3,5,\ldots}^{u-1}\{|\boldsymbol{A}_{i,j}(\Omega_\lambda)|^2-|\boldsymbol{B}_{i,j}(\Omega_\lambda)|^2\} + 2\sum_{\lambda=2,4,6,\ldots}^{u-2}\{|\boldsymbol{A}_{i,j}(\Omega_\lambda)|^2-|\boldsymbol{B}_{i,j}(\Omega_\lambda)|^2\} + \{|\boldsymbol{A}_{i,j}(\Omega_\chi^{i,j})|^2-|\boldsymbol{B}_{i,j}(\Omega_\chi^{i,j})|^2\}\Bigg\} \tag{28}$$

The index combination (i, j) that gives the minimum error is the required optimum combination. It should be noted that if the search algorithm returns only one seed function combination then it is not necessary to perform the integration.

7 Design example

A simple theoretical design example will now be given, using the methodology outlined in this paper, to demonstrate the optimal selection of a seed function combination in CF filters. Consider a band-pass filter for autonomous cruise control radar, having a centre frequency $f_0 = 76.5$ GHz, ripple bandwidth BW = 500 MHz, and worst-case pass-band RL = −25 dB. The CF total order is chosen to be $n_T = 7$, the seed function minimum order should be $n^r_{\min} = 1$ and maximum order $n^r_{\max} = 5$. The overall design is required to have 20% reduction in the minimum acceptable Q-factor than a conventional sixth-order Chebyshev filter, as well as to be able to provide a larger relative frequency separation of its RL zeros. The primary goal is to satisfy the overall filter quality factor, then the RL zero-frequency separations $\delta\omega_{\min}$ and, finally, the seed function orders. A FORTRAN-77 program was written to implement the optimal selection of seed function combinations [6]. The output of this algorithm was verified using Mathematica™.

For this simple theoretical example, and from (4), $P(7) = 15$ and, thus, the number of possible seed function combinations, δ_7, should be 13 (i.e. $\delta_7 = P(7) - 2$). Table 3 shows the calculated results for (4). These sets need enumerating and sorting using the design restrictions. The enumerated partitions, which can be calculated using the algorithm of Section 3.1, are as follows: $M_{2,2} = \{2, 5\}$, $M_{2,3} = \{3, 4\}$, $M_{3,1} = \{1, 1, 5\}$, $M_{3,2} = \{1, 2, 4\}$, $M_{3,3} = \{1, 3, 3\}$, $M_{3,4} = \{2, 2, 3\}$, $M_{4,1} = \{1, 1, 1, 4\}$, $M_{4,2} = \{1, 1, 2, 3\}$, $M_{4,3} = \{1, 2, 2, 2\}$, $M_{5,1} = \{1, 1, 1, 1, 3\}$, $M_{5,2} = \{1, 1, 1, 2, 2\}$, $M_{6,1} = \{1, 1, 1, 1, 1, 2\}$ and $M_{7,1} = \{1, 1, 1, 1, 1, 1, 1\}$. The CF parameters for every $\boldsymbol{M}_{i,j}$ need to be evaluated and inserted in the corresponding sets. The restriction set, for this example, would be completed as follows

$$\boldsymbol{R} = [\{1, 5, 1\}, \{6.1246, 3\}, \{0.2588, 2\}] \tag{29}$$

The search algorithm will return the solutions, $M_{4,1}$, $M_{4,3}$, $M_{5,1}$, $M_{5,2}$, $M_{6,1}$ and $M_{7,1}$. All these seed function combinations satisfy the given restrictions. The error evaluation routine would need to be used to indicate the optimal seed function combination for this example. As a result, the optimum solution will be the set $M_{4,1}$ (i.e. a fourth-order seed function chained with a cubed first order). Table 4 shows a comparison between sixth and seventh-order CC filters with the resulting CF filter. This seed function combination will provide an approximately 22% reduction in the minimum acceptable Q-factor $Q_{\min}$, as compared with the sixth-order CC, while it is almost half that required for a seventh-order CC. As a result, a much lower-Q fabrication process can be employed to realise the filter. The resulting pass-band losses will now be less for the CF filter, as compared with both CC filters [3]. The minimum frequency separation of the RL zeros $\delta\omega_{\min}$ for this filter combination will be double, when compared with the seventh-order CC, and almost 40% larger when compared with the sixth-order CC.

The mathematical synthesis of a lossless coupled-line filter [13] (which is considered to be one of the most susceptible to manufacturing errors [14]) was performed. Assuming random even and odd-mode impedance variations (to represent poor etching tolerances) of ±0.2% in all filter sections, it can be shown that when Monte-Carlo analysis

Table 3 Calculated results for (4) ($\kappa = 7$)

m	d_i	M	$\sigma_1\ (k-m)$	$P\ (m)$
0	{1, 7}	2	8	1
1	{1, 2, 3, 6}	4	12	1
2	{1, 5}	2	6	2
3	{1, 2, 4}	3	7	3
4	{1, 3}	2	4	5
5	{1, 2}	2	3	7
6	{1}	1	1	11

Table 4 Calculated results for (4) ($\kappa = 7$)

Function	$\frac{g_{\max}}{g_{\min}}$	Σ_g	R_o	$Q_{\min}$	$\delta\omega_{\min}$
sixth-order CC	2.3576	7.7445	1.1192	6.1246	0.2588
seventh-order CC	2.1349	9.6916	1	8.439	0.1931
$M_{4,1}$ CF	3.429	7.6924	1	4.7826	0.3827

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is performed the worst-case pass-band RL responses for the CC filter will depart from its designated value by approximately 15 dB, while the deviation for the CF filter is approximately 5 dB. This results in a 10 dB difference, between the two filter responses, as can be seen in Fig. 2. In addition, the CC function seems to have a higher variation in the filter's selectivity, as indicated in Fig. 2*a*, while the CF filter proves to be more robust.

We have discovered a fundamental Monte-Carlo margin that can be exploited. This represents an upper-bound on the worst-case pass-band RL responses that cannot be exceeded, no matter how many trials are performed in the analysis. Therefore a Monte-Carlo RL margin is proposed to compensate for the degradation in worst-case performance. For this particular example, a margin of 5.5 dB was determined. As can be seen in Fig. 2*b*, such a margin is appropriate for the CF filter, but not for the CC filters [4].

To demonstrate the reduced sensitivity to manufacturing tolerances with the CF filter, a practical implementation was previously reported in the open literature [4]. Here, microstrip parallel-coupled line filters have a centre frequency of 37 GHz and a ripple BW of 2 GHz. The target minimum input/output RL was −13 dB, so a pre-determined Monte-Carlo margin of 5 dB (this value was obtained using the equations presented) was included to give a design level of minimum input/output RL of −18 dB. The Rogers RO4003 laminate was used, having a dielectric constant of 3.38, substrate thickness of 200 μm and a maximum dissipation factor of 0.035. The filters were fabricated using a standard printed circuit board (PCB) etching technique. The masks were printed onto a standard

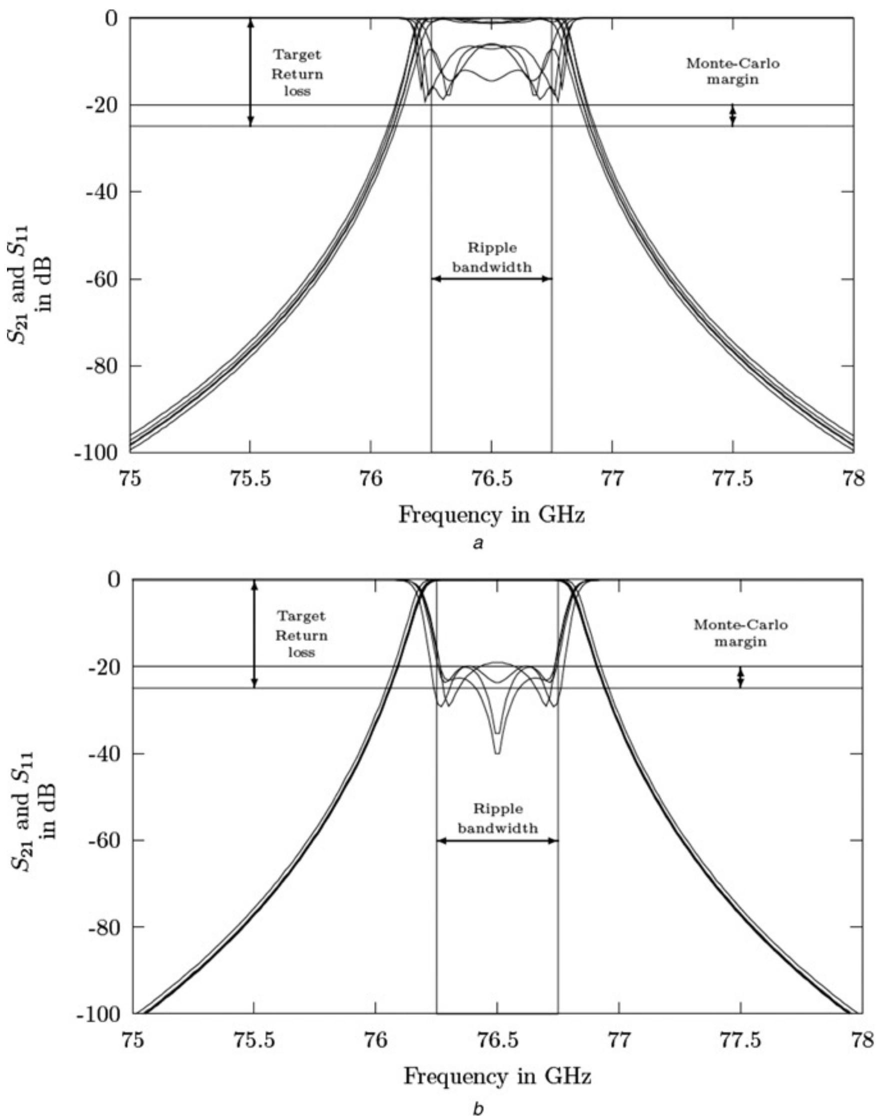

Figure 2 *Calculated frequency characteristics for the band-pass filters using Monte-Carlo analysis*

a Conventional sixth-order Chebyshev
b $M_{4,1}$ chained function

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overhead projector transparency, using a standard 600dpi laser printer. This was then used in conjunction with standard PCB photolithography and etching techniques [4].

8 Conclusions

An optimal CAD algorithm for implementing CF filters has been introduced for the first time. The major feature of this algorithm is the flexibility that it allows for the designer to impose various restrictions on the selection of the candidate seed function combinations, according to the design requirements. CAD software for implementing CF filters was developed for the European Space Research and Technology Center (ESTEC), based on the methodology described in this paper, and successfully used to demonstrate the synthesis against measured performances for a sixth-order filter at 37 GHz, manufactured under extreme conditions [4].

Combinatorial analysis and, in particular, the partition functions have been demonstrated for the first time for filter optimisation routines. This algorithm may also have many other CAD applications. For example, it could be used in the design of microwave active filters, having cascaded low-order sections. Here, parameters such as the active device parasitics can restrict the candidature of seed function selection. Since the complete formulation is generated, one has to modify the algorithm structure according to the design requirements. This can provide a very useful tool for engineers involved in filter design, for operation at any frequency and without any restrictions on the implementation technology.

CF filters are the key to low-cost, high-quality, microwave and millimetre-wave band-pass filters. The reduced sensitivity to manufacturing tolerances and the ability to generate different seed function combinations, for different fabrication technologies, can be used to extend the state-of-the-art in tuningless filter implementations towards higher frequencies or smaller fractional BWs or, alternatively, to lower the accuracy and manufacturing cost requirements for a given set of filter specifications. Even though this paper has formulated CC amplitude approximation polynomials, the approach can be easily adapted to any non-Chebyshev approximation.

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